Long Term Dependence of Popular and Neglected Stocks

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Abstract

In this study, we establish a connection between the levels of market attentions of a stock with its long memory features. We construct two portfolios of US equities based on Doyle et al's (2006) criteria for neglected and popular stocks and measure the degrees of persistence for their daily returns from January 1, 2003 to December 31, 2007. We find that all stocks except for one display anti-persistence in the neglect portfolio; while the popular portfolio stocks uniformly display random walk returns. This suggests that there is a connection between the persistence features of stock return series and the levels of "neglect" of stocks. We use book to market ratio, analyst coverage, and transaction frictions to classify the levels of market neglect of stocks. Based on our study, while these criteria combined appear to contribute to the long memory features of daily returns of stocks, we also suspect the presence of other factors driving the persistence of stock returns.

I. Introduction

Stock performance following the reporting of unexpected earnings has been of considerable interest in finance and accounting literature as studies consider this is a field where substantial inefficiencies can be unearthed. For example, Doyle et al. (2006) identify several criteria presented in firms with the tendency to report surprise earnings and construct a portfolio of firms with big earnings surprises. They observe that the portfolio beats the market by 15% over the subsequent two years after the earnings surprise.

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Doyle et al.'s criteria in firms with tendency to report surprise earnings include such variables as higher book-to-market ratios; higher transaction costs proxied by bigger bid-ask spread as a percentage of share prices; and fewer number of analysts following. These criteria reflect a lack of market interest in the security and an insufficiency of information available about the company.

Identifying specific company features, as the approach followed by Doyle et al.'s study, is one approach to discover inefficiencies in stock markets. Although Doyle et al. document consistent variables present in surprise earnings firms and summarize the phenomena as a product of neglect, they leave the door open to further research in the hopes of unveiling the mechanics of inefficiency.

In recent years, there is also development in the literature on measuring the inefficiency level of a financial time series using Hurst exponent (H). Developed for uses in hydrology, the Hurst exponent measures the predictability or the memory of a series by examining the change in the series over a rescaled range of the process. In the context of stock prices, the Hurst exponent measures the tendency of a financial time series to either trend in a direction, or revert to a mean. Hurst exponent is constrained by $0<H<1$. When $0.5<H<1$, indicating there is more than 50% chance an event will be followed by a similar event, the series is persistent with long memory (LM). When $0<H<0.5$, indicating there is less than 50% chance an event will be followed by a similar event, the series is anti-persistent with LM. In other words, in a persistent time series, if change has been up (down) in the prior period, then the chances are that it will continue to move up (down) in the next period. On the other hand, anti-persistent time series is considered mean-reverting. If change has been up (down) in the prior period, then the chances are that it will move to the opposite direction in the next period. When $H=0.5$, indicating there is a 50% chance an

event will be followed by a similar event, the series is a stationary Geometric Brownian Motion (GBM) with short memory, or a random walk.

When a financial time series is a random walk, the market is then considered efficient and past performance cannot predict future performance. When the series is a long memory series, the market is then considered not efficient and past performance can be used to predict future performance. Hurst exponents have been used in various studies in finding out the efficiency levels of different markets. For example, Kyaw et al. (2006) analyze the degree of long-term dependency of Latin American stock and currency markets and find that market returns in these markets are long-term dependent, nonnormal, and nonstationary.

Though there are a lot of studies examining the efficiency level of various markets at the aggregate level, not many studies have looked at the efficiency levels of individual stocks using Hurst exponent. Especially, one gap existed in the literature is that not many studies have linked the criteria used to identify neglect of stocks to the inefficiency levels of stocks as measured by Hurst exponent. For example, Cajueiro and Tabak (2005) estimate the Hurst exponents for the 30 stocks included in the Dow Jones Industrial Index. The focus on the 30 stocks in the index does not have any theoretical justification and to some extent is quite arbitrary. Our study tries to fill in the gap in the literature by linking the criteria used to identify stock neglect to the efficiency of individual stocks. We will breakdown the market inefficiency by investigating the financial time series of firms that embody the neglect criteria observed by Doyle et al. (2006) for degrees of persistence. The exploration of a relationship between the parameters of surprise earnings documented by Doyle et al. and the long memory features of share prices may contribute to the development of a comprehensive understanding of the market inefficiency. Previous studies examining the long memory feature of aggregate market series usually use policy factors to

explain the long memory feature existed in various markets (Kyaw et al., 2006; Los and Yu, 2008), not many studies have looked at what are the possible driving factors that affect the long memory feature for individual stocks. Our study will seek to understand if the underlying criteria of the neglected stock consistently correspond to either persistence or anti-persistence in stock prices and returns series.

The paper is organized as follows. Next section provides a review on the related literature on long memory of time series data and market efficiency. Section 3 presents the methodologies and discusses the test results. The final section concludes the paper.

II. Literature

The large body of research exploring the varying degrees of efficiency in capital markets through theoretical arguments is matched in volume by the body of research exploring market efficiencies through direct quantitative measurements. An area of particular focus is gauging the level of persistence in a given market by analyzing the time series of the market performance at aggregate level.

Persistence or anti-persistence is an importance property of fractional integrated process. Fractal is a self-similarity character of irregular objects across many scales. It is originally identified in many scientific phenomena such as the shapes of coastlines and the distribution of rivers (Mandelbrot, 1999). In recent years, there is a wide application of fractal analysis in economic and financial time series data. For example, Cheung and Lai (1993) find that deviation from the purchasing power parity is a fractionally integrated process such that it takes several years for the deviation to converge back to the parity. In determining the presence of persistence

or anti-persistence in a time series, the Hurst exponent is the accepted parameter. Although many techniques can be used to calculate the Hurst exponent, they vary greatly in efficacy.

Chamoli et al. (2006) compare the power spectrum, roughness length, semi-variogram, wavelet transform and rescaled range methods of calculating the Hurst exponent of a fractional Brownian motion time series. Using the successive random addition method, Chamoli et al. create fractional Brownian motion time series with known Hurst values. Through applying the different methods of Hurst calculation to the time series, Chamoli et al. compare the methods' results with the known Hurst values. They determine that regardless of the length of the time series, the rescaled range analysis and wavelet transform yield the most accurate results. They find this result to be robust for fractional Brownian motion time series of Hurst values from 0.4 to 0.9.

