Modular Capacitated Maximal Covering Location Problem for the Optimal Siting of Emergency Vehicles

By: Ping Yin, Lan Mu

Yin, P. and Mu, L. 2012. Modular Capacitated Maximal Covering Location Problem for Optimal Siting of Emergency Vehicles. *Applied Geography* 34: 247-254

Made available courtesy of Elsevier: http://www.dx.doi.org/10.1016/j.apgeog.2011.11.013

Abstract:

To improve the application of the maximal covering location problem (MCLP), several capacitated MCLP models were proposed to consider the capacity limits of facilities. However, most of these models assume only one fixed capacity level for the facility at each potential site. This assumption may limit the application of the capacitated MCLP. In this article, a modular capacitated maximal covering location problem (MCMCLP) is proposed and formulated to allow several possible capacity levels for the facility at each potential site. To optimally site emergency vehicles, this new model also considers allocations of the demands beyond the service covering standard. Two situations of the model are discussed: the MCMCLP-facility-constraint (FC), which fixes the total number of facilities to be located, and the MCMCLP-non-facility-constraint (NFC), which does not. In addition to the model formulations, one important aspect of location modeling—spatial demand representation—is included in the analysis and discussion. As an example, the MCMCLP is applied with Geographic Information System (GIS) and optimization software packages to optimally site ambulances for the Emergency Medical Services (EMS) Region 10 in the State of Georgia. The limitations of the model are also discussed.

Keywords: Modular capacitated MCLP | Spatial demand representation | GIS | Emergency vehicle

Article:

1. Introduction

Given a covering standard for a service, such as a distance or travel-time maximum, the objective of the maximal covering location problem (MCLP) is to locate a fixed number of facilities to provide the service to cover as many demands as possible. MCLP modeling, after being put forward by Church and ReVelle (1974), has been a powerful and widely used tool in many planning processes to optimally distribute limited resources to maximize social and economic benefits, such as the placement of emergency warning sirens (Current & O'Kelly, 1992), fire stations (Indriasari, Mahmud, Ahmad, & Shariff, 2010), distribution centers for humanitarian relief (Balcik & Beamon, 2008), health centers (Bennett et al., 1982, Griffin et al., 2008, Ratick et al., 2009 and Verter and Lapierre, 2002), and ecological reserves (Church, Stoms, & Davis, 1996). Among many different versions of MCLP models that have been proposed, a basic underlying assumption is that the facilities to be sited are uncapacitated. Under this assumption, the demand will be served as long as it is within the service covering standard of any facility. However, this assumption of uncapacitated facilities severely limits the application of covering models (Current & Storbeck, 1988). Many service facilities have finite capacities to ensure an acceptable level of service and spatial equity (Liao and Guo, 2008 and Murray and Gerrard, 1997). For example, an ambulance base can only respond to a limited number of demands within its service covering standard (e.g., 8-min driving distance) at one time because of the availability status of the ambulances stationed at the base. Therefore, the capacity limit—the main constraint addressed in this article—is an important consideration in location problems, especially for the siting of emergency facilities.

Chung, Schilling, and Carbone (1983) and Current and Storbeck (1988) published two early papers dealing with the capacitated versions of the MCLP. Both groups of authors added maximum capacity constraints into the mathematical formulations of the MCLP to ensure that the demands allocated to a facility will not exceed the capacity of that facility. However, these two capacitated MCLP models only consider the allocation of the demands within the service covering standard of facilities. Many systems, particularly public services, are typically available to all demands within their jurisdiction. For example, even if a demand is located in an area where no ambulances can reach the demand within a time standard, the demand must still be responded to and be counted as part of some facility's workload. Therefore, Pirkul and Schilling (1991) proposed an extension of the capacitated MCLP where all demands are assigned to facilities, regardless of whether that demand lies within the service covering standard. Such an idea of allocating all demands to facilities is also shown in some uncapacitated MCLP models, such as the generalized maximal covering location problem of Berman and Krass (2002). Following the work of Pirkul and Schilling, 1991 and Haghani, 1996 proposed a multi-objective capacitated MCLP model where the objective function maximizes the weighted covered demand while simultaneously minimizing the average distance from the uncovered demands to the located facilities. He showed how to ensure the maximization of the weighted covered demand to be the primary objective in the model by adjusting its weight in the objective function.

In all of the above capacitated MCLP models, only one fixed capacity level of the facility is considered for each potential facility site. However, many situations arise where each potential facility site could have several possible maximum capacity levels for a facility to choose. For example, the capacity limit of an emergency facility (e.g., ambulance base or fire station) can be assumed to be determined by its stationed emergency vehicles (e.g., ambulances or fire trucks). Therefore, varied numbers of emergency vehicles will provide a series of possible maximum capacity levels for the emergency facility to choose. Correia and Captivo (2003) called the location problems with such capacity constraints modular capacitated location problems. However, their model is an extension of the capacitated plant location problem, the objective of which is to minimize total costs, including fixed costs and operating costs, associated with plant and transportation costs, among others. For emergency services, the objective is often stated as the minimization of losses to the public, which is equivalent to the maximization of benefits (Indriasari et al., 2010). Cost is usually not the first consideration in these services. Therefore, the capacitated MCLP is more suitable than the capacitated plant location problem for emergency services. Although Griffin et al. (2008)considered three capability levels for each type of health care facility in their capacitated MCLP model, there is no composing relationship for the capacity levels of facilities, such as that between emergency vehicles and emergency facilities. In addition, their model did not consider the allocation of demands outside the service covering standard.

To apply the capacitated MCLP model to the emergency facility siting problem in which an emergency facility could have different possible capacity levels with varied numbers of stationed emergency vehicles, we propose an extension of the MCLP called the modular capacitated maximal covering location problem (MCMCLP). Similar to the multi-objective function in the model of Haghani (1996), the MCMCLP aims to maximize the weighted covered demand while simultaneously minimizing the average distance from the uncovered demands to the located facilities.

