# THREE ESSAYS IN REAL OPTION MODELS OF REAL ESTATE DEVELOPMENT

by

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ABSTRACT

YIYING CHENG. Three essays in real option models of real estate development.
(Under the direction of DR. STEVEN P. CLARK)

Real estate developments make great applications for real option theory. However, current real option models for real estate development do not pick up the important features of real estate development such as the land-use right as an on-going lease agreement or the entitlement processes in every development project. We build and solve mathematically parsimoneous yet plausible stochastic-control models to capture these features and the models yield rich implications. We then test these implications empirically using manually-collected data.

In chapter one, we model the real estate development process as a compound real option in a parsimonious continuous-time feedback control framework. The acquisition of a land-use right is the first option, and the decision to begin construction is the second option. We model the cost of maintaining the land-use right as a running cost during the waiting period before construction. This feature allows the running cost to be stochastic and interacts with both the decision to obtain the right and the decision to start construction. We obtain a closed-form solution for the value of the compound option and demonstrate rich implications using numerical examples. A higher running cost squeezes the two decisions together while a lower running cost encourages the acquisition of the land-use right and delays construction simultaneously.

In chapter two, we recognize that entitlement process is highly risky and out of control of the investors in reality. Moreover, the real estate market in the U.S. has shown a trend towards more stringent entitlement regulation. Therefore, we model entitlement process as

a separate stage in development process in which the investor has little control. In particular, we model the entitlement stage as a European style real option with a stochastic entitlement cost. We solve the model analytically. Our main result implies that, to the contrary of tradition real option theory, higher entitlement risk urges the investors to start entitlement process earlier in order to counter the lack of control.

In chapter three, we test the empirical implications of entitlement in the previous chapter using hand-collected Charlotte local data of rezoning petitions. In particular, we collect waiting time, number of revisions, size of lot, decision outcome as well as other characteristics for rezoning petition from 2001 to 2012, published on Charlotte-Mecklenburg City Planning website. The results of negative binomial regressions confirm our earlier theoretical prediction that the investors start earlier when facing more difficult and riskier entitlement process. Moreover, house price is overall negatively correlated with the entitlement riskiness, which aggravates the hastening effect of entitlement risk.

In conclusion, our real option models of land-use right and entitlement in real estate development prove to be mathematically novel, economically insightful, and show potential for wide applications.

# DEDICATION

This work is dedicated to my loving husband, Zeping Xu, who kindly encouraged me along the way and helped me in many critical points.

I also dedicate this work to my father, who has always supported me to pursue knowledge and a balanced life style.

# ACKNOWLEDGEMENTS

I am grateful for my advisor, Dr. Steven P. Clark, for numerous discussion sessions and kind suggestions. At critical points, he always pointed me towards ways to efficiently solve the problem.

Thanks to Dr. Steven H. Ott for introducing me to real options theory, and for his inspiring idea especially for the later two chapters about entitlement.

Lastly, I want to thank Dr. Mingxin Xu and Dr. Dmitry Shapiro, my best professor friends. Dr. Xu gently introduced me to stochastic control models and sparked my interest in this area. Dr. Shapiro taught me to think over all matters scientifically and independently. They serve as my role models for successful and balanced life style, while enjoying research as a lift-time vocation.

# INTRODUCTION

Land development has been one of the most researched applications of real option theories because much of the land value is derived from embedded real options. Following seminal papers of Brennan and Schwartz (1985), McDonald and Siegel (1986), Dixit and Pindyck (1994) and others, many scholars have studied real option value and the investment decision in real estate development, both analytically and empirically. For example, Williams (1991) analyzed the real option value and investment decision when developers have the flexility to choose when to start construction as well as the density of the construction. Grenadier (1996) developed the market equilibrium for multiple competitors and explained the bursts in construction activity. Empirically, Leishman et al. (2000) and Capozza and Li (2001) demonstrated the real option effect in construction by testing the investment response to interest rate changes.

In addition to this stream of literature, recent studies have explored the impact of individual options embedded in the development process. For example, Lai et al. (2004) and Buttimer et al. (2008) studied the effect of the pre-sale option as a risk management technique and demonstrated the risk transfer achieved by the pre-sale option. Chan et al. (2008) explored the impacts of pre-sale option on investment decisions with financing constraints. Leung et al. (2007), on the other hand, studied the risk-premium of the pre-sale option under asymmetric information.

However, current real option models for real estate development often treat real estate development as a uniform process and ignore the different features in different stages of development. In my dissertation, we aim at modelling two important features before the construction stage of real estate development.

First, in Chapter one, we model a specific land-use right structure in which investors pays a running cost to secure land-use right for arbitrary length of time. This structure is often seen in real world practice as an effective legal structure to separate the land ownership and land-use right. Yet, the real option model that includes a running cost remains unsolved due to mathematical difficulties. We solve the model using a clever work-around, and get implications that are consistent with what we see in the real world. In particular, the model indicates that investors waiting longer before getting land use right and start construction earlier if the running cost to maintain the land-use right is high, and vise versa. The model has a timely application in explaining the hydraulic fracturing, which is a popular economic technology to extract natural gas from ground. Consistent with the model prediction, we see oil companies start collecting the land-use right from local residents years before they begin to construct the gas well, because the cost of maintaining and collecting the land-use right is fairly low compared to the economic benefit of the gas well.

Second, in Chapter 2, we model the entitlement process, which is the process of getting all permits from regulators. Every developer has to get through this process before they start construction. The entitlement process is fundamentally different from the other stages of development because the investors do not have control over the timing. Therefore, traditional real option models that assume full control of investors predicts wrong behavior of investors'. We are the first to model entitlement process as a separate stage of real estate development and take away investors' control by modeling it as a European option. We also take the novelty to aggregate the entitlement cost from longer waiting time and more difficult process and model the total entitlement cost as a stochastic process, with its volatility standing for the entitlement risk. The model indicates that higher entitlement risk leads to investors' earlier decision to apply for entitlement, and this surprising hastening-effect is more prominent when the market house price is negatively correlated to the entitlement cost.

We test these implications in Chapter 3. To do so, we manually collect information on rezoning petitions published on Charlotte-Mecklenburg City Planning website. We define waiting time as the number of days between application date and decision date to proxy for the timing of investors' decisions. We argue that a longer waiting time indicates earlier or premature application for entitlement if we properly control for the entitlement difficulty. The results of a whole sample negative binomial regression shows that investors apply early for entitlement when facing higher entitlement difficulty or entitlement risk. We then group the rezoning petitions monthly by application date and use standard deviation of waiting time as additional proxy for the entitlement risk. The results again display strong positive relationship between entitlement risk, entitlement difficulty and early application. Finally, we group the rezoning petitions monthly in two ways to investigate how current available information affects the number of applications. Specifically, we group petitions by decision date to calculate petition-specific information; and we group the petitions by application date to get the number of applications for each month. We found that, given the information on petition cases concluded recently, the investors would make more applications if the entitlement is riskier or more difficult.

Our work makes significant contribution to the literature. The first chapter not only presents and solves a rigorous model for this legal structure, but also present rich implications that can be seen in the real world. The second and third chapters embark on the unexplored territory of entitlement process. We not only present a plausible stochastic control model, but also empirically test and confirm the implications of theoretical model.

The rest of dissertation is organized as follows. In Chapter 1, we present the model for land-use right and discuss its implications and real world application, hydraulic fracturing. In Chapter 2, we present a theoretical model for entitlement process in real estate development and discuss its implications, which is empirically tested in Chapter 3 using manually collected data.

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# CHAPTER 1: LAND-USE RIGHTS IN A COMPOUND OPTION MODEL

## 1.1 Introduction

The land-use right, the right to use the land without ownership, is an important form of land usage and is ubiquitous all over the world. For example, it has been the major form of land usage in China for the last three decades. In fact, few developers construct exclusively on land under their ownership. Instead, they obtain a land-use right from the land owner by leasing or amortizing, and developers sometimes share profits with land owners as a part of the compensation. Obtaining a land-use right allows a developer to start construction at his choice of time and blocks out competition before construction. Therefore, it is indeed a compound real option: an option to reserve the right to choose when to start construction.

We model a land-use right as a compound real option in a continuous-time stochastic feedback control framework. Rather than modeling the cost of the land-use right as an up-front lump-sum payment, we model it as the running cost during the waiting period between obtaining the land-use right and the beginning of construction. As a result, the total accumulated amount of the running cost becomes a stochastic variable that interacts with the two investment decisions. We present a closed-form solution and use numerical examples to demonstrate the interaction between the running cost and the two investment decisions, among other relationships.

Our analysis implies that, as manifested in all real option analyses, the option value increases with a higher volatility of the underlying price. A higher volatility also delays both the decision to obtain a land-use right and the decision to begin construction. Interestingly, a change in the running cost affects the two decisions simultaneously. In particular, a lower running cost encourages the developer to acquire a land-use right earlier and to delay the construction decision, while a higher running cost would squeeze the two decisions together. When the running cost is high, the developer would obtain the right and start construction immediately. However, if the running cost is too high, it may never be optimal to obtain the right.

We model convenience yield as two separate values before and after the beginning of construction due to the nature of land development. The convenience yield after construction begins has a similar effect as the running cost: a lower convenience yield leads to higher option values and separated investment decisions. On the other hand, the convenience yield before construction begins affects only the decision to obtain the land-use right, leaving the decision to construct unchanged.

Land development has been one of the most researched applications of real option theories because much of the land value is derived from embedded real options. Following seminal papers of Brennan and Schwartz (1985), McDonald and Siegel (1986), Dixit and Pindyck (1994) and others, many scholars have studied real option value and the investment decision in real estate development, both analytically and empirically. For example, Williams (1991) analyzed the real option value and investment decision when developers have the flexility to choose when to start construction as well as the density of the construction. Grenadier (1996) developed the market equilibrium for multiple competitors and explained the bursts in construction activity. Empirically, Leishman et al. (2000) and Capozza and Li (2001) demonstrated the real option effect in construction by testing the investment response to interest rate changes.

In addition to this stream of literature, recent studies have explored the impact of individual options embedded in the development process. For example, Lai et al. (2004) and Buttimer et al. (2008) studied the effect of the pre-sale option as a risk management technique and demonstrated the risk transfer achieved by the pre-sale option. Chan et al. (2008) explored the impacts of pre-sale option on investment decisions with financing constraints. Leung et al. (2007), on the other hand, studied the risk-premium of the pre-sale option under asymmetric information.

However, few studies have investigated the land-use right as an additional development feature in a stochastic real options framework. Sheen (2004) investigated the land-use right as a compound real option on land and empirically tested the model using data from Shanghai, China. Yet, our model differs substantially from this work. First of all, they used a discrete time binomial model that describes a market with limited possible states for prices. In contrast, we examine the compound option in a rigorous continuous-time stochastic feedback control framework. Secondly, they modeled the cost of the land-use right as an upfront lump-sum cost, while we model the cost of the land-use right as a running cost that interacts with the optimal investment decision. Most importantly, they resorted to a backward numerical procedure for the analysis, while we obtain a closed-form solution of the model and demonstrate our results in various parameter settings.

The rest of this chapter is organized as follows: In Section 2, we describe the model and investigate various properties of the value function. In Section 3, we present the closed-form solution to the problem. Section 4 demonstrates the implication of our model using various numerical examples, and Section 5 concludes the study.

## 1.2 Model

We begin by describing the development process, which is shown in Figure 1.1. At time 0, a developer is considering the option to obtain, from the land owner, a land-use right to construct commercial or residential properties. Obtaining and keeping the land-use right incurs a running cost $c$, which buys him the right to start the construction any time. The construction finishes in a fixed duration $T_c$. Because $T_c$ is fixed, we do not lose any insight by incorporating the running cost after the beginning of construction into the holding cost and/or convenience yield. Upon completion of construction, the developer sells all the developed properties and receives a pre-negotiated proportion, $m$, of the purchase price. We assume that the developer can successfully sell all the properties at market price $P(\tau_2 + T_c)$ upon completion. Because $T_c$ is fixed, we simplify the problem by aggregating the present value of all construction costs at $\tau_2$ as $h$.

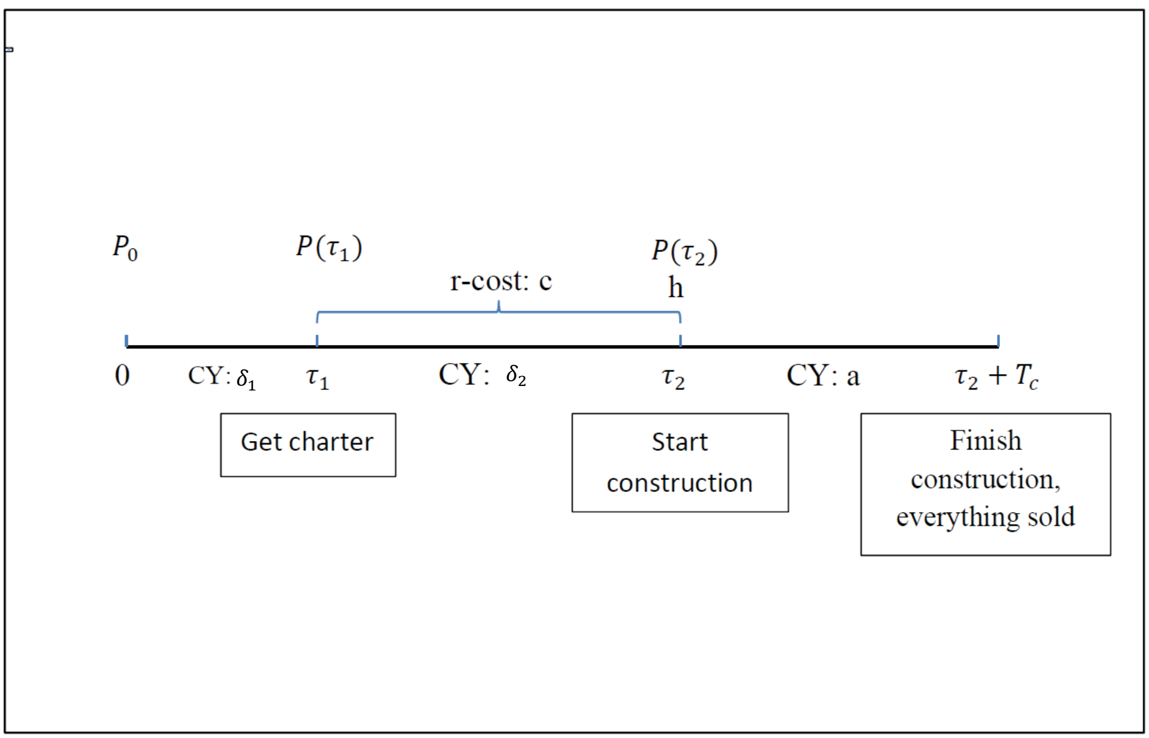



Figure 1.1: A time line of the compound option

This is a compound real option: by exercising the first option $F_1$ and obtaining the right from land owner at $\tau_1$, the developer takes a long position in the second option $F_2$, an option to start construction. The developer will choose the optimal time $\tau_2$ to start the construction, incurring construction costs along the way, and harvest the market value of the completed construction at $\tau_2 + T_c$.

For simplicity, we assume that this right can be renewed recurrently. Namely, as long as the developer is willing to pay the running cost, he can keep this project open forever. This assumption allows us to omit the time variable because the resulting option value and decision criteria remains the same even if the developer considers this option at a different time point. Namely, the options are perpetual.

We assume that the property price follows a geometric Brownian Motion:

$$dP/P = \alpha dt + \sigma dW_p,$$

We aggregate the convenience yield, the holding cost and running cost after the beginning of construction, and denote it as $a$. We denote the convenience yield before the exercise of the first and second options, *i.e.* before getting the land-use right and before construction begins, as $\delta_1$ and $\delta_2$ respectively. $\delta_1$ and $\delta_2$ are the opportunity costs of holding the options instead of exercising and holding the payoff. $\delta_1$ includes the implicit holding costs such as the research costs to keep the project in the development inventory, or monitoring costs to keep an eye on the competitors that could be saved by holding a land-use right. $\delta_2$ includes the convenience yield of being able to start construction at any time point and block out all other competitors, the inconvenience brought by holding the option-to-construct instead of actual buildings, constructed or under-construction, and the explicit costs of building infrastructure to ready the land for construction.

### 1.2.1 Second option $F_2$

We first solve for the value of the second option, and then use this to solve for the value of the first option. After risk adjustment, the drift rate of price process is $r-\delta_2$ before construction begins and $r-a$ after; and the discount rate is the risk-free rate $r$. The discounted expectation value of future revenue at the exercise time of the second option, $\tau_2$, is: $e^{-rT_c}\mathbf{E}_{\tau_2} mP(\tau_2+T_c)$. Since we aggregate the present values of all construction costs during $[\tau_2, \tau_2+T_c]$ into a single lump sum cost, $h$, at $\tau_2$, the payoff of $F_2$ is $\pi(P) = mP(\tau_2)e^{-aT_c} - h$.

