THE EFFECTS OF CONCRETE-REPRESENTATIONAL-ABSTRACT SEQUENCE
OF INSTRUCTION ON SOLVING EQUATIONS USING INVERSE OPERATIONS
WITH HIGH SCHOOL STUDENTS WITH MILD INTELLECTUAL DISABILITY

by

Jennifer Jo Cease-Cook

A dissertation submitted to the faculty of
The University of North Carolina at Charlotte
in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in
Special Education

Charlotte

2013

Approved by:

_____
Dr. David Test

_____
Dr. Charles Wood

_____
Dr. David Pugalee

_____
Dr. David Gilmore

ABSTRACT

JENNIFER JO CEASE-COOK. The effects of concrete-representational-abstract sequence of instruction on solving equations using inverse operations with high school students with mild intellectual disability. (Under the direction of DR. DAVID W. TEST)

This study used a multiple probe across participants design to examine the effects of concrete-representational-abstract sequence of instruction on solving equations using inverse operations with high school students with mild intellectual disability. Results demonstrated a functional relation between the Abstract sequence of instruction and students ability to solve equations using inverse operations. Students were also able to maintain the skills learned up to four weeks post-intervention. Implications for practice and recommendations for future research are described.

ACKNOWLEDGEMENTS

I would first like to thank God who gave me the grace and privilege to purse this program and successfully complete it in spite of many challenges faced.

I would like to express my profound gratitude to my committee members, Drs. David Test, Charles Wood, David Pugalee, and David Gilmore for their support and guidance throughout the course of this work. I am grateful to Dr. Test, my advisor, my dissertation committee chairperson, and mentor for giving me opportunities, confidence, and guidance to pursue my goals. I appreciate the teachers and students who participated in this study without whose persistence, this research would not have been possible.

I also want to acknowledge the assistance and emotional support of my colleagues and friends Lauren Bethune, Flo Parkhill, Drs. Audrey Barthlomew, Valerie Mazzotti, Catherine Fowler, Ozalle Toms, and Amber McConnell. When times were tough, you gave me the confidence and strength to keep pressing on to achieve my goals. God Bless you all.

The sacrifice of my family throughout my years here are simply ineffable. The belief you have in me and support you gave me instilled the values and virtues that see me through on a daily basis. I thank my parents for helping me develop a strong work ethic and for instilling a love of education and learning. I would like to acknowledge the patience, support, and dedication of my children. Kelssie, Gavin, and Payton, you each inspired me to complete this task and without your love and support, it would not have been possible.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: INTRODUCTION

Statement of the Problem

Although standards-based reform has been evolving over a period of 40 years, it is relatively new to the field of special education (Browder, Trela, et al., 2012). It has been 20 years since special educators began anticipating how standards-based reform would increase expectations for learning for students with disabilities (McDonnell, McLaughlin, & Morison, 1997). The purpose of standards-based reform is to better align special education programs and policies with larger national school improvement efforts (Nolet & McLaughlin, 2000). For only the last decade, special educators have been feeling the impact of this reform. Starting with No Child Left Behind (2001) and more recently, the proposed reauthorization of Elementary and Secondary Education Act (ESEA) reinforced the standards-based reform for students with disabilities. In March, 2010, the Obama administration released recommendations for reauthorizing the (ESEA) in a document titled "Blueprint for Reform" (http://www2.ed.gov/policy/elsec/leg/blueprint/index.html). The blueprint provides incentives for states to adopt academic standards that prepare students to succeed in postsecondary education and the workplace. The document asserts that "every student should graduate from high school ready for college and a career. Every student should have meaningful opportunities to choose from upon graduation from high school" (Blueprint, p. 7). This standards-based reform has become known as the College and

Career Ready movement. To help achieve the stated goal of College and Career Readiness, the proposed reauthorization of ESEA calls for raising standards for all students in English language arts and mathematics, developing better assessments aligned with college and career-ready standards, and implementing a complete education through improved professional development and evidence-based instructional models and supports (United States Department of Education, USDOE; 2010). The focus on college and career readiness has been a result of (a) four out of every 10 college students, including those at two-year institutions, needing to take remedial courses in college; and (b) many employers commenting on the inadequate preparation of high school graduates (USDOE, 2010).

To address these issues, in June 2010 the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA) released final versions of the Common Core State Standards (CCSS) for Language Arts and Mathematics. CCSS describe the knowledge and skills in English Language Arts and Mathematics students will need when they graduate, whatever their choice of college or career. These sets of standards define the knowledge and skills students need to succeed in entry-level, credit-bearing, academic college courses, as well as in workforce training programs. The standards are based on the best national and international standards, giving students a competitive advantage in the global economy (USDOE, 2010). Together, the CCSS initiative and proposed ESEA reauthorization recommendations are relevant for all students, including students with disabilities at the secondary level, because they have implications for curricula, instruction, and assessment. To date, 49 states/territories have

fully adopted the CCSS, one state has provisionally adopted the standards, and one state has adopted the ELA standards only.

CCSS currently addresses Mathematics and English Language Arts (ELA) in Literacy in History/Social Studies, Science, and Technical Subjects. The standards articulate rigorous grade-level expectations in the areas of Mathematics and ELA and identify the knowledge and skills students need to be successful in college and careers (National Governors Association Center for Best Practices, 2010). According to National Governors Association Center for Best Practices (2010), these standards provide a "historic opportunity to improve access to rigorous academic content standards for students with disabilities (Application to Students with Disabilities, pg. 1)." In order to be successful in the general curriculum, students with disabilities should be provided additional supports and services such as (a) instructional supports for learning based on the principles of universal design for learning (UDL), (b) instructional accommodations which include changes in materials or procedures but not changes to the standards, and (c) assistive technology and services (National Governors Association Center for Best Practices, 2010).

Historically, for students with disabilities, there have been multiple pathways to graduation, including IEP diplomas, alternative track diplomas, and certificate of completion (Thurlow & Thompson, 2000). Since the adoption of CCSS, a major shift in graduation requirements has resulted in students with disabilities being required to pass more rigorous courses and exams. Recently, the Center on Education Policy reported 28 states in 2010 and 25 states in 2011 required an exit exam for graduation. Thirty states do not have diplomas that were available only to youth with disabilities, eight states used the

certificate of attendance only for youth with disabilities, six states used the certificate of achievement only for youth with disabilities, and three states used their occupational diploma only for youth with disabilities. As the term implies, the IEP/Special Education diploma is used by eight states only for youth with disabilities (Johnson, Thurlow, & Schuelka, 2012).

This major shift in academic rigor has led many states to require students to pass algebra in order to graduate high school. Algebra is a branch of mathematics in which symbols, usually letters of the alphabet, represent numbers or members of a specified set and are used to represent quantities and to express general relationships that hold for all members of the set (Maccini, Mulcahy, & Wilson, 2007). CCSS in mathematics and algebra were built on progressions: narrative documents describing the progression of a topic across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics. These documents were spliced together and then sliced into grade level standards (USDOE, 2010).

While these rigorous standards were released in June 2010, almost all states have adopted them. In fact, the federal government has spent billions of dollars in grants, and two consortia of states have been awarded $330 million to develop new assessments; however, many teachers have not seen any changes. The standards may have reached their districts, but not their classrooms (Gates Foundation, 2012). Even in states that have begun to provide professional development and support, teachers are still struggling with the progression of these complex math skills across grade level and across disabilities (Gates Foundation, 2012). Unfortunately, only a few teaching strategies are available to teach students with disabilities content that links to the common core

standards or other state standards in math, specifically in algebra (Browder, Jimenez, et al., 2012).

Because algebra is a gatekeeper to postsecondary education, it is important students learn these skills in high school (National Mathematics Advisory Panel, 2008). Given this emphasis, the performance level of students with disabilities is a concern. There is a need to develop more efficient and effective algebra interventions for students with disabilities. Only a small number of studies address algebraic concepts and skills for students with disabilities.

Algebra Instruction for Secondary Students with Disabilities

Mastering mathematics depends on continuous development and blending of a combination of various critical skills. Gaps in any of these component skills cause students to struggle in many aspects of their mathematics education. This is especially true for students with disabilities. The deficits in mathematics for students with disabilities are found in the areas of basic facts, computation procedures, fractions, and solving word problems. Since many of these deficit areas are prerequisite skills for algebra, it is no surprise that students with disabilities struggle with algebraic concepts (Witzel & Riccomini, 2009). The majority of research involving teaching algebra to students with disabilities has focused on students with learning disabilities.

Students with learning disabilities. Maccini, Mulcahy, and Wilson (2007) conducted a meta-analysis on algebra instruction for secondary students with learning disabilities (LD). Their review included 14 studies with participants with learning disabilities published between 1995 and 2006. Effective strategies for teaching secondary algebra skills to students with learning disabilities have used schema-based instruction

(Jitendra, Hoff, & Beck, 1999 & Xin, Jitendra, & Deatline-Buchman, 2005), mnemonic strategy instruction (Maccini & Ruhl, 2000), Concrete-to Representational to-Abstract (CRA) sequence of instruction (Maccini & Hughes, 2000; Witzel, Mercer, & Miller, 2003; Witzel, 2005), contextualized instruction (Bottge, 1999; Bottge, Henruchs, Chan, Mehta, & Watson, 2003; Bottge, Henrichs, Mehta, & Hung, 2002; Bottge, Henruchs, Mehta, & Serlin, 2001), and explicit inquiry routine (Scheuermann, Deschler, & Schumaker, 2009). The most common strategy used for students with learning disabilities is CRA. As its name implies, the CRA instructional sequence systematically and explicitly instructs students through the three levels of learning (a) concrete, (b) representational, and (c) abstract. Each transition is purposeful through each of the stages and encourages students to learn concepts, as well as the procedures and computations that are so important in mathematics.

Students with intellectual disability. Research on mathematics instruction for students with intellectual disability has primarily focused on functional curricula. Functional curricula help students with disabilities acquire adaptive skills needed for post-school life that they would not develop unless explicitly taught (Snell & Browder, 1987). Students with intellectual disability often have difficulties acquiring daily living skills, vocational skills, or community access skills without direct instruction in those areas (Bouck, 2009). Because secondary students with disabilities may not be prepared adequately for post-school life, some educators and researchers have recommended increased use of functional curriculum (Bigge, Stump, Spagna, & Silberman, 1999; Bouck, 2004; Cronin, 1996; Dever & Knapczyk, 1997; Knowlton, 1998; & Patton, Pollaway, & Smith, 2000). For example, Pollaway, Patton, Smith, and Roderique (1991)

suggested students' post-school environments should determine the curricular approach in school. Although a functional curriculum includes math, language arts, science, and social studies it is not the same content or for the same purpose (Bouck, 2009). For example, a functional mathematics curriculum may include how to balance a check book, use an ATM machine, or shop for groceries, but not algebra. However, if a student plans to graduate with a standard diploma, he/she must take and pass algebra.

Browder et al. (2008) conducted a meta-analysis of strategies used to teach mathematics to students with intellectual disability. Their review included 68 studies of mathematics instruction and strategies used with students with intellectual disability published between 1975 and 2005. Of those 68 studies, only two studies focused on skills related to algebra and involved teaching students to solve word problems using addition and subtraction and quantifying sets (Miser, 1985; Neef et al., 2003). Since this meta-analysis, three additional studies have been conducted on teaching algebra to students with intellectual disability using systematic instruction with a concrete representation (Jimenez, Browder, & Courtade, 2008), schema-based instruction (Browder, Trela, et al., 2012), and read-alouds of math problems with task analytic instruction to find the solution (Browder, Jimenez, et al., 2012).

Significance of Study

There are many factors to consider when choosing an instructional practice for teaching algebra skills to students with disabilities. Those factors include grade alignment and generalization. One must consider if the instruction and skill is grade aligned. For example, Jitendra, Hoff, and Beck (1999) used schema-based instruction to teach algebraic word problem solving. The students in that study were working on addition and

subtraction which was several grades behind typical algebra classes. In addition, Witzel, Mercer, and Miller (2003) suggest that while CRA was effective, it was important to make sure the components of algebra were age-level appropriate.

Another important factor for teaching algebra to students with intellectual disability is generalization. Although general education often assumes generalization to real life activities, students with intellectual disability may need opportunities to apply their skills to daily living (Browder, Jimenez, et al., 2012). Browder, Trela et al. (2012) and Browder, Jimenez, et al. (2012) recommend that future research examine the extent to which students with intellectual disability are able to generalize the academic concepts to real life activities. Browder Jimenez, et al. (2012) recommended that future research address evaluations by stakeholders to validate that (a) the skill learned is algebra, (b) skill has social validity to the student, and (c) the skills are applicable to students' real life.

Because of its abstract nature, teachers have struggled to help students comprehend algebraic concepts (Witzel, et al., 2003). For students to understand the initial algebra concepts, it is important students to learn those abstract concepts in a concrete manner first (Devlin, 2000). One suggestion for simplifying complex concepts is to transform them into concrete manipulations and pictorial representations (Witzel, et al., 2003). An effective approach in making algebra more accessible through the use of concrete materials that develop into representational and eventually abstract thought is known as the concrete-representational-to abstract sequence of instruction (CRA; Witzel, et al., 2003).While this approach has been effective in teaching algebraic concepts to students with learning disabilities, it is important that research continue to be conducted

to determine if CRA can be used to teach algebra to students with intellectual disability as well.

Therefore, the purpose of this study was to examine the effects of using CRA instruction on solving equations using inverse operations with high school students with mild intellectual disability. An example would be $3n = 15$; using the inverse or opposite operation to solve for the single variable. This study extended the literature for CRA by examining the strategy with high school students with mild intellectual disability and collecting data on student ability to generalize to functional skills performed in a community context. The specific research questions were:

1. What are the effects of CRA instruction on high school students with intellectual disability ability to solve solving equations using inverse operations?

2. What are the effects of CRA instruction on high school students with intellectual disability ability to maintain skills learned on solving equations using inverse operations?

3. What are the effects of CRA instruction on high school students with intellectual disability ability to generalize skills learned to a skill performed in a community context?

4. What were students' perceptions of CRA instruction as a method for increasing their ability and to what extent it had an effect on solving equations using inverse operations?

5. What were teachers' perceptions of the use of CRA instruction as a method for increasing students' ability to solve equations using inverse operations and to what extent do they feel it had an effect on their ability?

Delimitations

       This study used a single-subject research design. As with any single subject study, the small number of participants may hinder the generalization to other teachers and students. All participants were at the high school level, this may also limit generalization.

Definition of Terms

       The terms that were used in this study and their definitions are presented in this section. The terms chosen for defining in this section are critical for comprehending the implementation procedures and results of the study.

       Algebra: Algebra has been defined as a branch of mathematics in which symbols, usually letters of the alphabet, represent numbers or members of a specified set and are used to represent quantities and to express general relationships that hold for all members of the set (Maccini, Mulcahy, & Wilson, 2007).

       Concrete-Representational-Abstract (CRA) instruction: Concrete to Representational to Abstract Sequence of Instruction has been defined as including concrete which is expeditious use of manipulatives, pictorial Representations, and abstract procedures (Witzel, 2005).

          Concrete: The concrete level of understanding is the most basic level of mathematical understanding. It is also the most crucial level for developing conceptual understanding of math concepts/skills. Concrete learning occurs when students have ample opportunities to manipulate concrete objects to problem-solve (Allsopp, 1999).

          Representational: At the representational level of understanding, students learn to problem-solve by drawing pictures. The pictures students draw represent the concrete objects students manipulated when problem-solving at the concrete level. It is

appropriate for students to begin drawing solutions to problems as soon as they demonstrate they have mastered a particular math concept/skill at the concrete level. While not all students need to draw solutions to problems before moving from a concrete level of understanding to an abstract level of understanding, students who have learning problems in particular typically need practice solving problems through drawing. When they learn to draw solutions, students are provided an intermediate step where they begin transferring their concrete understanding toward an abstract level of understanding. When students learn to draw solutions, they gain the ability to solve problems independently. Through multiple independent problem-solving practice opportunities, students gain confidence as they experience success. Multiple practice opportunities also assist students to begin to "internalize" the particular problem-solving process. Additionally, students' concrete understanding of the concept/skill is reinforced because of the similarity of their drawings to the manipulatives they used previously at the concrete level (Witzel & Riccomini, 2011).

Abstract: Abstract understanding is often referred to as, doing math in your head. Completing math problems where math problems are written and students solve these problems using paper and pencil is a common example of abstract level problem solving (Allsopp, 1999).

Contextualized instruction: Contextualized instruction has been defined as conception of teaching and learning that helps teachers relate subject matter content to real world situations (Bottge, 1999).

Explicit inquiry routine (EIR): EIR has been defined as integrating general education mathematical teaching practices such as inquiry and dialogue and special

education teaching practices such as intensive and explicit instruction to engage students across multiple methods including concrete, representation, and abstract (Scheurman, Deshler, & Schumaker, 2009).

Intellectual disability: Intellectual disability has been defined by the American Association on Intellectual and Developmental Disabilities as Intellectual disability is a "disability characterized by significant limitations both in intellectual functioning and in adaptive behavior, which covers many everyday social and practical skills. This disability originates before the age of 18. One criterion to measure intellectual functioning is an IQ test. Generally, an IQ test score of around 70 or as high as 75 indicates a limitation in intellectual functioning. Standardized tests can also determine limitations in adaptive behavior" (www.aaidd.org) .

Inverse operations: Inverse operations, often called single-variable operations, involves solving for an unknown number or set of numbers. (Common Core Math Standards, 2010).

Schema-based instruction: Schema-based instruction has been defined as teaching the student how the semantic structure of the word problem translates into elements of the schema used in the problem-solving process. A schema is a cognitive framework or concept that helps organize and interpret information (Jitendra, et al., 1999).

Metacognition: Metacognition refers to higher order thinking which involves active control over the cognitive processes engaged in learning. Activities such as planning how to approach a given learning task, monitoring comprehension, and evaluating progress toward the completion of a task are metacognitive in nature (Flavell, 1979).

Mild Intellectual Disability: Mild intellectual disability has been "characterized by significantly subaverage intellectual functioning, existing concurrently with related limitations in two or more of the following applicable adaptive skills areas: communication, self-care, home living, social skills, community use, self-direction, health and safety, functional academics, leisure, and work (Polloway, Patton, Smith, & Buck, 1997, p. 298).

Mnemonic Strategy: Mnemonic Strategy instruction has been defined as a memory device that can help students recall the sequential steps from familiar words using first letters from the beginning of other words (Maccini & Ruhl, 2000).

CHAPTER 2: REVIEW OF LITERATURE

The following review of the literature is designed to provide a comprehensive overview of empirical research to provide the rationale for the need for the current study. This review will reflect both recent and seminal research that are key components to the development of the proposed study. The review will review characteristics of students with mild intellectual disability, teaching students with intellectual disability, standards based reform, and teaching algebra.

Characteristics of Students with Mild Intellectual Disability

For the last 50 years, the term most frequently used to identify students with mild intellectual disability was mental retardation. Despite the negative connotation associated with mental retardation, it is the most commonly used descriptor among educators and parents (Polloway et al., 2010). In 2000, 26 state departments still continued to use the term mental retardation. For the purpose of the proposed study, the term mild intellectual disability will refer to a student that is "characterized by significantly subaverage intellectual functioning, existing concurrently with related limitations in two or more of the following applicable adaptive skill areas: communication, self-care, home living, social skills, community use, self direction, health and safety, functional academics, leisure, and work" (Polloway, Patton, Smith, & Buck, 1997, p. 298).

Students with mild intellectual disability were once considered the largest focus in special education and the category is often portrayed as the foundation of the field (Bouck, 2007; Edgar, 1987; Polloway, 2006). Unfortunately in research and in

identification for special education, students with mild intellectual disability are declining (Polloway, 2006). As a result, students with mild intellectual disability have been referred to by some as the forgotten generation (Fujiura, 2003). These students are receiving other classification for services and being put into high incidence disabilities, despite not having mild needs (Polloway, 2004; Smith, 2006). This misidentification has led to a lack of attention, research, and advocacy for students with mild intellectual disability in terms of curriculum, instruction, and post-school outcomes (Bouck, 2012; Polloway, 2004). The majority of reports and research on in-school experiences of students with mild intellectual disability aggregate data with students with moderate/severe disabilities (Newman et al., 2009; Yu, Newman, & Wagner, 2009), resulting in a lack of knowledge on the educational programs of students with intellectual disability (Bouck, 2012).

