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VARIATION OF RADIO SOURCE COUNTS WITH DIRECTION, FOR THE 3CR AND 4C SURVEYS

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SUMMARY

In an investigation of variations with direction of the radio source counts $N(S)$ for the 3CR and 4C surveys, no anisotropy has been found which is not explicable in terms of random statistical differences between limited samples of sources; in particular, there is no significant difference between the slopes of the source counts in the northern and southern galactic hemispheres.

1. INTRODUCTION

The counts of extragalactic radio sources have been studied extensively, as they reflect both the geometry of the Universe and the spatial distribution of the sources within it, although these two properties cannot be separately determined from the source counts alone. In interpreting the counts it is usually assumed that the sources are isotropically distributed in an isotropic metric, and it is important to determine whether the source counts themselves support this assumption.

It has been established (e.g. Ryle 1968; Longair 1971) that at high flux densities ($S_{178} \gtrsim 10^{-26}$ W m^{-2} Hz^{-1}) the function $N(S)$, the number of sources in unit solid angle with flux density greater than S, does not agree with the predictions of any isotropic cosmological model in which radio sources are distributed uniformly; all such models predict that the logarithmic slope of the source counts

$$\beta = -d(\log N)/d(\log S)$$

should be less than 1·5, whereas β is observed to be about 1·8. Recently, however, it has been suggested that β varies with direction, and that the value of 1·8 is not typical of the whole sky.

Pauliny-Toth & Kellermann (1972) have found that in the NRAO survey at 5 GHz this 'anomalously' large value of β is confined to sources with steep spectra ($\alpha > 0{\cdot}5$, where $S \propto \nu^{-\alpha}$) in the northern galactic hemisphere. Yahil (1972) has compared estimates of β in the northern and southern galactic hemispheres for two of the NRAO surveys, two Parkes surveys (at 2·7 GHz and 408 MHz) and for the Third Cambridge survey (3CR, 178 MHz). In all cases he found, in agreement with Pauliny-Toth & Kellermann, that β is larger in the northern than in the southern hemisphere, though the difference is not of great statistical significance in most cases. Katgert *et al.* (1973) have suggested that there may be variations in $N(S)$ on a scale of about 10 square degrees (1·4 GHz).

This paper describes a statistical investigation of variations in the shape of $N(S)$ and the value of β between different parts of the 4C survey. This covers a larger area than the NRAO surveys, and has a lower limiting flux density than the

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3CR and the Parkes surveys. If, therefore, the anisotropy detected in other surveys is present at low frequencies and low flux densities, this analysis should detect and locate it. For comparison, the methods have also been applied to the 3CR survey.

Section 2 describes the relevant details of the 3CR and 4C catalogues, Section 3 the statistical methods used and Section 4 the results. In Section 5 some instrumental properties which might affect the results are considered, and the conclusions are presented and discussed in Section 6.

2. THE SOURCE CATALOGUES

The 3CR and 4C catalogues were both compiled from surveys made at 178 MHz. 3CR is complete for $S_{178} \geq 9 \times 10^{-26}$ W m^{-2} Hz^{-1}, and 4C for $S_{178} \geq 2{\cdot}0 \times 10^{-26}$ W m^{-2} Hz^{-1} (Table I).

TABLE I

The source catalogues

Survey	Region of completeness	Limiting flux density S_{178} (10^{-26} W m^{-2} Hz^{-1})	Number of sources with $\lvert b \rvert \geq 20°$
3CR (Bennett 1962a)	$\delta > -5°$	9·0	212
4C (Pilkington & Scott 1965; Gower, Scott & Wills 1967)	$-7° < \delta < 80°$	2·0	3161
	(1)	9·7 (2)	153

(1) A number of small areas in which the sidelobes of bright sources raise the limiting flux density have been excluded. These sources comprise about 3 per cent of the area surveyed and contain 35 sources.
(2) For comparison with 3CR.