The remainder of this article is organized as follows: In the next section, the concepts, formulations, and related issues of the MCMCLP are introduced and discussed in terms of two situations. The first situation involves a fixed total number of facilities to be located; in the second situation, the total number of facilities is not fixed. Subsequently, we briefly review the approaches for spatial demand representation that could influence the accuracy of the problem solutions. The method called service area spatial demand representation (SASDR) is briefly described. Next, the MCMCLP and the SASDR are applied to the optimal siting of ambulances for the Emergency Medical Services (EMS) region 10 in the State of Georgia (GA). Finally, a discussion and conclusions are provided.

2. Modular capacitated maximal covering location problem (MCMCLP)

Because of the capacity limit of a facility, the allocation problem (i.e., how to allocate demands to facilities) sometimes must be solved in conjunction with the location problem (i.e., where to site facilities) (Haghani, 1996). Under the assumption that one demand can only be allocated to, at most, one facility, we define three demand types and use them in the following part of this article: 1) *unallocated demand*, which is not allocated to any facility (e.g., the demands d_a and d_b in Fig. 1); 2) *covered allocated demand*, which is located within the service covering standard of a facility and is allocated to that facility (e.g., the demand d_c in Fig. 1); 3)*uncovered allocated demand*, which is located beyond the service covering standard of a facility but is allocated to that facility (e.g., the demand d_d in Fig. 1).

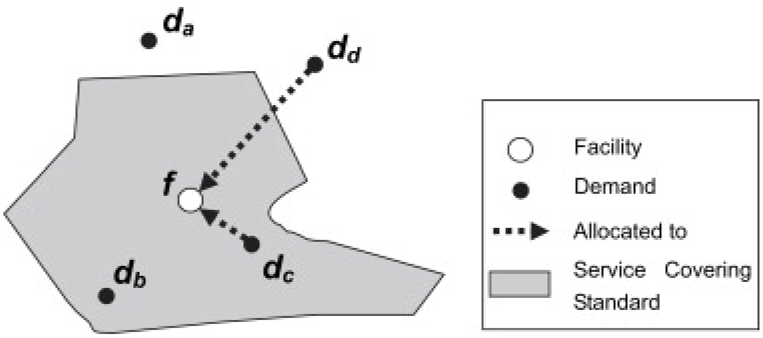

Fig. 1. Illustration of three demand types: unallocated demand (d_a and d_b), covered allocated demand (d_c), and uncovered allocated demand (d_d).

Following the work of Pirkul and Schilling (1991) and Haghani (1996), and in light of a different perspective of the capacitated plant location problem of Correia and Captivo (2003), we present an extension to the capacitated MCLP called MCMCLP and utilize it for siting emergency services. In addition to the basic concept of the MCLP that the covered allocated demands should be maximized by optimally siting a fixed number of facilities, the MCMCLP also includes the following considerations: 1) the facility at each potential site has a maximum capacity, which will be chosen from a finite and discrete set of available capacity levels; 2) all demands need to be allocated to facilities (i.e., no unallocated demands exist), and the uncovered allocated demands could be assigned on the basis of their proximity to facilities; 3) the demands within a demand object, which is a spatial point or areal unit derived by abstracting or partitioning continuous demand space, may be divided and allocated to multiple facilities.

An area with a larger population usually has a higher frequency of calls for emergency service than an area with a smaller population. In addition, one emergency vehicle can only respond to one call at a time and will be available only after that task is finished. Therefore, the larger population an ambulance serves, the higher the busyness probability it usually has, the longer the average response time for a call is, and the poorer the service it will provide. To ensure an acceptable average response time for a call, each emergency vehicle can be thought to have a maximum population that it can serve. In this article, we take population as demands, and the upper limit of the population served by an emergency vehicle is defined as the capacity of that vehicle. In fact, the calculation of an emergency vehicle’s capacity needs to consider multiple factors, including the requirement for the average response time, the average frequency of calls in the population that it will serve, and the average treatment time for a task, among others. For simplicity, in this article, all emergency vehicles are assumed having the same capacity, and the capacity of a facility can be assumed as the total capacities of all vehicles stationed in that facility. For example, if there could be at most *p*vehicles stationed in a facility, there are *p* possible levels of capacity from which to choose. A facility will not be established in a location unless at least one emergency vehicle needs to be stationed there.

There are two situations for the MCMCLP. If there is no constraint on the total number of emergency facilities that will be established to station vehicles, then we call such a non-facility-constraint problem MCMCLP-NFC. This situation mainly focuses on how to allocate a given number of vehicles to a set of predefined potential facility sites. If the total number of facilities is fixed, such facility-constraint problem is termed MCMCLP-FC. This situation needs to select the sites for a given number of facilities and then allocate a given number of vehicles to these facilities. Consider the following notation:

I = the set of demand objects $\{1, \cdots, i, \cdots, m\}$;

J = the set of potential facility sites $\{1, \cdots, j, \cdots, n\}$;

S = the service covering standard of facility (i.e., maximum distance or time);

d_{ij} = the travel distance or time from potential facility site j to demand object i;

J_i = the set of potential facility sites j within the service covering standard of which demand object i lies, i.e., $\{j|dij \leq S\}$;

a_i = the amount of service demands at demand object i;

p = the total number of emergency vehicles to be located;

c = the capacity of one emergency vehicle (assuming all vehicles have the same capacity);

w = the weight associated with all the uncovered allocated demands;

x_j = the number of emergency vehicles stationed at potential facility site j; a facility is located on site $j$when $x_j > 0$;

y_{ij} = the percentage of demands at demand object i that is allocated to the facility on site j.