Students with mild intellectual disability exhibit several common academic and behavioral characteristics that may affect their success in standard-based mathematics activities. In addition to performing several grade levels below their peers without disabilities (Wagner et al., 1995) other characteristics that may hinder performance include difficulty attending to key dimensions of tasks (Kauffman, 2001) and deficits in metacognitive strategies (Gallico, Burns, & Grob, 1991). Other characteristics often associated with students with mild intellectual disability include difficulty transferring and generalizing information, putting information into memory, and retrieving information from memory (Belmont, 1966; Stephens, 1972).

According to the National Longitudinal Transition Study (NLTS; 1996) and National Longitudinal Transition Study- 2 (NLTS-2; 2011), students with mild intellectual disability have the poorest post-school outcomes of all disability categories

(Blackorby & Wagner, 1996). The original NLTS (1985-1993) collected data from parents, youth, and schools to provide a national picture of individuals was they transition into adulthood. This data were collected from 1985 until 1993.Results showed a 35% employment rate for students with intellectual disability and that only 14.1% reported living independently. They also found that for students with intellectual disability only 15.4% attended postsecondary education with 11.0% attending a vocational school, 5.1% attending a community or two-year institution, and 0% attending a four-year university (Blackorby & Wagner, 1996). Results from the NLTS-2 (2000-2020) follow-up of the original study reported no improvement in post-school outcomes for students with intellectual disability (Newman et al., 2011). When compared to other disability categories, young adults with intellectual disability were less likely to be enrolled in postsecondary education (61% compared to 75%). While young adults had employment rate of 57%, they worked fewer hours per week and earned less per hour ($11.10 vs $7.90) compared to youth with other disabilities (Newman et al., 2011).

Given these poor post-school outcomes, the educational programming and instruction students with mild intellectual disability receive must be addressed. Teachers reported 28.4% used a special education curriculum, 19% used a functional curriculum, 15.3% used a general education curriculum, and the remaining teachers used small frequencies of other models (e.g., lower grade level, vocational education, or no curriculum; Bouck, 2004). Teachers in this study reported a high level of dissatisfaction with the curricular options and noted the greatest improvement need for students with intellectual disabilities was a more appropriate curriculum.

Summary of characteristics of students with mild intellectual disability. Students with mild intellectual disability have experienced poor post-school outcomes for decades. Because students with mild intellectual disability are characterized as performing several grade levels below their peers without disabilities and have deficits in metacognitive strategies, it is important to address the curricular options for these students. Students with intellectual disability primarily have been taught using special education, general education, and functional curricula. The next section focuses on the functional curricula used to teach students with mild intellectual disability.

Teaching Functional Curricula to Students with Mild Intellectual Disability

One curricular approach for students with intellectual disability has been to use a functional curriculum (Edgar, 1987; Kaiser & Abell, 1997; Patton, et al., 1989). A functional curriculum includes "functional skills and applications of core subject areas, vocational education, community access, daily living, financial, independent living, transportation, social/relationships, and self-determination" (Bouck, 2012, p. 140). Research supports the use of a functional curriculum. For example, students participating in the Youth Transition Program, which is a program that involves life skills, had increased post-school outcomes in education, employment, and independent living (Benz, Lindstrom, & Yovanoff, 2000). Alwell and Cobb (2009) found students benefited from functional curriculum but that research has primarily focused on students with severe or low incidence disabilities. However, recent analyses of NLTS-2 data reported receipt of a functional curriculum did not influence post-school outcomes for students with mild intellectual disability (Bouck & Joshi, 2012) or students with severe disabilities (Carter et al., 2012).

Functional skill instruction. Many evidence-based practices have been identified

to teach students with intellectual disability functional life skills by the National

Secondary Transition Technical Assistance Center (2009). They include using backward

chaining to teach treating a cut, burn, and insect bite (Gast et al. 1992) and purchasing

fast food and grocery items (McDonnell & Laughlin, 1989). Second, using constant time

delay to teach crossing a street and mailing a letter (Branham, Collins, Schuster, &

Kleinert, 1999), washing and drying clothes (Miller & Test, 1989), and cleaning a sink

and folding clothes (Wolery, Aust, Gast, Doyle, & Griffen, 1991). Third, using forward

chaining to teach food preparation skills (Horsfall & Maggs, 1986) and home

maintenance skills (McDonnell & McFarland,1988). Fourth, using progressive time

delay to teach reading warning labels (Collins & Stinson, 1994), how to use the

payphone, (Collins, Stinson, & Land, 1993), purchasing (McDonnell, 1987), and price

comparison (Sandknop, Schuster, Wolery, & Cross, 1993). Fifth, using self-monitoring

instruction to teach leisure skills in the community (Mahon & Bullock, 1992) and

increasing physical activity (Todd & Reid, 2006). Sixth, simultaneous prompting has

been used to teach opening a locker with a keyed lock (Fetko et al., 1999), grocery

shopping (Singleton et al., 1999), and restaurant terminology (Smith, Schuster, Collins, &

Kleinert, 2011). Seventh, system of least to most prompting has been used to teach

making a grocery list for nutritious meals (Arnold-Reid, Schloss, & Alper, 1997), using a

washer and dryer at laundromat (Bates et al., 2001), making toast (Steege, Wacker, &

McMahon, 1987), using a cell phone to call for help when lost (Taber et al., 2002) and

using speed dial on a cell phone when lost (Taber et al., 2003). Eighth, using most to

least prompting to teach ATM usage (McDonnell & Ferguson, 1989), purchasing skills

(McDonnell & Laughlin, 1989), exercise routine (O'Conner & Cuvo, 1989), and bowling and pinball skills (Vandercook, 1991). Finally, using total task chaining to teach changing a sanitary napkin (Ersoy, Tekin-Iftar, Kiracaali-Iftar, 2009), laundry skills and using a soap dispenser (McDonnell & Farland, 1988), purchasing fast food items (McDonnell & Laughlin, 1989), and bowling in the community (Vandercook, 1991). These practices have primarily been used with students with moderate to severe disabilities. Of those evidence-based practices, only three practices included students with mild intellectual disability. They were self-monitoring instruction, system of least-to-most prompts, and total task training.

Self-monitoring instruction has been defined as a procedure whereby a person observes his behavior systematically and records the occurrence or nonoccurrence of the target behavior (Cooper, Heron, & Heward, 2007). Mahon and Bullock (1992) used self-monitoring instruction through self control techniques to teach students with mild intellectual disability to teach decision making on leisure skills in the community.

Next, system of least-to-most prompts was used to teach students with mild intellectual disability to make grocery lists (Arnold-Reid et al., 1997) and laundromat skills (Bates et al., 2001). System of least to most prompts is a method used to transfer stimulus control from response prompts to the natural stimulus whenever the participant does not respond to the natural stimulus or makes an incorrect response. Least-to-most prompts begin with the participant having the opportunity to perform the response with the least amount of assistance on each trial. Greater degrees of assistance are provided with each successive trial without a correct response (Cooper et al., 2007). Students with mild intellectual disability were taught using least-to-most prompting in combination

with a meal chart organizer to teach making a grocery list to prepare nutritionally sound meals (Arnold-Reid, Schloss, & Alper, 1997) and was used during community-based instruction with social praise reinforcers to teach washing and drying clothes in a laundromat (Bates, Cuvo, Miner, & Korabek, 2001).

Lastly, students with intellectual disability were taught using total task chaining paired with time delay to teach changing sanitary napkins (Ersoy et al., 2009). Total task chaining is defined as a variation of forward chaining in which the learner receives training on each step in task analysis during each session (Cooper et al., 2007).

Functional math instruction. Historically, strategies to teach mathematics to students with intellectual disability have focused on teaching mathematics within daily living activities, such as shopping (Browder et al., 2008). For students with mild intellectual disability, the majority of research has focused on purchasing and money management (Xin et al., 2005). For example, Xin et al. (2005) conducted a meta-analysis on purchasing skill instruction for students with intellectual disability. Their review included 28 studies with participants with intellectual disability published between 1978 and 2003. The most effective strategies for teaching purchasing skills have been response prompting, simulations, constant time delay, community based instruction, and one-more-than strategy. However, only simulation training, community based instruction, and the one-more-than strategy have been found effective for students with mild intellectual disability.

First, community-based instruction (CBI) has been used to teach purchasing skills. Community-based instruction is a best practice in the education of students with moderate and severe disabilities (Alberto, Cihak, & Gama, 2005). Bates, Cuvo, Miner,

and Korabek (2001) examined simulated and community-based instructional arrangements across four functional living tasks with students with mild and moderate intellectual disabilities. Students ranged in age from 14 to 21. The four functional living skills were (a) purchasing at a grocery store, (b) use of a commercial laundromat, (c) purchasing a soft drink in a restaurant, and (c) janitorial skills in a restroom. Simulated instruction was used in the classroom with picture prompts of a student performing the steps of the task analysis in the community-based training sites. Each student received 10 trials in the classroom and then 10 trials in the community. Using a group experimental design, results for the two purchasing tasks indicated students made statistically significant improvements from pretesting to post-testing on these tasks.

Second, one-more-than strategy has been used to teach the functional math skills of counting money. This strategy involves teaching individuals to pay one more dollar than requested (e.g., if a salesperson says "$3.29", the student would provide four one dollar bills; Denny & Test, 1995). This strategy is also referred to as the dollar more, next dollar, and counting-on strategy. Denny and Test (1995) replicated the use of the one-more-than strategy with cents-pile modification with one, five, and 10-dollar bills to teach purchasing skills to students with mild and moderate intellectual disabilities. Students were all 17 years in age. Using a multiple- baseline design across students, the three students were evaluated on the number of steps performed correctly on a 12-item task analysis of the one-more-than strategy. Results demonstrated a functional relationship between training procedures, skill acquisition, and maintenance of the one-more-than strategy with cents-pile modification. Students were also able to use the techniques in non-trained community settings.

Finally, simulation training has been used to teach functional skills of using an ATM machine and tracking expenses. This strategy is defined as using materials and situations in the classroom that approximate the natural stimulus conditions and response topographies associated with the performance of functional skills in community settings (Bate et al., 2001). Rowe, Cease-Cook, and Test (2011) used a multiple probe design across participants to examine the effects of classroom simulation using static picture prompts on students' ability to acquire, maintain, and generalize skills necessary to use a debit card to make a purchase and track their expenses. Results demonstrated a functional relationship between simulated instruction and students' ability to use a debit card to make a purchase and track their expenses in a check register.

Summary of teaching students with mild intellectual disability. Given the poor post-school outcomes for students with mild intellectual disability, it is imperative researchers and educators consider the educational programming these students receive. Research has shown effective methods for teaching functional skills to secondary students with mild intellectual disability including self-monitoring instruction, system of least-to-most prompts, and total task training. Teaching functional math skills to students with mild intellectual disability has primarily focused on teaching purchasing skills. Community-based instruction, simulation training, and the one-more-than strategy have been shown to be effective methods with secondary students with mild intellectual disability. Community-based instruction combined with classroom instruction and community-based instruction combined with static picture prompts (Bates, et al., 2001) have shown positive results in teaching purchasing skills. Second, the one-more-than strategy using one-dollar bills only, one, five, and 10-dollar bills, and a cents-pile

modification (Denny & Test, 1995) have shown positive results in teaching purchasing skills.

Standards-Based Education Reform

Standard-based education reform has made promises to the country for the future of education. Some of those promises include (a) more accurate overall picture of education with comparing schools and districts, (b) providing benefits for students with disabilities who take part in state assessments, and (c) promoting high expectations for students (Thurlow, 2002). Federal laws (IDEA, Title 1) require access to the general curriculum and the participation of students with disabilities in state assessments. The greatest promise of standard-based reform and the development of the Common Core State Standards (CCSS) for students with disabilities is that it will result in programmatic and instructional improvements.

School-based reform in mathematics. National Commission on Excellence in Education (1983) declared the United States to be a "nation at risk…whose educational foundations are presently being eroded by a rising tide of mediocrity that threatens our very future (p. 17)." This report entitled, A Nation at Risk (1983) criticized American students and said they were not learning enough and there was a need to improve the national educational results significantly (Manno, 1994). This publication is credited as the initiating event of the current standards-based movement (Marzano & Kendall, 1996). In 1993, it was reported that historians would undoubtedly note the last decade of this century as the time when the need for national standards emerged (Glaser & Linn, 1993).

A Nation at Risk (1983) reported the decline in the nation's academic achievement dating back to 1963 in mathematics. Between the years 1975 and 1980,

there was a 72% increase in college remedial mathematics courses. The report recommended an increase in college math skills, training of qualified teachers, and more rigorous texts. With the publication of this report, policy makers began demanding educators be accountable for student results (Buttraam & Waters, 1997).

Math standards. In response to The Nation at Risk report, National Council of Teachers of Mathematics (NCTM) responded by appointing a committee to develop recommendations for school systems. NCTM (1989) released the first standards document for mathematics called Curriculum and Evaluation Standards for School Mathematics. These standards advocated for fundamental changes in secondary school mathematics curricula, instruction, and assessment (National Research Council, 1989). For secondary mathematics, the proposed changes had implications for teaching algebra (Huntley et al., 2000). Traditional algebra was primarily taught in college preparatory courses, but the shift envisioned a curriculum that integrated strands of algebra, functions, geometry, statistics, and probability for all students (Huntley et al., 2000). Those original math standards were updated and called Principles and Standards for School Mathematics (NCTM, 2000). These content standards included (a) number and operations, (b) measurement, (c) data analysis and probability, (d) geometry, and (e) algebra.

Common core state standards. Today, educators views the skills set needed for students today are vastly different than what their parents were taught a generation ago (Daggett, 2012). Recognizing this changing nature of work, technology, and competition in the global job market, the federal government placed mandates on schools that received funding through the American Recovery and Reinvestment Act (ARRA; 2009),

which allocated $100 billion for school improvement efforts. These mandates called for new steps to better align the Elementary and Secondary Education Act (ESEA) to support college-and-career ready standards. While standards have been around for decades, the incentive to implement national standards has recently changed with these mandates. The new school-based reform movement is called the College and Career Ready movement. President Obama's administration released a Blueprint for Reform that asserts that all students should graduate college and career ready. Race to the Top (RTT) fund, a section of ARRA, targeted $4.3 billion dollars to reform education. Council of Chief State School Officers and the National Governors Association (NGA) released Common CCSS for Language Arts and Mathematics that would be aligned to the RTT goals for educational reform (2010). While adoption of the standards was voluntary, states could be awarded grants from RTT funding for their adoption of CCSS and commitment to an implementation schedule. CCSS currently addresses Mathematics and English Language Arts (ELA) in Literacy in History/Social Studies, Science, and Technical Subjects. The standards articulate rigorous grade-level expectations in the areas of Mathematics and ELA, these standards identify the knowledge and skills students need in order to be successful in college and careers (NGA Center for Best Practices, 2010). CCSS (2010) conceptual framework for secondary math included (a) number and quantity, (b) geometry, (c) functions, (d) algebra, and (e) statistics and probability (Common Core Mathematics). Currently, 45 states, the District of Columbia, four territories, and the Department of Defense Education Activity have adopted CCSS (CCSS, 2013). The differences within the new standards are increased rigor for student achievement and alignment with higher order thinking (Daggett, 2012). According to NGA Center for

Best Practices (2010), these standards provide a "historic opportunity to improve access

to rigorous academic content standards for students with disabilities (Application to

Students with Disabilities, pg. 1)."

Algebra. This major shift in academic rigor, has also led to changes in high-stakes

assessment and graduation requirements. Along with the demands of high-stakes testing,

many states now require the completion of algebra courses, in addition to standardized

algebra assessment, as requirements of high school graduation (Witzel, Smith, &

Brownell, 2001). Mathematics, especially algebra, is a common area of difficulty for

students with disabilities. Algebra is a branch of mathematics in which symbols, usually

letters of the alphabet, represent numbers or members of a specified set and are used to

represent quantities and to express general relationships that hold for all members of the

set (Maccini, Mulcahy, & Wilson, 2007). The CCSS in mathematics and algebra were

built on progressions which are narrative documents describing the progression of a topic

across a number of grade levels, informed both by research on children's cognitive

development and by the logical structure of mathematics. These documents were spliced

together and then sliced into grade level standards. From that point on the work focused

on refining and revising the grade level standards (USDOE, 2010).

Students with disabilities in math have been described as having "deficits in the

ability to represent or process information in one or all of the many mathematical

domains in one or a set of individual competencies within each domain" (Geary, 2004,

p.4). Other characteristics include (a) difficulty with math language, (b)

misunderstanding multi-step problems, (c) inability to recall facts, and/or (d) failure to

verify and check answers (Bryant, Bryant, & Hammill, 2000; Maccinni & Gagnon,

2000). All of these characteristics have an impact on students' ability to learn and apply mathematics skills, which is why research is needed to identify effective instructional strategies for students with disabilities.

Summary of standards-based reform. Since the greatest promise of standard-based reform and the development of the CCSS for students with disabilities is that it will result in programmatic and instructional improvements, there have been changes in requirements that impact students with disabilities. One of those changes is refining and revising grade level standards in mathematics. Students with disabilities struggle with mathematics and the concepts of algebra which is why it is important to identify effective strategies for teaching these standards.

Teaching Algebra to Secondary Students with Disabilities

Maccinni et al. (1999) found students with disabilities were being taught new algebraic concepts while lacking fluency with basic mathematical terms and operations . To be successful in algebra, students must be able to recognize and use mathematical terms, perform basic mathematical operations to represent problems, solve problem, and self-monitor their problem-solving skills (Hutchinson, 1987). Hynd, Marshall, and Gonzalez (1991) report the limited success for students with disabilities in algebra may be contributed to a history of academic difficulty or inappropriate interventions. Most research involving teaching algebra has been with students with learning disabilities while research that focused on functional mathematics has been the focus with students with intellectual disability. Only a few studies have addressed teaching students with intellectual disability algebraic concepts.

Teaching algebra to students with learning disabilities. The majority of research involving effective teaching strategies for algebra have been focused on students with learning disabilities. For example, Maccini, Mulcahy, and Wilson (2007) conducted a meta-analysis on algebra instruction for secondary students with learning disabilities (LD). Their review included 14 studies with participants with learning disabilities published between 1995 and 2006. Results indicated instruction in algebra included mnemonic strategy instruction, graduated instructional approach, cognitive strategy instruction involving planning, schema-based instruction, and contextualized video-disc instruction showed significant gains.

Effective strategies for teaching secondary algebra skills to students with learning disabilities have used schema-based instruction (Jitendra, Hoff, & Beck, 1999 & Xin, Jitendra, & Deatline-Buchman, 2005), mnemonic strategy instruction (Maccini & Ruhl, 2000), contextualized instruction (Bottge, 1999; Bottge, Henruchs, Chan, Mehta, & Watson, 2003; Bottge, Henrichs, Mehta, & Hung, 2002; Bottge, Henruchs, Mehta, & Serlin, 2001), and explicit inquiry routine (Scheuermann, Deschler, & Schumaker, 2009), Concrete-to Representational to-Abstract (CRA) sequence of instruction (Maccini & Hughes, 2000; Witzel, Mercer, & Miller, 2003; Witzel, 2005). Most of the research focused on teaching algebra to students with disabilities involve teaching students to solve word problems.

Schema-based instruction. One successful strategy for teaching solving algebraic word problems is using schema-based instruction or schemata instruction. Schemas are the knowledge structures that an individual uses when recognizing or operating with aspects from their own environment (Jitendra et al., 2010). Schema-based instruction

includes teaching a student how the semantic structure of the word problem translates into elements of the schema used in the problem-solving process. Several schema-based instruction strategies for teaching algebra word problems to secondary students with learning disabilities have been shown effective (Jitendra, Hoff, & Beck, 1999; Xin, Jitendra, & Deatline-Buchman, 2005). First, Jitendra et al., (1999) conducted a study using schema-based instruction to teach middle school students with disabilities to solve algebraic word problems. They used a multiple-probe across participants and behaviors design to examine the effects of schemata instruction on solving word problems. Results indicated a functional relation between schemata instruction and participants ability to solve one-and two-step word problems involving addition and subtraction.

Second, Xin, Jitendra, and Deatline-Buchman (2005) conducted a study using schema-based instruction to teach middle school students with learning disabilities to solve algebraic word problems. They used a pretest–posttest comparison group design with random assignment to measure the effects of both the schema-based instruction and a general strategy instruction on solving algebraic word problems. Results showed a statistically significant change with students in the schema-based instruction group performing better than the general strategy instruction group. In addition, students in the schema-based instruction group were able to acquire, maintain, and generalize the skills.