The 4C survey was made with an aperture-synthesis interferometer of spacing 469 λ, oriented east–west. The response of the interferometer varied with zenith angle, and hence with declination; the small statistical corrections to the results that this necessitates are discussed in Section 5.

Holden (1966) has investigated the isotropy of the 4C catalogue by comparing the total numbers of sources in equal areas of the sky. There were no significant departures from the expected Poisson distribution in areas of 25 deg^2 or more. This result indicates that the *normalization* of the source counts does not vary significantly with direction, but it does not exclude the possibility of small variations in the *shape* of the curve $N(S)$, in particular at high flux densities. The present investigation, which is not concerned with normalization, is thus complementary to Holden's, like that of Yahil (1972).

Holden's result has been extended to lower flux densities ($0{\cdot}2 < S_{178} < 1 \times 10^{-26}$ W m^{-2} Hz^{-1}) by Hughes & Longair (1967) who studied the original records of the 4C survey using the statistical method of Scheuer (1957) and Hewish (1961). They found no evidence for variations of N on scales of 20° or more. Thus there is no evidence for clustering of sources at 178 MHz at source densities comparable with those considered by Katgert *et al.* (1973) at 1·4 GHz.

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3. STATISTICAL METHODS

3.1 *The function $N(S)$*

Over the range of flux density covered by 3CR and 4C, the function $N(S)$ is conventionally represented by a power law

$$N(S) \propto S^{-\beta} \tag{1}$$

and the observed sources are assumed to constitute a random sample from a (hypothetical) population with this distribution. Then the probability density function for a source, taken at random from a survey complete for $S \geqslant S_0$, to have flux density $S = sS_0$ is

$$p(s) = \beta s^{-(\beta+1)}$$

and the variable $s^{-\beta}$ is uniformly distributed between 0 and 1.

The power law hypothesis (1) can be tested by the usual statistical goodness-of-fit methods. Crawford, Jauncey & Murdoch (1970) suggest that, if ζ is their maximum-likelihood estimate of β, then one should compare the distribution of $s^{-\zeta}$ with the expected uniform distribution. For grouped data, as in the 3CR and 4C catalogues, the appropriate test is the χ^2-test (e.g. Eadie *et al.* 1971). The maximum-likelihood estimate of β for 3CR ($|b| \geqslant 20°$) is $\zeta = 1{\cdot}90$ (in agreement with Bennett 1962b) and the χ^2-test shows no departure from a power law at the 5 per cent level of significance. For the 4C survey, the fit to a power law is not so good (β varies slightly with S), but the statistic ζ should nevertheless be characteristic of the distribution.

3.2 *Comparison of zones using a single statistic*

We want to compare the functions $N(S)$ derived from observations of two different parts of the sky. If both samples of sources follow a power law, then *any* statistic calculated from the samples should reflect the values of β, and a difference in the statistics implies a difference in β. In principle, the statistic should be chosen to contain as much as possible of the information in the sample, and it is this criterion which leads us to choose the maximum-likelihood estimate, ζ, as the best estimate of β (if the population follows a power law). Yahil (1972) used the statistics $\langle x \rangle$ (where $x = s^{-3/2}$) and F (the fraction of sources with $x \geqslant 0{\cdot}5$). Here, all three statistics have been used to compare the zones, although they are not independent and are not all equally good for the purpose. In particular, F contains little information about the sample and is consequently a poor estimator of β. $\langle x \rangle$ gives more weight to the faint sources than ζ does, so we can expect differences between ζ and the estimate of β derived from $\langle x \rangle$. The behaviour of the statistics has been investigated by a Monte-Carlo method and will be discussed further in Section 6. Some properties of the three statistics are given in Table II.

In the limit of large samples, the sampling distribution of each statistic approaches a normal distribution, and in this limit a difference of more than 2σ (where σ is the standard error in the difference, estimated as shown in Table II) between the statistics obtained from two samples constitutes evidence for rejecting, at the 5 per cent confidence level, the hypothesis that the two samples are taken from identically distributed populations.