The value of $F_2$ at time $\tau_2$ is then

$$\left[e^{-rT_c}\mathbf{E}_{\tau_2} mP(\tau_2+T_c) - h\right]^+ = \left(mP(\tau_2)e^{-aT_c} - h\right)^+.$$

The Bellman equation states that the value of this perpetual American option at any time is always the greater of the two choices: terminating by exercising the option and getting the payoff instantaneously, and continuing to incur the running cost plus the present value of the option at a later time point. That is, at any time $t$ and price $P$:

$$F_2(P,t) = \max\left\{\pi(P,t), \quad -cdt + e^{-rdt}\mathbf{E}[F_2(P+dP, t+dt)|P(\tau_2)]\right\},$$

where the first branch in maximization is the termination payoff, and the second branch is the continuation payoff. Because this is a perpetual option, the time variable can be suppressed:

$$F_2(P) = \max\left\{\pi(P), \quad -cdt + e^{-rdt}\mathbf{E}[F_2(P+dP)]\right\}.$$

Expanding the second branch by Itô's Lemma, and subtract both sides of the equation by $F_2(P)$ to get the return, we get:

$$0 = \max\{\pi(P) - F_2(P), \quad \frac{1}{2}\sigma^2 P^2 F_2''(P) + (r - \delta_2) P F_2'(P) - rF_2(P) - c\}. \qquad (1.1)$$

### 1.2.2 First option $F_1$

The payoff of $F_1$, the option to obtain land use right, is the difference between discounted value of option to construct $F_2$ and the present value of expected total running cost, which in turn depends on how long the developer holds the right before construction. At $\tau_1$, the expected present value of this running cost is

$$\mathbf{E}_{\tau_1}\left[\int_{\tau_1}^{\tau_2} ce^{-r(s-\tau_1)} ds\right].$$

Therefore, the payoff of $F_1$ evaluated at $\tau_1$ is:

$$\mathbf{E}_{\tau_1} F_2 e^{-r(\tau_2-\tau_1)} - \mathbf{E}_{\tau_1}\left[\int_{\tau_1}^{\tau_2} ce^{-r(s-\tau_1)} ds\right].$$

Substituting the value of option $F_2$ into the above formula, the payoff of $F_1$ becomes:

$$\mathbf{E}_{\tau_1}\left[e^{-r(\tau_2-\tau_1)}\left(mP(\tau_2)e^{-aT_c} - h\right)^+ - \int_{\tau_1}^{\tau_2} ce^{-r(s-\tau_1)} ds\right].$$

Before the exercise of $F_1$, Bellman equation indicates that the compound option value is the maximum between the terminating value, payoff of $F_1$, and the continuation value, the discounted future $F_1$ value:

$$
\begin{aligned}
F_1(P) = \max\Bigg\{ & \mathbf{E}_{\tau_1}\left[e^{-r(\tau_2-\tau_1)}\left(mP(\tau_2)e^{-aT_c}-h\right)^+ - \int_{\tau_1}^{\tau_2} ce^{-r(s-\tau_1)}ds\right], \\
& (1+rdt)^{-1}\mathbf{E}[F_1(P+dP,t+dt)|P(\tau_1)]\Bigg\}.
\end{aligned} \tag{1.2}
$$

Solving the above Bellman equation involves evaluating a expectation of a convex function of two optimal stopping time, $\tau_1$ and $\tau_2$, which is difficult as indicated by Jensen's inequality. Following Asmussen and Taksar (1997), we argue that the exercise of $F_1$ is in fact a transform from the continuation region of $F_1$ to the continuation region of $F_2$. As a result, we circumvent the evaluation of expected time duration between two stopping time and obtain the solution from the following HJB equation:

$$
\begin{aligned}
0 = \max\{ & \frac{1}{2}\sigma^2P^2F_2''(P)+(r-\delta_2)PF_2'(P)-rF_2-c, \\
& \frac{1}{2}\sigma^2P^2F_1''(P)+(r-\delta_1)PF_1'(P)-rF_1\}.
\end{aligned} \tag{1.3}
$$

### 1.2.3 Value function and its properties

For a given pair of stopping times $(\tau_1,\tau_2)$ and current market price for new construction, $P$, the value of the performance functional $J$ at time 0 is:

$$
\begin{aligned}
& J(P,\tau_1,\tau_2) \\
= \; & \mathbf{E}_0 e^{-r\tau_1}\mathbf{E}_{\tau_1}\left[e^{-r(\tau_2-\tau_1)}\left(mP(\tau_2)e^{-aT_c}-h\right)^+ - \int_{\tau_1}^{\tau_2} ce^{-r(s-\tau_1)}ds\right] \\
= \; & \mathbf{E}_0 e^{-r\tau_1}\mathbf{E}_{\tau_1}\Bigg[e^{-r(\tau_2-\tau_1)}\left(mP(\tau_1)e^{(r-\delta_2-\sigma^2/2)(\tau_2-\tau_1)}e^{\int_{\tau_1}^{\tau_2}\sigma dW_P}e^{-aT_c}-h\right)^+ \\
& - \int_{\tau_1}^{\tau_2} ce^{-r(s-\tau_1)}ds\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathbf{E}_0\left[e^{-r\tau_2}\left(mPe^{(r-\delta_1-\sigma^2/2)\tau_1}e^{(r-\delta_2-\sigma^2/2)(\tau_2-\tau_1)}e^{\int_0^{\tau_1}\sigma dW_P}\mathbf{E}_{\tau_1}\left[e^{\int_{\tau_1}^{\tau_2}\sigma dW_P}\right]-h\right)^+\right.\\
&\quad\left.-\int_{\tau_1}^{\tau_2} ce^{-rs}ds\right]\\
&= e^{-r\tau_2}\left(mPe^{(r-\delta_2-\sigma^2/2)\tau_2+(\delta_2-\delta_1)\tau_1}\mathbf{E}_0\left[e^{\int_0^{\tau_2}\sigma dW_P}\right]-h\right)^+ - \int_{\tau_1}^{\tau_2} ce^{-rs}ds.
\end{aligned}
$$

Then the value function of this compound option is:

$$
\begin{aligned}
V(P) &= \sup_{0\le\tau_1\le\tau_2} J(P,\tau_1,\tau_2)\\
&= \sup_{0\le\tau_1\le\tau_2} e^{-r\tau_2}\left(mPe^{(r-\delta_2-\sigma^2/2)\tau_2+(\delta_2-\delta_1)\tau_1}\mathbf{E}_0\left[e^{\int_0^{\tau_2}\sigma dW_P}\right]-h\right)^+ - \int_{\tau_1}^{\tau_2} ce^{-rs}ds,
\end{aligned}
$$

In a feedback control framework, we argue that the optimal exercising criteria can be represented by a critical price point for each option: $P_1^*$ and $P_2^*$. This requires that the difference between continuation value and termination value be monotone. In our case, one can imagine that a higher price would make exercising (termination) more attractive, therefore continuation value minus termination value should be monotonic decreasing.

We show that the value function demonstrate the following properties.

1. The value function is non-decreasing.

   Suppose $P_1$ and $P_2$ are two possible current market prices of the property, and $P_1 \ge P_2$. Further suppose that $(\tau_1', \tau_2')$ and $(\tau_1'', \tau_2'')$ are the optimal exercise time point sets that correspond to $P_1$ and $P_2$ respectively. Then:

$$
\begin{aligned}
V(P_1) &= J(P_1;\tau_1',\tau_2') \ge J(P_1;\tau_1'',\tau_2'')\\
&= e^{-r\tau_2''}\left(mP_1e^{(r-\delta_2-\sigma^2/2)\tau_2''+(\delta_2-\delta_1)\tau_1''}\mathbf{E}_0\left[e^{\int_0^{\tau_2''}\sigma dW_P}\right]-h\right)^+ - \int_{\tau_1''}^{\tau_2''} ce^{-rs}ds\\
&\ge e^{-r\tau_2''}\left(mP_2e^{(r-\delta_2-\sigma^2/2)\tau_2''+(\delta_2-\delta_1)\tau_1''}\mathbf{E}_0\left[e^{\int_0^{\tau_2''}\sigma dW_P}\right]-h\right)^+ - \int_{\tau_1''}^{\tau_2''} ce^{-rs}ds
\end{aligned}
$$

$$= \quad J(P_2, \tau_1'', \tau_2'') = V(P_2)$$

Therefore, the value function $V(P)$ is non-decreasing.

2. The value function is measurable.

   Since the value function is non-decreasing, it is measurable.

3. The value function is convex. Namely, $V(x)$ satisfies :

$$V\left(\frac{x+y}{2}\right) \le \frac{V(x)+V(y)}{2}.$$

   Suppose for two different current market situations $P_1$ and $P_2$ ($P_1 \ge P_2$), there are two sets of optimal exercising time sets $(\tau_1', \tau_2')$ and $(\tau_1'', \tau_2'')$. *i.e.* $V(P_1) = J(P_1, \tau_1', \tau_2')$, and $V(P_2) = J(P_2, \tau_1'', \tau_2'')$. Moreover, suppose the optimal exercising time set is $(\tau_1, \tau_2)$ if the current market price is $(P_1+P_2)/2$.

   Then

$$\begin{aligned}
&\frac{V(P_1)+V(P_2)}{2} \\
=\ & \frac{1}{2}\left(J(P_1,\tau_1',\tau_2') + J(P_2,\tau_1'',\tau_2'')\right) \\
\ge\ & \frac{1}{2}\left(J(P_1,\tau_1,\tau_2) + J(P_2,\tau_1,\tau_2)\right) \\
=\ & \frac{1}{2}\left(e^{-r\tau_2}\left(mP_1 e^{(r-\delta_2-\sigma^2/2)\tau_2+(\delta_2-\delta_1)\tau_1}\mathbf{E}_0\left[e^{\int_0^{\tau_2}\sigma dW_P}\right]-h\right)^+ - \int_{\tau_1}^{\tau_2} ce^{-rs}ds\right) \\
&+\frac{1}{2}\left(e^{-r\tau_2}\left(mP_2 e^{(r-\delta_2-\sigma^2/2)\tau_2+(\delta_2-\delta_1)\tau_1}\mathbf{E}_0\left[e^{\int_0^{\tau_2}\sigma dW_P}\right]-h\right)^+ - \int_{\tau_1}^{\tau_2} ce^{-rs}ds\right) \\
\ge\ & e^{-r\tau_2}\left(m\frac{P_1+P_2}{2} e^{(r-\delta_2-\sigma^2/2)\tau_2+(\delta_2-\delta_1)\tau_1}\mathbf{E}_0\left[e^{\int_0^{\tau_2}\sigma dW_P}\right]-h\right)^+ - \int_{\tau_1}^{\tau_2} ce^{-rs}ds \\
=\ & V\left(\frac{P_1+P_2}{2}\right).
\end{aligned}$$

## 1.3 Solution

We obtain the solution by solving the two ODEs in equation (1.3) and using value matching conditions and smooth pasting conditions as boundary conditions.

$$0 = \frac{1}{2}\sigma^2 P^2 F_1''(P) + (r-\delta_1) P F_1'(P) - rF_1 \tag{1.4}$$

$$0 = \frac{1}{2}\sigma^2 P^2 F_2''(P) + (r-\delta_2) P F_2'(P) - rF_2 - c \tag{1.5}$$

These two ODEs share the form of the following characteristic function with two roots:

$$\frac{1}{2}\sigma^2 \beta(\beta-1) + (r-\delta_1)\beta - r = 0.$$

$\delta_2$ should substitute for $\delta_1$ for equation (1.5). Listed below are the roots of the characteristic functions, with $\gamma_{1,2}$ the roots for equation (1.4) and $\beta_{1,2}$ the roots for equation (1.5).

$$\gamma_1 = \frac{1}{\sigma}\left[-\left(\frac{r-\delta_1}{\sigma} - \frac{\sigma}{2}\right) + \sqrt{\left(\frac{r-\delta_1}{\sigma} - \frac{\sigma}{2}\right)^2 + 2r}\right] \tag{1.6}$$

$$\gamma_2 = \frac{1}{\sigma}\left[-\left(\frac{r-\delta_1}{\sigma} - \frac{\sigma}{2}\right) - \sqrt{\left(\frac{r-\delta_1}{\sigma} - \frac{\sigma}{2}\right)^2 + 2r}\right] \tag{1.7}$$

$$\beta_1 = \frac{1}{\sigma}\left[-\left(\frac{r-\delta_2}{\sigma} - \frac{\sigma}{2}\right) + \sqrt{\left(\frac{r-\delta_2}{\sigma} - \frac{\sigma}{2}\right)^2 + 2r}\right] \tag{1.8}$$

$$\beta_2 = \frac{1}{\sigma}\left[-\left(\frac{r-\delta_2}{\sigma} - \frac{\sigma}{2}\right) - \sqrt{\left(\frac{r-\delta_2}{\sigma} - \frac{\sigma}{2}\right)^2 + 2r}\right]. \tag{1.9}$$

The solutions for the two ODEs take the following form:

$$F_1(P) = A_1 P^{\gamma_1} + A_2 P^{\gamma_2} \tag{1.10}$$

$$F_2(P) = B_1 P^{\beta_1} + B_2 P^{\beta_2} - c/r. \tag{1.11}$$

To determine the coefficients of the two options, we consider the following conditions:

1. When price $P$ is zero, the two option value should both be zero. As a result, the coefficients for the negative power terms should be zero.

$$A_2 = 0, \qquad B_2 = 0.$$

2. Value matching: At the exercising thresholds for $F_1$ and $F_2$, $P_1^*$ and $P_2^*$ the option value should equal to the payoff when exercised. This means that:

   - at $P_1^*$,

$$F_1(P_1^*) = F_2(P_1^*), \tag{1.12}$$

   - at $P_2^*$

$$F_2(P_2^*) = \pi(P_2^*) = mP_2^* e^{-aT_c} - h. \tag{1.13}$$

3. Smooth pasting:

$$F_1'(P_1^*) = F_2'(P_1^*), \tag{1.14}$$

$$F_2'(P_2^*) = me^{-aT_c}. \tag{1.15}$$

From (1.13) and (1.15), we can get

$$P_2^* = \frac{e^{aT_c}}{m}\left(h - \frac{c}{r}\right)\frac{\beta_1}{\beta_1 - 1} \tag{1.16}$$

$$B_1 = (me^{-aT_c})^{\beta_1}\left(h - \frac{c}{r}\right)^{(1-\beta_1)}\frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1}}. \tag{1.17}$$

From (1.12) and (1.14), we can get:

$$A_1 P_1^{*\gamma_1} = B_1 P_1^{*\beta_1} - \frac{c}{r} \tag{1.18}$$

$$A_1 \gamma_1 P_1^{*(\gamma_1 - 1)} = B_1 \beta_1 P_1^{*(\beta_1 - 1)}. \tag{1.19}$$

Solving these two equations, we get:

$$P_1^* = \left(-\frac{c}{r}\cdot\frac{\gamma_1}{\beta_1 - \gamma_1}\right)^{\frac{1}{\beta_1}}(B_1)^{-\frac{1}{\beta_1}} \tag{1.20}$$

$$A_1 = \left(-\frac{c}{r}\cdot\frac{\gamma_1}{\beta_1 - \gamma_1}\right)^{\frac{\beta_1 - \gamma_1}{\beta_1}}\cdot\frac{\beta_1}{\gamma_1}\cdot(B_1)^{\frac{\gamma_1}{\beta_1}}. \tag{1.21}$$

where $B_1$ is solved above.

As a result, the optimal exercising time for $F_1$ and $F_2$ can be represented as the following first hitting time:

$$\tau_1(P) = \inf\{t \in T | P = \left(-\frac{c}{r}\cdot\frac{\gamma_1}{\beta_1 - \gamma_1}\right)^{\frac{1}{\beta_1}}(B_1)^{-\frac{1}{\beta_1}}\} \tag{1.22}$$

$$\tau_2(P) = \inf\{t \in T | P = \frac{e^{aT_c}}{m}\left(h - \frac{c}{r}\right)\frac{\beta_1}{\beta_1 - 1}\}, \tag{1.23}$$

where $T$ is the time space.

Notice that $P_1^*$ is a power function, with its power the reciprocal of $\beta_1$. $\beta_1$ is an increasing function on the convenience yield $\delta_2$ with minimum value 1 (when $\delta_2 = 0$), hence its reciprocal $1/\beta_1$ is always between 1 and 0. Therefore the first bracket $\left(-\frac{c}{r} \cdot \frac{\gamma_1}{\beta_1 - \gamma_1}\right)$ has to be positive in order for $P_1^*$ to have meaningful value and not be imaginary. As a result, $\beta_1 < \gamma_1$, and $\delta_2 < \delta_1$ has to hold.

Intuitively, $\delta_1$ is higher because reserving the land use right brings many benefits including blocking out the competitors and the ability to start construction at any time point, while costing only the minimum research effort to keep the project open. After the developer starts the construction, however, the aggregate convenience yield/holding cost decreases to $\delta_2$ to reflect the increased holding cost of building under construction.

## 1.4 Numerical examples

### 1.4.1 Base case

In Figure 1.2, we show graphically the analytical solution of the compound option values $F_1$ and $F_2$ as well as their optimal exercising points $P_1^*$ and $P_2^*$, in our base case scenario. Following Buttimer et al. (2010), we set the risk-free rate $r = 0.05$. We set the aggregate convenience yield/holding cost $\delta_1$ between the charter registration and the beginning of construction to be 0.04. This is relatively high because, while the land use right costs only the minimum research effort to keep the project open (excluding the explicitly modeled running cost $c$), it brings many benefits including blocking out the competitors and ability to start construction at any time point. Therefore, the opportunity of not keeping the option open and not get the land use right is relatively high. After the developer starts the construction, however, the aggregate convenience yield/holding cost $\delta_2$ drops to 0.02. We set the Running cost $c$ to be 1; volatility $\sigma$ to be 15%, construction duration $T_c$ to be one year, present value (at beginning of construction) of construction cost $h$ to be 100. Since

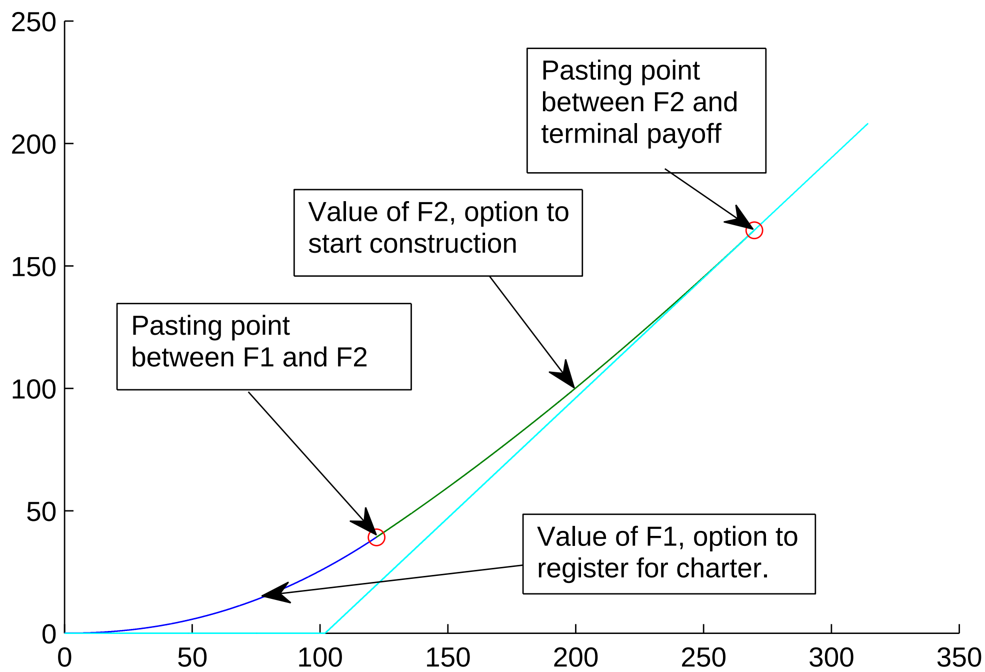



Figure 1.2: Plot of the value function in base case scenario.
X-axis is the current market price of the properties. Y-axis is the value of options. Parameter settings are as following: $\delta_1 = 0.04, \delta_2 = 0.02$. $a = 0.01$ ; $r = 0.05$. $\sigma = 0.15$, $T_c = 1$, $h = 100$, $c = 1$. The first optimal exercising happens at $P_1^* = 120.93$, while the second optimal exercising at $P_2^* = 267.17$.

changing the proportion value $m$ is equivalent to changing the scale of price $P$, we set it as 1.0 for the base case. X-axis is the current market price of the properties. Y-axis is the value of options.

Figure 1.2 manifests that the value function of the compound option is non-decreasing and convex. Notice that in this setting, even if the construction takes only one year to finish and the developer is sure to sell all the properties upon finish, he would require a current price of more than the current construction cost $h = 100$ to obtain the land use right. Construction decision happens when the current property price is about 2.7 times all the construction cost, at $P_2^* = 267.17$.