Mnemonic strategy instruction**. Another strategy for teaching algebraic skills involves the use of mnemonic strategy instruction. Mnemonic strategy instruction is defined as using memory devices to help students recall the sequential steps from familiar words using first letters from the beginning of other words. Mnemonic strategy instruction has been effective for teaching subtraction of integers (Maccini & Ruhl,

2000). Maccini and Ruhl (2000) evaluated the effects of mnemonic strategy instruction with middle school students with learning disabilities. They used a multiple-probe across participants design to measure the impact of mnemonic strategy "STAR" on solving word problems using subtraction with integers. STAR stands for search the word problem, translate from equation to picture form, answer the problem, and review the solution. Results indicated a functional relation between the STAR mnemonic strategy instruction and solving word problems using subtraction with integers.

Contextualized instruction. Contextualized instruction is defined as conception of teaching and learning that helps teachers relate subject matter content to real world situations (Bottge, 1999). Contextualized instruction has been used to teach solving word problems with fractions (Bottge, 1999), pre-algebraic word problem solving skills (Bottge et al., 2001), multi-step equations (Bottge et al., 2002), and adding and subtracting mixed numbers in word problems (Bottge et al., 2003). First, Bottge (1999) conducted a study to teach middle school students with learning disabilities to solve word problems with fractions. The author used a pretest-posttest comparison group design with random assignment to measure the effects of contextualized video-disc instruction on solving word problems with fractions. Results were statistically significant indicating students in the treatment group outperformed students in the control group and were able to generalize skills to plan and build two skateboard ramps.

Second, Bottge et al. (2001) conducted a study to teach middle school students with learning disabilities to solve pre-algebraic word problems. They used a pretest-posttest comparison group (groups were the remedial pre-algebra class and the pre-algebra class) to determine the effects of contextualized instruction on skills for solving

pre-algebraic word problems. Results were statistically significant showing students in the remedial pre-algebra class using contextualized instruction were able to successfully solve pre-algebraic word problems and their scores matched scores of students in the pre-algebra class.

Third, Bottge et al. (2002) conducted a study to teach middle school students with learning disabilities to solve word problems involving adding and subtracting mixed numbers. They used a quasi-experimental control group design to measure the effects of contextualized instruction on solving word problems involving adding and subtracting mixed numbers in general education settings. Results were statistically significant showing students in general education with and without disabilities improved their problem solving performance following contextualized video-disc instruction.

Finally, Bottge et al. (2003) conducted a study to teach middle school students with learning disabilities and students without disabilities to solve algebraic word problems. They used a repeated measures design with staggered baselines to determine the effects of contextualized instruction on computation problems and word problems. Results were statistically significant showing students' performance in both groups improved after instruction.

Explicit-inquiry routine. Another teaching routine has been used to teach algebra skills to students with disabilities called explicit-inquiry routine (EIR). EIR is defined as integrating general education mathematical teaching practices such as inquiry and dialogue and special education teaching practices such as intensive and explicit instruction to engage students across multiple methods including concrete, representation, and abstract (Scheurman, Deshler, & Schumaker, 2009). Scheurman et al. (2009)

conducted a study to teach middle school students with learning disabilities to solve one-variable equations. They used a multiple-baseline across students to measure the effects of EIR on students' scores on solving one-variables, results showed a functional relation between EIR and students ability to solve one-variables. Students were able to generalize skills to textbook problems and standardized mathematics exams.

Concrete-representational-abstract (CRA). The CRA instructional sequence involves systematically and explicitly instructing students through the three levels of learning: (a) concrete, (b) representational, and (c) abstract. Each transition is purposeful through each of the stages and encourages students to learn concepts, as well as the procedures and computations that are so important in mathematics. CRA uses concrete materials to introduce the concept, then the students design representations of the concrete materials on paper, and eventually use abstract thought. This sequence of instruction has been used to teach problem solving skills with integers (Maccini & Hughes, 2000), algebra transformation equations (Witzel, Mercer, & Miller, 2003), and solving linear algebraic functions (Witzel, 2005). Concrete to Representational to Abstract Sequence of Instruction includes concrete which is expeditious use of manipulatives, pictorial Representations, and abstract procedures (Witzel, 2005). CRA is three level learning process in which students problem solve mathematics through physical manipulation of concrete materials followed by learning through pictorial representations of the concrete manipulations, and ending with solving mathematics through abstract notation (Witzel, 2005). Other terms have been associated with this sequence of instruction such as concrete to semiconcrete to abstract and graduated instruction (Gagnon & Maccinni, 2007). Teaching students through CRA has been shown

to be beneficial to secondary students with disabilities (Maccini & Hughes, 2000; Witzel, Mercer, & Miller, 2003; Witzel, 2005).

First, Maccini and Hughes (2000) conducted a study to teach high school students with learning disabilities to solve problems using addition, subtraction, multiplication, and division with integers. They used a multiple probe across participants design to measure the impact of CRA on the rate of accuracy in solving the problems. Results indicated a functional relation between CRA and solving problems with integers.

Second, Witzel, Mercer, and Miller (2003) conducted a study to teach middle school students with learning disabilities to transform equations using single variables. They used a pretest–posttest comparison group design with random assignment to measure the effects of both the CRA instruction and a general strategy instruction on transforming equations using single variables. Results showed a statistically significant difference in between students in CRA instruction and general strategy instruction group.

Finally, Witzel (2005) conducted a study to teach middle school students with learning disabilities to solve linear algebraic functions. They used a pretest–posttest comparison group design with random assignment to measure the effects of CRA and a general strategy instruction on solving linear algebraic functions. Results showed students in both groups learned skills but students who were taught with CRA scored statistically significantly higher on the post and follow-up tests.

Teaching algebra to students with mild intellectual disability. Little research is available on teaching algebra to students with intellectual disability. For example, Browder et al. (2008) conducted a meta-analysis of strategies used to teach mathematics to students with intellectual disability. Their review included 68 studies of mathematics

instruction and strategies used with students with intellectual disability published between 1975 and 2005. Of those 68 studies, only two studies focused on skills related to algebra and involved teaching students to solve word problems using addition and subtraction and quantifying sets (Miser, 1985; Neef et al., 2003). Since this meta-analysis, three additional studies have been conducted on teaching algebra to students with intellectual disability using systematic instruction with a concrete representation (Jimenez, Browder, & Courtade, 2008), schema-based instruction (Browder Trela, et al., 2012), and read-alouds of math problems with task analytic instruction to find the solution (Browder, Jimenez, et al., 2012).

First, Jimenez et al. (2008) conducted a study to teach high school students with moderate/severe intellectual disability how to solve a simple linear algebraic equation. They used a multiple-probe across participants design to determine the effects of systematic instruction with concrete representation on student acquisition of an algebra skill. The intervention included: (a) concrete representation with wooden spoons, red, and green markers; (b) task-analysis for the steps to solve the problem; and (c) systematic prompting to promote errorless learning. Results indicated all students learned to solve simple linear equations, two students maintained the skills, and one student generalized the steps across materials. No functional relation was reported.

Second, Browder, Trela, et al. (2012) conducted a study to teach high school students with moderate/severe intellectual disability to solve algebraic problems involving intersection of figures in a coordinate plane. They used a quasi-experimental design to measure the effects of schema-based instruction on student ability to solve problems involving coordinate algebra. The intervention included a literacy-based

approach that embedded math problems in a story context which provided a schema for students to organize facts (Anderson, Spiro, & Anderson, 1978; Zambo, 2005). Key components of the algebraic intervention included story-based activities they were familiar with, a graphic organizer and manipulatives, and step-by-step training in a task analysis to identify and organize facts to solve the problem. Results indicated a statistically significant difference between the intervention and control groups on mathematic gain scores, providing support for the impact of the math intervention on learning each of the problem-solving skills.

Finally, Browder, Jimenez, et al. (2012) conducted a study to teach middle school students with moderate/severe intellectual disability to solve problems for different types of standards (algebra, geometry). They used a multiple-probe across standards with concurrent between participant replications to measure the impact of the mathematic intervention on students' acquisition of math responses. The math intervention included (a) math story word problems based on activities familiar to student, (b) graphic organizer and manipulatives, and (c) step-by-step training in a task analysis to identify and organize facts to solve the problem. Results indicated a functional relation between math instruction and student behavior with an overall increase in independent correct responses.

While most research has focused on strategies to teach algebra to students with learning disabilities, it is important to identify ways to promote access to algebra for students with intellectual disability. Recently, while research has begun to be conducted to identify effective math instructional practices for students with intellectual disability, for algebra instruction, it is limited.

Summary of teaching algebra to secondary students with disabilities. Because of its abstract nature, teachers have struggled to help students comprehend algebraic concepts (Witzel, et al., 2003). While students with learning disabilities have been taught algebra through schema-based instruction (Jitendra, Hoff, & Beck, 1999 & Xin, Jitendra, & Deatline-Buchman, 2005), mnemonic strategy instruction (Maccini & Ruhl, 2000), contextualized instruction (Bottge, 1999; Bottge, Henruchs, Chan, Mehta, & Watson, 2003; Bottge, Henrichs, Mehta, & Hung, 2002; Bottge, Henruchs, Mehta, & Serlin, 2001), and explicit inquiry routine (Scheuermann, Deschler, & Schumaker, 2009), Concrete-to Representational to-Abstract (CRA) sequence of instruction (Maccini & Hughes, 2000; Witzel, Mercer, & Miller, 2003; Witzel, 2005), there is less research available for teaching math to students with intellectual disability. Those effective strategies include using systematic instruction with a concrete representation (Jimenez, Browder, & Courtade, 2008), schema-based instruction (Browder, Trela, et al., 2012), and read-alouds of math problems with task analytic instruction to find the solution (Browder, Jimenez, et al., 2012).

Summary of Literature Review

While students with mild intellectual disability are often given other disability category labels, such as learning disabilities or another high incidence disability, they continue to have educational needs that are not mild (Bouck & Joshi, 2012). Over the past decade, as standards-based reform has changed expectations for all students, the expectation of academic achievement for students with intellectual disability has grown. With this, many states have now adopted the Common Core State Standards. As mandated by legislation, students with intellectual disability must have access to general

curriculum (i.e., CCSS) and demonstrate proficiency in core content areas including algebra (CCSS, 2010; NCLB, 2001). Researchers and educators are now in need of instructional strategies to teach students with intellectual disability algebra (Cushing, Clark, Carter, & Kennedy, 2003).

To date, research on teaching algebra to students with disabilities, has primarily involved students with learning disabilities. The most widely used strategies for teaching algebra for students with learning disabilities are CRA and contextualized instruction.

Despite the disability category, there are many factors to consider when choosing an instructional practice for teaching algebra skills to any students with disabilities. First, one must consider if the instruction and skill is grade aligned. For example, Jitendra, Hoff, and Beck (1999) used schema-based instruction to teach algebraic word problem solving. The students with learning disabilities in that study were working on addition and subtraction which was several grades behind typical algebra classes. In addition, Witzel, Mercer, and Miller (2003) suggest that while CRA was effective, it was important to make sure the components of algebra were age-level appropriate. While algebra has been taught to students with intellectual disability primarily through using math story word problems, graphic organizers, and a task analysis to solve the problem, only three studies taught grade-aligned mathematics to secondary students with moderate/severe intellectual disability.

Next, an important factor for teaching algebra to students with intellectual disability is generalization. Although general education often assumes generalization to real life activities, students with intellectual disability may need opportunities to apply their skills to daily living (Browder, Jimenez, et al., 2012). Browder, Trela et al. (2012)

and Browder, Jimenez, et al. (2012) recommend that future research to the extent to which students with intellectual disability are able to generalize the academic concepts to real life activities. Browder, Jimenez, et al. (2010) recommended that future research address evaluations by stakeholders to validate that (a) the skill learned is algebra, (b) skill has social validity to the student, and (c) the skills are applicable to students' real life.

Finally, for students to understand the initial algebra concepts, it is important for students to learn those abstract concepts in a concrete manner first (Devlin, 2000). One suggestion for simplifying complex concepts is to transform them into concrete manipulations and pictorial representations (Witzel, et al., 2003). An effective approach in making algebra more accessible through the use of concrete materials that develop into representational and eventually abstract thought is known as the concrete-representational-to abstract sequence of instruction (CRA).This approach has been effective in teaching algebraic concepts to students with learning disabilities, it is important that research continue to be conducted to determine if CRA is truly an evidence-based method for teaching algebra to students with intellectual disability as well. Therefore the purpose of this study will be to investigate the effects of CRA instruction on solving equations using inverse operations with high school students with mild intellectual disability.

CHAPTER 3: METHOD

The purpose of this study was to examine the effects of Concrete-Representational-Abstract (CRA) sequence of instruction on solving equations using inverse operations with high school students with mild intellectual disability. A multiple-probe across participants design was used to determine if students learned to solve equations using inverse operations using CRA. Social validity and generalization data were also collected. Solving equations using inverse operations can be defined as "deriving the value of unknown variables by applying appropriate operations to solve the goals of the problem" (Maccini & Hughes, 2000, p. 10).

Participants

Participants for this study included three high school students with mild intellectual disability. To be included in this study, participants had to be (a) be in high school (grades 9-12; ages 14-17); (b) be enrolled in Occupational Course of Study; (c) be identified as having a mild intellectual disability; and (d) have prerequisite math skills including addition, subtraction, multiplication, and division of one digit numbers. In North Carolina, intellectual disability is defined as significantly subaverage general intellectual functioning that adversely affects a child's educational performance existing concurrently with deficits in adaptive behavior and manifested during the developmental period. Specifically, mild intellectual disability is two standard deviations below the mean plus or minus one standard error of measure for standardized testing (Rules Governing Exceptional Children, 2011).

Caleb. Caleb was a Caucasian 15 year old male identified as having a mild intellectual disability. His IQ was 68 as measured by Wechsler Intelligence Scale for Children (WISC-IV) and psychological assessment report stated his adaptive behavior skills were in the low range. Caleb was a high school sophomore in participating in an occupational course of study. He scored below criterion on the state alternate assessment for Math. Formal academic assessments conducted while Caleb was in eighth grade indicated his math skill grade equivalent was 4.1. Caleb had the prerequisite math skills needed to participate in this study.

Donna. Donna was an African American 16 year old female identified as having a mild intellectual disability. Her IQ was 66 as measured by Wechsler Intelligence Scale for Children (WISC-IV) and psychological assessment report stated her adaptive behavior skills were in the very low range. She scored below criterion on the state alternate assessment for Math. Formal academic assessments conducted while Donna was in eighth grade indicated her math computation grade equivalent was 2.6 and math reasoning was 2.9. Donna had the prerequisite math skills needed to participate in this study.

Amanda. Amanda was a Caucasian16 year old female identified as having a mild intellectual disability. Her IQ was 63 as measured by Wechsler Intelligence Scale for Children (WISC-IV) and psychological assessment report stated her adaptive behavior skills were in the very low range.. Amanda was a sophomore in high school participating in an occupational course of study. She scored below criterion on the state alternate math assessment. Formal academic assessments indicated that Amanda's math computation

grade equivalent was 3.5 and her math reasoning grade equivalent was 3.1. Amanda had the prerequisite math skills needed to participate in this study.

The researcher met with the classroom teachers prior to the start of the study to discuss inclusion criteria. The researcher asked the teachers to identify a minimum of three students who meet the inclusion criteria. The researcher obtained all informed consents (i.e., teachers and parents) and student assents using the format provided by the Institutional Review Board (IRB) at the University of North Carolina at Charlotte (UNC Charlotte).

Institutional review board. Prior to data collection, approval to conduct this study was obtained by the researcher for research with human subjects. The researcher also obtained written consent from the school's principal, parents or guardians, and students indicating willingness to participate in the study. Informed consent was obtained by using a parent or guardian consent form. Once the parents consented, the students were asked to complete assent forms indicating their willingness to participate in the study. Only students, for whom parents sign the consent forms, were presented with assent forms. Setting

All training and intervention sessions were conducted in a public high school located in the southeast United States. The school is located in a rural town with a population of 5,712 people. The school has three special education teachers who serve 98 students with disabilities. There are 31 students in the Occupational Course of Study diploma track and the 9-12 population is 602 students. This school is a Title I school with over 47% of the population on free or reduced lunch. The intervention sessions were

conducted in a special education classroom. The study was conducted in the spring semester of 2013 school year.

Researcher

The researcher was a fourth year doctoral student in special education at UNC Charlotte. She has a Master's of Arts degree in Teaching Special Education and was a teacher of high school students with intellectual disability enrolled in the Occupational Course of Study for four years. She has co-authored several manuscripts that have been published in peer-reviewed special education journals. The researcher was responsible for (a) gaining IRB approval, (b) coordinating agreement with the school district to do the study, (c) developing the contextualized instruction intervention and materials, (d) implementing the intervention (d) training a second observer to collect interobserver reliability data, and (e) communicating plans and progress with her dissertation committee.

Materials

Five CRA lessons were adapted from *Solving Equations* (Witzel & Riccomini, 2003) by the researcher. The adapted lessons were reviewed and found to be valid by a co-author of Solving Equations. The target population for these materials was originally students with learning disabilities or at risk in mathematics. The lessons were adapted to be shorter and the skills were broken into smaller chunks for students with intellectual disability. Only one new skill is presented in a lesson. The concrete lessons include hands-on interactions by teacher and student with the concrete objects or manipulatives. While no set of absolute materials are needed for this intervention, the following items were selected for the purpose of this study. Materials that will be used are (a) 20

toothpicks that will represent the ones place value, (b) four popsicle sticks that will represent tens place value, (c) 12 red plastic cups that will represent the variables (letters), (d) two multiplication symbols (x and /) drawn on cardstock and laminated, (e) one piece of string to represent the equal sign, (f) eight minus signs drawn on green cardstock and laminated, and (g) eight plus signs drawn on red cardstock and laminated.

Dependent Variables

There were two dependent variables in this study. They were student (a) ability to solve equations using inverse operations as measured by the percentage of correct answers and (b) performing the steps correctly in solving problem, as measured by percentage of steps performed correctly. This dependent variable was measured as percentage of correct answers to a 12-item probe given following each of the five lessons. The 12-item probes required the students to solve equations using inverse operations using the four operations and developed by the researcher. The researcher chose problems from Solving Equations and generated new problems that required the students to solve equations using inverse operations. The probes were developed to cover addition, subtraction, multiplication, and division. Three problems required students to solve equations using inverse operations using addition, three problems using subtraction, three problems using division, and three problems using multiplication. Finding the solution involved finding the solution into sequential steps. Percentage of steps performed correctly in solving equations using inverse operation was also collected in addition to correct solutions. For each operation, there were two steps leading to the answer to perform in each of the problems regardless of the operation (i.e. addition, subtraction, multiplication, or division) being used resulting in a total of 24 possible steps. The

students are taught three questions to ask themselves to choose the operation and solve

the equation. These were mathematically one-step equations, however when collecting

data for steps correct, the researcher included solving the equation as the second step. For

example, $2x = 88$. The first step is dividing both sides by 2, the second step is $x = 44$.

The validity of all probes was evaluated by an algebra content expert. The content expert

also evaluated the assessment in terms of academic alignments to grade-aligned algebra

(see Appendix A for Sample Scoring Sheet).

       Interobserver reliability. Interobserver reliability data were gathered by the

classroom algebra teacher. The researcher scored the student on the both dependent

variables (i.e student ability to solve equation using inverse operation and percentage of

steps performed correctly) during each probe session. The classroom algebra also scored

the student on both dependent variables. Interobserver reliability data were collected for

100% of probes collected during baseline, 98% of probes collected during intervention,

100% of probes collected in maintenance and 100% of probes collected in generalization;

an average of 99.5% across all phases. An item-by-item comparison of agreements and

disagreements was conducted between the probe scored by the researcher and the probe

scored by the classroom teacher. Agreements were divided by 12 for the primary

dependent variable, the total number of problems, and multiplied by 100 to yield a

reliability coefficient. For the second dependent variable, agreements were divided by 24,

the total number of steps performed, and multiplied by 100 to yield a reliability

coefficient.

Generalization Data

Data were collected to determine if participants could generalize skills learned into a new setting, specifically a grocery store. For example, at the store students were told to buy a magazine and four notebooks and only given $25. Students were only scored as correct or incorrect for solving equation using inverse operation at the grocery store. Data were not collected on steps completed correctly. Students were given paper and a pen to take with them to the store and told they could use their cell phones if they needed to use the calculator. Generalization data were collected once during baseline and again after the intervention has ended. Students were given two scenarios in baseline and two scenarios post-interventions. Scenarios represented one of the four operations (i.e., addition, subtraction, multiplication, and division) and were chosen randomly. When participants met mastery criteria on the probes for solving equation using inverse operations, the intervention had stopped, and maintenance data were collected, then post-intervention generalization data were collected in a local grocery store.

