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BYSTANDERS' GUIDE TO SOCIATIVE LOGICS

RICHARD SYLVAN

BYSTANDERS' GUIDE TO SOCIATIVE LOGICS:

A short interim edition

by

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PREFACE

As the working typescript of *Directions in Relevant Logic* (DRL) grew too long to publish at all feasibly in one book, as it far surpassed the generous but strict page limits I had been given, it suddenly became evident that there was a book within the book, which stood on its own, and offered something that would be lost were it buried within the larger text. That book within the book is this book, which comprises the original introductions to the larger DRL. What this overflow offers that would have been buried is simply this: a philosophical guide to sociative logics, their variety and range, their motivation, their history and development, their features, their strengths and weaknesses, their prospects.

In a *sociative* logic, premisses and conclusion of an argument, or correspondingly antecedents and consequent of a valid implication, are associated; they characteristically have *enough* to do with one another. To that extent any such logic is broadly relevant. But what have become known as relevant logics, the best known of which are the relevance logics largely forged in dirty Pittsburgh, comprise only a quite proper subclass of the broader class of sociative logics, many kinds of which have a much longer and more substantial history than relevant logics. If the story elaborated in this guide is correct early sociative logics did not arise in reaction to noxious irrelevant products; these logics were the original logics. The oversimplification and excessive power of irrelevant systematisation only *came later*; and then especially in the later middle ages and contemporary times when such systematisation came to dominate, there was a due, though substantially ineffectual, reaction against it.

What the later sociative logics that developed in reaction have in common is primarily the aim to avoid the most obvious paradoxes of dominant logical theorising. The plurality of logical theories and sketches that now make up sociative logics share little else however. There is no common commitment, for instance, to supply an account of entailment, or a theory of relevance, or a technical story of the use of premisses in argument, though there are bound to be commitments to some more central logical enterprises or other, such as elaboration of a theory of argument or inference, an account of reasoning, explication of conditionals, and so on. What these different commitments were and are, and ought to be, will begin to unfold as the story proceeds.

Naturally, the notation, referencing conventions, and so forth, deployed in this overflow are uniform with those of DRL. But, where these details are not sufficiently self explanatory, the aim has been to explain them as the text proceeds. So it is hoped that the book, although an overflow, is accessible independently of DRL, indeed that it stands on its own to the usual limited extent that such books do.

I wish to thank Frances Redrup and David Bennett for their considerable efforts in the preparation of this book, and Jan Srzednicki for his constant encouragement.

CHAPTER 1

INTRODUCTION TO THE GUIDE

Sociative logics can stand on their own. They do not require the framework of classical logic, or of an earlier alternative dominant logic, in order to be explained or understood. First degree relevant logic, for instance, a central sociative system, could be taught directly, on its own, as a first course in logic; contemporary classical logic could be displayed then a degenerate case. It could even begin in a common fashion with standard truth tables with two truth-values, t and f say (as will be shown below). The combination of these values yields four effective values: t only, f only, both t and f, and neither t and f; that is, under one intended reading: true, false, both (overcomplete), and neither (incomplete). What classical logic does is to remove this desirable independence of value, and thereby incompleteness and overcompleteness, whence classical degeneracy. Such meritorious educational practice is, of course, nowhere exemplified; classical logical theory now has a virtually complete monopoly in logical education. This guide is offered in the hope that it will help in breaking that monopoly. It is intended, among other things, as a further step on the way to making sociative logic a curriculum subject.

Given, however, the now entrenched position of classical logic, it is a reasonable expectation then that most of those who dip into this guide will have some passing acquaintance (at least) with elements of classical logic, and that many will find that ideology more familiar than any of the alternatives to be considered here. Lack of much acquaintance is not however a serious handicap; on the contrary, too much immersion in classical theory could be a serious impediment to thinking sociatively, and to speaking sociatively with a good accent. More important than any knowledge of a specific logical theory, such as classical theory, is some appreciation of features of logical-formalist methods. For elements of these are presupposed; in particular what is taken for granted is the notion of a formal or logistic system (patiently explained in Church for instance) and the idea of symbolic formulation of sentences drawn from natural language and duly regimented. But so that this guide too can stand on its own, sentences represented symbolically will, to begin with, be expressed in (logicians') English.

In this guide two significant trends in relevant logics are further advanced: a *broadening* of the range of logics admitted and studied, and a historical and philosophical *deepening* of the investigations of these logics. Above all, the guide attempts to give a wider impression than previous work of directions and unmapped regions. It is also intended to accomplish several of the things required to put relevant routes and themes on less esoteric maps. It aims to show the extent of the broader region, something of its importance and range of concerns, as well as to reveal difficulties confronted there. The history matters too, because it helps to show up the arbitrariness and artificiality of the presently enforced and channelled mainstream. Foucault's emphasis upon '... the necessity of excavating our own culture in order to open a free space for innovation and creativity' applies to reasoning and logical

practice, as well as elsewhere in mainstream culture.[1]

Sociative logics have begun to make an increased, if still small, appearance and noise on the logical scene - in certain cases a comeback - through the confluence of several recent developments: logically, the confluence of various relevant and paraconsistent investigations and excavations, more practically, the rise of computer and cognitive science and such subdivisions as artificial intelligence.[2] Although sociative logics can stand on their own, their place in relation to other logics, relevant and paraconsistent logics especially, needs to be indicated, to establish bearings and to link into the larger logical whole, to show what is happening and what the noise is about, and so on. Let us begin by exploring a little along the relevant fork.

Most of the sociative logics we shall encounter are Northern products, combining European origins with American ingenuity and technology. So it is with relevant logics, the ideas for which were conceived in medieval Europe, but then forgotten. Connectional ideas were revived however with the provocative flaunting, by Russell and Lewis especially, of irrelevant implications, formal and strict respectively, as supplying all there was logically to fundamental logical notions such as entailment. The first formalisation of relevance logic - then called, by contrast with strict implication, *rigorous* implication - was accomplished, as recently as 1956, by Ackermann, an underrated member of the very Germanic Hilbert school (a school itself responsible for much, and much that is too restrictive, in prevailing ideas about formalisation). It took a mere two to three years before relevance enterprise was underway in the U.S. of A. The exact origins of the American packaging term 'relevance logic' are presently lost in the shrouds of contemporary history. However it became an easy case of transference once Belnap proved the weak relevance (i.e. the variable-sharing property) of system E, "of entailment", a modification of Ackermann's system, and of the related system soon after to be called R, for relevant implication. The contentious name of 'relevant implication' for the main implication $\rightarrow$ of system

R, and even for R itself (or at least for its pure implication subsystem $R_{\rightarrow}$), was established by 1964 (cf. ENT p.20). Anderson and Belnap certainly went on to encourage the dubious idea that systems E and R were "relevance logics" by beginning to refer to them as 'logics of relevance', and to R as the logic of 'relevant implication' (appellations entrenched with ENT, e.g. § 28). Meanwhile, Meyer and Routley introduced the alternative title 'relevant logics' for a much wider spectrum of logics than those favoured in the Anderson-Belnap stable; nor were they neglecting other reasons such as those of topicality (nonetheless their reasons only overlapped, Meyer invariably promoting system R, Routley always preferring deep relevant logics, the D systems of RLR).

In the light of later theoretical developments, these labels for easily forgettable systems have not proved particularly suitable; but poor labels are the order of the day in this underdeveloped area of science. There is, for example, nothing very classical about "classical logic", since it is primarily a turn-of-the-twentieth-century development, that would (rightly)

have appalled most thinkers of all classical periods. Though the Philonian conditional was contemplated along with other conditionals and logics in classical times, it remained a minority position, rightly ridiculed, in the long debate upon conditionals. There is nothing very strict about "strict implication", though it is no doubt strict by Philonian standards. There is nothing particularly intuitive about much of "intuitionistic logic", but indeed much that is arbitrary, not least Heyting's inclusion of the scheme of *ex falso quodlibet*, i.e. A & ~A → B, that a contradiction, A and not ~A, implies any statement B at all. At least relevance logics are weakly relevant, even if only as an epiphenomenon. So let us stick with more or less established titles, which in any case it is hard to change (despite recent taxonomic efforts). 'Relevance logics' will refer to systems in the Anderson-Belnap stable, primarily *E*, *R* and *T*. Thus relevance logics form a ("small") subclass of relevant logics, which are characterised in turn as those which, while retaining lattice logic, avoid the implicational paradoxes essentially by rejecting Disjunctive Syllogism, i.e. A & (~A ∨ B) → B, and its variants (a more detailed but narrower characterisation was attempted in RLR p.153 ff.).

These relevant logics are however by no means the only, or earliest studied, systems which in fact meet technical requirements of relevance of one sort or another, which are broadly relevant. This wider class of logics will be called *broadly relevant* or, to adapt an older term, *sociative*. "Broadly relevant" is thus intended to cover that stretch in the term "relevant" increasingly made nowadays,[3] but different terminology is preferable, to reduce confusion; hence the reoutfitting of the term *sociative*.

1. The sociative enterprise, the different sociative enterprises. The sociative enterprise is an even looser-knit one than the relevant enterprise which it includes. The relevant enterprise itself encompasses a variety of logics, and also of objectives (some of them, such as constructivity, complexity, efficiency and the like, separate and apparently remote from the original directions). Disturbingly, much of what now passes under the banner of relevant logic has rather little to do with relevance, despite the now conventional title for a main band of the logics concerned. So *what* is it all *about*?

A short answer, implied by sociativity, is *connection*, relational bonding of statements. Paradigmatically, one statement *implies* another only if it is connected with it, only if the statements have *enough to do with* one another; in symbols, if A → B then B is connected with A (here *implies* serves as a representative con-junction; for other statemental relations also carry and demand con-nection). The connection must be genuine; it cannot be determined from features of (one of) the parts alone, as with material-implication or strict-implication. The type of connection involved is often put - though it doesn't have to be, and sometimes oughtn't to be, so put - in terms of relevance.

The broad relevant enterprise has much the same focus as the logical enterprise itself. What differentiates it is the now divisive contention that central logical notions satisfy connectional requirements that present mainstream logics neglect, to their serious cost. The enterprise concentrates on a bundle of fundamental logical notions, which remain, after more than 2000 years of investigation,[4] still much confused and ill-explicated. This situation

corresponds to, indeed is an integral part of, almost 2000 years of neglect of relations,[5] and repeated attempts to reduce those that would not go quietly away to their components and to functions and properties (e.g. of implication to the property of logical falsehood or impossibility applied to the pair of components comprising the antecedent and the negation of the consequent). These fundamental notions comprise deducibility and its near equivalents (e.g. entailment, logical consequence, fully demonstrative reasoning), sound argument, valid inference, implication and content inclusion, conditionality, logical commitment, and the like. Investigation of the central notions is of course combined with the study of other connectives and functions, in *combination* with which the logical features of the original notions are especially revealed. The further operations include, in particular, connectives such as those of conjunction (&), disjunction (v) and negation (~), and quantifiers such as those of universality (U) and particularity (P), but are not confined, by *any* means, to this now conventional set, or *reducible* to this set.

Bound up with the analytic attempt to exorcise connection is a fatal assumption of much contemporary logic: that the *meaning* of core logical notions can be given in *isolation*. This separation assumption often appears in variant forms; when not expressed in terms of meaning - which is itself often boiled down, with serious loss in value, just to reference or to truth - like notions substitute for it, such as sense or content. Such an assumption is built into much of what is taken for granted in semantics, and what gets called and passes for "semantic analysis", where logical labour largely stops with a model-theoretic truth-definition of some sort, with key notions defined through a functional analysis into components. Indeed analysis itself, not merely logical and semantical analysis, depends upon such shedding of interrelations for its success. Otherwise the essence of a structure would not be revealed by just working down into components; it would depend also upon the relations of the structure to other structures.

Similar isolationist ideas are at work in the misplaced contemporary emphases on pure systems, such as pure implicational systems, behind which lies the faulty assumption that the (logical) properties of implication can be captured in splendid isolation stripped of its connections with other functors; they are also at work in the insistence upon combinations conceding minimal properties at most to the interrelations of the isolated pure forms. These ideas have even crept insidiously into the development of logics that are supposed to be about (re)introducing connection, and relevance in particular, to a honorable place in logic - thus further encouraging the quest for un-duly strong systems (cf. RLR p.240). Indeed programs like that of Curry, which has been accorded high honour in the halls of relevance logic, incorporate just such ideas: that a core objective, especially proof-theoretically (said to be the heart of the logical matter) but also semantically, is to specify the role of each connective in isolation, shorn of interconnections. As regards the separation of implication at least, the idea effectively fails. For, contrary to the appearances of connective purism, in order to supply rules (which fall far short of meaning rules) for connectives such as *and* or *or*, what amount to principles of a first degree implication (e.g. formulated through a sequent relation) are required. Atomistic purism has no doubt played a significant part in the development of

cruder (mostly first attempt, but entrenched) logical theories, which substantially dispense with connection. But the underlying individualistic assumptions are seriously astray, especially in the idea that meaning can be completely explained in such a way; the assumptions are far from compulsory; and, in the course of reaching satisfactory connectional theories, they are better avoided. Put bluntly, many very fashionable approaches to logic, including those transplanted to broadly *relevant* logic, should be junked. As will become apparent, too, much of the sociative enterprise is not very radical at all; some of it does not pretend to offer anything other than protective adjuncts to classical logical theory, to help it out of some of its difficulties. Many of the (reductionistic) assumptions and analyses elaborated with the rise of the classical logical paradigm are rather uncritically accepted.

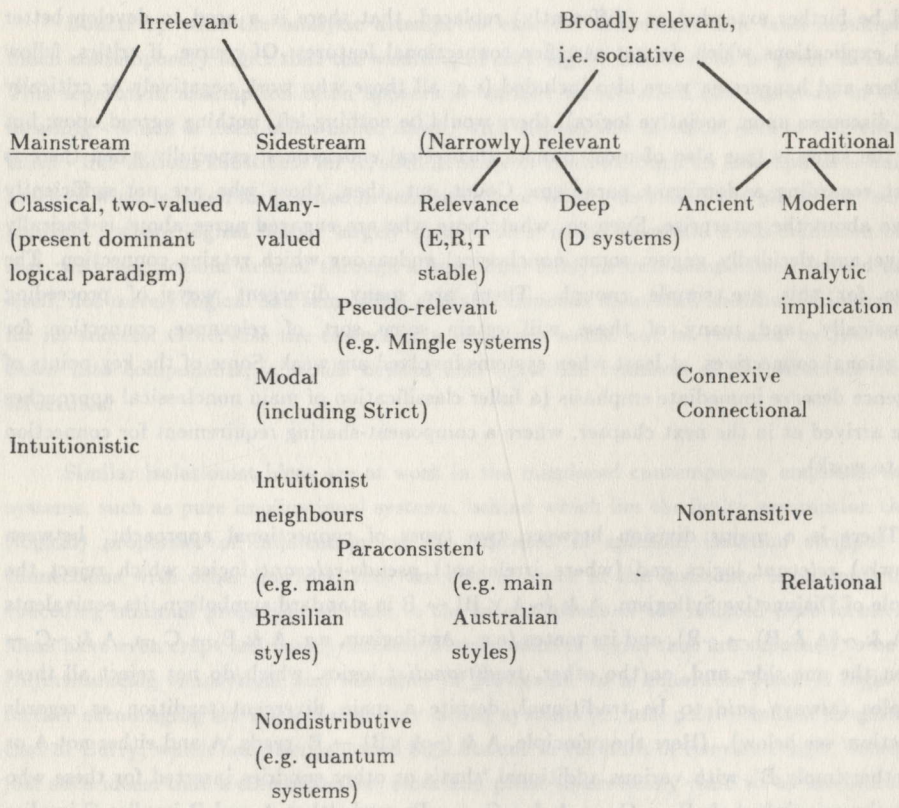
Within the *broad* relevant enterprise there is little agreement to anything *except* a certain nonclassical connectional orientation: namely, that classical logic and its usual extensions are inadequate as they stand to some of the main notions under investigation, and should be further extended or (differently) replaced, that there is a need to develop better logical explications which do not sacrifice connectional features. Of course, if critics, fellow travellers and hangers-on were also included (e.g. all those who work negatively or critically on, or discourse upon, sociative logics), there would be nothing left, nothing agreed upon; but much the same is true also of most human theoretical endeavours, especially when there is conflict regarding a dominant paradigm. Count out, then, those who are not sufficiently positive about the enterprise. Even so, what those who are engaged agree about is basically negative and decidedly vague: some *non*classical endeavour which retains connection. The reasons for this are simple enough. There are many divergent ways of proceeding nonclassically, and many of these will retain some sort of relevance connection for implicational connectives, at least when systems involved are weak. Some of the key points of divergence deserve immediate emphasis (a fuller classification of main nonclassical approaches will be arrived at in the next chapter, where a component-sharing requirement for connection is put to work).

There is a major division between two types of connectional approach: between (narrowly) *relevant* logics and (where irrelevant) *pseudo-relevant* logics which reject the principle of Disjunctive Syllogism, $A \,\&\, (\sim A \vee B) \rightarrow B$ in standard symbolism, its equivalents (e.g. $A \,\&\, \sim(A \,\&\, B) \rightarrow \sim B$), and its mates (e.g. Antilogism, e.g. $A \,\&\, B \rightarrow C \rightarrow .\ A \,\&\, \sim C \rightarrow \sim B$) on the one side; and, on the other, *traditionalist* logics, which do not reject all these principles (always *said* to be traditional, despite a main divergent tradition as regards disjunction: see below). (Here the principle, $A \,\&\, (\sim A \vee B) \rightarrow B$, reads 'A and either not-A or B together imply B', with various additional 'that's or other sundries inserted for these who prefer; the principle $A \,\&\, B \rightarrow C \rightarrow .\ A \,\&\, \sim C \rightarrow \sim B$, reads 'that A and B implies C implies that A and it is not the case that C together imply that it is not the case that B'; and so on). The reasons for the division, if not already evident, will become clear as the introductory discussion proceeds. Traditionalist logics include connexive logics, containment logics and various nontransitive logics (such as usual relational logics). Most of these logics are of course not historically authentic, but are recent concoctions, with however some viable historical

roots. Those, such as connexive logics which retain Antilogism, follow the main Aristotelian tradition in a more straightforward and honest way than those which, while insisting upon Disjunctive Syllogism principles, reject Antilogism.[6] For (as fn 6 shows) these latter systems are bound, given their other commitments, to reject one or other of the firmest historical rules: Transitivity and Contraposition. The main research thrust in contemporary connectional resurgence, though not historically rootless, has (like much American-dominated research) not been strongly historically oriented; it has been in narrowly relevant logics, and specifically in relevance logics (grouped around E and R). Traditionalist logics are presently a much more minor affair, though they will feature quite prominently in this guide.

The main divisions in terms of which the introductory discussion is set are depicted in the following diagram.

Diagram 1. A working classification of (statemental) logics.



Notes

1. The classification does not pretend to be exhaustive. For example, nonponible logics (avoiding the modus ponens rule) which are not considered in the interim text, are not included. Like nonmonotonic logics, they form a quite minor sidestream, so far (eventually they will be very important).

2. Nor is the classification, as shown, duly exclusive. For instance, stronger systems of the types listed as sociative may be irrelevant, and sidestream systems of types listed as irrelevant may be relevant, e.g. certain functionally incomplete many-valued logics.

3. Under several classes, there are subclassifications, with subtypes best developed in the case of modal logic. The subtypes (which depend on the way the $\Box$ modality, in particular, is constrained and interpreted) include these: alethic (logical necessity, logical truth, provability); physical (natural necessity, lawlikeness, causality); epistemic, doxastic, assertoric; deontic, volitional; tense, chronological; conditional; etc. Then there are two-place (dyadic) modalities, such as those of change, conditional obligation, conditional realisability; and more generally n-place modalities. Moreover the subtypes can be multipled up, as in multiply modal logics. Similar developments can be made of other types, notably of relevant logics, as in DRL (see esp. chapter 19).

4. Further explanation of most systems classified will be found in RLR or failing that ENT. The same goes for technical notions applied but perhaps insufficiently explained herein, such as those of degree of an expression, conservative extension, etc.

No attempt will be made here to explain irrelevant logics; there is a multitude of texts available doing that, especially presenting and promoting the dominant classical logic. Suffice it to say that classical logic results from relevance logic by imposing either of the strong paradoxes of implication - $A \rightarrow . B \rightarrow A$ or equivalently $\sim A \rightarrow . A \rightarrow B$ - from which irrelevance in such forms as $B \rightarrow . A \vee \sim A$ and $A \,\&\, \sim A \rightarrow B$ is immediate. Intuitionistic logic essentially weakens the negation structure of classical logic, most notably removing $A \vee \sim A$ (excluded middle) and $\sim\sim A \rightarrow A$ (half of double negation); but the adjustment takes other principles involving implication and disjunction with it, notably $((A \rightarrow B) \rightarrow A) \rightarrow A$ (Peirce's notorious law) and $A \,\&\, (\sim A \vee B) \rightarrow B$ (DSyll $\vee$-form). Usual relevance logics are properly included in classical logics, but not in intuitionistic as they have a more classical negation logic. But of course there are relevant intuitionistic logic, relevantly weakening intuitionistic logic (and its variant minimal logic), and there are relevant logics which include nonclassical principles (e.g. relevant connexive logics, and relevant Andersonian logics which add principles like $C \not\rightarrow D \not\rightarrow . A \rightarrow B$, i.e. nonimplications do not imply implications; on both see RLRII).

The usual narrowly relevant logics result from classical logic essentially by removing principles in the relevant orbit of Disjunctive Syllogism. Since theory deletion does not generally lead to unique results, to say the least, and since a range of other principles, apart from irrelevant ones and Disjunctive Syllogism associates, come up for (re)examination, a wide range of relevant logics emerges (for which see RLR), and therewith a interesting and tricky choice-of-systems problem. The other sociative logics so far introduced all retain Disjunctive Syllogism and generally its normal strengthening Antilogism. An important exception comprises analytic implicational systems, or more comprehensively containment logics, which jettison all principles that introduce new parameters, or more generally new "content" into consequents, a paradigmatic example being Addition, $A \rightarrow . A \vee B$ (but the strong paradoxes such as $A \rightarrow . B \rightarrow B$ are other imputed examples).

As containment logics throw out lattice principles (specificially $A \rightarrow . B \vee A$

corresponding to $a \leq a \cup b$), so, in one way or another, do all other *trad* logics cited. Lattice logic, the logic essentially axiomatising lattice principles (discussed in RLR), did not enjoy the logical grip in historic times that it has recently acquired. Nontransitive logics inflict different damage on lattice structure from containment logics, striking at lattice order requirements, namely (as the name suggests) Transitivity: $A \rightarrow B$, $B \rightarrow C$ / $A \rightarrow C$. Relational logics, new-fangled in technique, old-fashioned in style, follow either nontransitive or containment lines, depending upon the conditions imposed upon the operational relation governing implicational connection (details come in later chapters; this is a mere introduction). Connexive and connectional logics (which unlike containment logics retain Contraposition and also typically Antilogism) reject, indeed are more or less obliged by Contraposition to reject, the contraposed image of Addition, namely Simplification, $A \& B \rightarrow A$ and $A \& B \rightarrow B$. Connexive logics go beyond connectional logics (which merely jettison lattice decrease and increase principles, namely Simplification and Addition) in requiring certain distinctive nonclassical principles, the bedrock of which is Aristotle's principle, $\sim(A \rightarrow \sim A)$. Connexive logics are perhaps nowadays the most radical of *trad* logics, though once (notably in the 12th century) part of the dominant logical paradigm. Their dominance collapsed at the end of the 12th century with the emergence of lattice principles incompatible with them (see the historical chapter); and these logics have certainly been recessive since, though connexive genes keep reappearing in philosophical logicians.

2. Rival logical programs and paradigms. The history of logic can be revealing explained, if in a rough and ready way, in the same fashion as the history of other sciences - in terms of a succession of paradigms and programs, dominant and recessive (for a fuller account, an apposite adaption of the well-known Kuhn-Lakatos story, see Priest in PL). In this century the formerly dominant traditional logical paradigm, based on the theory of syllogism, has been gradually displaced by the classical paradigm, based on classical two-valued statemental logic.[7] That, now very mathematical, logic did not have an easy time, by any means, in becoming established. For example, in USA, now a heartland of mathematical logic, classical symbolic logic was, even until the second world war, 'simply one important contemporary school of logic' - though one with 'high hopes of its supplanting all other types, a position which only lovers of symbols are ready to take' (Robinson pp.340-1). Nonetheless Robinson was able to write, in 1924 (from the University of Indiana, an institution now linked with relevance enterprise), about the great advances in logic; but, astoundingly, he was referring to the work of 'that great logician, Bosanquet', whose theory of inference was 'was bound sooner or later to revolutionize logic' (Robinson p.vii, reiterating Muirhead). Robinson did not read the evidence of a weather change in logic well.

Certainly, the traditional logic was ripe for take-over and asset stripping, and for the insertion of some fresh logical enterprise. For, except in the later medieval period, when a theory of strict implication became widely accepted,[8] the traditional position was not coupled with now expected adjuncts, such as even an expressly formulated statemental logic. The theory of "immediate inference" and of syllogistic transformation and reduction of modern (pre-Boolean) traditional logics could, however, have been supplied by a range of competing

statemental systems, both relevant and irrelevant. Under "modern traditional" logic (or 'traditional formal logic' as the modern synthesis is often called), syllogism was the central part of logic; other parts reduced to it or were supplementary to it. The new (and narrower) classical paradigm inverted this position entirely. Statemental logic supplemented by quantification was central, and the theory of syllogism (insofar as it was "correct") reduced to this (or, on a later more relaxed approach, was a minor supplement to quantificational theory).

The dominant twentieth century paradigm, though it began in a narrow crusading way, is no longer a monolithic structure. In particular, it is important to distinguish a *narrower* classical approach, which is hostile to intensional, inexistential, and other extensions and adjustments of classical theory, and a more *liberal* approach, which is rather more tolerant of modal logics, free logics, and other nonstandard logics that can be recast as extensions of classical quantification theory. The more liberal development views such extensions not necessarily as antagonistic, not as a real threat to classical enterprise, but as perhaps useful (or more often, useless but harmless) elaborations of it.

Thus, for example, the early, and initially radical, twentieth-century challenge to the narrow classical paradigm mounted by modal logic (from which the first, Harvard, wave of sociative logics grew) was soon co-opted under a liberalised classical paradigm. Modal logic was reformulated as a straightforward extension of classical logic. Modal logic continues to afford a threat *only* to the narrower extensionalist program (a program the main philosophical positions underlying classical logic *do* however yield: see JB p.56ff.). The wider, more generous classical paradigm, which includes extension programs, is now being ringed and shielded by a protective belt of supplementary theories and pragmatical appendages, such as modal logics, conditional logics, probability logics, etc.

Things *look* just fine, but are not. The wider paradigm, while apparently much increasing the invulnerability of the classical position, begins to white-ant the paradigm from within. For the *justification* of the classical program lies in the narrow program, which is extensional, existential, and generally referential. But that program is inadequate, as the wider program starts to reveal.

Nor was all opposition to the classical paradigm easily, or at all, co-opted. Intuitionism, which continues to present a genuine threat, was not so easily accommodated. As a result of much effort, however, significant posits of the original intuitionist critique have been incorporated into the burgeoning classical picture, as for instance constructivity through a theory of effectiveness, or else have been given broadly classical representation, as for example with the rival logic itself, through semantical and category-theoretic modellings. Those ill-fitting substitutes do not however satisfy bona-fide intuitionists. Nor will relevant logics, which join with intuitionism in discarding Disjunctive Syllogistic principles, be easily co-opted. Still less do paraconsistent logics, which run directly antithetical to classical thinking, admit of co-option; indeed the reverse is more likely in the longer run; they will absorb a

further liberalised classical logic (see PCL).

3. Paraconsistent relevant logics and relevantism. Sociative and relevant logics divide in another way into two groups: those which are paraconsistent, and so are highly resistant to classical appropriation (e.g. as extensions), and those which are not, such as Parry's analytic implication and Ackermann's rigorous implication. A (*genuinely*) *paraconsistent logic*, to be more explicit about that recurring notion, is one which can provide the logical basis for an inconsistent but (genuinely) non-trivial theory. A theory is inconsistent if it (eventually) yields a pair of contradictory statements such as A and its negation ~A, i.e. it has A and ~A as consequences. A theory is trivial if it yields all statements in its field; it is genuinely nontrivial if it does not yield all statements of some given merely syntactical type. Minimal logic, for instance, fails the latter requirement, because given some contradictory pair A and ~A it supplies all negated statements, i.e. ~B, whatever B. A crucial test for paraconsistency of a logic is the nonderivability of spread principles such as A, ~A $\vdash$ δComp, where δComp is some syntactical nontheorematic function of its components. In considering *genuinely* paraconsistent logics, the letter but not the spirit of some previous accounts of paraconsistent logic has been violated.[9] Few there were, however, who wished to hail minimal logic as a paraconsistent find, even though it met the letter of a narrow law, or who would wish to exclude the medieval theory of *obligationes* as paraconsistent, because it vacuously "satisfied" the crucial test, the pair A, ~A never explicitly appearing within one side or the other of a discussion (except perhaps terminally).

There is a point, moreover, in pushing the notion of genuineness still further, to *authenticity*, so as to exclude systems which, while technically paraconsistent, are useless for fully logical inconsistent theories. Systems thus excluded as authentic paraconsistent ones include both main relevance logics R and E.[10] The argument which flunks R shows that, where RL is a *logic* extending R (*hence* closed under substitution upon variables), should A and ~A be theorems of RL, then an arbitrary B is also a theorem, i.e. RL is then trivial (for details see ENT p.462). With E the situation is like that for minimal logic; while an arbitrary B is not a theorem, all statements of a given syntactic class are (by an argument like that for R, but using a permutable-forward implicational expression of form $r \rightarrow r$). The main relevance logics do not make a sufficient break from classical limitations, from mainstream inability to accommodate reasoning in the precincts of inconsistency.

Those equipped with adequate logical tools, with genuine and especially authentic paraconsistent systems, are strategically placed to investigate logical reasoning concerning classes of principles, and involved in types of argument, which outrun mainstream approaches and even destroy mainstream logical tools. The principles include, in particular, unqualified abstraction and characterisation principles (see PL); the types of argument comprise all those which genuinely circuit through inconsistency. For such important logical purposes, intuitionistic machinery is little better than classical, i.e. useless. For while intuitionistic apparatus can deal, in an enthymematic way, with incompleteness, it is in no way equipped to cope with its dual, inconsistency. The issue of adequacy of logical equipment for the full range of reasoning situations brings out especially sharply the limitations of classical logic (cf. RLR

introduction). It becomes evident that classical logic is not simply inadequate in a limited, rectifiable way. It is not merely that it set out with a rather minimal and impoverished set of connectives (a hammer and hand-saw logical technology), which was correct so far as it went and could be fixed up by additions. The approach through the extended classical program has been to try to rectify it by additions, by adding on further "compatible" apparatus (compatible at least with the extended classical theory, which relaxes extensional constraints and tolerates a certain, often high, level of platonic pollution). The apparatus includes both the approved syntactic, proof-theoretic equipment and the certified semantic, model-theoretic machinery (what get certified, and applauded, are of course the modellings that can be *absorbed* within the expanded program, e.g. platonic set-theoretical representations of complete possible worlds and of other "non-existent" objects). It is thus but a somewhat liberalised version of what keeps reappearing in different thin disguises (like the recognisable movie-star trying to play different roles): the old reductionistic strategy, through an underlying canonical (or deep) structure or ideal language, supplemented by logical (or linguistic) constructions; in short, the old destructive ideal language program.

A significant part of the emerging relevant program, that committed to authentic paraconsistency, rejects such an approach, and is highly resistant to co-option under it (to be sure, there are classical-looking modellings of basic relevant theories, supplied from the program itself, but they do not get certified). For it contends that extension, though important, is not nearly enough. For the core structure from which extensions are made is not merely ramshackle, but seriously defective, and properly condemned. To be both blunt and quite specific about it, the canonical structure, embodying classical logic, is incorrect. It is rotten at the core.

There are several major defects in all classical programs, two of which are especially important in what follows (others, such as unwarranted ontic commitments, are documented elsewhere, e.g. JB).

D1. The basic rule of Material Detachment, in standard symbols A, $A \supset B/B$ (or A, $\sim A \vee B/B$), is incorrect. Its scope is restricted to certain consistent situations.

D2. The (derived) rule of Strict Replacement, i.e. intersubstitutivity everywhere of provable material equivalents (e.g. $A \equiv B/ \Phi(A) \supset \Phi(B)$) is incorrect. Its correct scope of application is restricted to narrowly modal contexts, a rather diminutive sub-class of those of genuine logical interest.

Because of D1, classical logic and its extensions are worthless for main paraconsistent purposes. Because of D2, classical logic and its extensions are ultimately worthless for logical investigation of irreducibly intensional functors, and so of natural languages which are rich in such functors.

Within the sociative enterprise there is major disagreement about such contentions, about such radicalism or atheism as regards the established classical faith. Undoubtedly the majority of those interested in the sociative enterprise, especially those in North America, are

either theists, believers in a substantial part of the extended classical program and in its basic correctness, or agnostic hangers-on, for example logical technicians who have a comfortable living and no wish to disturb the classical equilibrium. Most of those who have, unlike the usual technicians, serious philosophical interests in traditional sociative logic, are, underneath the liberal classical facade, theists, committed to a classical program. No, the main divisions over the correctness of classical logic can be found alive and thriving within the narrowly relevant reaches of sociative logic, where a fascinating intellectual dispute (overlapping that between bourgeois classicists and non-conformists with relevant commitments) is currently running.[11]

The main issue, within narrowly relevant logical theory, has been put in terms of relevantism. Relevantism rejects classical logic as incorrect, and adopts instead a relevant logic as supplying the basis of a theory of correct argument. In significant respects relevantism is like intuitionism; it is likewise anti-classical, but bases its program on relevant rather than intuitionist logic. Like intuitionism, relevantism sets a substantial theoretical program: that of reworking logic and what hinges materially upon it, such as the foundations of mathematics and science (much of the program is outlined in PL p.369, p.523; some is looked at in the concluding chapter of DRL). Part of the close connection of relevantism - or *relism* as is more easily and elegantly said - with paraconsistency is immediately appreciated. For one main reason, the adoption of paraconsistent *relevant* logic as a most satisfactory type of paraconsistent framework, itself a required anti-classical selection of framework, leads directly to relevantism (both claims are argued for in detail in PL p.177ff.). The argument for paraconsistent relevant logics as a superior choice, involves D1 and its restriction, essentially for paraconsistency, crucially for relevance; and adjustment of D2 is intricately tied up with replacement of classical and intuitionist logics by relevant logics in improved explications of main logical notions, especially those of philosophical and linguistic import. Relevantism does not, of course, exclude adoption of other logics for limited or for special purposes; for instance, use of classical logic as a shortcut technique in certain recognisably consistent situations such as those of sentential metatheory, use of irrelevant finite-valued logics, such as *RM3*, for preliminary investigations in inconsistent mathematics, and so on. Nor does it block attempts at synthesis, for instance, explication of relevance logics such as E and R as relevance preserving enthymematic systems. Nor does it exclude relevant adaptation of leading items of analytic implication and relational logical theory and thus a certain relevant co-option in explication of auxiliary notions, such as relevant containment, adaptations designed to resolve better and relevantly some of the problems rival broadly relevant logics were introduced to meet (e.g. frame problems, of many types, difficulties in fallacy theory, etc: see later chapters, and for detail RCR). Nor, certainly, need relevantism militate against decent pluralistic admission of other, different or rival positions - while pointing to what it sees as as their respective limitations.

Even when the requirements of relism are heeded, a rather embarrassing variety of (deep[12]) relevant systems remain, a variety offering some hard choices. In the face of these riches, this proliferation of logics, how is one to proceed, to choose where choice seems

inescapable? Should one shrug off the requirements of relism, while still decently adhering to connectivism, choice becomes increasingly difficult.

6. On choice of systems, and correct choices. Choice is characteristically directed, it is for some purpose or other. Choice of logic may be for rather local purposes, as in a game, or for a logical exercise; or it may be more comprehensive or even global, as when a serious philosopher is trying to select some fairly general all-purpose logic which is philosophically adequate (as distinct from mathematically convenient), for instance, it does not bring with it a series of gratuitous problems or constraints. The choice of system is never entirely a technical matter, but is, in important cases, an *ideological* matter. Nevertheless, a system may be selected *for* more local investigation because it presents some technical problems of interest, of an appropriate level of difficulty, etc.

A careful choice of system for some given, perhaps rather general, purpose will naturally involve weighing up considerations of several sorts, including pragmatic features such as simplicity, familiarity, strength, adaptability, and so on. Carried out in due detail choice will require application, much better explicit, of some model of rational choice-making. A satisfactory choice of system, for the explication of suppression-free implication or entailment, leads, so it has already been argued in much detail, using a rational choice model, to a choice of a (deep) relevant logic. The arguments (the main details are set out in UC and RLR) also lead to a liberal relevantism.

A preferred outcome assumes the following lines:- A deep relevant logic is chosen as basic, as supplying what is needed for characterizing valid argument for instance, a suppression-free implication; and its logical necessitation is taken as representing entailment, i.e. full logical sufficiency (see RLR). Then the usual relevance logics appear as examples of relevant enthymematic logics, i.e. systems which permit some suppression while maintaining relevance. Mainstream logics and modal logics simply go much further in condoning suppression (e.g. in modal logics sustaining positive suppression of all recognised necessary truths, and negative suppression of all recognised contradictions). But naturally these logics, which are part of the pluralistic mix, can be used in contexts where the restrictive conditions for their application are satisfied. Furthermore, deep relevant theory is supplemented, much like more liberal classical theory, by a range of extensions and special purposes logics. Thus, a select relevant logic can be extended by alethic and tense functors and by relevant conditionals (on all these see DRL chapter 19); among special purposes logics are nontransitive logics to designed to capture immediate and finite-step inference, and relevant containment logics to be applied in the many circumstances where containment notions are required (see again RCR).

While there are certainly constraints on choices for most purposes, it is rare that these fix a choice uniquely. Sometimes they will exclude any choice at all; often they will permit a range of choices. Moreover, the choice made is commonly free, the end-results are not compulsory, and other researchers of good-will may make other choices. The choices ultimately made accordingly form part of the pluralistic mix (of which DRL and RLR represent

small logical samples). There need be no conflict between the ideal of a universal logic and such pluralism. A universal tool (so-called) can be one tool in a basket of tools, a rather inefficient or awkward tool for many special purposes. Universalism, by contrast with universal devices, insofar as it involves exclusiveness, insofar as it would not countenance rival positions or logics, is different, and dangerous (as Galtung has explained). Unfortunately, it is universalism that choice of a relevant logic, whether as universal or not, has often to combat.[13]

The arguments which have been presented in the literature for adopting relevant logics as the correct logics of nonenthymematic implication and entailment have not so far taken the philosophical or logical worlds by storm. That does not mean that the arguments lack merit, but rather that well-entrenched positions can so far afford to ignore them, or even jeer at them. But increasingly raiding parties are sent out from the classical citadel, bent on inflicting substantial damage. Objections to relevant logics from the perspective of classical logic are thus now commonplace and nothing particularly new. Newer, for contemporary times, are styles of disputes within relevant logics themselves: as to the place of relevance, over the appropriate strength of "natural" negation, over the respective merits of different proof-theoretic methods and, especially, different semantics. Concurrently the external debate has altered; the level of the on-going debate between classical logic and relevant logic recently exhibited has moved to a more sophisticated level than the initial debate during the 60's, largely as a result of technical developments. The development of a formidable semantical theory has made the arguments about relevant logic considerably more complex and, for the most part, better informed, but has also opened the door to new areas of choice and disagreement. Coupled with the more developed stage of relevant logics are - or ought to be - more developed and sophisticated critiques.[14]

NOTES

1. Elsewhere in the same text, Foucault remarks, even more to the present point, that 'What reason perceives as *its* necessity, or rather [pluralising], what different forms of rationality offer as their necessary being, can perfectly well be shown to have a history; and the network of contingencies from which it emerges can be traced, ... they reside on a base of human practice and human history; and since these things have been made, they can be unmade ..., these form of rationality [are] put to work in the process of domination...'.