In order to have small standard errors, the samples must naturally be large. For example, if a 10 per cent difference in β is to be significant at the 2σ (5 per cent) level in a comparison of two equal samples, then each sample must contain rather

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TABLE II
Statistics

Statistic	Definition	Value, for $N(S) \propto S^{-\beta}$	$\beta = 1{\cdot}5$ (1)	Asymptotic estimates of: Standard error	Asymptotic estimates of: Standard error in the difference between two samples
$\langle x \rangle$	Mean value of $x \equiv (S/S_0)^{-3/2}$	$\frac{\beta}{\beta+\frac{3}{2}}$	0·5	$\frac{\sigma}{M^{1/2}}$ where $\sigma = \langle x^2 \rangle - \langle x \rangle^2$	$\left(\frac{\sigma_1^2}{M_1}+\frac{\sigma_2^2}{M_2}\right)^{1/2}$
F	Fraction of sources with $x \geqslant 0{\cdot}5$	$1-2^{-2\beta/3}$	0·5	$\left(\frac{F(1-F)}{M}\right)^{1/2}$	$\left\{P(1-P)\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\right\}^{1/2}$ $P = \frac{(M_1F_1+M_2F_2)}{(M_1+M_2)}$
ζ	Maximum-likelihood estimate of β: (2) $\frac{M-1}{M\langle \ln (S/S_0) \rangle}$	β	1·5	$\frac{\zeta}{M^{1/2}}$	$\left(\frac{\zeta_1^2}{M_1}+\frac{\zeta_2^2}{M_2}\right)^{1/2}$

M = number in sample; suffixes 1, 2 refer to the two samples.
(1) $\beta = 1{\cdot}5$ in a static, Euclidean universe.
(2) Over the range of flux density $S_0 \leqslant S < \infty$.

more than 200 sources (the standard error in the difference being about $\beta(2/M)^{1/2}$). For the surveys considered here a difference of 2σ corresponds to a difference in β of about 30 per cent (3CR), or 8 per cent (4C), for the subdivision into northern and southern galactic hemispheres.

4. RESULTS

The statistics of Section 3.2 were calculated for each of the following subdivisions of the sky, and the differences found between the zones are expressed in terms of the estimated standard errors in Table III. In all cases sources with $|b| < 20°$ were excluded (to limit the analysis to extragalactic sources) and the sources in the 4C sidelobe areas were also excluded. In some cases the 4C survey was analysed with a limiting flux density $9{\cdot}7 \times 10^{-26}$ W m^{-2} Hz^{-1} (corresponding to 9·0 on the scale of 3CR) for comparison with the 3CR survey. It should be emphasized that the statistics presented in the Table were calculated from the flux densities quoted in the catalogues, taking no account of the corrections necessary for the varying response of the instruments. It will be shown in Section 5 that to include the corrections would not alter significantly the *differences* between the zones.

(a) *Northern and southern galactic hemispheres* (*Table III(a)*)

There are no significant differences between the two hemispheres, contrary to the result of Yahil (1972) and (at 5 GHz) of Pauliny-Toth & Kellermann (1972) that β is consistently greater in the northern than in the southern hemisphere.

For 3CR, the three different statistics give widely different results. In particular, the maximum-likelihood estimate of β is greater in the southern than in the northern hemisphere, contrary to the trend shown by Yahil's two statistics. This can be attributed largely to the small size of the samples (see Section 6).