Simonsen et al. (1998) devise a method of calculating the Hurst exponent of a time series using what they term the average wavelet coefficient method (AWC). By plotting the average wavelet coefficient of transformed data into the wavelet domain against a log-log plot, Simonsen et al. are able to calculate the Hurst exponent of time series characterized by a small number of samples with increased precision compared to Fourier transform based methods. They also conclude that the AWC method of calculating the Hurst is acceptable for experimental and noisy data.

Jones et al. (1995) compare the wavelet packet method with the root-mean-square roughness and second-moment methods for calculating Hurst values. Similar to Chamoli et al., Jones et al. test the methods on fabricated fractional Brownian motion time series with known Hurst values. They conclude that the wavelet pack method is comparable to the root-mean-square roughness and second-moment methods in accuracy, but its ability to apply different filter

functions thereby changing the resolution scale allows for the focusing on local features and an increased understanding of how "energy depends on position and scale" (2509).

While the majority of markets have been empirically proven to follow a random walk as per Fama's theses, instances of financial markets or individual securities within the markets displaying inefficiency have been observed. Factors contributing to the existence of non-random walk financial markets vary from the physical age of the market to the role of external forces. However, more important to note is the confirmation of market characteristics that deviate from the assumed and idealized form of market efficiency pioneered by Fama.

Eom et al. (2008) observe an example of a non random-walk market movement. They look to confirm the correlation between the weak form efficient market hypothesis and the predictability of changes in stock prices. Due to its property of reflecting long-term memory capabilities of a time series, Eom et al. calculate the Hurst exponent for 27 indices of varying nationality and age. They observe that more efficient and mature markets exhibit a higher degree of randomness and short memory with Hurst exponent closer to 0.5. Conversely, less efficient emerging markets are characterized by larger Hurst exponents and exhibit long-term memory properties.

Los and Yu (2006) examine the persistence characteristics of the Chinese stock markets by calculating the Hurst exponent for the Shanghai, Shenzhen A and Shenzhen B indices using multiresolution wavelet analysis. Specifically, they investigate the correlation between Chinese government regulation and the persistence within the country's capital markets. They note that the multiresolution wavelet analysis allows them to not only calculate overall Hurst exponents for time series, but also Hurst exponents for various sub periods of varying macro-economic conditions. During times of heightened government regulation, they calculate Hurst exponents of

0.63, 0.56 and 0.64 for the Shanghai, Shenzhen A and Shenzhen B indices respectively. In recent years with less frequent intervention they calculate Hurst exponents of 0.54, 0.54 and 0.55 respectively. They conclude that the uniform transformation of all three indices from persistent markets to near Brownian motion indicates an increase in market efficiency within the country.

Cajueiro and Tabak (2005) estimate the Hurst exponent for the Dow Jones Industrial Index and for 30 companies included in the index. They calculate the exponent for two different sets of data, the open-open returns and the closed-closed returns, citing documented differences due to market structure leading to greater volatility among open-open returns. With data spanning from January 2, 1990 to December 19, 2003, Cajueiro and Tabak find evidence of long-term dependence in many of the individual stocks composing the index. The majority of stocks correspond to Hurst exponent values less than 0.50 for both open-open returns and closed-closed returns suggesting greater degrees of anti-persistence. They also observe much greater levels of long-term dependence when using open-open returns, 83% displaying anti-persistence compared to 50% for closed-closed returns, reinforcing the notion of market structure leading to overreaction and heightened volatility among open-open returns. Cajueiro and Tabak suggest that the aggregation of individual stocks with long-term dependency into indices obfuscates their individual levels of persistence and creates an incorrect view of market return movements. Additionally, they note the significance of all long-term dependent stocks having Hurst values less than 0.50, suggesting previous studies were overly focused on persistent movement, or Hurst exponents greater than 0.50.

Lento (2009) investigates the correlation between the profitability of several technical trading rules applied to 15 largest global equity markets and the Hurst exponents of the indices. He observes excess returns over buy and hold strategies for technical trading rules when they are

applied to indices with Hurst values greater than 0.56. But technical trading rules underperform buy and hold strategies when Hurst values are less than 0.53, suggesting that profits from technical analysis are at least partially driven by persistence.

As outlined above, many studies have empirically isolated inefficiencies and evaluated markets for their degrees of persistence using techniques to find evidence of non-random walk movements. However, a disconnection still exists between measures of market efficiency via the Hurst exponent and the criteria for neglected stocks as studied in Doyle et al. (2006). While many have either measured the long-term persistence of various markets and individual securities (Eom et al., 2008; Cajueiro and Tabak, 2005; Los and Yu, 2006), or have researched surprise earnings announcements caused by investor neglect (Doyle et al., 2006), a link between individual stock's level of neglect and its corresponding Hurst exponent, has yet to be established. This study serves to establish the link by directly measuring the degrees of persistence for stocks whose underlying companies are characterized by Doyle et al.'s criteria of neglect. The benefit of bridging the gap between the bodies of research that quantitatively measure market efficiency via the Hurst exponent and the criteria of neglected stocks is a better understanding on the exact causes of market efficiency at the individual stocks level.

III. Empirical Tests

Data and Methodology

The dataset we use for this study encompasses active US stocks traded from January 1, 2003 through December 31, 2007. We choose this period to avoid the impact of market crashes in the 2001/2002 and 2008/2009 periods.

To identify stocks that are likely to be described as neglected, we sort the stocks using three parameters based on Doyle et al.'s findings, the stock's book to market ratio, the number of analysts following the stock, and the transaction costs proxied by bid-ask spread as a percentage of share prices. Adjusted daily prices are retrieved from CRSP database; book to market ratios are retrieved from the COMPUSTAT database; analyst coverage are from I/B/E/S database; and daily closing bid-ask spreads are from the CRSP database.

Stocks are divided into ten deciles based on their book to market ratios and transaction costs with equal numbers of observations in each decile. The highest decile corresponds to scores of 1 for stocks that are most neglected as reflected in higher book to market ratios and transaction costs. In the grouping process based on analyst following, for stocks followed by 0-10 analysts, their respective scores are equal to the number of analysts. For stocks with more than 10 analysts, we divide them into two groups with equal numbers of observations and assign the scores of 11 and 12 respectively. The process of assigning scores based on the parameters is iterated from 2003 to 2007.