2. This important applied branch will not be followed here. Computing science may, as a contingent matter, have arisen largely within classical logical confines, but in several important respects (beginning with negation, and modality), it is having to break out of these confines, as classical theory proves inadequate and classical restrictions generate gratuitous problems.

3. Acquisition of the term 'relevant' has been contemplated or even made by logical entrepreneurs; contemplated by Woods and Walton for relational logics, actually made by Tennant for his own nontransitive system.

4. In much of that time, however, there was no fundamental investigation, but, when logic

was studied, it was largely repetition of what had been accomplished earlier, with perhaps minor correction (and perhaps accumulating error). Thus, for instance, the tiresome sequence of pontifical and empty texts expanding upon the received theories of syllogisms and fallacies. The traditional paradigm certainly needed ousting; but from the point of view of fundamental notions and logical freedoms, the outcome of the "classical" revolution was, like too many revolutions, decidedly suboptimal.

5. For fuller discussion of this crucial neglect, pointed out by Russell and again by Austin, see JB p.753ff. The persistent neglect of relations is also seen, to take yet another example, in the so-called calculus of individuals, where any two individuals, however related, are said to make a further individual. But in the ordinary sense, only suitably *related* individuals compose to yield individuals. The neglect is seen, somewhat differently, in such group and collective activities as decision-making, where there are repeated attempts to reduce group relations to properties of individual members of the groups, such as their individual preferences.

In modern times, a sustained attempt has been made to compensate for the damage the persistent exclusion of relations (other than a few unavoidables such as conjunction, resemblence and membership), by the qualified admission of functions and deployment of functional reductions.

6. Antilogism is often seen, quite inaccurately, as merely generalising Contraposition. It would be nearer the mark to say that it amalgamates Contraposition *and* Disjunctive Syllogism (DSyll), as will now be shown. (Symbolism and labelling of principles straightforwardly adapts that regularly used for relevant logics, as in DRL and ENT.) Firstly, given these principles Antilogism can be derived, in an innocuous implication setting. Consider these derivations in the first degree, *rule* setting, where the interrelations are more perspicuous. Then

$$\begin{array}{lll} A \,\&\, B \rightarrow C & / \sim C \rightarrow \sim(A \,\&\, B) & \text{Rule Contraposition} \\ & / A \,\&\, \sim C \rightarrow A \,\&\, \sim(A \,\&\, B) & \text{Rule Factor} \\ & / A \,\&\, \sim C \rightarrow \sim B & \text{DSyll, Rule Transitivity} \end{array}$$

Secondly, for the converse, there are two derivations. DSyll is but a one-step application of Rule Antilogism (formulated as above), using Identity, $A \rightarrow A$. Further,

$$\begin{array}{lll} A \rightarrow B & / \sim B \,\&\, A \rightarrow B & \text{Rule Monotonicity} \\ & / \sim B \,\&\, \sim B \rightarrow \sim A & \text{Rule Antilogism} \\ & / \sim B \rightarrow \sim A & \text{\&-Idempotence} \end{array}$$

The first and last steps use Rule Transitivity, the first step just that and Simplification. Rule Transitivity is critical for following through the implications involved.

It is a trifle puzzling nowadays that the obvious problems with full Antilogism were made nothing of in ancient times. For, as Duncan Jones nicely observed, Antilogism yields $p \,\&\, q \rightarrow p \leftrightarrow .\ p \,\&\, \sim p \rightarrow \sim q$, 'which is one of the paradoxes we are trying to avoid' (p. 77). But was it? Maybe it was supposed that paradoxical content rubbed off on $p \,\&\, q \rightarrow p$; certainly Simplification enjoyed no routine following in former times (see the historical chapter).

7. As remarked, the now established label 'classical' is singularly unfortunate. To make matters worse, the markedly *non*formal logic of the post-medieval period (fifteenth to seventeenth centuries) has been called 'the classical logic'; thus e.g. Bocheński, chapter 36, on what he scathingly describes as 'the so-called "classical" logic' (p. 254).

8. Although adoption of some form of strict implication became the dominant position by the fourteenth century, it was certainly not the only position. Furthermore, a unique strict system was not supplied, though a common first degree system and certain second degree principles can be extracted.

9. For steps towards an improved, but still inadequate, account (and typology) of genuinely paraconsistent logics, see Batens 80, pp.201-2. Named and investigated logics which are (weakly) paraconsistent but not genuinely paraconsistent include Curry's system D, which delivers all negated wff, and the Arruda-da Costa J systems, which yield all implicational wff (see Urbas).

10. Under a tempting strengthing of authenticity, they also include, among irrelevant paraconsistent logics, the positive-plus C systems of da Costa, which succumb to the Curry objection, trivialising upon addition of unrestricted comprehension principles (see PL p.176).

11. On the issues surrounding relevantism, see Routley 84 and work cited and criticised therein. Other fascinating disputes, overlapping relevantist issues, are mentioned in the concluding chapter of DRL.

12. Depth is explained in the next chapter. Very roughly, a deep relevant logic is relevant all the way down (though implicational nesting).

13. All these issues obtain some of the further detailed consideration they deserve in a forthcoming series on correcting mainstream logic and logical ideology. There too some of the themes on universal and natural logics (of UU and UC) are appropriately reset, in terms of pluralism and of satisizing (rather than maximizing) choices.

14. The author apologizes for the overlap between this interim introduction and the introduction to DRL. The hard fact is that the same ground has to be traversed.

CHAPTER 2

AN ORIENTATIONAL SURVEY OF SOCIATIVE LOGICS

1. Types of connection, and sorts of sociative and relevant logics.

A *sociative* or connectional logic involves a definite connection in its implicational or inferential part, it is broadly relevant.[1] Trivial connections (such as being either the same or different, or as implying or not implying) are not definite; they provide no genuine linkage of components. Nor, though the claim does not go undisputed, do the implicational connections exhibited in the basic paradoxes of implication - $(A \,\&\, \sim A) \rightarrow B$ and $B \rightarrow (A \vee \sim A)$ in standard notation - afford definite connections. For A and B may have nothing whatsoever to do with one another, share no content. Later we will look hard at the implausible theme that the modal ecology of the universe is such, the modal web spun so tight, that impossible and necessary statements, whatever they concern, have definite connections with everything else. To begin however, we make the preanalytic assumption that implication requires connection. The main type of connection that has been emphasized recently is relevance, whence "relevance logic". But relevance does not constitute the only relation of definite connection, or the only one of historical importance - consider inclusion of content, sharing of terms, etc. Moreover as is well known and will appear from later chapters, relevance in general (not pinned down to specific determinates) is an elusive notion.

Because they include implicational and inferential paradoxes, the mainstream logics of the twentieth century, classical and intuitionistic, are not sociative, but *dissociative*.[2] By contrast the main relevance logics, *E*, *R* and *T*, are sociative because their implicational connective, $\rightarrow$, meets the requirement of weak relevance; that is, they have no theses of the form $A \rightarrow B$, where A and B fail to share a (sentential) parameter. But in their standard presentations, within only partly formalised metatheory (or epitheory), the rule structure or inferential component of these logics is not sociative (being classically expressible); for example, the rule, from B to infer $A \rightarrow A$, is a derived rule of *R* (though not a *normal* one).

The classification introduced depends of course on being able to distinguish connectives and meta-connectives as representing implication and inference. Typically this is given in advance or can be done. To circumscribe discussion let us concentrate, in the usual way, upon sentential logics which include, as well as truth-functional connectives &, $\vee$, $\sim$, an identifiable implicational or conditional connective, $\rightarrow$ say. (The usual formal-theoretic apparatus, of syntax and proof, is thus largely taken for granted.) Such a logic will be *sociative* (in $\rightarrow$) if it contains no thesis of the form $A \rightarrow B$ where A and B are not appropriately connected. That is, to display the principle involved for subsequent comparison purposes,

R. $A \rightarrow B$ is a theorem only if r(A,B), i.e. there is a definite connection between A and B.

There is more than one way to treat such a definite connection, r: for instance, to introduce it into the object language, as in relational logics, or to leave it in the metatheory, as typically with relevant logics. That issue (addressed later) is not of immediate moment. More

important, definite connection can be guaranteed in a variety of ways, most obviously in these technological days syntactically, but by no means only in this way. There is, however, a long-persisting tendency, linked with the antagonism towards relations, for connections to vanish, if not entirely, at least into the *sharing* of factors.[3] Definite connection of statements, for instance, is reduced, firstly to some sharing of content (substance, sense, etc.), and then, in conveniently circumscribed contexts, content is represented syntactically (so there is something hard and definite that logicians or computing machinery can go to work upon). In a third popular stage of reduction, the residual requirements reached are declared undesirable, unworkable or unnecessary, and thereupon junked. This latter stage of irrelevant degeneracy, often reached in these latter days, lacks any decent justification, as will become apparent; but the earlier stages of reductive explication are both important. The first stage issues in the rather traditional *content* principle

CR. $A \rightarrow B$ is a theorem (or obtains, to put it in more traditional form) only if A and B share content (sense, use, or some such).

The sharing can take various different forms - inclusion, overlap, etc. - depending on what the presupposed account of *content* (or *use*) permits. Though content is at base a semantical notion, use is not; and in principle the further analyses of sharing can be accomplished in either semantic or pragmatic forms or reflected syntactically. All have been attempted, but syntactical reduction, with which we begin, has been strongly favoured.

The sentential situation, upon which we continue to focus, is crucial. Not only does it have to be included in any comprehensive story, but the rest can be seen as expanding it or can be contracted to it. For, in principle, quantification can be dealt with in an analogous fashion, for instance with predicate parameters generalising in a familiar way on sentential parameters (which serve as zero-place predicates). Or otherwise, further enlargements can be handled by the important strategy of *conservative* extension, that is, by the requirement that they do not induce new (irrelevant) results back on the sentential ground floor.

1.1. Syntactical connection, where statements share syntactical components. To render the connection nontrivial and stable (under elementary transformations) the components concerned must be more than punctuation, such as brackets. For suppose the formation rules of sentential logic were written so that they began with the clause that a sentential parameter enclosed in brackets is a wff. Then every wff of the logic would involve brackets, and A and B would always share syntax, much as all written English declarative sentences share full stops. Connection would be then trivially guaranteed - when there may be none of substance. Less artificially, any sentential logic where *variable* sharing holds at the first degree (i.e. the part of the logic where no nested $\rightarrow$s occur), will exhibit a syntactical connection because higher degree wff will always share brackets. But sharing punctuation is a trivial connection; it affords no guarantee that A and B have anything to *do* with one another. Furthermore the notion would not be stable, because bracketing vanishes upon syntactical transformation, for instance upon re-expression in Polish notation. It is in part for these sorts of reasons that it is sometimes insisted that requisite connections must be semantical ones of some kind, of meaning or content. But in suitably restricted contexts, such connections may well be reflected in syntactical linkages, which *do* duly avoid triviality.

Such triviality problems are avoided in the most thoroughly investigated syntactical connection requirements, by restricting sharing to categorematic elements, to sentence variables at the sentential level. Thus the weak relevance principle:
WR. $A \rightarrow B$ is a theorem only if A and B are weakly relevant to one another, i.e. A and B have some variable in common.
Thus also the stronger proscriptive principle:
PR. $A \rightarrow B$ is a theorem only if all variables in B already occur in A.
The first reflects, in an approximate syntactical fashion, content overlap, the second, content inclusion.

Principles of these types enjoy a respectable history. The idea of weak relevance, rediscovered by Belnap in application to sentential systems, traces backs at least to the 15th century, and reappears periodically thereafter. To quote Gaulincx from the 17th century: 'in a logical argument some term recurs and is stated both in the antecedent and in the consequent'. It is an easy argument from the formation rules for traditional syllogism, that valid syllogistic arguments meet this requirement. Similarly, the proscriptive principle of Parry was anticipated by the Jesuits of Coimbra who defined a class of *argumentationes* where every variable in the conclusion is in the premisses but some variable in the premisses may not be in the conclusion (see further Thom p.28).

These syntactical relevance principles are far from trivial requirements; they are requirements which many systems fail. Evidently these principles, which are presented as but *necessary* conditions for theorem-status, do not exhaust the types of variable sharing principles of some technical interest: a strong proscription principle, for instance, requires that A and B share all variables.[4] Another more jejune principle makes variable sharing sufficient (or almost sufficient) as well as necessary; and so on.[5] Each such relevance principle determines a corresponding class of systems; e.g. WR determines *WR* systems, weakly relevant sentential systems. The classes will include some strange systems, unless there are other controls. Some of them will have "implications" which fail to preserve values such as truth, necessity or meaning, or to meet other customary desiderata. Such strangeness only emphasizes however the severe inadequacy of the principles as *sufficient* conditions.

Leaving out syncategorematic expressions such as connectives from these relevance principles has an element of arbitrariness, technically convenient though it may be. Connectives are not without syntactically reflected meaning and content, which may afford connection. Nor is the omission altogether satisfactory, as the curious struggle to find grounds for excluding the Mingle principle $A \rightarrow . A \rightarrow A$ has revealed. Yet is is evident enough that where A is zero degree (i.e. contains no occurrences of $\rightarrow$), $A \rightarrow A$ contains an occurrence of connective $\rightarrow$ when A does not. The result is that where Mingle holds a nonimplication implies an implication, a result Ackermann, and following him Anderson and Belnap, certainly wanted to proscribe (see ENT p.237ff.).

Obviously variable and connective sharing principles can be combined in a mix of ways,

some of them are worth setting down. An ultra-weak relevance, or basic syntactic connection, is provided by the simple disjunction:

UR. $A \rightarrow B$ is a theorem only if A and B share a variable or a connective.

The *I* systems (studied by Routley, e.g. 72, 88) are sociative inasmuch as they meet this weak necessary condition. For these systems have the same first degree logic as relevance logics *E* and *R*, but diverge from them at higher degrees in ways that violate WR. However the higher degree theorems involved in the divergence, such as $p \rightarrow p \rightarrow . q \rightarrow q$, always share $\rightarrow$ connectives, largely by virtue of being higher degree.

There is a case for adopting UR as a *minimal* requirement for a sociative logic, and thus rendering *UR* systems *minimally sociative*. For it provides *a* basic connection of content syntactically-reflected. It also marks out an important watershed. Then not only are classical and intuitionistic logics and surrounding systems dissociative, not only are strict logics (with implication connective that of strict implication) and contemporary "logics of conditionals" dissociative; so also are systems such as *RM*, i.e. relevance logic *R* together with Mingle, and their extensions (though sometimes accounted "semi relevant", e.g. in ENT p.375). For *RM* has, like "logics of conditionals", theorems such as $p \,\&\, \sim p \rightarrow . q \rightarrow q$, where antecedent and consequent share neither variables nor connectives.[6]

It is one thing to guarantee some syntactical connection, which is all UR does at bottom; it is quite another to obtain intuitively *satisfactory* linkage of components. Basic connection not only fails to block higher degree irrelevance and suspect formulae such as $r \rightarrow s \rightarrow . p \rightarrow p$ or $r \rightarrow r \rightarrow . p \rightarrow p$. It also fails on its own to exclude defective principles like Mingle which may induce no dissociativity in weaker logical settings than *R*. Worse, it fails to block the addition of bizarre principles such as $p \,\&\, \sim p \rightarrow . q \vee \sim q$. In order to be more exclusive connective combinations need to be scrutinized. Consider, for instance, the more discriminating mixed principle,

MR. $A \rightarrow B$ is a theorem only if A and B share a variable and all intensional connectives in B occur in A.

While MR duly excludes Mingle, the somewhat ad hoc restriction to intensional connectives avoids wiping out also such often-enough questioned themes as $A \rightarrow \sim\sim A$, $A \rightarrow . A \vee B$, etc. (given of course that $\sim$ and $\vee$ are not intensional).[7] But it is a crude and rather unsatisfactory way of getting at what is - independently - sought. For while it of course removes Mingle, it does not touch substitution instances of Mingle such as $A \rightarrow B \rightarrow . A \rightarrow B \rightarrow . A \rightarrow B$. (Nor would such a connective device discriminate this from what is often taken, in these warped export-oriented days, as even paradigmatically correct, namely $A \rightarrow A \rightarrow . A \rightarrow A \rightarrow . A \rightarrow A$).

More subtle connection principles must somehow combine degree layering of wff with variable sharing. This can be accomplished in various ways; but only some quite specific ways, answering to given relevant logics, have been investigated. One, established for relevance logics such as E, is Maximova-relevance:

MXR. $A \rightarrow B$ is a theorem only if some variable occurs as an antecedent part of both A and B or as a consequent part of both A and B (ENT p.253).

But, on its own, owing to the cross-over fashion by which antecedent and consequent part are defined, the requirement fails to exclude weakly irrelevant wff involving negation, such as the Minglish formula $\sim(p \rightarrow p) \rightarrow .\ q \rightarrow q$ (ENT p.253). MX "relevance" too has to be combined with *other* requirements of relevance, to exclude Minglish erosion for instance.

A much more demanding necessary condition for systemic relevance, not open to these sorts of problems, is depth relevance:
DR. $A \rightarrow B$ is a theorem only if A and B share a variable at the same depth, where what determines depth is numerical extent of intensional nesting, typically of occurrences of $\rightarrow$.
To be more exact about this important notion, let wff contain just connectives from the standard set $\{\&, \vee, \sim, \rightarrow\}$. *Depth of occurrence* of a sub wff of wff A is defined inductively as follows: The sub wff A of wff A is of depth 0 in A. If $\sim$B is a sub wff occurrence of depth n in A then this occurrence of B is of depth n in A. If B & C [similarly B $\vee$ C] is a sub wff occurrence of depth n in A then these occurrences of B and C are both of depth n in A. If $B \rightarrow C$ [similarly other 2-place intensional connections] is a sub wff occurrence of depth n in A then these occurrences of B and C are both of depth n+1 in A. Then A and B share a variable p at the same depth iff for some nonnegative integer n there is an occurrence of sub wff p in A at depth n and in B at depth n. Depth relevance of course implies weak relevance. *Deep systems* satisfy the depth relevance requirement, or (if they are super-propositional) conservatively extend systems which satisfy the depth relevance requirement.

Depth relevance enjoys, by contrast with some of the other rather arbitrary relevance conditions considered, significant theoretical linkages. A main result, adduced by Brady, is that the depth relevance condition affords a way, which is not *ad hoc*, through the logical and semantical paradoxes. Deep systems can nontrivially carry *unrestricted* comprehension principles and the like, while retaining extensionality principles in set theory (for full details see Brady 84). The condition also turns out to circumscribe those relevant logics - the D systems - arrived at (as in RLR) on independent adequacy grounds, as candidates for entailment and deducibility proper. Thus depth relevance exhibits features that make for an important distinction of systems.

Although depth relevance is a natural extension of weak relevance to combinations involving levels of intensionality, which does have its desired effect of ruling out Ackermann formulae such as Mingle and $A \rightarrow .\ A \rightarrow B \rightarrow B$, it runs into trouble with (now well established) relevance logics, and so will not appeal to exponents of these systems. For it also takes out much of what is distinctive about systems E and T, namely both *Exported Syllogistic* (ESyll) *forms*, $A \rightarrow B \rightarrow .\ B \rightarrow C \rightarrow .\ A \rightarrow C$ and $B \rightarrow C \rightarrow .\ A \rightarrow B \rightarrow .\ A \rightarrow C$, and *Contraction principles* like Absorption, $A \rightarrow (A \rightarrow B) \rightarrow .\ A \rightarrow B$, Importation, and Assertion, $A \& (A \rightarrow B) \rightarrow B$.[8] Deep systems reject both classes of contested principles. The questioning of these contemporary principles, most of which enjoy little historical standing, had certainly begun when Lewis was puzzling over whether S2 or S3 came closer to capturing entailment (and wisely, given his limited irrelevant options, settled for S2). The critical investigation is taken much further in recent work on deep relevant theory, which much extends the typology of relevant systems in terms of the classes of principles they meet. In

short, then, depth relevance removes practically all those parts of system E elsewhere argued to be in serious doubt for the intended explicandum, deducibility (see RLR chapter 3). Thus depth relevance looks like exactly what the entailment doctor *should* have prescribed. But, because of its sheer exclusive power, such a requirement is inevitably controversial. Syntactical relevance requirements themselves require justification, nonsyntactical justification, for example in terms of what the requirements do do.

Syntactic connection certainly has its limitations. It depends (too) heavily on the syntactical forms of the systems investigated, and on form being appropriately displayed (and not condensed or hidden, definitionally or otherwise). While syntactic specificity is advantageous for proof purposes, too much may depend on a specific formulation of a system and not extend to equivalent systems (e.g. with different primitives); such defects afflict even the account offered of minimal sociativity. Despite what was said earlier, moreover, notions of syntactical connection do not always extend so easily or uniquely to more comprehensive logics, for instance to applied predicate logics. More important, syntactic connection is not the only type of connection, but, so to say, a surface representation (which syntactic forms may in fact hide) of other sorts of less superficial connection, such as connection of content or of argument. In particular, morphological linkages only indirectly, superficially and obliquely, represent what is crucial to implication and deducibility: preservation of (truth-)value and of content.

1.2. Semantic connection, through content or meaning. Much less work has been attempted on semantic and pragmatic connection of components of genuine implications than on syntactical linkages, despite repeated and regular traditional and twentieth century emphases on meaning or content coupling for conditionality, entailment and the like. But what has been attempted on the semantic front, primarily for relevant logics (but also to some extent for neighbours such as relational logics), can be extended, for what it is worth, to other sorts of sociative logics (those equipped with a tractable world semantics). Main attempts amount in fact to adaptations of what positivistic logicians (Carnap especially, but also Wittgenstein and others) tried less successfully to carry out on the bases of irrelevant logics (typically systems such as S5). These irrelevant explications of semantic notions such as sense, content, information, and so on, of course simply transferred the paradoxes of implication over to the semantical notions, and accordingly led to serious violations of the content principle, CR, and its analogues. While sociative explications can avoid such damaging outcomes (as that all necessary truths have the same content, usually and wrongly said to be none), the explications are sometimes not particularly helpful, in the way that syntactical principles can be, but partly parasitic on the notions of which a better understanding is sought.

The point applies especially to the first type of analysis, which characterises some notion of sense or content through implication itself, or a relation in its circle such as consequence. Typical of these *consequence* types is the account of the sense of A, sence(A), in terms of what A implies, e.g. $\lambda C(A \rightarrow C)$ or, under popular extensionalisation, $\{C\colon A \rightarrow C\}$. (The neologism *sence* is used, because this explication doesn't quite exhaust sense in the way too

often supposed.) *Every* logic with an implication can furnish such an account (one for each different implication). The accounts will be satisfactory, for instance in meeting a sence analogue of CR, to the extent that the logic already has a decent implication, and so is appropriately sociative.

A second superior type of analysis, which does make genuine use of semantics, ultimately encounters trouble also. Fairly representative of these *situational* types is the account of the content of A in terms of the situations, or worlds, A excludes. Most simply, cont (A) = λ a(A fails to hold at a), i.e. extensionally {a: I(A,a) $\neq$ 1}, where I is a situation relativised evaluation function i.e. an interpretation function. Such accounts work very nicely for a *large* class of relevant logics. They incidentally vindicate, in a straightforward way, by contrast with modal logics, content principle CR (see UU, FD; and for further roles for content relevance, Routley 82). But, in a way, they work a little too well. They offer little or no discrimination between marketed relevant logics, and they extend to a wide range of other sociative logics, indeed to any - and that means extensive classes of virtually all types - that can be rigged out with suitable situational semantics. Yet, despite their lack of discriminatory power, such analyses have considerable point. They show, for example, that what has often been alleged cannot be done, can be done; that important traditional requirements upon implication as to (meaning) inclusion and sharing of content are sound, and can be made good, and so on (these points are elaborated in FD and RLR).

Mistaken claims abound in the philosophical literature about what relevant logics cannot achieve in the way of truth-preservation semantics for implication, interrelations of these with content semantics, and so forth. The fact is that a truth-preservation analysis of relevant implication can now be straightforwardly provided, and straightforwardly integrated with an inclusion of content analysis, on the lines sketched and referred above. Of course, the analyses (better presented in terms of *holding* than *truth*) will make use of "impossible situations" or some substitute for them or construction of them; but, as is now at last being realised, that is quite alright.

1.3. Use and argument connection. Prior cleverly claimed (as Bennett 54 reported) that arguments to the basic paradoxes of strict implication simply show that necessary and impossible statements *are* connected to everything else. Of course the arguments do not show that much (without a difficult further climb upward through the hierarchy of modal systems, using supports and ropes which give way in better systems, which duly separate implicationally distinct principles); for the paradoxes work in such weak modal systems as S0.5 and S1, which do not sustain the more sweeping modal ecology. A less ambitious orthodox retort, whose attempt to bag intuition will be disallowed, takes the following lines:- What, after all, could be better than linkage through intuitively valid and simple arguments such as those, known on and off from medieval times, for (A & $\sim$A) $\rightarrow$ B and B $\rightarrow$ (A $\vee$ $\sim$A)? Because of these arguments, (duly broken below), establishing appropriate nonparadoxical connection by way of argumentative linkage is an uphill effort. Circumscribed, or canonical, forms of argument - which however leave out arguments widely (if mistakenly) accepted

nowadays as valid - have to be invoked. Whilst this can be done (and will straightforwardly emerge from the systematisation of sociative logics), the business of separating out the arguments which are good, from those which are not, involves appeal to other criteria, so leading beyond pure argument connection.

A more promising approach is that concerning *use* of argument components. Again there is a scattered tradition, but the approach has been pursued most diligently by Anderson and Belnap (with their strenuous work brought together in ENT). For there is a clear sense in which B is not used in the following sort of derivation leading to paradox:

A	$\Vdash$	A	pristine axiomatic form
A,B	$\Vdash$	A	weakening, or thinning
A	$\Vdash$	$B \to A$	deduction principle
	$\Vdash$	$A \to . B \to A$.	deduction principle

Given the way B is infiltrated, as an *extra* (which is merely conjoined and doesn't undercut other premisses), there is little excuse for then assigning B an implicational role as the deduction principle proceeds to do. In such simple cases, it is easy to see how to stop the rot; premisses not *used* in the argument cannot be assigned deductive (exportative) rules. One early problem is how to extend this simple and often obtained insight, even to expected full systems of sentential logic.

It soon becomes evident that the insight does not extend either easily or uniquely, even under some "natural" controls. Indeed there are several ways to proceed, some of them fairly natural, especially once further connectives are introduced; and certainly with controls relaxed a plethora of systems can result (e.g. to begin with, subscripted natural deduction systems or intensionalized Gentzen systems for hundreds of relevant logics: cf. RLR II chapter 11). Worse, the insight does not lead, without much complication (e.g. through two types of coupling of premisses, or else through subscription) in expected, *independently* motivated directions; and so far it has not yielded expected nice results for main systems (e.g. those that Gentzenisation often yields regarding proof structures, cut-elimination, and so on).

Use approaches, like argument approaches, do not live up to their initial promise. Though important, though a significant part of an integrated investigation, they cannot serve as a satisfactory starting point. For they do not stand on their own. Nor do they disclose motivation, or reflect semantical motivation, sufficiently well.

1.4 Pragmatic connection, and relevance. Appeals to pragmatics, and injunctions to be relevant, can work in two very different ways. On the one side, they can be invoked to wave away demands for connection in logic, and for sociative logics. Everything is fine, or fairly fine, with mainstream syntax and semantics; connection is a pragmatic froth on top of the deeper structure. That foam can be satisfactorily skimmed off and taken care of along with other complicating froth of pragmatics. Such appeals to pragmatics are made not merely by defenders of the classical faith; but they also come from linguists who wish to use classical logic as deep structure, from computer scientists who want to program their machines with a

classical frame, from communication theorists, from cognitive scientists, and so on. To those who have glimpsed some relevant light on the scene, such appeals are unconvincing. They are unsatisfactory because they do not get things near enough right at a semantical level, or on more radical sociative thinking because they get things decidedly wrong (cf. chapter 1). The paradoxes of implication are paradoxical because of semantical features, the semantics of implication especially, not because of some neglected nicety of speaker or hearer use not touching semantics. The pragmatic buffer-zone shielding mainstream logic does not render it impervious to sociative criticism. Yet, on the other side, much sociative argument relies upon pragmatic considerations. But from this side the pragmatic features are not semantically superficial; they reach down to touch such issues as validity, content, and so forth, and down again to affect a properly-reflective syntactical analysis, whence requirements of syntactical connection.

Arguments to syntactical requirements of relevance are commonly of this sort. They begin with how discourse is used, how mathematicians talk and reason, above all from how individuals argue. They point out that these are not procedures which operate in irrelevant ways; if they should they are open to legitimate criticism (as the famous paradox-applying mathematical author in ENT pp.17-8 is wide open to editorial criticism). Moreover, these constraints are not mere conversational conventions, they concern the context-independent meanings of key notions involved. Fallacies in argument, such as those involving relevance, cannot be set aside as pragmatically infelicitous, and immaterial to validity; fallacies bear directly on validity.

Such, in very broad outline to be filled out subsequently, are some of the reasons why requirements of connection - now pretty generally conceded at the pragmatic level, because hard to dismiss there - should be transmitted downwards to narrower semantical and syntactic levels. They by no means exhaust the arguments for connection at lower levels, as should now be becoming evident. Motivation for connection is commonly directly at the semantic level, already abstracted from rules for discourse users. And it is partly for this sort of reason that very little investigation has been made of specifically pragmatic criteria for connection. A different excuse, from the other side, is that it is very easy or convenient to neglect items shoved up into the pragmatic attic. In any event, there is little so far to consider in the way of accessible criteria for connection at the pragmatic level. What criteria are suggested drop down to ones like those looked at already; and the same applies to tests for relevance.

2. A working classification of sociative logics.

The paradoxes of implication do have the virtue of delivering an exhaustive classification of sociative logics. For there are at least as many styles of sociative logics as there are distinctive ways of halting these arguments to dissociation, and of course no more. A little more precisely, an initial classification of sociative logics can be made in terms of the characteristic steps in derivations of the basic paradox arguments of strict implication at which these arguments are said to be broken.[9]

For an initial classification, the derivation of the more persuasive negative paradox, $A \,\&\, {\sim}A \rightarrow B$, affords a more revealing breakdown. The manifestly implausible expansion step, $A \rightarrow .\ A \,\&\, (B \vee {\sim}B)$, and its mate $A \rightarrow .\ A \,\&\, B \vee .\ A \,\&\, {\sim}B$, of the standard positive paradox argument, is rather uniformly rejected by quite different types of sociative logics (though it is retained by certain nontransitive and relational logics).

Apart from substitution - not to be neglected - one revealing derivation involves the following steps and principles:

Steps	Principles applied	Principle Names
$A \,\&\, {\sim}A \rightarrow {\sim}A$	$A \,\&\, B \rightarrow A$	Simplification (L)
${\sim}A \rightarrow .\ {\sim}A \vee B$	$A \rightarrow .\ A \vee B$	Addition (R)
$A \,\&\, {\sim}A \rightarrow .\ {\sim}A \vee B$	$A \rightarrow B, B \rightarrow C / A \rightarrow C$	(Rule) Transitivity
$A \,\&\, {\sim}A \rightarrow A$	$A \,\&\, B \rightarrow B$	Simplification (R)
$A \,\&\, {\sim}A \rightarrow .\ A \,\&\, ({\sim}A \vee B)$	$A \rightarrow B, A \rightarrow C / A \rightarrow .\ B \,\&\, C$	(Rule) Composition
$A \,\&\, ({\sim}A \vee B) \rightarrow B$	$A \,\&\, ({\sim}A \vee B) \rightarrow B$	Disjunctive Syllogism
$A \,\&\, {\sim}A \rightarrow B$	$A \rightarrow B, B \rightarrow C / A \rightarrow C$	(Rule) Transitivity

L abbreviates 'on the left'; R 'on the right'. But the distinction will do little work; for, from antiquity onwards, commutativity and also associativity of connectives & and v have been almost invariably, and correctly, taken for granted. The rule/thesis distinction is an entirely different matter, and most important.

Near enough, rejection of *any* of the principles involved yields a class of logics which halt the argument; and if the rejections are carried through in a systematic way, they yield classes of logics which exclude all such paradox arguments. Thus there is a class of non-Ψ logics for each principle label Ψ. That is, more colloquially, there are logics of all the following types, at least:- **nonsimplifying, nonadjunctive, nontransitive, noncomposing, nondissembling** (the latter where Ψ is Disjunctive Syllogism). Rejection of some principles makes room, in turn, for others. So logics of the classes discerned are by no means all subsystems of the strict systems which reject none of the principles involved. For example, nonsimplifying logics include as a fascinating subclass, connexive logics which underwrite principles like ${\sim}({\sim}A \rightarrow A)$; nondissembling logics make room for a decent investigation of properties of nonimplication, and of such principles as that nonimplications do not imply implications, e.g. the Anderson theme $A \not\rightarrow B \not\rightarrow .\ C \rightarrow D$ (see further RLR II chapter 10). Indeed there are numerous interesting, often undreamt of, nonclassical principles for future logicians to play about with (the recently investigated $A \rightarrow B \rightarrow B \rightarrow A$ of Abelian logics is but a foretaste of this future: see PL p.245ff). The serious investigation of such radically nonclassical systems remains however in its infancy. A main focus of nonclassical (or nonmodal) logics has simply been on busting paradoxes, by logical means, rather than slumming it with them, and for this end rejections are what matter – *well*-motivated rejections if the idea is to obtain a good start.

Virtually every way of breaking the paradox argument by abandoning one of the principles cited has been attempted, except the rejection of historic commutative features such

as A & B → B & A connecting left and right forms, and the dropping of compositional principles such as A → B, A → C/A →. B & C. In effect, the latter too has been tried, though in a paraconsistent setting, with discussive logics, a leading feature of which is abandonment of Adjunction: B, C/B & C. It is not difficult to show that (as based on normal modal logics), discussive logics also drop, what is closely allied, Rule Composition. The motivating idea is that what is guaranteed in separate discusions cannot necessarily be amalgamated (under the one guarantee) in a coherent discussion. But standard discussive logics, though paraconsistent, are not sociative, and indeed *retain* the negative paradox, A & ~A → B, for discussive implication. While it appears no difficult logical feat to design sociative discussive logics (parasitic, on relevant rather than modal logics), and there is plausible motivation for attempting such a feat, the exercise has yet to be followed through properly. While all other paradox-smashing ways have been tried in recent times, mostly the attempts have been made with rather special forms of systems within the types distinguished. There is much more room for logical experiment within the sociative sphere. As a result of the specialisation, most of recent attempts have at least isolated or embryonic historical antecedents, some of them to be enlarged upon in a subsequent historical introduction. But (as with much earlier intellectual endeavour) even some of the attempts tried have not been pushed or explored very far, usually because they lack satisfactory motivation; and within the range of tried procedures there are some highly favoured moves. Thus, for example, while the idea is much favoured, especially by novices, of a logic which fails or qualifies uniform substitution (i.e. in effect limits applications of principles to cases), the idea has not been advanced very far, and often intended qualifications to the principle remain unarticulated. When duly clarified, the idea tends to fade away, or to disappear into other paradox resolutions.

Nonsubstitutional logics reject, or rather (if they are to get anywhere) qualify, the principle of uniform substitution upon sentential variables; not all substitutions are admissible. (So rejected thereby is the implicit generality interpretation of variables, the free algebra assumption, etc.) Thus schematic formulations of sentential logics, such as those adopted here, tend to be eschewed, in favour of presentations which make the role of uniform substitution explicit. But equivalently these logics can be seen as prohibiting or restricting rewriting or relettering of principles, especially when the results identify components. The envisaged qualifications typically affect Simplification or Rule Transitivity, blocking their operation as regards inconsistent antecedents. For example, qualification of substitution (or rewriting) applied to Simplification, a quite popular move, stops derivation of A & ~A → A from p & q → p (or from A & B → A). Similarly, restriction of Transitivity (i.e. Conjunctive Syllogism), (p → q) & (q → r) →. p → r, blocks in particular substitution of A & ~A upon p. Restriction of Rule Transitivity (normally presented in schematic form) stops analogous rewriting with an explicitly inconsistent antecedent.

Main motivation for both these sorts of restrictions characteristically derives from a severe unease with contradictions, from the idea that there are serious limitations upon where contradictions like A & ~A can figure and what, if anything, they can imply. This disease terminates in the limit in the very difficult (anti-classical) position, not lacking a stream of

philosophical adherents however, that contradictions entail nothing, are logically sterile. In part, this comes from overreaction to mainstream folly, with the net result a false all-or-nothing dichotomy: contradictions either entail everything or else nothing. In between these extremes lie more discriminating positions, like relevant ones, that contradictions entail some propositions (those that they do) but not others. But in part it derives from a traditional widely-held picture of negation, a neutralisation or cancellation picture diametrically opposed to the contemporary mainstream explosion picture (as explained in NC), according to which $\sim$A cancels out A leaving nothing. But from nothing, it is always said, nothing follows. The picture is one thing, and easily grasped in outline; its formal development is quite another. Proposed substitutional restrictions, which are *one* way of trying to neutralise contradictions, have not been well-motivated, or closely linked to underlying negation pictures.

Substitutional restrictions tend to be tricky to manage formally, though not impossible as mixed experience with higher order predicate logic has shown. But as that experience has also come to show, substitutional restrictions can typically be removed by reaxiomatisation (e.g. restrictions on substitution rules become qualifications on comprehension schemes). So it is also with nonsubstitutional logics; upon clarification these transform into logics of other types.

What is proposed in nonsubstitutional logics is *qualification* of substitution, not its total elimination - which would stop logic in its tracks. But qualification amounts to substitution subject to provisos, i.e. to the restricted rule

$$A \quad \sqrt{} \quad S^{p_1 \cdots p_n}_{B_1 \cdots B_n} A \quad \text{, provided C} \qquad \text{(i)},$$

where $S^{p_1 \cdots p_n}_{B_1 \cdots B_n} A$ is the result of uniformly replacing variable p_1 by wff B_1, p_2 by B_2,...,p_n by B_n throughout A. As is standard, the rule form $A_1,...,A_n \sqrt{} B$ records that where $A_1,...,A_n$ are theorems so also is B, whence B can be added to a proof sequence.[10] But the provisoed rule, in which C may depend on both A and $B_1,...,B_n$, has a similar force to

$$C, A \quad \sqrt{} \quad S^{p_1 \cdots p_n}_{B_1 \cdots B_n} A \qquad \text{(ii)},$$

that is, an "unrestricted" substitution rule with a further premiss. Of course to so introduce C into the object language *may* require additional symbolism. Furthermore, in many logical settings the additional premiss of (ii) can be shunted away, allowing restoration of the usual unadorned substitution rule, $A' \sqrt{} S^{p_1 \cdots p_n}_{B_1 \cdots B_n} A'$ (which then typically reduces to an iteration of single substitutions $A' \sqrt{} S^{p}_{B} A'$). This may be achieved by adding an appropriate form of C to *other* rules as a further premiss. Alternatively, in certain cases, the result may be achieved by conditionalising the axiom schemes to which the qualifications are supposed to apply. Then the proviso of (i) is distributed across the rule forms, yielding $C \rightarrow A \sqrt{} S^{p}_{B}(C \rightarrow A)$, which is simply a case of the usual rule. Looking at the main example where restriction of substitution is tempting in this setting should help to clarify the more abstract argument. A common

objective (perhaps with a history stretching back to before the twelfth century) is to ensure that Simplification, C & D → C, only operates provided the addition of D is compatible with C. Instead of curtailing substitution everywhere, it is enough to restrict applications to p & q → p, where however the intended effect can be obtained, in the form C ∘ D, p & q → p $\surd$ $S^{pq}_{C\,D}$p & q → p, where ∘ is a standard compatibility (or consistency) symbol. In some settings a conditional form may prove satisfactory, e.g. p ∘ q > p & q → p, for some conditional >, whereupon qualified substitution can be replaced by usual substitution. For many purposes (but not some finitary ones) it is preferable to move to schematic formulation (which gives some of the effect of substitution across rule signs), for instance to A ∘ B $\surd$ A & B → A. In these sorts of ways, through formal precisification, nonsubstitutional logics can be transformed into logics with unqualified substitution.