Social Validity Data

At the conclusion of the study, participants and their teachers were given questionnaires to assess their perceptions of the study procedures and outcomes. The classroom teacher read the questionnaire to the students. Students were asked if they (a) felt they learned to solve equations using inverse operations, (b) liked the strategy used to teach solving equations using inverse operations, and (c) would like to learn other algebra concepts using this instruction (see Appendix C for Student Social Validity Questionnaire). Teachers were given their survey to complete and return to the researcher. They were asked if (a) they felt the students learned to solve equations inverse

operations (b) if the strategy was useful, (c) if they would like to use this instruction with other algebra concepts (see Appendix D for Teacher Social Validity Questionnaire).

Experimental Design

The experimental design was a multiple-probe across participants design (Tawney & Gast, 1984) to evaluate the effects of CRA instruction on solving inverse operations. In a multiple-probe across participants design, baseline data are collected initially on all participants for a minimum of five sessions, and the participant with the lowest most stable baseline data enters intervention first. Data are collected repeatedly for the participant in intervention, but for the other participants, probes are conducted intermittently providing the basis from which to determine behavior change (Cooper et al., 2007). Using this design, a functional relation between the independent variable and change in behavior will be demonstrated if baseline levels remain stable and low, and participants show a change in level and trend only as a result receiving the independent variable (Tawney & Gast, 1984), which in this study will be CRA instruction.

During baseline, a minimum of five data points were collected from each participant to determine level of performance prior to the intervention. After all five date points were collected for each participant, the participant with the lowest, most stable baseline data for percentage of correct answers entered the intervention phase first. Once the first participant reached mastery for two consecutive sessions during intervention (i.e. correctly solving 10 out of 12 problems on each probe), another baseline data probe were administered to the remaining participants to determine if their levels of performance remained stable and low for ability to solve equations. Based on the remaining students' performance on percentage of solving equations using inverse operations on baseline data

probes, the next student who shows the greatest need entered the intervention phase. The remaining participant will enter intervention phase using the same method as the second participant. When participants met mastery criteria on the probes for all five lessons, the intervention stopped, maintenance data were collected once per week.

Procedures

General procedures. Students participated in the intervention individually for approximately five 45- minute sessions. Lesson times ranged from 20-minutes to 90-minutes.Thes sessions included time for students to complete independent practice on their learning sheets. Following each intervention lesson, a probe was conducted to evaluate the student's ability to solve equations using inverse operations and perform steps correctly to solve problems.

Baseline. During baseline, data were collected using the 12-item probe for five sessions. In addition, generalization data were collected in a grocery store. The researcher gave the probe sheet to the students and told them to solve the problems and show their work. Students were scored based on their ability to solve the problem and perform the steps correctly to solve the problem.

CRA intervention procedures. The most common approach for teaching solving equations using inverse operations is called "change sides, change signs," (Pirie & Martin, 1997). The understanding is built on the assumption that the integrity of the original equation is preserved. For example, $ax + b = cx + d$ becomes $ax = cx + d - b$ becomes $ax - cx = d - b$ becomes $x (a\text{-}c) = d - b$ becomes $x = \frac{(d-b)}{(a-c)}$. Students solve the equation that is not in the original form, but altered to a simplified equation. In the original example, the variables include addition, once they are changed to the other

"side" the signs change to subtraction. Before students can solve for unknown variables, the student needs to be able to effectively simplify expressions. By simplifying expressions, students can see that unknowns and numbers cannot be added, subtracted, multiplied, or divided while the unknown exists. For example, the expression $4x + 3b + x$ can be simplified to $5x + 3b$, which is its most basic form. The first three lessons of the intervention were on simplifying expressions. CRA instructional sequence systematically and explicitly instructs students through the three levels of learning (a) concrete, (b) representational, and (c) abstract. Each transition is purposeful through each of the stages and encourages students to learn concepts, as well as the procedures needed to solve equations using inverse operations.

    During intervention, the teacher followed an instructional script for each lesson. CRA instruction began with students using manipulative objects to display and solve math problems. Once students understand the topic concretely, they worked with the same concept using pictorial representations and then eventually abstract thought (Witzel et al., 2003).

    Lesson one. This lesson included concrete instruction to teach reducing terms in an equation. Materials needed for this lesson included popsicle sticks for tens place values, solo cups for coefficients, toothpicks for ones place value, strings for equal signs, letter cards for variables, multi-colored cards for operations, and sentence strips for division. The teacher followed the instructional script (see Appendix E) for the lesson. The teaching script followed a demonstrate/model, guided practice, and independent practice format. During the lesson, data were collected on the number correct on the independent practice component of the lessons. An item analysis was conducted to

determine what error correction procedure was necessary. If students solved fewer than five correctly, then they received more guided practice and completed more independent practice. If students solved fewer than five correctly again, the entire lesson was repeated. This lesson ranged from 20-minutes to 65-minutes.

Lesson two. This lesson used representational/pictorial instruction on reducing terms in an equation. The teacher followed the instructional script for the lesson. In this lesson, students were taught to draw or represent the concrete examples for the expression in order to simplify. For example, in the problem $5n - \frac{2}{x} + n - 12$, students were taught in lesson one to represent the 5n with 5 cups and a "n" card. In lesson two, students were taught to draw the representations of the concrete items. In the same example, $-5n - \frac{2}{x} + n - 12$, students would represent "-" by drawing the minus sign. To represent the $\frac{2}{x}$, students would draw two toothpicks over an x. Now, to represent the + n, the students would draw a plus, a cup, and an n. The cup would be placed in there because there is only one n. Finally to represent a minus and a 12, students will draw a minus sign and sticks to represent 12. Since 12 has one ten and two ones, those would be drawn by adding a long line for the 10 and two dashes for the two. This is the same equation as students were taught in lesson one, just in a pictorial representation. This lesson followed describe/model, guided practice, and independent practice format. Instructional decision making followed the same guidelines as in lesson one. Lesson two times ranged from 35-minutes to 80-minutes.

Lesson three. This lesson used abstract instruction on reducing terms in an equation. The teacher followed the instructional script for the lesson. In this lesson, students were taught to move from the pictures representation to using numbers only.

This lesson followed demonstrate/model, guided practice, and independent practice format. Instructional decision-making followed the same guidelines as in lesson one and two. Lesson three times ranged from 30-minutes to 70-minutes.

Lessons four and five focused on solving equations using inverse operations, often called single-variable equations (e.g. $x - 5 = 14$). To establish the concept of a variable, all variables in this unit represent one specific number. The usefulness of solving for single unknowns using inverse operations becomes apparent when students work with practical math problems to find a missing quantity. Following these lessons, the probe data show a change in trend and level.

Lesson four. This lesson used concrete instruction on solving equations using inverse operations. Materials needed were popsicle sticks, cups, toothpicks, strings, letter cards, multi-colored cards, and sentence strips. The teacher followed the instructional script for the lesson. Students were taught three questions to ask themselves when solving for equations using inverse operations. Those questions were (a) what is the variable, (b) what operation is being performed, and (c) what is the opposite operation. This lesson followed demonstrate/model, guided practice, and independent practice format. Instructional decision making followed the same guidelines as in lesson one, two, and three. Lesson four times ranged from 40-minutes to 90-minutes.

Lesson five. This lesson used representational instruction on solving equations using inverse operations. The teacher followed the instructional script for the lesson. Students were taught to use the pictorial representation strategies to solve equations using inverse operations. This lesson followed demonstrate/model, guided practice, and independent practice format.

After completion of lesson five, all three students met mastery criteria (10 out of 12) on probes. Students were showing an increase in number correct on their probes after lesson four and five. However, they had not met mastery for two consecutive sessions, additional guided practice and independent practice occurred based on the type of errors made. All three participants met mastery criteria a second time after additional practice was provided after lesson five. None of the participants required the booster session using ISOLATE (Witzel, 2003). ISOLATE stands for (I) identify the variable and the equal sign, (S) something has to be done to isolate the variable, (O) organize the equation to equal out what I have done (using operators), (L) let the calculations fly, (A) answer the calculations around the variable, (T) total the other side, and (E) evaluate if it makes sense.

Maintenance. When participants met mastery criteria of 10 out of 12 correct on the probes for solving equations using inverse operations for two days in a row, the intervention stopped, maintenance data were collected on students' ability to solve equations using inverse operations. Maintenance data were collected once per week to determine participants maintained gains from the intervention once the intervention is removed.

Procedural fidelity. Procedural fidelity refers to the extent that intervention procedures are implemented as intended (Cooper et al., 2007). A procedural fidelity checklist (see Appendix F) was used for the intervention procedures. An observer and the classroom teacher were trained on how to collect procedural fidelity data. Two instructional sessions per student were video-taped and scored by the trained observer. The remaining sessions were scored by the classroom teacher. The observer of each

session checked the "yes" box for a correctly completed step and checked the "no" box for a step not completed or incorrectly completed. The number of "yes" boxes were added, divided by total, which varied per lesson, and multiplied by 100 to calculate the percent of procedural fidelity during sessions.

CHAPTER 4: RESULTS

Findings of the study are presented below. First, results for interobserver reliability and procedural fidelity are presented, followed by results for each research question.

Interobserver Reliability

Interobserver reliability data were collected for 100% of probes conducted during baseline, 100% of probes during intervention, and 100% of maintenance and generalization probes conducted post-intervention, an average of 100% across all phases of the study. The interventionist scored the student on the primary dependent variable (i.e. correct scores on the probes) during each probe. At the same time, the classroom algebra teacher scored the student on the primary dependent variable. An item-by-item comparison of agreements and disagreements was conducted between the probes scored by the interventionist and the probes scored by the classroom algebra teacher. Interobserver agreement was 98% for all phases of the study.

Procedural Fidelity

Procedural fidelity were gathered on 100% of sessions during instruction for all three students. During instruction, 33% of sessions were video-taped and scored by a trained observer using the procedural fidelity checklist. The other sessions were scored by the classroom teacher during the instruction. Procedural fidelity was 100% during intervention.

Research Question 1: What was the effect of CRA instruction on high school students
with intellectual disability ability to solve equations using inverse operations?

Research Question 2: What was the effect of CRA instruction on high school students
with intellectual disability ability to maintain skills learned on solving equations
using inverse operations?

Research Question 3: What is the effect of CRA instruction on high school students with
intellectual disability ability to generalize skills learned to a skill performed in a
community context?

Results for each participant are presented in Figure 1. The graph shows the results
for all three participants. Results indicated a functional relation between the CRA
sequence of instruction and teaching students with mild intellectual disability to solve
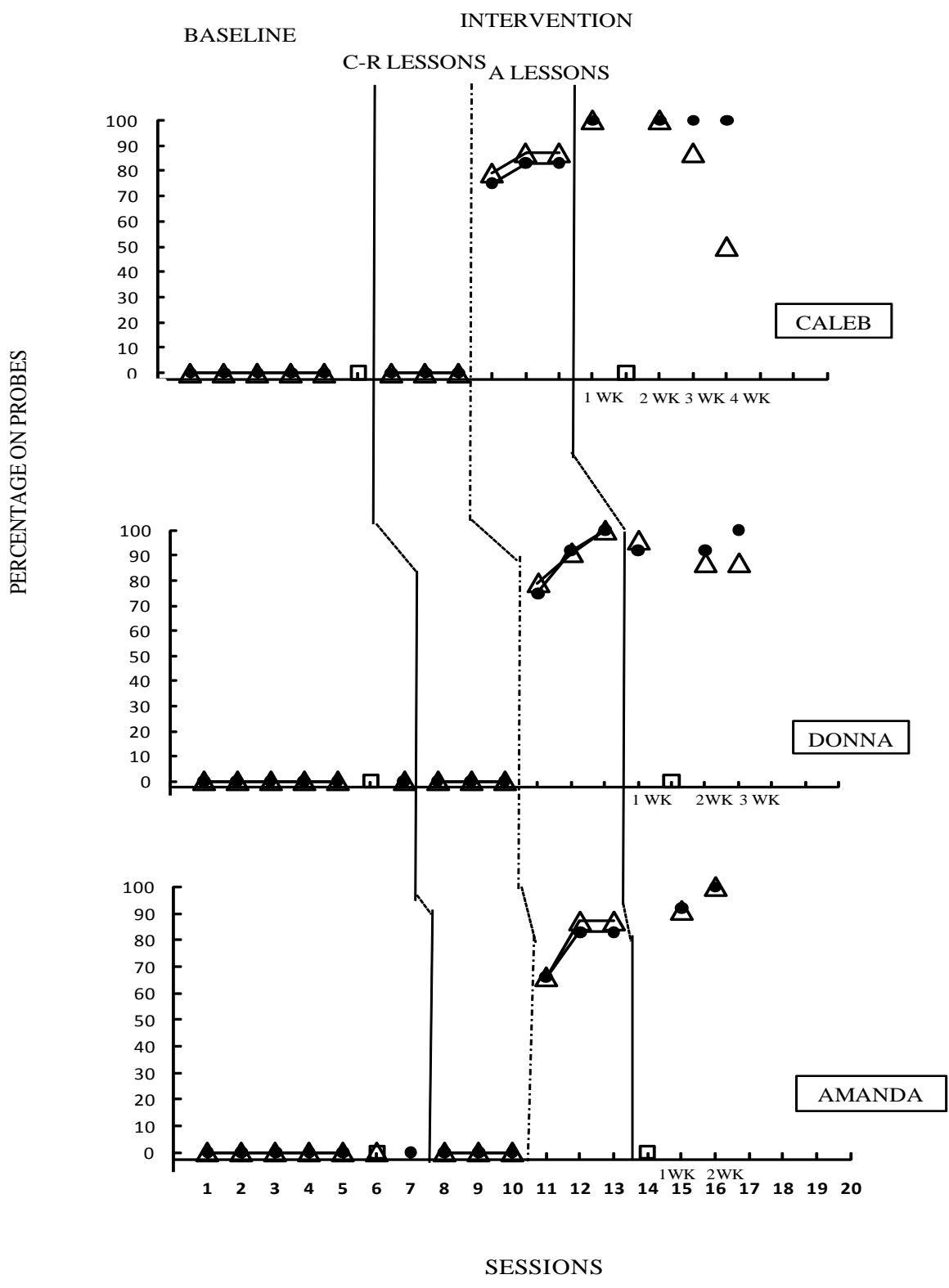equations using inverse operations.

FIGURE 1: Student data on CRA intervention

Note: Closed circles represent correct answers. Triangles represent steps completed correctly. Open squares represent correct answers in generalization setting.

Caleb. During baseline, Caleb scored 0 on correct responses as well as steps performed correctly on all five probes. During intervention in Concrete and Representational lessons, Caleb scored 0 on all probes. During intervention in the Abstract lessons, Caleb's scores ranged from 75%to 83% with a mean of 80.3%. Item analysis of all probes indicated Caleb incorrectly solved four equations using division and three equations using multiplication. During maintenance, Caleb's scores two weeks, three weeks, and four weeks post-intervention were all 100% . During generalization, in baseline and post-intervention, Caleb scored 0.

Donna. During baseline, Donna scored 0 on correct responses as well as steps performed correctly on all five probes. During intervention during the Concrete and Representational lessons, Donna scored 0 on all probes. During intervention during the Abstract lessons, Donna's scores ranged from 75%to 100% with a mean of 89%. Item analysis of all probes indicated Donna incorrectly solved three equations using division and one equation using multiplication. During maintenance, Donna's scores two weeks, three weeks, and four weeks post-intervention ranged from 92% to 100% with a mean of 94.6%. During generalization, in baseline and post-intervention, Donna scored 0.

Amanda. During baseline, Amanda scored 0 on correct responses as well as steps performed correctly on all five probes. During intervention during the Concrete and Representational lessons, Amanda scored 0 on all probes. During intervention during the Abstract lessons, Amanda's scores ranged from 66%to 83% with a mean of 77%. Item

analysis of all probes indicated Amanda incorrectly solved six equations using division, one equation using multiplication, and one equation using subtraction. During maintenance, Amanda's scores two weeks and three weeks post-intervention ranged from 92% to 100% with a mean of 96%. During generalization, in baseline and post-intervention, Amanda scored 0.

Table 1: Lesson times per student

Student	Lessons					
	One	Two	Three	Four	Five	Total
Caleb	20	35	30	40	45	170
Donna	45	65	40	45	80	275
Amanda	65	80	70	90	90	395

Table 1 shows a summary of lesson times per student. Lesson One times ranged from 20-65 minutes with an average of 43 minutes. Lesson Two times ranged from 35-80 minutes with an average of 60 minutes. Lesson Three times ranged from 30-70 minutes with an average of 47 minutes. Lesson Four times ranged from 40-90 minutes with an average of 58 minutes. Lesson Five times ranged from 45-90 minutes with an average of 72 minutes. Total time for all five lessons ranged from 170-395 minutes with an average of 280 minutes.

Research Question 4: What are students' perceptions of CRA instruction as a method for increasing their ability and to what extent it had an effect on solving equations using inverse operations?

To evaluate the acceptability of the intervention, social validity data were collected from the students. To validate the importance of the effects of the intervention, data were collected by surveying students using a questionnaire. The researcher sat with

each participant and read each question from the questionnaire. Students were asked to be honest with their responses. Table 2 provides a summary of their responses.

Table 2: Intervention acceptability survey for students

Question	Caleb	Amanda	Donna	Average
1. This instructional strategy helped me learn how to solve equations using inverse operations	2	1	1	1.3
2. I liked this strategy instruction.	1	1	1	1.0
3. I would like to use this strategy to learn other algebra concepts	2	1	1	1.3
4. I feel that I will use this strategy in the future to help me solve algebra problems.	1	1	2	1.3

Note. Scale for questions 1-4: 1=strongly agree, 2=agree, 3=disagree, 4=strongly disagree

Overall student ratings for the intervention were high (i.e., 1.0). Averages for student ratings of the intervention ranged from 1.0-1.3 with an average of 1.22.

Research Question 5: What are teachers' perceptions of the use of CRA instruction as a method for increasing students' ability to solve equations using inverse operations and to what extent do they feel it had an effect on their ability?

To validate the appropriateness of the procedures, the algebra and special education teachers were asked to review the materials, contents of the instructional manual, and provide feedback related to the acceptability of the intervention. Table 3 provides a summary of teacher responses. Teacher ratings for the intervention were high with both teachers scoring "strongly agree" to all questions.

Table 3: Intervention acceptability for teachers

_____

Question	Special Education Teacher	Algebra Teacher	Average

_____

Question	Special Education Teacher	Algebra Teacher	Average
1. The CRA instruction helped my students learn how to solve inverse operations.	1	1	1.0
2. I liked the CRA strategy instruction.	1	1	1.0
3. I would like to use the CRA strategy to teach other algebra concepts.	1	1	1.0
4. I felt the CRA instruction was useful	1	1	1.0

Note. Scale for questions 1-4: 1=strongly agree, 2=agree, 3=disagree, 4=strongly disagree

Overall, both the special education teacher and algebra teacher rated this intervention as an acceptable intervention for high school students. Both teachers liked the intervention and agreed that the intervention would be useful to teach other algebra concepts. The special education and algebra teachers both strongly agreed the intervention was effective in teaching students how to solve equations using inverse operations.

CHAPTER 5: DISCUSSION

The purpose of this study was to examine the effects of Concrete-Representational-Abstract (CRA) sequence of instruction on students' ability to solve equations using inverse operations. A multiple probe across participants design was used to determine the effectiveness of the independent variable (i.e., CRA) on the dependent variable (i.e., student ability to solve equations using inverse operations). The intervention was implemented with three 10th grade students with mild intellectual disability. Results indicated a functional relation between the Abstract sequence of the CRA and students' ability to solve equations using inverse operations. All three participants maintained skill up to four weeks post-intervention. Finally, social validity data indicated students and teachers liked the intervention, felt it was useful, and would use it for other algebra concepts. Findings are described by research question. Lastly, limitations of the study, suggestions for future research, and implications for practice are provided.

Effects of the Intervention on Dependent Variables

Research Question 1: What was the effect of CRA instruction on high school students with intellectual disability ability to solve equations using inverse operations?

Research Question 2: What was the effect of CRA instruction on high school students with intellectual disability ability to maintain skills learned on solving equations using inverse operations?