### 1.4.2 Comparisons

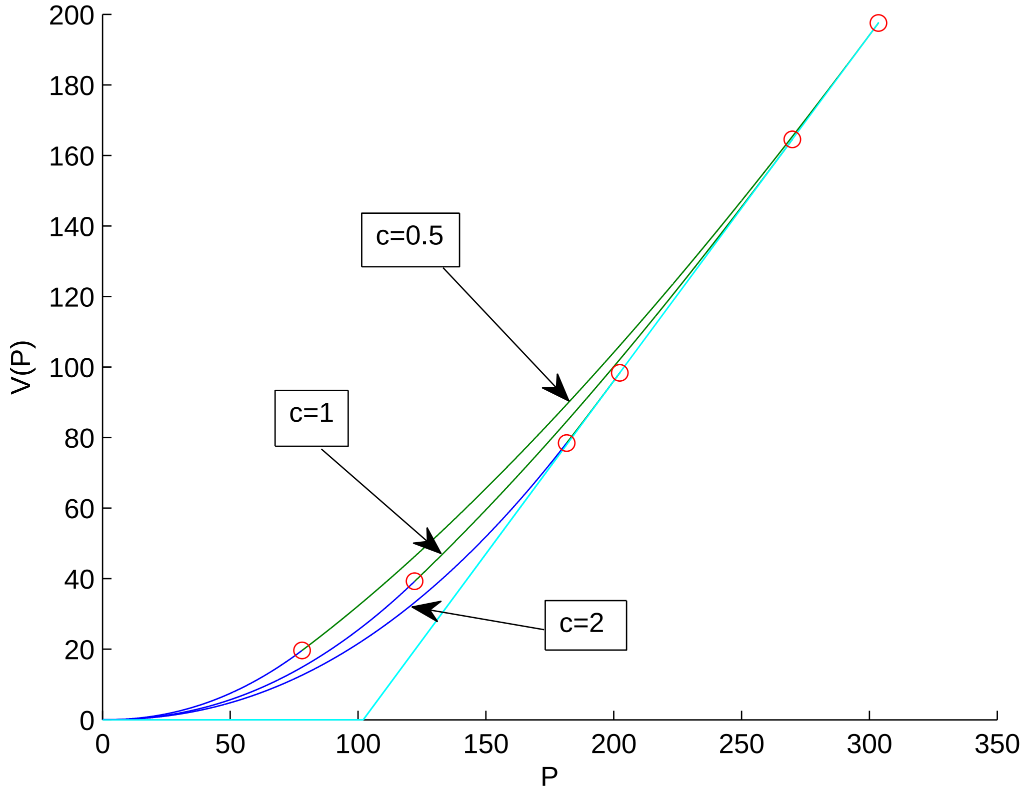



Figure 1.3: Plot of value function for different c values.
Plot of value function for different c values: $c = 0.5, 1, 2$. X-axis is the current market price of the properties, $P$. Y-axis is the value of options, $V(P)$. Other parameter settings are as following: $\delta_2 = 0.02$ $\delta_1 = 0.04$ $a = 0.01$; $r = 0.05$. $\sigma = 0.15$, $T_c = 1$, $h = 100$.

First, we examine how the running cost affects the compound option value and the decision points. Figure 1.3 shows the value function for various running cost settings. From top to bottom, the three lines shows the value function for $c = 0.5, 1, 2$ respectively. A higher running cost would cut down the option value as well as change the decision points substantially. In particular, for a higher running cost, the developer requires a higher property current price to be willing to obtain the land use right, but a lower property price trigger to start construction. Namely, the developer is more reluctant to obtain the land-use right, and would hasten the start of construction.

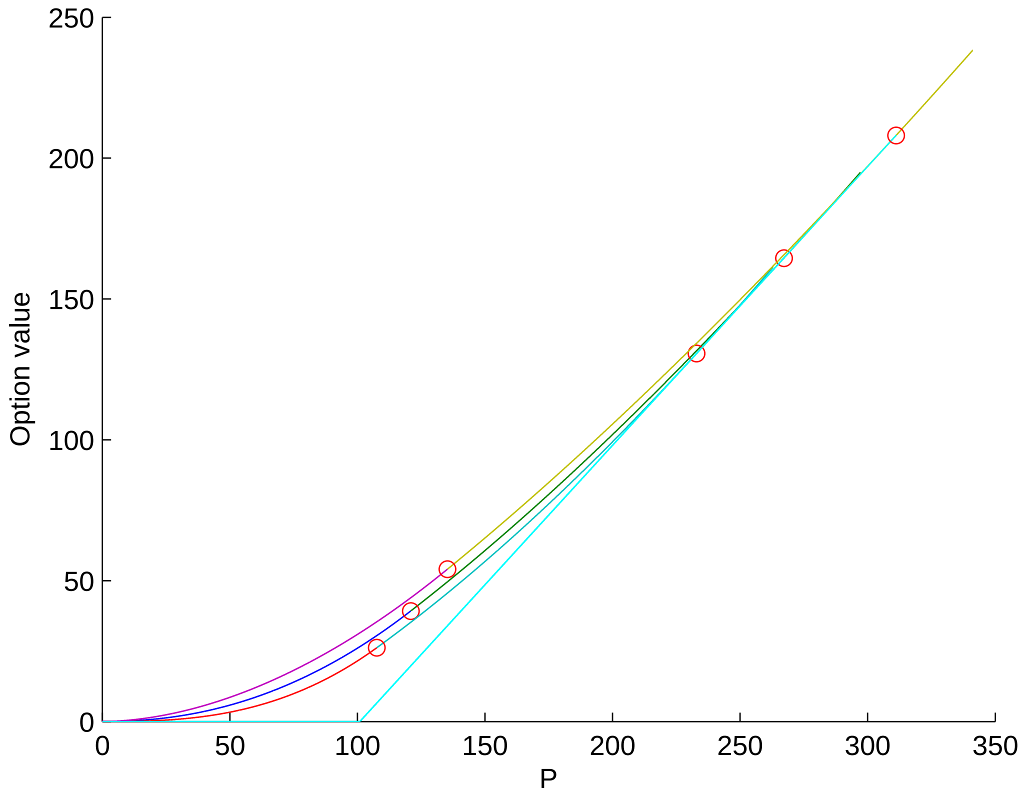



Figure 1.4: Plot of value function for different volatility

Plot of value function for different volatility: $\sigma = 0.10, 0.15, 0.20$, $(T_c = 1)$ from top to bottom. X-axis is the current market price of the properties, $P$. Y-axis is the value of options, $V(P)$. Other parameter settings are as following: $\delta_2 = 0.02$, $\delta_1 = 0.04$ $a = 0.01$, $r = 0.05$, $h = 100$, $c = 1$, $T_c = 1$.

Second, we look at the changes brought by varying price volatility $\sigma$ and the construction duration $T_c$ in Figure 1.4. As expected, the option values increase with a higher volatility value. The decision points for both options are pushed higher. That is, in a more volatile market, developers would delay both obtaining the land use right and starting construction. Moreover, the delay in obtaining right is less significant than the delay in construction.

Third we look at the impact of the convenience yields $\delta_1$ and $\delta_2$. A lower holding cost/convenience yield $\delta_2$ after obtaining the land use right indicates higher benefit of hold-

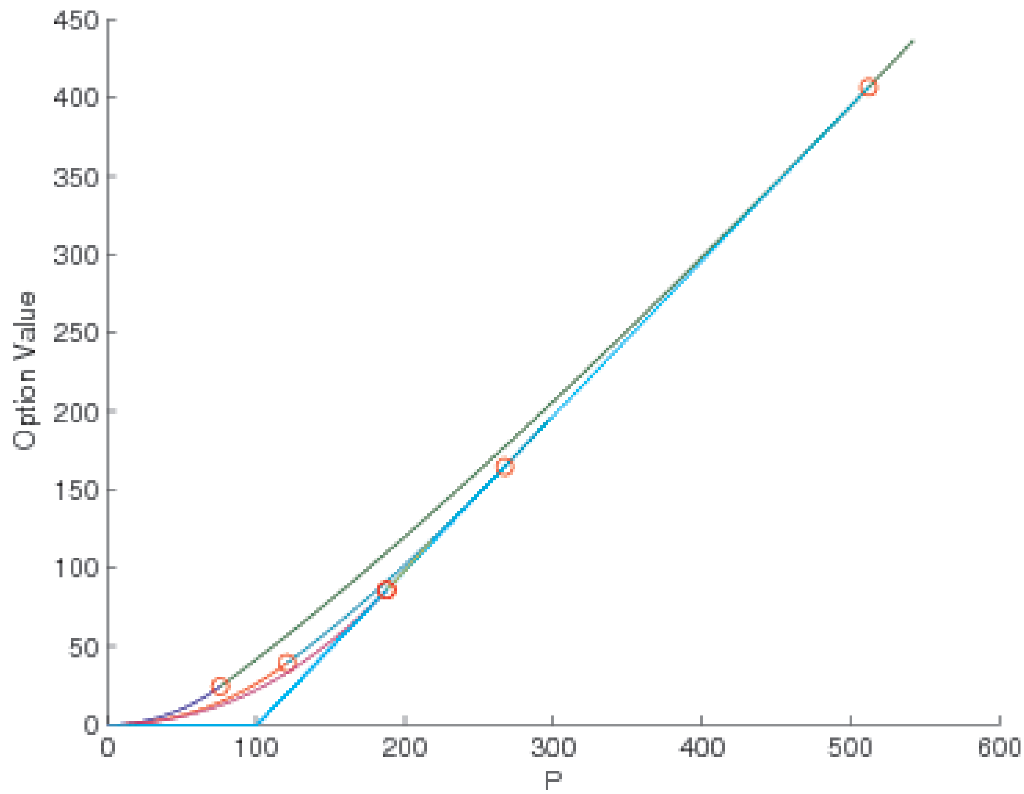



Figure 1.5: Plot of value function for different convenience yield
Plot of value function for different convenience yield $\delta_2$ while fixing $\delta_1$. From top to bottom, three lines depict $\delta_2 = 0.01, 0.02, 0.03$ respectively. X-axis is the current market price of the properties, $P$. Y-axis is the value of options, $V(P)$. Other parameter settings are as following: $\delta_1 = 0.04$, $r = 0.05$, $\sigma = 0.15$ $T_c = 1$, $h = 100$, $c = 1$, and $a = 0.01$

ing the right, hence significantly lengthens the wait until construction by changing both option exercising points. In particular, it hastens the purchase of the land-use right and delays the start of construction. We can see in Figure 1.5 that, in the case of highest $\delta_2$ value of the three, $\delta_2 = 0.03$, the two option exercise points are almost squeezed together. That is, if the benefit of holding the land use right it high enough, the developers would prefer to obtain the land-use right early, and keep the option to construct open for a longer period. On the other hand, if the land use right do not bring much convenience yield,the developers will obtain the land-use right and start construction almost simultaneously. In Figure 1.6

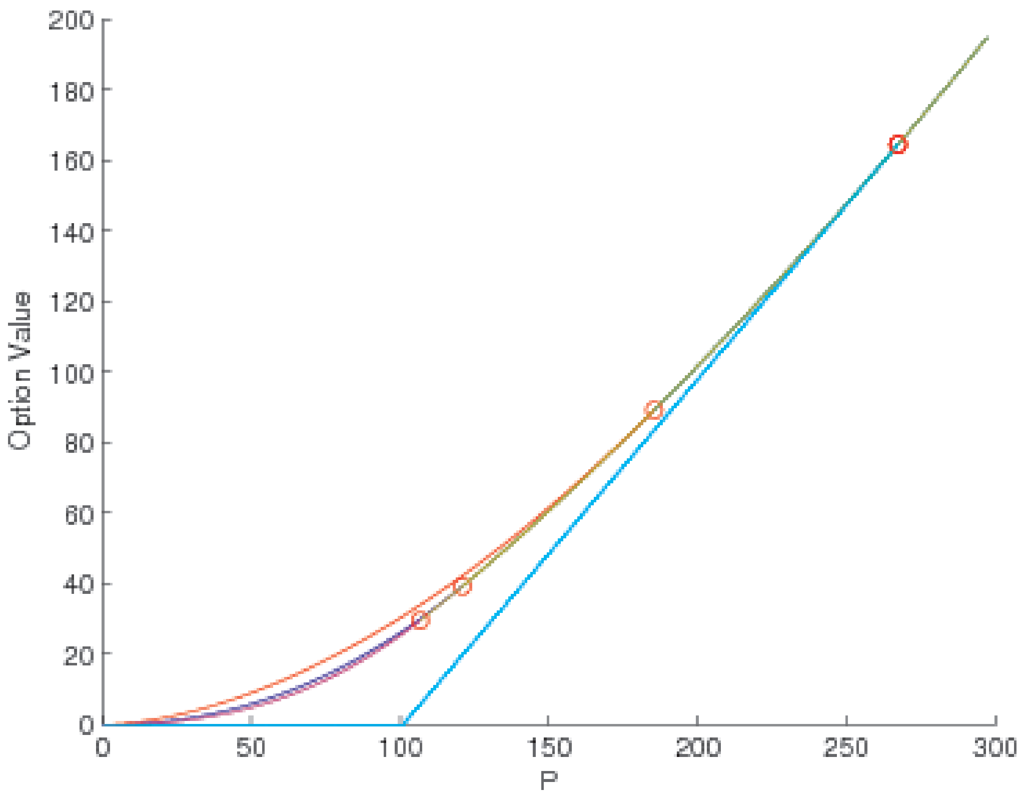



Figure 1.6: Plot of value function for different convenience yield
Plot of value function for different convenience yield $\delta_1$ while fixing $\delta_2$. From top to bottom, three lines depict $\delta_1 = 0.03, 0.04, 0.045$ respectively. X-axis is the current market price of the properties, $P$. Y-axis is the value of options, $V(P)$. Other parameter settings are as following: $\delta_2 = 0.02$, $a = 0.01$, $r = 0.05$, $\sigma = 0.15$ $T_c = 1$, $h = 100$, $c = 1$.

we can see that a lower first-stage convenience yield $\delta_1$ delays the registration for charter, while leaving the construction time point unchanged. In both graphs a higher convenience yield corresponds to a higher option values, because it lowers the discount rate.

Last, we see how the construction cost $h$ affects the option value and decision points (Figure 1.7). From left to right, the three sets of graphs depict the value function and decision points at $h = 100, 300,$ and 500 respectively. An increase in construction cost does not affect the decision of land-use right purchase, but delays the construction substantially.

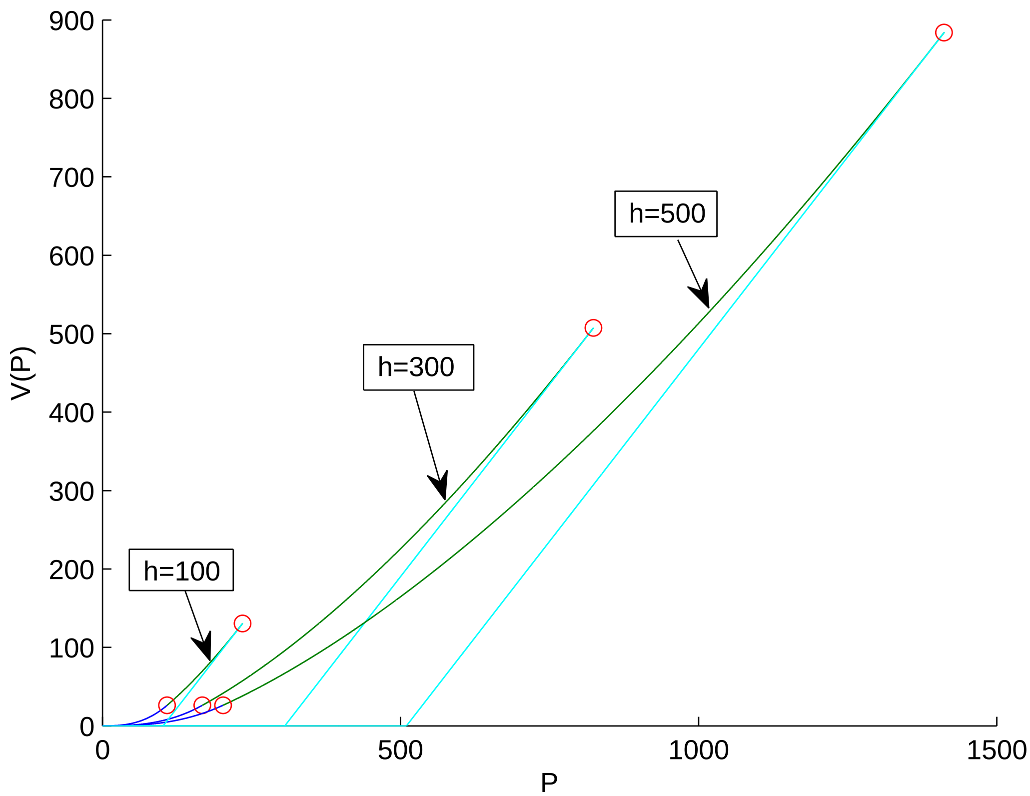



Figure 1.7: Plot of value function for different construction cost
Plot of value function for different construction cost $h$ with a fixed running cost $c$: $h = 100, 300, 500$, from left to right. X-axis is the current market price of the properties, $P$. Y-axis is the value of options, $V(P)$. Other parameter settings are as following: $a = 0.01$, $\delta_2 = 0.02$, $\delta_1 = 0.04$, $r = 0.05$, $\sigma = 0.15$ $T_c = 1$, $c = 1$.

## 1.5 Model Application: Hydraulic Fracturing

Hydraulic fracturing technique has been used to collect natural gas for more than fifty years. The whole process consists of the following the steps. First, a long concrete pipe is extended into the ground layer that contains the natural gas. Then local explosions are conducted at the end of the pipe extending horizontally along the shale layer to ``fracture'' the rock and to allow the natural gas to move freely from the rock pores. Finally, the ``hydraulic'' chemical mixture is pumped into the rock layer through the pipe, pushing the natural gas to the producing well, where it is collected.

This highly economic process produces some 30% of the total energy supply in the U.S, according to a report by the Independent Petroleum Association of America issued in 2008, making a great contribution to the relatively low energy price in the U.S. However, the process has raised environmental concerns and heated discussion in the social media.

Supporters of Hydraulic Fracturing practice claim that the process is environmentally safe and economically efficient. It is environmentally safe because drinking water locates at a different level of the ground, separated from the fracturing layer by a buffer layer of hard rock/shale that is not affected by fracturing. All pipes going through the water layer are fortified by concrete and steel structure to prevent leakage. It is so economically efficient that less oil/gas wells need to be drilled, and the oil/gas price can be kept low and affordable.

On the other hand, the protestor's concerns are multifold. First, they fear that the fracturing process would intoxicate the drinking water. One concern is that the fracturing (``or fracking"), namely the local explosions along the shale, may shake the earth enough that the buffer layer above is also affected allowing the toxic hydraulic mixture to leak through the buffer layer into water level. The other major concern is about the concrete/steel pipe. It maybe improperly installed, or it could wear down over the years and let the hydraulic mixture leak into water.

Second, they are concern that the large scale shale fracturing will change the ground structure over the years, leading to more frequent earth quake that can be a direct threat to the local residents' living environment. The indirect threat can be equally hazardous. Even the smaller scale earth quake that does not produce much casualty can still form cracks between the rock layers that unknowingly leak hydraulic mixture into the water. In this sense, the latent threat of hydraulic mixture is compared to bombs under the ground.

Finally, in west America where water is scarce, the water usage in hydraulic fracturing also compete directly with the agriculture usage. Above all, the culprit of the whole problem is the lack of proper regulation. Current regulation imposes no standard on the company's environmental compliance report, namely the company voluntarily reports what they think is appropriate.