Nonsubstitutional logics designed to halt the "hard" paradox argument by qualifying substitution upon Simplification give way to nonsimplifying logics and very typically to connexive logics, while those qualifying substitution upon Transitivity give way to nontransitive logics. As to the first, the grounds for qualifying substitution ensure that such characteristic principles as ~(A → ~A) hold. For if A were to imply ~A it would have ~A as part of its content, and so would amount to the null content A & ~A which does not imply ~A; so A cannot imply ~A. (Such an argument does not pass uncontested however; there are problems, for example, with such reductio methods in connexive settings.)

Nonsimplifying logics include not only connexive logics, but a variety of other sorts of systems as well. They include all sociative logics which are thoroughly traditional in adhering to the rule applied in indirect reduction of syllogism, now known as Antilogism: A & B → C/A & ~C → ~B (cf. chapter 1). For given Simplification, A & ~A → ~B is but a single step. So, not surprisingly, many of the systems implicit in ancient logical theory, by no means all of them connexive, are nonsimplifying (cf. the historical introduction). Nonsimplifying logics also include all those which find something wrong with going anywhere logically from or within an explicit contradiction, such as A & ~A - indeed this is a main motivating idea behind such logics. They thus include logical elaborations of positions as seemingly remote as those of orthodox medieval *obligationes* theory, which prohibited posit of such assumptions, and those of Wittgenstein, Körner and others, which called for logical procedures to halt (and backtrack) at such contradictions (see e.g. PL on Wittgenstein). A common basis for most of those sorts of positions derives, once again, from the traditional cancellation picture of negation, that ~A in some fashion cancels or erases A, leaving no A-content, whence A & ~A can neither include nor imply A, and similarly A & ~A cannot involve or sustain ~A. Of course, such a cancellation picture (elaborated in NC) leads back to connexivism; for A cannot include what cancels it, its negation ~A.

Among the oldest ways of trying to resolve the paradoxes are such connexive ways, which supply some of the oldest sociative logics (as will emerge in the historical chapter). **Connexive logics** are distinguished by two connected features: their restriction of

Simplification, and their espousal of nonclassical connexive principles such as $\sim(\sim A \rightarrow A)$ and $\sim(A \rightarrow \sim A)$. It is easy to see that such principles trivialise mainstream logics and dialethize (i.e. render inconsistent) various sociative logics. A typical argument (already known in medieval times) to such a result need use only some of the most venerable of logical principles: Contraposition, Double Negation and Transitivity, along with Simplification. From Simplification, $A \,\&\, \sim A \rightarrow A$ and also, as in the paradox argument, $A \,\&\, \sim A \rightarrow \sim A$, whence by Contraposition (and Double Negation) $A \rightarrow \sim(A \,\&\, \sim A)$. So by Transitivity, $A \,\&\, \sim A \rightarrow \sim(A \,\&\, \sim A)$, contradicting Abaelard's connexive principle $\sim(C \rightarrow \sim C)$ for C of the form $A \,\&\, \sim A$. Consistency generally - with the principle of Noncontradiction a pre-eminent example - was a major requirement of the dominant traditional paradigm; something had to give: Simplification. Simplification of redundant components has, and had, no traditional standing. (For example, the number of premisses of an argument, as well as their genuine use, was an important consideration in both Peripatetic and Stoic logics.) The way was clear for an obvious traditional exit from implicational paradoxes worked out to some extent in the twelfth century, then partly abandoned under strict occupation of central medieval logical territory - namely, qualification of Simplification.

Whereas Simplification and its effects have been under suspicion since ancient times, Transitivity, the other likely causality of rectified nonsubstitutional approaches, enjoyed unquestioned standing in the ancient classical world. The leading nontransitive idea is that the trouble in the paradox argument comes not from the immediate steps, but with chaining them together by Transitivity. The trouble, or alleged trouble, with transitive chaining is regularly traced to epistemic sources, especially by those mainstream proponents who regard the "trouble" not as genuine logical trouble, but as trouble for certain users and uses of the arguments. Such questions about Transitivity, though with medieval antecedents (e.g. in Strode and in Sermonete), are largely modern issues, flowing from the epistemologization of logic and of thought generally (the broad source of intuitionisms and idealisms). As typically doubts about Transitivity arise from epistemological springs, so normally epistemological currents motivate **nontransitive logics**. Thus, for instance, it is claimed that a "good implication" enables natural (epistemological) transit from the antecedent to the consequent, not transit simply (as with material implication) on the strength of knowing the falsity of the antecedent or knowing the truth of the consequent; but a "good" implication, so epistemologically characterised, does not conform generally to transitivity. With Frege and others came, however, a strong reaction to the modern psychologization of logic. Objections to classical logic generally were seen by Russell and most logical empiricists, quite erroneously, as drawn from epistemological sources (the way they were disposed to view modality also), and were accordingly dismissed - from *within* the broader epistemologization setting - as part of the damaging epistemologization of logic. With notable exceptions such as intuitionism, wrongly so dismissed. Still, one of the regular and tedious reactions from tougher promoters of classical logic to objections to the "paradoxes of implication", remains of course that these objections rest on a confusion: a logical/epistemological confusion.[11] Such an argument from dichotomous confusion, worthless against most sociative targets, is too blunt an instrument to cope with the finer features of nontransitivism. While there are,

naturally, perfectly good logical routes, by deducibility, along long classical chains, which epistemically ideal creatures would grasp without hesitation, ordinary mortals might well not grasp these chains as a whole or from end to end. But the chain effect is achieved by Transitivity. Accordingly, iterated uses of Transitivity fail epistemologically (the theme obtains reinforcement from Sorites paradoxes, where a chain effect multiplies indiscernable or barely discernible differences into conspicuous differences). While obviousness undoubtedly gives out under long chaining - hence one source of the considerable capacity of deductive arguments to yield new knowledge (as opposed to new logical content) - it is hardly convincing to claim that a five-step argument from an explicit contradiction to any proposition whatsoever is so long that evidence has been irretrievably lost along the way. While it is true that the implication (or inference) from A & $\sim$A to B is not *immediate*, except in provocative resystematisations, explication of immediate implication (or of one- or three- or five-step inference) is not the objective. Implication just is the closure of immediate implication under such operations as chaining.

There has been a series of more recent attempts to repair such an epistemic grounding for nontransitive logics; amusingly, main earlier approaches as elaborated by Lewy in fact ended up underwriting first degree relevant logics, i.e. non-dissembling logics, rather than nontransitive ones (see RLR on Lewy). These earlier approaches started once again from some account of an epistemically acceptable implication A $\rightarrow$ B, as being one where an epistemic agent could not come to know A $\rightarrow$ B, recognise its truth or whatever, simply through coming to know B or through coming to know $\sim$A. As it happens, by a little variation, such an account (already doctored to avoid the assumed unknowability of contradictions) can be doctored to yield different sociative outcomes, e.g. parts of relational logics.

Though relational logics tend to fall among nontransitive systems, as that is where the main forms recently advanced belong, strictly they yield a cross-classification of the typology offered. For how such logics break the paradox arguments, *if* they do, turns on the condition the relation involved satisfies. If the relation is transitive as well as reflexive and symmetric, then a relational logic based on a modal (or classical) logic collapses, by the paradox arguments, back into modal logic. Something else has to give, e.g. symmetry, in which case resulting relational logics may well resemble containment logics, i.e. nonadditive forms.

The nonadditive idea is that the trouble with the paradox arguments sets in with the tacking on of B which may be no part of the content of A; with the addition of an arbitrary, perhaps irrelevant B (as with A $\rightarrow$. A $\vee$ B, and with the widely repudiated A $\rightarrow$. A & B $\vee$. A & $\sim$B). The main forms of **nonadditive logics** so far investigated are systems in the vicinity of Parry's systems of analytic implication. It is now known that these systems, directly tied to modal logics, form a rather special class of nonadditive systems (a somewhat more general class of containment logics is introduced in RCR). The motivation for these systems is also tied to what would have been counted, in the bad old days, as epistemological considerations, and would perhaps now rank as semantic or pragmatic matters. Whether the motivation, in terms of containment of content and concepts, set out first by Parry for analytic implication,

ranks as epistemological or not, it does not withstand careful examination if intended for a theory of deducibility, implication or conditionality (see RLR p.96ff.).

Even so, a theory of containment type is important for other purposes, as will start to appear, both traditional logical purposes and new purposes. That is not to suggest that "analytic implication" is a theory with ancient historical roots. While some of its concerns, such as content, are old, it is a modern theory; its rejections are modern. For as well as jettisoning Rule Antilogism, it rejects Contraposition, an implication principle largely unquestioned until modern times. Of course a nonadditive logic, different from analytic implication, can retain Contraposition; but then either it will be a nonsimplifying logic also, and so already spoken for (connexive logics typically reject or qualify Addition), or it will break now standard connections between disjunction and conjunction through negation, i.e. $\sim(\sim A \vee \sim B) \leftrightarrow A \,\&\, B$ and the like ("extended De Morgan" laws). There are many precedents, reaching back apparently to Stoic times, for abandoning the latter linkage. Apart from a period in later medieval logic, it is only really in this century that disjunction has regularly been treated truth functionally along with conjunction. But while an intensional construal of disjunction makes for interesting complications and defences (e.g. that in ENT of Disjunctive Syllogism as involving an intensional linkage), it does not materially affect the generality of the discussion, or treatment of paradox removal. For the whole previous argument and typology can simply be repeated with a new connective, $\cup$ say, defined $A \cup B =_{Df} \sim(\sim A \,\&\, \sim B)$. At most there will be requirements on negation (now in doubt under intuitionism and its duals) if $\cup$-disjunction principles are to be derived from conjunctive mates.

Nondissembling logics are not entirely a contemporary innovation; Disjunctive Syllogism (DSyll) does not enjoy the uncontested history the (broad) classical paradigm mythologically ascribes to it. At many times in the history of logic, it was at the very least under a cloud (as the historical chapter below begins to reveal). There is a significant contemporary literature questioning or rejecting DSyll. It is questioned, for example, in Jeffreys in 42, for essentially the right reasons, the same as those advanced in Cologne and elsewhere in the fifteenth century. Namely, if both A and $\sim A$ are assumed, then A cannot also be used to knock out $\sim A$ in $A \vee B$ to arrive at B. Nor, in twentieth century systematisations of non-syllogistic logic is the rejection of DSyll something recently concocted by Ackermann and seized upon by Anderson and Belnap and others. DSyll was, rather accidentally, omitted from an early relevant logic formulated by Lewis, and it was rather more deliberately omitted from the system of Halldén of 1948. Halldén's observations, which elaborate upon Duncan-Jones, hit important sociative nails on the head. But Halldén did not follow through on his formal innovations, which in fact adopted a combination strategy, of rejecting both Adjunction and DSyll principles, specifically Antilogism.[12] Thus his very weak system, *S0*, meets both relevant and containment requirements. It is thus a relevant connectional logic, a type of much interest (further investigated in RCR), but representing overkill from a paradox-removal objective. As it happens, DSyll is not rejected just by relevant logics; in its main form it is also contested by both minimal and intuitionist systems, in a way that interestingly

parallels relevant rejection (cf. Sylvan 87), and it is jettisoned in main discussive systems (cf. OP p.50).

Standard relevant logics represent a maximal-style of nondissembling logics - maximal in the inexact sense that all first degree principles other than those in the immediate vicinity of DSyll are retained. A more exact characterisation of relevant logics, as conservatively extending Distributive Lattice Logic (system *DLL*), comes to much the same (the characterisation is elaborated in RLR p.153ff.) Plainly, then, there are nondissembling logics which are not standard relevant logics. But, apart from the significant class of relevant nondistributive logics (which have achieved their main development along with relevant logics), these other types all fall under classes already considered, as for instance relevant connectional logics fall into the nonadditive class.

The classification of sociative logics elaborated is primarily orientational, not critical. Requisite critical work can be found elsewhere (see especially again RLR). On some of the types of sociative logic discerned, little or no investigation has been carried out. This is true not only for types which are admittedly not of great interest other than for limited technical purposes (such as those dropping commutativity of conjunction), but also for types of long-standing historical interest, such as connexive logics.

FOOTNOTES

1. Much searching went into trying to find a satisfactory term to distinguish these logics. The eventually adopted term, *sociative*, derived through French from the Latin *sociare*: 'to combine, unite, etc.', and *socius*, 'companion'. It now means 'expressing or denoting association, conjunction, union': see OED. Occasionally, the now obsolete English noun, verb and participle, *sociate*, will also be deployed.

2. These mainstream logics admit, however, through limitation of material detachment rules, of restricted sociative extensions. But in the process they are transformed into modus ponens modifying or abandoning systems.

3. The explication of *relation* through sharing is not a peculiarity of philosophy; it runs right across the present reductive scientific disciplines. For example, in philosophy both identity and similarity (to which many other relations are collapsed in empiricism) are explained through sharing of properties; in physics forces are dealt with in terms of sharing of appropriate (sometimes specially concocted) particles; in chemistry bonds are accounted for through sharing; and so on.

   As sharing itself is a relation, there is really no prospect of eliminating relations in this fashion. But it does make sharing into a, or even *the*, *fundamental* relation (an idea highly appealing to those certain ideological persuasions). For there are evidently prospects for reducing the whole of relation theory to sharing, after the crude fashion that passes muster in classical logical theory. In fact if we can whittle sharing down to the determinate case of sharing elements, e.g. α shares element γ with β, then we can define membership $\gamma \in \alpha$ through such a connection as α shares γ with α, or an extensionalisation thereof, and then rely on the reduction of relations set theory is supposed to afford.

4. The strong proscription principle can be satisfied in various weaker connexive logics. But in fact it is straightaway satisfied in the equivalential parts of Parry logics, i.e. the parts with connective set $\{\leftrightarrow, \&, \vee, \sim\}$ where implication is now construed as equivalence.

5. Copeland, to take an isolated example, has been interested in making relevance sufficient as well as necessary for implication - leaving out, or approximating, the further standard requirements on implication of truth or content preservation. But sufficiency would exonerate such paradoxes as $A \rightarrow .\ B \rightarrow A$ and $A \rightarrow .\ \sim A \rightarrow C$, but fail Detachment or Substitution, since these "relevant" principles, yield irrelevance immediately. Other less jejune forms of variable sharing have been considered by several other authors, e.g. Cleave 74, Kielkopf 77, Weingartner and Schurz 86. Kielkopf in 77 presents a valuable, and duly critical, survey of several of these types of requirements (now quite fashionable in Europe).

6. A proof sketch can begin from *RM* theorem $\sim(p \rightarrow p) \rightarrow .\ q \rightarrow q$, established in ENT p.429. In *R*, and so in *RM*, $r \rightarrow s \rightarrow .\ \sim(r \,\&\, \sim s)$, whence contraposing $p \,\&\, \sim p \rightarrow .\ \sim(p \rightarrow p)$. Then use Transitivity. Note that this proof depends critically upon Commutation. In logics without forms of Commutation (i.e. Permutation), basic connection is restored (at least in negation weakened logics of this sort). For similar *RM*-style irrelevance in "logics of conditionals", see recent surveys of dissociative "conditional" logic, e.g. Chellas 75, Nute 84. For some of the belatedly burgeoning sociative theory, see DRL ., and also Hunter.

7. Containment logics, of interest in Artificial Intelligence, conform to the requirement
AR. $A \rightarrow B$ is a theorem only if all variables and connectives in B occur in A.
Obviously there is a intermediate requirement of passing interest, viz.
NR. $A \rightarrow B$ is a theorem only if A and B share a variable and all connectives of B occur in A.

8. There are already problems as regards the mere omission of Contraction in a logical setting as strong as system R. For there, Contraction is tantamount to Reductio, $A \rightarrow \sim A \rightarrow .\ \sim A$ - surely a mistaken equation of principles of different logical sense and force - and these in turn to the much less obnoxious *LEM*, $A \vee \sim A$!

9. A similar approach was earlier adopted in RLR chapter 1, where however some forms of sociative logic are given only cursory discussion. The different emphasis in the essays that follow will do something to compensate for what that one-sided approach neglected.

10. In settings with appropriate apparatus available $A_1,\ldots,A_n \surd B$ gets defined as $\vdash A_1,\ldots,\vdash A_n / \vdash B$. The intended contrast between / and $\surd$ is of course this: whereas / may operate hypothetically, $\surd$ does not. Most of the book follows, however, the usual practice of papering over this important contrast, assigning symbol / to cover both types of rules.

11. The distinction is heavily deployed, for instance, by Iseminger against relevance logic. Of course that isn't the only supposedly dichotomous and heavy distinction that is regularly deployed in accommodating or in attacking critics; the object language/metalanguage distinction, subsequently concocted (though again with isolated historical anticipation), was also worked very hard. Any established paradigm gets surrounded by a battery of distinctions, which are regularly deployed by those faithful to the paradigm.

12. In early contemporary days there was considerable confusion, well exemplified by Duncan-Jones, as to how much of classical (or modal) logic had to be "given away" to avoid paradox. (Thus too the approach, which should have placed more emphasis on working upwards from correctness, was defective.) Such oppositional muddlement, characteristic of Anglo-American philosophy when Moore was influential, of course made the crude mainstream cause so much the easier. Duncan-Jones requires a meaning connection for entailment but says nothing explicit about relevance; where p entails q then 'q arises out of the meaning of p'. While in principle a modal account of "entailment" is not so excluded, Duncan-Jones is rightly and Moorishly certain that q does not arise out of the meaning of p & ~p. He is also certain that Antilogism must go, but quite unclear as to where to stop the modal rot, the "independent arguments" of Lewis. In fact he attempts obliteration bombing of the logical landscape, doing *all* the following things: exhibits strong connexivist leanings with the claim that p & ~p does not entail ~(p & ~p), aligns himself with nonadditive practice in forcing out Contraposition along with Antilogism - practice he reneges upon at the very end by conceding that it may be possible to get cases of Contraposition back again - and leaves room for a nondissembling approach, by not endorsing Disjunctive Syllogism. More clarity about entailment, its "analysis" (or "nonanalysis") and properties, had to await a different philosophical tradition.

CHAPTER 3

ON THE RELEVANCE OF RELEVANCE:

in general, in logic, in argument, in fallacies

Much has come to be expected from relevance, in an astonishing variety of theoretical areas: too much. Relevance will never supply what many of those investigating entailment and conditionality, for example, have seemed to expect of it. It does not even afford a route to a unique style of entailment logic, since relevance requirements amenable to formal treatment can in general be met in a variety of ways. For instance, the paradoxes of implication, which do lead to irrelevance, can be avoided in a range of ways. Suppose, moreover, to indicate how bad things can become even *with* relevance guaranteed, that the object is to eliminate the main paradoxes of strict implications, ex falso quodlibet and its mates; and suppose further that these paradoxical "entailments" are relevance (or relationally) restricted, under some decent notion of relevance. Plainly that still leaves *far* too many claims intact, with, for instance, a contradiction entailing all relevant statements. Thus, a contradiction in spherical geometry would entail all statements of spherical geometry, assuming that they are relevant (similarly, one theorem would entail all others in the relevant field). That gives quite insufficient control of inconsistency (and correspondingly of necessity). But control of inconsistency (without losing control through incompleteness) is a major goal in deeper investigations of relevant logics, more important than some weak, or loose, analysis of relevance.

But even though the charge, that relevant logics do not have quite as much to do with relevance as some enthusiasts have supposed, can be sustained, the charge does not tell decisively against relevant logics, which stand on their own and other merits. Many of those who have advanced or who fancy logics of broadly relevant sorts have never claimed that relevance is crucial or even central, but have been interested in the logics for other reasons, above all because they offer the prospect of decent explications of main logical notions: deducibility, conditionality, commitment, and so forth. Relevant logics do not founder on the problematic rock of relevance. Still less do sociative logics, as relevance is but one sort of connection.

Nonetheless, problematic though it is, relevance is an important notion in a wide range of intellectual and theoretical endeavours. Such an ubiquitous notion merits investigation in its own right, even should the investigation eventually reflect the ragbag character of the notion itself. Claims and judgements of relevance, or irrelevance, are made in the course of appraising processes, steps, data and so on, in many reaches of intelligent or goal-directed activity, including reasoning. The philosophical and formal elucidation of such a prominent and widely deployed notion is thus an important task, irrespective of whether it casts much light on entailment. And undoubtedly it casts some. Conversely, relevance is a notion upon which more adequate logics than those we have inherited may shed some light.

1. What is relevance?

Relevance is a notion that enters, welcome or not, not only at surprisingly many places in subjects like philosophy and logic, but in several other fields as well, in artificial intelligence and in the theory of the dance, but especially in law and in literature.[1] Yet, apart from some special cases, there are few attempts at explication, and there is practically no work directed to seeing if some overall synthesis can be accomplished. Given the paucity of real work, it would be cavalier, at least, to dismiss the notion of relevance as irrelevant and of no interest at the outset. Certainly special cases of the notion, such as that of sharing components, are of much technical interest.

There is an immediate obstacle, which we can easily go around, to such attempts at explication and synthesis, at putting relevance together and making it relevant; namely, the recent philosophers' black-ban on such *What is*? questions. Thus, to adapt Popper[2]: 'If asked the essentialist question, *What is relevance*?, I would be inclined to say that I do not know, and that I do not care to answer any *what-is*? questions'. Not for us such indolence, or such a high-handed brushing aside of genuine philosophical questions. We are not essentialists, and like many others we can ask *What is*? without expecting, or presupposing, essentialist answers. Perhaps we shall be lucky enough to discern some necessary conditions for relevance (and certainly for special cases thereof). But we need not expect useful tight necessary and sufficient conditions for what is evidently a fuzzy notion, and so shall be well satisfied with a rather loose and open-ended cluster characterisation. What controls such a characterisation is of course the received meaning of the terms in the relevance vocabulary as supplied by ordinary and technical usage.

The basic term semantically in the relevance nexus is the adjective 'relevant', rare in its main English sense before 1800, meaning 'bearing upon, connected with, pertinent *to*, the matter in hand' (OED). The term, which came through medieval Latin, derives from the present participle of the verb 'relevare' meaning 'to raise up', a verb which has also been transformed into 'relieve' (thus a rare obsolete sense of 'relevant' is 'relieving; remedial').

What can be distilled from this? Relevance is a relation, syntactically at least a two-place relation. An obvious logical starting point is then the form: α is *relevant to* β. Even where the adjective 'relevant' applies, whether predicatively or attributively, there is always some *matter at hand*, perhaps contextually defined and supplied, to which what is relevant relates. So the notion is always implicitly relational, and given its dictionary meaning, the relation involved - a type of *connection*, affording evaluative control - is hardly hidden. (Nothing in this initial relational analysis excludes the neat suggestion, sometime made, that relevance is, is really, a three-place connection: that α is relevant to β in respect, or as regards, γ. The third component of this respect-*modified* relevance, which can be factored in, is plugged up for the present.)

The relation of relevance where it holds, already implies that α is connected with β, but perhaps in a weak sense. There are two familiar problems with this. Firstly, there are millions

upon millions of relations, so which one(s) is it? Secondly, unless more, much more, is said, relevance already encounters what will prove to be its bane, *triviality*. Everything is related, in the sense of weakly connected, to everything else. But relevance is weak connection. Therefore, relevance interconnects everything, and is trivial. Philosophers can rightly forget about relevance: relevance is irrelevant! But what sustains the main premise? An odd collocation of arguments. Firstly, the (so-called) ecological law: everything is related to everything else. Really however, this "law" requires relevance restriction itself, to *within* smaller ecosystems, since interference with something in one forest (a tree in Queensland) will not extend across oceans (e.g. to trees in Finland). Secondly, abstraction principles in relation theory, which yield relations for universal connections; e.g. since either α is identical with β or different from β, there is a relation between and α and β (i.e. that of identity or difference). What this again indicates however is the need also for a more discriminating theory, for instance a theory of relevance-bearing or *relevant relations*, a quite proper subclass of relations (cf. Dunn 87). Thirdly, implication principles in logic, which in mainstream theories connect too much to too much. What implies something is relevant to it, so by the chain principle, $\alpha \rightarrow \beta \vee \beta \rightarrow \alpha$, α is relevant to β or β is relevant to α. But relevance is symmetrical, so for any α and β, α is relevant to β. Again the remedy is evident: find implications which, unlike material-implication for instance, *are* relevance-bearing. And this time the results are well known, namely that more discriminating theories of implication can be furnished, theories of relevant implication, which will, in particular, exclude the chain principle and the paradoxes of implication. For this argument generalizes a theme of Prior's: that what the Lewis paradox arguments demonstrate is that every contradiction *is* connected to everything else[3] and, pari passu, every necessary truth with everything else. In each of the three cases, then, the bane of relevance turns things around - to the search for relevant connections. The bane of relevance reveals further places for the use of relevance.

What is distinctive about the relation of relevance is that, where α is relevant to β, α bears upon β in such a way as to affect β's evaluation, or more weakly, where β is assessed α cannot simply be left out of account. Relevance thus takes, like various *action* notions, two forms: commission and omission. In the stronger commission form, where β is to be evaluated α should be taken into account[4], while in omission form, α should not be left out. Similarly for relevant considerations, relevant respects, and so on; a stronger relevant whatnot is one that *should* be taken into account in the case or situation at hand; a weaker relevant whatnot is one that *should not* be left out of account. The *reason* for this involvement is that the whatnot makes, or may make, a difference. In this way, what is relevant narrows, controls a situation, a sphere of investigation, a field. This is a main function of relevance considerations.

These points, which also indicate the common normative character of relevance assessments, extend to modified relevance, as in α is socially/morally/legally/situationally/etc. relevant to β, i.e. generally α rel_γ β. Then α rel β iff, for some γ, α rel_γ β. Some important examples, tying directly with the Latin origins of the term, illustrate the matter of commission. In these cases the evaluation is semantical, of truth or extent of truth

or of probability.

Consider first the basic idea of (positive) relevance in probability theory. There α is relevant to β if α "raises up" the probability of β ; the probability assigned to β increases when α is taken into account, or, on the standard modern explication, the probability of β given α exceeds the probability of β, $p(\beta,\alpha) > p(\beta)$. Put differently, in a way that leads into the next special case, α probabilifies β, in symbols $\alpha \multimap \beta$. Once established as raising up probability value, obvious extensions of the idea of relevance were made in probability theory, not merely to relativised probability, but to any sort of variation induced in probability value. Hence negative relevance, where α decreases the value β has.

Consider next the related idea of relevance in Scottish law (assigned a separate listing in OED). What is particularly interesting here is that the limiting case of sufficiency is explicitly included. When α is sufficient for β, α is (legally) relevant to β. For components α and β of the right type then, α implies β. The more general case of relevance in Scottish law can be seen as that of partial sufficiency, or as some would reexpress this, *partial implication*. Again then the relation can be symbolised, $\alpha \multimap \beta$, in implication style.

The illustrations bring out further features of the relevant nexus, and lead to others. Firstly, even outside explicitly implicational settings, relevance is often bound up with an implication of some sort. Indeed, the investigation of relations like $\multimap$ will lead to lots of toy logics, all of them nonclassical on pain otherwise of triviality, most of them little investigated, some of them of much interest.[5] In many cases, moreover, relevance is explained through implication, and more generally through relations quite *like* implication (but e.g. not ensuring detachment or other common features, relations called elsewhere *plications*). The explanation connection does not run, it should be noted, in the reverse direction. Should it do so, an explanation of relevance is liable to be circular, with implication elucidated through relevance which is explained through implication, which ... . With a notion as basic as relevance, such circularity charges are however not so damaging. Because it is pretty fundamental, any deeper explanation of relevance risks ultimate circularity; it may nonetheless be informative.

Secondly, relevance can come in grades or amounts, as extent of relevance, in both probability and partial inclusion, shows. One item can be more relevant than another, nearer, more weighty, more demanding of attention, and so forth. If relevance is so ranked or graded, what is relevant is what has some, or enough, relevance (there is a related degrees-of-implication notion, likewise so far little investigated). For the present, we will concentrate however, on the difficult enough qualitative notion.

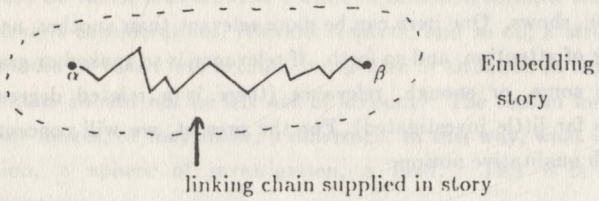
Thirdly, the illustrations emphasize, what hasn't escaped notice, the determinable character of relevance. With more definiteness go determinate sorts of relevance: statistical relevance, probabilistic relevance, evidential relevance, legal relevance, and so on. Relevance appears in fact to function as a *logical determinable* (a notion already treated in some detail elsewhere, e.g. JB p.249, p.920), as a kind of connection determinable. But a determinable

n-place relation can be viewed as amounting to a disguised (n+1)-place relation, where the further place, which holds places for determinates, is typically left unspecified or to be *contextually* specified (often because hard to specify precisely). The determinable connection α rel β accordingly reflects a 3-place relation α rel$_\gamma$ β with γ the determinable parameter.

The main technical investigations involving relevance, for instance in statistics, concern specific determinates. Our main concern also - though not the only logical concern, since relevance is critical as well in dialogue, conversation, communication, and information transmission generally - is with certain relevance determinables, those tied up with sharing of content or topic, especially as reflected in common standard components in logical forms. The determinable stands to these specific determinates rather as meaning stands to criteria, under the familiar meaning/criteria distinction, exploited for instance in attempted explications of high-level notions such as truth and causation. Some determinates such as technical ones, yield specific criteria for relevance, detailed tests for evaluation, for example of what gets into a given class (putative evidence for something, potential consequences of something, etc.). There are no such tests for the determinable. For the criteria will vary from determinate to determinate, context to context. Structural relevance criteria, such as sharing of syntax, are useless for main statistical purposes, statistical tests inapplicable in sentential logic. By contrast, the determinable meaning is invariant; it concerns what enters into an evaluation or assessment, as opposed to what gets left out or discounted. Thus, too, the determinable is fuzzy, as well as value laden. Precision enters with certain determinables, where evaluative features are also typically absorbed under conventional assumptions.

Despite the evaluative haze surrounding the determinable, more can be said. Items are relevant where there is a connecting *story* (considerations, theory), not going on too long, not going too far afield, linking components, a story which furthermore preserves some contextually supplied parameter, such as a topic. That is, α is relevant to β where there is such a value-preserving embedding story connecting β with α. The story story diagrammed is pretty vague: that is as it should be.

Diagram 1. *The story story.*



Evidently, then, standard logical properties bottom out at *zero* for the determinable; for properties enjoyed by some determinates do not extend to others. Even symmetry does not obtain generally. A story that runs well in one direction, bearing information or increasing evidence or whatever, may fare badly or break down in reverse. Reflexivity fails where

enhancement of the supplied parameter is required, and is often only concessional otherwise. Nor does a neat alternative, such as irreflexivity, hold. Transitivity eventually fails, by virtue of length cut-off; value-preserving connection peters out.

While the story story is a convenient prop - for this reason too the notion of story is sometimes taken as primitive in semantics - it can hardly be a final resting point. For a story is itself a complex structure, which accordingly can be submitted to further analysis. Moreover, it is a notion that does itself require some limitation. For we are all familiar with "stories" that go on too long or ramble too widely. A *story* is a certain sort of structured discourse, which can itself be subject to discourse analysis (and which appears amenable to an analogue of phrase structure analysis). What holds the structure together? As always certain connections. What is important from a relevance angle about the type of relational structure involved is that the relations connecting the components are relevance-bearing. Thus unless an independent account of the structural coherence of a story is forthcoming, we shall have come full circle. Even so it will have been an informative loop journey.

What an embedding story does is to connect items by relevance-bearing relations. *Relevance is* thus *explicated through relevance-bearing relations.* An item β is relevant to α only if there is a chain of relevance-bearing relations, where relevance has not given out, linking β with α. Some relations are relevance-bearing, but many are not. Genuine probabilification and plausibilification are; arbitrary conjunction and juxtaposition are not on their own. Implication is a relevance-bearing relation; material-implication is not.

Because not all relations carry relevance, symmetrical relevance is not obtained by symmetrising an arbitrary relation. Good implications carry (content) relevance, because they guarantee content inclusion; but material- implication, $\supset$, does not carry relevance, and its symmetrisation trivializes relevance through the chain principle, $\alpha \supset \beta$ or $\beta \supset \alpha$. The intended notion of *carrying* implies sufficiency; implication is a sufficient connection for relevance. Hence if $\alpha \rightarrow \beta$ then α rel β. Since converse implication is also sufficient (whence symmetry of relevance in this case), if $\beta \rightarrow \alpha$ then α rel β. Therefore, if $(\alpha \rightarrow \beta \vee \beta \rightarrow \alpha)$ then α rel β. Thus any logic, such as classical logic or *RM3* or Abelian logic *A* (of PL), which contains the chain principle is useless for reflecting the relevance of implication, incorrectly trivialising it, with everything relevant to everything else, α rel β, for every α and β.

Although a relevance-bearing relation (a genuine connexion) such as implication is sufficient for relevance, it is certainly not necessary, as many other different connexions also suffice. Accordingly, it is unsatisfactory to define relevance in terms of implication, as some have attempted to do. At best such definitions would characterise *implication*-relevance, for the implication involved. Such attempted definitions do however highlight an awkward circularity in trying to select, or filter out, an implicational connection from among extensional trash using relevance - for instance, in most simple-minded terms, by tacking a relevance relation on to material-implication. For what is sought or to be selected is already presupposed in the selection criteria.

The problem is more general. If relevance is determined through a short chain of relevance-bearing relations, connecting components, and one of the relations *is* implication, then relevance can hardly be fully explained independently of implication. Efforts to get at implication through relevance have got things back-to-front. Implication comes first.

The quest for relevance does not simply break down because of circularity; rather it begins to break *up*, to get displaced by other investigations, namely these:- Firstly, into circumscription of (central) relevance-bearing relations. At worst those relations can be given by a list (such as can be compiled from the Appendix). Secondly, the detailed work of explication is transferred to these bearer relations, and thus in effect to relevance determinates, from which the determinable can be assembled. The main relation upon which much else devolves is implication. Implication is crucial among relevance-bearing relations because it is a basis for or a guide to the elucidation of other logical relevance-bearing relations, such as commitment, confirmation, reason, evidence, cause, probabilification, and so on (as explications in other chapters help to show). The route then to a decent account of relevance is through satisfactory accounts of relevance-bearing relations, first and foremost implication (or more generally, plication). In more usual narrower logical settings, such as we shall turn to, implication is in fact the only relevance-bearing relation to consider; for extensional connectives and quantifiers, such as *and*, *or*, *not*, *every* and *some* (perhaps unlike their natural language sources) are not relevance-constrained. Even so relevance cannot be satisfactorily defined in terms of implication as some medieval and moderns have supposed (e.g. in terms of there being implicational relations between A, or differently A or not-A, and B or not-B). For the contraction to implication is contingent upon quite special contexts. And even there relevance provides only an important control, an essential but insufficient condition, upon a good nonenthymematic implication.

2. Relevance in relevant logics.

The position and standing of relevance has turned through virtually a full circle in contemporary relevant logics, from relevance being virtually everything to relevance being almost nothing. There certainly has not been a single answer, even within the relevant enterprise, to the trouble-making question: What have relevant logics got to do with relevance? The suggested answer, by an unsympathetic opposition, is: nothing. The same deflating answer is no doubt encouraged by much literature on or appealing to relevance, even on its central role in communication and cognition, which ignores relevant logics entirely. The same opposition answer has even been chorussed by some assisting in, and white-anting, the relevant enterprise. The answer is wrong. Relevant logics are a main route to the explication of relevance. But the route is indirect, and, in the present state of research, quickly degenerates into a mere track. Relevant logics afford a characterisation, through implicit definition and the like, of main relevance-bearing connections such as relevant implication and conditionals. These connections are then applied in turn in characterising, in similar fashion, other relevance-bearing relations. Finally ... relevance is characterised, as indicated, through intimate-enough (and thus value-preserving) linkage through relevance-bearing relations.

Demand for relevance, in logic (as elsewhere), is an old reaction, but now formally decked out, to an old reductionism, newly and impressively formally attired. The new ("classical") logic of Frege, Russell and others, heavily reductionistic, had no place for relevance. But from the beginning, much criticism, along with outrage and disbelief, greeted the "new" classical logic, in particular the crucial theories of implication and entailment. Unfortunately, a main early critic, Lewis (after accidentally producing some relevant systems), soon lapsed into strict implicational irrelevance, supposedly ideologically justified by the rediscovered "independent arguments" - arguments known from the twelfth century - for the destructive paradoxes of implication. Moreover, with the admission of these contagious paradoxes, paradox, and damaging irrelevance, was assured in virtually every area where argument counts. Unlike later medieval times, these strict systems, which themselves offered an embarrasing choice (e.g. of strength of system coupled with extent of nastiness), gained no new consensus (nor have they obtained much real use); instead the even cruder "classical" position gradually became dominant. Indeed the strict systems have in turn been subjected to heavy criticism, from both sides, classical and unconventional.

Many of the early criticisms of strict implication as a theory of entailment incorporated the theme that the strict "connection", scarcely better than the classical "connection" it was intended to upgrade, did not preserve relevance. Anderson and Belnap, in their synthesis of earlier work, took this over, and made relevance a central showpiece, an ingredient essential to their approach. Insistence upon relevance was not however uniformly a feature of the work Anderson and Belnap built upon so elegantly. In particular, Ackermann, who first formulated what amounts to the system E of entailment, did not explicitly appeal to relevance, though he aimed for some sort of *connection* between antecedent and consequent in a correct implication, and though a certain relevance was thus a *by-product* of his theory. Relevance thereby diminishes in importance from an essential to an epiphenomenal status. More recently, there has been a further reaction against relevance, especially by some of the freer-wheeling technicians involved in the relevance logical enterprise, to the effect that relevance is irrelevant and an unnecessary restriction (e.g. on investigations of inconsistent theories conveniently based on simpler but irrelevant systems such as RM).

There are accordingly three types of views on the position of relevance in relevant-style logics aimed at the explication of entailment and associated notions:-

1. Relevance as a prior essence. Such an assumption dictated the strategy of *Entailment* and of earlier work of Anderson, Belnap and others that it built upon. Relevance, along with necessity, was imposed as a filter, on what was assumed to be pretty much a *natural*, but irrelevant, logic; namely, "absolute" logic or Hilbert's system. (It remains difficult to discover what sustains this European-conceived and American-propagated illusion that absolute logic, so-called, is very natural.)

One major trouble with this strategy is serious nonuniqueness; that is, it can lead to very different things, depending on both starting points and how the approach is followed through (similarly such quixotic quests as that for "absolute" logic). Part of the trouble

(which didn't bother A & B since they knew in advance where they were going) is that the underlying notion is insufficiently stable. For example, a simple-minded imposition of the requirement of a relevant connection upon classical logic as a starting point, leads to relational logics, which can be relevant enough, but are very different from systems in A & B's direction of business. For weakly relevant relational logics retain that horror of horrors Disjunctive Syllogism, A & (~A ∨ B) → B (which commonly *looks* relevant enough to the uninitiated), while, crime of crimes, rejecting transitivity of implication (for details see e.g. OP pp.137-8, pp.143-4). But the situation is considerably worse than the thin spectre of relational logics indicates. For even the imposition of decent use requirements or other relevance-reflecting conditions on pure implicational theory (without necessity) leads to several alternatives to system R_I, both well inside R_I and also comprehending it, which can meet the conditions imposed (such as those of ENT pp.18-20; cf. Batens 87). Worse was to follow, as systems expanded beyond unsatisfactory pure implicational settings; various relevance tests split apart, and systems multiplied (cf. RLR p.234ff.). At the statemental level then, proposed relevance tests were too various, too indiscriminating, and left too many candidates (most of them duds) in the field; while at higher levels the applicability of the tests not only often gives out or is dubious, but when they apply they look like knocking out star relevance systems. The instability and lack of statemental selectivity of relevance remains, even so, a lesser reason for striking out relevance as a prior essence.