Findings indicated a functional relation between the Abstract sequence of instruction and students' ability to solve inverse operations. All students exhibited an immediate change in level from baseline to intervention after receiving the Abstract lesson. All three students reached mastery criteria (i.e., 10 out of 12 problems solved correctly for two consecutive sessions) after participating in CRA sequence of instruction for solving inverse operations. While data show students' ability to solve inverse operations did not increase until they participated in the Abstract lesson, it is probable that without the prerequisite skills learned in the sequence of instruction of Concrete to Representational to Abstract this would not have happened. For example, during the Concrete lesson students learn about coefficients, variables, and simplifying expressions. During the Representational lesson, students use multiplication with parenthesis and learn to multiply across the equal sign. During these lessons, students learn to move manipulatives and draw pictures to solve for unknown variables and to use opposite operations. Students must know that if a variable does not have a coefficient the number is one. For example, in the problem $x - 10 = 3$; students must know the coefficient is one. In the Concrete lessons, cups are used with paper to represent the coefficient. A number paper always has to go with the cup during this lesson. However, in $3y = 24$; students need to know that the coefficient is 3 and you would use division on both sides of the equal sign. In the Representational lesson, students draw to represent what is being done with operations. While the Representational sequence of instruction is typically referred to as the student "internalizing" the process (Witzel, 2003), an alternate interpretation could be students are showing signs of covert behaviors. While overt behaviors are observable and measurable, such as observing the students write out the steps needed to

solve an equation. Covert behaviors are not observable, but they are measureable such as the students saying the steps to themselves to solve an equation. In this study, two students appeared to demonstrate covert behavior in maintenance data probes. During instruction, all three students wrote down the steps needed to solve an equation using inverse operations. However, during maintenance Caleb did not write down any of the steps to solve equations on his last maintenance probe and his answers were all correct. Donna did not write down all of the steps to solve equations and her answers were also correct. These two students began to do those steps in their heads. Therefore, we can infer the covert behavior of doing the steps in their heads was occurring because of the overt behavior they demonstrated during instruction of having to write down the steps to solve the equation.

Overall, results of the study support previous research related to using CRA instruction to increase students' ability to solve equations using inverse operations with students with learning disabilities. Previous studies using CRA focused on teaching secondary students to solve linear equations (Witzel, 2005), transform equations (Witzel et al., 2003), and to perform operations with integers (Maccini & Hughes, 2000). This study extended CRA literature by making unique contributions to the body of research by (a) selecting participants who were not previously represented in literature related to CRA and solving equations using inverse operations (i.e. students with mild intellectual disability) and (b) including algebra standards from the Common Core State Standards (CCSS).

First, since previous research has focused on strategies to teach algebra to students with learning disabilities, it is important to identify ways to promote access to

algebra for students with intellectual disability. Students with intellectual disability need information presented in smaller chunks and in shorter lessons. Previous research using CRA has been shown to be effective with secondary students with learning disabilities (Maccini & Hughes, 2000; Witzel, Mercer, & Miller, 2003; Witzel, 2005). The current study extended the literature related to using CRA to teach algebraic concepts by including students with mild intellectual disability.

Second, previous studies taught algebra concepts that were either not grade aligned skills or they were linked to alternate achievement standards. For example, Jitendra et al. (1999) used schema-based instruction to teach algebraic word problem solving. However the students were working on addition and subtraction, which was several grade levels behind typical algebra class and therefore not grade-aligned.

Algebra skills, such as solving simple linear equations have been taught to students with intellectual disability through math story word problems, graphic organizers, and a task analysis to solve the problems (Browder, Trela, et al., 2012; Jimenez, et al., 2008). The mathematics standards in these two studies were linked to alternate achievement standards. The development of the alternate achievement standards are used for the students with the most significant cognitive disabilities (34 C.F.R. Part 200). According to the regulations, these students should be identified within one or more of the existing IDEA categories and have a "cognitive impairment preventing them from attaining grade-level achievement standards, even with the best instruction" (USDOE, 2005, p. 26). For example, Browder, Trela, et al. (2012) taught an alternate geometry standard. NCTM standard states that students will "specify locations and describe spatial reasoning using coordinate geometry and other representational systems" and the

alternate standard that they taught states "identify and describe the intersection of figures in a plane." Although the standard may be a alternate achievement standard, it must be linked in meaningful ways to their grade level (US DOE, 2005). However, in alternate achievement, the depth, breadth, or complexity of grade level content is reduced (US DOE, 2005). This study extended the literature by including an algebra standard taken directly from the Common Core State Standards (2010) strand instead of teaching it as an alternate achievement standard. The standard was from High School-Algebra Reasoning with Equations and Inequalities "CCSS.Math.Content.HSA.REI.B.3. Solve linear equations in one variable, including equations with coefficients represented by letters." Research Question 3: What is the effect of CRA instruction on high school students with intellectual disability ability to generalize skills learned to a skill performed in a community context?

In previous research, suggestions were made to focus on the extent to which students are able to generalize the academic concepts to real-life activities (Browder, Trela, et al., 2012). Although general education often assumes generalization to real life activities, students with intellectual disability may need opportunities to apply their skills to daily living (Browder, Jimenez, et al., 2012).

Cooper et al. (2007) define setting/situation generalization as "the extent to which a learner emits the target behavior in a setting or stimulus situation that is different from the instructional setting" (p.617). This study planned for situation/setting generalization of the skill to a setting other than the instructional setting (i.e., grocery store). Generalization was planned for by teaching word problems in each lesson by training to generalize (Stokes & Baer, 1977). The tactic used in this study was instructing the learner

to generalize (Cooper et al., 2007). The students were explicitly told that solving equations using inverse operations was a skill used in real-life. Students were taught how to set up and solve problems using inverse operations based using word problem scenarios. There were two problems during both guided and independent practice that students completed in during each lesson.

Item analysis of student work during independent practice indicated all students missed all the word problems. Donna and Caleb set the problems up and tried to solve the equations using inverse operations, they used the wrong operation every time. However, Caleb chose to use addition on every word problem, in the one independent practice problem that used addition, he set the problem up incorrectly. Item analysis of the incorrect answers on probes show that 70% were division, 27% were multiplication, and 3% were subtraction problems.

Given the poor performance of students during independent practice on setting up word problems, it is not surprising that students were not able to generalize this skill to real-life setting. In this study, students were taken to a local grocery store and given two real-life scenarios that involved solving equation using inverse operation. Students were given a total of four scenarios, two during baseline and two post-intervention. None of the students demonstrated the ability to correctly solve equations using inverse operations to untrained community setting post-intervention. While only data on correct answers were collected, anecdotal evidence from the two visits to the grocery store suggests that the students were trying to solve the problems following intervention. During the post-intervention probe, all students used their calculators to try to solve the problem. For both post-intervention problems, Caleb correctly followed the steps for addition, since it was

the wrong operation, his answer was incorrect. Neither Donna nor Amanda used the steps or solved the problems correctly.

While setting/situational generalization did not occur, Donna and Caleb's maintenance data may indicate some possible response generalization. Response generalization is defined as "extent to which a learner emits untrained responses that are functionally equivalent to the trained target behavior" (Cooper et al., 2007, p.621). During maintenance data, Donna maintained her ability to solve equations using inverse operations by scoring 92% and 100%. However for steps completed, she scored 87% for both. On her last maintenance probe, she correctly solved all equations using inverse operations, but only completed 87% of the steps, or 21 out of 24 steps. Donna began to follow the steps to solve equations using inverse operations without having to write the steps down. This also happened for Caleb. During maintenance Caleb scored 100% on all three probes, however for steps completed he scored 100%, 87%, and 50%. For his last probe, Caleb only completed half of the steps and still correctly solved the equations using inverse operations. Students were not trained to do this, they were taught to follow the steps to solve for equations using inverse operations. The students began to look at the problems and follow steps without having to write them down, as a result it appears response generalization may have begun to occur.

Research Question 4: What are students' perceptions of CRA instruction as a method for increasing their ability and to what extent it had an effect on solving equations using inverse operations?

Research Question 5: What are teachers' perceptions of the use of CRA instruction as a method for increasing students' ability to solve equations using inverse operations and to what extent do they feel it had an effect on their ability?

One of the quality indicators of single subject research identified by Horner et al. (2005) relates to the social validity of an intervention. While quality indicators must be met related to participants, setting, dependent variable, independent variable, procedures, and results, to be considered high quality, it must also adhere to certain standards regarding social validity. First, the dependent variable must be socially important (Horner et al., 2005). In this study, social validity data were collected from students and teachers to determine the usefulness of CRA. All three participants agreed the strategy taught them to solve equations using inverse operations, they liked it, and they would use it again. Both the algebra teacher and special education teacher strongly agreed they liked the strategy, found it useful for teaching solving equations using inverse operations, and would use it teach other algebra concepts.

Next, the magnitude of change in the dependent variable resulting from the intervention must also be social important. Based on the functional relation demonstrated in this study, it appears the magnitude of change in the students' ability to solve equations using inverse operations was socially important. Also, implementation of the independent variable must be described by the author as practical and cost effective. In this study, the researcher spent less than $3.00 to purchase the materials used in the Lesson one and two. Therefore, the intervention did not incur high cost. Finally, the quality indicators suggest that social validity is enhanced by implementation of the independent variable by typical

intervention agents, in typical and social context (Horner et al., 2005). This was not done in this study.

 To date, only one of the five studies (i.e., Maccini & Hughes, 2000), which used CRA to teach algebra to students with learning disabilities collected social validity data. Maccini and Hughes (2000) collected social validity from students and teachers by using a survey with Likert scale and open-ended questions. Results showed that the teachers and students thought the strategy was effective, worth their time, and helped them understand algebra concepts. In addition, only one of the three studies that included students with intellectual disability included a measurement of social validity. Browder, Trela, et al. (2012) collected data through a profile given to students and teachers to indicate their level of satisfaction with the training and instructional materials. Results showed that respondents agreed the intervention was useful, practical, and beneficial to students. This study reported similar social validity data results as Maccini and Hughes (2000) therefore extending the literature on social validity for using CRA for students with mild intellectual disability.

Contributions of this Study

 This study was the first to teach students with mild intellectual disabilities algebra skills aligned to the CCSS. This study contributed to the literature for the use of CRA sequence of instruction. The first contribution was teaching the algebraic concept of solving equations using inverse operations to students with mild intellectual disability. Previous students included only individuals with learning disabilities (Bottge, 1999; Bottge, et al., 2001; Bottge, et al., 2002; Bottge et al., 2003, Jitendra, Hoff, & Beck, 1999; Maccini and Hughes, 2000; Scheurermann, Deschler, & Schumaker, 2009; Witzel,

2005; Witzel, Mercer, & Miller, 2003; Xi, Jitendra, & Deatline-Buchman, 2005). The next contribution was a response to limitations and recommendations in previous research (Browder, Trela, et al., 2012; Browder, Jimenez, et al., 2012; Witzel, Mercer, & Miler, 2003) that the instruction and skill are grade-aligned. This study chose a high school algebra standard from the Common Core State Standards. While the What Works Clearinghouse (WWC; 2009) has identified CRA as an effective practice for teaching mathematics for elementary and middle school students, this study extends the literature by including high school students.

Limitations and Implications for Future Research

As with any study, there are limitations. First, this study included two Caucasian students aged 15-16 and one African American student age 16 who lived in a small rural community in the southeastern part of the United States. Therefore, results can not be generalized to others due to the small number of participants and limited geographical location. Future research should include students from all cultural backgrounds and other geographical locations.

Next, the overall results are limited by the lack of data showing students could generalize skills learned to grocery store. Students were unable to solve equations using inverse operations in a grocery store. In the intervention, students were required to set up and solve a word problem during the instruction. However, the probes were based on the students' ability to solve the equations using inverse operation. This study did not explicitly teach word problem solving or strategies to solve word problems. In the grocery store, students had to set up and solve based on scenario given to them. Future research should focus on explicitly teaching students how to set up and solve word

problems involving inverse operations. In this study, there were two word problems in guided practice and two word problems in independent practice for the students to solve. In this study, students were explicitly taught how to solve equations using inverse operations, however in the word problems students had to know which operation was being used in the word problems already. This skill would require inference, something that was not taught in this study. Future research should use specific strategies for promoting generalization to help students solve word problems that require them to solve equations using inverse operations. Strategies to include for future research include (a) teaching the full range of relevant stimulus conditions and response requirements and (b) making the instructional setting similar to the generalization setting (Cooper et al., 2007). Future research should also include more examples that sample the full range of solving word problems. By teaching the full range of examples, students would have the opportunity to respond to possible stimulus and response examples for solving equations using inverse operations. For example in this study, students were taught two problems in guided practice per lesson, these were not enough. Future research should also make the instructional setting similar to the generalization setting. For example, using a simulated grocery store to teach students to solve equations using inverse operations in real life scenarios.

Future research could do a component analysis to collect data on all components of the sequence of instruction. Future research could investigate if students could solve equations using inverse operations with only receiving the Abstract lesson. Also, future research is needed to extend this research to other strands of algebra. Within a strand like algebra, high school has numerous topics and objectives. Much more research is needed

to explore the extent to which students with intellectual disability can achieve these skills aligned with the CCSS. For example, this study only taught the third out of twelve standards in algebra for Reasoning with Equations and Inequalities. Other algebra standards for Reasoning include the ability to solve quadratic equations with one variable, completing a square to transform quadratic equations, and solving quadratic equations by inspection. Another topic in algebra is Seeing Structure in Expressions and a strand of that topic includes interpreting the structure in terms of quantity. Future research is needed in all topics and strands of algebra with students with mild intellectual disability.

Finally, social validity can be enhanced by implementing the independent variable by typical intervention agents, in typical physical and social context. This study was implemented by an outside researcher in the special education classroom. As a result, this study only met three out of four of the social validity quality indicators suggested by Horner et al. (2005). Future research should use the classroom teacher to implement the independent variable for stronger social validity.

Implications for Practice

The results of this study lead to several implications for practice. First, addressing CCSS is a relatively new concept for all teachers, not just special education teachers. Teachers are struggling to find effective strategies for teaching algebra. Because algebra is such an abstract concept, CRA provides teachers a way to break down the individual concepts by starting with concrete examples using manipulatives, then pictorial representations, before moving to abstract concepts. Students with intellectual disability may need to learn in a smaller subset of skills, more teaching trials, and specific

procedures (Gardill & Browder, 1995). This strategy that teaches with concrete representations as prerequisite skills make it possible for students to acquire strands of algebra that are directly linked to the CCSS. As the algebra teacher in this study reported, students should be able to use this strategy for other concepts in algebra.

Next, in this time of standard-based education, teachers struggle to find the opportunity to teach students transition-related and academic skills to help students achieve their postsecondary goals. While this study did not show through generalization data that students generalized this skill to a real life setting, it does show that students with mild intellectual disability need to be taught those real life skills explicitly. It cannot be assumed that the general education algebra concepts will be useful for independent living unless those concepts are taught explicitly. Teachers could utilize both simulation training and community -based instruction. By using simulations in the classroom, teachers could provide a full range of examples of word problems that would be applicable in a community context and provide more practice for students. By using community based instruction, students would be given the opportunity to practice skills in the natural environment. Therefore, teachers could utilize simulation training combined with community-based instruction to teach solving equations using inverse operations in a meaningful way.

Another implication for practice is that this study provided teachers with a cost-effective strategy for teaching solving equations using inverse operations. The scripts used for instruction were easy to follow. There are only five lessons and times ranged from 20-90 minutes per lesson and the total time for all five lessons ranged from 170-395 minutes per student. Since a typical high school class is structured for 90-minute blocks,

each of these lessons could be implemented in one class period/block. Most of the materials are available in a school building (paper, index cards, string) and cups are very inexpensive to purchase. Because of the limited amount of time for instruction and the cost-effectiveness of the materials, this intervention could be incorporated into any algebra classroom.

Finally, social validity data from the classroom teacher indicated the instructional materials were very easy to use. The lessons were scripted for the researcher to follow. This script provided the researcher with a systematic way to provide instruction and provide feedback. The instructional scripts were also used as procedural fidelity checklist, ensuring that the researcher was following procedures. Teachers could use this to manage progress on students as well as ensuring procedures are followed for the sequence of instruction. According to both the algebra and special education teacher in this study, they would like to use these procedures to teach other algebra concepts.

Summary

The shift in education to implementing more rigorous standards has led to states requiring all students to pass algebra as a graduation requirement. As a result, students with mild intellectual disability need to pass algebra to obtain a high school diploma. While previous research on teaching grade-aligned algebra skills has focused on at-risk students and students with learning disabilities (Bottge, 1999; Bottge, et al., 2001; Bottge, et al., 2002; Bottge et al., 2003, Jitendra, Hoff, & Beck, 1999; Maccini and Hughes, 2000; Scheurermann, Deschler, & Schumaker, 2009; Witzel, 2005; Witzel, Mercer, & Miller, 2003; Xi, Jitendra, & Deatline-Buchman, 2005). The few studies that taught

algebraic concepts to students with intellectual disability are aligned to alternate

achievement standards (Browder, Trela, et al., 2012; Browder, Jimenex,et al., 2012).

The purpose of this study was to examine the effects of using CRA instruction on

solving equations using inverse operations (a grade-aligned algebra skill from the CCSS)

with high school students with mild intellectual disability. Results indicated a functional

relation between the Abstract sequence of the CRA and students' ability to solve

equations using inverse operations. Results of this study extended the research on

teaching algebra concepts to students with disabilities by including students with

intellectual disability, the addition of CCSS grade-aligned skills, using a simple

instructional method that is cost-effective, and collecting social validity from

stakeholders.

REFERENCES

Alberto, P. A., Cihak, D. F., & Gama, R. I. (2005). Use of static picture prompts versus video modeling during simulation instruction. *Research in Developmental Disabilities, 26,* 327–339.

Allsopp, D.H. (1999). Using modeling, manipulatives, and mnemonics with eighth-grade math Students. *Teaching Exceptional Children, 32*, 74-81.

Alwell, M., & Cobb, B. (2009). Functional life skills curricula intervention for youth with disabilities: A systematic review. *Career Development for Exceptional Individuals*, *32*, 82-93.

American Recovery and Reinvestment Act of 2009, 111 U.S.C. 5 § 1512 *et seq.* (2010).

Anderson, R.C., Spiro, R.J., & Anderson, M.C. (1978). Schemata as scaffolding for the representation of information in connected discourse. *American Educational Research Journal*, *15*, 433-440.

Arnold-Reid, G. S., Schloss, P. J., & Alper, S. (1997). Teaching meal planning to youth with mental retardation in natural settings. *Remedial and Special Education, 18,* 166–173.

Bates, P. E., Cuvo, T., Miner, C. A., & Korabek, C. A. (2001). A simulated and community-based instruction involving persons with mild and moderate mental retardation. *Research in Developmental Disabilities, 22,* 95–115.

Belmont, J.M. (1996). Long term memory in mental retardation. *International Review of Research in Mental Retardation*, *4*, 219-255.

Benz, M.R., Lindstrom, L., & Yovanoff, P. (2000). Improving collaboration between schools and vocational rehabilitation: The youth transition program model. *Exceptional Children*, *66*, 509-529.

Bigge, J. L., Stump, C. S., Spagna, M. E., & Silberman, R. K. (1999). *Curriculum, assessment, and instruction for students with disabilities*. Belmont, CA: Wadsworth.

Blackorby, J., & Wagner, M. (1996). Longitudinal post school outcomes of youth with disabilities: Findings from the national longitudinal transition study. *Exceptional Children*, *62*, 399-413.

Bottge, B. (1999). Effects of contextualized math instruction on problem solving of average and below average achieving students. *The Journal of Special Education*, *33*, 81-92.

Bottge, B., Heinrichs, M., Chan, Shih-Yi, & Serlin, R.C. (2001). Anchoring adolescents' understanding of math concepts in rich problem-solving environments. *Remedial and Special Education*, *22*, 299-314.

Bottge, B., Heinrichs, M., Chan, S.Y., Mehta, Z.D., Watson, E. (2003). Effects of video-based and applied problems on the procedural math skills of average-and low achieving adolescents. *Journal of Special Education Technology, 15*, 5-22.

Bottge, B., Heinrichs, M., Mehta, Z.D., & Hung, Ya-Hui. (2002). Weighing the benefits of anchored math instruction for students with disabilities in general education classes. *The Journal of Special Education*, *35*, 186-200.

Bouck, E. C. (2004). The state of curriculum for secondary students with mild mental retardation. *Education and Training in Developmental Disabilities, 39,* 169–176.

Bouck, E. (2007). Lost in translation? Education secondary students with mild mental impairment. *Journal of Disability Policy Studies*, *18*, 79-87

Bouck, E. C. (2009). Functional curriculum models for secondary students with mild mental impairment. *Education and Training in Developmental Disabilities*, *44*, 435-443.

Bouck, E.C. (2012). Secondary students with moderate/severe intellectual disability: Considerations of curriculum and post-school outcomes from the NLTS-2. *Journal of Intellectual Disability Research*, *56* (12*)*, 1175-1191.