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TABLE III

Estimates of β and other statistics from 3CR and 4C. These statistics have been calculated using the formulae of Table II (ignoring the grouping of the data)

		Zone	M	$\langle x\rangle$	$\Delta\langle x\rangle/\sigma$	F	$\Delta F/\sigma$	ζ	$\Delta\zeta/\sigma$	β (from $\langle x\rangle$)	β (from F)
(a) 3CR	$S_0 = 9·0$	North	150	0·593	0·85	0·633	1·58	1·90	−0·36	2·18	2·17
		South	62	0·558		0·516		2·00		1·89	1·57
4C	$S_0 = 9·7$	North	106	0·575	−0·81	0·623	0·07	2·11	−0·69	2·03	2·11
		South	47	0·611		0·617		2·39		2·35	2·08
	$S_0 = 2·0$	North	2144	0·556	−0·07	0·565	−0·60	1·89	−0·29	1·88	1·80
		South	1017	0·557		0·576		1·91		1·89	1·86
(b) 3CR	$S_0 = 9·0$	Centre	73	0·562	−0·74	0·603	0·08	1·58	−2·30	1·93	2·00
		Anticentre	139	0·593		0·597		2·18		2·19	1·97
4C	$S_0 = 9·7$	Centre	58	0·599	0·50	0·655	0·68	2·20	0·03	2·24	2·30
		Anticentre	95	0·578		0·600		2·19		2·05	1·98
	$S_0 = 2·0$	Centre	1196	0·573	2·53	0·595	2·38	1·98	1·88	2·01	1·96
		Anticentre	1965	0·547		0·552		1·84		1·81	1·74
(c) 4C	$S_0 = 2·0$	$b \geqslant 45°$	1148	0·555	−0·21	0·562	−0·30	1·88	−0·06	1·87	1·79
		$20 \leqslant °b < 45°$	996	0·558		0·568		1·89		1·89	1·82
		$-20° \geqslant b > -45°$	691	0·567	1·71	0·590	1·34	1·97	1·47	1·97	1·93
		$b \leqslant -45°$	326	0·535		0·546		1·79		1·73	1·71

Column:

1 Survey: S_0 = limiting flux density (10^{-26} W m^{-2} Hz^{-1}).

2 Zone: North $b \geqslant 20°$
South $b \leqslant -20°$
Centre $-90° \leqslant l < 90°$ } $|b| \leqslant 20°$
Anticentre $90° \leqslant l < 270°$ }

3 Number of sources in zone, M.

4 $\langle x\rangle$.

5 Difference of $\langle x\rangle$ in units of the standard error.

6 F.

7 Difference of F in units of the standard error.

8 ζ.

9 Difference of ζ in units of the standard error.

10 For comparison, an estimate of β calculated from $\langle x\rangle$:

$$\beta \doteq \frac{1·5\langle x\rangle}{1-\langle x\rangle}.$$

11 An estimate of β calculated from F:

$$\beta \doteq \frac{1·5 \log (1-F)}{\log (0·5)}$$

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For the large samples derived from 4C the agreement between the statistics is much better, and the differences between the hemispheres are insignificant.

(b) *Two zones at different galactic longitude* (*Table III*(*b*))

This subdivision is orthogonal to that of (a) and may detect any asymmetry associated with the Galaxy.

In 3CR there are again large differences in the behaviour of the three different statistics: ζ shows a significant difference between the zones, while $\langle x \rangle$ does not.

For 4C, there are negligible differences between the zones for $S_0 = 9{\cdot}7$, but for $S_0 = 2{\cdot}0$ the differences are more than 2σ for two of the statistics, significant at the 5 per cent level. β is larger in the 'centre' region than in the 'anticentre'.

(c) *Dependence on galactic latitude* (*Table III*(*c*))

Each hemisphere was divided into two parts, at $|b| = 45°$. (Only the 4C survey was analysed, in order to have large samples.) There are no significant differences, but the estimates of β are consistently smaller for the zone $b \leqslant -45°$ than for the rest of the southern hemisphere. There is no comparable effect in the northern hemisphere.

(d) *Dependence on declination*

Fig. 1 is a histogram showing ζ as a function of declination for the 4C survey. Any systematic variation of ζ with declination is seen to be much smaller than the statistical uncertainty in ζ.