Since the horizontal fracturing process in hydraulic fracturing affects a wide local area that does not belong to the oil company, common practice of oil company is to collect land use rights from local residents before the construction of the well. This separate stage of collecting land use rights can be approximated by the running cost stage in our model for two reasons. First, the cost of collecting and maintaining the land use rights occurs throughout this period. Second, overall the running cost of this stage is approximately level. At the beginning, the cost of collecting land use right through negotiation with local residents is higher, while towards the end of this stage the cost of maintaining the existing land use rights is higher.

On the other hand, given the current technology in hydraulic fracturing practice, the oil reserve in the potential well can be accurately predicted, leaving price uncertainty of gas/oil the only uncertainty. Therefore, it is reasonable to model the fracturing process with underlying being the stochastic oil/gas price process as in our main model. The oil company's option here is to choose the best timing to begin collecting land use rights and to begin constructing the well, facing the uncertain oil/gas price.

Our model predicts that the company will start the running cost stage early if the benefit of blocking out the competitors is high and the running cost is relatively low compared to the project value. Since there is a high benefit of blocking out the competitors and a relatively low cost of collecting and maintaining the land use right, compared to the value of gas/oil well, our model predicts that the optimal strategy for oil companies is to start collecting land use right early and delay the construction of the gas/oil well. This is exactly what we see in the real world. In areas like Akra, Ohio, where the land price is low and land use right is relatively cheap, the on-going negotiation of land use right with the local residents often starts years ahead of the well construction.

## 1.6 Conclusion

We present a parsimonious continuous-time feedback control model that analyzes the land-use right as a compound real-option. We model the cost of obtaining a land-use right as a running cost, which makes total running cost a stochastic variable that interacts with the investment decision. We obtain a closed-form solution and demonstrate rich implications using numerical examples. Our model can be applied to many situations where the cost of one or both part of a compound option is a running cost. In particular, the option value increases and both decisions are delayed with a higher volatility of the underlying price. A lower running cost and a lower convenience yield have similar effect: they both lead to higher option values and pushes apart the investment decisions. Convenience yield before obtaining the land-use right affects only the decision to obtain the land-use right, whereas the convenience yield after getting land-use right affects both decisions simultaneously.

# CHAPTER 2: REAL OPTION MODEL OF ENTITLEMENT RISK

## 2.1 Introduction

The entitlement is the process of obtaining all approvals for the right to develop property for a desired use (Miles et al. (2007)). In the real world, the majority of real estate developers, especially land developers, have to obtain entitlement before they can start any construction. Unlike other stages in real estate development, the entitlement process is mostly out of control of the investors. Moreover, it has become increasingly costly and risky in the past ten years, due to many political and environmental concerns. (Kelley (2007) and Gyourko et al. (2008)). The significant increase in entitlement risk has gained an increasing amount of attention in the real estate literature. However, most of the existing real option models do not identify the entitlement process as a separate stage in development.

We model the entitlement process as a separate stage in a compound option model of real estate development. Our model emphasizes the lack of investors' control in two ways. First, we model the entitlement process as an European real option instead of the traditional perpetual American option. Fixing the expiry of the option takes away the investor's control on investment timing in an American option.

Second, we limit investors' control by introducing a random entitlement cost, which leads to an exogenous entitlement risk. In the real world, there are two representative sources of entitlement risk: one is the high running cost of hiring a ``dream team'' of various expertise during the uncertain waiting period, and the other is the sometimes ``wild'' requirements (eg. to build a school for potentially increased population) from the local municipalities in exchange for the entitlement (Miles et al. (2007)). In order to capture both sources of entitlement risk, we model the entitlement cost as a stochastic process whose value is revealed at the end of the entitlement stage. In essence, we model the entitlement risk as an uncertain cost over a fixed time period instead of a fixed cost over an uncertain waiting period.

This modification allows us to get a closed-form analytical solution with the effect of entitlement risk explicitly demonstrated. First, our solution predicts that a higher entitlement risk will induce the developers to start the entitlement process earlier. Indeed, when the entitlement process is completely out of control, the developers would start early to ensure that they have a good inventory of entitled land to capture the potential shock in market demand. This result is surprising compared to those of the traditional real option theory. Yet, it is consistent with the result of Grenadier (1995), Bar-Ilan and Strange (1996), and Bar-Ilan and Strange (1998) in that out-of-control risk hastens the investment decision.

Second, a higher building price volatility, by itself, would decrease the entitlement project value and delay the entitlement application. This is because a higher price volatility significantly delays the construction decision, therefore deeply discounting the payoff at the expiry of the entitlement option and lowering the probability that the investor would pay the entitlement cost.

In addition, our model allows the entitlement cost to be correlated with the developed property price. Therefore, it allows us to explore the interaction between the entitlement risk and the investors' decision. In reality, local municipalities often adjust the entitlement risk and waiting time to control the real estate market. For example, when the market has oversupply, the government slows down the entitlement process and decreases the entitlement success rate to discourage construction, creating an artificially negative correlation between land/house price and entitlement cost. This interaction between policy makers and land/house prices has been extensively studied. Turnbull (2005), Turner et al. (2011), and Ihlanfeldt (2007) provide evidence that stringent land-use regulation creates an undersupplied market. Our model allows us to look at this interaction from a dynamic perspective. Our solution predicts that the hastening effect of higher entitlement risk is most pronounced when the house market is relatively volatile and when the government control of entitlement artificially makes the entitlement cost and building price negatively correlated. Therefore, the government who wants to discourage the development activities by tightening the entitlement process may find more applications filed instead of waning construction.

This paper is related to the long stream of studies on real option value embedded in land development. Following seminal papers of Brennan and Schwartz (1985), McDonald and Siegel (1986), Dixit and Pindyck (1994) and others, recent studies have explored the impact of individual options embedded in the development process. As an early real option study that recognizes entitlement risk, Riddiough (1997) demonstrates that the political risk in the entitlement process can adversely affect the land value as well as development activity. Recently, Buttimer et al. (2010) developed a reduced-form land valuation model in which they explicitly model each phase of the general development process, identifying the entitlement risk as a distinct source of risk. Our model differs significantly from Buttimer et al. (2010) in that we model entitlement risk as a separate, European style real option in a compound option model of real estate development.

The rest of this chapter is organized as follows. In section two, we briefly describe the entitlement process. Section three presents the model and the analytical solution. In section four, we present a numerical analysis in various settings, and section five concludes.

## 2.2 Entitlement process

Entitlement in real estate development refers to the process of working with local regulators to obtain all the permits for development. This constant renegotiation process involves various parties, often begins right after the formation of a first tentative design, and lasts all the way until the beginning of actual construction. Sometimes, even after the construction begins, there can be changes in regulations that render the project totally undoable.

In the 1920s and 1930s, most states enacted legislation to grant the local government authority to regulate real estate development for purposes of health, safety and general welfare. Since then, the local government feels entitled to the right to regulate the land use. Nowadays, the government is asserting an ever increasing role in managing the development process. The entitlement process therefore has become more costly not only in time and effort, but also in the form of unpredictable requirements. Local municipalities may demand the developer or the land owner build extra infrastructure that is solely for benefit of the local public and is utterly irrelevant to the project itself.

Various interested groups also pressure the local official and community residents to restrain the development. Increasing environmental concern of the local organizations have stated the new higher requirement for development. The trend of the public opinion is gearing towards smart growth, new urbanism, transition-oriented development and green development. These higher and more complicated standards mean that more regional public agencies are stepping into the land development process to make additional demands on both local government and developers.

The hardened process of entitlement process has changed the development process vastly. Nowadays, more the entitlement cases are being handled by professional entitlement teams. In some area, entitlement process itself has become a highly risky investment process that yields great return as the difference between the entitled land and raw land. This trend also brings a cohort of multi-regional and national developers, who hold various entitlements in their inventory to meet the potential real estate market demand surge.

## 2.3 Model and solution

### 2.3.1 Model

We model the entitlement and development process as a multi-stage compound option. The scenario can be explained as following. The developer is considering initiation of a real estate project, for which many permits have to be obtained from the local jurisdiction. We call this whole process of obtaining various required permit the entitlement process. The developer can start the entitlement process any time. He makes the decision to start with the observation of two underlying state variables: 1) current entitlement cost, and 2) current market price of a finished property similar to the one he intends to build. We assume that the entitlement cost and the property price follow two correlated stochastic processes, with both of them geometric Brownian Motion for simplicity:

$$dP_t = (\mu_P - \delta_P)P_t dt + \sigma_P P_t dW_P, \tag{2.1}$$

$$dL_t = (\mu_L - \delta_L)L_t dt + \sigma_L L_t dW_L, \tag{2.2}$$

with $\rho$ the correlation between them: $dW_P dW_L = \rho dt$.

Figure 2.1 displays the whole process graphically. At $\tau_1$ the developer decides to start

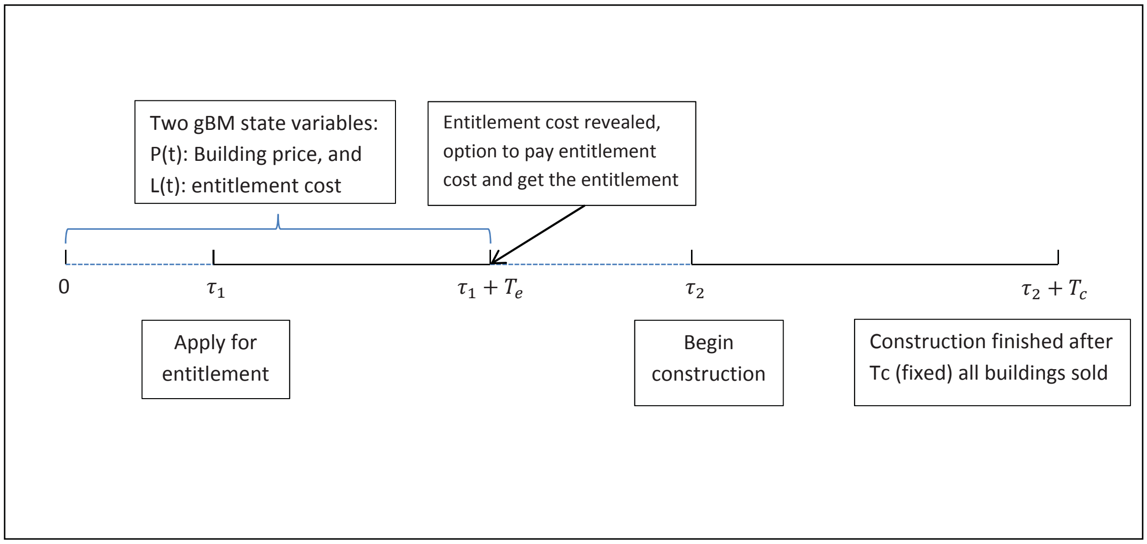



Figure 2.1: Timeline of the compound option.
Investor start the entitlement process by paying the fixed entitlement application cost upfront. After a fixed period, $T_e$, the stochastic entitlement cost is revealed. The investor then has the option to abandon or pay and get the entitlement, which is a perpetual option to construct. He chooses a time point $\tau_2$ to start construction, and finishes construction after fixed time period $T_c$, when all buildings are sold.

the entitlement process. He pays a part of entitlement cost $C$, a fixed cost, upfront but does not know the amount of remaining entitlement cost, the stochastic part, until the end of entitlement process. After a fixed time period $T_e$, the stochastic part of the entitlement cost is revealed. The developer then has the option to pay the stochastic part of the entitlement cost, $L$, and get the entitlement, or to abandon the project. If he gets the entitlement, then he has the option to start the development process any time in the future. During the whole process, there are two underlying state variables: the building price $P_t$ and the stochastic part of the entitlement cost $L_t$, although the entitlement cost is only relevant before the exercise of the entitlement option at $\tau_1 + T_e$.

Our model consists of three compound options: the option to start the entitlement process, the option at the end of the entitlement process to pay the revealed entitlement cost in exchange for the option to construct, and the option to construct the building at last. Here, we focus on the first two options.

Among the three options, the first option, the option to start the entitlement process, is a perpetual option. Since the situation is exactly the same at any point in time, the option value to start the entitlement process does not depend on time. On the other hand, the second option, entitlement option -- the option to pay the revealed entitlement cost and get the option to construct -- is a European option, as manifested by the fixed time period of entitlement process. The payoff of this option is the entitlement, which is essentially the option to begin construction at time of choice. The strike is the stochastic entitlement cost.

According to CAPM, the return of a non-tradable but replicable project should reflect only the systematic risk. Following Constantinides (1978), we first replace the drift $\mu$ by a risk-adjusted drift $\mu^*$, and then evaluate the compound option as if the market price of risk were zero.

$$\mu_P^* = (\mu_P - \phi\rho_{PM}\sigma_P) \tag{2.3}$$

$$\mu_L^* = (\mu_L - \phi\rho_{LM}\sigma_L), \tag{2.4}$$

where $\phi = (\alpha_M - r)/\sigma_M$ is the market price of risk, and $\rho_{PM}$ and $\rho_{LM}$ are the correlation between the building price and market, and entitlement cost and market, respectively.

At $\tau + T_e$ the developer pays for the entitlement, then he has a perpetual real option to start construction at time of his choice. The only state variable in this stage-- finished building price $P$ -- follows the a geometric Brownian Motion (GBM) with two different convenience yields: $b$ before the construction begins, and $a$ after the construction begins. At the optimal time point $\tau_2$ the construction begins. When it finishes after a fixed time period $T_c$, the developer sells portion $m$ of the building and get $mP(\tau_2 + T_c)$ in revenue.

When investors begin the entitlement application process, they often need to overcome an initial hurdle. For instance, this initial cost can be a fixed nominal application fee required at the beginning of the entitlement applications, or it can be the non-trivial cost to hire a professional team to prepare for the entitlement application. The initial fixed entitlement cost $c$ represents such a hurdle.

Although in reality the entitlement waiting time is often flexible, we model this unknown waiting-time to entitlement approval, the major risk in entitlement, as the uncertainty in entitlement cost instead of in time. Namely, we model entitlement costs as a stochastic cost $L$ that is not revealed until the end of entitlement cost, besides the fixed initial cost $c$. This stochastic entitlement cost represents more than just the real world entitlement cost. First of all, the real world entitlement itself is uncertain. It can be anything between a smooth standard procedure and a bumpy process that requires multiple re-design and re-filing. Moreover, more and more local communities try to assert themselves, and can sometimes make the entitlement negotiation unproductive for years. Therefore, both the time and monetary cost are uncertain in the entitlement process. Since the time cost eventually means monetary cost to the investors in the form of higher labor cost (of hiring the professional team for a longer period) and higher opportunity cost, we argue that our stochastic entitlement cost successfully captures both the time cost and the monetary cost. Secondly, this stochastic entitlement cost captures the possibility of entitlement failure since $L$ can be unbearably high. Such failure happens when the government demands extraordinary cost in exchange for the approval, often a certain facility to be built. Thirdly, with a stochastic entitlement cost coupled with a fixed waiting time, our model emphasize that the entitlement process is out of investor's control. Indeed, the investor cannot choose when the entitlement gets approved. Finally, our model also intrinsically allows the entitlement risk and the market building price to be correlated. This correlation is seen in many market conditions where the local municipality adjust the entitlement processing speed and success rate to help regulate the real estate market. After all, our parsimonious setting -- stochastic entitlement cost in fixed waiting time-- buys us the insight of interaction between entitlement risk and investor behavior with minimum distortion of reality.

### 2.3.2 Option to construct

Let's start with the last option, the option to construct, $F_c$. If the developer pays the entitlement cost, he would then have the option to start construction at any time of his choice. This is a perpetual real option, namely the value of this perpetual option does not depend on time.

When the developer exercises the construction option, (*i.e.* begins construction), he receives discounted expectation value of future revenue:

$$e^{-rT_c}\mathbf{E}_{\tau_2} mP(\tau_2 + T_c).$$

After risk adjustment, the drift rate of price process is $r - a$ after construction begins; and the discount rate is the risk-free rate $r$. For simplicity, we aggregate the present values of all construction costs during $[\tau_2, \tau_2 + T_c]$ into a single lump sum cost, $h$. Therefore, exercised at the optimal time point with optimal exercising price $P_2^*$, the option gives a payoff of $F_c(P_2^*) = mP_2^* e^{-aT_c} - h$. Denote the time between getting the entitlement and the beginning of construction as $\tau_2$. The value function of the construction option is then $e^{-r\tau_2}(mP_2^* e^{-aT_c} - h)^+$.

Since the return on building price $P$ should also follow CAPM, we take the risk-neutral measure and retain the convenience yield $\delta_P$. The ordinary differential equation that it satisfies is:

$$0 = \frac{1}{2}\sigma_P^2 P^2 F_c''(P) + (r-\delta_P)PF_c'(P) - rF_c$$

The characteristic function for this ODE is :

$$\frac{1}{2}\sigma_P^2 \beta(\beta-1) + (r-\delta_P)\beta - r = 0,$$

with two roots:

$$\beta_1 = \frac{1}{\sigma_P}\left[-\left(\frac{r-\delta_P}{\sigma_P} - \frac{\sigma_P}{2}\right) + \sqrt{\left(\frac{r-\delta_P}{\sigma_P} - \frac{\sigma_P}{2}\right)^2 + 2r}\right] \tag{2.5}$$

$$\beta_2 = \frac{1}{\sigma_P}\left[-\left(\frac{r-\delta_P}{\sigma_P} - \frac{\sigma_P}{2}\right) - \sqrt{\left(\frac{r-\delta_P}{\sigma_P} - \frac{\sigma_P}{2}\right)^2 + 2r}\right]. \tag{2.6}$$

And $\beta_1 > 0 > \beta_2$. The option value would take the form of :

$$F_c = AP^{\beta_1} + BP^{\beta_2}$$

Since the option value should not explode if the underlying property price is zero, the coefficient $B$ in the second term of option value should be zero. Namely,

$$F_c = AP^{\beta_1}.$$

When the developer exercises the construction option at a price $P_2^*$, he receives the payoff (value matching):

$$F_c(P_2^*) = AP_2^{*\beta_1} = mP_2^* e^{-aT_c} - h.$$

On the other hand, the changing rate of option value and the payoff function should match too (smooth pasting):

$$F_c(P_2^*) = A\beta_1 P_2^{*\beta_1 - 1} = me^{-aT_c}.$$

Solving these two equations together allows us to determine jointly $P_2^*$ and $A$:

$$P_2^* = \frac{\beta_1}{\beta_1 - 1}\frac{h}{m} \cdot e^{aT_c} \tag{2.7}$$

$$A = \left(\frac{m}{\beta_1}\right)^{\beta_1} \left(\frac{h}{\beta_1 - 1}\right)^{1-\beta_1} e^{-a\beta_1 T_c}. \tag{2.8}$$

### 2.3.3 Entitlement option as a power-exchange option

The entitlement process is an European option. The developer enters the option by applying for the entitlement at $\tau_1$, and decides if he wants to exercise this option -- pay the entitlement cost and get the construction option -- after a fixed time period $T_e$. The entitlement cost in this process is stochastic and follows a geometric Brownian Motion. Its actual value is not revealed until the end, $\tau_1 + T_e$.