Bouck, E.C. & Joshi, G. (2012). Functional curriculum and students with mild intellectual disability: Exploring postschool outcomes through NLTS2. *Education and Training in Developmental Disabilities*, *47*, 139-153.

Branham, R.S., Collins, B.C., Schuster, J.W., & Kleinert, H. (1999). Teaching community skills to students with moderate disabilities: Comparing combined techniques of classroom simulation, videotape modeling, and community-based instruction. *Education and Training in Mental Retardation and Developmental Disabilities*, *34*, 170-181.

Browder, D. M. (2000). *Curriculum and assessment for students with moderate and severe disabilities.* New York, NY: Guilford.

Browder, D.M., Jimenez, B.A., & Trela, K. (2012). Grade-aligned instruction for secondary students with moderate intellectual disability. *The Journal of Special Education*. Advance online publication. doi:10.1177/0022466910369942

Browder, D.M., Spooner, F., Ahlgrimm-Delzell, L., Wakeman, S., & Harris, A. (2008). A meta-analysis on teaching mathematics to students with significant cognitive disabilities. *Exceptional Children*, *75*, 33-52.

Browder, D.M., Trela, K., Courtade, G.R., Jimenez, B.A., Knight, V., & Flowers, C. (2012). Teaching mathematics and science to students with moderate and severe developmental disabilities. *The Journal of Special Education*, *46*, 26-35.

Bryant, D. P., Bryant, B. R., & Hammill, D. D. (2000). Characteristic behaviors of students with LD who have teacher-identified math weaknesses. *Journal of Learning Disabilities*, *33*(2), 168-177.

Buttram, J. L., & Waters, T. (1997). Improving America's Schools through Standards-Based Education. Introduction. *NASSP Bulletin*, *81*(590), 1-6.

Carter, E.W, Austin, D., & Trainor, A. (2012). Predictors of postschool employment for young adults with severe disabilities. *Journal of Disability Policy Studies*, *23*, 50-63.

Collins, B.C., & Stinson, D. (1994). Teaching generalized reading of product warning labels to adolescents with mental retardation through the use of key words. *Exceptionality*, *5*, 163-181.

Collins, B. C., Stinson, D., & Land, L.A. (1993). A comparison of in vivo and simulation prior to in vivo instruction in teaching generalized safety skills. *Education & Training in Mental Retardation*, *28*,128-142.

Cooper, J. O., Heron, T. E, & Heward, W. L. (2007). *Applied Behavior Analysis* (2nd Ed.). Upper Saddle River, NJ: Pearson.

Council of Chief State School Officers (2010). *Common Core State Standards (Mathematics).* National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C.

Cronin, M.E. (1996). Life skills curriculum for students with learning disabilities. Journal of *Learning Disabilities*, *29*, 53-68.

Cushing, L. S., Clark, N., Carter, E. W., & Kennedy, C. H. (2003). Peer supports and access to the general education curriculum. *TASH Connections*, *29*(10), 8-11.

Daggett, W. (2012). Common Core State Standards Initiative: Classroom Implications for 2014. Retrieved from http://www.leadered.com/pdf/Common%20Core%20Standards%20Paper%20FINAL.pdf

Denny, P. J., & Test, D. W. (1995). Using the one-more-than technique to teach money counting to individuals with moderate mental retardation: A systematic replication. *Education and Treatment of Children, 18,* 422–432.

Dever, R.B., & Knapczyk, D.R. (1997). *Teaching persons with mental retardation: A model for curriculum development and teaching.* Madison, WI: Brown & Benchmark.

Devlin, K. (2000). Finding your inner mathematician. *Chronicle of Higher Education*, *46*, B5.

Edgar, R. (1987). Secondary programs in special education: Are many of them justifiable? *Exceptional Children, 53*, 555-561.

Elementary and Secondary Education Act (2010). *Blueprint for Reform*. Retrieved from http://www2ed.gov/policy/elsec/leg/blueprint/index.html

Ersoy, G., Tekin-Iftar, E., & Kiracaali-Iftar, G. (2009).Effects of antecedent prompt and test procedure on teaching simulated menstrual care skills to females with developmental disabilities. *Education and Training in Developmental Disabilities*, *44*, 54-66.

Fetko, K., Schuster, J.W., Harley, D., & Collins, B.C. (1999). Using simultaneous prompting to teach a chained vocational task to young adults with severe intellectual disabilities. *Education and Training in Mental Retardation and Developmental Disabilities*, *34*, 318-329.

Flavell, J. (1979). Metacognition and cognitive monitoring. *American Psychological Association*, 34*(10)*, 906-911.

Fuijura, G. & Taylor, S. (2003). Continuum of intellectual disability: Demographic evidence for the forgotten generation. *Mental Retardation*, *41*, 420-429.

Gagnon, J.G., & Maccini, P. (2007). Teacher-reported use of empirically validated standards-based instructional approaches in secondary mathematics. *Remedial and Special Education*, *28*, 43-56.

Gallico, R., Burns, T. J., & Grob, C. S. (1991). *Emotional and behavioral problems in children with learning disabilities.* San Diego,CA: Singular.

Gardill, M. C., & Browder, D. M. (1995). Teaching Stimulus Classes to Encourage Independent Purchasing by Students with Severe Behavior Disorders. *Education and Training in Mental Retardation and Developmental Disabilities*, *30*(3), 254-64.

Gast, D. L., & Winterling, V. (1992). Teaching first–aid skills to students with moderate handicaps in small group instruction. *Education & Treatment of Children, 15,* 101–125.

Gates Foundation (2012). *Supporting Teachers Investing in Our Future*. Retrieved from: http://www.gatesfoundation.org/highschools/Documents/supporting-instruction.pdf

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of learning disabilities*, *37*(1), 4-15.

Goldberg, M., & Harvey, J. (1983). A nation at risk: The report of the national commission on excellence in education. *The Phi Delta Kappan*, *65*(1), 14-18.

Horsfall, D. & Maggs, A. (1986). Cooking skills instruction with severely multiply handicapped adolescents. *Australia & New Zealand Journal of Developmental Disabilities*, 12(3), 177-86.

Horner, R. H., Carr, E. G., Halle, J., McGee, G., Odom, S., & Wolery, M. (2005). The use of single-subject research to identify evidence-based practice in special education. *Exceptional Children, 71,* 165–179.

Huntley, M. A., Rasmussen, C. L., Villarubi, R. S., Sangtong, J., & Fey, J. T. (2000). Effects of Standards-based mathematics education: A study of the Core-Plus Mathematics Project algebra and functions strand. *Journal for Research in Mathematics Education*, 328-361.

Hutchinson, N. (1987). Strategies for teaching learning disabled adolescents algebraic problems. *Reading, Writing, and Learning Disabilities*, *3*, 63.74.

Hynd, G. W., Marshall, R., & Gonzalez, J. (1991). Learning disabilities and presumed central nervous system dysfunction. *Learning Disability Quarterly*, 283-296.

Jimenez, B.A., Browder, D.M., & Courtade, G.R. (2008). Teaching an algebraic equation to high school students with moderate developmental disabilities. *Education and Training in Developmental Disabilities*, *43*, 266-274.

Jitendra, A. K., George, M. P., Sood, S., & Price., K. (2010). Schema-based instruction: Facilitating mathematical word problem solving for students with emotional and behavioral disorders. *Preventing School Failure*, *54*, 145-151.

Jitendra, A.K., Hoff, K., & Beck, M.M. (1999). Teaching middle school students with learning disabilities to solve word problems using a schema-based approach. *Remedial and Special Education*, *20*, 50-64.

Johnson, D. R., Thurlow, M. L., & Schuelka, M. J. (2012). *Diploma options, graduation requirements, and exit exams for youth with disabilities: 2011 national study* (Technical Report 62). Minneapolis, MN: University of Minnesota, National Center on Educational Outcomes.

Kaiser, D., & Abell, M. (1997). Learning life management in the classroom. *Teaching Exceptional Children*, *30*, 70-75.

Kauffman, J. M. (2001). *Characteristics of emotional and behavioral disorders of children and youth* (7th ed.). Upper Saddle River, NJ: Merrill/ Prentice Hall.

Knowlton, E. (1998). Considerations in the design of personalized curricular supports for students with developmental disabilities. *Education and Training in Mental Retardation and Developmental Disabilities*, *33*, 95-107.

Maccini, P., & Gagnon, J. C. (2000). Best practices for teaching mathematics to secondary students with special needs. *Focus on Exceptional Children*, *32*(5), 1-22.

Maccini, P., Mulcahy, C.A., & Wilson, M.G. (2007). A follow-up of mathematics interventions for secondary students with learning disabilities. *Learning Disabilities Research & Practice*, *22*, 58-74.

Maccini, P., McNaughton, D., & Ruhl, K. L. (1999). Algebra instruction for students with learning disabilities: Implications from a research review. *Learning Disability Quarterly*, *22*(2), 113-126.

Maccini, P., & Ruhl, K.L. (2000). Effects of a graduated instructional sequence on the algebraic subtraction of integers by secondary students with learning disabilities. *Education and Treatment of Children*, *23*, 465-489.

Manno, B. V. (1994). *Outcome-Based Education: Miracle Cure or Plague?*. Hudson Institute.

Mahon, M. & Bullock, C. (1992). Teaching adolescents with mild mental retardation to make decisions in leisure through the use of self-control techniques. *Therapeutic Recreation Journal*, *26* (*1*), 9-26.

Marzano, R. J., & Kendall, J. S. (1996). *A Comprehensive Guide to Designing Standards-Based Districts, Schools, and Classrooms*. Association for Supervision and Curriculum Development, 1250 N. Pitt St., Alexandria, VA 22314-1453.

McDonnell, J. (1987). The effects of time delay and increasing prompt hierarchy strategies on the acquisition of purchasing skills by students with severe handicaps. *Journal of the Association for Persons with Severe Handicaps*, *12*, 227-236.

McDonnell, J., & Ferguson, B. (1989). A comparison of time delay and increasing prompt hierarchy strategies in teaching banking skills to students with moderate

handicaps. *Journal of the Association of Persons with Severe Handicaps*, *22*, 85-91.

McDonnell, J., & Laughlin, B. (1989). A comparison of backward and concurrent chaining strategies in teaching community skills. *Education and Training in Mental Retardation*, *24*, 230-238.

McDonnell, J., & McFarland, S. (1988). A comparison of forward and concurrent chaining strategies in teaching laundromat skills to students with severe handicaps. *Research in Developmental Disabilities, 9,* 177–194.

McDonnell, L. M., McLaughlin, M. J., & Morison, P. (Eds.). (1997). *Educating one and all: Students with disabilities and standards-based reform.* Washington, DC: National Academy Press. Retrieved from ERIC database. (ED409677)

Miller, U.C., & Test, D.W. (1989). A comparison of constant time delay and most-to-least prompting in teaching laundry skills to students with moderate retardation. *Education and Training in Mental Retardation*, *24*, 868-870.

Miser, A.L. (1985). Number abstraction processes in mentally retarded adults (concepts, quantification processes, counting). (Doctoral dissertation, The University of Connecticut, 1986). *Dissertation Abstracts International, 46*, 3241B.

National Council of Teachers of Mathematics (NCTM; 1989). *Curriculum and evaluation standards for school mathematics*. Reston. VA: Author.

National Council of Teachers of Mathematics (NCTM; 2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the national mathematics advisory report*. Washington, DC: U.S. Department of Education. Retrieved from http://www.ed.gov/MathPanel.

National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common Core State Standards (Application for Students with Disabilities).* National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C.

National Research Council. (1989). Everybody counts: A report to the nation on the future of mathematics education. Washington, DC: National Research Council.

Neef, N.A., Nelles, D.E., Iwata, B.A., & Page, T.P. (2003). Analysis of precurrent skills in solving mathematics story problems. *Journal of Applied Behavior Analysis*, *36*, 21-33.

Newman, L., Wagner, M., Cameto, R., & Knokey, A. (2009). *The post-high school outcomes of youth with disabilities up to 4 years after high school. A report of*

findings from the National Longitudinal Transition Study-2 (NLTS-2)(NCSER 2009-3017). Menlo Park, CA: SRI International. Retrieved from http://www.nlts2.org/reports/2009_04?nlts2_report_2009_04_complete.pdf

Newman, L., Wagner, M., Knokey, A.-M., Marder, C., Nagle, K., Shaver, D., . . . & Schwarting, M. (2011). *The Post-High School Outcomes of Young Adults With Disabilities up to 8 Years After High School. A Report From the National Longitudinal Transition Study-2 (NLTS2)* (NCSER 2011-3005). Menlo Park, CA: SRI International.

No Child Left Behind Act of 2001, 20 U.S.C. 70 § 6301 *et seq.* (2002).

Nolet, V., & McLaughlin, M. (2000). *Accessing the general curriculum.* Thousands Oaks, CA: Corwin Press.

O'Conner, Y. M., & Cuvo, A. J. (1989). Teaching dancercise to persons who are mentally handicapped: Programming transfer of stimulus control to a community setting.*Behavioral Residential Treatment*, *4*, 289–311.

Odom, S., Brantlinger, E., Gersten, R., Horner, R., Thompson, B., & Harris, K. (2005). Research in special education: Scientific methods and evidence-based practices. *Exceptional Children, 71*, 137-148.

Patton, J.R., Cronin, M.E., Polloway, E.A., Hutchinson, D., & Robinson, G. (1989). Curricular considerations: A life skills orientation. In G.A. Robinson, J.R. Patton, E.A. Pollaway, & L. Sargent (Eds.), *Best practices in mild mental retardation* (pp. 21-38). Reston, VA: Division on Mental Retardation, Council for Exceptional Children.

Patton, J.R., Polloway, E.A., & Smith, T.E.C.(2000). Education students with mild mental retardation. *Focus on Autism and Other Developmental Disabilities*, *15*, 80-89.

Pirie, S. E., & Martin, L. (1997). The equation, the whole equation and nothing but the equation! One approach to the teaching of linear equations. *Educational Studies in Mathematics*, *34*(2), 159-181.

Polloway, E.A. (2004). A eulogy for MMI. *DDD Express*, *14*, 1-8.

Polloway, E.A., (2006). Mild mental retardation: A concept in search of clarity, a population in search of appropriate education and supports, a profession in search of advocacy. *Exceptionality*, *14*, 183-190.

Polloway, E.A., Patton, J.R., Smith, T.E.C., & Buck, G. (1997). Mental retardation and learning disabilities: Conceptual and applied issues, *Journal of Learning Disabilities*, *30*, 297-308.

Polloway, E.A., Patton, J.R., Smith, J., & Rodrique, T. (1991). Issues in program design for elementary students with mild mental retardation: Emphasis on curriculum development. *Education and Training in Mental Retardation*, *26*, 142-150.

Polloway, E.A., Smith, J.D, Lubin, J., & Patton, J. (2010). Mild intellectual disabilities: Legacies and trends in concepts and educational practices. *Education and Training in Autism and Developmental Disabilities*, *45*, 54-68.

Rowe, D.A., Cease-Cook, J., & Test, D.W. (2011). Effects of simulation training on making purchases with a debit card and tracking expenses. *Career Development and Transition for Exceptional Individuals*, *34*, 107-114.

Sandknop, P. A., Schuster, J. W., Wolery, M., & Cross, D. P. (1992). The use of an adaptive device to teach students with moderate mental retardation to select lower priced grocery items. *Education and Training in Mental Retardation*, *27*, 219-229.

Scheuermann, A. M., Deshler, D. D., & Schumaker, J. B. (2009). The effects of the explicit inquiry routine on the performance of students with learning disabilities on one-variable equations. *Learning Disability Quarterly*, *32*, 103–120.

Shimabukuro, S. M., Prater, M. A., Jenkins, A., & Edelen-Smith, P. (2000). The effects of self-monitoring of academic performance on students with learning disabilities and ADD/ADHD. *Education and Treatment of Children, 22,* 397–414.

Singleton, D.K., Schuster, J.W., Morse, T.E., & Collins, B.C. (1999). A comparison of antecedent prompt and test and simultaneous prompting procedures in teaching grocery store words to adolescents with mental retardation. *Education and Training in Mental Retardation and Developmental Disabilities*, *34*, 182-189.

Smith, B., Schuster, J.W., Collins, B., & Kleinert, H. (2011). Using simultaneous prompting to teach restaurant words and classification as non-target information to secondary students with moderate to severe disabilities. *Education and Training in Autism and Developmental Disabilities*,*46*, 251-266.

Snell, M., & Browder, D. (1987). Domestic and community skills. In M. Snell (Ed.) *Systematic instruction of persons with severe handicaps* (pp. 390–434). Columbus, OH: Charles E. Merrill.

Steege, M., Wacker, D., & McMahon, C. (1987). Evaluations of the effectiveness and efficiency of two stimulus prompt strategies with severely handicapped students. *Journal of Applied Behavior Analysis*, *20*, 293-299.

Stephens, W.E. (1972). Equivalence information by retarded and nonretarded children at different mental ages. *American Journal of Mental Deficiency*, *77*, 311-313.

Sundel, M. (2005). *Behavior change in human services*. New York: Sage Publications.

Taber, T. A., Alberto, P. A., Hughes, M., &. Seltzer, A. (2002). A strategy for students with moderate disabilities when lost in the community. *Research and Practice for Persons with Severe Disabilities*, *27*, 141-152.

Taber, T. A., Alberto, P. A., Seltzer, A., & Hughes, M. (2003). Obtaining assistance when lost in the community using cell phones. *Research and Practice for Persons with Severe Disabilities*, *28*, 105-116.

Tawney, J. W., & Gast, D. L. (1984). *Single subject research in special education*. Columbus, OH: Merrill.

Thurlow, M. (2002). Positive education results for all students: The promise of standards-based reform. *Remedial and Special Education*, *23*, 195-202.

Thurlow, M., & Thompson, S. (2000). *Diploma options and graduations policies for students with disabilities* (Policy Directions No. 10). Minneapolis, MN: University of Minnesota, National Center on Educational Outcomes. Retrieved from: http://education.umn.edu/NCEO/OnlinePubs/Policy10.htm

Todd, T., & Reid, G. (2006). Increasing physical activity in individuals with autism. *Focus on Autism and Other Developmental Disabilities*, *21*, 167-176.

United States Department of Education (USDOE; 2010). Grants to improve assessments for students with disabilities. Retrieved from http://www.ed.gov/news/press-release/us-education-department-awards-grants-improve-assessments-students-disabilites

U.S. Department of Education (2005). *Alternate Achievement Standards for Students with the Most Significant Cognitive Disabilities Non-Regulatory Guidance*. Retrieved from….August 2005, p. 23

Vandercook, T. L. (1991). Leisure instruction outcomes: Criterion performance, positive interactions, and acceptance by typical high school peers. *The Journal of Special Education*, *25*,320-339.

Westling, D. L., & Fox, L. (2004). *Teaching students with severe disabilities* (3rd ed.). Englewood Cliffs, NJ: Merrill.

What Works Clearinghouse (WWC; 2009). Assisting students struggling with mathematics: Response to intervention for elementary and middle schools. Retrieved from http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=2

Witzel, B. (2005). Using CRA to teach algebra to students with math difficulties in inclusive settings. *Learning Disabilities: A Contemporary Journal*, *3*, 49-60.

Witzel, B., Mercer, C.D., & Miller, M.D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice*, *18*, 121-131.

Witzel, B., & Riccomini, P.J. (2009). *Computations of integers: Math intervention for elementary and middle grades*. Upper Saddle River, NJ: Pearson.

Witzel, B., & Riccomini, P.J. (2011). *Solving Equations.* Upper Saddle River, NJ: Pearson.

Witzel, B., Smith, S., Brownell, M. (2001). How can I help students with learning disabilities in algebra?. *Intervention in School and Clinic*, *37*, 101-104.

Wolery, M., Ault, M.J., Gast, D., Doyle, P.M., & Griffen, A.K. (1991). Teaching chained tasks in dyads: Acquisition of target and observational behaviors. *The Journal of Special Education*, *25*, 198-220.

Xin, Y.P., Jitendra, A.K., & Deatline-Buchman, A. (2005). Effects of mathematical word problem-solving instruction on middle school students with learning problems. *The Journal of Special Education*, *39*, 181-192.

Yu, J., Newman, L., & Wagner, M. (2009). Secondary school experiences and academic performance of students with mental retardation. Menlo Park, CA: SRI International Retrieved from http://www.nlts2.org/fact_sheets/nlts2_fact_sheet_2009_07.pdf

Zambo, R. (2005). The power of two: Linking math and literature. *Mathematics Teaching in Middle School*, *10*, 394-400.

APPENDIX A: SAMPLE SCORING SHEET

Show the steps of your work.