The formulation of the MCMCLP-NFC is

Maximize $\sum_{i \in I} \sum_{j \in J_i} a_i y_{ij} - w \sum_{i \in I} \sum_{j \notin J_i} d_{ij} a_i y_{ij}$ equation(1)

Subject to:

$$\sum_{i\in I} a_i y_{ij} \le c x_j \quad \forall j \in J$$ equation(2)

$$\sum_{j\in J} x_j = p$$ equation(3)

$$\sum_{j\in J} y_{ij} = 1 \quad \forall\, i \in I$$ equation(4)

$$x_j = 0, 1, 2, \ldots, p \quad \forall\, j \in J$$ equation(5)

$$0 \le y_{ij} \le 1 \quad \forall\, i \in I$$ equation(6)

Among Equations (1), (2), (3), (4), (5) and (6), (1) is a multiple objective function that seeks to maximize the amount of the covered allocated demands $(\sum_{i\in I}\sum_{j\in J_i} a_i y_{ij})$ while simultaneously minimizing the total distance between the uncovered allocated demands and the sites to which they are assigned $(\sum_{i\in I}\sum_{j\notin J_i} d_{ij} a_i y_{ij})$. In this function, the weight $w \ge 0$ can be varied to adjust the preference on each objective. Constraints (2) ensure that all demands allocated to any facility cannot exceed the maximum capacity of that facility (i.e., the total capacities of the emergency vehicles stationed there). If no facility (i.e., no vehicle) is located on a site, no demand will be allocated to that site. Constraint (3) specifies the total number of emergency vehicles to be located. Constraints (4) ensure that all demands at each demand object will be allocated to a facility. Constraints (5) indicate that the decision variable x_j is a non-negative integer. Constraints (6) restrict the continuous decision variable y_{ij}, which ranges from 0 to 1.

We use $\min\{p, n\}$ to denote the smaller value between the total number of emergency vehicles, p, and the total number of potential facility sites, n. In the MCMCLP-NFC, emergency vehicles could be stationed in the facilities located on the sites as many as $\min\{p, n\}$, whereas the MCMCLP-FC considers fixing the total number of facilities to be sited. To present the formulation of the MCMCLP-FC, we need to introduce additional notations:

q = the total number of facilities to be sited;

K = the set of possible facility sizes (i.e., the number of vehicles) on each potential facility site $(1, \cdots, k, \cdots, p)$;

$$x_{jk} = \begin{cases} 1 & \text{if a facility with } k \text{ vehicles is located on potential facility site } j \\ 0 & \text{otherwise} \end{cases}$$

The MCMCLP-FC has the same objective function (1) and constraints (4) and (6) as the MCMCLP-NFC formulation. The other constraints include:

$$\sum_{k \in K} x_{jk} \le 1 \quad \forall j \in J$$ equation(7)

$$\sum_{i \in I} a_i y_{ij} \le \sum_{k \in K} kc x_{jk} \quad \forall j \in J$$ equation(8)

$$\sum_{j \in J} \sum_{k \in K} k x_{jk} = p$$ equation(9)

$$\sum_{j \in J} \sum_{k \in K} x_{jk} = q$$ equation(10)

$$x_{jk} \in \{0, 1\} \quad \forall j \in J, k \in K$$ equation(11)

Constraints (7) ensure that no more than one facility can be located on each potential facility site. Constraints (8) ensure that all the demands allocated to a facility cannot exceed the maximum capacity of that facility. Constraint (9) specifies the total number of emergency vehicles to be stationed. Constraint (10) specifies the total number of facilities to be sited.
Constraints (11) impose integrality restriction on the decision variable x_{jk}.

In objective function (1) for both MCMCLP models, the weight w associated with uncovered allocated demands can be varied to trade off the two objectives: the maximization of covered allocated demands and the minimization of the total distance of uncovered allocated demands to facilities. When $w = 0$, the model considers only the former objective, and the service level for the uncovered allocated demands will not be assured because they may be allocated to a further facility instead of to a nearer one. With w increases, the service level for the uncovered allocated demands will improve because more preference is given to the latter objective while the covered allocated demands may not be maximized by as many as demands as when $w = 0$. In general, maximization of the covered allocated demands would be the primary objective in emergency service planning, which means that, for a model with an appropriate weight w, the optimal solution will provide as good or better coverage of the covered allocated demand than any other feasible solutions (Haghani, 1996). With the similar proof given by Haghani (1996), we can prove that, to ensure maximization of the covered allocated demands is the primary objective, the weight w must meet the following condition when assuming integer demands:

$$0 \le w \le \frac{1}{A(d_{\max} - d_{\min})}$$ equation(12)

where A is the total demands $\sum_{i \in I} a_i$, and $d_{\max}$ and $d_{\min}$ are the maximum and minimum distances, respectively, between any pairs of demand object i and potential facility site j.

3. Spatial demand representation

Taking residents as demands, the aggregated census data may be the spatial information of demands that we can easily obtain. When information on individual activity or tracking data is not available, a practical consideration is to assume that the demands are distributed continuously within the census units. For such continuous area demands, some spatial demand representation has to be adopted so that the MCLP model can be applied. The widely used point-based abstractions may be prone to measurement and coverage errors (Murray and O'Kelly, 2002 and Tong and Murray, 2009). The areal representations with census units or grids of regular polygons often complicate the model because of the explicit processing of partial coverage caused by the mismatch between the boundaries of service covering areas and the demand areal units. To maintain both the simplicity and the high degree of accuracy of the maximal coverage model, the SASDR, which was proposed by Yin and Mu (submitted for publication), is used in this article to represent demand space.

The SASDR is a polygon-overlay-based representation for continuously spatial demands. In this representation, the demand objects are created by using the service areas of all potential facility sites to partition the whole demand space. Fig. 2(a) shows an example where a square demand space U will be partitioned into the SASDR by two potential facilities f_1 and f_2 with circular service areas S_1 and S_2.Fig. 2(b) shows the four resulting demand objects in the final SASDR, which includes $U-(S_1 \cup S_2)$, $(U-S_1) \cap S_2$, $(U-S_2) \cap S_1$, and $U \cap S_1 \cap S_2$. The biggest advantage of the SASDR is that all the demand objects lie either within or beyond the service covering standard of any potential facility site, which can avoid partial coverage in the model. With the basic functions in GIS software packages, such as buffer, overlay and network analysis, the SASDR can be easily realized.