Apparently the strike of the entitlement option, the entitlement cost $L$, is stochastic. The payoff is also stochastic: it is the beginning value of the construction option $AP_{\tau_1+T_e}^{\beta_1}$. Both of them are power functions of an underlying state variable. Therefore, the entitlement process is indeed a European power exchange option.

Blenman and Clark (2005) solved the power exchange option and gave the closed-form solution. In our case the payoff of the entitlement option is $(AP^{\beta_1}_{\tau_1+T_e} - L_{\tau_1+T_e})^+$, in which:

$$\beta_1 = \frac{1}{\sigma_P}\left[-\left(\frac{r-\delta_P}{\sigma_P} - \frac{\sigma_P}{2}\right) + \sqrt{\left(\frac{r-\delta_P}{\sigma_P} - \frac{\sigma_P}{2}\right)^2 + 2r}\right] \quad (2.9)$$

$$A = \left(\frac{m}{\beta_1}\right)^{\beta_1}\left(\frac{h}{\beta_1 - 1}\right)^{1-\beta_1} e^{-a\beta_1 T_c}. \quad (2.10)$$

Compare this payoff function with the one presented in Blenman and Clark (2005), and incorporate the risk adjustment in the CAPM fashion, we see that the two geometric Brownian Motion processes $S_1, S_2$ are replaced by

$$dP_t = (\mu_P - \phi\rho_{PM}\sigma_P - \delta_P)P_t dt + \sigma_P P_t dW_P, \quad (2.11)$$

$$dL_t = (\mu_L - \phi\rho_{LM}\sigma_L - \delta_L)L_t dt + \sigma_L L_t dW_L, \quad (2.12)$$

respectively, where $\phi$ is the market price of risk $\phi = (\alpha_M - r)/\sigma_M$. Without the convenience yield $\delta_P$ and $\delta_L$, by CAPM we can obtain $\mu_P - r = \phi\rho_{PM}\sigma_P$, which implies $\mu^* = r$. Then Eq. (2.11) becomes risk-adjusted processes:

$$dP_t = (r - \delta_P)P_t dt + \sigma_P P_t dW_P, \quad (2.13)$$

$$dL_t = (r - \delta_L)L_t dt + \sigma_L L_t dW_L, \quad (2.14)$$

Compare to the setting in Blenman and Clark (2005), then $\lambda_1 = A$, $\lambda_2 = 1$ $\alpha_1 = \beta_1$ and $\alpha_2 = 1$. Therefore, if we denote $t$ as the time elapsed from the beginning of entitlement application, the entitlement option value at time $0 < t < T_e$ with parameter $\xi = (r, \beta_1, 1, A, 1, \delta, 0)$ is:

$$PE(t, P, L, \xi; T) = \Upsilon_1(t, P, \xi; T_e)N(d_1) - \Upsilon_2(t, L, \xi; T_e)N(d_2),$$

where

$$d_1 = \frac{\ln\left(\frac{AP^{\beta_1}}{L}\right) + \left[\frac{1}{2}\beta_1^2\sigma_P^2 + \beta_1(r-\delta-\frac{\sigma_P^2}{2}) - (r-\delta_L-\frac{1}{2}\nu^2)\right](T_e-t)}{\nu\sqrt{(T_e-t)}}, \quad (2.15)$$

$$d_2 = \frac{\ln\left(\frac{AP^{\beta_1}}{L}\right) + \left[\frac{1}{2}\beta_1^2\sigma_P^2 + \beta_1(r-\delta-\frac{\sigma_P^2}{2}) - (r-\delta_L+\frac{1}{2}\nu^2)\right](T_e-t)}{\nu\sqrt{(T_e-t)}}, \quad (2.16)$$

and

$$\nu^2 = \beta_1^2\sigma_P^2 + \sigma_L^2 - 2\beta_1\rho\sigma_P\sigma_L \quad (2.17)$$

$$\Upsilon_1(t,P,\xi;T_e) = AP^{\beta_1}\exp\left\{\left[(\beta_1-1)r - \beta_1\delta_P - \beta_1(1-\beta_1)\frac{\sigma_P^2}{2}\right](T_e-t)\right\} \quad (2.18)$$

$$\Upsilon_2(t,L,\xi;T_e) = L\exp\left[-\delta_L(T_e-t)\right]. \quad (2.19)$$

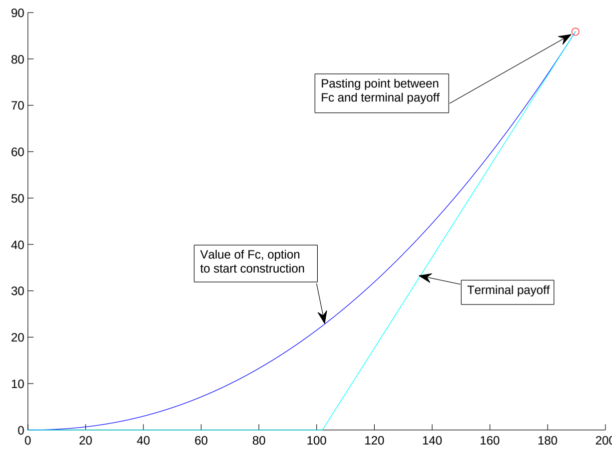
Decision point: the developer decides to apply for the entitlement if value of the power exchange option is greater than the initial fixed cost

$$PE(t,P,L,\xi;T) \geq c.$$

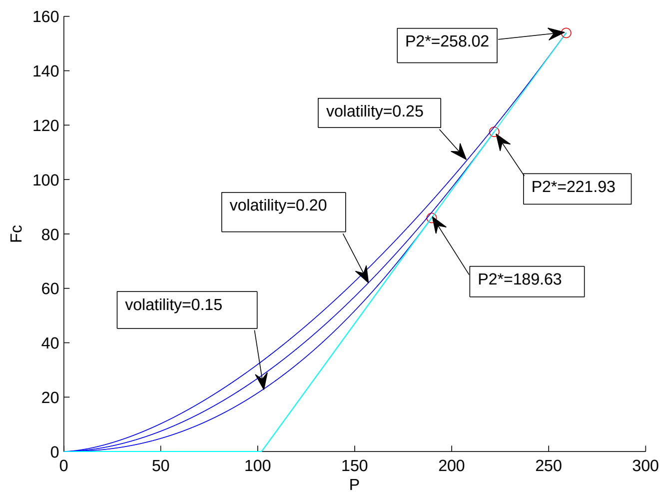
## 2.4 Numerical comparisons

### 2.4.1 Option to construct

In Figure 2.2, we show graphically the value the of construction option. Figure 2.2(a) shows the option value $F_c$ in base case, as well as their optimal exercising point $P_2^*$ in our base case scenario. Following Buttimer et al. (2010), we set the risk-free rate $r = 0.05$. We set the aggregate convenience yield/holding cost $\delta_P$ between the charter registration and the beginning of construction to be 0.04. After the developer starts the construction, however, the aggregate convenience yield/holding cost $a$ drops to 0.02 to reflect the increased holding




(a) Plot of construction option value in base case scenario. X-axis is the current market price of the properties. Y-axis is the value of option. Parameter settings are as following: $a = 0.02$ $\delta_P = 0.04$; $r = 0.05$. $\sigma = 0.15$, $T_c = 1$, $h = 100$. The optimal construction decision happens at $P_2^* = 189.63$




(b) Plot of construction option value for different volatility: $\sigma = 0.25, 0.20, 0.15$, from top to bottom respectively. X-axis is the current market price of the properties. Y-axis is the value of option. Other parameter settings are as following: $a = 0.02$ $b = 0.04$; $r = 0.05$. $\sigma = 0.15$, $T_c = 1$, $h = 100$. The optimal construction decision happens at $P_2^* = 189.63, 221.93, 259.02$, from lowest volatility to highest volatility respectively.

Figure 2.2: Plot of construction option value.

cost of building under construction. We set the volatility $\sigma$ to be 15%, construction duration $T_c$ to be one year, present value (at beginning of construction) of construction cost $h$ to be

100. Since changing the proportion value $m$ is equivalent to changing the scale of price $P$, we set it as 1.0 for the base case. X-axis is the current market price of the properties. Y-axis is the value of options.

Figure 2.2(a) manifests that the value function of the compound option is non-decreasing and convex. Notice that in this setting, even if the construction takes only one year to finish and the developer is sure to sell all the properties upon finish, he would require a current price of more than the current construction cost $h = 100$ to obtain the land use right. Construction decision happens when the current property price is about 1.9 times all the construction cost, at $P_2^* = 189.63$.

Figure 2.2(b) demonstrate the impact of volatility on construction option value as well as the decision points. As the volatility increases from 0.15 to 0.25, the construction option value function becomes less convex and takes higher value. The investment decision point also becomes higher, which means investors require that the property market price pass a higher threshold for them to start construction. Namely, the investment decision is delayed.

### 2.4.2 Entitlement option

#### Base case

Figure 2.3 is the plot of entitlement project value in base case scenario. Risk-free rate $r$ is set to be 0.05, with convenience yield $\delta_P$ and $\delta_L$ set to be 0.03 and 0.04, receptively. For the base case, we set the volatility of building price $\sigma_P = 0.20$, and volatility of entitlement cost $\sigma_L = 0.15$, with the correlation between them slightly negative, $\rho = -0.1$. Construction time is $T_c = 1$, and the waiting time for entitlement is $T_e = 0.5$. This base setting resembles a scenario where the entitlement is not very hard to get. This can be the case for residential entitlement in many small towns, where small scale residential rezoning or permit application are approved in a regular process.

From the side view plot we can see that the developer applies for the entitlement if the entitlement option value exceeds the initial cost $C$. The entitlement option value increases with the current building price, $P$, and decreases with the entitlement cost. The surface is convex, which means a unit increase in the current entitlement cost has to be offset by an increasing increment of building price.

The bottom view plot shows the shape of the exercising boundary curve in P-L plane. The exercise region boundary carved out by the initial cost, $C = 100$ is where the darkest blue starts to turn to bright blue. We can see that a 200 increase in building price will be offset by roughly 400 decrease in entitlement cost, about double the price increment.

### Varying the volatility

From the real options theory, we know that higher uncertainty generally increase the option value and delays the option exercise. The effect is manifest in Figure 2.4. In all three graphs, base case plot is the bottom surface. It shows that a higher entitlement risk leads to a higher entitlement project value. Usually investors have an objective trigger of project value that, if exceeded, he would begin the entitlement process. This trigger is a horizontal plain that cuts off the surface of entitlement value. At any state of $P$ and $L$, the trigger would cut off the higher surface at a lower $P$ value. Because the higher entitlement risk leads to a higher surface, the investor then would begin the entitlement process earlier if the entitlement risk is higher.

This result differs from the classical real option theory, which implies that the investors will exercise the option to construct later when facing a higher price volatility. Instead, our result shows that the investors will apply for entitlement earlier when facing a higher entitlement risk, because the process is totally out of his control and he cannot ``wait out'' part of the risk by collecting more information. We find our result similar to those of Bar-Ilan and Strange (1996) and Bar-Ilan and Strange (1998), who find that an artificial delay in construction process, as well as a separation of investment into two stages, will lead to earlier investment.

A comparison across the top to bottom in Figure 2.4 shows that the effect of higher entitlement risk is most pronounced at an extremely negative correlation, $\rho = -0.9$. In many areas, the local municipality tries to encourage development when the market supply is low and price of building is too high, by streamlining the entitlement application and raise the approval rate. Then the correlation between price process and the entitlement risk is synthetically negative. Our analysis shows that this artificial low correlation between building price risk and the entitlement risk amplifies the effect of entitlement risk.

Comparing the sub-figures in Figure 2.4 on left side and right side, we also see that the entitlement risk effect is smaller for a lower building price volatility. When the market price of buildings are more volatile, the investors will rush to start the entitlement process to make sure that he has a sufficient inventory of entitled land to meet the potential demand surge.

### Varying entitlement waiting time

Although most of the uncertainty in entitlement waiting time is modeled as entitlement risk, here we can still see the effect of a prolonged entitlement process. From Figure 2.5 we can see that a longer waiting time would increase the entitlement project value, and hence the developer would start the process earlier. In reality, many developers try to counter the longer waiting time by starting early. Notice however the change is asymmetric. When entitlement waiting time change from $T_e = 1$ to $T_e = 0.25$, the entitlement project value decreases by 58.6 from 375.4 to 316.8; when entitlement waiting time increase from $T_e = 1$ to $T_e = 1.75$, the value increase by 50.3 from 375.4 to 425.7. The longer the waiting time, the smaller the change brought by additional variation in waiting time.

### Varying convenience yield

We can see from Figure 2.6 that the entitlement project value increases with a higher convenience yield and decreases with a lower one. Since it is a change in the building price process, the effect is most prominent at the side of highest building price, and the change is even across different values of entitlement cost. The graph also displays an apparently asymmetric change. While 0.01 decrease in convenience yield lower the highest option value (at $P = 1000$ and $L = 0$ ) from 1305.8 to 1007.2, 0.01 increase in convenience yield raise the option value for the same $P$ and $L$ from 1305.8 to 1906.6. The closer it is to the risk-free rate, the more prominent its effect. Contrast to the big impact of building price convenience yield, the impact of entitlement cost convenience yield is almost unrecognizable.

### Related initial cost and entitlement risk

In the entitlement application practice, there is a subtle yet epidemic phenomenon: investors who invest extra resources to prepare a detailed, state-of-art application package, often get entitlement easier. In our model, we can easily accommodate this feature by setting the initial cost to be inversely related to entitlement risk.:

$$c = \frac{\alpha \cdot h}{\sigma_e},$$

where $\alpha$ the scaling parameter, and $h$ is the present value of the construction cost to ensure that the initial cost is proportion to the project size. With this setting, a higher initial cost $C$ would lower the entitlement risk $\sigma_e$ significantly.

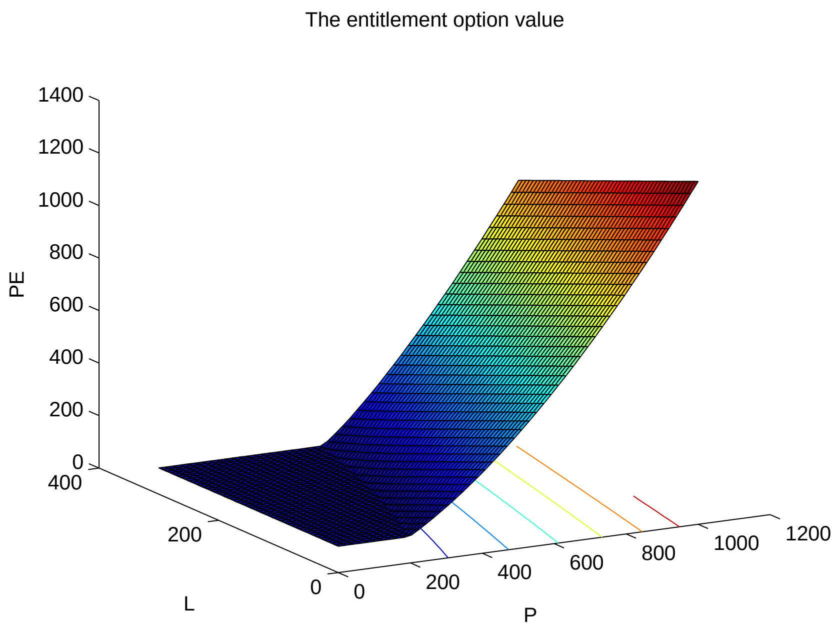
1. Base case

Figure 2.8 demonstrates how the project value and entitlement application decision changes if the initial cost is inversely related to entitlement risk. The three surfaces intersect each other. The surface with the highest initial cost, i.e. the surface with the highest horizontal plateau, has the lowest power-exchange option value in the exercise region (the tilted surface is lowest). This is because a higher initial cost is linked to a lower entitlement risk, which decreases the option value. Holding the entitlement cost level $L$ unchanged, an investor will demand that the building price meet a higher trigger before starting the entitlement process for a lower entitlement risk, or equivalently, a higher initial cost. That is, the investor would wait longer before the application decision facing a lower entitlement risk or a higher initial cost, feeling more in-control of the situation.

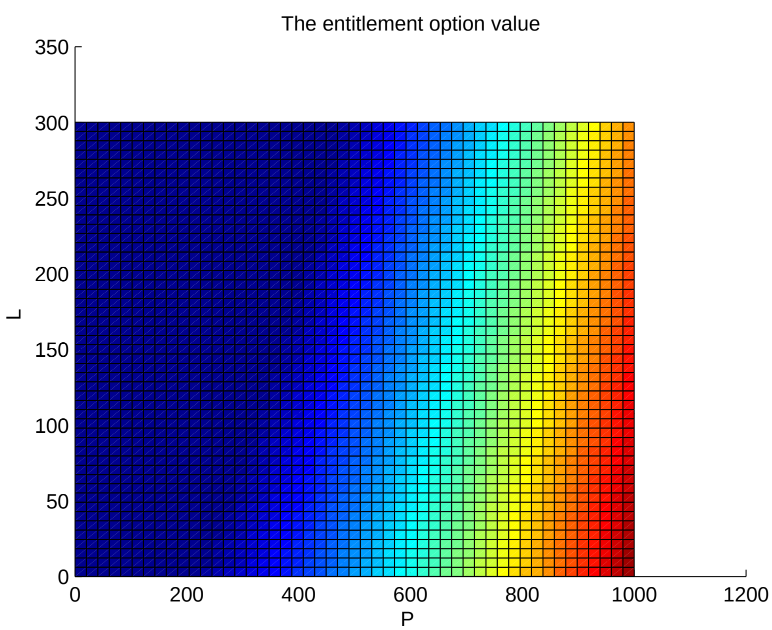
This shows that the inverse link between the initial cost and the entitlement risk would exaggerate the ``urging-effect'' of the higher entitlement risk. When investors face higher entitlement risk and choose not to pay extra initial cost to lower this risk, not only the high and uncontrollable entitlement risk would urge them to start project earlier, the lower initial cost would also encourage them to start early. On the other hand, if they have the willingness and ability to invest extra initial cost and be compensated with a lower entitlement risk in the unknown entitlement cost, they will feel more in control and will delay the application.

## 2.5 Conclusion

We build a real option model of development in which the entitlement process is modeled as a European style real option with stochastic entitlement cost being its strike. Our model captures important features of the entitlement process. In particular, the entitlement process is out of control of developers, with its cost exogenous and random. The entitlement risk is correlated with the building price risk, as government often intends to regulate the market by easing or tightening the entitlement process. The model implies that a higher entitlement risk will induce the developers to start the entitlement process earlier, and this effect is most pronounced at more volatile house market, and when the government's control makes the building price risk and entitlement risk negatively correlated. This indicates that developers start early to counter the uncontrollable delay and exogenous risk, to ensure that he has a good inventory of entitled land to capture potential demand surge and price increase. A higher building price risk, with unchanged entitlement risk and correlation, would lower the option due to a significant delay in the actual construction and hence a thinner chance to pay for the entitlement. If the initial cost is inversely related to the entitlement risk, the urging effect is exaggerated: investors who face a high entitlement risk will start even earlier.