The means of the scores for each of the three grouping parameters are then combined to determine a stock's overall score. Stocks with an overall score of 2, which is the lowest possible score with scores of 1 for book to market ratios and transaction costs parameters and 0 for analysts coverage parameter, are considered the neglect portfolio. Stocks with an overall score of 30 or greater make the popular portfolio.

To estimate the degrees of persistence of the stocks in these two portfolios, we use the adjusted daily closing prices of the stocks from the beginning of 2003 to the end of 2007. Several methods have been proposed to test long-term dependency. Most of the empirical studies rely on the frequency domain method originated by Geweke and Porter-Hudak (1983). However, this

approach cannot present satisfactory estimation of asymptotic properties. The other widely used method is R/S (Range/Scale) based Hurst exponent (Hurst, 1951). Empirical studies based on Hurst's R/S statistics generate contradicting results (Sadique and Silvapulle, 2001; Cheung and Lai, 1995). Among the different methods estimating Hurst exponent, wavelet Multiresolution Analysis (MRA) method is more powerful than other tools. Los (2003) indicates that wavelet MRA methodology can identify and measure the degrees of long-term dependency of financial time series on a more localized level. In this study, we use the wavelet MRA approach. We also implement an R/S test proposed by Mandelbrot (1972) and extended by Lo (1991) to compare the results generated by these two different methods.

Wavelet MRA results are generated by using the online ION Script Research Systems Interactive Wavelet Program.[†] The very general Morlet-6.0 wavelet is used to calculate the localized wavelet resonance coefficients, which are visualized in the wavelet scalogram with colorized resonance coefficients. The scalogram provides a time-frequency localized analysis of the time series. The time-average analysis is presented in a scalegram, which shows the ACF of the time series in the scale domain.

There are three parts in each wavelet scalogram and scalegram plot. Part a is the original time series together with the illustration of the Morlet-6.0 wavelet used in the analysis. Part b is the scalogram, which reflects the localized wavelet power spectrum and is a colorized plot of the squared value of the wavelet resonance coefficients. Part c is the global wavelet scalegram, which plots the variances of the zero-mean wavelets against the scales and forms the statistical time-average of the scalogram. The negative slope of the scalegram of the level time series (β) has the following relationship with the monofractal Hurst exponent (H), $\beta = 2H + 1$; and the

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[†] The program is available at http://ion.researchsystems.com/IONScript/wavelet/.

negative slope of the scalegram of the differences of level series equals $\beta = 2H - 1$. The two scalegrams should provide identical monofractal Hurst exponent value.

In empirical studies, many financial time series have been identified to be multifractal, meaning monofractal Hurst exponent is not sufficient to reflect the change of Hurst exponents against the scales or time periods (Stavroyiannis *et al.*, 2010; Engelen *et al.*, 2011). In other words, there is marked evidence for the existence of heterogeneous Hurst exponents, each applicable for the frequency scaling in a sub-period. When comparing the scalegram plots of level series and their corresponding differences series, the plots of difference series can reveal more localized risk properties than the level series. In the following analysis, we will focus on the scalegram plots of the difference series, or return series, to examine the localized risk properties of each time series.

For comparison purpose, we also simulate a benchmark series, a random walk series and show its corresponding difference series (white noise) in Fig. 1. The white noise series consists of 1200 daily returns, generated using Microsoft Excel's random number generator. In this randomly generated series, there are 619 positive returns among a total of 1200 returns. In Part b of the wavelet analysis plot (wavelet power spectrum), the spectral peaks are evenly scattered at various scales and across the whole time period. In Part c of plot, the global wavelet scalegram is nearly perfectly vertical with only small variances being seen on either side.

<Figure 1 about here>

The slope of a scalegram of returns can indicate persistence, anti-persistence or a random walk as shown in Figure 2. When the negative slope of the Global Wavelet Scalegram (β) for the return series is negative, the time series will be anti-persistent. Because when β is smaller than 0, the monofractal Hurst exponent H will be smaller than 0.5, because $H=(\beta+1)/2$ in difference

series. Similarly, when the negative slope of the Global Wavelet Scalegram (β) for the return series is positive, the time series will be persistent. Because when β is bigger than 0, the monofractal Hurst exponent H will be bigger than 0.5, because $H=(\beta+1)/2$ in difference series.

<Figure 2 about here>

Results

Table 1 shows the compositions of the neglect and popular portfolios, the overall ranks of the securities using the three neglected stock criteria. As mentioned in the previous section, the approximate slope of a scalegram of returns can indicate whether the return series is a persistent or anti-persistent series, or a random walk.

<Table 1 about here>

Comparing the scalegrams of returns for the neglect portfolio stocks and the popular portfolio stocks we see the emergence of a pattern. The neglect portfolios' scalegrams of returns consistently show anti-persistence (Figure 3); whereas the popular portfolios' scalegrams more closely and uniformly resemble a random walk, or a vertical slope (Figure 4).

<Figure 3 about here>

<Figure 4 about here>

By noting the consistent anti-persistence among the neglected stocks' returns, we can begin to see a potential explanation on the long memory of individual stocks that are characterized by Doyle et al.'s (2006) criteria.

The lack of long-term memory for the popular stocks' returns is consistent with previous studies findings, most prominently Cajueiro and Tabak (2005), who estimate Hurst exponents

near 0.50 for the daily returns of all the components of the DJIA. In line with Doyle et al.'s

neglect stock hypothesis, prominent securities such as the Dow Jones 30 or popular portfolio in

our study are prone to greater investor interest, both institutional and individual, and are subject

to increased attention from analysts and other forecasters. The greater demand for the security

results in a near inexhaustible amount of available information more closely aligning with

Fama's efficient market hypothesis. Given the abundance of information, the daily returns

observed are best described as white noise driven by unpredictable random events, with one

day's gain or loss completely independent of every other day's gain or loss.