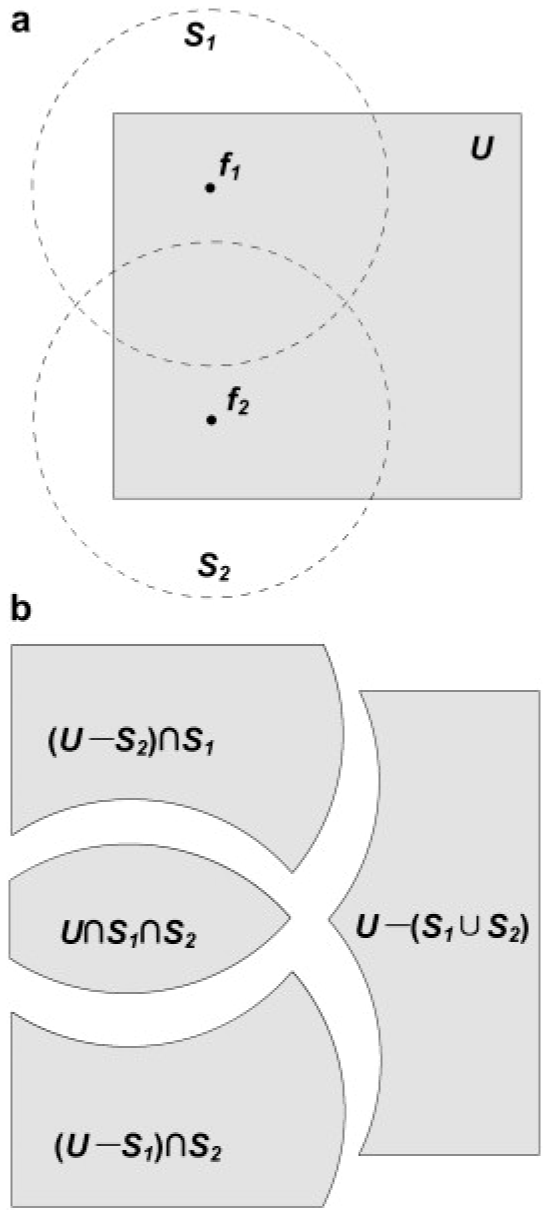

Fig. 2. Example of the SASDR with circular facility service area (a) demand space U (the square) and two potential service areasS_1 and S_2 (the circles) (b) four demand objects in the SASDR result of demand space U partitioned by service areas S_1 andS_2.

4. Applications: optimal siting of ambulances

Because of its important social and economic objectives, the ambulance location problem has been widely studied over the past 40 years (Adenso-Díaz and Rodríguez, 1997, Brotcorne et al., 2003, Daskin and Dean, 2005, Eaton et al., 1985 and Henderson and Mason, 2005). Because ambulances are usually stationed in fire departments or parking lots with little additional construction or administrative costs, it is unnecessary to limit the total number of facilities to be sited. Given this practical consideration, the MCMCLP-NFC model may be more appropriate than the MCMCLP-FC model. However, to better compare the performances of these two models, we here apply both MCMCLP-NFC and MCMCLP-FC to the optimal siting of ambulances for EMS Region 10 in GA.

4.1. Study area and data

EMS Region 10 is one of the 10 EMS regions in GA, which is in the northeastern section of GA and is composed of 10 counties (Fig. 3). The region serves 405,231 people (2000 census data) in a 3006 total square mile area with 13 licensed ambulance services and 58 vehicles (OEMS, 2006). The population in 2010 was 460,189, and the quartile map of the population density (persons/km^2) by census block group is shown in Fig. 3. The population data, boundary maps of census units, and street map are all taken from US 2010 census data because we need to reflect well the variation in demand across the study area with the population data at a relatively low spatial aggregation level, such as at the block group or block level, which are only available in census years. The Georgia EMS stations data from 2005 to 2007 are the only EMS data that we can obtain thus far; these data come from the Homeland Security Infrastructure Program (HSIP) and were downloaded from the website of the Georgia Department of Community Affairs (DCA, 2011). These data consist of the information of the locations where the EMS personnel are stationed or based, or where the equipment that such personnel use in performing their jobs is stored for ready use. According to these data, a total of 82 EMS stations provide ambulance service in our study area (Fig. 3). Among these stations, only two (Madison County Emergency Medical Services Station 4 and Greene County Emergency Medical Service) are not stationed in the fire departments. The count of EMS stations (82) is larger than the count of ambulances (58). This result may be due to the inconsistency in the time periods for which the data were collected. In addition, it is common for ambulances to be periodically relocated among facilities to insure a good coverage at all times, which is an important difference between the operations of emergency medical services and other emergency services, such as those of fire departments or police departments (Brotcorne et al., 2003). Therefore, some EMS stations may not site the vehicles all the time. Although the population data and EMS data for different time periods are used, the time interval between these data is short; the time inconsistency is therefore ignored in this application until better-quality data become available. This data input is not the critical part of the models and should not significantly influence the illustration and validation of our models and their applications.