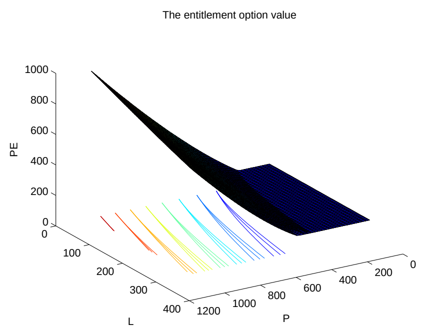



(a) 3D-plot side view

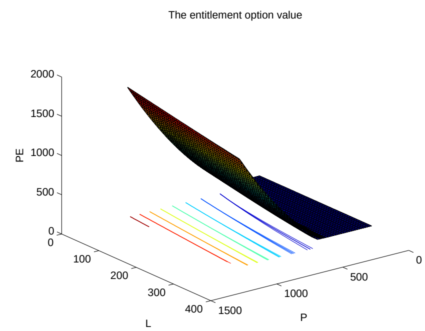



(b) 3D-plot bottom view

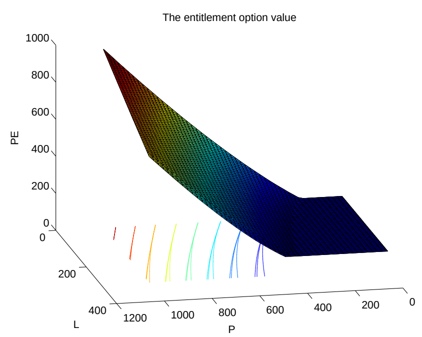
Figure 2.3: Plot of entitlement project value in base case scenario.
X-axis and Y-axis are the current constructed building price and the current entitlement cost, respectively. Z-axis is the value of option. Parameter settings are as following: $a = 0.02$, $r = 0.05$, $\delta_P = 0.03$, $\delta_L = 0.04$. $\sigma_P = 0.20$, $\sigma_L = 0.15$, and $\rho = -0.1$. $T_c = 1$, $T_e = 1$, and $C = h = 100$. The optimal construction decision happens at $P_2^* = 189.63$.



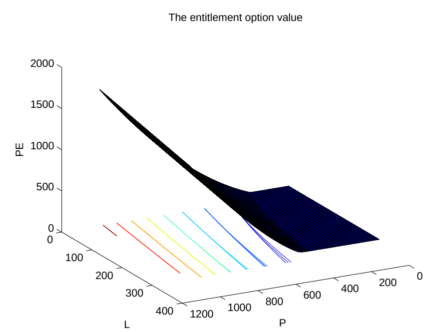

(a) Changing $\sigma_L$ at high $\sigma_P$; $\rho = -0.9$



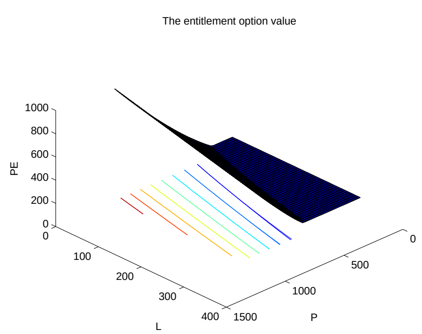

(b) Changing $\sigma_L$ at low $\sigma_P$; $\rho = -0.9$



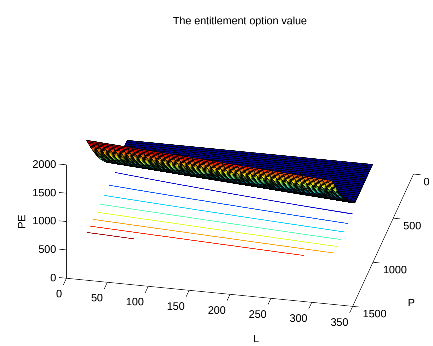

(c) Changing $\sigma_L$ at high $\sigma_P$; $\rho = -0.1$




(d) Changing $\sigma_L$ at low $\sigma_P$; $\rho = -0.1$




(e) Changing $\sigma_L$ at high $\sigma_P$; $\rho = 0.9$




(f) Changing $\sigma_L$ at low $\sigma_P$; $\rho = 0.9$

Figure 2.4: Plot of entitlement project value at various entitlement risk

Plot of entitlement project value at various entitlement cost volatility, with three correlation settings and two building price volatility settings. Plot 2.4(a) and 2.4(b) : $\sigma_P = 0.35$ on left and $\sigma_P = 0.15$ on right, both $\rho = -0.9$; plot 2.4(d) and 2.4(c): $\sigma_P = 0.35$ on left and $\sigma_P = 0.15$ on right, both $\rho = -0.1$; and plot 2.4(e) and 2.4(f) $\sigma_P = 0.35$ on left and $\sigma_P = 0.15$ on right, both$\rho = 0.9$. In each plot, the entitlement cost volatility $\sigma_L$ is $0.5, 0.35, 0.15$ from top surface to bottom surface, respectively. Other parameters take the base case setting: $a = 0.02$, $r = 0.05$, $\delta_P = 0.03$, $\delta_L = 0.02$. $T_c = 1$, $T_e = 1$, and $C = h = 100$. The optimal construction decision happens at $P_2^* = 441.01$.

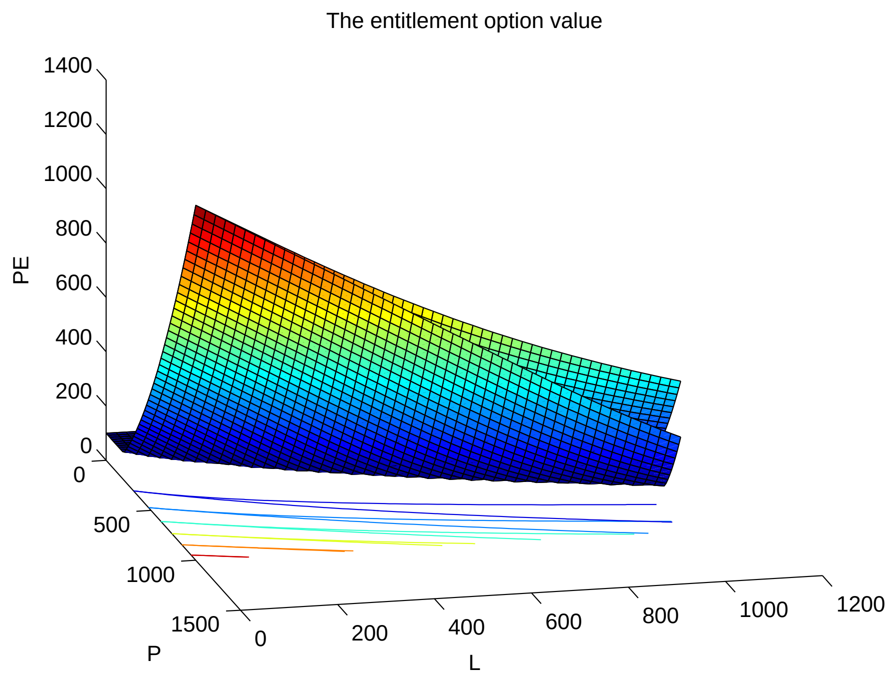



Figure 2.5: Plot of entitlement project value with different waiting time
Plot of entitlement project value with short and long entitlement waiting time. Upper surface is the plot of $T_e = 1.75$ where the lower surface is the plot of $T_e = 0.25$. Other parameters take the base case setting: $a = 0.02$, $r = 0.05$, $\delta_P = 0.03$, $\delta_L = 0.02$. $\rho = -0.1$, $\sigma_P = 0.20$,$\sigma_L = 0.15$. $T_c = 1$, and $C = h = 100$. The optimal construction decision happens at $P_2^* = 189.63$.

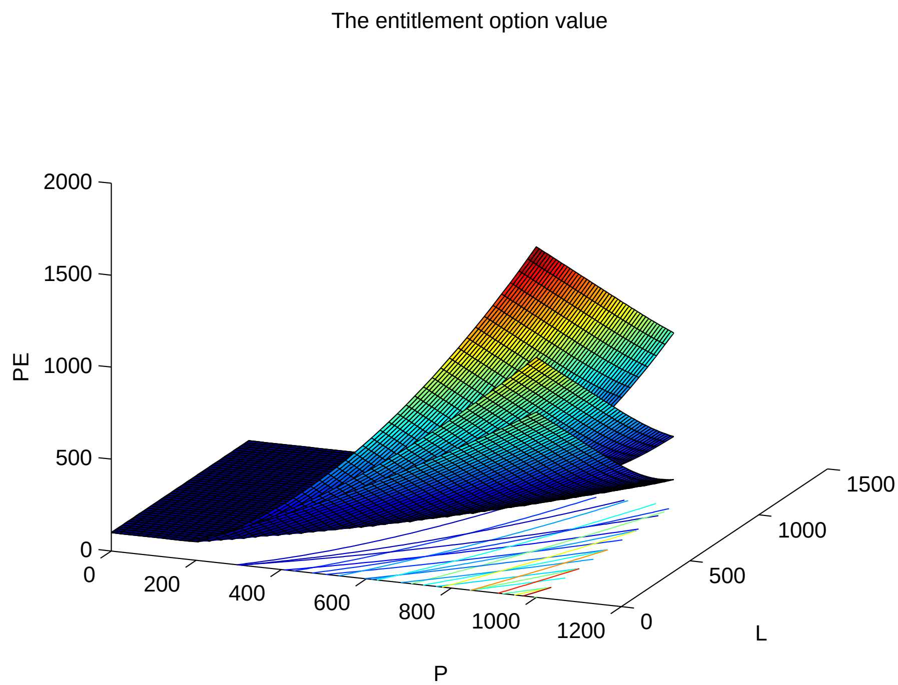



Figure 2.6: Plot of entitlement project value with various $\delta_P$

Plot of entitlement project value with various $\delta_P$ value. Upper surface is the plot of $\delta_P = 0.04$, the middle surface $\delta_P = 0.03$, and the lower surface $\delta_P = 0.02$. Other parameters take the base case setting: $a = 0.02$, $r = 0.05$, , $\delta_L = 0.02$. $\rho = -0.1$, $\sigma_P = 0.20$,$\sigma_L = 0.15$. $T_c = 1$, $T_e = 1$ and $C = h = 100$. The optimal construction decision happens at $P_2^* = 189.63$.

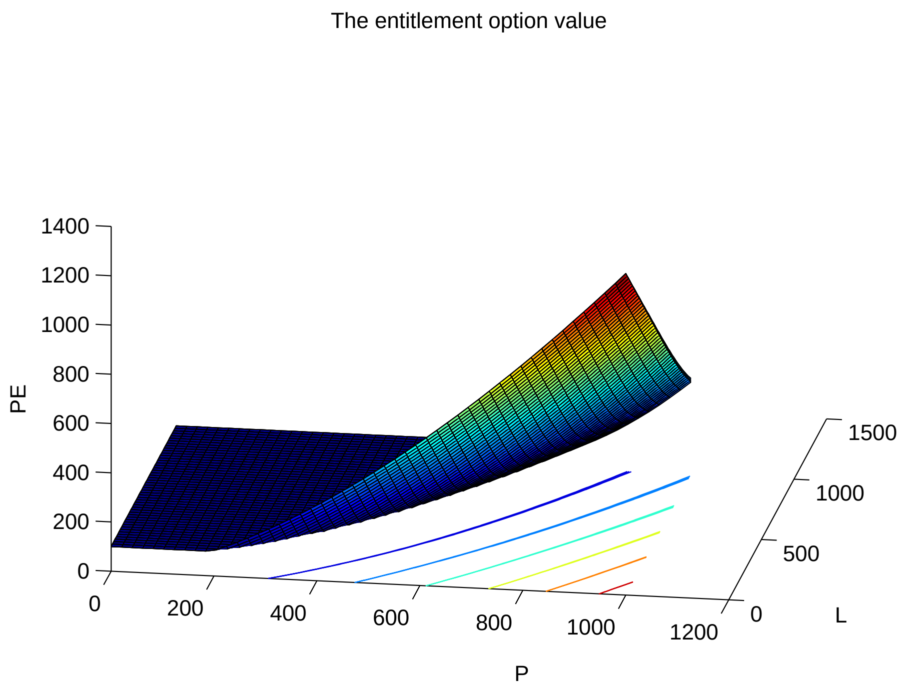
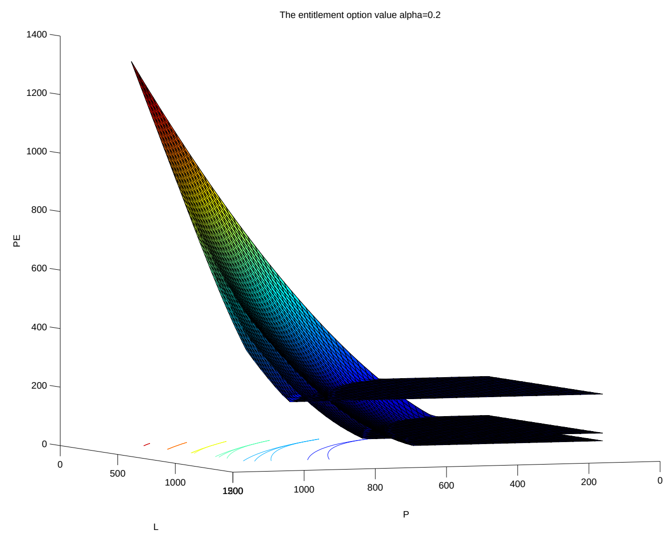
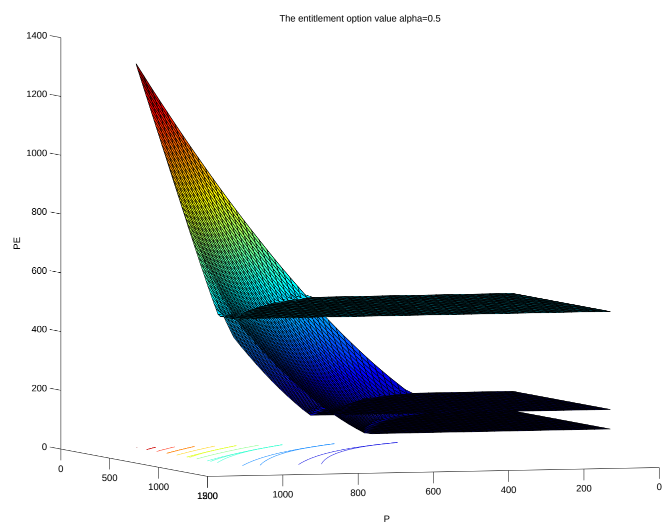



Figure 2.7: Plot of entitlement project value with various $\delta_L$

Plot of entitlement project value with $\delta_L = 0.02, 0.03, 0.04$. Difference is almost unrecognizable. Other parameters take the base case setting: $a = 0.02$, $r = 0.05$, , $\delta_P = 0.03$. $\rho = -0.1$, $\sigma_P = 0.20$,$\sigma_L = 0.15$. $T_c = 1$, $T_e = 1$ and $C = h = 100$. The optimal construction decision happens at $P_2^* = 189.63$.




(a) Changing initial cost $c$; $\alpha = 0.2$ $\rho = -0.5$




(b) Changing initial cost $c$; $\alpha = 0.5$ $\rho = -0.5$

Figure 2.8: Initial cost relates to entitlement risk

Plot of entitlement project value where extra initial cost buys lower entitlement risk $\sigma_L$. From top to bottom, the entitlement risk settings are: $\sigma_L = 0.1, 0.3, 0.5$. Scaling parameter: $\alpha = 0.2$ for the top graph, and $\alpha = 0.2$ for the bottom graph. And the correlation coefficient between entitlement cost process and the price process is $\rho = -0.5$. Other parameters take the base case setting: $a = 0.02$, $r = 0.05$, $\delta_P = 0.03$, $\delta_L = 0.02$. $T_c = 1$, $T_e = 1$, and $h = 100$.

# CHAPTER 3: TESTING THE EFFECT OF ENTITLEMENT RISK

## 3.1 Introduction

Standard real option theory indicates that developers delays investment decision when facing a higher underlying volatility. The intuition is that a higher volatility results in a higher option value, due to the increased probability of favorable return to the developers. The investor then chooses to wait for the option value to paste into a higher payoff level. An implicit assumption here is that the investors have full control over the exercise of this real option. It is not true, however, for all stages of the real estate development. One example is the entitlement process, the process of getting all permits from the local municipalities that precedes almost all actual construction stage. While it is the investor's decision to apply for the entitlement, the waiting time, the total cost and the final decision outcome are all out of the investor's control.

In the theoretical model of Chapter 2, we single out this utterly different process in real estate development and model it as a separate stage. In particular, we model the entitlement process as a European style real option whose expiration is exogenous to the investors, with its strike being the entitlement cost and its payoff being the option to construct. To model the uncertain amount of effort put in the entitlement process, as well as the cost from unforeseeable requirements from the negotiation with municipalities and local community organizations, we model the entitlement cost as a stochastic process that reveals its true value at the expiry of the European entitlement option.

Our theoretical model has interesting implications. To counter a higher out-of-control entitlement risk, developers starts the entitlement process earlier. Therefore, a higher entitlement risk results in earlier exercise of the option to begin entitlement process, rather than delaying the exercise of a real option as suggested by classical real option theory. From an capital budgeting point of view, the developers would initiate more entitlement applications where entitlement process is riskier, to ensure that they get at least some projects approved. This gives advantage to larger development firms who have access to more capital. Furthermore, we conjecture that developers could also diversify the entitlement risk by distributing their project geographically scattered. The result is a raise of many national developers who have large scale capital and develop projects in various locations.

In this chapter, we test the above implications from Chapter 2 empirically using manually collected data of the rezoning petitioning, which is the most difficult part of the entitlement process. For each rezoning petition published on Charlotte-Mecklenburg City Planning official website, we manually collect its application- and decision-dates, number of revisions, size of lot, decision outcome as well as other characteristics for rezoning petitions from 2001 to 2012. We define waiting time as the number of days between application date and decision date. Naturally, a longer waiting time is a result of either an earlier/premature application for rezoning, or a more difficult rezoning process. If we control for rezoning difficulty level by the number of revisions requested for each application, then the waiting time becomes a clean proxy for investors' early or premature decision to apply.

Using a full sample negative binomial regression, we first regress waiting time on the number of revisions using negative binomial regression, controlling for project size using the size of lot, the decision outcome using a set of dummy variables, and the local housing market condition using Case-Shiller Home Price Index (HPI) monthly for Charlotte city. The number of revisions proxies for aggregate rezoning difficulty and rezoning/entitlement risk. Across the board, the results indicate that higher entitlement risk/difficulty leads to earlier timing of application, which confirms our theoretical implication: a hastening effect of entitlement risk.

In addition, we group all petitions monthly by their application date, and use average waiting time as a proxy for investment timing. The standard-deviation of waiting time for each month group becomes an additional proxy for entitlement risk. Regression results of average waiting time on number of revisions, standard deviation of waiting time, as well as other control variables further confirms the hastening effect of entitlement risk. Noticeably, the correlation between housing market condition (HPI) and the monthly average waiting time is significantly negative. Since higher waiting time translate to a higher entitlement cost, this also indicates a strong negative correlation between house price and entitlement cost, which is a situation that is shown to exaggerates the hastening effect of entitlement risk in Chapter 2.