The main reason is that already noticed, that the attempt to obtain implication from relevance gets things back to front. Though relevance is carried by good nonenthymematic implications and conditionals, it is of little help in explicating such notions. The linkages proceed rather the other way around. The notion of connection, of which implication is one (but only one) important type, is required in explaining relevance.

2. Relevance as an epiphenomenon. Relevance is not an essential prior item in characterising entailment, which is differently delineated - through *sufficiency* for instance - but is a by-product of any satisfactory explication (e.g. by way of broad truth-preservation or content inclusion). The theory of sufficiency, avoidance of suppression or the like, itself delivers the requisite connections. It is a mistake then to try to impose a filter on some inferior irrelevant implication relation and work down. The relevance approach of Anderson and Belnap and the subsequent relational logic approach of Walton and others, are just two different aspects of the one defective approach. However once entailment is properly characterised - by way of conditions it should meet in advance - relevance of the appropriate types will emerge: relevance is an epiphenomenon. Such is the position taken and defended elsewhere (notably in RLR e.g. p.232). To be sure, an explication which did not lead to relevance, for instance at least to variable sharing linkage at the statemental level, would be inadequate: relevance does not vanish as a necessary condition for entailment, and for worthwhile implication.

From an epiphenomenal viewpoint, it is in *no way* a damaging criticism of sociative logics that a satisfactory characterisation or theory of relevance is hard to come by or circularly attained.[6] Relevance is no longer an obstacle or an embarrassment. For relevance is not, any more than on an irreverent irrelevant approach, a priority. Irrelevance is not thereby

admitted. Relevance remains a necessary condition; but finding and verifying weak and tractable forms of relevance is no longer a difficult problem.

The opposition line, that relevance should not feature even to this qualified extent, is not confined to the enemies of entailment and relevant implication. It is to be found even within the much divided relevance camp.

3. Relevance as irrelevant. The main features of relevant logic are independent of relevance, which was simply a mistaken desideratum introduced in setting up relevant logics. The irrelevant combination of relevant logics with classical negation - often seen, somehow, as real negation, and so as bound to be included in a full sentential logic - has encouraged this approach. Indeed often nowadays there is resistance to the idea that anything worthwhile can emerge from, or be made of, the notion of relevance (thus e.g. Meyer 85). It is the same sort of idea that has delayed or stopped so many innovations; it is like the vulgar assumption in logic, strikingly enunciated by Smiley (writing after Ackermann, but taking no account of Ackermann's technological break-through), that there is no promised land other than classical logic, paradoxes, pustules and all (59 p.234). These sorts of innovation-stopping paradigm-entrenching assumptions are rejected here. A notion as persistent and ubiquitous as relevance cannot be set aside so easily, simply on such bases that no explication so far has emerged, especially when comparatively little real effort has in fact been expended. Even if the notion is eventually cast aside, its wide role calls for explanation, in rather the way that the paradoxes of implication for example, though rejected, require explanation (which they obtain, e.g. through a theory of suppression given in RLR p.140ff.).

These divisions over the role of relevance are closely tied up with attitudes concerning the relation of relevant logic to classical logic (and other logics), where too there are competing approaches. An initial classification of positions takes the following shape:

A. Relevant logic is a rival to classical logic.

A*. Relevant logic is not a rival; rather

B. Relevant logic stands simply as an extension of classical logic, in the way modal logic has proved to be.

B*. Relevant logic is not simply an extension, but is, for example, one of a relativistic basket of logics.

Position A is more or less bound to admit that relevance is of some significance, if only as a by-product; the avoidance of paradoxes of implication, which exhibit irrelevance, is a main objective of genuinely relevant logic, and it is from this that a crucial part of the rivalry with mainstream logics derives. Position B, by contrast, is bound to downplay relevance. For to make the extension line work, classical negation has to be added to relevant logic, rendering it irrelevant. A key issue, then, is the standing and scope of classical negation, or, as it is often called, Boolean negation.[7] Some exponents of position B take the line that classical negation just is negation, as classical logicians and their acolytes have insisted, and that relevant (De Morgan) negation is a funny sort of negation, if negation at all. Certainly

relevant negation is not particularly fundamental, but can be adjoined, as an optional extra, to what is fundamental, namely, classical logic extended by an arrow, $\rightarrow$. Such a line is entirely rejected by position A, and is also rejected by position B*. It runs counter to all initial motivation for relevant logic (it is a line emerging through some loopholes which semantics for relevance logic disclosed). To take position B is to abandon much of the original relevant enterprise, and most of what was valuable in it.

A major complaint lodged against relevance (*part* of what underlies B and view 3) is that there has been no "satisfactory" explication of relevance. But many major notions continue in use, profitably, perhaps over a very long time, without a satisfactory general explication. Sometimes, as with deducibility, probability, and effectiveness, they eventually receive explications, of varying calibre, and perhaps competing ones; sometimes, as with plausibility, process, and, so it is alleged, relevance, even such initial explications are lacking. To gain a grasp on such still elusive notions, it helps to look at what they do, what uses they are put to, what roles they play. It helps here to move from semantics to pragmatics, to ask for use and work, rather than (what is different) meaning.

It is easier to come to appreciate what relevance *is* from what relevance *does* in various settings. The use and point of relevance is, as earlier indicated, to delimit and control an area or field, in order to render data managable or a problem tractable or in order to avoid paradox. Many of its uses are thus like those suggested for significance. Certainly sometimes appeal to relevance is put to diabolic purposes; for instance, where it is used to obtain a ruling, at a meeting or in a court, against an opposition, that some of their considerations are irrelevant, and are accordingly inadmissible. But such uses can be separated out; there remains a difference between what is said to be or is ruled relevant, and what is really.

It is not difficult to say what relevance is supposed to be doing in logical theory. It is supposed, in particular, to control and to limit where one can get from where, by such access relations as implication, and thus for instance to effect field delimitation and to prevent the commission of certain fallacies (for elaboration on the *many* such roles of relevance, see the Appendix). Indeed Sidgwick applied such an accessibility picture in the fallacy that came to be called "irrelevant conclusion", suggesting that committing the fallacy resembled taking the wrong train. Following out the work that relevance has been supposed to do, and in fact does, in the business of argument and the associated business of fallacies, is *one* relevant direction to take. From the epiphenomenal standpoint adopted, the subsequent critical and constructive sections amount however to supererogation. The "hard work" of sufficiently accommodating relevance has, as regards the central case of implication, already been accomplished.

3. Relevance in argument: further criteria for relevance, and relevance as a connective.

Because of the difficulties in assessing relevance in wider settings, the lack of uniform criteria and the paucity of effective tests, a shift is commonly made, early on, to *workable*

criteria for relevance determinates in circumscribed contexts, such as restricted logical settings, for instance those of argument or implication, of functionality or dependence. This has been a main approach in deductive logic, where the contexts have generally been sentential logics, or at most predicate logics, the criteria typically syntactical, though occasionally semantical, and the target primarily implicational connectives (and derivatively functional dependence). But it has not been the only systematic approach.

Other approaches to relevance in logical systems can be divided into two:

- systemic inclusion of relevance, typically through an operator which represents relevance - as usually opposed to
- extrasystemic criteria, to which a system, or certain parts of it, conforms (through imposition, by emergence, etc.)

The second *criterial* approach, which has historical priority in deductive logic, can in special cases (such as Parry logics) be reduced to the first *connective* approach; a relevance-type connective representing some relevance criterion can be defined in the system. Conversely, efforts to gain more control, and strength of system, under connective approaches lead naturally to criterial approaches. Thinner connective approaches will be now investigated; more informative criterial approaches have already been considered (in the previous introduction). The problems with imposing too much weight upon either approach, especially the first, can again be avoided by treating relevance epiphenomenonally.

It is evident that relevance can be represented as a sentential connective, along with disjunction and negation. For the functor '(that)... is relevant to (that)...' is sentence forming on sentences. Let us symbolise it now by infix 'r', i.e. A r B is a wff where A and B are wff. It is also evident that A r B enjoys none of the prized value-functional properties of, for instance, partial truth-functional complexes; in particular, truth or falsity of components never determines truth-value of the relational complex A r B. Indeed like many determinables (e.g. notably the English determinable 'if': RLR p.42), the relevance determinable has, as we have already glimpsed, few or no distinctive logical properties. Reflexivity and symmetry are sometimes supposed to be categorical properties relevance always enjoys. But consider probabilistic relevance; then reflexivity, for example, is qualified (on classical explications) because A does not raise the probability of A when A is logically true or inconsistent.

What logical properties connective r has depends on the systemic setting in which r appears. If it appears within a deep relevant system, as will now be supposed, then more can reasonably be expected: not only should entailment guarantee r relevance, but a rule of intersubstitutivity of coentailment should hold, i.e.

SE. $\quad A \leftrightarrow B, C \leftrightarrow D \;/\; A \mathrel{r} C \rightarrow B \mathrel{r} D$.

With an irrelevant implication those principles will break down. As r is technically cryptosystemic if SE holds, a semantical analysis can be furnished at once (as in chapter 22). The analysis will extend immediately to relevant (relevant) entailment, defined $A \mathrel{-\!\!\gg} B =_{df} A \rightarrow B \;\&\; A \mathrel{r} B$. But as $A \mathrel{-\!\!\gg} B$ and $A \rightarrow B$ are interderivable, introduction of such relevance adds nothing. In short, there is only virtue in so introducing connective r if the

theory starts out *ir*relevant.[8] With such a poor start r might be profitably applied to upgrade an implication connection to a relevant one. But it is a difficult route, as the history of sieve methods reveals. Sieve methods also take this definitional form, along with relational logics, but offer an analysis of sieve relation r, which sieves out irrelevance in terms of specified procedures (originally A r B was read as "there is no way of coming to know B without coming to know A", something that was stage by shifting stage reconstrued in terms of proof procedures: for details see Dunn 80a).

For more interesting logical properties to obtain than the determinable supplies, it is necessary to drop down to relevance determinates, and typically also (as, e.g., for structural requirements such as a variable or symbol sharing to work) to drastically restricted contexts. Among the determinates, it is worth considering some of those that have begun to make their way into more generous contemporary logical agendas, beginning with (context independent) relevance in content or subject matter.

Though the idea of adding a relevance connective to the syntax of relevant logics goes back to the early days of relevance logic, it was brought to prominence by an apparently rival camp, that of relational logic. Within that setting, connection of subject matter is put up as the crucial ingredient missing from classical logic for a satisfactory analysis of fallacies. 'By adding a relation to the basic building blocks of classical logic, we can construct an alternative formal system that does take into account connections of subject matter. Here at last is the heralded logic of topical relations' (Woods & Walton pp.viii-ix; they claim they would also have called the heralded logic 'relevance logic' had that title likewise not already been grabbed).

Relatedness of subject matter is characterised in the same fashion as the structural form yielded by weak relevance: in terms of overlap. A is related to B iff 'there is at least one common element of subject-matter between' A and B (p.192). Though overlap is an obvious choice, plainly there are, depending upon purpose, other choices. A favoured one, if relatedness is to be tacked onto an irrelevant implication, is inclusion. For implication is commonly taken to preserve content strictly, so the subject-matter of a consequent should be included in that of an antecedent. Such an alternative approach leads in the direction of containment logics, rather than the nontransitive direction the relatedness logic with overlap (of Woods and Walton) takes. Overlap, by contrast with inclusion, is a reflexive and symmetrical relation; it is not transitive.

There is no having it both ways, with both symmetry and reflexivity (the latter naturally extended, as in Walton, to A r f(A), where f(A) is some (truth-)function of A) on the one hand, and transitivity on the other. For the result is restoration of the paradoxes of implication, by essentially the standard medieval and Lewis arguments. Overlap combined with inclusion of content (e.g. as represented in sentential variables) leads not merely to a partition into equivalence classes, itself implausible enough, but to collapse, with everything related (through non-contingencies) to everything else. As always with multiple assumptions,

there are various ways to go. Containment and Parry logics opt for inclusion and abandon symmetry. The promoted relational logics, on which we now focus, abandon transitivity maintaining symmetry and extended reflexivity. Abandoning transitivity carries a heavy penalty, sacrifice of even the transitivity rule, A $-\gg$ B, B $-\gg$ C / A $-\gg$ C, and thereby any claim to be explicating entailment or full implication (see RLR p.70ff.). Still the loss is not entirely fatal; for there *are* connectives of this transitivity-shedding sort, e.g. one-step implication, obvious implication, partial implication and certain related conditionals. Unfortunately for relational logic, these are not the sorts of things proponents say *it* is about.

While relational logics are pretty little systems, which do exclude the most blatant of implicational paradoxes, they fail badly on most other intended or expected fronts as well. For, firstly, they admit straightaway variable sharing analogues (i.e., under interpretation, content overlapping analogues) of the standard paradoxes. Thus, in particular, (A & $\sim$A) & A r B $-\gg$ B and A & (A r B) $-\gg$. B $\vee$ $\sim$B. So, wherever A r B, both A & $\sim$A/B and A/B $\vee$ $\sim$B. But A r B can be ensured apparently by mere overlap of concepts; so, for instance where f(a) and g(a) are *any* two propositions about a, f(a) r g(a). Hence f(a) & $\sim$f(a)/g(a) and f(a)/g(a) $\vee$ $\sim$g(a). Similarly f(a) $-\gg$ g(a) $\vee$. g(a) $-\gg$ f(a), and so on, for other notorious principles. Since, letting f be a propositional predicate, f(C) r g(C), and in particular on promoted relational logics, f(C) r C, the following evidently rotten principles and many others like them hold: A & (C $\vee$ $\sim$C) $-\gg$ (B $\vee$ $\sim$B) $\vee$ C, (A & $\sim$A) & (B $\vee$ $\sim$B) $-\gg$ B, (A & $\sim$A) & (B $-\gg$ B) $-\gg$ B, etc. Plainly, as leading proponents of relational logics seem prepared to recognise, A & $\sim$A & (B $\vee$ $\sim$B) $-\gg$ B, a relationalised paradox, is not much better than A & $\sim$A $-\gg$ B. Its inferential role is certainly as damaging. A corollary of importance is then that relational logics are, like related Parry logics, useless for paraconsistent purposes.[9] Anything at all with overlapping content emerges from a contradiction.

The problems arise, in large measure, from trying to tack filtering conditions onto classical logic or to a classically-approved logic, which supplies the deductive power. Logics like classical logic cannot however be repaired in this simple way (or by similar pragmatic patching, with assertional filters, in the style of Grice). The reason is at bottom that material-implication, $\supset$, does not meet correct truth or valuation conditions for implication or conditionality even when content relevance is fully assured. Consider, to illustrate how far it deviates, appropriate substitutions upon such variable-sharing anomalies as (p $\supset$ q &. r $\supset$ s) $\supset$. (p $\supset$ s $\vee$. r $\supset$ q), putting say A(p,s) for p and B(q,r) for q. In this way relatedness tautologies result that are every bit as implausible for implicational and conditional interpretations as corresponding classical tautologies. (The implausibility of these relevance-preserving tautologies is explained in RLR p.6ff.) A corollary is that the canvassed relational view that we just define a relevant conditional or implication by adding a relational filter to material-implication is unacceptable. (Similarly the analogous view, that a bit of assertional patching in conditional contexts will do, is unacceptable.) The view doesn't even get the first degree of implication right; at higher degrees it is seriously astray, as variable-sharing anomalies show.

To avoid these problems, relational or assertional conditions need to be tacked onto a more satisfactory starting system than classical, or modal, logic (as can be done with the theory of content and content inclusion developed for relevant logic, for which see UU). Then however the conditions tend to become otiose for relevancizing purposes; but worse, the whole underlying motivation for such approaches collapses. For a core background requirement, seldom explicitly stated, is to stay within the orbit of the received classical paradigm. One way of trying to meet the requirement is by approved extensions to the classical scheme of logical things: through adjuncts, like modal logics and tense logics, to the protective belt of buffer theories shielding the classical paradigm from objections and refutation. Relational logic, like recent conditional logic, is a further such addition to the broader classical paradigm. (Such interesting extensions are of course assigned a very low status, if not disowned, by hardline priests of the orthodox unreformed classical ideology.) Relational logic aims to bring a certain systemic relevance (intended to represent overlap of subject matter) within the sphere of classical logic, and thereby to enable broadly classical theory to make some sort of room for such classical casualities as the traditional "theory" of fallacies. As a little investigation shows, however, relational logic fares only marginally better on fallacies than it does with paraconsistency and paradox avoidance. Relational logic, like classical logic, performs poorly and makes but few inroads even on those fallacies for which it was supposedly fashioned - fallacies of relevance, whichever they are.

4. Fallacies of relevance in the history of logic.

Though relevance considerations did feature significantly in earlier logical investigations, they did not enter in the way that contemporary tales would make it appear. Relevance was not an explicit concern in the pre-modern theory of fallacies (of course it was bound to appear through indirection). But it was an issue - though under different headings, such as connection and pertinence - in theories of implicationals and conditionals and in the connected medieval theory of *obligationes* (see further chapter 9).

There certainly was an ancient requirement of connection on a good implication, but it early became controversial (see chapter 9). Violation of the requirement, which went beyond that of relevance, was not a recognised fallacy. Although relevance came to figure in various ways in medieval logic, particularly in *obligationes*, it was not in ways that help recent appeals to a tradition concerning fallacies of relevance at all. For, on the contrary, the eventual shaky consensus in later medieval logic appears to have included a commitment rather like that in mainstream contemporary logic, that entailment amounts to some kind of strict implication, irrelevance and all. Accordingly, there were, under a main tradition, many valid inferences violating relevance.

Yet a consideration much advanced in favour of relevance has been - what appears to be - an argument from tradition, that irrelevant logics sanction fallacies of relevance. One traditional fallacy in support of another, so to say. Even such an appeal to tradition is however less than decisive, since tradition competes against tradition; in particular, the alleged tradition favouring relevance conflicts with the alleged traditional validity of

Disjunctive Syllogism. Unless read with uncommon care, tradition isn't entirely coherent.

Anderson and Belnap have to take a large part of the responsibility for catapulting "traditional" fallacies of relevance to their present prominent position, and for the provocative allegation that Disjunctive Syllogism is a fallacy of relevance. In several places (e.g. notably ENT p.17), they have advanced the following bold historical themes:
AB1. 'For more than two millennia logicians have taught that a necessary condition for the validity of an inference from A to B is that A be relevant to B'. The invited inference is that a failure of relevance produces not merely invalidity but fallaciousness (p.17, p.30).
AB2. 'Virtually every logic book up to the present century has a chapter on fallacies of relevance, and many contemporary elementary texts have followed the same plan' (p.17. A weakened version of AB2 appears in Rescher, 64a p.78, who ambiguously asserts that 'the fallacies of relevance have been studied by logicians for many centuries...'.).

Despite the importance Anderson and Belnap attach to such themes in motivating their case, they amass no evidence at all for them. Nor, when examined, does the available evidence support the themes. A straightforward search of the literature hardly sustains AB1, and undermines AB2, revealing it as a considerable exaggeration. While many pre-twentieth century logic books included chapters or appendices on fallacies, not all did even in the nineteenth century (e.g. Boole's works). Moreover, these chapters were not on, or devoted to, fallacies of relevance, and before Whately even relevance obtained scant, or no, notice. The idea of a specific group of fallacies of relevance is a twentieth century invention.

The first theme AB1 refers, presumably, back to Aristotle. Aristotle himself says very little concerning relevance.[10] He certainly does not endorse a condition like that of AB1. Nor does the subsequent history of disputes over implication serve to sustain AB1. In the first place, the issue was not set in terms of relevance, but rather in terms of connection and containment - from which however relevance and, what Anderson and Belnap sometimes equate with it, logical dependence, may be (though shakily) inferred. Secondly, the issue was, apparently throughout, in serious dispute, with many logicians rejecting any requirement of connection for validity (thus Philo and followers in Stoic times; thus the Parvipontani and the eventual mainstream in medieval logic).

The long history of fallacies and the late intervolvement of the separate strand of relevance with fallacies is much more complex, and interesting, than Anderson and Belnap's slick and slight treatment indicates. Of the fallacies that are included in Aristotle's ramshackle but oft-repeated classification, there are two, in particular, that might be accounted "fallacies of relevance": *ignoratio elenchi* and *non-cause as cause*, hereafter dubbed respectively IC (for "irrelevant consequent") and IA (for "irrelevant antecedent"). Neither fallacy was accounted a fallacy of relevance by Aristotle (the term 'relevant' creeps into English translations of the later gloss the Philosopher gives on IA, but not into any account of IC). There is no subsequent historical development of the idea that IA (which is examined in chapter 9) is a fallacy of relevance. The idea that IC is a fallacy of relevance of

some sort, and the label 'irrelevant conclusion' for *ignoratio elenchi*, appears to be a nineteenth century innovation, due to Whately. (The innovation is explicitly ascribed to Whately by Mill, p.542, and the label does not appear to appear before Whately's text. For instance, Reid, who follows Aristotle closely, does not mention irrelevance; *ignoratio elenchi* is called 'mistaking the question', p.707.)

Ignoratio elenchi is first introduced by Aristotle as a type of fallacy that is 'independent of language', namely 'that which depends upon ignorance of what "refutation" is' (note that for such independence it must be ignorance or misconception of conception, not of language). Aristotle explains the fallacy as follows (166b 25: 167a 21):

> Other fallacies occur because the terms 'proof' or 'refutation' have not been defined, and because something is left out in their definition. For to refute is to contradict one and the same attribute - not merely the name, but the reality - and a name that is not merely synonymous but the same name - and to confute it from the propositions granted ... in the same respect and relation and manner and time in which it was asserted. A 'false assertion' about anything has to be defined in the same way. Some people, however, omit some one of the said conditions and give a merely apparent refutation, showing (e.g.) that the same thing is both double and not double: for two is double of one, but not double of three.

In the course of this explanation, Aristotle slides from theory to practice, from a failure to provide or grasp definitions to a practice which neglects requisite (or, if you like, *relevant*) details such as respect, manner and time; from understanding or knowing *that* or what, to appreciating and practicising *how*. The confusion is endemic to Aristotle's attempt to reclassify all fallacies as types of *ignoratio elenchi* (*all* fallacies as fallacies of relevance?). The sketch (168a18-169b17), that 'it is possible to analyse all ... fallacy into breaches of the definition of refutation' (and associated notions such as *proof*, *argument*, etc.), involves repeated sliding from [in]adequate definition of *refutation* (or the like) to [in]adequate refutation. But what is at issue is not (just) definition - not all mistakes are mistakes of definition or theory - but arguments or refutations which appear adequate but are not (the starting account given: 169a 24). Furthermore, the attempt to reclassify all fallacies under the ragbag *ignoratio* heading is to assume that all discussants are well-intentioned or else stupid. But frequently, as with the sophists Aristotle had clearly in view, they are neither. Fallacious arguments are taken advantage of by those who have some reasonable grasp of the concept (as well as it is so far ordinarily characterised); there is in such cases no real ignorance or misconception.

Ignoratio elenchi does, then, require redefinition, in terms of erroneous practice, whether well-intentioned or not. But what distinctive argument practice? As the examples Aristotle gives show, and as some of the tradition recognised, those that involve (incremental) amendment of the intended conclusion, e.g. in manner or respect or time. Thus they do involve a certain (limited) *shift* of point and ground in the course of an argument (or a proof or a refutation, a refutation being 'a proof of the contradictory', of the negation). But none of Aristotle's examples affects irrelevance in main contemporary senses. In particular, they involve nothing like the irrelevance which paradoxes of mainstream implications facilitate.

All are in fact *relevant* in preserving topical links and so on, but *point* is altered. So, as Hamblin says, the modern title for the fallacy "irrelevant conclusion" 'is not very apt' (p.88). But the main reason for inaptness is not at bottom that which Hamblin offers, that Aristotle is concerned with *misunderstanding* of definitions; it is that an amended conclusion may not be irrelevant.

What is more, the modern idea that *ignoratio elenchi* is a fallacy of irrelevant conclusion - when it did appear, in Whately, more than 2000 years after Aristotle - does little to enhance Anderson and Belnap's argument from tradition against various propositional principles. Indeed at first sight what Whately is about seems to have little relevance to Anderson and Belnap's concerns with relevance. For, according to Whately, 'the Fallacy of irrelevant-conclusion [*ignoratio elenchi*] is' that of 'shifting ground'; it 'is nowhere more common than in protracted controversy, where one of the parties, after having attempted in vain to maintain his position, *shifts his ground* as covertly as possible to another, instead of honestly giving up the point' (p.193). Almost all of Whately's examples of so (as it happens, *invalidly*) shifting ground (given on p.144) involve contingent components, and could be accounted fallacies under mainstream theory, when duly transposed from a dialogue setting to propositional formulation. What Whately has to say about *ignoratio elenchi* in his initial classification of fallacies (p.106) may look more promising; for he alludes, in a footnote, to the *connexion* between premisses and conclusion, and he describes the fallacy as that 'when the conclusion is not the one required, but irrelevant'. But unfortunately for Anderson and Belnap's historical story (and also for Whately's purported examples of *ignoratio elenchi*[11]), the argument is supposed *valid*! It is classed as a 'Material, or Non-logical' fallacy (material fallacies taking up for the most part Aristotle's fallacies 'outside language') 'where the Conclusion *does* follow from the Premisses'![12]

Whately's reclassification of "fallacies" - to include "valid fallacies" (see his tree of Fallacies, p.108), in violence to the notion of fallacy - has proved very convenient to subsequent authors, who have wished to persist with an irrelevant implication. For then fallacies, such as those of irrelevance, can be set aside, not as cases of invalidity, but as involving other infelicities or "unsoundness". The trick has been exploited not only by defenders of the contemporary classical faith (it underlies, for instance, Grice's combination of relevance directives with material-implication), but also by late exponents of traditional logic. Thus Robinson (in 1924) follows the "modern" classification of fallacies according to which the fallacy of irrelevant conclusion is a material fallacy (which has now come to mean, at least since Joseph, a fallacy which 'depends ... on acquaintance with a particular subject matter'). 'This fallacy occurs when, in a disputation, one *proves* a conclusion which is not relevant to the argument. It *may* be *valid* in the sense that it really follows from the premisses, but it is *not germane*, and does not touch the main point at issue in the controversy' (p.196; italics added).[13]

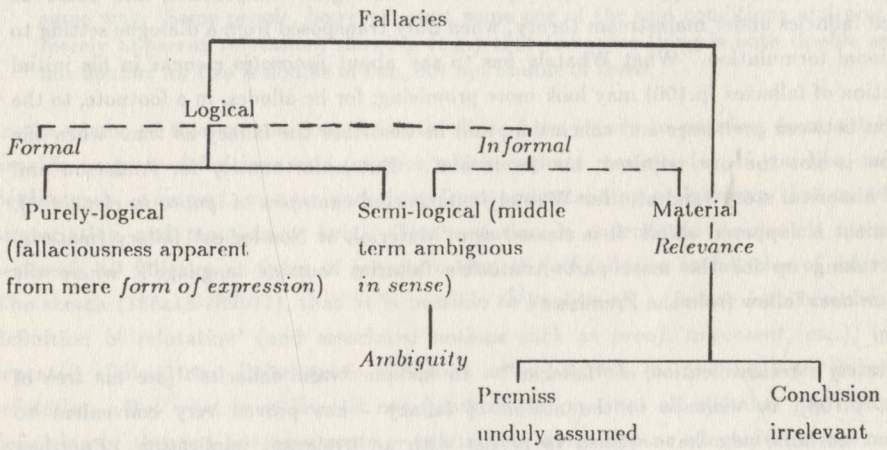
Though considerable, sometimes unscrupulous, advantage has been taken of the device of low redefinition of 'fallacy', it has been little noticed. For example, it passes unheralded in

the few texts on fallacies which sketch the history of the notion.

5. Contemporary fallacies of relevance (with marginalia on fallacy theory).

The introduction of the expression 'fallacy of relevance' to encompass a substantial class of fallacies, usually accounted as *material* in some sense under most classifications since Whately, is a contemporary American innovation, with but shallow historical roots. Worse, as we shall soon see, it is a seriously defective one, decidedly ill-characterised. Though now in widespread use, owing to the popularity of textbooks such as Copi's (and Rescher's) which adopt such a classification, the notion has little historical depth and little adequacy. Historically, it appears to originate from Whately's classification, which grouped many fallacies (most fallacies *ad*) under "irrelevant conclusion". Copi's main classification, for example, is simply an inferior adaption of Whately's, as the following tree comparison begins to indicate:

Diagram 2. *Popular classifications of fallacies.*



Key: Whately classification in solid lines, names in Roman script.
Copi reclassification in broken lines, names in italics.

A classification like Copi's gets repeated in Rescher (64a p.70), the only difference being that Whately's 3-fold division of informal fallacies is restored. Rescher's account, like Copi's, presents old, already much questioned wine in new bottles. The details of the classifications are not entirely the same, something unremarkable given that the authors are working within different logical paradigms; but in virtually every place where they differ, Whately's assignments appear superior to Copi's, though made more than a century earlier. In one relevant respect, Copi's new division (surely influenced, like Rescher, by Eaton) is particularly regressive, namely in reclassifying Whately's material fallacies as fallacies of relevance. Copi's principle of division of fallacies of relevance is supposed to be this: 'Common to all arguments which commit fallacies of relevance is the circumstance that their premisses are irrelevant to, and thus incapable of establishing the truth of, their conclusions. The irrelevance here is logical rather than psychological ...' (pp.87-8). But as Copi is obliged to concede in a footnote

(in later editions of his text), some of the fallacies classed as "of relevance" do not satisfy the principle, petitio principii in particular.[14] But the same point really applies as regards *all* Whately's premiss-unduly-assumed cases, most important non-cause (i.e. IA), which Whately tries to reduce, by finding a suppressed premiss, to 'false premiss' (p.107) and Copi to 'false cause' (p.97). For the premiss that remains in such arguments may be relevant enough to the intended conclusion, as for instance in a Humean "cause" (pace Copi), though inadequate in the absence of a false suppressed premiss (pace Whately's claims of following). Nor need fallacies of accident (and converse accident) - classed by Whately as semi-logical, but reclassified by Copi as fallacies of relevance - involve evident irrelevance; rather they involve failure of a generalisation to apply (or follow) in a particular, perhaps twisted or unusual, instance.

Copi's principle of division for fallacies of relevance gives rise to other evident difficulties. First, the fact that the premisses are irrelevant does not entail, at least according to mainstream logic, that they are incapable of establishing the truth of their conclusion. If the conclusion is a logical one, to take a worst case, any premisses should do, relevant or not. Did Copi have a different logic, which takes relevance seriously, in view? But then maybe some of the fallacies are not informal, but representable in the logic? What is formal ("purely-logical") depends after all on the *assumed* logical theory and how much "structure" is exposed. Secondly, is it that the premisses are irrelevant, as Copi says but some examples belie (e.g. p.89), or the conclusion, as Whately says? Or is it the interrelation? Or doesn't it matter, because if the premisses are irrelevant to the conclusion then the conclusion is irrelevant, and vice versa? Neither Whately nor Copi (nor others) carry the requisite presentation of the arguments involved far enough to see what is involved; and neither offers any requisite story on relevance.

Again Whately does a little better than Copi in giving "technical" information on the structure of the arguments supposedly involved. Whether "formal" or not, what are under consideration are fallacious *arguments*, which accordingly have an argument structure (though one typically embedded in a dialogue form), with premisses and intended conclusion and interconnecting argumentation linkages. As Whately indicates (p.142), in the case of the circumstantial *ad hominem* fallacy (discussed in Copi pp.89-90), what the proponent argues is not intended conclusion B (e.g. "such and such is the fact"), but ΨB, with Ψ (= Ψ_a) an intensional functor concerned with the character or circumstances of the adversary (e.g. "*this person* [a] is bound to admit B, in conformity with his principles, or principles of reasoning, or in consistency with his own conduct, situation, etc").[15] What should be given is the argument $A_1,\ldots,A_n \vdash B$; instead what is supplied is $A_1,\ldots,A_n, [D_1,\ldots,D_k] \vdash \Psi B$, with assumptions $D_1,\ldots,D_k$ used in obtaining ΨB infiltrated, and $A_1,\ldots,A_n$ perhaps rendered redundant. What is incorrect in the proceedings is the replacement of the intended conclusion B by the *presented* conclusion ΨB, which does not guarantee it. There may nonetheless be reason, in the dialogue context, for expecting the adversary to concede B, though it is not usually implied; but in most cases there will not. (If there is a correct route from ΨB to B, then the

ad hominem is *not* fallacious.) The granted premisses $A_1,...,A_n$ may, or more likely may not, be relevant to ΨB; ΨB is (loosely) relevant, under such determinates as common subject matter or parameters, to B. A failure of relevance is not highly relevant to the diagnosis; and 'shifting ground' is quite insufficiently explicit. What has occurred is *conclusion replacement*, with intended conclusion B illicitly replaced by allied conclusion ΨB with B covered by an intensional functor Ψ (which typically does not sustain it). A beginning can next be made on distinguishing types of *ad hominem* arguments, by unscrambling different classes of intensional functors Ψ, of *ad hominem* functors. The detour makes a small beginning on refuting Hamblin's claim that 'there is precious little chance of a formal account of *ad hominem*' (p.218). The emerging account by-passes relevance; so far as it enters, relevance is, once again, a by-product.

The state of the part of the theory of fallacies we have been examining is indicative of the situation with the theory generally: as with relevance, there is so far no decent theory. It does not follow, of course, that there cannot be a satisfactory theory - what De Morgan suggested on other, now generally rejected, grounds (pp.237-8). But what De Morgan (a forerunner of American entailment theory in humorous style, if not in alleged substance, such as the mis-named negation) also indicated, still obtains:- After 2000 years of reflection and a stream of books, most of them admittedly largely copying predecessors, there is still no satisfactory theory of fallacies - and no proof that the area is not amenable to theory (which however it apparently is). The best English language texts to date, those of Hamblin and of Woods and Walton fall far short of such a theory; rather they too pick up tradition and carry it a little further. Hamblin's text ends, so its author in effect concedes, in failure; and the dialogue model offered is, rather evidently, inadequate as it stands. Woods and Walton's book, which builds on Hamblin, does not develop the dialogue theory, leaving it murkily and unnecessarily informal, and incapable of handling some significant fallacies (as Walton's subsequent unsuccessful effort at closing some gaps makes plain). But Woods and Walton do make significant advances upon Hamblin, in genuinely including epistemic functors and in introducing their version of relevance logic, relatedness logic, to accommodate cases where classical logic manifestly fails. But relatedness logic is a primitive and unsatisfactory approach to relevance (as we have observed above and elsewhere), imposing only a crude filter rigged on classical logic, which it leaves untouched as starting point. Yet a main problem with the whole contemporary theory of fallacies appears to be excess adherence to the classical paradigm, which is a source of major difficulties, in particular forcing so much of the theoretical data into the "informal" domain.

A reasonable prediction - again there is nothing definitive, just enough indication for a promising conjecture - is that a satisfactory theory is not attainable within the confines of either old or new paradigms, within either traditional or classical logic frameworks (even an amalgamation of these frameworks, as opposed to standard reductions one way or the other, would offer some improvement). Better contemporary work is having to overstrain these frameworks (to breaking point) and to proceed beyond them; for example, in the revival and elaboration of dialogue (or dialectic, in this sense), in the inclusion of many intensional

functors previously denied any special treatment, and in the inclusion of relevance (even if in degenerate forms). But there is more to it than this: what counts as a fallacy turns on what counts as correctness, and thus, in logic, crucially on validity. But there are now different competing logical theories supplying rival accounts of validity. As the notion of *logic* has begun a relativistic shift, so the coupled notion of *fallacy* must follow. Inferring any (irrelevant) necessary truth from any other is not a logical fallacy on mainstream logic. But on relevant perceptions, as still on preanalytic ones, it *is*; and there is no need to try to resort to *extra*-logical explanation of what has gone wrong or seems to have gone wrong. Mainstream logic simply does condone what are, in a straightforward if recent sense, fallacies of relevance. Unfortunately the straightforward notion was almost immediately stretched and warped.

The straightforward examples are those of entire surface disconnection, such as occurs in paradoxes of implication. Few would want to say, if they felt they could easily avoid it, that A implies B, or that "if A then B" is valid, where A and B are totally disparate in content. The straightforward illustrations Anderson and Belnap start out from are of this type. The germ of the well-known mathematical editor example (ENT p.9) is that editorial practice is to reject implications of the form $A \rightarrow B$, when A and B have nothing to do with one another, i.e. are totally disparate in content. In their further characterisation of relevance (62 p.46), Anderson and Belnap assert that *fallacies of relevance* sanction the inference from A to B even though A and B may be totally disparate in meaning. The implication $A \rightarrow B$, where A and B are *any* irrelevant truths, available in applications of many fashionable logics, undoubtedly delivers fallacies, which could reasonably be accounted fallacies *of relevance*. As Woods and Walton observe, such an implication licences virtually any change of subject (p.51).

Such requirements of relevance (as for instance, no implication where components are totally disparate in content) would eradicate gross disconnection only. Certainly such requirements put us in the sociative logic arena, but they don't discriminate among these logics. One evident reason is that several steps are involved in paradox arguments; responsibility for paradox may be, and erroneously (by a location fallacy) has been, variously laid upon almost every step. Another reason is that the logical ground supporting a fallacy of some type need not involve a fallacy of that type. In particular, the steps in arguments issuing in paradoxes of implication or in other relevance fallacies need not themselves manifest irrelevance. $A \,\&\, {\sim}A \rightarrow B$ is undoubtedly a fallacy, a fallacy of relevance, where B is irrelevant to A and ~A. It is a case of "irrelevant conclusion", certainly a traditional fallacy. Accordingly, any arguments *leading* to this conclusion, any hard argument, is fallacious. Presumably some step or combination of steps must incorporate a fallacy, a fault of some sort. But it *doesn't* follow that *that* fallacy must be a fallacy of relevance. A fallacy of one sort can be responsible for a fallacy of another (with paradoxes of implication, the fallacious assumptions are those of assuming consistency and completeness too extensively). There are some corollaries, the first of which is that whichever principle is selected[16] as source of the paradoxes in sociative logics, it is not automatically, or even at all, a fallacy of relevance.

Another corollary is that the paradoxes are not simply *due* to lack of relevance (as exponents of the tack-a-relevance-relation-onto-classical-logic fashion are wont to assume); this, like breakdown of variable-sharing, is a sign of trouble, a symptom, but not the source. The lack of relevance is, like the patent paradoxes, the product of a deeper cause (namely, loss of connection due to suppression, the argument correlate of consistency and completeness assumptions).

Accordingly, a main principle applied by Anderson and Belnap in arriving at their controversial fallacies of relevance gets rejected. The principle is that if B is deducible from A in system E and B violates formal relevance, then A commits or involves a fallacy of relevance. There are two things wrong with this principle: the elevating of what E-implies a breakdown of relevance to a fallacy thereof (thus Peirce's principle $((p \rightarrow q) \rightarrow p) \rightarrow p$ emerges as such a fallacy, 62a pp.49-50, though antecedent and consequent share variables); and the presumption that E is the canonical "correct" system in terms of which fallacy status can be resolved. What A & B call 'fallacies of relevance' on their low redefinition, are at best E-xtended fallacies.

Until Anderson and Belnap came along, no one had the clever idea of trying to force Disjunctive Syllogism (DSyll) into the defective and ill-characterised category of fallacies of relevance. To be sure, A & B could beat up a case, applying the "almost universal" definition of fallacy. Disjunctive Syllogism certainly seemed to be valid - wasn't that what all the fuss was about? - yet wasn't because system E of entailment informed us of what was a properly valid argument form and DSyll was not among these forms. While this *may* serve to show that DSyll is an E(formal) fallacy, it assumes a large question, the acceptability of E as appropriately canonical in characterising valid arguments in a more system independent way. In any event, the case made would only show that DSyll was a fallacy, not a fallacy of *relevance*. And the case itself undermines appeals to established usage to justify the label. For on the Anderson-Belnap story, DSyll is invalid, the fallacy is a *formal* one. Accordingly the fallacy is not one of relevance on either nineteenth century British or twentieth century American senses, whatever precisely 'relevance' or 'fallacy of relevance' was supposed to mean. The need to specify with any exactitude was not felt by most authorities from Whately to Copi. Yet A & B depend heavily, in their explanation of what fallacies of relevance are intended to be, on appeal back to older logic texts (e.g. 62a p.33).