Next we also implement an R/S test proposed by Mandelbrot (1972) and extended by Lo

(1991) to compare the results generated by using wavelet MRA. We estimate the fractional

differencing parameter d for each of series by adopting a GPH estimator proposed by Geweke

and Porter-Hudak (1983). In calculating the GPH estimates, the Fourier frequency m is set to be

$T/2$, where T is the total number of observations in the time series. Table 2 lists the estimation

results for the $\hat{d}$'s. In a FBM series, the differencing parameter d has the following relationship

with H: $H_{FBM} = d - 0.5$. The Hurst exponents are calculated and shown in Table 2 too.

<Table 2 about here>

The GPH estimates $\hat{d}$'s for popular stocks are not significantly different from 1 and their

corresponding Hurst exponents are not significantly different 0.5. This result indicates that

popular stocks are random walk series. Whereas in neglected stocks, except for APSA, which

has a GPH estimates $\hat{d}$ that is not significantly different from 1 and a Hurst exponent that is not

significantly different from 0.5, all other stocks have demonstrated a anti-persistence with Hurst

exponents significantly smaller than 0.5. This result is consistent with that of the wavelet MRA.

Discussion

To better understand why neglect portfolio experiences anti-persistent returns rather than a random walk as observed for popular portfolio, we further look at the differences in the underlying fundamentals of the portfolios.

One of the distinguishing features of the neglect portfolio stocks is the lack of interest in the marketplace. Daily volumes are often zero as minimal attention is given to the securities by market participants. As observed through the portfolio rankings, no analysts publish I/B/E/S earnings forecasts and market makers are extremely hesitant to create liquidity evident by the high transaction fees. The vacuum of available information leads to a greater level of ambiguity concerning the stocks' intrinsic values, creating the potential for a mispricing, or market inefficiency to persist. Intuitively, and given the abnormal returns observed by Doyle et al. (2006) for their neglected stocks, we would expect our neglect portfolio stocks to exhibit degrees of long term persistence. However, our neglect portfolio stocks are measured as having anti-persistent returns and prices. The bigger question to ask is why our portfolio of neglected stocks, whose selection is based on Doyle et al's criteria for surprise earnings, displays the mean reversion tendencies.

The most logical answer traces back to the number of shares of the neglect portfolio stocks that are traded each day. As Table 3 shows, the average daily volumes are extremely low and most stocks have a price lower than $5.00, falling into the penny stock classification.

<center><Table 3 about here></center>

The low volume causes multiple efficiency related phenomena to occur. The first volume related issue is the lack of investor attempts to price the security through trading. Unlike more liquid securities that are constantly being bid up and down by market participants, eventually

forming a consensus opinion on price and value, the neglect portfolio stocks are sometimes only subject to one trade per day, essentially leaving the pricing entirely to the buyer and his or her counterparty. Rather than a marketplace full of investors with a combined mosaic of information to be reflected in the security's price, the daily closing price of a neglected stock is determined by the paltry number of individuals who chose to trade the stock on a given day, resulting in a considerably less amount of information being reflected in the price.

The second volume related issue is the potential for the buyers and sellers of the neglected stocks to inadvertently move the market with their orders. The illiquidity of the stocks means buyers will have to pay a premium or sellers will have to suffer a discount in order to move their shares.

An investor looking to unload a large block of shares will therefore drive the price of the neglected shares down. Other investors may observe the decline. Lacking any substantial information, they may assume the stock to be trading at a discount and purchase shares looking to make a quick profit. The purchase of shares will then drive the price back up as the buyer's counterparty profits on the large spread. As a result, there is potential for a large amount of the daily movement observed in the neglected securities to be caused by the previously described circumstances.

Another feature of our results is a better understanding of the reliability of global Hurst exponent as a means of measuring persistence in a financial time series and the importance of the date range of the data used in the estimations. The date range of the data is simply the period in time from which the closing prices of the stocks are taken. The conundrum is as follows, choose a range too short and there is an insufficient amount of data to form robust conclusions, choose a range too long and the markets are prone to experiencing structural changes, thus skewing the

resulting measurements of memory. As mentioned in previous section, the range of January 1, 2003 to December 31, 2007 is selected for its position between the dotcom bubble of the early 2000s and the credit crisis of 2008. The date range provides a relatively large number of data points (1258 closing prices) and a "normal" period of economic expansion.

However, despite choosing a range free of market-wide aberrations and/or fundamental changes such as bubbles or recessions, in evaluating the effectiveness of our data range we must be cognizant of the risk of greater changes to underlying characteristics of the stocks being analyzed that accompany multi-year periods. Changes in features such as analyst coverage, book to market ratios, or daily volumes of shares traded will decrease the reliability of the link between the calculated persistence and the hypothesized drivers of dependency. Similarly, if we choose a range too short we run into the risk of not having a sufficient sample size to generate a statistically reliable conclusion. Fortunately, the consistency in our results, namely the correlation between anti-persistent returns and our neglected portfolio stocks versus the random-walks of our popular portfolio stocks, suggests that a data set of approximately 1300 points is capable of accurately describing global trends of securities' prices.

Limitations of our findings must also be mentioned, as they are both relevant to our conclusions and to stimulating of future studies. While the size of our data set is capable of generating interpretable results, there exists a distinct possibility that the results are exclusive to the years 2003 to 2007. As described above, macroeconomic trends stand to alter the behavior of equities considerably, which could in turn permute the connection between the criteria of neglect and the anti-persistence of returns. Figure 5 illustrates the sensitivity of a stock's (MSFT) memory to changes in the period being analyzed. Examining the two different power spectrums

and scalegrams, we can see a stark difference between the memories of daily returns from July 7, 2005 to December 31, 2007 and from July 7, 2008 to December 31, 2010.

<Figure 5 about here>

The scalegram of MSFT's daily returns from the earlier range (a subset of the range used to calculate MSFT's scalegram in figure 4) appears relatively vertical and is concurrent with the random-walk observed for our original 2003-2007 range. The scalegram for the later period deviates from a vertical line and instead is slightly positively sloped suggesting anti-persistence. This counter-example strives to illustrate the effect market-trends can have on a stock's global Hurst exponent. We see extreme volatility of returns from days 70-150 for MSFT Returns (LATER) and large variances or "shocks" in its power spectrum for the same sub period. The period of returns for MSFT Returns (LATER) begins on 7/7/08. We conclude that the shocks in the power spectrum correspond to the effects of the credit crisis and the conflagration of losses in the financial markets. This serves to demonstrate the power of systematic trends or events on the global Hurst exponent. While our results, concurrent with previous findings, suggest that popular stocks such as MSFT likely have random-walks for returns, if certain data sets are used for analysis there is the potential for non-consistent results.