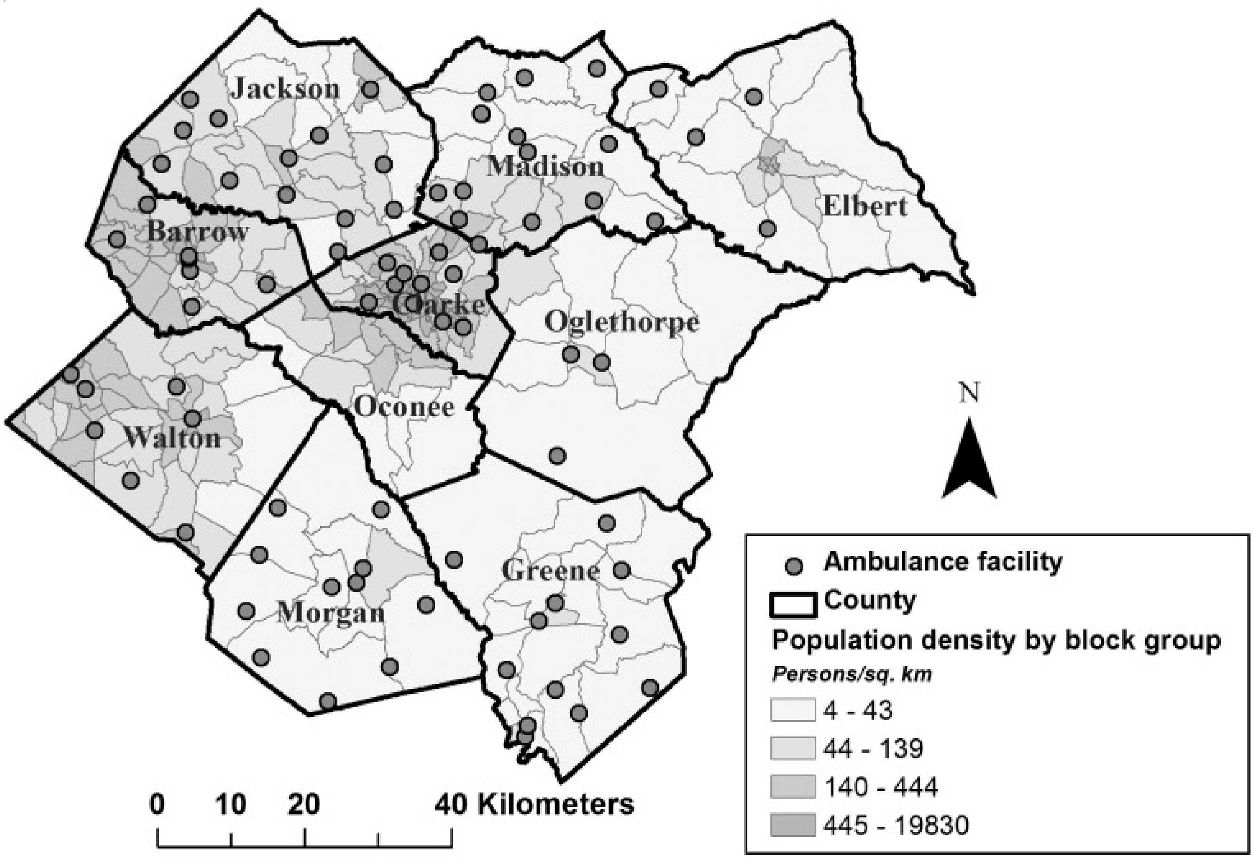

Fig. 3. Population density of Georgia EMS Region 10 (study area) by census block group and existing ambulance facility locations.

4.2. Tasks

To test the application of the MCMCLP for emergency services, a total of 58 ambulances will be allocated to maximize the covered allocated demands within 8-min driving distance from the facilities. The locations of 82 existing EMS stations are regarded as the potential facility sites. The demands are represented by the census population in 2010 by census block group. To ensure the existence of a feasible solution to the problem, we define the capacity of each ambulance as 8000 persons so that 58 ambulances have total capacity of 464,000, which exceeds the total demand of 460,189. We assume that the capacity of 8000 persons per ambulance can meet the requirement of the average response time to the calls for service in this region. In the MCMCLP-NFC model, the 58 vehicles could be allocated to, at most, 58 facility sites. In the MCMCLP-FC model, only 20 potential facility sites will be chosen, and the 58 vehicles will be allocated to these 20 sites. ArcGIS™ v9.3.1 is used to realize the SASDR. Programming with Visual Basic for Applications (VBA) for ArcObjects in ArcGIS™ v9.3.1 is used to structure the optimization model files. The optimization problems are then solved using the commercial mixed integer programming (MIP) software package CPLEX v12.2. All analyses are performed on a personal computer equipped with an Intel Core Quad 2.4 GHz CPU and 3 GB of RAM.

4.3. Results

4.3.1. Realization of SASDR

In the realization of SASDR, three types of roads are used to create the road network and then to create the 8-min service area for each potential facility site. The information for roads is listed in Table 1 and includes the MAF/TIGER Feature Class Codes (MTFCC) defined in the census data, road descriptions and hypothetical speed limits. Fig. 4 shows the road network in the study area.

Table 1. Information for roads.

MTFCC	Description	Speed limit (miles/h)
S1100	Primary road	70
S1200	Secondary road	55
S1400	Local neighborhood road, rural road, city street	40

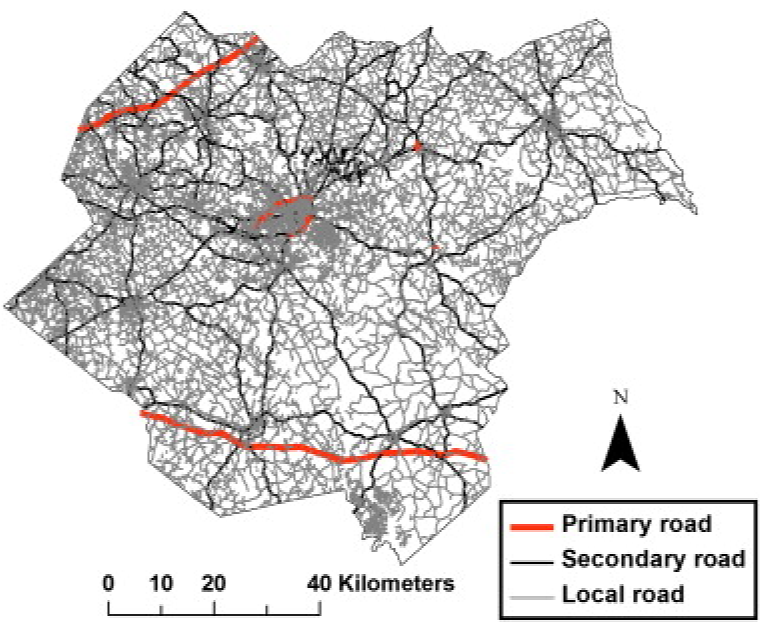

Fig. 4. Road network in the study area.