Given, furthermore, that B appears in subformulae on both sides of the arrow in the familiar form of DSyll, $A \ \& \ (\sim A \vee B) \rightarrow B$, and that the antecedent certainly appears used (and needed) in getting to the consequent, Anderson and Belnap and followers have had a difficult time in persuading anyone else that DSyll is a fallacy of relevance, even by their own standards for relevance. Not surprisingly, they have become a favourite and easy target for predatory philosophical logicians. The theme (advanced in these introductions) that DSyll is a fallacy of consistency, not of relevance, is not such an easy target. For it is then readily explained what goes wrong - neglect of inconsistent situations (which block elimination of $\sim A$ by A) - and why it is a fallacy. It *seems* valid because it is correct in consistent settings to

which it is typically restricted, those of mainstream logic; but it is *not valid* because it fails in situations which are inconsistent, situations not improperly omitted in canonical relevant theory. DSyll, so explained as a fallacy, sets the pattern, furthermore, for a more comprehensive treatment of fallacies.

For a more comprehensive treatment, a *two tier* or double system *theory* is suggested (as in RCR). The basic idea is a simple adaptation of the almost universal account of fallacies as arguments which seem valid but are not valid. For each putative fallacy, there is *one* system or framework, a correct "canonical" framework in which the argument, if fallacious, is not valid, and *another* related duplicious system, which supplies seeming-validity (i.e. validity in "seeming" models). The adaptation makes due allowance for logical relativity, for the canonical framework can vary according to type of logician or reasoner. Given that systems are made rich enough in logical equipment - they would need, in general, to be able to cope with dialogue and polylogue to accommodate the range of fallacies - there is a fair prospect of rendering all fallacies formal. Of course the theory so far suggested leaves a certain amount to be explained, in particular, the selection and appeal of various "seeming" models (e.g. those treating what is a conditional as a biconditional). At worst those matters could be dealt with in a fallacy-by-fallacy way, much as in the traditional theory; but there is reason to expect, as with the *ad hominem* and other examples already developed,[17] something rather better and more general. The two tier proposal offers the real prospect of such a general, depsychologized theory of fallacies.

APPENDIX. Beyond the implicational setting: a survey of some of the manifold roles and uses of relevance.

While this preliminary survey will concentrate upon logical and philosophical applications of relevance, application of quite varying degrees of success, it is worth recording some of the other disciplinary areas where relevance figures prominently. Plainly there is much overlap. The question of relevant evidence in the law is not disjoint from that of relevant evidence as investigated in the philosophy of science, the notion of relevant data or relevant search areas does not fluctuate wildly from field to field.

Outside logic, methodology of science and philosophy, relevance is of particular importance and interest in

- Law, notably in the theory and practice of evidence, but also in the procedure and administration of law. Relevance in law does not differ materially from, but reflects, relevance outside law (in ordinary life, as refined a little in science, etc.); that appears to be the prevailing view in philosophy of law, though there is occasional dissent (see e.g. Twining p.76, p.145, and for undersubstantiated dissent p.143). As outside law, so within, relevance is carried by certain nondegenerate relations such as implication, probabilification, proof, argument, evidence, and the like. As elsewhere, relevance is epiphenomenal.
- Administration, for example in the conduct of meetings or committees or for proceedings of any kind. As with rules of order for meetings, so with procedure in law and elsewhere, the

point of an irrelevance appeal or objection is to remove an item from account, to have it taken out of consideration. Such relevance is what is sometimes called, for obvious reasons, consideration relevance (see Goddard and Sylvan).

- Communication and informational sciences, including communication theory, cognitive science and artificial intelligence (e.g. in the frame problem: see RCR).
- Probability theory, and statistics.
- Linguistics.
- Literature, and the arts, including performing arts such as the dance.

But relevance can enter anywhere, in any field, for instance with the question of the relevance of this or that. However relevance is never a final court of judgement or appeal; wherever proper, claims or assessments of irrelevance can be supported by reference back to infringement or violation of relevance-bearing, and derivatively field delimiting, relations.

Within philosophy the roles of relevance appear initially almost as diverse as the areas where relevance features. But the diversity can again be contained. Very many cases can be seen, in one way or another, as cases of *field delimitation*. The important role of relevance in *paradox avoidance* can in turn be seen as special forms of field delimitation. If the removal of paradoxes by type restrictions is kept in view, this is easy to see. It is also easy to see however that a range of paradoxes can be at least cut off (even if the root cause is not dealt with) by limiting where one can get from where, as with derived paradoxes of implication such as those in deontic logic, epistemic logic and so on.

It is easy also to appreciate why field limitation is increasingly important: because of limitation of time, information and energy (both total time or energy available to an individual or group; time before action is required or a deadline arrives, as in decision and frame problems; etc). Bound up with these limitations are questions of effectiveness and of cost. Limiting things to those that are relevant is evidently important for both.

Field delimination can be given a uniform treatment, as in the text, through a theory of relevance-bearing relations centred upon and patterned upon a relevant theory of implication and reasoning. How this is achieved will now be illustrated, starting at the centre with implication type connections and proceeding outwards.

In the centre region are relevant logics which supply a class of fundamental relevance-bearing relations: entailment, relevant implication, conditionality (how those work, and fit together harmoniously in a single framework, is explained in other essays). These connectives satisfy relevance conditions tight enough to smash all the standard paradoxes of implication-like relations (and also their variable-sharing analogues). To shed most of the derived paradoxes that extensions of modal logic to deontic, doxastic, epistemic, assertoric, and other areas have induced, it is often enough to replace strict implication (or provable implication) by entailment, by a relevance-bearing deducibility relation. (For a detailed example in the deontic case, see PL, in the doxastic case, see JB.)

Close by or at but a few steps out from the central region are relevance-bearing relations, analysed as a combination of relevant implication with further process or containment relations; examples are reason, cause and relevant containment (all considered in RCR). Some of the applications overlap those of more central relevant relations. (There is nothing sinister in this, but simply disagreement as to whether, for example, a believes [or wants] A implies that a believes [or wants] A or B; and likely there are different notions at work). The applications can be divided, roughly, into two groups:-

1. Cases and logics where tight relevance is required, or at least paradoxes and like puzzles are resolved through such relevance considerations. As useful surveys of such cases are presented elsewhere (see especially Weingartner and Schurz), a brief listing can serve:

• doxastic logics, assertoric logics, epistemic logics and so forth, where again superfluous additions are in much dispute;

• deontic logics, preference logics, and volitive logics (with operators like 'desires that', 'wishes that', 'feels that');

• legal reasoning (where Ross's "paradox" has some exposure), and relevant reasoning more generally (cf. Bollen for some of what is to be varied);

• logics of explanation and confirmation, where superfluity again generates several puzzles and gratuitous paradoxes. For a partial survey see Weingartner and Schurz, whose classical tack-on resolutions are *not* however endorsed (for reasons essentially given in the text, or, as directed against relational logics, in Kielkopf 77 and in RLR). Moreover, such relational strategies make quite insufficient impact upon inductive puzzles, like Goodman's paradox of disjunctive predicates such as 'is grue', which are neatly taken out by properly relevant logics. As Bass has observed, the key damaging argument concerning grue, the argument to the prediction that an emerald observed after time t and is blue, depends essentially upon Disjunctive Syllogism. The argument, for an arbitrary emerald a, involves an implication like: a is grue & a is not observed before t $\rightarrow$. a is blue when observed after t, where a is grue $\leftrightarrow$. a is green & observed before t $\vee$ a is blue & observed after t. No doubt the puzzlement generated also relies, like several moves to dis-ease in confirmation theory, upon tack-on implications such as A $\rightarrow$. A $\vee$ B, in moving from "all emeralds observed before t are green" to "all those emeralds are grue". Thus, a full resolution may again look to relevant containment theories.

2. Discourse, cases and logics involving tightly circumscribed contexts, where additional (irrelevant) information is, in some way, inappropriate or otherwise excluded:

• logics of fiction and stories, where additions are in serious doubt. According to the story story, stories don't introduce new content (see e.g. Daniels, pp.221-2). Such operators as 'In the story it is the case that' are only closed under tight relevant containment assertions;

• communication networks, and polylogue theory. For communication and dialogue, like stories upon which they generalise, are characteristically information circumscribed. (For the many places in polylogue theory where relevant containment logics tend to enter, see Sylvan 85).

• meaning, inclusion of meaning synonymy, analysis analysis. For example, even if inclusion-of-meaning theory and Tarski-Davidson meaning-through-truth theory are respectively rectified by setting the theories on relevant logic foundations (see RLR for the first and chapter

26 for the second), odd puzzles at least remain. For, primarily by virtue of Addition, relevant logics permit the introduction of superfluous meaning, through such biconditionals as $p \leftrightarrow .\ p\ \&\ (p \vee q)$, $p \leftrightarrow .\ p\ \&\ (p \vee q)\ \&\ (p \vee r)$ ..., etc., where q, r, ..., may have nothing to do with p. But these relevantly equivalent expressions don't have the same meaning. So in particular a "meaning-giving" T-schema of the form, s is true $\leftrightarrow$ p, should not deliver, s is true $\leftrightarrow .\ p\ \&\ (p \vee q)\ \&\ (p \vee r)$. By switching to relevant containment logics (which are derivationally adequate for the purposes at hand), *these* difficulties are avoided.

• dependence, functional dependence, "relevant" or dinkum predication. Among classically well-formed or well-defined predications, many genuinely depend upon or intervolve their arguments; but some do not, such as vacuous predication (as in 'Hobbes is such that uranium is radioactive'), and perhaps constricting complexification, as with $R = \lambda x \lambda y$ (fx and gy) where f and g may have nothing to do with one another. As with predicates, so correspondingly with functions; many genuinely depend on their arguments, but some (artificial ones) do not. In North American relevance circles, such a contrast has been construed, though without impressive grounds, as one of relevance, and various ways of marking out the contrast technically have been proposed, some of them cumbrously inductive from questionable atomistic bases (for a survey, see Dunn 87). However an appealingly simple way of making good the contrast is suggested by Dunn, namely using the (relevant) failure of full identity substitution to define irrelevant predication. What emerges can be summed up here in the biconditionals;

> $(\rho x\ fx)a$, read: a relevantly has the property of being (an object) such that f, i.e. a relevantly has f, iff $(x)(x = a \rightarrow .\ fx)$;
> $(\rho xy\ gxy)ab$, a relevantly has g (stands in the relation of being such that g) to b, iff $(x,y)(x = a \rightarrow .\ y = b \rightarrow .\ gxy)$.

Observe that these characterisations of "relevant predication", which have relevance carried by relevant implication and identity, are effectively nothing but relevant transcriptions of Russellian definitions of *transparency* of predicates with respect to their argument places. If however there is anything the matter with, any 'funny business' in, opaque predication, it is surely not irrelevance. The predicate 'George IV wished to know whether the author of *Waverley* was ...' certainly appears relevant to the argument, Scott; but in Dunn's account it does not count as a relevant predication. Unless Dunn wishes to join forces with notorious reactionaries, his proposed explication fails, and therewith his ambitious project of using 'the apparatus of relevant predication [earlier linked, p.348, with no funny predication] ... to stop once and for all, the "funny business" that fuels a whole philosophical industry of a "Chisholming" kind' (p.374),. What Dunn may have captured is a certain tight dependence feature, *tependence* let's say, dinkum predication some might be tempted to say (but only 'real' or 'natural' predication from the reactionary stance). There are other troubles with tependence (concerning its compounding features) which we can pass over here. Some of the exclusions Dunn rightly wants to effect can again be made by switching to relevant containment logic; in particular, the problem of vacuous predication (and complexification) can then be handled by requiring that the predicates do have (and are built-up from those with) due places for free variables (cf. p.351). There are other grounds for using such a

tighter containment logic for explicating genuine dependence, for instance that the presence of Addition leads to such apparant anomalies as that the disjunction of any old formula with a relevant predication for a is also a relevant predication for a (cf. p.373). (But, more generally for the "object language" explication of such notions as logical dependence, logics offering even more control than relevant containment logics are called for: see RLR II.)

• frame problems. In important respects, general frame problems (investigated in some detail in RCR) typify this whole class of adjacent problems.

A little further out again are relevance-bearing relations, fundamental in philosophy of science, such as probabilification and evidence; but the semantical theory may nonetheless be patterned after that of more central relations. A leading strategy consists in replacing possible world analyses or their equivalents by semantical elucidations which include, as well as complete possible situations ("possible worlds"), both incomplete and inconsistent situations ("other worlds"). That is, the procedure corresponds exactly to the type of semantical transition that leads from provable material-implication or first degree strict implication to first degree entailment. Algebraically, the transition amounts to that from Boolean algebras to De Morgan lattices. Such an expansion strategy appears to work well in supplying (first degree) accounts of relevant probability and relevant information, and can also be applied to such notions as relevant preference (for details, see JB, appendix). For relevant evidence, elucidation will presumably be patterned after relevant reason; that for relevant confirmation will build upon this and relevant cause. And so on, or at least that is the idea. For there is plainly *much* to be accomplished in these outer regions and further out.

What is in no doubt is that most of the notions encountered meet relevance requirements, so that adequate elucidations *must* deliver relevance features (at least) as a by-product. The point applies in fact to many of the main relations in the philosophy of science. As with implication, the standard procedure - of starting with classical connections, and then tacking on further relevance requirements or the like, to remove anomalies and paradoxes - is most unsatisfactory, and gets things back to front (as already argued in the text in the central case of implication).

As it is with implication, so it is with other relevance-bearing connections, such as causation. A satisfactory analysis of cause delivers relevance as a by-product (as that in RCR does). But a standard mainstream approach, favoured by those empiricists who acknowledge the problem, has been to try to hook relevance conditions on to a constant conjunction analysis. Thus relevance has come to feature in (attempted) analyses of causation, especially in trying to patch up invariable sequence analyses done in terms of "similar antecedent - similar outcome". To avoid counter-examples, the similarities need to be *relevant* similarities. The trouble with such a repair is, of course, that it has proved difficult, to say the least, to characterise relevance in this context without circularity, without appeal back to causality. (These issues are discussed in some detail in Taylor.)

As causation is to be patched up by tacking on relevance requirements, so, on

mainstream approaches, are explanation and confirmation. Restriction is rendered more difficult because mainstream efforts begin from stripped-down theories of deducibility (or at most deducibility and probability) from which relevance connections have been removed. So somehow they have to be added back again. Consider explanation, 'Any scientific explanation [explanans] must be *relevant* to the thing needing explanation [explanandum]' (thus Mannoia, p.75, in a quite orthodox introductory philosophy of science text). The deployment of classical logic in the main deductive model of explanation runs in outright defiance of this relevance requirement. For, in any explanation involving general laws, the archetypal case according to Hempel (a staunch advocate of the deductive model), the laws can be suppressed, since always true, leaving perhaps irrelevant conditions supplying the supposed explanation. Hempel aims to escape problems of this sort by tacking on a *use* condition, strongly reminiscent of use requirements imposed for relevant implication derivations. Thus his requirement:
'(R2). The explanans must contain general laws, and these must be actually required for the derivation of the explanandum' (p.248). This represents, once again, a jerry-built way of joining on relevance conditions that should have been incorporated front end in.

It is the same with mainstream approaches to confirmation and evidence. Most unfortunately, basic to most endeavour in these reaches, is classical probability theory - a severe impediment to more illuminating, relevant connections (see UU). Classically, positive-relevance is defined in terms of increase in classical probability, perhaps relative to some background information; and negative relevance defined through decrease (see e.g. Carnap 62). Evidence, confirmation, and the like are then defined in one way or another, through classical positive-relevance. In fact there are problems even with the initial classical account of relevance (as Keynes realised and Gärdenfors explains). Notwithstanding, in a typical reductive attempt, Mackie offers what he calls 'a relevance criterion of confirmation or evidence'. The familiar-looking criterion is this: e is evidence that h given b, iff p(e, h & b) > p(e,b), i.e. h is positively relevant to e given b. But such a paradox-prone criterion provides neither a necessary nor a sufficient condition for evidence (as a little reflection shows, and as Achinstein argues, p.352).[18] Much classical energy has gone into complicating the criterion to avoid its problems. Straightforward sociative repairs can however be made by substituting for the underlying classical probability theory a relevant or sociative theory. The differences that would result can be indicated by recasting the matter in implicative terms. Since positive-relevance can be represented as an implication of a sort, all the dependent accounts can be given a similar implicational representation. With an improved probability theory, matching a more satisfactory implication, improved accounts would result, as with entailment. Similarly many other applications of positive-relevance could be upgraded, by going appropriately nonclassical. But even if an improved theory of evidence should result from a relevant theory of probabilistic argument, a superior approach to start out from, an approach, tending rightly to eschew reductions, is that suggested by Glymour, where relevance of evidence is an outcome of satisfactory test and evidential procedures. Of course it remains to circumscribe and formally represent these procedures (a large business at which Glymour does not enjoy much success). But, so long as relevance is not (wrongly) part of that detail, the matter of relevance has been met; it will fall out of the relevance-bearing relations critical to

the procedures involved.

It is thus evident, what is often observed, that requirements of relevance enter as conditions of adequacy upon the explications of several fundamental notions in the philosophy of science, and are neglected only at peril. It may look then as if "relevance" ought to be a central concept in the philosophy of science', what Gärdenfors claims, on the flimsy grounds that irrelevant information is discounted (p.351). In fact Gärdenfors proceeds to make relevance a complex derivative notion within the defective classical tradition; he too proposes in the end (p.362) to reduce relevance, but in an epicyclic, pretty unusable, and questionable fashion (making inconsistent information irrelevant, and admitting irrelevant components as relevant), through classical probability. Such industry illustrates another evident moral emerging from our partial survey of the intrusion of relevance into the philosophy of science: namely, the poorness of standard approaches, of beginning with irrelevant classical logical bases (usually classical quantification and classical probability logics) and subsequently trying to define in or to tack on relevance fixes. It is rather like trying to insulate and render passive solar a house after it has been built, instead of making these features an integral part of the building. A much superior approach is to begin with logical systems that build appropriate relevance into the structure integrally.

Matters of relevance are by no means confined to the easily accessible philosophical surrounds so far surveyed. They reach further out still, tight over some philosophers' philosophical horizons. Relevance admits, for instance, of practical, moral and political modification and application. A convenient bridge between philosophy of science and such reaches is now afforded by decision theory, where relevance certainly enters, for instance in the distinction of irrelevant and relevant alternatives. But, quite independently of decision theory, and often in disdain or ignorance of it, moral and political relevance are topical. Moral relevance, in particular, is much discussed, if seldom decently characterised. According to Hare, in a typical pronouncement, the notion of moral relevance is crucial to moral argument; yet, also typically, he offers nothing more than an exceedingly loose characterisation of the notion. What is morally relevant to some act or situation comprises those features, individuals and so on, that enter into (or should enter into) its moral appraisal; what is irrelevant is what is (or should) be left out of assessment (p.73ff.).[19] It is thus a kind of field-delimiting relevance, carried by appraisal and (value) assessment relations.

Relevance also figures prominently in some recent, though generally disappointing, assaults in the fashionable area of communication and cognition. For example, *Relevance* is the title of a sizeable recent book (subtitled 'Communication and Cognition') by Sperber and Wilson. Many writers on the topics, including Sperber and Wilson, make heavy use of Grice's work on logic and conversation (more of which has long been forthcoming in a book of *that* title). An important, but uninvestigated, maxim in Grice's work is that of relevance. The maxim entitled 'Relation' (under a quaint Kantian classification of the principles required for *cooperation* in conversation) is stated succinctly enough: '*Be relevant!*' A similar maxim is of course embedded in highly structured interchanges such as organised meetings, where the

rules of order embody and include, in some fashion, requirements of relevance (cf. Roberts' rules). But what do these maxims amount to, mean, require?

Though the question of relevance is crucial to his enterprise, Grice himself has apparently not found time to address himself to the business. In 'Grice's account ... essential concepts mentioned in the maxims are left entirely undefined. This is true of *relevance*, for instance' (S & W p.36). By contrast, Sperber and Wilson (S & W, for short) aim to advance the analysis beyond 'dressed-up appeals to intuition' and a 'commonsense view towards theoretical sophistication' (pp.36-7). They are by no means alone in their proclaimed endeavour (needed while there are still bounders about who say it can't be done). Lakoff, for example, sees the maxims deriving from Grice's "cooperative principle" as falling within the purview of "natural logic" (what in this work doesn't?). He proposes 'restating' the maxim of relevance as follows: 'If x is cooperating with y, then x will do only what is relevant to the enterprise at hand, unless his actions make no difference to the enterprise', and suggests that this seeming platitude 'should follow from the meaning of *cooperate*' (75 p.270)! Even if it does, even if fuller cooperation is relevance-bearing, the principle Lakoff offers hardly serves as a useful or relevant restatement of Grice's "cooperative principle". In fact, it is not a restatement, because Grice's principle permits application (whether intended or not) to antagonistic conversations such as debates, quarrels, etc., which involve *limited* cooperation. In a legal conversation, for example, deployment of fallacies, irrelevance, delays, and the like, may prove useful strategies - whence the need for further principles beyond limited cooperation, such as those of relevance, for "well-conducted" conversations. No doubt relevance is a *derivative* feature, once again, upon other features of more satisfactory, better conducted or friendlier, conversations. So at least Lakoff did not err in saying nothing at all about what relevance consists in, and in claiming that 'no separate set of pragmatic principles should be necessary'. But therewith he destroys much of Grice's not so subtle enterprise: that aiming to sustain the part of classical paradigm by a pragmatic ring of defences.

Sperber and Wilson do try to do something direct about relevance. But their attempt to fill the yawning gap in Grice's work is far from successful (to report in a very charitable way on the adequacy of their effort). They do not even improve upon the 'platitude' Strawson had earlier 'dignified' by the title 'the Principle of Relevance'; namely, 'We do not, except in social desperation, direct isolated and unconnected pieces of information at each other, but on the contrary intend in general to give or add information about what is a matter of standing or current interest or concern' (64 p.115), a requirement of *topic* relevance for normal communication. In S & W, relevance is characterised only for a unduly limited case, certainly inadequate for Grice's purposes, that of processing new information. It is easy to gain the impression (from p. 48 and elsewhere) that old information is never relevant!, that (compliments of defective models of communication) relevance only enters where new information gobbling is proceeding apace. 'When the processing of new information gives rise to ... a multiplication effect: information which could not have been inferred [otherwise], we call it *relevant*' (p.48, rearranged). No doubt the intention was to define relevance along *measure increase* lines, in the fashion of probabilistic relevance: A is relevant ... if its

obtaining increases some measure in some appropriate context. But intentions are one thing, results another. The actual result is worthless, because *any* genuinely *new* information, whether relevant or not, will increase what can be derived. S & W's illustrative example indicates they meant something different, not exactly new information but information that is worth attention, that a processor 'should pay attention to' (p.49). But that is not to step out of the relevance circle towards an explication.

Not too long afterwards they say, differently, that 'relevant information ... is information that modifies and improves an overall representation of the world' (p.71). Removing 'improves' would make an improvement, since information can be relevant but misleading; their later vague and relevance-undiscriminating account makes *this* clear: something is relevant if it has a contextual effect, makes a difference in the context (cf. p.119). There are still a couple of major problems with this adjusted global account, which represents S & W's acme, apart from the fact that it doesn't really correspond to local conversational relevance. First, the requirement doesn't distinguish relevant information from news, or from certain confirmation notions. Secondly, highly irrelevant information, for instance interjected, can modify a world representation. Later again, S & W speak of *relevance* as a technical notion, in cognitive science, for 'a property of mental processes' (p.119)! But many of the relevance determinates already examined are *not* of this sort.

The general (but expected) failure of their various explications of relevance has a serious effect on S & W's wide project - as well, of course, as producing a damaging flow-on effect on associated pragmatic theories such as Grice's. For S & W's main thesis concerns a principle of relevance: 'The main thesis of this book is that an act of ostension carries a guarantee of relevance, and this fact - which we call the *principle of relevance* - makes manifest the principle behind the ostension. We believe that it is this principle of relevance that is needed to make an inferential model of communication explanatory' (p.50). None of this is perlucid: maybe relevance is only incited, not guaranteed; maybe a good deal more is needed to rescue the inferential model; but much of the assessment turns on what is conspicuously lacking, in S & W as in Grice, a decent account of relevance. Moreover, some awful if very American accretions will have to be removed from S & W's picture of human interaction and communication for such an account to become plausible. One is the claim, tied in with 'maximising the relevance', 'that all human beings automatically aim at the most efficient information processing possible, ... whether they are conscious of it or not ...' (p.49). Like much of the preceeding material on goals of cognitive efficiency, this appears false to much of the human predicament, and to the goals of many unAmerican players. There is no 'general goal', 'a crucial factor in human interaction', of 'maximising the relevance of information processed' (p. 49). Whatever that means exactly, here as elsewhere, enough is enough.

The topics sketchily considered by no means exhaust the places where relevance enters, or is supposed to enter, in philosophy and its surrounds. Some out-standing examples, regularly thrown up in literature searches, will supply a fitting conclusion to this survey. Goodman, for instance, makes it look as if he is about to present a discussion of relevance,

something parts of his theory could surely use, by his eye-catching heading "relevance" (regularly picked up in computer searches and by literature-knowledgeable colleagues). No doubt good signification and reference relations, like good implication ones, carry relevance. But Goodman proceeds, erroneously, to equate relevance with the issue of what a statement is *about*, a significant question which he too quickly contracts to the issue of what a statement says something about (p.241), relevance disappearing from view. In short, relevance - which is much more comprehensive than such statemental aboutness - features primarily as a misleading advertising heading; expectant readers could justly feel they had been conned.

Differently, philosophers in the idealist tradition suppose that there is a major problem of relevance in metaphysics, a problem that is often, as for example in Schutz, hard to fathom. "The problem of relevance" Schutz appears to be grappling with is but dubiously a problem primarily of relevance. It comprises rather a bundle of problems, genuine enough, as to choice, prominence, importance. For "the problem" concerns the selection, organisation and ranking of phenonema, especially in perceptual fields (but also in "relevant" sets), which renders some items prominent, problematic or whatever, and thereby organises and structures the field (cf. p.26). Too often however the alleged idealist problem of relevance is based on an over-assiduous application of erroneous ontological assumptions - as happens in Whitehead, who appeals to his ontological principle to justify the proposition that relevance lies in suitable relation (togetherness) with the actual, a proposition making trouble for unrealised abstract forms (p.27). But relation with what is actual or exists is neither necessary nor sufficient for relevance (or for relationality, to note an idealist variation); and the underlying ontological assumptions on which these so-called problems of relevance rest involve major metaphysical confusion (see JB). The numerous counterexamples to ontological assumptions, especially those drawn from theory and from fiction, make it evident that relevance has very little to do with actuality.

To sum up, relevance is an important desideratum, in many areas. But it is not something to be sought directly. Its satisfaction, like that of other major desiderata (e.g. goodwill, happiness), is to be obtained by indirection. The recipe, at least, is straightforward. For relevance is derivative upon the composition of relevance-bearing relations. If these are properly explicated (so far as required), relevance will emerge, as an epiphenomenon. In this obliqueness of relevance lies its elusiveness.

NOTES

1. Assembled in the Appendix are main applications and special explications of relevance, not only in logic and philosophy and associated areas, but also from other more remote areas. An useful initial bibliography of relevance can be readily assembled from entries in *The Philosopher's Index*.

2. The adaption consists in the replacement of one word, '*rationality*' by '*relevance*': see Popper 74 p.1088. Popper here appears to follow in the steps of Wittgenstein and more ordinary language Oxbridge philosophers - though he would not care to acknowledge it, and does not. However it could be that he arrived at the erroneous result on his own. It

is a result he could easily have avoided, since his recently-perceived world 3 (a Renaissance idea wheeled out afresh) offers a natural home for such scientific essences as relevance and rationality.

3. Prior was fond of this ploy in discussion; it is reported in Bennett, and it is discussed in the previous essay.

4. The student demands for relevance (of courses, *to their* lives and *their* interests) took a stronger version of the stronger form.

5. Where these toy logics can be closed under modus ponens and, more easily achieved, reflexivity and transitivity, they resemble logics already familiar from elsewhere (e.g. RLR chapters 2 and 3 or, where latter closures fail, RCR - or under a different representation, non-normal modal analogues of topological logics). But where, more realistically, they cannot be closed under modus ponens, they belong to a new logical area, little scarred by human enterprise. For an initial survey of some of these systems, see Sylvan 87.

6. But criticisms of relevance logic along these lines just keep on reappearing. A grosser example is provided by Diaz, who, after trying to dispose of Anderson & Belnap & Co., proceeds on his own unsuccessful pursuit of relevance - which remains a pot of gold at the end of a connectional rainbow. Diaz, who starts off in a wrong direction by effectively equating meaning connection with relevance (p.8), advances as his main criticism of A & B that their theory requires - but lacks - an analysis of relevance (p.9, p.57). Likewise, of system *R* itself, he claims that 'the system literally begs for an analysis of relevance to serve as a foundation for their "analysis" of relevance', but 'it lacks a foundation in a theory of relevance' (p.57). Such criticisms fall when relevant logic is differently motivated, supplying relevance as a by-product.

7. Although the labels, *Boolean* negation for classical negation, and *De Morgan* negation for relevant negation (or its algebraic analogue), have some currency, they appear to have little historical justification. De Morgan, who encouraged Boole in his logical endeavours, did not appear to hold conspicuously different views on negation. But, as it happens, Boole's theory does not, without supplement, deliver full classical logic. However, the distinctive features of "De Morgan" negation, which distinguish minimal relevant negation and separate it from intuitionistic negation (as in RLR chapter 6), are the De Morgan laws: $\sim(A \,\&\, B) \rightarrow . \sim A \vee \sim B$ and $\sim(A \vee B) \rightarrow . \sim A \,\&\, \sim B$ - principles clearly stated by Ockham, who *may* have entertained a relevant position, subsequently rediscovered by De Morgan.

8. With different objectives in view, such as imposition of a tighter relevance or containment connection than entailment supplies, there is of course a point, and considerable virtue, in proceeding in such a way. For such an approach see RCR.

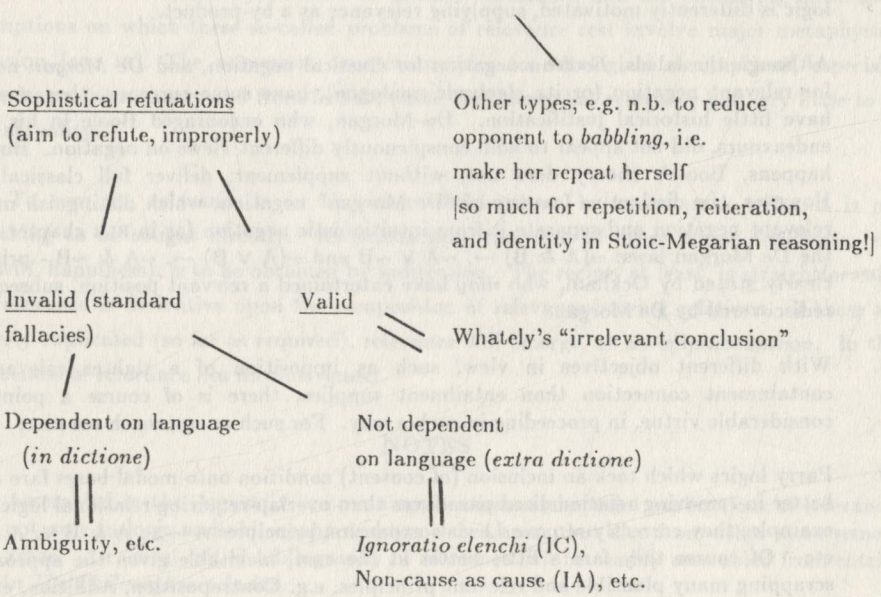
9. Parry logics which tack an inclusion (of content) condition onto modal bases fare a *little* better in removing relationalised paradoxes than overlap-requiring relational logics. For example, they correctly remove Lewis's expansion principles $A \,{-\!\!\gg}\, . A \,\&\, B \vee . A \,\&\, \sim B$, etc. Of course they fare a little better at the cost, inevitable given the approach, of scrapping many plausible and relevant principles, e.g. Contraposition, Addition, etc. As to the uselessness of Parry logics for paraconsistent purposes, see Kielkopf 77.

10. While that does not mean that the notion is unimportant - Aristotle also says very little about effectiveness, for instance - it would have served to reduce considerably its historical thrust. If the Philosopher *had* made something of relevance, had explicitly

required relevance of a good conditional, it would have been logically newsworthy for much of recorded history, and would have significantly altered that history, in a direction better conforming to relevance mythology.

11. There are other things Whately should be taken to task for as regards *ignoratio elenchi*, including, presumably, his misconception of what is involved in a refutation or rendered fallacious by virtue of it. Given this misconception it is unremarkable that he fallaciously concludes that 'it might be desirable therefore to lay aside the name of "*ignoratio elenchi*", but that it is so generally adopted as to receive some mention be made of it' (p.107). On the classification of fallacies, Whately is properly taken to task by Joseph, who however, instead of offering some genuine improvements, prefers to revert to Aristotle's antiquated classification!

12. There is both some logical basis for what Whately does, and a historical basis going back to Aristotle, which renders his procedure more intelligible. The logical basis is that a person, reasoning sophistically, who shifts ground *may* nonetheless argue validly. Arguments to the wrong (to an "irrelevant" conclusion) may be either valid or invalid. The historical basis is more intriguing. Aristotle investigates in *Sophistical Refutations* not just the commission of fallacies (by opponents in discussion) but the much more extensive area of "contentious reasoning", and not just sophistical refutations which are fallacies, but a wider class of such practices. The classificatory picture is as follows:



In short, Aristotle included under "sophistical refutations" not only traditional fallacies of reasoning, but a *third* category 'sophistical refutations by valid

arguments inappropriate to the subject matter', which in fact includes Whately's "irrelevant conclusion". 'By a sophistical refutation and syllogism I mean not only a syllogism or refutation which appears to be valid but is not [i.e. a standard fallacy], but also one which though it is valid, only appears to be appropriate to the thing in question. These are those which fail to refute and prove people to be ignorant according to the nature of the thing in question ...' (169b 20-24). In the subsequent confusion of the original fallacy of *ignoratio elenchi* with this third category (almost juxtaposed with it) appears to lie the genesis of the modern "fallacy" of *ignoratio elenchi*. None of this historical muddlement helps lift those who claim to find a tradition of fallacies (or sophistries) of relevance off the hook.

13. Robinson contrasts "irrelevant conclusion" with *non-sequitur*, 'in which the conclusion is not only irrelevant to the point at issue, but is also invalid in the sense that it does not even follow from the premisses' (p.198). His prime examples, far removed from contemporary fallacies of relevance, do not strictly conform to his account. 'The most vicious form of *non-sequitur* is to infer that a certain proposition [the existence of God] or theory [evolution] has been refuted because some argument in favour of it has been exploded' (p.198). Topical relevance is not violated.

14. Copi subsequently tries to maintain that though the premiss of a petitio argument is logically relevant, it is irrelevant for '*proving* or *establishing* the conclusion'. Such a new principle of division may remove some difficulties, but introduces others.

15. An *ad hominem* strategy may be tried by either party in a disputation against the other. Here we have simply selected the case where it is applied against the adversary by the proponent. But it could be the other way around.

16. It could be claimed that no principle is the source of trouble; it is the whole combination of them. While this is correct in a sense - paradoxes and weak relevance are systemic features - the larger holistic principle involved is difficult to put into practice, and rests in any case on the complex of sociative logics.

17. For a neat treatment of petitio principii along similar double system lines, see E. Martin 83.

18. Other similar linkages, characterising evidence in terms of classical probability, are also damagingly criticised by Achinstein.

19. In utilitarian terms, such as Hare is committed to, an individual is morally irrelevant to a situation if that individual's utility would make no different to utility summations.

CHAPTER 4

A PRELIMINARY WESTERN HISTORY OF SOCIATIVE LOGICS

Contemporary logical investigations enjoy the advantage of vastly improved logical technology as compared with all earlier terrestrial times. Yet, by comparison with earlier periods of high logical activity, the twentieth century is anomalous in its heavy mainline concentration upon classical logic, and, as a result, appears stodgy and unadventurous. For the deadening effect of the wide educational imposition of a narrow and intellectually disastrous dominant logical paradigm, classical logical theory, has (again) destroyed much logical expression and adventure. Rival logics have become very much a minority and esoteric activity, not even incidental to the serious affairs of life; no longer do even the city crows converse over logical issues. The full flourishing of sociative logics, in their rich variety, has yet to occur.

So far as we know, there have been three main periods in the long history of Western logic when the central issues of logic, as to what makes an argument valid, when deducibility obtains, and whether these connections can be captured in true or necessary conditionals, have been vigorously discussed. The periods are these: around the third century BC when Stoic logic flourished, in the medieval period, especially the twelfth century AD, and in the present century.

The logical investigations carried out in these three significant periods are, thus far, substantially independent. The Stoic enterprise of the third century exerted little or no influence on medieval thought, and indeed details (such as they are in Sextus Empiricus) were not available until after the seminal work of Abaelard's school and rival schools had already been accomplished. Of the main contemporary strands of sociative logic, only connexive and nontransitive logics have clear historical representation, and even there main investigations have proceeded substantially independently of historical inputs. When history has been appealed to in support of relevance logics, for example, it has been rather peripheral and, too often, historically dubious.

Though the main historical settings for sociative logic presupposed a heavy consistency assumption (in particular throughout medieval times), and though paraconsistent logics tend to be missing in expected areas of application (such as treatment of semantical paradoxes), nonetheless the history of sociative logics is deeply interwoven with that of paraconsistent logics. One major reason for the intertwining is of course that a crucial issue for sociative logics is what - by contrast with strict and classical spread and collapse - happens with impossible premisses and assumptions. How is loss of connections to be avoided there? A special section of the theory of obligationes (or suppositional reasoning and commitment) was devoted to this issue in the Middle Ages; and a similar division of research, plainly parasitic on classical logic however, can be seen in contemporary North American research (such as that of Rescher and Brandom and of Woods and Walton). These pretty unsatisfactory ways

of shunting off, and sidetracking, significant logical problems fortunately by no means exhaust feasible lines of approach, as the rich history of paraconsistent logics helps disclose. That history has already been documented, admittedly also in a very preliminary fashion, elsewhere.[1] The entertaining story, which overlaps and complements the history of sociative logics, will not be repeated, but elements of it will be drawn upon where appropriate (and readers who seek a fuller picture to begin upon their own investigations should consult that story, as well as, of course, but cautiously, standard texts).

Influences in historical investigations run, so to say, two ways. Not only can historical work influence later work, though almost invariably in a highly selective way; thus, for example, the influence of Aristotle's work and Stoic logic on the contemporary path of connexive logic (e.g. in Angell and in McCall). Later work can also have a heavy impact on the interpretation and construction of earlier work, which is often read as anticipating the moderns or is boxed in by later conceptions of the alternatives. The mid-twentieth century interpretation of Stoic logic affords a striking example. Both Mates and Bocheński attribute to Chrysippus a theory of strict implication - despite his explicit requirement of connection - seemingly because that was the only box they had available into which to fit him! And Mates' tale that Chrysippus is committed to 'strict implication' (p.4) is simply reiterated in a sequence of subsequent writers (e.g. Bocheński, De Rijk on Abaelard, Gould). Now that a much wider range of logics is recognised (than was apparently available to Mates), rather more justice can be done to original logicians like Chrysippus. But, of course, while we still lack any satisfactory picture of the scope of logics, and while many quite basic logical notions remain without adequate investigation, the distinct possibility of pushing ancients, and especially original thinkers, into preconceived boxes which they do not fit, remains.[2] Stoic logicians are by no means the only casualties from classical times. Aristotle himself, subsequently the towering figure in Western logic (also "the author of darkness" according to followers of Ramus), may also have been a casuality, as will soon appear.