Our results recognize the vulnerability of the monofractal Hurst exponent to both changes in market conditions and underlying characteristics but also establish a precedent for connecting select fundamentals of stocks with their degrees of persistence for daily returns. We surmise that our results demonstrate the potential to establish interconnection and are an important first step in the effort to devise a veritable solution.

IV. Conclusion

In this study we construct two portfolios of US equities based on Doyle et al's criteria for neglected and popular stocks and measure the degrees of persistence for their daily returns from January 1, 2003 to December 31, 2007. We find that all stocks except for one display anti-persistence in the neglect portfolio; while the popular portfolio stocks uniformly display random walk returns. This suggests that there is a connection between the persistence feature of stock return series and the levels of "neglect" presented by Doyle et al. However, while book to market ratio, analyst coverage, and transaction frictions appear to contribute to the memory features of daily returns of stocks, we also suspect the presence of other factors driving the persistence of these time series.

Our study focuses on 2003-2007, a period of relative financial tranquility. Other periods containing large events or disruptions to normal market behavior may create a structural change in a stock's monofractal Hurst exponent. Improvements should be made in future studies to search for more relevant fundamental factors that can affect the persistence of stock return series.

A final point worth noting is the potential limitations of a trading strategy based on the exploitation of inefficiencies present in neglected stocks. As observed by Doyle et al. (2003), while their portfolio of neglected stocks does exhibit abnormal returns, large frictions present a likely obstacle to exploitation. We observe similar transaction expenses driven primarily by a lack of liquidity in our neglect portfolio and reason that operational inefficiencies in the marketplace prevent the opportunity for exploitation. Our results suggest that our neglected stocks' price changes are driven by market moving trades, rather than the dissemination of positive news into the market, and a reluctance of neglected securities to absorb the information as posited by Doyle et al.

That being said, future studies still stand to benefit from further investigation of the

relationship between the levels of neglect and the persistence of stocks. Adjusting such

parameters as the range of dates used could lead to more conclusive results through an increased

understanding of the effect of the time period on a stock's memory. Additionally, while we

interpret high book to market ratios, low analyst coverage, and high transaction frictions as

possible drivers of anti-persistence, a more comprehensive combination of factors would make

the results more applicable. These factors would provide identifying mechanisms for stocks

poised to follow relatively predictable trends. Thus, the documenting of criteria for persistence

combined with an increase in the generality of a global Hurst exponent's applicability could truly

begin to set the stage for the development of a profitable trading strategy based on stocks'

measured memories and a corresponding tendency to outperform the market.

References
Cajueiro, O. D., & Tabak, M. B. (2005). Long-range Dependence and Market Structure. *Chaos, Solitons and Fractals, 31,* 995-1000.

Chamoli, A., Bansal, R. A., & Dimri V. (2006). Wavelet and Rescaled Range Approach for the Hurst Coefficient for Short and Long Time Series. *Computers and Geosciences, 33,* 83-93.

Cheung, Y., & Lai, K. (1993). A fractional cointegration analysis of purchasing power parity. *Journal of Business and Economic Statistics , 11*, 103-112.

De Bondt, F. M. W., & Thaler, R. (1984). Does the Stock Market Overreact? *The Journal of Finance, 40,* 793-805.

Doyle, T. J., Lundholm, J. R., & Soliman, T. M. (2006). The Extreme Future Stock Returns Following I/B/E/S Earnings Surprises. *Journal of Accounting Research, 44,* 849-887.

Eom, C., Oh, G., & Jung, W. (2008). Relationship Between Efficiency and Predictability in Stock Price Change. *Physica A, 387,* 5511-5517.

Fama, F. E. (1965). Random Walks in Stock Market Prices. *Financial Analysts Journal, September-October 1965,* 55-59.

Fama, F. E. (1969). Efficient Capital Markets: A Review of Theory and Empirical Work. *The Journal of Finance, 25,* 383-417.

Jones, L. C., Lonergan, T. G., & Mainwaring, E. D. (1995). Wavelet Packet Computation of the Hurst Exponent. *Journal of Physics A: Mathematical and General, 29,* 2509-2527.

Kyaw, N. A., Los, C. A., & Zong, S. (2006). Persistence characteristics of Latin American financial markets. *Journal of Multinational Finance Management , 16*, 269-290.

Lento, Camillo, (2009). Long-term Dependencies and the Profitability of Technical Analysis. *International Research Journal of Finance and Economics, 29, 126-133.*

Los, A. C., & Yu, B. (2008). Persistence Characteristics of the Chinese Stock Markets. *International Review of Financial Analysis, 17,* 64-82.