After the road network is created, a service layer that includes the 8-min service polygons for the 82 potential facility sites is created from the road network using the network-analysis functions in ArcGIS (Fig. 5). The white areas indicate that no vehicles can reach these locations within 8 min from any potential facility location. Each service polygon was identified by the ID of its corresponding facility site.

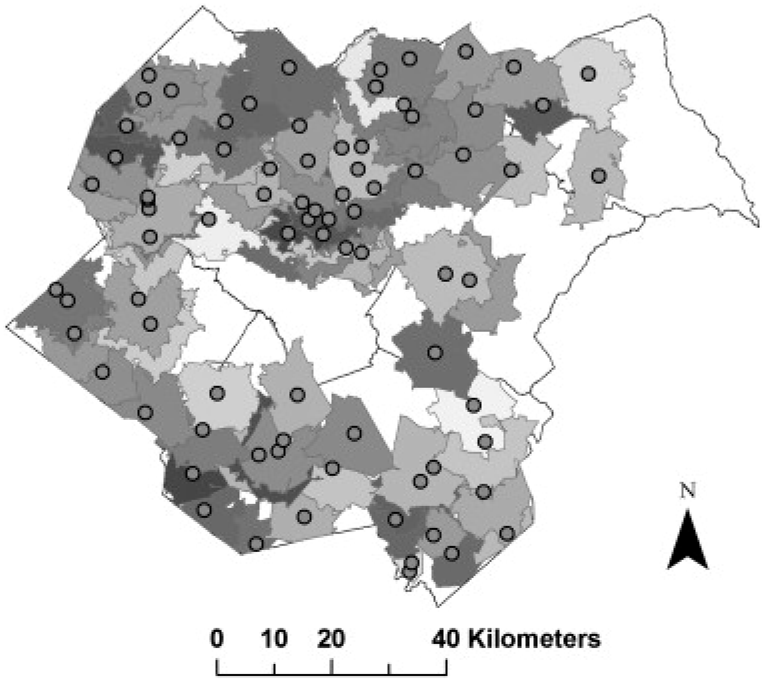

Fig. 5. Eight-minute service areas (non-white polygons) of all potential ambulance facility sites (red points) based on the road network. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

With the polygon overlay tool "Identity" in ArcGIS, the service layer is used to partition the study area to derive the partition layer that includes all intersecting units among the service polygons and the study area. Because of possible overlap among the service polygons, the partition layer may include duplicate intersecting units that have the same location and shape but different facility site IDs. A new field, "DO_ID", is created in the partition layer, and the "Field Calculator" function in ArcGIS with VBScript is used to compare the centroid coordinates and the area of each unit to identify the duplicate units. All units that represent the same demand object will be assigned the same demand object ID in the field "DO_ID". In the attribute table of the partition layer, both facility site ID and demand object ID now exist in each record. The facility site j in the record of the demand object i indicates that the demand object i can be completely covered by the service from the potential facility site j. This information will later be used to construct the model input file for CPLEX to solve the problem. A total of 2721 demand objects are obtained for the study area. We export them from the partition layer to create the demand object boundary layer.

The next step for the realization of SASDR is to calculate the amount of demands in each demand object, which will be interpolated from the census block group population data and assumed to be distributed uniformly within the demand object. When the polygon overlay tool "Intersect" in ArcGIS is used to overlay the layer of population density by block group on the demand object boundary layer, many intersecting units will emerge. The population in each unit is calculated by timing its population density with the size of that unit. Finally, the population of the intersecting units is aggregated to the demand objects. Fig. 6 shows the final SASDR result for the study area with demand (i.e., population) distribution. Because of the round-off error, a total aggregated population of 460,219 in the study area is obtained, which is then used as the total amount of demands in the subsequent model. There are 623 demand objects with no people because of their small sizes and low population densities. These zero-population demand objects are first excluded from the optimization problem to reduce the computing complexity. After the optimization problem is solved by CPLEX, these demand objects will be brought back and allocated to their nearest facilities.

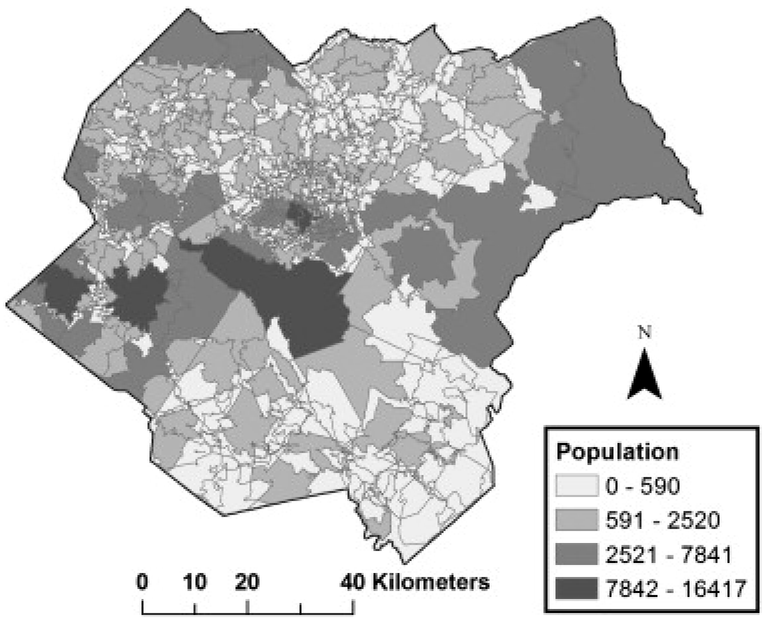

Fig. 6. SASDR result for the study area with demand (population) distribution.