The misrepresentation of prominent figures in the history of logic is one aspect of mythologic; another is the setting up of a myth supporting dominant logic, confirming its supremacy and all-encompassing character, and illustrating its historic progress. The history of medieval logic, especially, has suffered from this sort of Whiggish treatment. Yet another aspect of mythological distortion in favour of the conventional wisdom consists in the elevation of thinkers, perhaps much more minor in their times, who appear to anticipate elements of the present dominant framework, and to neglect or demote those pursuing rival themes. Naturally imperialism in logic has much in common with imperalism elsewhere; mythologic, an approved progressive (Whiggish) history of logic, is but a part of the pattern of domination. Control of or influence upon the subject and its presentation through appointments, patronage, journals, publications, grants and support, are other features, as old as the history of logic. While these devices are not always successful, particularly in the longer term, they have impacted on the lives of many significant logicians, especially alternative thinkers such as Abaelard and MacColl and dubious ones such as Ockham; those affected include, appropriately enough, Aristotle (for details, consult what information we have on

their lives).

I. The initial classical period: Aristotle and early Peripatetic logic

The Western history of sociative logics undoubtedly reaches back well before Aristotle. It is sociologically implausible to suppose that workaday logic sprang up, full-clad like the goddess Athena, in a single philosopher. More substantially, we have solid indirect evidence that many of the pre-Socratics and Sophists were bound to adhere to nonstandard logical theories, in order to support their strikingly unorthodox philosophical positions, positions which were, in several interesting cases, apparently paraconsistent (for details see PL p.5 ff.). Given the tight control Greek thinkers liked to keep upon assumptions and arguments, given that they did not appear to approve or tolerate regular suppression of assumptions in argument practice, it is likely that the implicit logics underlying many of their very distinctive positions were not merely paraconsistent, but sociative. The positions involved include not only those of several Sophists, but also of pre-Socratics like Heracleitus and of later schools such as the Megarians. But such claims as to sociative, or other, underpinnings seem bound to remain speculative; thanks partly to celebrated barbaric ancestors, there is insufficient literature surviving to enable much worthwhile logical reconstruction.

The significant surviving literature for the recorded history of sociative logics, begins, like much else, with Aristotle. Nothing as crude as classical logic (even in "natural deduction" disguise) is to be found in Aristotle's work. Rather there is a formally elegant and tightly controlled theory of syllogism, which meets demanding standards of relevance between premisses and conclusions; and there are some proposals for extending that theory, for example in intensional directions, the case of modal syllogisms being worked out to some considerable extent. But there is also, coupled with this syllogistic theory and reduction of figures, the rudiments of a theory of argumentation, of good and fallacious argument, several elements of which were very important for the subsequent direction of sociative logics.

Included among these elements are Aristotle's account of progressive reasoning and argument, his theory of topics, and his theory of fallacies. Another, which helped shape recorded logical work on implication, especially that of Boethius in the fourth century and the sustained investigation of twelfth century, is Aristotle's apparent commitment, certainly taken for granted in the twelfth century, to connexive principles, that is to certain nonstandard principles of conditional argumentation. To this limited extent, Aristotle is the originator of connexive logic.

While Aristotle offered no systematic treatment of conditional statements, despite heavy use of them in his logical theory, and no specific propositional logic, he was responsible for the directions taken in early medieval discussion. Most important was the connexive principle of the *Prior Analytics* (II4 57b43): '... it is impossible that the same thing should be necessitated by the being and by the not-being of the same thing. I mean, for example, that it is impossible that B should necessarily be great since A is white and that B should necessarily be great since A is not white'. In Boethius's formulation, the principle became, 'it is impossible

that the same thing is by necessity from something when it both is and is not'. Demodalised, statementalised, and put in implicational formulation, this is

$$\sim((A \rightarrow B) \,\&\, (\sim A \rightarrow B)) \qquad \text{AR1.}$$

Aristotle argued for this principle, apparently in a quite general way,[3] effectively as follows:- Suppose on the contrary, $A \rightarrow B \,\&\, \sim A \rightarrow B$. Then as $A \rightarrow B$, $\sim B \rightarrow \sim A$, by contraposition. Since also $\sim A \rightarrow B$, by transitivity, $\sim B \rightarrow B$, something Aristotle took as absolutely impossible. AR1 then follows by contraposition, from the simpler (equivalent) principle

$$\sim(\sim A \rightarrow A) \qquad \text{AR2.}$$

These difficult principles, too quickly set aside in contemporary logic, bedevilled early medieval investigations.

Aristotle's theory of syllogistic and its surrounds involved, implicitly, a propositional logic of some sort, which was applied, for example in deriving some arguments and figures from others. While it seems unlikely that any such logic can be precisely determined, especially as many inessential principles are optional extras (which might have been selected by Aristotle, *if* he had considered the matter), certain minimal systems can presumably be tentatively figured out, and some bounds imposed. A minimal logic in $\{\rightarrow, \&, \sim\}$ would no doubt include, in some form, the principles now tabulated:

TABLE 1. DISTILLING ARISTOTLE'S IMPLICIT STATEMENTAL LOGIC

Principle	*Certainly Applied Forms*	*Likely Accepted Forms*	*Unlikely Forms*
Transitivity$^{\circ\dagger}$	$A\rightarrow B,\ B\rightarrow C/A\rightarrow C$	$(A\rightarrow B)\&(B\rightarrow C)\rightarrow .$ $A\rightarrow C$	$A\rightarrow B \rightarrow .B \rightarrow C\rightarrow .\ A\rightarrow C$ $B\rightarrow C\rightarrow .\ A\rightarrow B\rightarrow .\ A\rightarrow C$
Contraposition*	$A \rightarrow B/\sim B \rightarrow \sim A$ and variants	$A \rightarrow B \rightarrow .\sim B \rightarrow \sim A$ and variants	
Connexivity*	$\sim(\sim A \rightarrow A)$	$(A \rightarrow \sim B) \rightarrow .\ \sim(A \rightarrow B)$ and variants	
Non-Contradiction*	$\sim(A \,\&\, \sim A)$		
Modus Ponens$^{\circ}$	$A,\ A \rightarrow B/B$	$A \,\&\, (A \rightarrow B) \rightarrow B$	$A \rightarrow .\ (A \rightarrow B) \rightarrow B$
Reductio Forms (Modus Tollens*)	$A \rightarrow B,\ \sim B/\sim A$ (from Contraposition)	$A \rightarrow B \,\&\, A \rightarrow \sim B \rightarrow .\sim A$	
		$A \rightarrow B,\ A \rightarrow \sim B/\sim A$ (from Contraposition and Transitivity)	$A \rightarrow B \rightarrow .\ A \rightarrow \sim B \rightarrow .\ \sim A$

Double Negation†	$A \rightarrow \sim\sim A$, $\sim\sim A \rightarrow A$		
Antilogism†	$A \,\&\, B \rightarrow C$,	$A \,\&\, B \rightarrow C / A \,\&\, \sim C \rightarrow \sim B$	
and "Disjunctive"	$A \,\&\, \sim C / \rightarrow B$	$A \,\&\, \sim(A \,\&\, \sim B) \rightarrow B$	
Syllogism			$A \,\&\, B \rightarrow C \rightarrow .\ A \,\&\, \sim C \rightarrow .\ \sim B$
Adjunction	$A, B / A \,\&\, B$		$A \rightarrow .\ B \rightarrow A \,\&\, B$
&-Replacement	$A \,\&\, B \rightarrow C,\ D \rightarrow A /$	$(A \,\&\, B \rightarrow C) \,\&\, (D \rightarrow A) \rightarrow .$	
	$D \,\&\, B \rightarrow C$	$D \,\&\, B \rightarrow C$	
	$A \,\&\, B \rightarrow C,\ D \rightarrow B /$		$A \,\&\, B \rightarrow C \rightarrow .\ D \rightarrow A \rightarrow .\ D \,\&\, B \rightarrow C$; etc.
	$A \,\&\, D \rightarrow C$		
Praeclarum	$A \rightarrow B,\ C \rightarrow D /$	$(A \rightarrow B) \,\&\, (C \rightarrow D) \rightarrow .$	
	$A \,\&\, C \rightarrow B \,\&\, D$	$A \,\&\, C \rightarrow B \,\&\, D$	
Antecedent Commutation†	$A \,\&\, B \rightarrow C /$	$A \,\&\, B \rightarrow C \rightarrow .B \,\&\, A \rightarrow C$	
	$B \,\&\, A \rightarrow C$		

Key

* Explicitly formulated principles (cf. Bocheński p.97, who also cites the modal principle $A \rightarrow B / \Diamond A \rightarrow \Diamond B$).

° Fairly general formulations are stated (cf. Bocheński p.89).

† Consciously presupposed rules, according to Bocheński (see p.77, p.78). (It has been suggested that some principles Aristotle used regularly, such as Antilogism, apply only to syllogistic reasoning and theory. But in Aristotle's terms that strictly makes no qualification, since syllogistic, as distinct from term logic, embraced the whole of logical reasoning.)

The "certainly applied" forms yield, ideally after a little organisation, a minimal logic - of a *sociative* sort. Moreover, this minimal logic represents, while connexivity stands, some sort of connexive logic. The point applies not merely to what it includes, e.g. also $\sim(A \rightarrow \sim A)$, but what is apparently omitted and *not* endorsed in Aristotle, e.g. cases of $A \rightarrow A \,\&\, A$, of $A \,\&\, B \rightarrow A$, and no doubt of $A \rightarrow A$. Thus the minimal logic differs in *two* important respects from contemporary connexive logic. Firstly, it appears to be progressive, or noncircular, excluding mere repetition - a *Peripatetic* feature. Secondly, Aristotle's implicit propositional logic appears to be significantly weaker and more rule-oriented than better-known connexive logics, in particular than those which have been shown to be adequate for the quantificational representation of the full theory of syllogistic (independently by McCall and by Angell). However, substantially weaker logics than those deployed would serve for such a contemporary reductionistic representation; and weaker logics again would suffice if, more authentically, syllogistic forms are taken as primitive and not quantificationally characterised.

There are several reasons, apart from connexivity, then, for inferring that Aristotelian propositional logic differs not only from strong connexive logics, but also differs markedly from mainstream logics. Firstly, Aristotle's presentation includes none of the higher degree principles, listed as unlikely forms. Indeed such forms scarcely enter the logical record, except by occasional accident, until contemporary times. The point is enough to defeat recent formal reconstruction of syllogistic theory. Secondly, there is Aristotle's much quoted account of what [conclusive] reasoning consists in, presented at the beginning of the *Topics* (100a 25): 'Now reasoning is an argument in which, certain things being laid down [perhaps as assumptions], something other than these necessarily comes about through them'. The account, repeated in essentials elsewhere, appears to make reasoning, the main type of argument upon which Aristotle concentrates, irreflexive. The striking result is that Identity, $A \rightarrow A$, fails. (Note that this implicational failure does not imply the breakdown of the traditional law of thought, $A = A$, Identity proper.) There is indeed considerable independent evidence that Aristotle is concerned with *progressive* reasoning, which would certainly reject $A \rightarrow A$. The fallacy of "begging the question", which he is concerned to emphasize, to involving *invalidity*, says as much. So does his stress on reasoning as leading to new knowledge. In fact the fallacy delivers more, leading to the rejection of degenerate syllogisms with an "identical" premiss (such as when A belongs to B and B belongs to B, then A belongs to B) as begging the question (65a 10). Thus Simplication in the form $C \,\&\, D \rightarrow C$ is also rejected, at least sometimes, as would be now expected on connexivist principles. Perhaps Rule Simplification, $C \,\&\, D \,/\, C, D$, is retained – it would simplify the theory of inference (while upsetting ancient requirements on the number of premisses) – perhaps not.

Consider, furthermore, the immediate historical development of Aristotle's account of reasoning, that by the Peripatetics. They almost certainly construed reasoning as progressive, and rejected such conditionals as $A \rightarrow A$ (e.g. 'If it is day, it is day': see below). The comparative complexity of a logic of progressive reasoning – a matter still by no means worked out properly now more than 2000 years later[4] – does something to account for the fact that the Peripatetics did not, so far as we know, formulate a propositional logic which can easily be compared with Stoic propositional logics.[5]

Nothing in Aristotle's theory commits him to the tenets of contemporary mainstream logic, such as the classical logic Łukasiewicz and many others of note would foist upon him in reconstructing and axiomatizing or natural-deductionizing syllogistic.[6] Not only is Aristotle's connexivism incompatible with such logics, and the room he makes for future contingents apparently incompatible with principles of classical logic; but more, the whole tenor of Aristotle theory of reasoning and argument, from initial definitions on, is inimical to the highly suppressive features of mainstream logics (see further RLR, p.141 ff.). The step-by-step progressive elaboration of necessary truths typical of elementary geometry, was a leading model for conclusive reasoning in classical Greece (cf. Kneales, p.3 ff.). It would not have been at all readily conceded by Aristotle, or by most other Greek logicians, that these demonstrations could be truncated, since any necessary truth yielded any other. Nor did Aristotle grant that a contradiction leads anywhere; to the contrary he argued, in his famous

defence of Non-Contradiction and of consistency themes, rather that contradictions led nowhere, nullifying themselves (see further Łukasiewicz 71 and OP p.28). In short, it seems clear that the paradoxes of implication would have been roundly rejected by Aristotle. Nor does his decidedly relevant theory commit him to them.

On the contrary, the notion of connection, as required in demonstrative and hypothetical reasoning, surfaces at several points in his work, particularly in this theory of fallacies and sophistical refutations. The fallacy of non-cause as cause in both its original and subsequent forms affords a striking example. According to Hamblin,

> *Sophistical Refutations* refers quite literally to the Fallacy as the one "about the non-cause as cause"..., but makes it clear that a logical interpretation is intended by, later in the book, referring to it merely as "insertion of irrelevant matter". Irrelevant matter can be inserted in an ordinary argument without prejudice, but it is methodologically dangerous to permit it in a *reductio*. However, some later writers objected generally to insertion of irrelevant matter: cf. the Stoic Fallacy of Superfluous Premiss (p.79, incorporating a fn).

The logical bearing of the fallacy is almost immediate if the wide sense of 'cause' is recalled, which encompasses reason. Reexpressed in terms of reason, the fallacy is, according to the initial reference, that of 'stating as reason what is not the reason' (166b 27). As Aristotle goes on to explain, insertion of what is not a reason (a 'false cause', "irrelevant matter") in an argument *ad impossibile* is fallacious. It is not just 'methodologically dangerous', but unsound. Hamblin's treatment here, as elsewhere (e.g. with begging the question), reveals his classical prejudices. For the type of argument Aristotle is rejecting is classically sound. Let B be false cause, inserted in an reductio from A, which is assured, to C, which proves impossible. Then the type of argument can be presented: $A \rightarrow C$ / $A \,\&\, B \rightarrow C$; insertion of irrelevance (which *is* connexively invalid). But now C is impossible, and A holds by assumption; therefore, as $A \,\&\, {\sim}C \rightarrow {\sim}B$ by antilogising, $\sim$B. (Of course there are classic ways out of this little hole, e.g. A can no longer be maintained, since it implies C without B.)

Contemporary selection procedures, for instance in the face of logical paradox, illustrate the original "non-reason as reason" fallacy quite neatly. These involve picking out one factor from a whole group of factors, as cause of the paradox, though it is perhaps irrelevant or not the reason as variants on the paradox show (thus e.g. Frege's "way out" of the Russell paradox). What the fallacy of non-cause as cause quickly and subsequently became, as opposed to what it originally was, so Hamblin further explains, is that of inflating a sequential (material) association into a cause or reason; it is the mistake of assuming that 'because B happens *after* A, it happens *because* of A' (*Rhetoric* 1401b 30). It is thus very similar to the fallacious inference from $A \supset B$ (or its quantified, formal analogue) to $A \rightarrow B$. Like most natural language connectives, *because of* requires connection beyond material "association".

The sort of progressive connexive reasoning that Aristotle's underlying propositional practice appears to lead to, has yet to obtain proper investigation.[7] The relevant theory is neither connexive logic, as normally presented, because that includes redundancies such as A

$\rightarrow$ A (and perhaps A & A $\rightarrow$ A and A $\rightarrow$ A & A, etc.), nor straight progressive reasoning, since that need not be connexive. Rather, it is a *fusion* of the these two separate sociative theories, both of which were to persist after Aristotle. The fusion itself also persisted and flourished, so it appears, firstly in the logical work of Theophrastus who seems to have elaborated and made more explicit Aristotle's implicit propositional, predicate and class logic, secondly in the theory of the long-maintained Peripatetic school, and, to jump further ahead, in variant forms in the later logical enterprises of Boethius and Abaelard.

II. The richer classical period: the great debate on implication.

The ancient development of sociative logics really blossomed, in Alexandria, with a major debate about conditionals, a debate which was continued through the main era of Stoic logic. Progressive reasoning of some sort appears to be one of the four positions cited in the famous debate on conditionals, a position now commonly, and appropriately, ascribed to the Peripatetics. 'The Stoic controversy over implication was by no means restricted to the Philonian [material implication] and Diodorean [tensed formal implication] views. In a very interesting and important passage Sextus states and illustrates four distinct definitions which were discussed by the Stoics' (Mates p.47). It is the third and fourth positions that matter for sociative logic:

> '[3] And those who introduce "connection" or "coherence" say that a conditional holds whenever the denial of its consequent is incompatible with its antecedent; so that, according to them, the above-mentioned conditionals do not hold, but the following is true: "If it is day, then it is day." [4] And those who judge by "suggestion" declare that a conditional is true if its consequent is in effect included in its antecedent. According to these, "If it is day, then it is day," and every repeated conditional will probably be false, for it is impossible for a thing itself to be included in itself.[8]

The full passage gives the further information that the third position rejects mere material juxtapositions of truths, such as 'if A then B' when both A and B are (contingently) true, and also rejects paradoxical features of Diodorean formal implications, namely conditionals which hold by virtue of always false antecedents or always true consequents. Regettably, the passage does not decisively inform us whether the third position, usually attributed to Chrysippus, also rejected, in its requirement of *connection*, other paradoxes of strict implication. Mates assumes not, and proceeds, on the basis of but scant evidence, to identify the third, genuinely Stoic position, with strict implication.[9]

But Diodorean implication constitutes a kind of *strict* implication. 'For, according to Diodorus, whatever is true for all time is necessarily true; thus, any conditional which would satisfy his requirements for truth would also satisfy his requirements for necessary truth' (p.47). Putting Diodorean components together, the Diodorean account is simply: "if A then B" holds iff it is impossible that both A and not B, i.e. $\Box \sim(A \ \& \sim B)$ for Diodorean necessity $\Box$, i.e. *a* strict implication. Mates, however, goes to some trouble to evade what is thus obvious, and to set aside the prevailing view that Diodorean implication is 'the ancient counterpart of strict implication' (pp.49-51): the distorting assumption that there is only one

strict implication is made throughout. He also tries to offset his own opinion that Diodorus's opinion would be that Diodorean implication is strict implication, by counterbalancing it with the claim that 'it is doubtful that any modern logician would recognise it as such' (p.47). Unfortunately for Mates, modern tense logic was in the making; it was not long before several competent modern logicians, Prior and his former students, were treating Diodorean implication as an interesting kind of strict implication.[10]

It seems decidedly unlikely that the great debate on conditionals was over *kinds* of strict implication, and much more likely that significantly different types of implication were under consideration. Most of the evidence points that way. Mates' case for treating 'the third type of implication' as 'the ancient version of strict implication' (p.49) is exceedingly weak. At bottom the case for a strict conditional consists in imposing a narrow *modal* interpretation on the crucial term translated as 'incompatible' or 'inconsistent'.

> Judging from the position of this type in the list, which obviously was intended to proceed from weakest to strongest [note: The examples make it obvious], we are led to suppose that "incompatible" is used in its ordinary sense, according to which incompatible propositions cannot both be true, i.e. their conjunction is logically false. The example bears out this interpretation (p.48).

Even granting the inferred ordering of strength (which is not obvious and is in doubt, especially as to the fourth position), it is more than a little obscure how this can lead to an allegedly ordinary sense of 'incompatible', a modal one! Perhaps it is taken to suggest that there is no evidence for 'incompatible' being construed in an unusual way, that everything said is compatible with a strict interpretation. While everything said, including the example given, is so compatible, it is also compatible with more interesting rival constructions, such as a connexive interpretation (proposed by McCall) and a relevant interpretation (suggested in RLR, p.83). Nor does the ordinary sense of 'incompatible' independently underwrite Mates' modal reduction, of compatibility to conjoint possibility (i.e. of $A \circ B$ to $\Diamond(A \,\&\, B)$). According to standard English dictionaries, 'incompatible' means 'inconsistent with something else; incapable of subsisting with something else; incongruous' (thus *Concise English*; similarly OED). Not only is incompatibility regularly of less than logical (or analytic) strength; more important, since the modal $\Diamond$ can also be construed as of less than logical strength, incompatibility (like incoherence) is thus ordinarily *relational* ('*with* something else'), in a way that the modal reduction, to a one-place functor $\Diamond$, removes. Let us accordingly represent incompatibility by the relational functor $\wr$, and correspondingly compatibility by the more familiar fusion connection $\circ$ (introduced by Lewis for consistency, but now mainly treated in a fashion that breaks free of his modal reduction). Incompatibility is uncontroversially analysed as the negation of compatibility, i.e. $\wr$ spills out as $\sim \circ$.

The third, Chrysippean position thus furnishes the connection

CD. $\quad A \rightarrow B \quad$ iff $\quad \sim(A \circ \sim B)$,

(given that denial is represented as negation). Then the example, which Mates claims bears out his interpretation, falls out of *any* interpretation, such as main sociative ones, in which

propositions are never compatible with their own negations. Moreover, Mates' assumptions that the ordinary sense of "A is incompatible with B" is "It is logically false that both A and B", $\sim\Diamond(A \& B)$ in standard symbols, is tantamount to the assumption that implication is, ordinarily, strict implication, and just as controversial (see further RLR, p.361 ff.) For $A \circ B$ iff $\sim(A \rightarrow \sim B)$ from CD, i.e. iff $\sim\sim\Diamond(A \& \sim\sim B)$ by the infiltrated strict equation, i.e. (by further traditional negation transformations) $\Diamond(A \& B)$. In sum, $A \circ B$ iff $\Diamond(A \& B)$. Mates' reasoning begs the interpretational question at issue.

The evidence against a strict interpretation of the third position stacks up much better than the slight evidence is its favour. In the first place, the Stoics, like many subsequent schools, considered that natural laws supplied true conditionals. Such conditionals do *not* admit of a strict interpretation (in the sense of Lewis and Mates) - though they may admit of a (still unsatisfactory) modal analysis when the modality $\Diamond$ is not interpreted logically, but rather naturally, e.g. as 'it is scientifically possible that', 'as far as natural laws go, it is possible that'. Evidence that the Stoics exceeded strict logical bounds, both in their treatment of conditionals, and in their accounts of incompatibility, is assembled in several authors (e.g. Long p.144, Gould). More detail than Long offers on the incompatibility typically being *empirical* is provided in Gould (p.154). Most of the sample Stoic arguments still extant involve contingent or lawlike connections, e.g. dark with night, milk with conception, etc., not logical ties. The incompatibility of 'Fabius was born at the rising of the dogstar' and 'Fabius will die at sea', for instance is not certainly logical. Gould calls it empirical, explains why it is contingent (since relying on an inductively established principle), but then ties it with natural connections and laws (pp.159-60).

In the second place, there is the fundamental requirement of connection, specifically imposed, which strict implication and all modal accounts wrongly abandon (except under low Priorean redefinition). Thirdly, interlinked, there is the crucial matter of invalid arguments. Two important classes of the four classes of invalid arguments comprise components which are strictly valid, i.e. valid according to the canons of modalism. These classes are important not only in countering strict interpretations and impositions, but also both in indicating the relevant character of Stoic logic and in delimiting its type. They include the following:-

> 1. Incoherent arguments ... are arguments which are invalid because there is no logical connection of the premisses with one another or with the conclusion.
>
> If it is day, then it is light
> Wheat is being sold in the market.
> Therefore, Dion is walking
>
> 2. Redundant arguments ... contain a premiss which is not necessary for drawing the conclusion.
>
> If it is day, it is light.
> It is day.
> Dion is walking.
> Therefore, it is light.
>
> If it is day, it is light.
> It is day.
> Virtue is beneficial
> Therefore, it is light.

> By all usual tests [sic!] these would be perfectly valid arguments, though inelegant. Perhaps Sextus made a mistake here, or perhaps he was following an inferior handbook (Mates pp.82-3).

In a similar caustic vein, Mates remarks that 'the principle, if any, which was used by the Stoics in their classification of invalid arguments is hard to detect' (p.82). Is it? Isn't the principle but a tight requirement of relevance, for use? Redundant or unnecessary or disconnected premisses are not needed, and need not be used, in reaching the conclusion; accordingly they are, under such a use criterion, irrelevant. (Nor are such extras always merely an inelegance, as mainstream positions would have; sometimes, as in connexive logics, they can result in triviality: see Montgomery and Routley). Interestingly, Thom uses such a lack of variable sharing, as in I, to characterise incoherence, a strong form of irrelevance, and specifically links it with one of the Stoic kinds of invalidity, incoherence, which he takes as a sort of relevance violation. But such a presentation in terms of relevance (which the Stoics do not explicitly mention) can be avoided. What matters is that arguments *without* connection of premisses with their conclusion, or even with one another, as in some forms of Simplification (e.g. cases of p & q ∴ p where q has nothing to do with p) or *with* redundancy of premisses, as in other forms of Simplification (e.g. p & p ∴ p and p & q ∴ p where q follows from p or vice versa), are *invalid*. As they are strictly valid, Stoic argument does not correspond to strict implication. Most important, arguments without connection of premisses with their conclusion are valid (most unfortunately no examples decisive against strict theory are given, though the principle is clearly stated). Thus Stoic logic is rendered appropriately sociative.

Much more congenial to a sociative story than the presently entrenched strict interpretation of the third position is a connexive interpretation. Such a very different interpretation of the third position has been pushed by McCall, who (in several publications) contends that Chrysippean logic is connexive, and even that 'connexive logic represents an attempt to formalise the species of implication recommended by Chrysippus' (ENT p.435). One countervailing fact is that many researchers arrived, rather independently, at distinctively connexive principles with little or no input from Stoic logic, in particular, the contemporary "founder" of connexive logic, Nelson. Another problem for McCall's proposal is that the (excessively) strong systems of connexive logic, which he presents in his full proposals, are substantially irrelevant, and thus include Stoically invalid arguments. Moreover, if Chrysippus did espouse connexive logic, that information, long lost, has only recently been disclosed.

How does McCall, who starts out correctly with the Chrysippean definition CD, manage to ascribe connexive principles to Chrysippus? He pulls off this feat by importing, from nowhere, 'the plausible thesis that if A implies B, A is compatible with B' (McCall in ENT p.435), i.e.

M^c. $\quad A \rightarrow B \rightarrow .\ A \circ B,$

a thesis which at least imposes an interesting constraint on the problematic connection:

namely, compatibility. From M^c it is an immediate step, applying CD (e.g. reformulated as a definition of $\circ$), to the connexive principle

BT. $\quad A \to B \to .\ \sim(A \to \sim B)$,

a principle found in Boethius and Abaelard, and called by McCall *Boethius's thesis* (though the more difficult converse, $\sim(A \to \sim B) \to .\ A \to B$, is what is really distinctive of Boethius's theory). What yields immediately a reputably implausible principle wears its plausibility rather tarnished; and indeed the proposition that M^c is plausible would not win much contemporary support (from those who understand it). M^c is itself a connexive principle. It yields at once, given identity, that *every* statement is self-compatible, *even* contradictions; and in any case, it tells us that whatever a contradiction implies is compatible with it! *It* stands in need of more independent support than an allegation of plausibility.

More relevant here is: what is the evidence that Chrysippus adhered to this "plausible" principle? McCall adduces none, and there appears to be none. Rejection of the principle is compatible with all the information we have on the third position or on Stoic logic (as a relevant modelling will show). Furthermore, if the fourth, Peripatetic position was, as already hinted, a connexive position, then it is unlikely that the third position was also (though again the contrast made between positions could just have been between progressive and inclusive connexivism).

Nor is it simply that there is little or no evidence that Chrysippus and other third positioners would have accepted M^c or the distinctive nonclassical theses, BT and $\sim(A \to \sim A)$, of connexive logics. It seems that such ancients as the Megarians were committed to non-connexive implicational principles like $A \,\&\, B \to A$ (Simp) and $A \to .\ A \vee B$ (Add), which quickly lead to the rejection of M^c. For then, as was known in the thirteenth century, $A \,\&\, B \to .\ A \vee B$, whence using normal negation principles again, $A \,\&\, \sim A \to \sim(A \,\&\, \sim A)$. Therefore, assuming M^c, $(A \,\&\, \sim A) \circ \sim(A \,\&\, \sim A)$, i.e. where C is $A \,\&\, \sim A$, $C \circ \sim C$, whence, $\sim(C \to C)$. But the Megarian position, like the third position, and by contrast with the fourth did accept Identity. Accordingly, McCall's assumption M^c has to be given up, on Megarian as on contemporary preconceptions. But it is unlikely that such a point of difference between Megarians and Stoics would have passed unnoticed. The type of connection Stoics like Chrysippus apparently insisted upon is not that of connexive logic so McCall-construed. What sort of connection was it?

An overlooked problem - overlooked because of the removal of *connection* with the strict equation - is this: *how* does Chrysippus's logic avoid the paradoxes and retain connection? It is a problem because the Stoics accepted, on the face of it, many of the principles involved in the derivation of the difficult negative paradox, ex falso quodlibet. Yet there is no indication that Stoic logicians confronted, or were aware of the problem, though the derivation was hardly beyond their considerable logical ingenuity. Why not? However it is approached, the matter calls for some extra-historical construction; various possibilities are open. What seems the most likely reconstruction will be developed here; other possibilities will merely be

indicated. It needs remarking that Stoic logical systems, in which demonstrations were carried out, comprised "indemonstrables", corresponding to axiom schemata and yielding rules, and "themata", corresponding to further inference rules. It is not known exactly what these themata were; but, until some better grasp is gained upon what they were, there is no certainly about what can - and, more important here, what cannot - be proved in logical frameworks such as that of Chrysippus. As a result, assessment of the Stoic claim of the "completeness" of their logic for instance, remains a difficult exercise.

The standard derivation of the paradoxical inference A, ~A/B, which circuits through disjunction, is in fact easily averted in the Stoic scheme of things. Certainly *a* rule form of Disjunctive Syllogism was adopted in the fifth Stoic 'indemonstrable' (i.e. axiomatic first principle), namely

5. *Either the first or the second; but not the second; therefore the first.* But much turns on how 'or' is construed. The evidence indicates that disjunction was *not* intended truth-functionally.

> A disjunctive proposition..., i.e., a proposition such as we express by the use of "or", was said to involve a complete opposition, ... of its disjunction ... . The expression we have translated by "complete opposition" must surely be understood to mean incompatibility, i.e., more than mere de facto separation (Kneales, pp.160-1; see also p.162-3).

Moreover, Galen stated that

> the disjunctive statement "Either it is day or it is night" is equivalent to "If it is not night it is day" (p.162).

So construed, the fifth indemonstrable amounts rather to a variant on Modus Ponens, i.e., the first indemonstrable. Under such an intensional construal of disjunction, Addition (e.g. A ∴ A ∨ B and its mates) fails, even strictly. For A does not at all guarantee that not-A implies B or that not-A is incompatible with any arbitrary statement. Thus the standard demonstration of A, ~A ∴ B breaks down, as with containment logics (Parry logics and others) and certain relational logics, on Addition.

Another wave at once forms to a overwhelm connectional interpretations of Chrysippean logic. For the paradox argument can be rerun without disjunction, intensional or other. The third indemonstrable

3. *Not both the first and the second; but the first; therefore not the second*, an orthodox equivalent of a Disjunctive Syllogism rule, does not on the face of it admit of similar intensional reconstrual. For, by contrast with disjunction, the Stoics characterised a conjunctive statement as one which is true if both components are true, and false if one component is false, i.e. essentially truth-functionally (cf. Kneales 62 p.160, Mates). It is important to observe that such an account does not make conjunction two-valued; the recipe is compatible with, and indeed adopted for, four-valued interpretations of relevant logic (under the American-plan for the semantics; RLR p.319). And it would equally fit with Stoic truth-theory, which appears to have been three-valued, allowing for neither true nor false

statements. Nor, even more critical, does the account of conjunction guarantee all the strict principles concerning conjunction. These depend also on the semantical accounts given of implication and inference. (For example, the account of conjunction does not guarantee, $A \rightarrow . B \rightarrow A \,\&\, B$, and quite rightly.) Still one fairly certain result has to be confronted: given known Chrysippean principles, not all strict, not even all normal, conjunction principles can be retained without sacrifice of connection. The paradox argument to this conclusion is given in implicational form, but it has an inferential analogue, which should be not lost sight of, because it is also damaging unless faulted.

The implicational form of argument to loss of connection runs as follows:

1. $\sim A \rightarrow \sim(A \,\&\, B)$, by Contraposing Simp, $A \,\&\, B \rightarrow A$
2. $A \,\&\, \sim A \rightarrow A \,\&\, \sim(A \,\&\, B)$, by Composition, Factoring, or Praeclarum, using 1
3. $A \,\&\, \sim(A \,\&\, B) \rightarrow \sim B$, by the third indemonstrable (*for* implication)
4. $A \,\&\, \sim A \rightarrow \sim B$, by Transitivity from 2 and 3.

The argument, together with what we think we know about what the Stoics accepted, leaves little room for manoeuvre. A form of Contraposition was the second indemonstrable, a principle that appears unquestioned throughout antiquity. Transitivity, although not an indemonstrable, was regularly applied and appears to have been a Stoic thema (cf. the third principle and accompanying discussion in Kneales 62, p.164). By elimination (Stoically admissible) then, the trouble lies either with composition principles (& introduction group) or simplification principles (& elimination group). While it is not entirely clear which must go, the latter seems the decidedly likely candidate.

There are several reasons for supposing that is & elimination (or Simp) that fails, beginning with a theoretical consideration, that the building-up character of composition, as distinct from decomposition, can hardly lead to trouble through suppression of information. A more direct consideration is that the Stoics appear to have accepted composition principles (see Kneales p.169). But there is considerable evidence indicating that they did not accept simplifying principles without some qualification. Firstly, there are matters concerning number of premisses of an argument and redundancy of premisses. Unrestricted Simplication would allow two premisses (generally required by Chrysippus) to be amalgamated to one, through $A \,\&\, B / A, B$, and would admit redundancy by allowing $A \,\&\, B$ where A sufficed as premiss. At least in inference, the Stoics took inclusion of redundant or otiose premisses to be fallacious. Indeed the Stoic theory of invalidity appears to be decisive again Simplification, as we have already seen. Secondly, there is the matter of content, a notion important in Stoic logic. Implication involved connection (and probably *inclusion*) of content, but in premisses like $A \,\&\, \sim A$, $\sim A$ undercuts the content of A, leaving no content to be transmitted in simplifying implications like $A \,\&\, \sim A \rightarrow A$. But those are Peripatetic ideas; it is not certain that Stoics like Chrysippus would have accepted all of them. One Aristotelian principle the Stoics did accept is however almost decisive against Simplification. That is the principle of Antilogism, the first of the Stoic themata, assumed by Aristotle in his indirect reduction of syllogisms: 'If two propositions entail a third, then either of those two together with the negation of the third entails the negation of the remaining one' (Kneales p.169, quoting

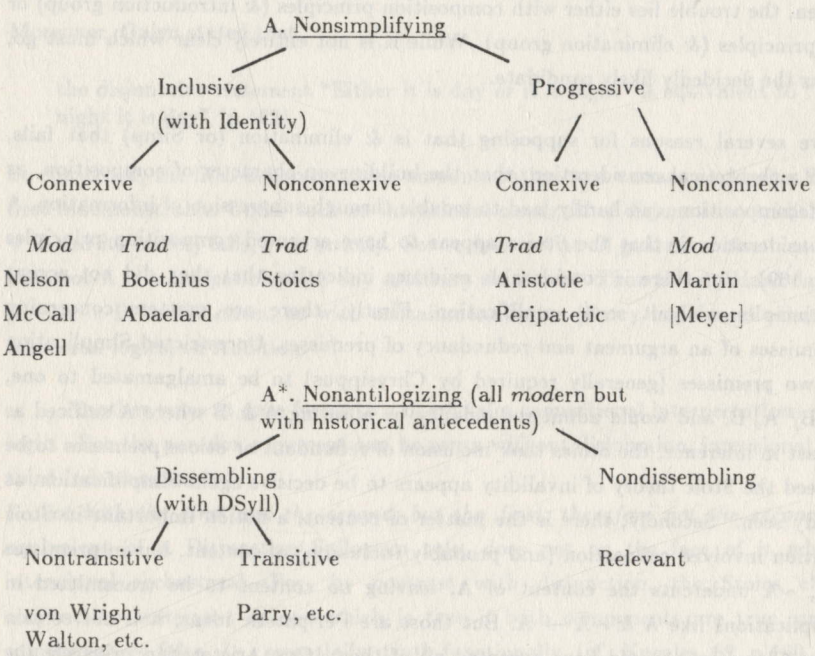
Apuleius), i.e. in intended inferential form.

$$A \,\&\, B \rightarrow C \;/\; A \,\&\, {\sim}C \rightarrow {\sim}B \qquad \text{(RAntil)}$$

But given Simplification it is an immediate inference to disconnection. It is an inference regularly rediscovered in recent times (e.g. Nelson, Duncan Jones), which could scarcely have escaped the astute Stoics, who constructed some much more elaborate propositional proofs (see especially the Kneales' reconstruction of Stoic theorems, pp.171-2). The argument simply takes C as A. Then, supposing $A \,\&\, B \rightarrow A$, $A \,\&\, {\sim}A \rightarrow {\sim}B$, and also $A \,\&\, {\sim}A \rightarrow B$ (putting ~B for B and applying double negation, i.e. "super negative" principles as they were called by the Stoics, who would have '1', '2', '3' for 'A', 'B', 'C' respectively). Whence disconnection. So by Contraposition, or by Reductio (of which the Stoics much approved), Simplification sometimes fails.

As the elementary *watershed* argument from RAntil and Simp shows, these are two strikingly different routes the quest for logical connection can take, the traditional Nonsimplifying direction, imposed historically by Aristotle's authoritative imposition of RAntil, and the very modern Nonantilogising direction. An interesting cross-classification of sociative logics takes the following shape:-

TABLE 2. A BINARY CROSS-CLASSIFICATION OF SOCIATIVE LOGICS.



Although virtually all of Chrysippus's extensive works have been lost, the general shape of his propositional logic has been roughly patched together from reports in subsequent, often ill-disposed or hostile, commentators (for conjectured details of this logic, see Kneales and work cited therein, especially Becker, and also Corcoran 74). It is commonly assumed, and

argued, that Chrysippus's logic represents both the third position and mainstream Stoic logic. These assumptions are not however essential in what follows, which mainly concerns what is ascribed to 'the Stoics', as distinct from other Schools. Traditional Stoic propositional logic appears to admit of the following sort of formalization:- As well as initial propositional parameters, named as 'the first', 'the second', 'the third' and so on, it included the (apparently independent) connectives: $\rightarrow$ (conditional), & (conjunction), $\cup$ (exclusive disjunction) and $\sim$ (negation), in modern representation. The conditional $\rightarrow$ is definable in terms of a further primitive, $\circ$, of compatibility and negation, thus: $A \rightarrow B =_{Df} \sim(A \circ \sim B)$. Inversely, $\circ$ is definable through $\rightarrow$. (It is sometimes suggested that $\cup$ can also be defined in term of $\circ$, but in the logic it appears to have played a significant independent role.) Standard formation rules for well-formed formulae (wff) are adopted. But the formation rules should strictly extend beyond those for well-formed formulae, to include rules. For there were special conditions upon rules, restricting basic rules to two premisses and one conclusion, which are not matched in contemporary free-wheeling procedures that permit much suppression of premisses. The two-premiss formation rule of Chrysippus seems to be as follows: where A, B and C are wff, then A, B/C is an (object) rule (with '/' read 'therefore'.) The best preserved part of Stoic logic is a list of object rules, the 'indemonstrable moods'.