Malkiel, B. G. (1973). *A Random Walk Down Wall Street.* New York: W. W. Norton and Company, Inc.
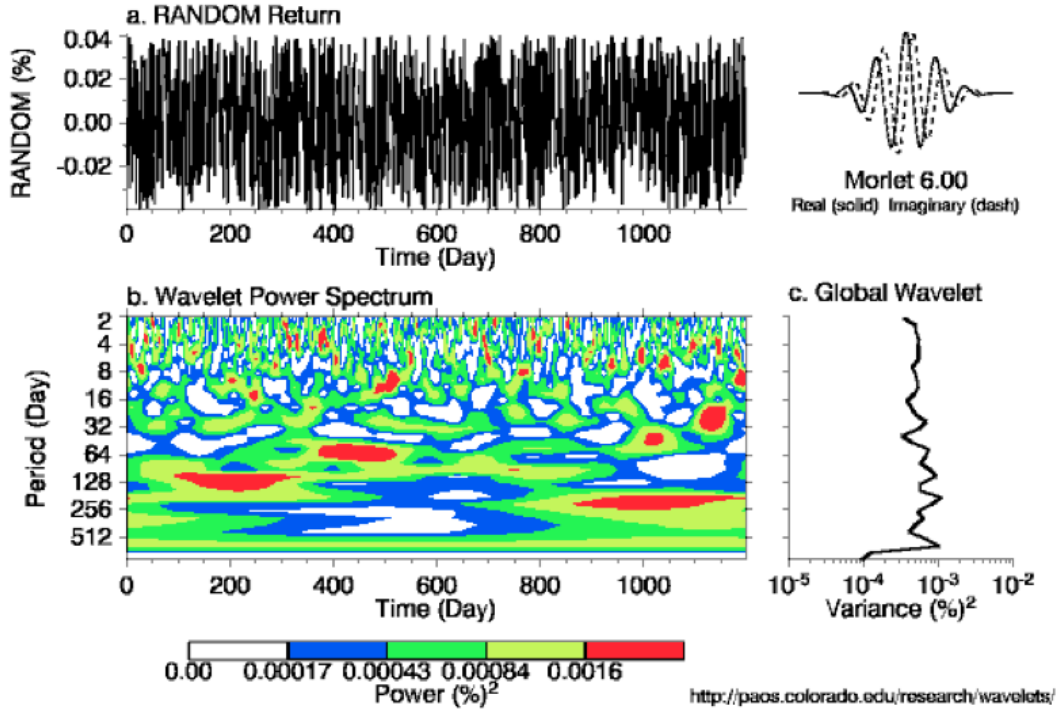
Shiller, J. R. (1981). Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? *The American Economic Review, 71,* 421-436.

Simonsen, I., Hansen, A., & Nes, M. O. (1998). Determination of the Hurst Exponent by use of Wavelet Transforms. *Physical Review E, 58,* 2779-2787.

Appendix

Figure 1: Wavelet scalogram and scalegram of white noise series
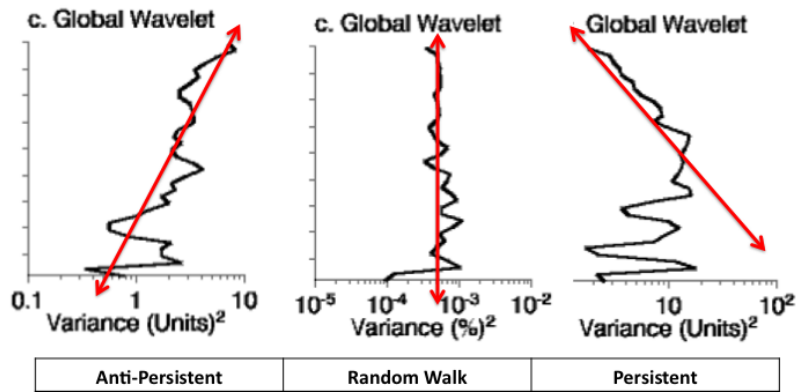

Notes: The white noise series consists of 1200 "daily returns", generated using Microsoft Excel's random number generator. In this randomly generated series, there are 619 positive returns among a total of 1200 returns. In Part b of the plot, the spectral peaks are evenly scattered at various scales and across the whole time period. In Part c of the plot, the global wavelet (scalegram) is nearly perfectly vertical with small variances being seen on either side.

Table 1: Popular and Neglected Portfolios, Criteria Ranks, and Hurst Exponents

Popular Portfolio

Name	Ticker	BM Rank	Analyst Rank	Transaction Cost Rank	Overall Rank
AMGEN INC	AMGN	8.2	12	10	30.2
BROADCOM CORP	BRCM	8.6	12	10	30.6
CISCO SYSTEMS INC	CSCO	8.6	12	10	30.6
ALTERA CORP	ALTR	8.8	12	10	30.8
EBAY INC	EBAY	8.8	12	10	30.8
MICROSOFT CORP	MSFT	8.8	12	10	30.8
QUALCOMM INC	QCOM	8.8	12	10	30.8
YAHOO INC	YHOO	8.8	12	10	30.8
SCHLUMBERGER LTD	SLB	9	12	10	31
GILEAD SCIENCES INC	GILD	9.6	12	10	31.6

Neglect Portfolio

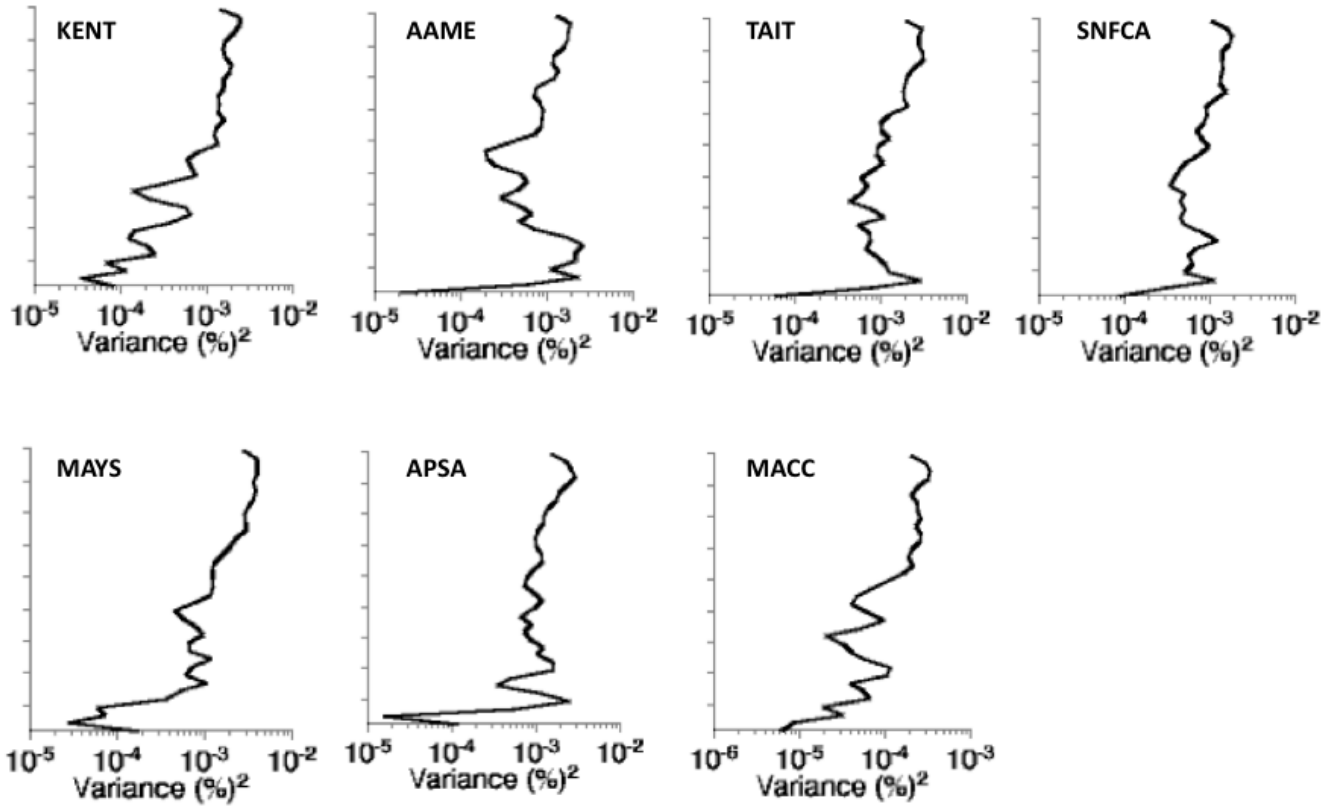| Name	Ticker	BM Rank	Analyst Rank	Transaction Cost Rank	Overall Rank
SECURITY NATIONAL FINANCIAL CORP.	SNFCA	1	0	1	2
ATLANTIC AMERICAN CORP	AAME	1	0	1	2
MAYS (J.W.) INC	MAYS	1	0	1	2
KENT FINANCIAL SERVICES INC	KENT	1	0	1	2
MACC PRIVATE EQUITIES INC	MACC	1	0	1	2
TAITRON COMPONENTS	TAIT	1	0	1	2
ALTO PALERMO S.A.	APSA	1	0	1	2