1. $x - 10 = 3$

Step 1:_____
Step 2: _____

Number of Steps Correct: ____/ 2
Solved Correct: ____ / 1

2. $13 = X + 5$

Step 1:_____
Step 2: _____

Number of Steps Correct: ____/ 2
Solved Correct: ____ / 1

3) $\underline{N} = 5$
   2

Step 1:_____
Step 2: _____

Number of Steps Correct: ____/ 2
Solved Correct: ____ / 1

4) $3Y = 24$
   2

Step 1:_____
Step 2: _____

Number of Steps Correct: ____/ 2
Solved Correct: ____ / 1

5) $5 = Y - 3$

Step 1:_____
Step 2: _____

Number of Steps Correct: ____/ 2

Solved Correct: _____ / 1

6) 7 + N = 8

Step 1:_____
Step 2: _____

Number of Steps Correct: ____/ 2
Solved Correct: _____ / 1

7) 3X = 21

Step 1:_____
Step 2: _____

Number of Steps Correct: ____/ 2
Solved Correct: _____ / 1

8) $\underline{N}$ = 9
  4

Step 1:_____
Step 2: _____

Number of Steps Correct: ____/ 2
Solved Correct: _____ / 1

9) 18 = Y + 5

Step 1:_____
Step 2: _____

Number of Steps Correct: ____/ 2
Solved Correct: _____ / 1

10) X - 11 = 2

Step 1:_____
Step 2: _____

Number of Steps Correct: ____/ 2
Solved Correct: ____ / 1

11) 3 = $\frac{X}{6}$

Step 1:_____
Step 2: _____

Number of Steps Correct: ____/ 2
Solved Correct: ____ / 1

12) 8C = 64

Step 1:_____
Step 2: _____

Number of Steps Correct: ____/ 2
Solved Correct: ____ / 1

Total Number of Steps Correct: ____/ 24
Total Solved Correct: ____ / 12

APPENDIX B: SCENARIOS FOR GENERALIZATION PROBES

1. One pound of hamburger meat makes four hamburger patties. Write and solve an equation to determine how many pounds of meat are needed to make 36 burgers.

2. Each recipe calls for 12 minutes to cook. You have 48 minutes to finish the meal. Write and solve an equation to determine the number of recipes you can finish in that time.

3. You are cooking muffins. The recipe calls for 7 cups of sugar. Someone already put in 2 cups. How many more cups do you need? Write and solve an equation to determine how many more cups you need.

4. How many boxes of envelopes can you buy with $12 if one box costs $3? Write and solve an equation to determine how many you can buy.

5. At a restaurant, Mike and his three friends decided to split the bill evenly. If each person paid $13, then what was the total bill. Write and solve an equation to determine the total bill.

6. Ozalle spent $56 on cereal. If they cost $7 box, how many boxes did she buy? Write and solve an equation to determine how many cereal boxes she bought.

7. Five candy bars are on sale for $26.05. Write and solve an equation to determine how much each candy bar costs.

8. How many packs of batteries can you buy with $45 if one pack costs $5? Write and solve an equation to determine how many batteries you can buy.

APPENDIX C: STUDENT SOCIAL VALIDITY QUESTIONAIRE

Questions	Responses			
This instructional strategy helped me learn how to solve inverse operations. (RQ # 5: Outcome)	1 STRONGLY AGREE	2 AGREE	3 DISAGREE	4 STRONGLY DISAGREE
I liked this strategy instruction. (RQ # 4: Procedure)	1 STRONGLY AGREE	2 AGREE	3 DISAGREE	4 STRONGLY DISAGREE
3. I would like to use this strategy to learn other algebra concepts. (RQ # 4: Procedure)	1 STRONGLY AGREE	2 AGREE	3 DISAGREE	4 STRONGLY DISAGREE
I feel that I will use this strategy in the future to help me solve problems. (RQ #4: Outcome)	1 STRONGLY AGREE	2 AGREE	3 DISAGREE	4 STRONGLY DISAGREE

APPENDIX D: TEACHER SOCIAL VALIDITY QUESTIONAIRE

Questions	Responses			
1. The Concrete-Representational-Abstract instruction helped my students learn how to solve inverse operations. (RQ # 6: Outcome)	1 STRONGLY AGREE	2 AGREE	3 DISAGREE	4 STRONGLY DISAGREE
2. I liked the Concrete-Representational-Abstract strategy instruction. (RQ # 7: Procedure)	1 STRONGLY AGREE	2 AGREE	3 DISAGREE	4 STRONGLY DISAGREE
3. I would like to use the Concrete-Representational-Abstract strategy to teach other algebra concepts. (RQ # 7: Procedure)	1 STRONGLY AGREE	2 AGREE	3 DISAGREE	4 STRONGLY DISAGREE
4. I felt the Concrete-Representational-Abstract instruction was useful for my students. (RQ # 6: Outcome)	1 STRONGLY AGREE	2 AGREE	3 DISAGREE	4 STRONGLY DISAGREE

APPENDIX E: SCRIPTED LESSONS AND LEARNING SHEETS

Teacher Procedures	Student Procedures	Teacher
This is a popsicle stick. This represents the number 10. If I hold up one stick I am showing the number 10. What am I showing when I hold up two?	Student's response: 20	If correct, "Good Job" and praise. If incorrect, review popsicle stick is 10 and two popsicle sticks would be 10 plus 10.
This is a toothpick. This represents the number one. If I hold up seven toothpicks, what number am I representing?	Student's response: 7	If correct, "Good Job" and praise. If incorrect, review the toothpick represents 1, and count out the 7 toothpicks to demonstrate for the student.
This is a cup. The cup will come before or after the variables. So this cup will cup before or _____ the variables.	Student response: after	If correct, "Good Job" and praise. If incorrect, restate the sentence. "The cup will come before or after the variables.
This is a string. The string is fun. We will use this string to split the equation across the equal sign.		
The multicolored cards are for the operations. The green side is for the positive or plus sign. So what do you think the other side represents (show red side)?	Student response: minus or negative	If correct, "Good Job" and praise. If incorrect, review the two sides again.
The last item is a strip of paper. This will be used when division is performed. Now that you have all the items, let's get started	Students will check to make sure they have all the items they need.	Teacher will double check the student does have all materials.
Demonstrate and Model		

Let's look at problem A on the learning sheet. This year I earned some money. I earned two payments for mowing the lawn. Teacher will show a N for mowing the lawn and two cups for two payments. $10 in gifts from my parents. Teacher will show a stick to represent 10. One more payment for mowing another lawn. Teacher will show another N and one cup. I had to give $5 to my sister. Teacher will hold up 5 toothpicks.		
Let's form an equation. The N represents mowing the lawn and the cups represent how many times. Since I earned $10 in gifts from my parents I should add them to the cups of N. Teacher will hold up the plus sign on the green side. Since I earned the next payment for the other lawn mowing I should add the one cup of N to 2N + 10. Teacher will show another green plus sign. Now I had to give my little sister the $5, so I need to subtract 5. Show a red minus sign.		
Teacher will model how to solve the equation using materials and then drawing the diagram on the board.	Student will complete A on the learning sheet by following along with teacher.	If correct, "Good Job" and praise. If incorrect, teacher will analyze the student's mistake and model the answer.
For Part B, I want you to follow along. We will write the equation this time. I have 5 cups of X. Teacher will hold up 5 cups and X card. Next I have a plus 5. Teacher will hold up a plus card and 5 toothpicks. Now I have to take away 6 toothpicks. Teacher will show a minus sign and 6 toothpicks. Teacher checks to make sure students have the same on their desk.	Student should have the same on their desk.	If correct, "Good Job" and praise. If incorrect, teacher will find which part is incorrect and review.
Which of these may I combine? I cannot combine the 5X and the toothpicks because the variable cannot be mixed with other variables or numbers alone. So I may combine the	Student will do the same as teacher. Student will complete B on the learning	If correct, "Good Job" and praise. If incorrect, teacher will find the mistake the student made with the materials and review.

5 and minus 6. Let's take one from each group at the same time. (Take away toothpicks from groups simultaneously. You will have 0 from the group of five and 1 toothpick next to the minus sign. Make sure students are duplicating your action.) Hmm. I have one toothpick next to the minus sign. I guess this means that I have a negative one. (Remove extraneous materials). So I only have left 5 cups of N and a minus 1 toothpick. This means I have 5N – 1 left. (Write 5N –1 on the board). Write down 5N –1 on your papers next to question (b).	sheet.	
Guided Practice		
Now let's try some together. For question (c) what do we lay down to represent 3Y?	Student says 3 cups and a Y card	If correct, "Good Job" and praise. If incorrect, review what represents the variable and coefficient.
What do we lay down to represent the plus sign?	Student says a green card	If correct, "Good Job" and praise. If incorrect, review the multi-colored cards.
Now how do we represent a Y?	Student says a y card, and one cup	If correct, "Good Job" and praise. If incorrect, review variables/cups.
Now, how do we represent the plus again?	Student says a green card	If correct, "Good Job" and praise. If incorrect, review multi-colored cards.
The five is represented how?	Student says five toothpicks	If correct, "Good Job" and praise. If incorrect, review toothpicks.

Now, what may we combine with this equation?	Student says add 3 cups of Y and one cup of Y to get 4Y	If correct, "Good Job" and praise. If incorrect, review like terms.
Can we add the toothpicks to anything?	Student says NO.	If correct, "Good Job" and praise. If student says yes, then ask them what they can add?
So what do you have in the end?	Student says 4Y + 5 and they write it on answer sheet.	If correct, "Good Job" and praise. If incorrect, look at where the mistake was made.
Teacher repeats guided practice for (d) with decreasing teacher input and think aloud.	Students complete (d).	Same level of error correction and praise for the (d).
For guided practice (e) and (f) follow the word problem with real life scenario and show student how to write the equation, like in Describe and Model (b). (e) While at work, you are cleaning the shelves you find 3 cans of soup and 2 boxes of macaroni out of place. Then on the floor you see 6 more cans of soup. Set up the simplest expression to show how many books you found. (f) While unloading the truck at work, the driver throws out 5 vegetables, 2 bags of vegetables, and then 3 more vegetables. Set up the simplest equation and show how many vegetables you took from the driver.	Students follow along with teacher and complete (e) and (f).	Same level of error correction and praise.
Independent Practice		

| Try the next problems on your own. | Students complete independent practice | Teacher will score the independent practice. If student doesn't get 3 out of 4 correct, item analysis will be done to find the error made by the student to make instructional decisions. |

Lesson One Learning Sheet

Describe and Model

a) 2N + 10 + N - 5   b) _____

Guided Practice

c) 3Y + Y +5   d) Y + Y + 2

e) While at work, you are cleaning the shelves you find 3 cans of soup and 2 boxes of macaroni out of place. Then on the floor you see 6 more cans of soup. Set up the simplest expression to show how many books you found.

f) While unloading the truck at work, the driver throws out 5 vegetables, 2 bags of vegetables, and then 3 more vegetables. Set up the simplest equation and show how many vegetables you took from the driver.

Independent Practice

g) N + N + 12   h) –X + 12 + 5   i) 5 + 2N + 7   j) 3X + 2 + 2X

k) 10th grade is going on a field trip to Carowinds. The trip will cost $18 per student. Write an expression to find the cost of the field trip for x students.

l) Lauren earns $22 per game as a referee for youth soccer games. Write an expression to find out how much money she will earn if she referees n games.

Lesson Two

Teacher Procedures	Student Procedures	Teacher
Yesterday we worked on reducing a list of variables and numbers to only the essential pieces. We will see that knowing how to do this will help us solve more complex looking equations in the future. In fact, you will encounter very long and frightening equations in a few weeks and by asked to solve for them. Unless you know how to reduce first, you may have difficulty.		
Remember how we reduced yesterday? What did we use to help us reduce those variables? .	Sticks, paper, toothpicks, cups, string, multi-colored paper, etcc.	That's right, we used sticks, paper, and toothpicks. Those items are easy to use in class but not everyone has access to these materials at home. To solve this problem we will learn how to draw these items to reduce similar equations. If incorrect, remind students of the materials used yesterday to elicit the other answers.
Demonstrate and Model		
1. Write the problem 5N – 2/X + N – 12 on the board. Yesterday to represent the 5N I used 5 cups and a N card.		
2. Today I will draw them out instead to look just like the cups and card. (Draw 5 cups and an N under the 5N). It looks just like the cups.		
3. To represent the "–" I will just draw a minus.		

4. To represent the 2/X I will draw two toothpicks over an X.		
5. Now, to represent the + N I will draw a plus, a cup, and an N. The cup is placed in there because there is only one N.		
6. Finally, to represent a minus and a 12 with a minus sign and sticks to represent 12. Since 12 has one ten and two ones I will draw those signs by writing a long line for the ten and two dashes for the 2.		
Now…Stand back and look at the equation.. Does this look like the equation up top? Yes, I think it does. Does it look like the cups, cards and objects I used yesterday? Yes, it does. Lets go on then. I see that the 5 cups of N and the 1 cup of N share the same variable of N so I want to combine them. Since the 5N is positive and the 1N is positive, I may add them together. (Draw 6 cups of N on the board under the first line of drawings). Now can I combine the 2/X with anything else in the equation? No, I don't think so because no other expression has an X. I will leave the 2/X alone. (Write - 2/X on the board next to the 6 cups of N). Now, about the –12, can I combine it with anything else? No, because nothing else has only numbers. (Draw it next to the 2 cups over X). I have left 6 cups of N, minus 2 cups over X and –12. What does that mean? Write out the expression 6N – 2/X –12.	Students complete (a) and (b) with assistance.	Teacher monitors students make the pictorial representations on their learning sheet, making error correction as needed.

Guided Practice		
Teacher repeats guided practice for (c), (d). Teacher repeats guided practice for word problems (e), and (f) with decreasing teacher input and think alouds.	Students complete d, e, and f.	Same level of error correction and praise for the (c), (d), (e), and (f).
Independent Practice		
Try the next problems on your own.	Students complete independent practice	Teacher will score the independent practice. If student doesn't get 3 out of 4 correct, item analysis will be done to find the error made by the student to make instructional decisions.

Lesson Two Learning Sheet

Describe/Model

a) $5N - \underline{2} + N - 12$
     X

b) $9 - 3X + 8 + X$

Guided Practice

c) $-6 + T + 4T + 2$

d) $2N - 3M + 12 + M$

e) You can carry only so many boxes of cereal to the checkout and they are out of grocery carts because it is a busy day. You found 2 big boxes of your favorite cereal and 4 small boxes for your brother. You notice that your sister's cereal is on sale, so you get 3 big boxes for her. Set up the expression the simplest expression to solve problem.

f) Kata has a savings account that contains $230. She decides to add $5 per month from her monthly earnings as a baby sitter. Write an expression to find out how much money Kata will have in her savings account after y months.

Independent Practice

g) $4U + 5U - W + 2W$

h) $6P + 4 - 3P + 7$

i) $\underline{W} + 4 - 12 + N$
    3

j) $2L + 6W - 2L + 13$

k) You are making a cake and icing. The cake takes 3 cups of flour and 1 cup of sugar. The icing takes 2 cups of sugar and 1 cup of water. What are the total ingredients? Set up the simplest expression to solve problem.

l)Your band wants to order t-shirts. The t-shirts cost $15 each plus a shipping fee of $10. Write an expression to find the total cost of c t-shirts.

Lesson Three

Teacher Procedures	Student Procedures	Teacher
Yesterday we learned how to attach pictures to the work we did using cards, toothpicks and such. You have shown that you now have the concepts to try these problems using numbers only. Throughout the lesson, remember, you are always able to use pictures to help answer difficult questions.		
Demonstrate and Model		
The first problem is 8 – F – 12X (2). Write the problem on the board .		
There is something new about this problem I have not seen yet.	Student says "parentheses"	If correct, say "good job" and praise. If incorrect, point to the parentheses and ask "have we seen this before" to elicit the correct answer.
The number in the parentheses needs to be multiplied to the 12X. I do this by multiplying 12 by 2. What is 12 times 2?	Student says 24	If correct, say "good job" and praise. If incorrect, write out multiplication.
I get 24. Now I have 24 X. (Write 24X under 12X(2).)		
Now can I combine anything in this expression?	Student says "no."	If correct, say "that's right, we can not combine anything." If incorrect, ask the students what can be combined to help with error correction.
So, the answer I end up with is 8-F-24X.		

Guided Practice		
Teacher repeats guided practice for (c), (d). Teacher repeats guided practice with word problems to set up their own expression for (e), and (f) with decreasing teacher input and think alouds.	Students complete c, d, e, and f.	Same level of error correction and praise for the (c), (d), (e), and (f).
Independent Practice		
Try the next problems on your own.	Students complete independent practice	Teacher will score the independent practice. If student doesn't get 3 out of 4 correct, item analysis will be done to find the error made by the student to make instructional decisions.

Lesson Three Learning Sheet

Describe/Model
a) 8 – F – 12X (2)   b) 9 / 3 + 8X + X + 1

Guided Practice
c) –N + 4M + 2W + 2N   d) $\frac{24Y}{6}$ + 6 + Y

e) Your uncle brought 4 red presents and 2 blue presents to the party. Your little brother then took 2 blue presents. Aunt Patty brought 2 more red presents. Set up the expression to figure out how many presents were left.

f) The football team passed the ball for 5 yards and then ran for 4 yards. They passed for 10 yards more and then ran for a loss of 2 yards. Set up the expression to show many yards were gained by the team through running and passing.

Independent Practice
g) 4U (5) – W + 2W   h) –3P + 7 + P – 12   i) $\frac{12W}{3}$ + 4 – 12

j) 2W + 6 – 2K + 2K

k) In PE, the teacher throws out 5 balls, 2 sacks of balls, and then three more balls. Set up the expression and show how many balls there were in PE.

l) To build a house you will need supplies. You bring 3 bundles of wood and 5 pallets of bricks. The foreman brings an additional 10 pallets of bricks and 5 bundles of wood. Set up an expression to show what was brought.

Lesson Four

Teacher Procedures	Student Procedures	Teacher
Today we are going to solve for single variables using operations such as adding, subtracting, dividing and multiplying. This will help us understand how to mathematically solve for something we do not know.		
Demonstrate and Model		
Now, my friend let me borrow money to buy a CD, but I forgot how much. (Place a cup and an X in a visible spot for the student to see).		
After I leave the store, I read the receipt. The receipt says I spent 10 dollars on the CD. (Show the class the popsicle stick).		
Now, since I *spent* the money, am I going to add the money to what I was lent or take away?	Student says "take away"	If correct, … Yes, take away. (Place a minus sign after the X and the 10 after the minus sign). If incorrect, repeat the question and say if "I spent " the money do I still have it?
Now in my pocket I have 3 one dollar bills left. Now since I have $3 dollars left does that go before or after an equals sign?	Student says "after"	If correct,… Right, after the string. (Lay the string and the 3 after that). If incorrect, reexplain the string and how to set it up.
The equation now reads X-10=3. This would be a lot easier to solve if the X was all alone on one side of the equation.		
Let's discuss the questions to ask ourselves to solve for x. First, question One: What is the variable?	Student says "x"	If correct, Good, x is the variable…we are solving for x. We want x to be alone or "isolated" on the side of the equal sign. If incorrect, say the variable here is "x", what is the variable? Students

		should say "x" then praise the correct answer.
Question Two: What operation is being performed?.	Student says "subtraction"	If correct, Good, subtraction is the operation If incorrect, review the operations.
Question Three: What is the opposite operation?	Student says "addition"	If correct, "Good, addition." If incorrect, review what opposite operations means.
So we need to use addition on both sides of the equal sign to solve for x. X = 13 Teach students that what you do to one side of equal sign, you must do to the other.		
Guided Practice		
Teacher repeats guided practice for (c), (d), (e), and (f) with think aloud with those three questions.	Students complete c, d, e, and f.	Same level of error correction and praise for the (c), (d), (e), and (f).
Independent Practice		
Try the next problems on your own.	Students complete independent practice	Teacher will score the independent practice. If student doesn't get 3 out of 4 correct, item analysis will be done to find the error made by the student to make instructional decisions.

Lesson Four Learning Sheet

Describe / Model
a) $X - 10 = 3$ b) $13 = X + 5$

c) $\dfrac{N}{2} = 5$ d) $3Y = 24$

Guided Practice
e) $5 = Y - 3$ f) $7 + N = 8$

g) In a basketball game, three players scored 15 points together. What is the average amount of points that each scored. Set up the equation <u>and</u> solve the equation.

h) Shaqwana sold cell phones. If she sold 4 cell phones she made $12. How much does she make per cell phone. Set up the equation <u>and</u> solve the equation.