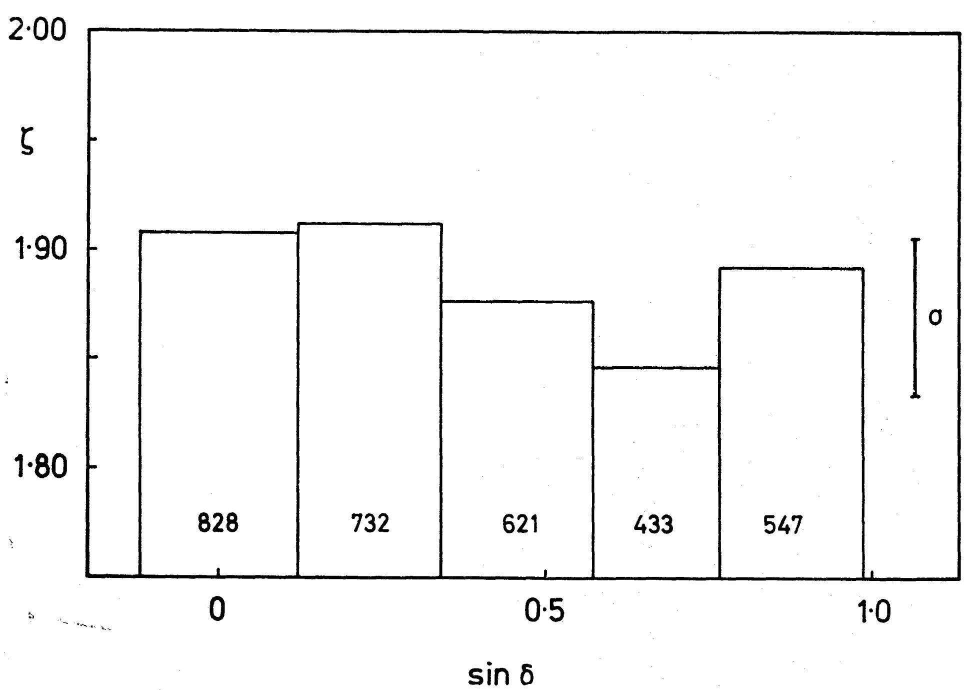

FIG. 1. *Maximum likelihood estimate* (ζ) *of the parameter* β *as a function of declination* (δ) *for the 4C survey. The number of sources in each declination range is shown on the diagram. The length of the vertical bar is approximately equal to the standard error,* σ, *in each value of* ζ.

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5. INSTRUMENTAL EFFECTS IN 4C

While the flux densities quoted in the 4C Catalogue are the best possible estimates of the flux densities of individual sources, the survey suffers from some unavoidable instrumental effects which introduce random errors into the measured flux densities. Gower (1966) has enumerated these effects and derived the necessary statistical corrections to the source counts. In the present context, the magnitude of the corrections is less important than their variation with direction; in particular, because the polar diagram of the 4C instrument varies with declination so also do the statistical corrections. In this Section it will be shown that the variation of the corrections with direction is small enough to be ignored.

(a) *Effect of confusion*

Gower used a Monte-Carlo method to determine the size of the overestimate of $N(S)$ due to confusion. His results, which are applicable for $4° < \delta < 80°$, are shown in Table IV, together with the corresponding overestimate of $N(S)$ at low declinations derived as described in the caption. The variation with declination of the correction to β is less than 1 per cent and, while it could account for part of the variation shown in Fig. 1, it is sufficiently small to be safely ignored in comparing the zones.

TABLE IV

Overestimates of $N(S)$ due to confusion

S 10^{-26} W m^{-2} Hz^{-1}	Percentage overestimate of $N(S)$ $4° < \delta < 80°$ (1)	$\delta = -7°$ (2)
2	13	14
3	13	13
5	10	12
7	8	10
10	5	6

(1) Gower (1966).

(2) Based on the assumption that the overestimate of $N(S)$ depends only on the number of sources (brighter than S) per beam area. The beam area varies as sec z; β is assumed to be 1·8.