Notes: BM Rank is the rank based on the security's book to market ratio. Analyst Rank is the rank based on the number of analysts issuing forecasts. Transaction Cost Rank is the rank based on the size of the bid-ask spread for the security. Overall Rank is the aggregate of the previous three scores.

Figure 2: Scalegrams of Anti-persistent, Random-walk, and Persistent Returns
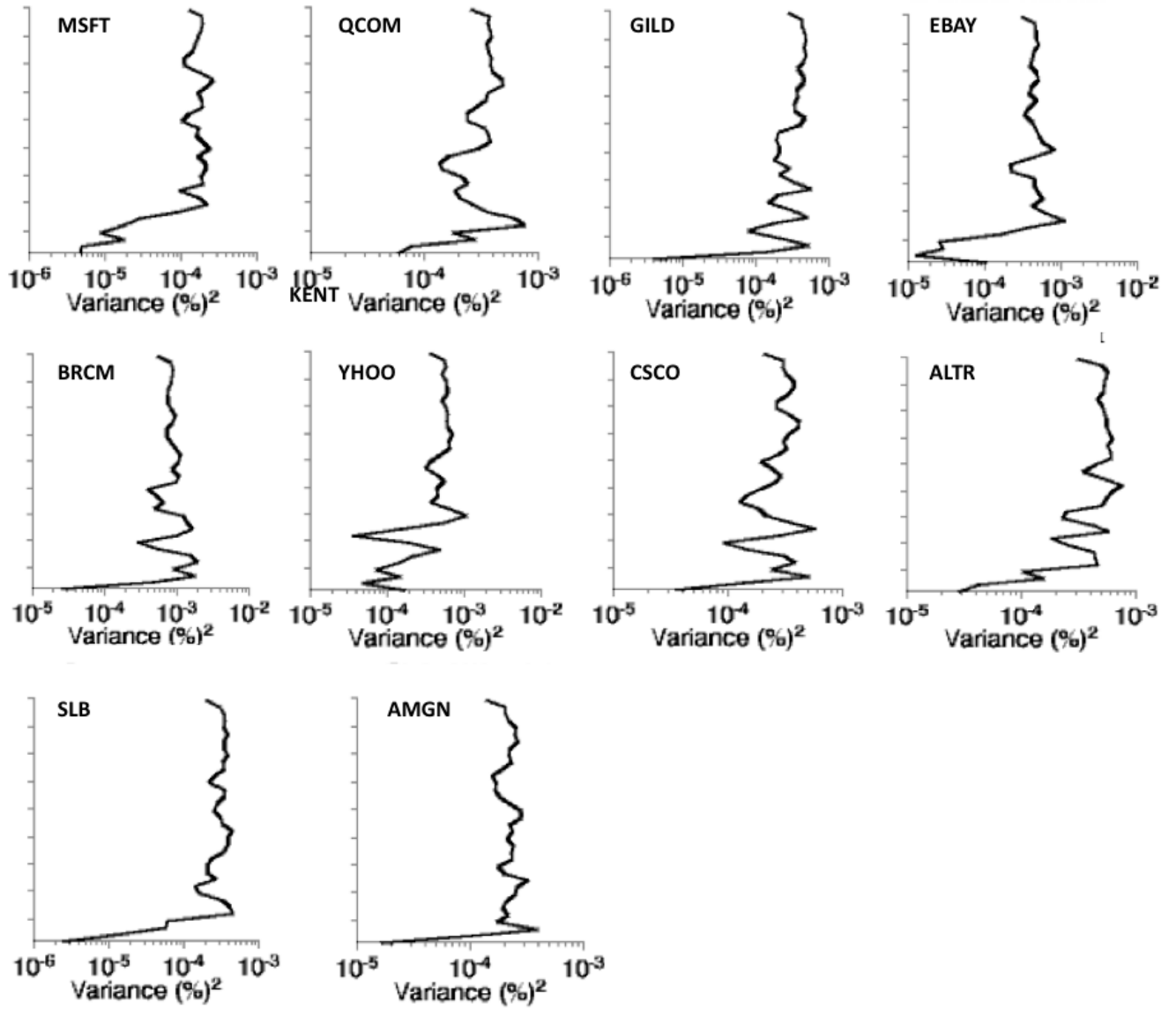

The return scalegrams for an anti-persistent, random walk and persistent time series are shown above. The red lines denote the approximate trends of the scalegrams with anti-persistent and persistent being positive and negatively sloped with respect to the vertical scalegram for a random walk. Notice that the lines are approximations of the scalegrams' trends.