Finally, we examine the indirect implication that a higher entitlement risk leads to more applications. To do this, we group all petitions monthly by two different criteria to ensure that all explanatory variables are based on information available at the time. In particular, we group the petitions by their decision date to get the statistics of petition-specific information, including average of waiting time, lot size, number of revisions and standard deviation of waiting time. We then regroup the whole sample monthly by application date to get the number of applications in each month and match it with the information of petitions concluded in the current month. We regress the number of applications on the petition-specific variables to see how investors form application decision given the information on recently concluded petition cases, controlling for current housing market condition by HPI. The results confirm that investors file more application when facing higher entitlement risk, which in turn indirectly confirms that investors apply prematurely when facing higher entitlement risk.

The literature of entitlement process as a separate stage of real estate development is non-existent. However, entitlement as an important part of land use regulated has been shown to affect house price and land price in various ways. For example, Mayer and Somerville (2000), Quigley and Raphael (2005)used data from Florida and California and showed that tighter regulation leads to higher house price, less market supply, and less price-sensitivity in the demand. While a stream of research looked at the effect in house price, few investigated the impact on land price. In reality, the cost of house structure itself, which depends on the material cost and labor cost, shows little variation. Therefore, one can reasonably argue that the majority of real estate volatility is absorbed by the land market, and land market should display the effect of regulation more clearly. Quigley and Raphael (2005) used data from 407 cities in California and showed that houses are more expensive, housing market supplies less stock and house supply is less price-sensitive in more stringently regulated area. Ihlanfeldt (2007) treated the restrictiveness of land use regulation as an endogenous variable. He used data on more than 100 Florida cities and found that greater regulation restrictiveness increases the house price and decreases the land price. Huang and Tang (2012) used a sample covering more than 300 cities and find that more restrictive residential land use regulation and geographic land constraints are linked to larger booms and busts in housing prices. In a survey paper, Turnbull (2005) summarized the empirical results and called for dynamic examinations of the relationship between regulatory risk and the housing market.

Our research contribute to the literature of land-use policy by providing the first dynamic examination between investors' decision timing and the land use policy maker's behavior. We are the first to separate different effects of land-use policy stringency and unpredictability, and provide empirical findings that are directed by theoretical modeling. Our results are interesting to policy makers as a warning to separate the regulation stringency and regulation unpredictability, since the two demonstrate opposite effects on investors' behavior.

The remainder of this chapter is structured as follows. Section 3.2 discusses the data collection methods and presents the sample statistics. Section 3.3 examines the main theoretical implication of the relationship between entitlement risk and investors' decision timing. Section 3.4 examines the indirect theoretical implication of entitlement risk effect on number of applications. Section 3.5 concludes.

## 3.2 Sample Description and Statistics

We manually collect from the official website of Charlotte-Mecklenburg (North Carolina) Planning Department the published information about the rezoning petitions listed between 2001 and 2012. Our initial collection yielded 1414 observations. For each observation, we collected the complete petition information including petition id number, name of applicant, location, size of lot, current and proposed zoning, public hearing date, decision which is the actual case outcome, date of decision, date of application and number of revisions. In addition, we collected monthly S&P/Case-Shiller Home Price Index for Charlotte, NC, from official website between 1999 and 2012.

The documentation style of Charlotte-Mecklenburg varies over the sample period. Before 2001 most identity variables are available, but there is no application date information available, nor is there a way to figure out the number of revisions. Starting from 2002, the Planning Department starts to publish the scanned final sumbitted site-plan for some cases. We collect the application date as the earliest date found on the map unless it is otherwise clearly indicated. After 2006, some cases have more complete documentations on file. If the application form is published and the application date is clearly written in the application form, we use this date in the place of the date on site-plan. For number of revisions, we collect the number of revisions by manually counting the revisions dates documented on the site-plan. If in some cases a list of revision documents are published in lieu with the comments from various municipalities, we take the maximum number of revisions among all revisions filed for different municipalities as the number of revisions because one revision could contain a set of revision documents for several municipalities. For each case I identify the outcome of the case, what we call "decision", as one of the following: Approval, Denial, Withdrawal and Unknown. If the case is "indefinitely deferred by applicant", we count it as withdrawal. Unknown is labeled only if there is no outcome information available. Along with the decision, we collect the decision date. Most approved cases list the decision date with the decision outcome, however many withdrawal cases don't have a clear date on file. We use the public hear date, if it is available, or the date of the last documented staff comments. We implicitly assume that the investor made the decision to withdraw as soon as he hears the last comment from the municipality. We then calculate the *waiting-time* as the number of calendar days between the application date and the decision date. Since application date is available only for part of cases after 2002, our final sample with waiting-time on file contains 982 observations that are listed from 2002 to 2012. Several early 2002 cases are initially applied in 1999 or 2001, therefore the application date ranges from 1999 to 2012.

Table (3.1) presents the descriptive statistics of waiting-time (W_TIME), number of revisions (N_REVISIONS), size of lot (SIZE) and Case-Shiller Home Price Index (HPI) for 982 rezoning petition cases with waiting-time on file. Panel A reports the statistics of the full sample. An average rezoning petition endured a 164 days' wait, 2 revisions, and has a lot size of about 24 acres. All three rezoning characteristics varies dramatically. Waiting time ranges from 4 days to 7.92 years and has a large standard deviation of 171.44 days. Number of revisions ranges from 0 up to 12 times, the standard deviation, 1.72 times, is also close to the magnitude its average. Lot size varies from a tiny 0.05 acre to 2140.97, showing that the sample covers different types of development projects.

To separate the various types of projects, we use size of lot as a proxy for the project size to classify the sample. We classify the petitions into: "tiny projects" with lot size less than 5 acres, " the small projects" with lot size between 5 and 10 acres, the "medium projects" with lot size between 10 and 25 acres, and "large projects" with lot size above 25 acres. Among the total 982 observations, 506 cases are tiny project, 150 are small, 173 are medium and 153 are large. Surprisingly, we do not see much difference in the waiting time on average. [1] Further more, it appears that the median waiting time and maximum waiting time are higher for smaller projects. For example, the small projects group median of 190.16 shows that half of the petition cases of this size waited more than half a year, and the longest waiting-time happens in the tiny/small project groups. On the other hand, the average, median and maximum of number of revisions all increases slightly from smaller projects to

[1] Because all characteristics have large standard deviation and large skewness as seen from the difference between mean and median, we refrain from testing the significance of difference in mean with the assumption of normal distribution.

larger project, which is reasonable since larger projects trigger more restrictions imposed by municipalities and the local communities and therefore are requested more revisions. Preparing more revisions without increasing (or even slightly decrease) the waiting time, it seems that the developers of larger projects are more efficient in following through the entitlement process.

We further classify the sample by the four decision outcomes: approval, denial, withdrawal, and unknown. Among all 982 observations, a majority of 906 cases were approved, 16 were denied after public hearing, 46 cases were withdrawn by applicants and the remaining 14 cases have unknown results. On average, the denied cases have a significantly longer waiting time compared to approved cases. This is not surprising since denials rarely happen after a short investigation, but often result from failure to converge after a long grueling negotiation. In addition, the denied cases seems to have a much smaller standard deviation in lot size, although its mean is not significantly different from the mean in approved cases due to the large variation of lot size in approved cases. Although withdrawn cases seem to have the least number of revisions, probably because withdrawn cases are the least documented, with missing revision counted as zero.

## 3.3 Waiting time vs. entitlement risk

### 3.3.1 Pooled sample test

In Chapter 2, we present a real option model of entitlement that implies a hastening effect of the entitlement application when facing higher entitlement risk. While entitlement is an all-inclusive comprehensive process, here we investigate one important part of it, the rezoning process, to see the predictions of the theoretical model. In particular, we aim at testing the interaction between investor's decision timing and the riskiness and difficulties

Table 3.1: Summary statistics

We collect the rezoning petitions from Charlotte-Mecklenburg City Planning website, excluding all cases for which either decision time or application time cannot be determined. The sample includes 982 petitions. We use the lot size to proxy the project size, with tiny projects having lot size of 5 acres or less, small projects between 5 and 10 acres, medium projects between 10 and 25 acres and large projects size above 25 acres. The four decision outcomes are approval, denial, withdrawn and unknown.

| Rezoning Statistics | Obs | Mean | Median | Min | Max | Std. Dev. |
|---|---|---|---|---|---|---|
| *Panel A. Full Sample* | | | | | | |
| w_TIME (days) | 982 | 164.03 | 126.00 | 4.00 | 2890.00 | 171.44 |
| N_REVISIONS | 982 | 2.12 | 2.00 | 0.00 | 12.00 | 1.72 |
| SIZE (acres) | 977 | 24.38 | 4.62 | 0.05 | 2140.97 | 122.17 |
| *Panel A. Lot size < 5 acres* | | | | | | |
| W_TIME (days) | 506 | 157.18 | 174.04 | 5.00 | 2890.00 | 123.00 |
| N_REVISIONS | 506 | 1.75 | 1.35 | 0.00 | 7.00 | 2.00 |
| SIZE (acres) | 506 | 1.82 | 1.34 | 0.05 | 4.96 | 1.50 |
| *Panel B. Lot size>=5 acres and <10 acres* | | | | | | |
| W_TIME (days) | 150 | 160.85 | 190.16 | 4.00 | 2093.00 | 122.00 |
| N_REVISIONS | 150 | 2.01 | 1.58 | 0.00 | 8.00 | 2.00 |
| SIZE (acres) | 150 | 7.10 | 1.42 | 5.00 | 9.86 | 6.97 |
| *Panel C. Lot size>=10 & <25 acres* | | | | | | |
| W_TIME (days) | 173 | 178.34 | 162.84 | 18.00 | 1400.00 | 138.00 |
| N_REVISIONS | 173 | 2.71 | 1.85 | 0.00 | 10.00 | 2.00 |
| SIZE (acres) | 173 | 15.99 | 4.44 | 10.00 | 24.98 | 15.12 |
| *Panel D. Lot size >25 acres* | | | | | | |
| W_TIME (days) | 153 | 173.62 | 152.07 | 28.00 | 1526.00 | 147.00 |
| N_REVISIONS | 153 | 2.79 | 2.32 | 0.00 | 12.00 | 2.00 |
| SIZE (acres) | 148 | 128.80 | 293.16 | 25.61 | 2140.97 | 55.27 |
| *Panel B. Approval Cases* | | | | | | |
| W_TIME (days) | 906 | 162.48 | 126.00 | 4.00 | 2890.00 | 163.85 |
| N_REVISIONS | 906 | 2.22 | 2.00 | 0.00 | 12.00 | 1.73 |
| SIZE (acres) | 901 | 23.54 | 4.67 | 0.05 | 2140.97 | 116.49 |
| *Panel C. Denial Cases* | | | | | | |
| W_TIME (days) | 16 | 273.25 | 169.00 | 112.00 | 1526.00 | 345.84 |
| N_REVISIONS | 16 | 1.56 | 2.00 | 0.00 | 3.00 | 0.89 |
| SIZE (acres) | 16 | 12.79 | 1.61 | 0.15 | 79.19 | 22.48 |
| *Panel D. Withdrawals* | | | | | | |
| W_TIME (days) | 46 | 153.39 | 110.00 | 14.00 | 1560.00 | 231.74 |
| N_REVISIONS | 46 | 0.72 | 0.00 | 0.00 | 3.00 | 0.89 |
| SIZE (acres) | 46 | 44.07 | 3.40 | 0.15 | 1542.00 | 226.46 |
| *Panel E. Decision Unknown* | | | | | | |
| W_TIME (days) | 14 | 174.64 | 140.00 | 54.00 | 452.00 | 99.33 |
| N_REVISIONS | 14 | 0.71 | 0.50 | 0.00 | 2.00 | 0.83 |
| SIZE (acres) | 14 | 26.62 | 11.68 | 0.64 | 116.04 | 35.84 |

of the rezoning process.

There is virtually no way to directly measure how early investors decide to start entitlement compared to the maturity of his development project. However, if there entitlement process was started pre-maturely, the whole process will last longer. Therefore, we use the rezoning waiting time (W_TIME) to proxy for the timing of investor's decision. Given the same level of difficulty of rezoning process, the earlier the starting point, the longer the waiting time. We use number of revisions (N_REVISIONS) to proxy for rezoning difficulty and expect that higher difficulty would lead to a earlier application according to our theory, which will be reflected in longer waiting time. As we report previously, the number of revisions does not significantly correlate with the waiting time, therefore this relationship is not obvious. We use lot size (SIZE) to proxy for the project size. One would imagine that the waiting time should be longer for bigger project since there are more details entailed. We control for the overall housing market condition using Case-Shiller monthly Home Price Index (HPI), matched by the month of rezoning application date.

The null hypothesis: waiting time has no relationship with entitlement risk; alternative hypothesis is: waiting time is positively related with entitlement risk, where entitlement risk is proxied by number of revisions. Figure 3.1 shows the histogram of waiting-time (in days). Waiting time is likely a over-dispersed count variable considering from its large standard deviation (close to mean). The histogram also confirms that variable waiting time is right-skewed and with large variation. Therefore, we model waiting-time as a negative binomial distribution and use negative binomial regression to investigate its relationship with proxy of entitlement risk factors and house price index. That is, we model the log of expected waiting time as a function of explanatory variables. The negative binomial

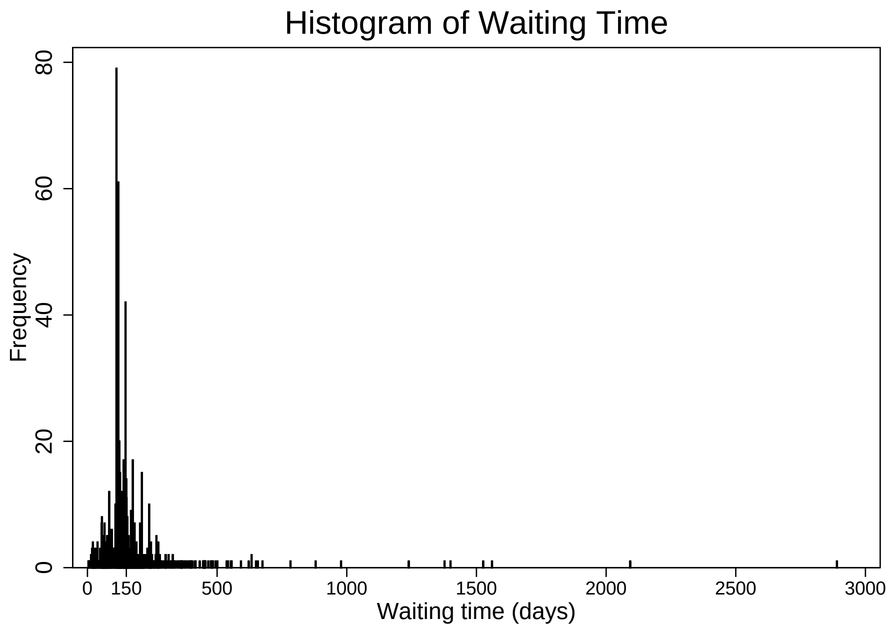



Figure 3.1: Histogram of waiting time in days.

regression model specification is:

$$log(W\_TIME) = \alpha + \beta_1 \times N\_REVISIONS + \beta_2 \times SIZE + \beta_3 \times HPI + \varepsilon. \tag{3.1}$$

We estimate equation 3.1 for the full sample and report the results under model (1) in Table 3.2. N_REVISIONS has a significant positive coefficient of 0.116, which suggests that, if on average investors are demanded one additional revision, the waiting time will be 12.3% longer, holding project size and house price the same. This is consistent with our theory that investors start earlier when facing more difficult and riskier entitlement process. Surprisingly, the size of the project does not significantly explain the length of waiting. We conjecture that larger projects, although more complicated and shown to endure more revisions in summary statistics, are often better prepared for initial application and employ more resources to follow through the process, therefore cancel the lengthening effect of

Table 3.2: Negative binomial regression

Dependent variable is waiting time (W_TIME) defined as the number of days between the application date and the decision date. Explanatory variables include the following. N_REVISIONS is the maximum number of revisions requested by all municipalities. SIZE is the size of lot reported in the site-plan. HPI is the Case-Shiller House Price Index monthly. Model (1) reports the result of full sample regression. Model (2), (3), (4) and (5) reports the regression results for each sub-group of different project size. In particular, tiny size means lot size < 5 acres; small size group has lot size >5 acres and < 10 acres; medium size group has lot size >10 acres and <25 acres; and large size group has lot size >25 acres. We report the coefficients with z-scores in parenthesis, and use * and ** and ***to denote significance at the 10% level, 5% level and 1% level, respectively. *alpha* and *lnalpha* are the over-dispersion parameter and the log-transformed over-dispersion parameter. We report the log-pseudolikelihood at the bottom of each model.

| W_TIME | (1)<br>whole set | (2)<br>tiny size | (3)<br>small size | (4)<br>medium size | (5)<br>large size |
|---|---|---|---|---|---|
| N_REVISIONS | 0.116*** | 0.149*** | 0.145** | 0.097*** | 0.077*** |
| | (6.38) | (4.58) | (2.46) | (2.67) | (3.26) |
| SIZE | -0.000 | 0.005 | 0.068* | -0.010 | 0.000 |
| | (-0.48) | (0.15) | (1.69) | (-0.84) | (0.06) |
| HPI | -0.006* | -0.006 | -0.014* | -0.013** | 0.005 |
| | (-1.79) | (-1.22) | (-1.96) | (-2.37) | (0.50) |
| cons | 5.595*** | 5.486*** | 5.879*** | 6.552*** | 4.345*** |
| | (13.17) | (9.83) | (6.96) | (9.20) | (4.13) |
| $ln\alpha$ | -1.048 | -1.047 | -0.916 | -1.166 | -1.238 |
| $\alpha$ | 0.351 | 0.351 | 0.400 | 0.312 | 0.290 |
| | (-11.46) | (-8.18) | (-6.56) | (-7.16) | (-4.51) |
| N | 977 | 506 | 150 | 173 | 148 |
| $Prob > \chi^2$ | 0.000 | 0.000 | 0.010 | 0.015 | 0.003 |
| Log pseudo-LL | -5713.431 | -2939.601 | -877.1547 | -1018.801 | -862.65435 |

project complexity. On the other hand, home price index has a weak but economically significantly negative effect on waiting time. Since full sample HPI average is 118.61, 1% increase of HPI from average is 1.18 units, leading to a 0.7% drop in waiting time. This negative relationship, along with the negative correlation between HPI and W_TIME, indicates a negative correlation between the entitlement risk and the house price. We have shown in Chapter 2 that this is exactly the situation shown to have maximum hastening effect of entitlement risk.

Since projects of different size are likely to show different dynamics between entitlement risk and decision timing, we also estimate equation 3.1 for each size group separately. Across the board, N_REVISIONS show significant positive effect on W_TIME. For the smaller two groups, one more revision on average would increase waiting time by about 16%, while for the two groups of bigger size projects, one additional revision merely costs 8 – 10% increase in waiting time. This can indicate two possibilities. First, it could be the case that the investors for smaller size projects start even earlier if they learn that rezoning process is more difficult; or second, as we conjectured, the investors of larger size groups simply employ more resources to push through and are therefore more efficient in making revisions. Within each size group, SIZE does not affect the W_TIME significantly, except for the small size projects (between 5 – 10 acres, for which one additional acre brings 7% increase in waiting time. Interestingly, home price index (HPI) have significant negative effect on W_TIME for middle two groups, yet is insignificant for the smallest group and the large size group.