4.3.2. Model construction and solution

The distance between demand object and facility location is measured from the centroid of the demand object to the facility location point in kilometers. The maximum distance in this study area is 33.377 km and the minimum distance is 2.683×10^{-2} km. According to Equation (12), the value of weight *w* should be within the range $[0, 6.515 \times 10^{-8}]$ to ensure that the maximization of the covered allocated demands is the primary objective. In fact, as long as the value of weight *w* falls in this range and does not equal zero, the solutions of each model will be the same, irrespective of the weight *w*. Therefore, we set $w = 6 \times 10^{-8}$ for both the MCMCLP-NFC and MCMCLP-FC models.

The model input files were constructed with the VBA program of ArcObjects in ArcGIS. These models were then solved in CPLEX, which uses a branch-and-cut technique to find the optimal solution (CPLEX Help, 2011). The run time is 3361 seconds for the MCMCLP-NFC model and

706 seconds for the MCMCLP-FC model. The solutions obtained from CPLEX were finally visualized as maps in ArcGIS.

Fig. 7 shows the results of two MCMCLP models using the choropleth maps overlaid with selected facility sites. In these maps, the facility and the demands allocated to it are represented in the same colors, and larger facility symbols indicate more ambulances. With such maps, the location-allocation patterns of the problem solution can be easily understood. For those demand objects whose demands will be divided and allocated to more than one facility, the strategy here is to split the demand object into multiple parts. For each facility that partially serves the demand object, there is a part in the demand object trying to be close to that facility, and its size is proportional to the percentage of demands served by that facility. In Fig. 7(a), in which the MCMCLP-NFC is applied, a total of 51 out of 82 potential sites are chosen to set up the facilities, and 402,365 demands (87.4% of total demands) are covered within the 8-min service covering standard. In Fig. 7(b), in which the MCMCLP-FC is applied, 20 facilities are required by the problem specification, and 358,477 demands (77.9% of total demands) are covered within the service covering standard. As expected, the amount of the covered allocated demands obtained by the MCMCLP-NFC is greater than that obtained by the MCMCLP-FC because more facilities in the MCMCLP-NFC provide greater flexibility for siting the ambulances. Because the proximity of the uncovered allocated demands to the facilities is considered in both models (i.e., $w = 6 \times 10^{-8}$), the demands allocated to a facility are generally distributed more compactly and more continuous than those in the models with $w = 0$ (results not shown). However, the allocations of many facilities are still dispersed into several parts that may be far away from one another. For example, there are two major demand patches with varied sizes (filled with diagonals) allocated to the facility at site 13 in Fig. 7(a). One reason for this allocation is that the primary objective of the models is to maximize the covered allocated demands instead of the proximity of the uncovered allocated demands to the facilities. The splitting operation of the demand objects to represent the partial coverage could also cause the noncontinuous demand allocations in the maps. Because of the smaller number of facilities established, the MCMCLP-FC shows a more compact and continuous distribution of the demands than the MCMCLP-NFC shows.