Figure 3: Scalegrams of Neglected Portfolio Returns

Notes: Scalegrams of daily returns for the neglected portfolio stocks. Most important to note is the anti-persistent trend among the seven securities. While the negative slope is more apparent in some (KENT, MACC) than others (SNFCA, APSA), for all neglected stocks, there is an observable non vertical slope.

Figure 4: Scalegrams of Popular Portfolio Returns

Notes: Scalegrams of daily returns for the popular portfolio stocks. Unlike the scalegrams of the neglected portfolio, the popular portfolio all exhibit near vertical scalegrams, indicating the random walk feature of the stock series.

Table 2 Fractional differencing parameter: GPH estimators of the popular and neglected stocks series

Popular stocks

		MSFT	AMGN	SLB	ALTR	CSCO	QCOM	GILD	YHOO	BRCM	EBAY
GPH Estimator, d	bandwidth = T/2	0.9940	0.9998	1.0008	0.9750	0.9989	1.0207	1.0009	1.0017	0.9832	1.0295
	Standard deviation	0.0245	0.0284	0.0173	0.0278	0.0266	0.0272	0.0158	0.0268	0.0288	0.0294
	Hurst Exponent	0.4940	0.4998	0.5008	0.4750	0.4989	0.5207	0.5009	0.5017	0.4832	0.5295

Neglected stocks

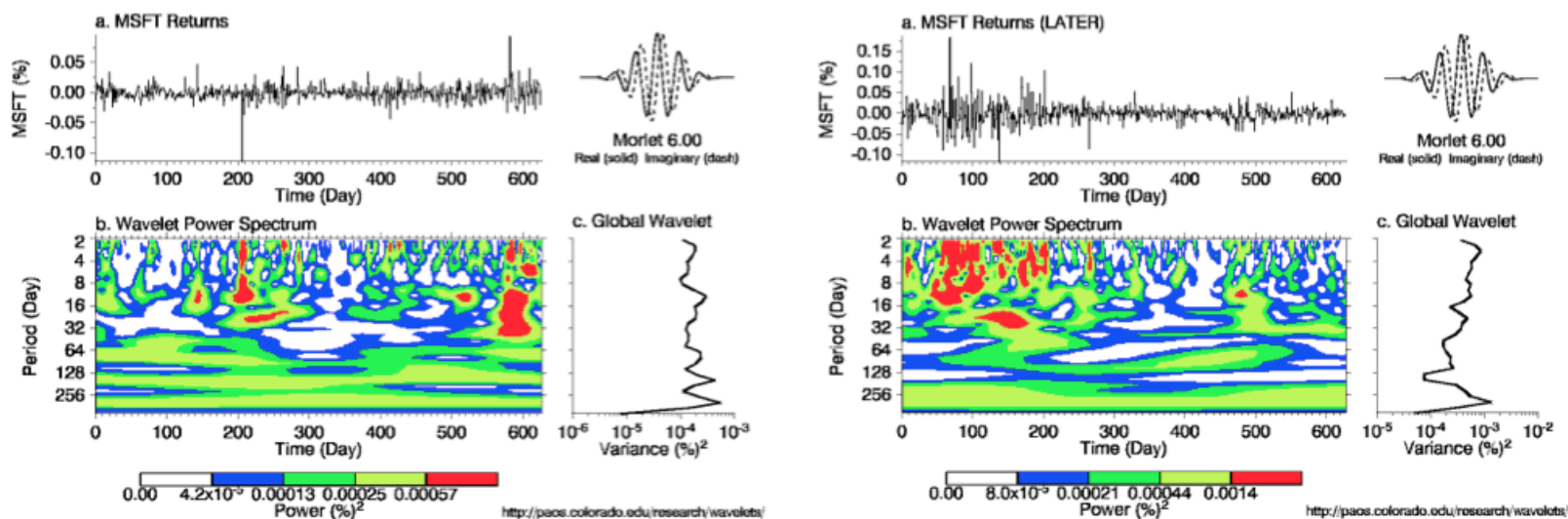|		SNFCA	AAME	MAYS	KENT	MACC	TAIT	APSA
GPH Estimator, d	bandwidth = T/2	0.8870	0.8357	0.9381	0.8191	0.6279	0.8023	0.9767
	Standard deviation	0.0323	0.0279	0.0273	0.0275	0.0272	0.0298	0.0247
	Hurst Exponent	0.3870	0.3357	0.4381	0.3191	0.1279	0.3023	0.4767

Table 3: Neglect Portfolio Average Daily Closing Prices and Volume

Neglect Portfolio

Name	Ticker	Exchange	Avg. Daily Volume	Avg. Daily Closing Price
SECURITY NATIONAL FINANCIAL CORP.	SNFCA	NASDAQ	5,702	$3.76
ATLANTIC AMERICAN CORP	AAME	NASDAQ	7,496	$2.85
MAYS (J.W.) INC	MAYS	NASDAQ	290	$16.75
KENT FINANCIAL SERVICES INC	KENT	NASDAQ	11,304	$2.16
MACC PRIVATE EQUITIES INC	MACC	PINK	2,686	$2.68
TAITRON COMPONENTS	TAIT	NASDAQ	4,718	$1.79
ALTO PALERMO S.A.	APSA	NASDAQ	1,837	$10.17

Notes: Average daily volumes are the mean values from January 1, 2003 to December 31, 2007.

Figure 5: Scalegrams of Returns for MSFT for Varying Data Ranges

On the left is the chart of daily returns, the power spectrum and the scalegram for MSFT from 7/2/05-12/31/07 and on the right is the same plots for MSFT from 7/7/08-12/31/10. One thing to note is the relative verticalness of the two scalegrams with the earlier data set resulting in a random-walk and the later data set suggesting anti-persistence in the returns. The volatility occurring from approximately day 70-150 on the chart and power spectrum of *MSFT Returns (LATER)* correspond with the financial crisis and shocks to the overall market. We posit that this event is a driver of the observance of anti-persistence in MSFT's daily returns and note the susceptibility of global Hurst exponents to be influenced by such occurrences.