It is natural to expect different dynamics for different decision outcomes. That is, for cases that are approved, denied, withdrawn and result known, the impact of entitlement

Table 3.3: Whole sample negative binomial regression with dummy.

Dependent variable is waiting time (W_TIME) defined as the number of days between the application date and the decision date. Model (1) includes dummy variables for different decision outcomes (APPROVAL, DENIAL, WITHDRAW and UNKNOWN). Model (2) includes both the dummy variables for decision outcomes and interaction term between the dummy variables of decision outcomes and number of revisions (APPRO-NREV, DEN-NREV, WITHD-NREV, UNKN-NREV). Other explanatory variables include the following. N_REVISIONS is the maximum number of revisions requested by all municipalities. SIZE is the size of lot reported in the site-plan. HPI is the Case-Shiller House Price Index monthly. We report the coefficients with z-scores in parenthesis, and use * and ** and ***to denote significance at the 10% level, 5% level and 1% level, respectively. $\alpha$ and $ln\alpha$ are the over-dispersion parameter and the log-transformed over-dispersion parameter. We report the log-pseudolikelihood, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) at the bottom of each model.

| | (1) | | (2) | |
|---|---|---|---|---|
| w_time | whole set | | interaction | |
| N_REVISIONS | 0.123*** | (6.66) | 0.117*** | (6.54) |
| SIZE | -0.000 | (-0.52) | -0.000 | (-0.76) |
| HPI | -0.008*** | (-2.61) | -0.009*** | (-2.66) |
| APPROVAL | 0.000 | (.) | 0.000 | (.) |
| DENIAL | 0.711** | (2.29) | 0.697** | (1.98) |
| WITHDRAWAL | 0.174 | (0.88) | -0.175 | (-0.89) |
| UNKNOWN | 0.281* | (1.76) | 0.465** | (2.45) |
| APPRO-NREV | | | 0.000 | (.) |
| DEN-NREV | | | 0.007 | (0.03) |
| WITHD-NREV | | | 0.373** | (2.02) |
| UNKN-NREV | | | -0.319*** | (-2.63) |
| cons | 5.778*** | (15.13) | 5.808*** | (15.34) |
| lnalpha | -1.080 | | -1.102 | |
| alpha | 0.3397 | | 0.332 | |
| N | 977 | | 977 | |
| Prob > $\chi^2$ | 0.000 | | 0.000 | |
| Log pseudolikelihood | -5696.898 | | -5685.425 | |
| AIC | 11.680 | | 11.665 | |
| BIC | 4729.614 | | 4734.205 | |

Table 3.4: Sub-sample Negative binomial regression with dummy.

Dependent variable is waiting time (W_TIME) defined as the number of days between the application date and the decision date. Model (1), (2), (3), (4) reports the negative binomial regression with dummy variables for decision categories, APPROVAL, DENIAL, WITHDRAW and UNKNOWN, for tiny, small, medium and large size projects. In particular, tiny size means lot size < 5 acres; small size group has lot size >5 acres and < 10 acres; medium size group has lot size >10 acres and <25 acres; and large size group has lot size >25 acres. Other explanatory variables are the following. N_REVISIONS is the maximum number of revisions requested by all municipalities. SIZE is the size of lot reported in the site-plan. HPI is the Case-Shiller House Price Index monthly.We report the coefficients with z-scores in parenthesis, and use * and ** and ***to denote significance at the 10% level, 5% level and 1% level, respectively. $\alpha$ and $ln\alpha$ are the over-dispersion parameter and the log-transformed over-dispersion parameter. We report the log-pseudolikelihood Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) at the bottom of each model.

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| w_time | tiny size | small size | medium size | large size |
| N_REVISIONS | 0.159*** | 0.147** | 0.102*** | 0.081*** |
| | (5.00) | (2.40) | (2.71) | (3.51) |
| SIZE | 0.005 | 0.049 | -0.011 | 0.000 |
| | (0.17) | (1.29) | (-0.89) | (0.73) |
| HPI | -0.009* | -0.013* | -0.013** | -0.003 |
| | (-1.95) | (-1.82) | (-2.44) | (-0.55) |
| APPROVAL | 0.000 | 0.000 | 0.000 | 0.000 |
| | (.) | (.) | (.) | (.) |
| DENIAL | 0.470** | | 0.061 | 1.465** |
| | (2.44) | | (0.65) | (2.28) |
| WITHDRAWAL | 0.376 | -0.736** | 0.161 | -0.065 |
| | (1.42) | (-2.37) | (0.68) | (-0.24) |
| UNKNOWN | 0.101 | 0.651* | 0.150 | 0.184 |
| | (0.68) | (1.68) | (0.79) | (1.00) |
| cons | 5.770*** | 5.887*** | 6.632*** | 5.196*** |
| | (11.40) | (6.87) | (9.05) | (7.71) |
| $ln\alpha$ | -1.077 | -0.979 | -1.170 | -1.470 |
| $\alpha$ | 0.340 | 0.376 | 0.310 | 0.230 |
| N | 506 | 150 | 173 | 148 |
| Prob > $\chi^2$ | 0.000 | 0.001 | 0.082 | 0.003 |
| Log pseudo-LL | -2931.318 | -872.042 | -1018.451 | -844.545 |
| AIC | 11.622 | 11.734 | 11.878 | 11.534 |
| BIC | 2768.046 | 1032.574 | 1191.762 | 994.477 |

risk is likely to be different. We therefore include dummy variable for each decision outcome (APPROVAL, DINIAL, WITHDRAW, UNKNOWN) and the interaction term between these dummy variables and the number of revisions (APPRO-NREV, DEN-NREV, WITHD-NREV, UNKN-NREV). Table 3.3 displays the result of full sample negative binomial regressions with dummy and interaction terms, and Table 3.4 displays the results of sub-sample regressions with dummy by each size group. Number of revisions still remains significantly positive across the board, and project size is still insignificant. Interestingly, home price index becomes negative significant at 1% level in full sample once we control for the dummy and interaction terms. 1% increase in home price index at the average level of 118.61 predicts a decrease of about 1% drop of waiting time.

Among various decision outcomes, denied cases on average has a significant increase in waiting time. In particular, an average denied case would have a 103.6% increase in waiting time compared to approved cases. Withdrawn cases does not have significant increase on waiting time. This result could be downward biased because we deliberately filled in the absent decision date of many withdrawn cases with the date of last document on-file, which could underestimate the amount of waiting time. On the other hand, the interaction terms are significant for the withdrawn cases and the unknown-result group. This indicates that denied cases does not show a significant change in coefficient of N_REVISIONS, while withdrawn cases has a significantly higher coefficient for N_REVISIONS and unknown cases have a significantly lower coefficient for N_REVISIONS. For withdrawn cases, one additional required revisions results in a 63.23% increase in waiting time compared to a 12.41% increase for approved cases, which means an extra 83 days spent in waiting for a sample average waiting time of 164 days.

To better understand the effect of decision outcome for various project size groups, we also predict the marginal effect of the decision dummies. That is, we calculate the number of waiting days at each decision outcome, holding other explanatory variables at sub-sample mean. While approved cases show almost uniform predicted waiting time, the predicted waiting time for denied cases and withdrawn cases vary vastly for different size group. We notice that the large-size denied projects are projected to wait 682 days, which is almost two years, for an average of 2.88 revisions. This wait is significantly longer than the predicted waiting time of full sample denied cases (315 days). On the other hand, a small size project shows a much shorter predicted waiting time of 73 days if it is withdrawn, a very big waiting time of 293 days for unknown group compared to the full sample prediction. The small size group does not have any denied cases. Again this could be an bias introduced by underestimating the waiting days for the withdrawn group if the withdrawn cases for the small size projects are particularly ill-documented.

### 3.3.2 Waiting time: monthly

To further smooth out the high variation in waiting time, we group the observations monthly by the application date of the observations, result in a sample of 137 observations. Table 3.6 reports the summary statistics of the monthly variables. AWTIME is the average waiting time of the month. The mean over the cases in all months is 193.896 days. ASIZE is the average lot size of all cases in the month, whose mean is 21.83. AREVISION is the average number of revisions of the month, which on average is 2.25 revisions. HPI is the Case-Shiller monthly home price index, and STD_WTIME is the standard deviation of the waiting time, which is on average 85.5 days.

We use average waiting time (AWTIME) for each month to proxy the investors' decision

Table 3.5: Marginal effect of decision dummies.

For full-sample and four different project size groups, we present the predicted waiting-time in days at each decision outcome (on left column) and the standard error of predicted waiting time (on right column), holding all other variables (N_REVISIONS, SIZE, HPI) at their means of the (sub-)sample. Decision_cat are: 1 for approvals, 2 for denials, 3 for withdrawals, and 4 for unknown results. tiny size means lot size < 5 acres; small size group has lot size >5 acres and < 10 acres; medium size group has lot size >10 acres and <25 acres; and large size group has lot size >25 acres.

| | Full sample | | Tiny size | | Small size | | Medium size | | Large size | |
|---|---|---|---|---|---|---|---|---|---|---|
| decision_cat | Margin | Std. Err. | Margin | Std. Err. | Margin | Std. Err. | Margin | Std. Err. | Margin | Std. Err. |
| 1 | 154.840 | 3.755 | 147.938 | 4.887 | 152.688 | 9.190 | 171.893 | 9.709 | 157.644 | 7.218 |
| 2 | 315.205 | 97.937 | 236.706 | 46.311 | | | 182.658 | 15.747 | 682.317 | 437.816 |
| 3 | 184.342 | 36.468 | 215.433 | 57.094 | 73.115 | 22.531 | 201.842 | 48.578 | 147.779 | 39.424 |
| 4 | 205.063 | 32.522 | 163.612 | 24.247 | 292.777 | 112.342 | 199.673 | 37.260 | 189.451 | 34.171 |

Table 3.6: Sample statistics for monthly data.

We group the entitlement applications by month of the application date, and calculate the average waiting time (AWTIME), standard deviation of the waiting time (STD_WTIME) and average number of revisions (AREVISIONS) for each month. HPI is Case-Shiller home price index.

| Variable | Obs | Mean | Median | Std. Dev. | Min | Max |
|---|---|---|---|---|---|---|
| AWTIME | 137 | 193.896 | 145.250 | 261.108 | 30.000 | 2491.500 |
| ASIZE | 137 | 21.839 | 12.133 | 43.176 | 1.110 | 364.294 |
| AREVISION | 137 | 2.254 | 2.250 | 1.054 | 0.000 | 7.000 |
| HPI | 137 | 116.630 | 114.460 | 9.004 | 99.240 | 133.850 |
| STD_WTIME | 128 | 85.505 | 61.987 | 82.024 | 0.000 | 563.564 |

timing. For explanatory variables, we use the standard deviation of waiting time within the month as an additional proxy for the entitlement risk to achieve cleaner identification. While the number of revisions emphasize on the difficulty of the rezoning process, the standard deviation of waiting-time is a clean proxy for how unpredictable is the process. The model specification is:

$$AWTIME = \alpha + \beta_1 \times STD\_WTIME + \beta_2 \times AREVISIONS + \beta_3 \times ASIZE + \beta_4 \times HPI.$$

Table 3.7 reports the result of the OLS regression with AWTIME as dependent variable, and ASIZE and HPI as control variables. Model (1) use AREVISIONS only to proxy the entitlement difficulty, model (3) uses both AREVISIONS and STD_WTIME, while model (3) uses STD_WTIME only. The significance and sign of coefficients across the three model are highly consistent: ASIZE is insignicant, AREVISIONS and STD_WTIME both have significantly positive effect on AWTIME. When included alone, one additional unit of AREVISIONS leads to 130.59 days of additional waiting on average; and 1 day increase in STD_WTIME results in 1.671 more days in waiting time. When both are included, the effect of AREVISION drops to 41.461 days' increase with weaker significance, while the effect of STD_WTIME drops only by 0.1 day on average and remains significant at 1% level. The increase of average waiting time can be the result of two factors: first is that investors start earlier and/or prematurely, and second is that the entitlement process gets more difficult and therefore takes longer. By controlling for AREVISIONS and therefore controlling for the rezoning difficulty level in model (2), we argue that the significant positive coefficient of STD_WTIME strongly suggest that investors start earlier when facing higher entitlement risk, which is the main implication of our model in Chapter 2. Notice that, similar to the whole sample results, HPI has a significantly negative relationship with AWTIME across the board. This indicates that the entitlement cost (which is directly related to waiting time in our model) is negatively related to the house price, which exaggerates the hastening effect of entitlement risk as shown in our theoretical model.

## 3.4 Number of applications: monthly

An indirect implication of our theory in Chapter 2 is an increased entitlement application when facing higher entitlement risk. Here we group the observations into monthly number of application by the application date, but group all other variables monthly by the decision dates. This data manipulation assumes that the investors file the application knowing information about recently concluded rezoning petitions. The sample contains 120 observations, and Table 3.8 reports a sample statistics similar to that of the previous section.

Table 3.9 reports the regression results of number of applications (NAPP) on: average waiting time (AWTIME), standard deviation of waiting time (STD_WTIME), average number of revisions (AREVISIONS), average lot size (ASIZE) and home price index (HPI). The model specification is:

Table 3.7: Regression using monthly data

We group the entitlement applications by month of the application date, and calculate the average and standard deviation of the waiting time, and average number of revisions for each month. OLS regression: Dependent variable is average waiting time for each month (AWTIME), explanatory variables include: average size of lot (ASIZE), average number of revisions (AREVISION), standard deviation of waiting time (STD_WTIME), and Case Shiller House Price Index of the month (HPI). We report the coefficients with t-statistics in parenthesis, and use * and ** and ***to denote significance at the 10% level, 5% level and 1% level, respectively.

| AWTIME | (1) | (2) | (3) |
|---:|---:|---:|---:|
| ASIZE | -0.671 | -0.488 | -0.374 |
| | (-1.56) | (-1.54) | (-1.17) |
| AREVISIONS | 130.585*** | 41.461** | |
| | (7.24) | (2.47) | |
| HPI | -4.961** | -4.519*** | -5.018*** |
| | (-2.36) | (-2.84) | (-3.11) |
| STD_WTIME | | 1.530*** | 1.671*** |
| | | (8.59) | (9.71) |
| cons | 492.806* | 489.355** | 622.590*** |
| | (1.90) | (2.52) | (3.26) |
| N | 137 | 128 | 128 |
| R-sq | 0.344 | 0.476 | 0.450 |

Table 3.8: Sample statistics for number of applcations

Sample statistics for number of applcations(NAPP) grouped by application date and other monthly observations grouped by decision date. AWTIME and STD_WTIME are the average and standard deviation of waiting time for all cases concluded in the month; AREVISIONS is the average number of revisions for each month, ASIZE is the average lot size, and HPI is Case-Shiller Home Price Index.

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---:|---:|---:|---:|---:|---:|
| NAPP | 119 | 7.513 | 3.826 | 1.000 | 20.000 |
| AWTIME | 120 | 159.414 | 62.809 | 32.000 | 451.000 |
| STD_TIME | 113 | 111.021 | 137.687 | 0.000 | 987.666 |
| AREVISION | 120 | 2.106 | 0.827 | 0.000 | 4.000 |
| ASIZE | 120 | 22.495 | 42.043 | 0.370 | 372.744 |
| HPI | 120 | 117.861 | 8.588 | 104.780 | 133.670 |

$$NAPP = \alpha + \beta_1 \times AWTIME + \beta_2 \times STD\_WTIME + \beta_3 \times AREVISIONS + \beta_4 \times ASIZE + \beta_5 \times HPI + \varepsilon.$$

The results indicate the following. First, the two proxies of entitlement risk, STD_WTIME and AREVISIONS, both have positive significant effect on the number of applications, which is consistent with our model implication that higher entitlement risk leads to more applications since investors will apply prematurely to secure a piece of entitled land to be ready for market demand increase. Second, average waiting time AWTIME has negative significant effect on the number of applications. This is not surprising because, interpreting longer average waiting time as higher entitlement cost, the real option value decreases and therefore investors are less likely to file application. Third, the home price index (HPI) has significant positive effect on number of applications. Indeed, increased house price also increases the the option value of starting the entitlement, therefore increase the number of applications.

## 3.5 Conclusion

To test the effect of entitlement risk on investors decisions, we manually collect information on rezoning petitions published on Charlotte-Mecklenburg City Planning website. We define waiting time as the number of days between application date and decision date to proxy for the timing of investors decisions. We argue that longer waiting time is a clean proxy for early or premature application for entitlement if we properly control for the entitlement difficulty. Using whole sample negative binomial regression, we find that investors apply early for entitlement when facing a higher entitlement difficulty or entitlement risk. We then group the rezoning petitions monthly by application date, and use standard de-

Table 3.9: Regression: number of applications on petition characteristics

We group the rezoning petitions by month of the decision date, and calculate the average and standard deviation of the waiting time, average number of revisions, and average lot size for each month. We then group the rezoning petitions by the month of decision date to calculate the number of applications for each month. OLS regression: Dependent variable is number of applications for each month (NAPP) identified by application date. Explanatory variables include: average waiting time for each month (AWTIME), average size of lot (ASIZE), average number of revisions (AREVISION), standard deviation of waiting time (STD_WTIME), and Case Shiller House Price Index of the month (HPI). We report the coefficients with t-statistics in parenthesis, and use * and ** and ***to denote significance at the 10% level, 5% level and 1% level, respectively.

| nappli_dlag | (1) | (2) | (3) |
|---:|---:|---:|---:|
| ASIZE | -0.011 | -0.009 | -0.009 |
| | (-1.39) | (-1.10) | (-1.09) |
| AWTIME | -0.028** | 0.001 | |
| | (-2.28) | (0.12) | |
| STD_WTIME | 0.014** | | 0.003 |
| | (2.57) | | (1.16) |
| AREVISIONS | 1.257*** | 1.100*** | 1.067** |
| | (2.95) | (2.72) | (2.51) |
| HPI | 0.100** | 0.119*** | 0.114*** |
| | (2.50) | (3.08) | (2.80) |
| cons | -3.656 | -8.680* | -8.178* |
| | (-0.71) | (-1.89) | (-1.69) |
| N | 112 | 119 | 112 |
| R-sq | 0.194 | 0.148 | 0.155 |

viation of waiting time as additional proxy for entitlement risk. The results again display strong positive relationship between entitlement risk, entitlement difficulty, and how early they start application. Finally, we group the rezoning petitions monthly by decision date to calculate petition-specific information, and group the rezoning petitions monthly by application date to get number of applications for each month. We found that, given the information on petitions concluded recently, the investors would make more applications if the entitlement is riskier or more difficult. Our research is first to dynamically examine how the investors' behavior relates to entitlement risk and stringency. The results are interesting to policy makers in two folds. First, we clear provide strong evidence that investors start early when facing higher entitlement risk; and second, we urge policy makers to separate the regulation stringency and regulation unpredictability, as the two demonstrate opposite effects on investors' behavior.

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