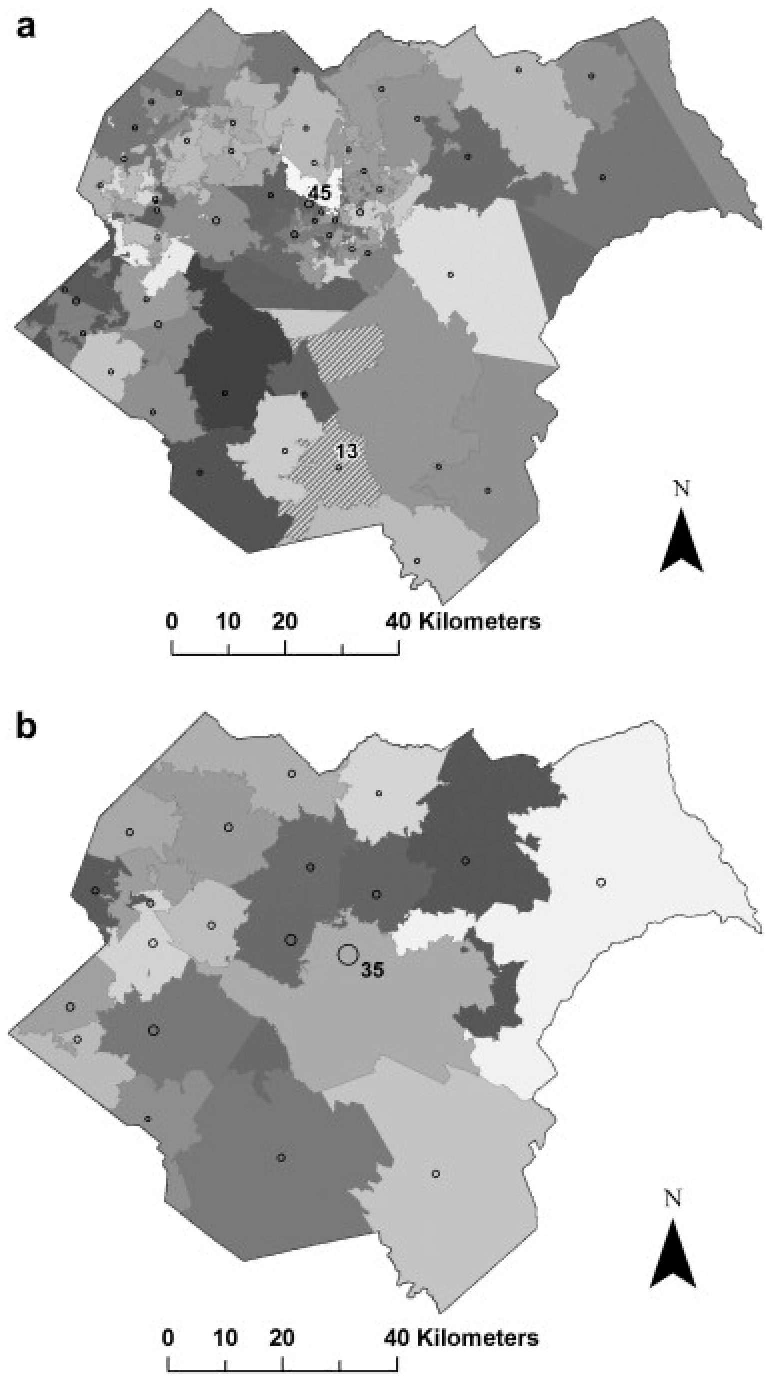

Fig. 7. Results of the MCMCLP models siting 58 ambulances in 82 potential facility locations with $w = 6 \times 10^{-8}$ (the facility location is rendered in the same color as its allocation area) (a) the MCMCLP-NFC model (b) the MCMCLP-FC model with 20 facilities.

Table 2 shows the counts of the facilities with varied numbers of ambulances in these two models. The maximum number of ambulances in a facility is 3 (site 45 in Fig. 7(a)) in the MCMCLP-NFC model and 12 (site 35 in Fig. 7(b)) in the MCMCLP-FC model.

Table 2. Count of the facilities with varied numbers of ambulances.

Number of ambulances in a facility	Count of facilities	
	MCMCLP-NFC	MCMCLP-FC
1	45	2
2	5	10
3	1	5
4	0	1
5	0	1
12	0	1
Total	51	20

5. Discussion

Several assumptions are made in this article to apply the MCMCLP models to optimally site emergency vehicles such as ambulances. One assumption is that a facility has a capacity that is related to the vehicles stationed there. This assumption is simple but reasonable. If the population in the jurisdiction of a facility is too large, one of the important indicators for the emergency service quality, the average response time to the calls for emergency service, will be too long. When the population exceeds a limit, the quality of the emergency service provided by that facility will be unacceptable. Given a requirement on the average response time to the calls, a facility with more vehicles may serve a greater population. In our application, for simplicity, we assume that each vehicle has the same capacity and that the capacity of a facility is equal to the total capacity of the vehicles located there. Admittedly, this is a very restrictive assumption because the capacity of an emergency vehicle actually depends on multiple factors, including the requirement on the average response time, the average frequency of calls in the population it will serve, and the average treatment time for a task, among others. A discussion of this problem exceeds the scope of this article. However, if the possible capacity levels of the facility at each potential site can be estimated and taken as a group of constants, the MCMCLP model can be easily modified to accommodate the situation. The location problems of emergency vehicles are, in reality, complex. The MCMCLP is a static model that does not consider the dynamic factors such as the daily population movement. Accounting for such factors will be the focus of our future work.

The MCLP has been proven to be nondeterministic polynomial time (NP)-hard (Megiddo, Zemel, & Hakimi, 1981), which means that no algorithm has yet been discovered to solve it in polynomial time in the worst case. As an extension to the MCLP, the MCMCLP is also NP-hard. Therefore, the use of exact methods (e.g., enumeration or linear programming with branch-and-bound) to solve a large-scale MCMCLP will be difficult. Seeking heuristic methods (e.g., genetic algorithm or Lagrangian relaxation) is important for promoting the applications of the MCMCLP. A potential heuristic method for solving the MCMCLP is a two-phase procedure, in which the locations of the facilities and the demand allocation are first determined under the assumption that the facilities are uncapacitated; the emergency vehicles are then allocated to each facility depending on the allocated demands. We note that this two-phase procedure does not consider that the second phase may change the demand allocation determined by the first phase, which will cause the configuration of facility locations determined by the first phase to not necessarily be the optimal solution for the whole problem.

Although model formulation and the optimization of algorithms are always the focus in location modeling, many other aspects of the location problem, such as the representation for spatial demands, also influence the accuracy of the modeling solutions and require attention. An effective visualization of the problem solutions will be helpful in understanding the location-allocation patterns and in making decisions by comparing different modeling results. One problem that we need to address for our MCMCLP models in the future is how to better represent in the map the demand objects served by multiple facilities.

In the MCMCLP model, GIS plays an important role. It is used to manage and organize the spatial data, to realize the spatial demand representation, to help construct the model input file for optimization software packages, and to visualize the problem solution with maps. In addition to these important functions, GIS also facilitates theoretical advances in current location science (Church, 2002 and Murray, 2010).

6. Conclusion

The MCMCLP that we proposed in this article is an extension of the capacitated MCLP to accommodate situations where the facilities to be sited have several possible capacity levels. For the optimal siting of emergency vehicles, the MCMCLP considers the modular capacity levels of a facility, the allocation of all demands, and the proximity of the uncovered allocated demands to facilities. Two situations–the MCMCLP-NFC and the MCMCLP-FC–can be used depending on the circumstances of the facility. In cases where the cost of a facility is low and maximization of the covered allocated demands is the main purpose, such as establishing bases for ambulances that are not always based in a building but are often at a very rudimentary location such as a parking lot (Brotcorne et al., 2003), the MCMCLP-NFC may be more useful because more covered allocated demands are generally obtained than with the MCMCLP-FC. If the cost of facilities is also an important consideration, such as with fire stations for fire trucks, the MCMCLP-FC may be better because we can incorporate information about how many facilities we can build in the location modeling.

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