(b) *Effect of partial resolution*

An extended source of diameter $\gtrsim 2'$ arc, or a double source with component separation $\gtrsim 1'{\cdot}5$ arc, is likely to have been observed with reduced intensity by the 4C interferometer. The lobe-spacing of the interferometer response does not vary with declination, so that this correction does not depend on declination.

(c) *Variation of gain with direction*

The gain of a phase-switched interferometer such as the 4C instrument depends on the brightness of the sky background in the beam. This could cause systematic variations with direction, especially where the galactic plane is seen by the instrument. Although the 4C catalogue was corrected as far as possible for these calibration errors, Gower estimated that there is a residual standard error in the quoted flux densities of about 5 per cent. The galactic plane has been excluded from the present analysis, and thus most of the systematic variation has been eliminated.

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The 3CR survey is affected much less than 4C by the effects discussed in this Section, and we can conclude from this discussion that for the present analysis these effects can be neglected in both 3CR and 4C.

6. DISCUSSION

6.1 *3CR*

It is notable that the results shown in Table III for the three different statistics differ widely and show no consistent asymmetry of β. To determine how much of this scatter might be due to statistical fluctuations and the small size of the samples, a series of Monte-Carlo trials was made, drawing samples of size 150 and 62 from a random population with a power law distribution ($\beta = 1{\cdot}9$). This showed that the differences between the statistics can be attributed almost entirely to statistical fluctuations. The results also showed that the estimate of β derived from F has a large variance, as predicted theoretically, confirming that F is a poor statistic. The variance of ζ is similar to that of $\langle x \rangle$, but while ζ gives equal weight to all the sources, $\langle x \rangle$ gives more weight to the faint sources. This accounts qualitatively for the differences between ζ and the estimate of β derived from $\langle x \rangle$.

Similar considerations apply to the samples with $S \geqslant 9{\cdot}7 \times 10^{-26}$ W m^{-2} Hz^{-1} drawn from the 4C catalogue. It is difficult to compare the magnitudes of β derived from these samples directly with those from 3CR because a number of sources are missing from 4C, either because they fall in the sidelobe areas or because they are resolved by the interferometer.

6.2 *4C*

There is no evidence from Table III for any asymmetry in 4C between the northern and southern galactic hemispheres. β is slightly greater in the southern than in the northern hemisphere, contrary to the trend shown at 5 GHz by the NRAO surveys, but the difference is only 1 per cent in β and is by no means comparable with the difference between the observed β and the Euclidean value of 1·5. For the NRAO survey at 5 GHz with a limiting flux density $S_0 = 0{\cdot}6 \times 10^{-26}$ W m^{-2} Hz^{-1} the difference between the two hemispheres in $\langle x \rangle$ (Yahil 1972) is 2·5 σ, and in ζ (for a larger sample of sources, Pauliny-Toth & Kellermann 1972) is 2·2 σ. If there were a real difference in β of this magnitude at all flux densities in the 4C survey, for which the samples are much larger, it would appear as a difference of about 8 σ in Table III(a).

The difference in β of about 10 per cent between 'centre' and 'anticentre' regions is, however, significant at the 5 per cent level, although again insufficient to require a new cosmological interpretation of the source counts. The difference in slope occurs mainly below 3×10^{-26} W m^{-2} Hz^{-1}, where the (uncorrected) source counts begin to flatten: a similar analysis with $S_0 = 3 \times 10^{-26}$ W m^{-2} Hz^{-1} showed that there is no significant difference in slope above this limit. The anisotropy is therefore confined to a small range of flux density near the limit of the survey, and could possibly be due to an instrumental effect. Even if the difference in slope is real, it is not remarkable that a significant result is found for *one* of the subdivisions.

6.3 *Conclusion*

This analysis has produced no evidence for anisotropy in the source counts at 178 MHz derived from the 3CR and 4C catalogues, and this result is consistent

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with that of Holden (1966). There is no evidence for the north–south asymmetry which has been found at 5 GHz from rather smaller samples of sources than those from 4C used here (Pauliny-Toth & Kellermann 1972; Yahil 1972).

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