The postulates of the logic can be divided into three groups:-

I. *Indemonstrata*, or primitive *object rules*:

I1. $A \rightarrow B, A \,/\, B$ I2. $A \rightarrow B, \sim B \,/\, \sim A$

I3. $\sim(A \,\&\, B), A \,/\, \sim B$

I4. $A \cup B, A \,/\, \sim B$ I5. $A \cup B, \sim B \,/\, A$

These schemes, traditionally stated with 'therefore' as conjunctor, are taken to supply both (normal) rules and theorem schemes, e.g. to consider the third, the theorem scheme, $\sim(A \,\&\, B) \,\&\, A \rightarrow . \sim B$. The connection is given through a relevantly correct principle of conditionalisation, namely, in many premiss form,

$$A_1,\ldots,A_n \,/\, B \quad \text{iff} \quad A_1\&\ldots\&A_n \rightarrow B,$$

or similar, e.g. with a meta-rule notation replacing 'iff' (for the connection, see Mates p.75, Kneales p.159). 'According to the Stoics any argument is valid if the conjunction of its premisses forms the antecedent and its consequent forms the conclusion of a true conditional' (Frede p.4, who cites many references). Now one half of this rule is straightforwardly derivable (metatheoretically), using equipment Stoic logic should have supplied given its oft-cited claim to "completeness". For suppose $A \,\&\, B \rightarrow C$, to take the appropriate doubleton case. Then

$A, B \,/\, A \,\&\, B$	by Adjunction
$A \,\&\, B \rightarrow C, A \,\&\, B \,/\, C$	by I1 (i.e. Modus Ponens)
$A, B \,/\, C$	by a Transitivity (or Cut) rule

Such a Transitivity rule was supplied (as a thema); and such a (two premiss) Adjunction rule is derivable from an Antilogism thema and I3, which yields $B, \sim\sim A \,/\, \sim\sim(A \,\&\, B)$, given Double Negation principles it is known the Stoics endorsed (e.g. Kneales p.169). Such little arguments also help reveal what else Stoic logic, as complete, no doubt included. Then too

the conditionalisation principle can be reduced, in the two premiss case, to the relevant principle:

CD. $\dfrac{A,B/C}{A\&B\rightarrow C}$

This rule is, it should be noted, very different from the contemporary mainstream rule $\dfrac{A,B/C}{A/B\rightarrow C}$ which is relevantly invalid, and in Chrysippus even ill-formed.

II. *Themata*, or *meta-rules* (sometimes described as those of 'exposition' or 'analysis', etc.):

Thema 1. 'If some third is deduced from two, one of the two together with the opposite of the conclusion yields the opposite of the other' (Bocheński p.127 citing Galen). Thema 1 thus comprises these two rules (given, in view of the separation of I4 and I5, ordering of premisses is respected):

$$\frac{A,B/C}{A,\sim C/\sim B} \qquad \frac{A,B/C}{\sim C,B/\sim A}.$$

Thema 1 yields, from the first indemonstrable I1, the Stoic theorem (of Counterexample): ~B, A / ~(A → B).

This simple example illustrates the grand Stoic thesis that 'any valid argument whatever is made up out of elementary, indemonstrable syllogisms' (i.e. arguments). Such reduction always involves, however, further rules: themata for central syllogisms, (unformulated) suppression rules for enthymemes or methodological conclusive arguments, and substitutivity of equivalence principles for "hyposyllogistic" arguments (see Frede p.5). In fact, as the simple example indicates, it would simplify presentation of the Stoic syllogistic system to introduce the supplementary thema

S1. $\dfrac{A,B/C}{B,A/C}$. Then half of Thema 1 and of Thema 3 could be simply derived. But Stoic logic no doubt contained enough resources (somewhere) to derive S1, as a metatheorem. (It would take a little work, since S1 permits some neat derivations. For instance, from Adjunction by S1, B, A / A & B, whence by CD, B & A → A & B.)

Thema 3. 'If some third is deduced from two and one (of the two) can be deduced syllogistically from others, the third is yielded by the rest and those others' (Bocheński p.128, Kneales p.169). The principle *appears* to yield such themata as $\dfrac{A,B/C \qquad E,F/A}{E,F,B/C}$, so violating a strict two premiss requirement. There are two escape routes: relax the two premiss requirement for a multiple premiss requirement (perhaps with $n \geqslant 2$); or assign further work to conjunction, &. The former, easier, course is needed if there is to be much prospect of proving implications like those of Associativity (and Distribution). The latter, tighter course would result in such themata as $\dfrac{A,B/C \qquad E,F/A}{E\&F,B/C}$, i.e. effectively rule CD is applied together with a prefixing principle,

$$\frac{D\rightarrow G \qquad G,B/C}{D,B/C}.$$

Details of the second and fourth Stoic themata are nowhere recorded, but apparently they comprised composition and cut rules rather like the third, Thema 3.

Taking advantage of CD, it is tempting then to adjust, or brazenly reformulate the thematic structure as follows: S1 together with the antilogism principle $\frac{A,B/C}{A,\sim C/\sim B}$ and the following composition principles:

$$\frac{D\rightarrow G \qquad G,B/C}{D,B/C} \text{ (Prefixing)} \qquad \frac{C\rightarrow E \qquad G,B/C}{G,B/E} \text{ (Suffixing)}$$

Note that the other Prefixing rule $\frac{D\rightarrow G \qquad B,G/C}{B,D/C}$

is derivable using S1 (and assumed iterability of rules). Also Suffixing would be derivable, were Contraposition available (as it no doubt should be, given ancient perceptions of negation), along with Double Negation elimination, thus:

$$\frac{C\rightarrow E \qquad G,B/C}{\sim E\rightarrow\sim C \qquad G,B/C}$$

$$\frac{\sim E\rightarrow\sim C \qquad \sim C,B/\sim G}{\sim E,B/\sim G}$$

$$\frac{\sim\sim G,B/\sim\sim E}{G,B/E}$$

But of course derivation of Double Negation elimination itself presupposes Suffixing in one case, or *else* an analogous replacement rule.

So far the systematic details available for Stoic logic - except for CD - provide *only* for a pure rule system (as Corcoran 74 p.179 has observed). There are initial rules, under I, and rules for deriving more rules, under II. Whence only rules result. But it also is known that the Stoics asserted several propositional principles, as well. They espoused not only those statements derived by rule CD, but such principles as Identity, Excluded Middle, Double Negation, and so forth. To capture these latter, a third class, of axiomata, is required.

III. *Axiomata*, or *truisms* (from assorted sources):

$A \rightarrow A$ $\qquad\qquad$ $\sim\sim A \rightarrow A$

$A \cup \sim A$ $\qquad\qquad$ $A \rightarrow \sim\sim A$

$A \rightarrow B \;\&\; B \rightarrow C \rightarrow .\; A \rightarrow C$ $\qquad$ $A \rightarrow B \rightarrow .\; \sim B \rightarrow \sim A,$

$A \rightarrow \sim B \;\&\; A \rightarrow B \rightarrow .\; \sim A$

No doubt the list, not particularly well organised, could be expanded. But there would be even greater danger of exceeding the meagre historical basis (which has already been

stretched, here and there). Nonetheless the general, and impressive, shape of the propositional system is evident. It is organised and determinate enough for several features of the logic, not hitherto made much of, to be remarked.

It is strikingly conspicuous that the propositional system so elaborated is not strict implication - however it is varied here or there in historically justifiable or plausible ways. It is commendably weaker than strict implication systems, in particular as regards paradoxes. It also involves an unusual disjunction, not present in Lewis modal logics, which do not validate rule I4. In fact, Stoic logic is a sublogic of several different sorts of sociative logics, and can perhaps be analysed as a certain intersection of those logics (as a doppelgänger logic in the fashion of RCR). For it can be mapped into sublogics of relevant logics and deeper connexive logics. An immediate corollary is: *mainstream Stoic logic is relevant.*

Consider the following connective mapping of Stoic logic into relevant logic (formulated at least in terms of connectives, $\rightarrow$, $\circ$, $\sim$, $\leftrightarrow$): $\rightarrow$ maps to $\rightarrow$, i.e. to itself; & maps to $\circ$, i.e. to fusion; $\sim$ maps to $\sim$, i.e. to itself; $\cup$ maps to antiequivalence, i.e. A $\cup$ B transforms to $\sim$ A $\leftrightarrow$ B (i.e. to strong fission). Finally transform A, B / C to A $\circ$ B $\rightarrow$ C (i.e. in effect A $\rightarrow$. B $\rightarrow$ C). It is a straightforward matter, mainly of inspection, to show that all the transformations of the principles of Stoic logic hold in (many) relevant affixing logics, e.g., for convenient definiteness, in system *R*. Only the transformations of & and $\cup$ are even distorting, and that for &, into intensional conjunction, is both familiar, and has some basis in Stoic theory. 'Chrysippus, with reference to the ... conditional, "If anyone is born under the Dog Star, then he will not be drowned in the sea," recommends that it be expressed as a negated conjunction, "Not both: someone is born under the Dog Star and he will be drowned in the sea".'[11] If the conditional is intensional and not material, as all the examples given in Cicero's text suggest, then the conjunction involved *is* an intensional conjunction. Only the transformation for Stoic disjunction, not heavily controlled by rules and regulations, is a trifle unusual, mainly because again of I4; otherwise $\cup$ could less deviously be represented as fission, intensional disjunction (on intensional conjunction and disjunction, see RLR chapter 5). But the biconditional rendition has a solid basis in Stoic logical theory (as will soon appear).

Now suppose Stoic logic voided decent relevance, to the extent that there was some Stoic thesis A $\rightarrow$ B where A and B failed to share a propositional parameter, or some proper Stoic rule C, D / E where C and D failed to share a parameter with E. Then $\vdash_R$ t(A) $\rightarrow$ t(B) or $\vdash_R$ t(c) $\circ$ t(D) $\rightarrow$ t(E), with t(F) the relevant transformation of F, where in the first case t(A) and t(B) fail to share a parameter and in the second case t(C) and t(D) fail to share a parameter with t(E). But both cases are impossible as system *R* is weakly relevant. So, Stoic logic does not violate minimum decent relevance. Since, furthermore, Stoic logic, by contrast with Peripatetic and connexive logic, admits such relevant embedding, it is not straightforwardly connexive.

Stoic logic was neither a single nor a static affair. Though it perhaps peaked in the extensive systematic work of Chrysippus, it was continued, varied and augmented, long after

Chrysippus. Two of the interesting developments were the enlargement of the acclaimedly complete set of indemonstrables (doubtless suitably complete only for the connectives they captured), and the further intensional interpretations offered of connectives such as conjunction and disjunction. Conjunction had been explained along truth-functional lines by the Stoics. That explanation applied of course only to actual situations (to the base worlds of the semantics); it did not determine the interaction of conjunction with intensional connectives, such as implication, where evaluation looks to other situations as well. But however conjunction behaved, disjunction was different. Disjunction did not inherit (De-Morgan-wise) the qualified truth-functionality conjunctions sometimes enjoyed.

Though the possibility of giving a truth-functional reading to disjunction has been realised since ancient time, for most of the long recorded history of logic, disjunction has not been construed merely truth-functionally, but intensionally in one way or another. This fact, shortly to be illustrated, has a significant bearing on recent appeal to the traditionality of Disjunctive Syllogism (DSyll). These appeals get revealed as fallacious. For what is to be defended is *tf* DSyll, i.e. DSyll with (inclusive) disjunction construed truth-functionally; but what tradition, from the Stoics on, more or less uniformly supports is *not tf* DSyll, but *nf* DSyll, i.e. an exclusive form with disjunction commonly intensionally construed. A corollary is that Chrysippus's famous and enlightened example of The Dog (laboriously evaded in ENT p.296 ff.) is no evident support for the *tf* DSyll at all.

Disjunction was usually regarded by the Stoics as at least exclusive. The fourth indemonstrable (A or B, A $\therefore \sim$B) was regularly put up along with the fifth, which is said to underlie The Dog's reasoning: thus not only earlier Stoics, but later writers in the tradition such as Cicero. But, further, disjunction was regularly taken as hypothetical, and normally as not truth-functional. However, there appears to have been a controversy over disjunction corresponding to that over implication, 'but unfortunately we do not know the details of this argument' (Mates p.51). It is known that 'there were some among the Stoics who did not regard a disjunction as true unless the components were incompatible', which Mates again tendentiously glosses in modal style, 'i.e. unless the components could not both be true' (p.52). The phrase 'some among the Stoics' is not just roundabout, but misleading also; for the mainstream Stoic position appears to have required not merely such incompatibility but rather stronger connecting conditions (as the passages from Gellius, Galen and others which Mates quotes make plain; pp.52-5). What emerges (from such later work) is that Stoic exclusive disjunction amounts to an antiequivalence: A or B is tantamount to, iff not A then B, i.e. $\sim$A $\leftrightarrow$ B, where connective '$\leftrightarrow$' is a non-truth-functional biconditional, often rendered as in English as a conditional (cf. the discussion in Mates p.56). Under this construal, the fifth indemonstrable becomes $\sim$A $\leftrightarrow$ B, $\sim$B $\therefore$ A, about which there is no issue, at least no issue similar to that about *tf* DSyll. The Dog can surely apply biconditional Modus Ponens. But The Dog would not have reasoned in accord with the truth-functional addition principle, A so A or B. Few before the thirteenth century did.

It is only in contemporary times that a truth-functional rendition of disjunction has

become entirely dominant, a rendition that does not however reflect at all well ordinary language uses of disjunction (as Strawson and others have explained). Even in Boole, one of the multiple progenitors of contemporary extensionally-ruinous logical theory, disjunction does not appear in extensional form (or implication really at all). Only in the later degenerating days of medieval logic does truth-functional disjunction come to have a recognisable role - a period to which contemporary apologists now look to draw (a pretty limited and insipid) inspiration and some historical underwriting for *tf* DSyll.

To gain an impression of the other unregimented unreconstructed side of the long story of disjunction, consider for instance Whately on disjunction. Whately refreshingly proceeds (p.70) to divide disjunctions into two classes:

i. those where there is a natural connection, where a disjunction states an alternative and will not be true *unless* one of the members of it is true; and ('on the other hand')

ii. those where there is 'no such natural *connexion* together as to warrant their being proposed as an alternative; as "either Britian is an island or a triangle is a square". Such a proposition would rather be called nugatory or absurd, than false ...' [!].

Then he appears to set type ii aside. 'Such propositions are often colloquially uttered in a kind of jest' (p.70), but not to be taken seriously logically. So much for the negative paradox argument depending on DSyll. Contradictions would only yield - what is still too much - all "naturally connected" (or relevant) propositions. Where there is a connection, i.e. type i obtains, that licences an inference. 'If, therefore, one of these categorials be denied (i.e. granted to be false), you may infer the remaining one, or (if several) *some one* of the remaining ones, is true' (p.70). Thus, 'it is evident that a disjunctive Syllogism may easily be reduced to a *conditional*, by taking as an Antecedent the *contradictory* of one or more of the members; e.g. if it is not spring or summer, it is either autumn or winter' (p.71). That is, disjunction is once again intensional; DSyll amounts once again to Modus Ponens for a suitable connecting conditional. Whately even advances within hailing distance of the mainstream Stoic position, in observing that disjunction is also mostly *exclusive*, so justifying in appropriate situations such inferences as: A or B, A; therefore not-B.

Not only was disjunction generally construed intensionally; conjunction sometimes was also. Herein lies a straightforward explanation of the problem of the two additional undemonstrated schemes that later Stoics such as Cicero added to the original five of Chrysippus. The additional schemes are these:

6.	Not both this and that. But this. Therefore, not that.	7.	Not both this and that. But not this. Therefore, that.

Cicero's list has presented a considerable problem, because it looks as if his sixth scheme is but a variable-amended version of the third scheme, while the seventh is patently invalid. But if instead Cicero has adjoined schemes for a further connective, a different (intensional) 'and' signalled by use of different variables, then there is no problem of redundancy. Moreover, for certain choices of conjunction connective, the puzzle of invalidity of scheme 7 disappears; both

schemes 6 and 7 can be validated.[12] Since there are few constraints on the connective involved but for 6 and 7 and some lax notion of "conjunctiveness", many choices are possible (including extensional ones). One obvious choice, with a fair pedigree, connects Ciceronean conjunction, symbolised $\cap$, with Stoic disjunction $\cup$, in an entirely standard way, i.e. $A \cap B$ iff $\sim(\sim A \cup \sim B)$. Thus, in the stronger setting of system *R*, $A \cap B$ is tantamount to $\sim(A \leftrightarrow \sim B)$, i.e. to a nonequivalence of one component with the negate of the other. Then, with such a Ciceronean conjunction both 6 and 7 follow, in effect, just by negation transformations from equivalential Modus Ponens. Or more simply, 6 and 7 reduce definitionally to schemes 5 and 4 respectively. This reduction suggests another straightforward choice, which obtains interesting historical confirmation:- Define a "conjunction" $\underline{m}$ thus: $A \underline{m} B$ iff $\sim(A \cup B)$, i.e. unscrambled as before, $\sim(\sim A \leftrightarrow B)$. Then schemes 6 and 7 formulated with $\underline{m}$ reduce at once to schemes 4 and 5 respectively, and so are valid for similar reasons. In weak relevant logic settings these are *equivalent* choices, furthermore, since $\sim(A \leftrightarrow \sim B) \leftrightarrow \sim(\sim A \leftrightarrow B)$, i.e. $A \underline{m} B \leftrightarrow . A \cap B$.

Revealing support for a two-conjunction resolution of the problem derives from Boethius's treatise *In Ciceronis Topica*, which however gives a striking slant to the matter: *both* conjunctions are intensional (as you no doubt already guessed). The Boethian conjunction of schemes 6 and 7 is the negation of the disjunction of schemes 4 and 5.

> The sixth and seventh modes are derived from the disjunctive proposition of the fourth and fifth modes by adjoining a negation, withdrawing the disjunction from the propositions which were inserted previously in the disjunctive proposition, in the following way. 'It is not the case that it is day and that is is night'; this was formerly a disjunction of this sort: 'Either it is day or it is night' (p.358).[13]

In short, $\sim(A \underline{m} B)$ iff $A \cup B$. It is a corollary, moreover, of the further data Boethius provides that the conjunction concerned is not truth-functional. For, on the one hand, 'the stipulations he places on propositions which can serve as the conjunctions in the last two modes show that the conjuncts in his view meet the conditions ... : one and only one of those conjuncts can and must be true' (Stump p.27). Thus the truth conditions for Boethian conjunction are those for exclusive disjunction, i.e. for nonequivalence; but according to Boethius's definitional account it is the negation of that, i.e. that for equivalence. It cannot be both. But, in any case (whether Stump is right or not about Boethian conjunction having such, or similar, conjunction properties), Boethius would not have accepted disconnected truth-functional complexes as supplying a conjunction or disjunction.

Naturally, the reduction of schemes rather diminishes their undemonstrated character (and destroys their indemonstrability), as does Boethius's reconstruction of the third scheme, which introduces distinctively hyper-connexive elements. For Boethius not only explains the conjunction of scheme 3 in terms of implication in a now familar intensional way, i.e. $A \circ B$ is tied to $\sim(A \rightarrow \sim B)$; but as well he imposes his strong connexive principle $\sim(A \rightarrow \sim B) \leftrightarrow . A \rightarrow B$, with the (disconcerting) result that intensional conjunction gets equated with a conditional, $A \circ B$ with $A \rightarrow B$. That is, $\sim(A \rightarrow \sim B)$ is equivalent not only to $A \rightarrow B$ but also to $\sim(A \circ \sim B)$, i.e. $\sim(A$ and $\sim B)$ (to put it the way Stump does, p.24; but note her suggested modelling is not satisfactory). Then the third scheme follows by negation principles

from the first scheme. The connexive interposition also explains Boethius's otherwise puzzling statements about conditionals showing connection better than equivalent conjunctions do. For instance, Boethius reformulates Cicero's example of the third scheme, which exemplifies the standard major premiss form, with the major premiss, 'It is not the case that if the silver was bequeathed, the coin was not bequeathed'. As he remarks

> Cicero himself, however, formulated the proposition in this way: 'It is not the case that the silver was bequeathed and the coin not bequeathed'; but one added the causal conjunction 'if' in order to show the genus of such a proposition. For an incompatible arises from a connected proposition with the addition of a negation. But no conjunction can show a connected proposition as well as 'if' can, although a copulative conjunction might produce the same proposition since things that are connected are also understood to be conjoined ... (pp.262-3).

Similar equations were envisaged by Alexander of Aphrodisias, who wondered whether the third indemonstrable was not the same as the first. He appears to have been toying with what amounts to the equation of $\sim$(A and B) with A $\rightarrow$ $\sim$B, an equation an intensional conjunction ($\circ$ of *R* for instance) could immediately supply.

Stoic logical theory , despite a marked reductionistic cast, included perceptive remarks about many connectives beyond standard contemporary sets, not only intensional conjunctions and disjunctions, but also for example on the behaviour of reasoning connectives like *since* and *because* (see e.g. Diogenes Laertius pp.180 ff.). Furthermore, the apparently restrictive canonical theory of hypothetical syllogism, examined above, was extended by a theory of enthymematic argument and as well by a theory of "hyposyllogistic" argument. Nor were Stoic logics confined (any more than other historic sociative logics) to the propositional level. They included also the rudiments of a *neutral* theory of quantification, apparently supplied through a combination of propositional logic with indefinite descriptors (for some discussion see Hay p.152 ff.). Certainly some such logical treatment of universal and particular judgements is required to justify the widely reported Stoic boast to be able to treat all valid arguments, including those of Aristotelian syllogistic. While the latter more modest claim may have some basis - since the confined forms of the "assertoric syllogism" are amenable to sociative analysis - the larger claim now looks mere bravado; it is a bit like boasting that quantificational logic is adequate for such purposes. It would be tempting to set the important matter of descriptors and quantifiers aside, to pretend that they make no difference to the range of Stoic sociative logics and their rivals. But such a convenient fiction simply is not true to the inconvenient facts still coming into view. Descriptors can work in surprising, and nonconservative, ways in sociative logics. Indeed the effects of descriptors in intensional logics are not yet much investigated or particularly well understood. There are further chapters on Stoic logics, unfortunately premissed again on very scanty data, to be written, or, where written, rewritten.

In this respect, what holds for Stoic logic, and the elaboration of the third position, applies also to its traditional rival, Peripatetic logic, to the retrieval of relevant details of Peripatetic logic and reassembly of fragments of the fourth position on implication. The

fourth position involved an inclusiveness or proper containment picture of conditionality. Under it the consequent is *implicitly* contained in the antecedent: 'judging by "suggestion"' is assessing by implicit containment. Evidently, proper containment was required, and mere repetition excluded; the analytic unravelling of what was implicitly contained in the antecedent had to yield different information. Accordingly the position was almost certainly coupled to the progressive reasoning which the Peripatetics opposed to Stoic hypothetical argument. If the Peripatetic coupling stands, it is the fourth position, not the third position, that there is a solid case for accounting as connexive. But the coupling is circumstantially based only. And the connexivism involved is progressive connexivism, deriving from Aristotle's theory of noncircular reasoning, not the reflexive connexivism, involving duplication, which McCall would attribute to the Stoics.

In any event, the fourth position was not a Stoic position. For the Stoics apparently accepted Identity (cf. Mates p.49), though they rejected - what is different - irrelevant premisses in arguments. Although the Stoics typically required (at least) two premisses in an argument, they allowed that arguments with redundant premisses are valid, for instance $A \rightarrow A$, A/A and $A \rightarrow . B \rightarrow A$, A, B/A. But the Peripatetics did not; they rejected such redundant arguments. Like Aristotle, the heavyweight source of the loosely-affiliated school, they insisted that the conclusion of a correct argument be different from any of the premisses.

The fourth position represents an early stage in the long and far from finished business of explaining inference and implication (or varieties thereof) in terms of containment or potential containment or proper potential containment. Its contrast with the third position indicates already that there are significantly different ways in which containment can be elaborated. It is tempting to speculate that the fourth position evolved as some variation upon the extended term logic and embryonic relational logic of Theophrastus (as pieced together by Bocheński), which was in turn an elaboration of 'Aristotle's hints about "syllogisms from hypotheses"' and his underlying theory of progressive reasoning, i.e. syllogisms (the quotation is from Bocheński p.104). It is also tempting to conjecture that the fourth position was not well developed technically because, in part, of the sheer difficulty of the business, even with contemporary logical technology (to reiterate an earlier point). Given the very little we presently know about the fourth position, many sociative explications, all much embroidering the position, are feasible (at least three different relevant explications are available, viz. RCR, RLR 11.2, and Meyer and Martin). It is a corollary of those explications that the fourth position does, like the other positions, have a non-vacuous representation of sorts in terms of worlds (pace Mates, p.49); for its approximate explications all do.

Given the different and apparently conflicting objectives underlying the third and fourth positions as tentatively expanded, and the different ways of effecting these objectives logically, it is hardly surprising that they contributed to a major dispute over general logical framework that broke out and persisted between Stoics and Peripatetics. So viewed, the long-running dispute, often portrayed as one over priorities, emerges not primarily or at all as one over priority (not a matter of such consuming importance to the ancients), but as one over logical

fundamentals, implication especially, and the proper development of these basics, over the correctness of apparently rival fundamentals, procedures, and methodologies (see further Frede 74). For instance, the Stoics insisted upon narrow canonical forms, such as the indemonstrables, and reduction by certain prescribed rules, such as the themata and substitution principles, to these circumscribed forms. By contrast, the Peripatetics tended to focus primarily or exclusively on the orthodox categorial syllogism, with the first figure as canonical; but, even where they *were* more generous in initial forms, they still only admitted argument forms which did show something, which yielded further information, which progressed.

The contrasting features pertaining to argument and conditionality get reflected in different demands imposed upon the *use* of premisses or antecedents. Whereas the Stoics operated a use condition in avoiding utter irrelevance, the Peripatetics adopted a use condition to exclude redundancy, to ensure that each premiss was separately used and pulling its weight. These constrasting use requirements, hardly much developed, were intimately tied to what it was supposed argument is about or doing. According to later Peripatetics like Alexander of Aphrodisias the role of argument is 'to show or establish something', something different from what is given or assumed. Whereas categorial syllogisms 'show or establish something', the Stoic 'hypothetical syllogisms, taken by themselves, do not show anything' (Frede pp.25-6, reporting Alexander). Genuine argument has to be progressive and not beg any questions.[14]

> ...Alexander's attitude towards non-categorical inferences and especially Stoic syllogisms, seems to be determined by two assumptions:
>
> (1) Only inferences of a form such that an inference of that form can show or establish something can be called syllogisms. To show or establish something is supposed to entail: what is supposed to be shown is not used as a premiss; what is supposed to be shown is not presupposed by any of the premisses in such a way that one has to accept or know the conclusion in order to accept or know the premiss; one can fail to know what is supposed to be shown even if one knows the premisses.
>
> (2) Hypothetical premisses, at least in hypothetical syllogisms, are not treated as assumptions about facts but as assumptions about the way one can argue which are to be exploited in the course of the argument in which they are made explicit. It is on the basis of these assumptions that Alexander may think that only Aristotelian categorical syllogisms are really syllogisms, though he is quite willing to admit that there are many other forms of valid, and even logically valid, inference (Frede, summing up, p.29).

What amounts to a distinction between facts and hypotheticals, which are like argument warrants, appears, in one form or another, in much Peripatetic theorizing. It appears for instance, in the proposition that hypotheticals must be supported by topical rules. These distinctions between facts and hypotheticals, along with special roles assigned to certain rules, topical rules or inference tickets and certain major premisses, broadly anticipate elements of Ryle's study, 'If, so and because', technically explicated, in one interesting way, in Anderson and Belnap's ticket entailment (ENT p.41 ff.). The explication, through labelling or

subscripting methods which track use of different types of statements, admits of interesting adaption and variation, from the implicationally strong ticket system T, to weaker and different theories of topical implication, irredundant argument, and unrepetitive conditionality.

Evidently then there are various contemporary ways in which significant features of main ancient theories of consequence can be further explicated. For one thing, the use-tracking methods can be applied at different points, for instance proof-theoretically or semantically (the latter would however be decidedly remote from ancient practices, *except* as adapted to containment models, whereas elements of subproof methods do appear in Peripatetic work). For the tricky Peripatetic requirements of separate use, not only would every hypothesis in a proof scheme, even if repeated, require a new (ticket) label, and the differentiation affected in T between major premisses and minor ones (or "facts") copied, but the rule of repetition would be sharply curtailed - so failing the "archetypal implication", $A \rightarrow A$, not a "foundation" for any sort of genuine Peripatetic reasoning. Indeed subscripting constraints are so severe, that the resulting reconstructed Peripatetic logic though not a subsystem of system T (because of different negation rules appropriate for RAntil), is bound to be sociative.

From the end of the fruitful period of Stoic and Peripatetic logic until the early Middle Ages there was little original Western work in sociative logic, so far as we presently know, except for that of Boethius. The long period may not have been a really creative one for logic of any sort, as Bocheński explains (61 p.134 ff.). But as Bocheński also observes it is 'a period into which hardly any research has been done'. What has been done since 1961, for instance by Ebbesen, appears to confirm Bocheński's main bleak conclusions (see e.g. the discussion of Aristotelian scholasticism, pp. 64-5). But Boethius is certainly one late exception; Apuleius, who influenced Boethius, may be an earlier exception (for some orthodox background see Sullivan); and there may well be other exceptions. For it seems sociologically unlikely that logical investigation sank for so long into an intellectual void, eventually to spring again from the void, especially when (unoriginal) work was undoubtedly proceeding in some centres.

Boethius, himself influenced by both Peripatetic and Stoic factions, served unwittingly as a major cultural transmitter, relaying ancient logic to the Middle Ages. He certainly exerted a significant influence upon twelfth century logical enterprise, largely but by no means only through his transmission of Aristotelian thought by way of detailed commentaries; for he also sketched out the rudiments a remarkable logical theory. By contrast with the Stoics, there is no well organised, clearly articulated framework of logic in Boethius, except for borrowed work. Like Wittgenstein, much of Boethius' output is substantially disorganised; like Wittgenstein too, it has proved very suggestive to subsequent researchers, while looking naive or simplistic to those trapped in mainstream thought. From Boethius' work likewise, an interesting collection of proposals can be pulled together, and a highly nonclassical protologic outlined. Features of especial sociative interest to be drawn from Boethius include these: the accounts offered of connectives, the association (or affective connection) view of

conditionality, and the strong connexive principles. There are other elements which may also turn out to be relevant as sociative investigation proceeds, for instance aspects of his historically influential but dense theory of topics.[15] As well there are matters we now tend to take for granted that may have originated with Boethius, like the distinction between logical and factual implication. Finally, of paraconsistent importance, there is the licence given for reasoning from impossibility. Reasoning from the impossible, which anticipates the medieval *positio impossibilis*, takes place when there is agreement to suppose that something which could never in fact occur holds. Boethius then describes how we may start from a recognised impossibility aiming not so much to derive an explicit contradiction but rather to explore the logical structure and features of the impossibility (for further details and some discussing of an elaborate original example see Martin 88 pp.37-8). Unfortunately for the larger paraconsistent history Boethius does not describe the logical procedures admitted and excluded.

Boethius is often looked down upon, by contemporary scholars of lesser stature, as a patient but rather dull scholar who simply laboriously assembled ideas of his predecessors, and was historically lucky (seen from *their* angle) to be in the right place at the right time. Boethius's astonishing elaboration of connexive logic, dismissed of course as a gross mistake, should have been enough to overturn such patronising approaches. Boethius presented as patterns of valid reasoning not only such connexive principles as $A \rightarrow B \rightarrow .\ \sim(A \rightarrow \sim B)$, and consequences of them such as $A \rightarrow (B \rightarrow C)$, $B \rightarrow \sim C\ /\ \sim A$ derived using Contraposition, but strikingly he strengthened the first to a biconditional, i.e. $A \rightarrow B \leftrightarrow .\ \sim(A \rightarrow \sim B)$, a hyperconnexive principle (already encountered) which he systematically applied. The logical theory thus indicated contains not only usual connexive principles, but as well stronger, not to say strange, nonclassical principles, most notably the connexive converse, $\sim(A \rightarrow \sim B) \rightarrow .\ A \rightarrow B$. These further principles of *hyper*connexive logic, occasionally reappearing in subsequent work, have so far substantially resisted contemporary attempts at explication and elucidation. They are technically interesting at least for that reason. But the principles of hyperconnexivism, straightaway plausible for an "equivalential" implication, also appear in other logical settings, old and new. For one thing, hyperconnexivism, whatever its (considerable) difficulties, allows for an entirely naive representation of syllogistic in propositional-quantifier form. Just as 'every A (item) is (a) B (item)' transforms directly into the form $(Ux)\ (Ax \rightarrow Bx)$, so 'some A (item) is (a) B (item)' transforms to $(Px)\ (Ax \rightarrow Bx)$. But by virtue of hyperconnexivism, this is tantamount to a *conjunctive* form $(Px)\ (Ax \circ Bx)$. Moveover, all the logical connections of the square of opposition are preserved without any hassle - including, obviously, inference from *every* to *some*, a traditionally valid principle knocked over in the modern mainstream rush.

Apart from this "naive" representation of syllogistic, a familiar castigated classroom temptation, there are two other places where hyperconnexivism is beginning to make a contemporary restricted appearance: in theories of subjunctive conditionals, and in the embryonic theory of *invited* inference. Boethius's biconditional can be reexpressed in the form, $(A \rightarrow B) \cup (A \rightarrow \sim B)$, which is a strong version of the principle of conditional excluded

middle (CED), which identifiable classes of conditionals are said to satisfy, at least in material form (including the Philonian conditional). But if $A \rightarrow B \equiv \sim(A \rightarrow \sim B)$, for example, holds in a certain logics of conditionals (e.g. Stalnaker systems), then it holds as a matter of logic; so the corresponding strict form should hold. But such a strict form is, it may be argued, yet another modal approximation to a proper implicational form, namely Boethius's biconditional. At first sight, the theory of invited inferences is far removed from that of certain everyday subjunctive conditionals. The idea there is that some statements, conditionals especially, *invite* inferences. For instance, the statement "If you mow the lawn I'll give you $5" invites the inference "If you don't mow the lawn I won't give you $5", "Dogs that eat Opla are healthy", invites the inference "Dogs that don't eat Opla aren't healthy", *and conversely.* Plainly, a theory beginning with a very weak logic of conditionals and closing under invited inference, would be a hyperconnexive theory ascribing equivalential features to its resulting conditional. Thus the theory would have salient features in common with Boethius's.[16]

Several characteristic connexive features merge in Boethius, along with presentation of strong connexive principles and projection of implication towards equivalence; namely, a containment picture of implication and conditionality, which is straightforwardly combined with an account (like the main Stoic position) of implication in terms of incompatibility; and a clear cancellation picture of negation (negation consists of removal, withdrawal, or the like). Both these intuitive modelling features were transmitted to the early Middle Ages, and clearly received. Furthermore, whatever the fuzzy shape of Boethius's logic, it can be conjectured with some confidence that it was sociative in intention as well as fact. For one reason, there were tight requirements of connection imposed not merely upon conditionals, but apparently upon all connectives. The demands often said to be made in everyday natural discourse upon connections of components of connectives, are to some extent matched in Boethius. For another, the propositional part of Boethius's logic appears to be embeddable in a fragment of equivalential-negation logic (connectives $\circ$, $\cup$, $\cap$ are definable). But even in this classical theory, of which Boethius's is only a quite proper part, significant restrictions upon parameter occurrence are met (see Prior 62 pp.306-7), which already suffice to remove some noxious paradoxes of implication ($\approx$ equivalence).

It is the modesty of explicit conditions upon connectives that makes Boethius's hyperconnexivism easy (by contrast with recent connexivism) to allocate to a sociative box. Seemingly, *none* of the standard propositional connectives get assigned their contemporary regimented roles in Boethius, whose theory takes decidedly more account of grammar and natural language than its recent successors. Even the connective *and* is not truth-functional and is subject to restriction in well-formation rules. Indeed Boethius appears committed to the proposition that no standard connectives are simply truth-functional (thus anticipating, but out-distancing, Strawson and van Dijk). Such a claim would not have astounded the ancients; for conjunction, perhaps along with negation, was the only connective regularly assigned to a truth-functional basket. Dispose of conjunction, as Boethius appeared to do (see Martin 87 p.34), and the main case for intensionality is done. Disjunction, for example, was

always coupled with conditionality, and (as noted) but few before the Parvipontani in the late twelfth century would have accepted the addition principles, $A \rightarrow . A \vee B$ and $B \rightarrow A \vee B$, that a truth-functional story (among others) characteristically underwrites. Conditionality itself was generally given a non-truth-functional rendition, though Philo's radicalism was well-known. Boethius himself, though he noted and perhaps incorporated a modal account, appears to have reached a rigorous association linkage.

According to Boethius, if A then B (e.g. "if it's day, it's light") 'does not assert that both are [the case] but rather that if one is, then the other follows, that both come together in a certain understanding' (Martin 88 p.110). Here and elsewhere what Boethius says suggests a double-banger analysis of implication (of a type concidered in RCR) which runs as follows: $A \rightarrow B$ iff $A \dashv B \,\&\, A \sim B$, where $A \sim B$ symbolises the association element: A is associated in understanding with B. Certainly Boethius makes *some sort* of strict requirement a necessary condition for the truth of a conditional, and he repeats the association condition (though sometimes as agreement in nature, rather than sense or understanding). The same double conditions are repeated in Boethius's explanation of *affective connection*, which appears to correspond to the (semantical) conditions for conditional statements. The double conditions for affective connections are again the strict conditions, the impossibility of one component without the other, and the association condition, the inevitable conjunction in the understanding of the components (cf. Martin 88 pp.131-2).[17] Given the equivalential cast of the auxiliary connective $\sim$, the analysis already has several of the right features for connexivism. But while the analysis eases the problem of validating connexive principles, it does not assist so well in vindicating hyperconnexivism. For all the impressive power of recent logical technology, a coherent synthesis of Boethius's sociative ideas on argument and conditionality remains to be achieved, if it can be.

III. The early medieval period: Abaelard, his rivals, and successors.

'Considerations of relevance (and connection) were important throughout the medieval period ... it is the 20th century that is aberrant in this respect is its treatment of conditionals'[18] Neither in Alexandrian times, nor during the medieval period when debates about the fundamentals of logics were intensive, did classical-style logic attain the dominance it has now achieved. Theories of entailment and conditionals were certainly an important issue in the twelfth century. There was a major debate, centred around Abaelard (as we now see it), over connexive logic. There was also an emerging theory of argumentation from false and impossible assumptions, the new supplementary theory of obligationes.

There are also other little remarked features of medieval investigations worth emphasizing. The logical principles deployed were almost invariably of low degree (mostly at first degree, occasionally at second). Conditionals were seldom nested; pure consequence statements never were. Coupled with this, there was no emphasis on strength of system, but a heavy stress on adequacy to apparent data. Thus, by contrast with contemporary system building, there was no pressure to formulate, for example, a logic of pure conditionals. Logical principles investigated mostly contained other connectives, such as 'and' and 'not', and very

often syllogistic material. (Not made were contemporary artificial separations of sentential and quantificational principles, divisions into syntactical, proof-theoretic and semantical theory, and so on.) The upshot was that what logical systems did emerge, or rather can subsequently be reconstructed, were very weak, at least by contemporary overpowered standards; indeed they were generally of lower power than that at which most recent theory functions well. Firstly, then, it is unlikely that the pure implicational theory would contain many principles beyond $A \rightarrow A$; that is fine, as there are interesting logics, such as those in the vicinity of relevant logic *B*, of this sort. Secondly, even where degeneration occurred and a modal theory was reached, as apparently happened under the dominant position in later medieval thought, the type of logic involved, hardly uniquely determined, would not have been "normal", but typically somewhere inside the weakest of Lewis systems *S1*. The general drive for strength is, like generalised will-to-power, and other universalization and maximization drives, a modern phenomenon. These drives are now built into the dominant social paradigm; they affect logic like all other intellectual endeavour.