Independent Practice
i) $18 = Y + 5$ j) $X - 11 = 2$

k) $9 = 3Y$ l) $\dfrac{X}{2} = 6$

m) Jose had 20 apples and gave some away. He had 3 left. How many did he give away? Set up the equation <u>and</u> solve the equation.

n) There are 11 employees at each store but many stores. If there are 33 employees, how many stores are there? Set up the equation <u>and</u> solve the equation.

Lesson Five

Teacher Procedures	Student Procedures	Teacher
(Write X- 14 = 12). Last time we worked on solving for a single variable using hands-on materials. This time we are taking a step further. This time we are going to draw out the parts to the problem we are given in the same manner we used the materials.		
Demonstrate and Model		
For example, what did we use to represent the variable?	Student says "paper letter"	If correct, .. Right. We used a paper letter. This time, instead of using a paper letter, we will represent variables by simply writing them down. (Write the X under the X in the equation). If incorrect, review.
What did the cup stand for?	Student says "groups or coefficients"	If correct, .. Groups (or coefficients) is correct. To represent groups we will use an empty circle next to our variable. (Write a circle next to the X in the equation). If incorrect, review and re-draw.
What did we use to represent the numbers? In this case the 14. …	Student says "popsicle sticks and toothpicks"	If correct, Popsicle stick and toothpicks is correct. Instead of sticks though, we are going to use tally marks. One small diagonal mark equals one (write a small dash), but to represent ten we will use a long straight mark (Write the long dash). In this problem, we must represent fourteen so what do I write? (Erase the previous dashes) … Correct, I make one long mark and four tally marks. If incorrect, review and re-draw
Now to represent the minus sign, I am just to make a minus sign and to represent the equals sign, I will draw a squiggly line overtop the equals sign, just as we had on our desks (Draw lines).		

To represent the 12 then, what do we write?	Student says "one long mark and 2 tally marks"	If correct, reaffirm and say One long mark and 2 tally marks is correct (Write the marks). If incorrect, review the materials and re-draw.
Now we could solve this problem. What is the first question we ask? What is the answer to that question?	Student says" What is the variable? " Student says "x"	If correct, yes, we ask what is the variable? What is the variable? X is the correct answer. If incorrect, say no the variable here is x.
What is the second question?	Student says "What operation is being performed?"	If correct, Good. If incorrect, repeat question two for the students and reask the question.
What operation is being performed?	Student says " subtraction"	If correct, good. It is subtraction. If incorrect, ask the students to look at the operation again and answer the question.
What is the last question we ask ourselves?	Student says "what is the opposite operation"	If correct, good. That is correct. If incorrect, ask the students what we have to do next to finish the problem after we identify the operation.
What is the opposite operation?	Student says "addition"	If correct, "Good, addition." If incorrect, review what opposite operations means.
So we need to use addition on both sides of the equal sign to solve for x. X = 26 Review with students that what you do to one side of equal sign, you must do to the other.		

Guided Practice		
Teacher repeats guided practice for (c), (d), (e), and (f) with think aloud with those three questions.	Students complete c, d, e, and f.	Same level of error correction and praise for the (c), (d), (e), and (f).
Independent Practice		
Try the next problems on your own.	Students complete independent practice	Teacher will score the independent practice. If student doesn't get 3 out of 4 correct, item analysis will be done to find the error made by the student to make instructional decisions.

Lesson Five Learning Sheet

Describe/Model

a) $T - 14 = 12$ b) $6 = Y - 23$

c) $3 = \dfrac{X}{6}$ d) $8C = 64$

Guided Practice

e) $2 = -15 + H$ f) $\dfrac{W}{4} = 3$

g) Lana had a large number of chocolate bars. He split them among 5 friends. After he split them, each friend had 2 chocolates. How many did he have total? Set up the equation and solve the equation.

j) Each grocery bag could carry 5 boxes of rice. How many grocery bags would you need to carry 25 boxes of rice? Set up the equation and solve the equation.

Independent Practice

k) $P - 6 = 13$ l) $14 = 2 + T$

m) $25 = 5Y$ n) $13 = X - 14$

o) You bake a cake with 18 pieces for a party. The pieces were divided among a few people. In the end, each person received 3 pieces of cake, how many people were at the party? Set up the equation and solve the equation.

p) You had $15. You paid the cashier for your groceries. You only have $2 left. What was the total for your groceries? Set up the equation and solve the equation.

ISOLATE Booster Session Lesson

Introduction of the mnemonic ISOLATE

Materials:
- Presentational device (Smartboard, Whiteboad, Overhead, etc)

<u>Student Objective</u>

Today instead of using pictures and materials to learn about solving problems, we are going to learn a mnemonic. Can you say the word ISOLATE? We are going to ISOLATE the variable on one side of the equation to solve for the variable. Let's get started.

<u>Demonstrate/Model</u>

Hand out learning sheet. Introduce ISOLATE.

I- identify the variable to be solved; point to the variable.

S-set up calculations to isolate the variable

O-organize the calculations to balance across the equal sign

L-list the calculations to happen in the same order on both sides

A-answer the calculations on the variables first side

T-total the calculations on the other side of the equation

E-evaluate that the answer is correct

1. The first problem is $\underline{N} = 2$

$\qquad\quad 4$

2. So let's use ISOLATE to solve. I want to solve for N.

3. S- There is no number aside from the coefficient on the same side as N, so I will work with the coeffieint, ¼.

4. O- organize across

5. L-let's calculate. Multiply4 on both sides

6. A-answer the unknown side. Divide by 1 on both sides

7. T-total the equations. y=12

8. E-evaluate that the answer is correct. Good job!

<u>Guided Practice</u>

Teacher can go through the steps for two more using think alouds. Go through the problems asking students what to do in each case. They should be able to push the teacher along while working the problems at their desk. Again, do not let students continue until you check their accuracy.

$6 = 3y$

$y - 10 = 14$

<u>Independent Practice</u>

Now you have a few problems to try on your own. Do your best to get correct answers.

Booster Session Learning Sheet

<u>Demonstrate/Model</u>

a) $\dfrac{N}{4} = 2$ b) $14 = 2 + T$

c) $64 = 4X$ d) $\dfrac{P}{6} = 7$

<u>Guided Practice</u>

e) $6 = 3y$ f) $y - 10 = 14$

<u>Independent Practice</u>

g) $54 = 14 + M$ h) $63 = 9Y$

i) $19 = X - 3$ j) $P - 18 = 8$

APPENDIX F: PROCEDURAL FIDELITY CHECKLIST

Teacher Procedures	Student Procedures	Teacher	Yes/No
This is a popsicle stick. This represents the number 10. If I hold up one stick I am showing the number 10. What am I showing when I hold up two?	Student's response: 20	If correct, "Good Job" and praise. If incorrect, review popsicle stick is 10 and two popsicle sticks would be 10 plus 10.	
This is a toothpick. This represents the number one. If I hold up seven toothpicks, what number am I representing?	Student's response: 7	If correct, "Good Job" and praise. If incorrect, review the toothpick represents 1, and count out the 7 toothpicks to demonstrate for the student.	
This is a cup. The cup will come before or after the variables. So this cup will cup before or _____ the variables.	Student response: after	If correct, "Good Job" and praise. If incorrect, restate the sentence. "The cup will come before or after the variables.	
This is a string. The string is fun. We will use this string to split the equation across the equal sign.			
The multicolored cards are for the operations. The green side is for the positive or plus sign. So what do you think the other side represents (show red	Student response: minus or negative	If correct, "Good Job" and praise. If incorrect, review the two sides again.	

side)?			
The last item is a strip of paper. This will be used when division is performed. Now that you have all the items, let's get started	Students will check to make sure they have all the items they need.	Teacher will double check the student does have all materials.	
Demonstrate and Model			
Let's look at problem A on the learning sheet. This year I earned some money. I earned two payments for mowing the lawn. Teacher will show a N for mowing the lawn and two cups for two payments. $10 in gifts from my parents. Teacher will show a stick to represent 10. One more payment for mowing another lawn. Teacher will show another N and one cup. I had to give $5 to my sister. Teacher will hold up 5 toothpicks.			
Let's form an equation. The N represents mowing the lawn and the cups represent how may times. Since I earned $10 in gifts from my parents I should add them to the cups of N. Teacher will hold up the plus sign on the green side. Since I earned the next			

payment for the other lawn mowing I should add the one cup of N to 2N + 10. Teacher will show another green plus sign. Now I had to give my little sister the $5, so I need to subtract 5. Show a red minus sign.			
Teacher will model how to solve the equation using materials and then drawing the diagram on the board.	Student will complete A on the learning sheet by following along with teacher.	If correct, "Good Job" and praise. If incorrect, teacher will analyze the student's mistake and model the answer.	
For Part B, I want you to follow along. I have 5 cups of X. Teacher will hold up 5 cups and X card. Next I have a plus 5. Teacher will hold up a plus card and 5 toothpicks. Now I have to take away 6 toothpicks. Teacher will show a minus sign and 6 toothpicks. Teacher checks to make sure students have the same on their desk.	Student should have the same on their desk.	If correct, "Good Job" and praise. If incorrect, teacher will find which part is incorrect and review.	
Which of these may I combine? I cannot combine the 5X and the toothpicks because the variable cannot be mixed with other variables or numbers alone. So I may combine the 5 and minus 6. Let's take one from each group at the same time. (Take away	Student will do the same as teacher. Student will complete B on the learning sheet.	If correct, "Good Job" and praise. If incorrect, teacher will find the mistake the student made with the materials and review.	

toothpicks from groups simultaneously. You will have 0 from the group of five and 1 toothpick next to the minus sign. Make sure students are duplicating your action.) Hmm. I have one toothpick next to the minus sign. I guess this means that I have a negative one. (Remove extraneous materials). So I only have left 5 cups of N and a minus 1 toothpick. This means I have 5N – 1 left. (Write 5N –1 on the board). Write down 5N –1 on your papers next to question (b).			
Guided Practice			
Now let's try some together. For question (c) what do we lay down to represent 3Y?	Student says 3 cups and a Y card	If correct, "Good Job" and praise. If incorrect, review what represents the variable and coefficient.	
What do we lay down to represent the plus sign?	Student says a green card	If correct, "Good Job" and praise. If incorrect, review the multi-colored cards.	
Now how do we represent a Y?	Student says a y card, and one cup	If correct, "Good Job" and praise. If incorrect, review variables/cups.	

Now, how do we represent the plus again?	Student says a green card	If correct, "Good Job" and praise. If incorrect, review multi-colored cards.	
The five is represented how?	Student says five toothpicks	If correct, "Good Job" and praise. If incorrect, review toothpicks.	
Now, what may we combine with this equation?	Student says add 3 cups of Y and one cup of Y to get 4Y	If correct, "Good Job" and praise. If incorrect, review like terms.	
Can we add the toothpicks to anything?	Student says NO.	If correct, "Good Job" and praise. If student says yes, then ask them what they can add?	
So what do you have in the end?	Student says 4Y + 5 and they write it on answer sheet.	If correct, "Good Job" and praise. If incorrect, look at where the mistake was made.	
Teacher repeats guided practice for (d), (e), and (f) with decreasing teacher input and think alouds.	Students complete d, e, and f.	Same level of error correction and praise for the (d), (e), and (f).	
Independent Practice			
Try the next problems on your own.	Students complete independent practice	Teacher will score the independent practice. If student doesn't get 3 out of 4 correct, item analysis will be done to find the error made by	

		the student to make instructional decisions.	

Lesson Two

Teacher Procedures	Student Procedures	Teacher	Yes/No
Yesterday we worked on reducing a list of variables and numbers to only the essential pieces. We will see that knowing how to do this will help us solve more complex looking equations in the future. In fact, you will encounter very long and frightening equations in a few weeks and by asked to solve for them. Unless you know how to reduce first, you may have difficulty.			
Remember how we reduced yesterday? What did we use to help us reduce those variables? .	Sticks, paper, toothpicks, cups, string, multi-colored paper, etcc.	That's right, we used sticks, paper, and toothpicks. Those items are easy to use in class but not everyone has access to these materials at home. To solve this problem we will learn how to draw these items to reduce similar equations. If	

		incorrect, remind students of the materials used yesterday to elicit the other answers.	
Demonstrate and Model			
1. Write the problem 5N – 2/X + N – 12 on the board. Yesterday to represent the 5N I used 5 cups and a N card.			
2. Today I will draw them out instead to look just like the cups and card. (Draw 5 cups and an N under the 5N). It looks just like the cups.			
3. To represent the "–" I will just draw a minus.			
4. To represent the 2/X I will draw two toothpicks over an X.			
5. Now, to represent the + N I will draw a plus, a cup, and an N. The cup is placed in there because there is only one N.			

6. Finally, to represent a minus and a 12 with a minus sign and sticks to represent 12. Since 12 has one ten and two ones I will draw those signs by writing a long line for the ten and two dashes for the 2.			
Now…Stand back and look at the equation.. Does this look like the equation up top? Yes, I think it does. Does it look like the cups, cards and objects I used yesterday? Yes, it does. Lets go on then. I see that the 5 cups of N and the 1 cup of N share the same variable of N so I want to combine them. Since the 5N is positive and the 1N is positive, I may add them together. (Draw 6 cups of N on the board under the first line of drawings). Now can I combine the 2/X with anything else in the equation? No, I don't think so because no other expression has an X. I will leave the	Students complete (a) and (b) with assistance.	Teacher monitors students make the pictorial representations on their learning sheet, making error correction as needed.	

2/X alone. (Write - 2/X on the board next to the 6 cups of N). Now, about the –12, can I combine it with anything else? No, because nothing else has only numbers. (Draw it next to the 2 cups over X). I have left 6 cups of N, minus 2 cups over X and –12. What does that mean? Write out the expression 6N – 2/X – 12.			
Guided Practice			
Teacher repeats guided practice for (c), (d), (e), and (f) with decreasing teacher input and think alouds.	Students complete d, e, and f.	Same level of error correction and praise for the (c), (d), (e), and (f).	
Independent Practice			
Try the next problems on your own.	Students complete independent practice	Teacher will score the independent practice. If student doesn't get 3 out of 4 correct, item analysis will be done to find the error made by the student to make instructional decisions.	

Lesson Three

Teacher Procedures	Student Procedures	Teacher	Yes/No
Yesterday we learned how to attach pictures to the work we did using cards, toothpicks and such. You have shown that you now have the concepts to try these problems using numbers only. Throughout the lesson, remember, you are always able to use pictures to help answer difficult questions.			
Demonstrate and Model			
The first problem is 8 – F – 12X (2). Write the problem on the board .			
There is something new about this problem I have not seen yet.	Student says "parentheses"	If correct, say "good job" and praise. If incorrect, point to the parentheses and ask "have we seen this before" to elicit the correct answer.	
The number in the parentheses needs to be multiplied to the 12X. I do this by multiplying 12 by 2. What is 12 times 2?	Student says 24	If correct, say "good job" and praise. If incorrect, write out multiplication.	
I get 24. Now I have 24 X. (Write 24X under 12X(2).)			

Now can I combine anything in this expression?	Student says "no."	If correct, say "that's right, we can not combine anything." If incorrect, ask the students what can be combined to help with error correction.	
So, the answer I end up with is 8-F-24X.			
Guided Practice			
Teacher repeats guided practice for (c), (d), (e), and (f) with decreasing teacher input and think alouds.	Students complete d, e, and f.	Same level of error correction and praise for the (c), (d), (e), and (f).	
Independent Practice			
Try the next problems on your own.	Students complete independent practice	Teacher will score the independent practice. If student doesn't get 3 out of 4 correct, item analysis will be done to find the error made by the student to make instructional decisions.	

Lesson Four

Teacher Procedures	Student Procedures	Teacher	Yes/No
Today we are going to solve for single variables using operations such as adding, subtracting, dividing and multiplying. This will help us understand how to mathematically solve for something we do not know.			
Demonstrate and Model			
Now, my friend let me borrow money to buy a CD, but I forgot how much. (Place a cup and an X in a visible spot for the student to see).			
After I leave the store, I read the receipt. The receipt says I spent 10 dollars on the CD. (Show the class the popsicle stick).			
Now, since I *spent* the money, am I going to add the money to what I was lent or take away?	Student says "take away"	If correct, … Yes, take away. (Place a minus sign after the X and the 10 after the minus sign). If incorrect, repeat the question and say if "I spent " the money do I still have it?	
Now in my pocket I have 3 one dollar bills left. Now since I have $3 dollars left does that go before or after an equals sign?	Student says "after"	If correct,… Right, after the string. (Lay the string and the 3 after that). If incorrect,	

		reexplain the string and how to set it up.	
The equation now reads X-10=3. This would be a lot easier to solve if the X was all alone on one side of the equation.			
Let's discuss the questions to ask ourselves to solve for x.

 First, question One: What is the variable? | Student says "x" | If correct, Good, x is the variable…we are solving for x. We want x to be alone or "isolated" on the side of the equal sign.
If incorrect, say the variable here is "x", what is the variable? Students should say "x" then praise the correct answer. | |
| Question Two: What operation is being performed?. | Student says "subtraction" | If correct, Good, subtraction is the operation
If incorrect, review the operations. | |
| Question Three: What is the opposite operation? | Student says "addition" | If correct, "Good, addition."

If incorrect, review what opposite operations means. | |
| So we need to use addition on both sides of the equal sign to solve for x. X = 13
Teach students that what you do to one side of equal sign, you must do to the other. | | | |
| | Guided Practice | | |

Teacher repeats guided practice for (c), (d), (e), and (f) with think aloud with those three questions.	Students complete c, d, e, and f.	Same level of error correction and praise for the (c), (d), (e), and (f).	
Independent Practice			
Try the next problems on your own.	Students complete independent practice	Teacher will score the independent practice. If student doesn't get 3 out of 4 correct, item analysis will be done to find the error made by the student to make instructional decisions.	

Lesson Five

Teacher Procedures	Student Procedures	Teacher	Yes/No
(Write X- 14 = 12). Last time we worked on solving for a single variable using hands-on materials. This time we are taking a step further. This time we are going to draw out the parts to the problem we are given in the same manner we used the materials.			
Demonstrate and Model			
For example, what did we use to represent the variable?	Student says "paper letter"	If correct, .. Right. We used a paper letter. This time, instead of using a paper letter, we will represent variables by simply writing them down. (Write the X under the X in the equation). If incorrect, review.	
What did the cup stand for?	Student says "groups or coefficients"	If correct, .. Groups (or coefficients) is correct. To represent groups we will use an empty circle next to our variable. (Write a circle next to the X in the equation). If incorrect, review and re-draw.	
What did we use to represent the numbers? In this case the 14. …	Student says "popsicle sticks and toothpicks"	If correct, Popsicle stick and toothpicks is	

		correct. Instead of sticks though, we are going to use tally marks. One small diagonal mark equals one (write a small dash), but to represent ten we will use a long straight mark (Write the long dash). In this problem, we must represent fourteen so what do I write? (Erase the previous dashes) … Correct, I make one long mark and four tally marks. If incorrect, review and re-draw	
Now to represent the minus sign, I am just to make a minus sign and to represent the equals sign, I will draw a squiggly line overtop the equals sign, just as we had on our desks (Draw lines).			
To represent the 12 then, what do we write?	Student says "one long mark and 2 tally marks"	If correct, reaffirm and say One long mark and 2 tally marks is correct (Write the marks). If incorrect, review the materials and re-draw.	
Now we could solve this problem. What is the first question we ask? What is the answer to that question?	Student says" What is the variable? " Student says "x"	If correct, yes, we ask what is the variable? What is the variable? X is the correct answer. If incorrect, say no the variable here	

		is x.	
What is the second question?	Student says "What operation is being performed?"	If correct, Good. If incorrect, repeat question two for the students and reask the question.	
What operation is being performed?	Student says " subtraction"	If correct, good. It is subtraction. If incorrect, ask the students to look at the operation again and answer the question.	
What is the last question we ask ourselves?	Student says "what is the opposite operation"	If correct, good. That is correct. If incorrect, ask the students what we have to do next to finish the problem after we identify the operation.	
\: What is the opposite operation?	Student says "addition"	If correct, "Good, addition." If incorrect, review what opposite operations means.	
So we need to use addition on both sides of the equal sign to solve for x. X = 26 Review with students that what you do to one side of equal sign, you must do to the other.			

Guided Practice			
Teacher repeats guided practice for (c), (d), (e), and (f) with think aloud with those three questions.	Students complete c, d, e, and f.	Same level of error correction and praise for the (c), (d), (e), and (f).	
Independent Practice			
Try the next problems on your own.	Students complete independent practice	Teacher will score the independent practice. If student doesn't get 3 out of 4 correct, item analysis will be done to find the error made by the student to make instructional decisions.	