An exasperating feature, from the system-design perspective, which medieval logic shares with much of less simplistic modern philosophy, is the marked lack of systematisation and the extensive qualification of principles and making of fine distinctions. The trouble is, as again with unsystematic contemporary work, that the massive qualification is frequently once only and for the case at hand, and that the distinctions go nowhere theoretically; these things do insufficient work and hardly justify their keep. These are the so-called "scholastic" elements displayed in much poorer linguistic philosophy, and copiously exhibited in medieval logic in syntactical qualification, over-qualification and hedging of principles. But in part it is because of this resilient complexity of detail that interesting medieval work is now beginning to break out of (early) contemporary Whiggish strait-jacketing and premature classification in available boxes. Thus too the satisfactory excavation and reconstruction of medieval logic calls for considerable patience (by primary researchers). But the effort that has recently been made is beginning to pay off, through the fascinating details, concerning implication especially, now being unearthed and put together. Few recent diggings have been richer than in Abaelard's work.

Abaelard accepted Aristotle's connexive principles AR1 and AR2, making them, along with two equivalent principles, central in his theory of conditionals. Any conditional which can be demonstrated, using acceptable principles, to imply conditionals which contravene them should be rejected. No doubt unqualified Simplification (i.e. $A \,\&\, B \rightarrow A$ and, by symmetry, $A \,\&\, B \rightarrow B$) with arbitrary and perhaps irrelevant or incompatible antecedents conjoined is a candidate for such rejection. For it yields immediately $B \,\&\, {\sim}B \rightarrow B$ and $B \,\&\, {\sim}B \rightarrow {\sim}B$ by unrestricted Substitution, whence, given only Adjunction, $(B^{\circ} \rightarrow B) \,\&\, (B^{\circ} \rightarrow {\sim}B)$ for $B^{\circ} = B \,\&\, {\sim}B$, in violation of Abaelard's first connexive principle

$${\sim}((A \rightarrow B) \,\&\, (A \rightarrow {\sim}B)) \qquad \text{AB1.}$$

Abaelard's second principle is likewise a contraposition of Aristotle's principles, and likewise equivalent using only minimal assumptions, to the first. It is that no statement implies its

own negation, i.e.

$$\sim(A \rightarrow \sim A) \qquad \text{AB2.}$$

As to statements infringing AB2 and implying their own negations, Abaelard remarks

> No one doubts that this is improper and embarrassing (*inconveniens*) since the truth of one of two propositions which divide truth not only does not require the truth of the other but rather entirely expels and extinguishes it (*Dialectica* p.290).

Thus is AB2 underpinned by a cancellation account of negation, with the content of A cancelling the propositional content of ~A (as explained more fully in NC). The response Abaelard could have made to Simplificational strategies directed against his principles, accordingly seem evident. The conditional A & ~A → A fails because in the antecedent the content of ~A cancels that of A removing the ground from A and so support for the consequent. Or, put differently, the content of the antecedent, since so nullified, no longer includes that of the consequent, as required however for a logical conditional. But though such a response accords well with Abaelard's account of negation and conditionals, Abaelard did not develop any such suggestions. In fact, unqualified Simplication appears not to have been taken very seriously as a general logical principle in the twelfth century; logicians were very ready to surround Simplification with qualifications and to require special justification for its application - Simplication rather than connexive principles, let it be emphasized. Interestingly, '... simplification as such does not appear in the 12th C canon of valid argument. In order to justify it an appeal would have to be made to an appropriate topical principle' (EA p.27).

Abaelard thought he could gain control over the wayward connexive principles, and avoid embarrassment, through a tight inclusion account of implication, coupled with a demanding theory of topical rules. A topical relation, or principle, provided a (natural) *connection* between antecedent and consequent elements, which would guarantee a true conditional. Topical rules regulated applications of Simplification and like nonuniversal principles, among other effects. Like Boethius, Abaelard rejects the standard modal account of the truth of a (necessary) conditional, accepting only one half of such necessary and sufficient conditions, namely that 'a necessary condition for the truth of a conditional is that the antecedent cannot obtain without the consequent'.[19] The modal requirement, he argues, is *not* sufficient. Abaelard works his way to alternative more rigorous necessary and sufficient conditions. Under the more rigorous condition for the truth of a conditional,

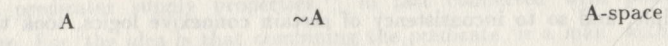
> not only can the antecedent not hold true without the consequent, but also the antecedent *of itself requires* the consequent. ... nothing can warrant the claim that "this is antecedent to that" other than "this *from itself* forces that" (*Dialectica*, p.283-4 italics added)

Thus Abaelard's tighter conditions give essentially the sufficiency-on-its-own account central to contemporary deep relevant theory (for instance, to the positive theory of *Relevant Logics and Their Rivals* and much of the work on which it was based). Not only are the tighter conditions those of a relevant account; what is more, Abaelard glosses his conditions with a

containment requirement: that the content of the antecedent is understood or *contained* in that of the antecedent (*Dialectica* p.284, cf. EA p.16). Exactly such a containment analysis has been much favoured by relevant thinkers, especially those who are (erroneously) prepared to concede a truth-preservation style of analysis to the strict or material opposition. Abaelard was clearly with the forces of truth and justice.

Implicit containment pictures of implication run all the way through logical history, from the Peripatetics well before Abaelard to Mill and Boole long after. But containment may be, and has been, explained in very varying ways, and subject to very varying conditions, from tight sense or property transmission conditions (such as are found in Abaelard and more formally in Priest 80a), through modal analyses (such as Carnap 56 supplies) to slack material or class calculus ones (such as classical logic offers). Abaelard explained logical conditionals (i.e. entailments) in terms of the consequents deriving necessarily just from their antecedents, and suggested one way of accounting for the necessity involved in terms of intensional containment, that in true conditionals the consequents are contentwise contained, or *understood*, in their respective antecedents. Sometimes (as in Priest 80a), this intensional (or sense) containment is taken as primitive, and simply assigned appropriate features; sometimes, as some of Boethius's remarks suggest and later traditional logic made explicit, containment was in turn accounted for through the transmission of suitable properties (for details and explication see RLR, especially p.216 ff.). Like other forms of intensional connection, containment amounts to the transfer or sharing of properties; here sense or content, already implicit in the antecedent, is transmitted to the consequent. Approached slightly differently with the same end result, containment is a partial identity (in one set of equations that were to become about axiomatic in the nineteenth century: $A \rightarrow B$, or $B \subseteq A$, iff $A \,\&\, B = A$, B is the same as part of A). But such partial identity, like identity proper and resemblance and interaction, is intensionally explained through the sharing of suitable properties.

A containment picture of implication, especially when combined with a cancellation account of negation, confirms basic connexive impressions. If $\sim A$ cancels A and vice versa, then A cannot be contained in or contain $\sim A$. So much is evident even on a Venn-style picture of "cancellation" (where what is cancelled is crossed out, not rubbed out, so remaining in view). Consider $\sim A$'s cancellation of A within A space, as illustrated:



It is evident that A is not contained in $\sim A$, and that $\sim A$ is not contained in A, which it erases. In short, $\sim(A \rightarrow \sim A)$ and $\sim(\sim A \rightarrow A)$ where non-implication is represented as non-containment. Similarly, both cancellation-wise and diagrammatically, as there are no null contents, where $A \rightarrow B$, i.e. B is contained in A, $\sim B$ is not contained in A, i.e. $\sim(A \rightarrow \sim B)$. But these appealing (first degree) pictures were not pulled together into a coherent modelling - not then, and, despite the immense recent increase in logical technology, not very satisfactorily since. Given an implicational setting as strong as that of system *E*, first degree

connexive principles spill over to higher degree connexivism; worse, in much weaker settings they spill into inconsistency. An argument for the first proceeds along those lines: $A \rightarrow B \rightarrow .$ $B \rightarrow \sim A \rightarrow .\ A \rightarrow \sim A;\ A \rightarrow B \rightarrow .\ \sim(A \rightarrow \sim A) \rightarrow .\ \sim(A \rightarrow \sim B)$; whence, as $\sim(A \rightarrow \sim A)$, by the Commutation rule of E, $A \rightarrow B \rightarrow .\ \sim(A \rightarrow \sim B)$. Arguments for the second were discovered - to the consternation (not of the multitude but) of many - in the twelfth century.

A satisfactory (relevant) account of the conditional is not, in a connexive setting, enough to avoid disaster, as Abaelard realised. Also necessary were further constraints on which materially-supplied conditionals held. Such control was particularly important in medieval theory where systems were always applied ones, with material examples rather freely imported into discussions of logical principles. Abaelard elaborated the theme, drawn from Boethius, that such a conditional is true, or an enthymeme valid, when there is a connection of topics guaranteed by a topical rule (a requirement which, Martin suggests, is rather like the modern idea of an inference ticket). Commonly the topical rules were presented as conditional connections. 'Many logicians of the 12th C ... [held] that there are real connections in the world, to which connections correspond natural conditionals' (p.32).

Abaelard sought to gain control, then, through a tightening of the topical rules, which justified material conditionals and licensed admitted enthymemes. What, in particular, Abaelard did was to apply his theory of topics to remove potentially damaging conditionals linking opposites, such as "If Socrates is a man, Socrates is not a stone", $A_1 \rightarrow \sim B_1$ for short.[20] For if such a conditional as $A_1 \rightarrow \sim B_1$ were admitted it could be shown that $A_1 \,\&\, B_1 \rightarrow \sim(A_1 \,\&\, B_1)$, contradicting connexive principles. The following embarrassing argument concerning opposites would do the damage:-

$A_1 \,\&\, B_1 \rightarrow A_1$	by admissible Simplification
$A_1 \,\&\, B_1 \rightarrow \sim B_1$	since $A_1 \rightarrow \sim B_1$, by Transitivity
$\sim B_1 \rightarrow \sim(A_1 \,\&\, B_1)$	contraposing Simplification
$A_1 \,\&\, B_1 \rightarrow \sim(A_1 \,\&\, B_1)$	by Transitivity.

This argument Abaelard's restrictive theory of inference licences deflects.

A decisive twelfth century beginning to sociative logic, in connexive form, was not however to be. The control Abaelard managed to achieve was inadequate. By cleverly varying Abaelard's own argument, Alberic produced an extremely embarrassing argument, meeting all Abaelard's requirements. The argument, which led to a counterexample to a connexive principle, and so to inconsistency of certain connexive logics, took the following form:

$A_\circ \rightarrow B_\circ$	*Any* conditional true according to Abaelard (e.g. "If Socrates is a man, Socrates is an animal").
$\sim B_\circ \rightarrow \sim A_\circ$	by (rule) Contraposition
$A_\circ \,\&\, \sim B_\circ \rightarrow \sim B_\circ$	by Simplification
$A_\circ \,\&\, \sim B_\circ \rightarrow \sim A_\circ$	by (rule) Transitivity
$\sim A_\circ \rightarrow \sim(A_\circ \,\&\, \sim B_\circ)$	by Contraposition from Simplification

$[A_\circ \& \sim B_\circ] \rightarrow \sim[A_\circ \& \sim B_\circ]$ by Transitivity.

Hence $C_\circ \rightarrow \sim C_\circ$, contradicting AR2. Both total-sufficiency and containment analyses of implication appear to straightforwardly validate Transitivity and requisite cases of Simplification; Contraposition (also easily guaranteed) was a given in medieval thought. Abaelard was in deep trouble. Abaelard's considered response to this simple and ingenious argument is not known! However Abaelard reacted (the opposition report is that he at once conceded, but more likely he imported further restrictions), the argument was a watershed one. For here was an argument

> on which everyone interested in the use of conditional - which meant any twelfth century logician - had to take a stand. A conditional apparently satisfying the most stringent requirement for necessity, in that being an animal is part of the definition of being a man, has been shown, by the use of apparently very basic and very plausible principles, to entail the contravention of a principle which, if not Aristotle's own, seems equally certain (EA p.23).

It was all extremely embarrassing. Connexive logic had suffered a major fall, from which it has never really recovered.

Many different neutralisations of the embarrassment were soon proposed, some of them anticipating contemporary moves, some of them rather surprising, but none abandoning conditionals as a bad lot or beyond redemption, in the fashion of the narrow contemporary paradigm. Some schools such as the Parvipontani 'essentially gave up on any refinement in the theory of the [demonstrative] conditional at this point' and adopted a modal theory of the broad type that was to become dominant in later medieval thought and is now so fashionable. More relevant, and interesting, are other resolutions, which apparently retained connexive principles as well as Contraposition. Essentially two types of option remain open: some restriction of Transitivity or some further muzzling of Simplification, and both were tried, though not in any thorough-going way. The options may be coupled, of course, with parallel options in the case of implicational paradox arguments.

The Albricini contested, like contemporary nontransitivity positions, Transitivity. But, unlike contemporary positions, they contested it on the grounds that the conditional 'expressed a "causal" relation between properties, and so failed where there was no corresponding property as in this case'. This nontransitivity approach, in denying that inconsistent predicates supply properties,[21] in fact connected with positions qualifying Simplification. For 'the idea is that combining the predicate 'is a man' with its contrary 'is not an animal' ... destroys the connection required for the natural conditional "if it's a man, then it's an animal" to be true' (EA). Thus the idea undercuts conditionals with impossible antecedents. Regulating or removing such conditionals, on one pretext or another, was, and remains, a popular move.

Several objected to the use of impossible conjunctions on the ground (which comes straight from the cancellation account of negation) that such conjunctions could not be posited without destroying part of the conditional (for the support the antecedent offers for

the consequent is removed, or the content of the antecedent collapses). The objection appears to imply some kind of (consistency) proviso on Simplication. Such an appealing restriction is ascribed in some texts to Abaelard's followers, the Nominales - but specifically the awkward view that from a conjunction of an affirmation and a negation only the affirmative conjunct follows. This would presumably admit $A_\circ$ & $\sim B_\circ \rightarrow A_\circ$, in the example, but rule out $A_\circ$ & $\sim B_\circ \rightarrow \sim B_\circ$, thus blocking Alberic's argument, but not others dangerously like it.

Closely related was the resolution of the Porretani, who argued (like contemporary connexivists beginning with Nelson) that Simplification from an arbitrary conjunction was inadmissible. But they claimed this (here in contrast with contemporary connexivists) because such Simplification involves fallacy of non-cause-as-cause, i.e. allows a type of irrelevance.[22] Like the Albricini, the Porretani required at least a causal *connection* between the antecedent and consequent of a true conditional. Thus when one conjunct of an antecedent is not used in obtaining a putative consequent, to take the whole conjunction as antecedent, as in unrestricted Simplification, *is* to commit the fallacy. Indeed they generalised their point, claiming that from

> no copulative do either of its parts follow. The reason is because it is a general principle both with regard to consecution and to inference [that it holds] only if the cause of the consequent or the conclusion is advanced for the consequent or conclusion... . What relevance does one of a pair of coupled antecedents have to the consequent when only the other is the cause. Thus in "if Socrates is a man and an ass, then Socrates is a man", isn't "Socrates is an ass", which is not a cause, advanced as a cause. (*Compendium Logicae*, II. 26, rearranged).

The resolution, because plainly relevantly based, is thus exceedingly interesting for the history of relevant logic.

A much more drastic amputation of consequences of inconsistent antecedents was adopted by the Meliduneses. They claimed not merely that nothing follows from the impossible (a sentiment congenial to connexivist ideas, and rather like modern views of Wittgenstein and others concerning halting at contradictions), but even that "nothing follows from the false".[23] Such strong themes need not be quite as restrictive as they at first appear to be, given for example procedures of default or suppositional inference and backtracking. Briefly, a reasoner proceeds from antecedents whose status is unknown but which are supposed possibly true at least until the evidently false is encountered, whereupon backtracking begins. But the approach, though now fashionable in computing theory and theories of theory change, is fraught with difficulties, yet to be decently resolved. Taken straightforwardly such approaches certainly underwrite distinctive features of connexivism. For if A is possible [or true] then A does not imply $\sim A$, else it would not be possible; but if A is impossible [or false], then as nothing follows from A, $\sim A$ does not. So, either way, $\sim(A \rightarrow \sim A)$.

IV. Later medieval theories of implication: further adjusting "history".

Although the great twelfth century debate did not persist undiminished into the thirteenth century, but gradually faded away, containment and connexive thinking did not disappear. Kilwardby, writing in the first half of the thirteenth century, appears to have adopted that characteristic combination: a meaning-inclusion view of implication, and a cancellation picture of negation, with a connexive outcome. Thus 'what is understood in some thing or things, follows from it or from them by a necessary and natural consequence; and so of necessity if one of a pair of opposites in repugnant to the premisses ... the other follows from them ... [In particular,] if one of the opposites stands, the other cannot', i.e. if $A \rightarrow B$ then $\sim(A \rightarrow \sim B)$ (see Bocheński p.199). Kilwardby's connexive theory of implication, not so far very thoroughly investigated, perhaps also included some stronger connexive principles, reminiscent of Boethius and flirted with in recent conditional logics, e.g. the principle $\sim(A \rightarrow B) \rightarrow . A \rightarrow \sim B$ (strengthening the material chain principle $\sim(A \rightarrow B) \supset . A \rightarrow \sim B$). But whether the final theory was hyperconnexive or not, it contained some serious tensions. For (by contrast with Abaelard and Boethius) Kilwardby also asserted that 'a disjunctive follows from each of its parts, and by a natural consequence; for it follows if you sit, then you sit or you do not sit' (Bocheński p.199). But, as Kilwardby was well aware, $A \rightarrow (A \vee \sim A)$ &. $\sim A \rightarrow (A \vee \sim A)$ counters Aristotle's and his own theme that the same result cannot follow both from a statement and its negative, i.e. $\sim(A \rightarrow B$ &. $\sim A \rightarrow B)$.

Kilwardby confronted two waves of objections to Aristotle's theme, objections apparently by that time standard. The first objection was based directly on the positive paradox: that where B is necessary (as e.g. "God exists") anything whatsoever implies it, in particular both A and $\sim A$ do. Kilwardby easily surmounted this objection, much as Abaelard had done before him, by distinguishing a natural (or natural and necessary) consequence operation, different from "accidental consequence" which is paradoxical. The positive paradox naturally fails for natural consequence. Such a relevant natural consequence connection is again characterised through content containment: the consequent is understood in the antecedent.[24] The second larger wave involved a relevant variation on the first; the argument replaced the arbitrary necessary truth by a specific one, Excluded Middle in the statement A concerned, i.e. it deployed non-expansive cases of the lattice principle of Addition. This wave may have dumped Kilwardby. For, by contrast with Aristotle, he conceded the parameter-sharing Addition principles as supplying natural consequences. Worse, he proceeded to commit Aristotle to these (apparently) new-fangled lattice principles. Aristotle had meant, Kilwardby maintained, only to deny that the same result could follow naturally in virtue of the *same part of itself* both from a given statement and its negation (see Kilwardby, quoted in Kneales pp.275-6). This amounts to special pleading, geared only to a counterexample based on a disjunctive conclusion like $A \vee \sim A$ where different parts, A and $\sim A$, may serve as antecedent. Kilwardby could more profitably have engaged in further division-chopping, distinguishing a *connexive* natural consequence under which $A \rightarrow . A \vee \sim A$ and its mate fail. An appeal to the ideas of Peripatetic progressive argument, for instance, could have made such a distinction both attractive and historically well-founded. Moreover, such a distinction would have saved his own connexive position.

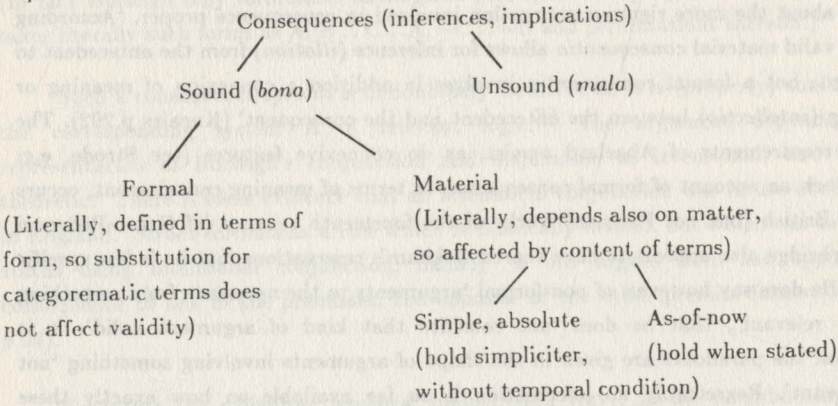
The vigorous debate of the twelfth century, partly focussed on connexivism, appears to have dissipated in the thirteenth century, when connexivism was slowly routed (with the advent, as above, of lattice connections). By the fourteenth century a strict position had, so it is usually said, become dominant, both as regards the consequence relation and as regards the coupled theory of conditionals. As to the coupling, of some sort, there is little doubt. 'There was always supposed to be a close connexion between the validity of a *consequentia* and the truth of a conditional statement' (Kneales p.293). For example, 'according to Ockham, a consequence is a hypothetical, conditional proposition [and it is true if and only if its antecedent implies (*infert*) its consequent]. That means that a consequence is composed of at least two categorical proposition which are joined by the syncategorematic terms "if-then" or their equivalents.... both parts may even by impossible' (Boehner p.55, insert from Moody p.66).[25]

What is taken to be the mainstream theory of formal (purely logical) consequence - presented clearly and simply enough by the shadowy Pseudo-Scotus (some-times identified as John of Cornubia) perhaps early in the fourteenth century - was, in broad outlines, a strict implicational one; but exact details of the theory were *much* debated. At bottom, Pseudo-Scotus required that for a consequence to be valid it is impossible that the first component (the antecedent) be true and the second (the consequent) be false.[26] Unlike earlier logicians who had used paradoxes of implication to discredit such strict accounts of formal consequence, Pseudo-Scotus tried to make the paradoxes appear inevitable and essential features of the notion of consequence - by establishing them through what amount to the modern "independent" arguments to them. Such modal views were, however, by no means as widely accepted as the recent conventional wisdom would have.

In a concluding chapter on consequence, Ockham sets down general principles for the notion - for *following from* - which were to be repeated with variation, and additions, in many later works on the topic. Unlike some of his commentators (notably Boehner), Ockham did not shrink from setting down, at the end of his resume, the evident paradoxes of strict implication: 'Other rules are given', says Ockham, as if he wanted to dissociate himself from these simple consequences, '(10) *Anything whatsoever follows from the impossible*; (11) *What is necessary follows from anything whatsoever*'.[27] Even so, Ockham evidently felt, like many medieval logicians, considerable reservations about these principles. For he proceeds to comment: 'But such consequences are not formal; neither should they be much used nor are they'. He was not claiming, in contemporary dishonest fashion, that these paradoxes are not or never could be used (but were, so to say, recorded, to complete the books honestly?). Of course they are used: in particular to halt discussion arriving at inconsistency, to exclude logical investigation of what is impossible. Ockham says they should not be used, much(?). More important, he appears, in saying they are not formal, to have been contending that they were not *correct*. For, firstly, '... in medieval latin *formalis* seems to have some of the associations which "proper" has in modern English' (Kneales p.292). But, in addition, Ockham specifically required in a formal consequence some necessary *connection*, for example through a topical rule. According to the Kneales, 'it is clear that Ockham wishes to call a

consequentia formal if, and only if, it involves a necessary connection in the stricter sense of which Abaelard spoke' (p.290). Really then, the paradoxes (10) and (11) hold at best for material consequence (such as lax enthymematic arguments), but not for consequence proper. Moreover, the way in which 'the material consequence [is] put aside' in the British medieval tradition 'can only be taken to mean that the material consequence is not considered a consequence in the strict or full sense of that term' (Green-Pedersen pp.286-7).

To arrive in the thick of what was at issue sociatively, it helps considerably to introduce certain of the basic classifications of types of consequence deployed in medieval theory. One important classification, made in varying competing ways, can be represented thus:-



The distinction between simple and as-of-now conditionals is roughly like that between Diodorian (temporally strict) and Philonian (material) "implication", at least according to mainstream perceptions (see Moody p.70 ff.). These material types were largely ceded to the forces of irrelevance. At least, there appears to have been fairly wide agreement that what amount to paradoxes of implication hold for material consequence types. What was primarily at issue was whether such paradoxes were *formally* valid, genuinely valid. But even as regards material consequence, simple consequence in particular, there were cross-currents of sociative significance, as a different classification of material consequence starts to reveal.

This interesting cross-classification was 'between consequences valid by reason of some connection of supposition or meaning between a term occurring in the antecedent and another term occurring in the consequent [materially relevant consequences], and consequences valid only because of the falsity or impossibility of the antecedent, or because of the truth or necessity of the consequent [materially irrelevant consequences]' (Moody p.73). Ockham and successors, such as Strode, considered such relevant consequences *adequate*, *formal* in their sense. But materially irrelevant consequences are dismissed as "merely material" by Strode, who illustrates his point with an example which soon became entrenched; "If some man is a stone, then a stick stands in the corner". The rules, for merely material consequences are, of course, the paradoxes themselves. Strode too says of such rules, 'But such consequences are not formal, so that those rules are not much used' (quoted in Moody p.74 ff.). Since the principles supposed to govern as-of-now consequences were also essentially material, 'not all

the medieval logicians accepted the consequence "as of now" as a valid consequence, and those who did recognise this sense of implication indicated that it was not of much importance for scientific purposes' (Moody p.79). This scientifically unimportant "implication" they realised could be defined truth-functionally, e.g. through $\sim(A \,\&\, \sim B)$.

Ockham was far from alone in his reservations and apparent dissent from the modal position, surprisingly veiled given some of his other political activities. Strode, for one, took the distinction between material consequence - which was conceded to be paradoxical, but neither much used or useful - and the stronger formal consequence - which was not paradoxical, but required connection - in much the same fashion as Ockham. But Strode is more precise about the more rigorous connection involved in consequence proper. 'According to him every valid material *consequentia* allows for inference (*illation*) from the antecedent to the consequent, but a formal *consequentia* involves in addition a connexion of meaning or understanding (*intellectio*) between the antecedent and the consequent' (Kneales p.292). The connectional requirements of Abaelard persist, as do connexive features (see Strode, e.g. f.12rb-va). Such an account of formal consequence, in terms of meaning containment, occurs frequently in British (but not Parisian) works of the fourteenth century (cf Green-Pedersen p.287). Ferrybridge also appears to have shared Ockham's reservations, but he is less specific than Strode. He does say however, of non-formal 'arguments to the necessary from something not specially relevant', that he does 'not consider that kind of argument valid'. It is interesting that the paradoxes are given in the shape of arguments involving something 'not specially relevant'. Regrettably no information is so far available on how exactly these medieval Englishmen proposed to break the known arguments to the paradoxes; but later commentators on Strode do offer suggestions.

It is not difficult, moreover, to ascertain how Burleigh's theory of consequences apparently avoided the paradoxes (thanks to recent recontruction). Burleigh, a contemporary of Ockham, who appreciated the logical priority of propositional logic and developed it to a considerable extent, elaborated a system of absolute (i.e. simple) consequence. The system included neither the paradoxes of implication nor (unqualified) simplification principles.[28] As it stands, it is a sociative system almost in direct line of (unhistorical) descent from mainstream Stoic logic (perhaps through twelfth century schools). It is very like Stoic logic as tentatively sketched earlier, but enriched by modal functors and an inclusive disjunction V, (with $A \;V\; B \leftrightarrow .\; \sim(\sim A \circ \sim B)$. Burley's demodalised system, formulated in terms of connectives $\rightarrow$ (for consequence), $\circ$, V, $\sim$, $\leftrightarrow$, with $\therefore$ indicating the rule linkages, takes the following lines (to adapt Boh's reconstruction, pp.313-4):

$A \rightarrow A$

$A \rightarrow \sim\sim A$ (the principles are

$\sim\sim A \rightarrow A$ implicit only)

$A \rightarrow B \therefore B \rightarrow C \rightarrow .\; A \rightarrow C$

$B \rightarrow C \therefore A \rightarrow B \rightarrow .\; A \rightarrow C$

$A \rightarrow B \therefore \sim B \rightarrow \sim A$

The implication-negation fragment thus appears to be exactly the same as that of basic system *B* of relevant logic. As with Stoic logic the differences (if any) come with $\circ$ and V.

$A, B \therefore A \circ B$

$\sim(A \circ B) \leftrightarrow . \sim A \vee \sim B$

$\sim(A \vee B) \leftrightarrow . \sim A \circ \sim B$

$A \rightarrow B, A \rightarrow C \therefore A \rightarrow B \circ C$ (Burley states only indicative special cases of this rule)

$\sim(A \rightarrow B) \leftrightarrow . A \circ \sim B$ (This latter principle is especially striking, and part of what justifies taking Burleigh's conjunction as intensional. Had he been working in a conventionally ascribed strict system his consequence system would collapse into a material one, i.e. absolute consequence would collapse to material, as-of-now).

$$A \circ B \rightarrow C \therefore A \circ \sim C \rightarrow \sim B, \text{ etc.}$$

(In fact Burleigh only formulates antilogistic principles for *syllogistic* consequence, where it takes literally such forms as $A, B \therefore C \ / \ A, \sim C \therefore \sim B$ and permutations thereon.)

Such a consequence system is undoubtedly sociative, as it is (properly) contained within the corresponding system *R* of relevant logic.[29] The argument depends upon the representation of Burleigh's conjunction and disjunction as intensional, and not lattice-theoretic. There is some evidence that an intensional conjunction was in use about that time in England. Strode formulates a rule which is classically invalid, but which can be given good status using intensional conjunction, namely 'If one argues from the opposite of the consequence to one of the premisses, the opposite of the other premiss follows' (see Moody p.93).

To locate the dwindling medieval mainstream, travel to the neighbouring Continent helps and may be required; perhaps best is a visit to Paris and to the harder-line modal school that formed around Buridan. Buridan shared none of those reservations felt earlier and elsewhere about the paradoxes as formal consequences. He offered the same "independent" proof of the negative paradox as Pseudo-Scotus, from which he concluded that the consequence was formally valid.[30] Influential logicians taught by Buridan felt none of Ockham's reservations or Strode's disbelief, perhaps as some sort of strict position became established in Paris. Thus Albert of Saxony, building on Buridan, felt sufficiently confident to present the paradoxes as first and second rules of (simple) consequence (see Bocheński p.200: note again however, the common qualification to *simple* $\approx$ *strict*). But even among such staunch supporters of the mainstream Parisian story as Buridan and as Albert, there were some interesting twists and variations (with truth playing an essential non-redundant role in Albert; see Boehner pp.71-2). All this adds to warranted speculation that the mainstream position, weakened by division and dissension within, held only an uneasy, dubious dominance - at least by comparison with the contemporary scene.

The contraction of the supposed mainstream to the outflow of a Parisian school gives a very different look to high medieval logic. It is worth briefly reassessing the standard contemporary argument, perhaps best assembled in Moody, for the modal picture of medieval logic. What Moody's interesting, but tendentious, depiction of the medieval logic of propositions as a strict implicational system - Lewis system S3 so it is claimed[31] - may do some justice to is only this: the consequence system of Buridan and his students at Paris, or

more narrowly the views of Buridan and Albert of Saxony. To arrive at his medieval strict system Moody proceeds by the evidently invalid method of selectively amalgamating the statements of several, disputatious, medieval logicians, namely Buridan, Albert of Saxony, Ockham, Burleigh, Strode and Paul of Venice, whose working lives furthermore spanned almost a century. He proceeds to compare "their" principles with those of Lewis and Langford (and also to a subset of those *Principia Mathematica*: see p.82 ff). The net result of such an exercise should really be, as it would be if attempted for contemporary logic, the trivial system (in the given morphology), not S3 or S2. For example, Burleigh's thesis, $\sim(A \rightarrow B) \leftrightarrow$. A and $\sim$B, yields classical logic, whereupon Strode's nonclassical principles yield triviality. But for reasons already advanced, Strode, Burleigh and Ockham should be counted out, for they all dissented, in one way or another, from such a modal pastiche; so also no doubt should Paul of Venice (as we shall see, and as Perreiah contends). That leaves Buridan and his former student Albert, i.e. the Parisian school.

It is evident, then, that rival positions to the dominant strict position were certainly not lacking; and these persisted through into later medieval times. Certainly there were many different positions adopted as regards the issue of correct conditionals, and these conditionals continued to be taken as virtually tantamount to valid consequences. But again appropriate details of many of these positions, who maintained them, the extent of their support, and so on, remain very scanty. Insufficient research into less orthodox positions and thinkers is no doubt one source of the trouble, of this information gap. Fortunately something, to build upon, is known.

By later medieval times, the number of interpretations of the conditional under discussion, but four or so in ancient times, had expanded to at least ten. In his *Logica Magna*, Paul of Venice listed, and summarily rejected, ten different, apparently extant, accounts of the truth of a conditional (and of the 'meaning of implication' as Bocheński puts it, pp.145-6), and these accounts do not appear to span all ancient positions. Several of the accounts cited certainly *admit* of sociative elaborations. Whether they actually attained such elaboration is another matter; however it seems likely, given the later logical ubiquity of the paradox arguments, and the apparent resistance to them. But how they were elaborated, or would have been - that sort of logically crucial information is again lacking. In typical medieval fashion Paul of Venice does not even give the names of medieval authors or schools who held the various views he mentions.

The variety of the views does however indicate both considerable dissatisfaction with the mainstream strict style of account, which had not stabilized to invariant agreed form, and also that no consensus as to the trouble with arguments like the paradoxes was emerging, as almost every partway plausible counterview was still being tried for size.

In Paul of Venice's list strict accounts tend to come first, with the first being in fact the first of the strict (modalist) views considered, and also counter-exampled, by Pseudo-Scotus (see Kneales p.286, p.293). According to this account, 'for the truth of a conditional is

required that the antecedent cannot be true without the consequent', 'without' no doubt being read as 'and not' (see the Buridan text elaborating the account, Bocheński p.196; otherwise, were 'without' read intensionally, a very different theory than the mainstream strict one could emerge). Among positions which readily admit of a sociative construal are at least these:-

- The third, where 'people have said that for the truth of a conditional it is required that it is not possible that the antecedent of that consequence be true unless the consequent be true': and
- The fifth, where 'people say that for the truth of a conditional it is required that if things are as is signifiable by the antecedent, necessarily things are as is signifiable by the consequent'. Among positions which almost certainly obtained some sociative elaboration are the last three given:

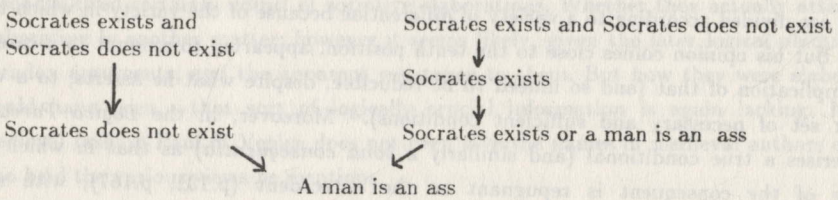
> Eighthly people say that for the truth of a conditional it is required that the consequent be understood in the antecedent...
> Ninthly people say that for the truth of a conditional it is required that the adequate significate of the consequent be understood in the antecedent.
> Tenthly people say that for the truth of a conditional it is required that the opposite of the consequent be incompatible with the antecedent...(Bocheński p.696).

The tenth position, which superficially resembles that of Chrysippus, is here already distanced from strict interpretations. The eighth position, which resembles that of Abaelard's school, is tantamount that of Strode. Evidently sociative accounts of implication and conditionality had by no means been entirely overwhelmed, or died out, by the fifteenth century, even if, as contemporary mythologic has it, they had been pushed into minority positions far down the lists.

The position of Paul of Venice, often represented as in the mainstream of medieval logic, is instructive, illustrating something of the complexity of later medieval theory. Having argued against *all* the positions listed, he says that 'my own opinion is that we cannot give any completely general account of what is sufficient or necessary for the truth of conditionals, for they are divided according to a variety of differentiae because of the variety in which they occur'. But his opinion comes close to the tenth position, appearing to amount to a case by case complication of that (and so indeed to be reducible, despite what he asserts, to a very complex set of necessary and sufficient conditions). Moreover, in the *Logica Parva* he characterises a true conditional (and similarly a *bona consequentia*) as that in which the opposite of the consequent is repugnant to the antecedent (p.131, p.167), with such repugnance glossed in terms of *unimaginability* of the denial of the consequent with the antecedent (thus a modal-style reduction of relation *is* reached, but through imaginability or conceivability). Paul of Venice's position is *not* a sociative one, tying the tenth account with connection; but *nor* is it a straightforward modal position by any means, among other things because imaginability differs from possibility (on medieval perceptions those properties only properly overlap one another, cf. p.51). Thus a different perspective would be expected towards implicational paradoxes. And indeed in the *Logica Magna*, he argues against the ninth account, a refinement of the eighth, that the popular post-Ockham conditional, "If you are something other than yourself, a stick is standing in the corner", 'is true and necessary,

and yet what its consequent signifies is not thought about in the antecedent because it is not signified by the antecedent or by any part of it'. Plainly the antecedent and consequent are not connected (except in that tenuous modal sense). Though Paul offers a formal deduction of "A stick is standing in the corner" from "You are other than yourself", he apparently does not hold the standard view that every consequence with an impossible antecedent is valid. Such a principle holds good only when 'impossible' is taken in a certain strong sense (presumably that of unimaginability), which the example given "You are other than yourself" meets.[32] Elsewhere however (*Logica Parva*, pp.167-8), Paul repeats the familiar paradoxes of implication (statements (10) and (11) of Ockham), and it certainly looks as if they can be derived from principles he gives, if not from the characterisation he offers. That Paul of Venice does not represent the acme of medieval logic is now conventional wisdom; but nor does he signal the later decline. Some of the most interesting medieval developments in sociative logic were still to come.

In the late (perhaps *post*-) medieval work of the Cologne school (active around 1493) the argument to the hard negative paradox through Disjunctive Syllogism was stopped in its tracks, in a way anticipating that of contemporary relevant logical theory. The argument to *ex falso quodlibet* $A \,\&\, \sim A \rightarrow B$ (or inferentially $A, \sim A \,/\, B$) was broken, as in relevant logics, by rejection of Disjunctive Syllogism (in inferential form licensing general inference from $\sim A$ and $A \vee B$ to B). It was destroyed on the elementary grounds that where both $\sim A$ and A are assumed, $\sim A$ cannot *also* be legitimately used to rule out A in the disjunction $A \vee B$. Failure to grasp this elementary point, easily explained to novices in logic or volunteered by them, has not merely diverted the whole course of contemporary logic into philosophically disastrous classical ways, but has also helped in substantially distorting the history of medieval logic, with the result that nonclassical positions are not taken very seriously or examined carefully or at all. The procedure of the Kneales, who, unlike many (unsympathetic) commentators, do give some small coverage to nonclassical positions, is instructive. They condense the main paradox argument from Pseudo-Scotus in the following tree of derivation:



> As the Pseudo-Scotus says, each of the *consequentiae* used in the derivation is formally valid; and so, paradoxical as it may seem, no one can reject it without rejecting something essential to primary logic (p.282).

After that supposedly compelling demonstration they have a difficult time giving any credence to accounts of consequence which dismiss or try to avoid paradox.

But suppose it holds or is granted (per impossibile, but what is impossible can be taken as an hypothesis) that Socrates exists and Socrates does not exist; but it does not hold or is

not conceded that a man is an ass (or *Baculus stat in angulo* or such like). Then, applying normal *and* and *or* principles, both "Socrates does not exist" (~A) and " Socrates exists or a man is an ass" (A ∨ B) hold, but "A man is an ass" (B) does not. Accordingly, the inference from ~A and A ∨ B to B fails, for it leads from what holds or is granted to what does not. The inference flunks an evident generalisation of the truth-preservation requirement (a generalisation which can of course be recast in truth-preservation form, as in RLR). The idea that ~A eliminates A in A ∨ B is based on that assumption that not both ~A and A can ever hold; but that assumption breaks down, because precisely what has been granted is that both do hold. Thus it is not true that Psuedo-Scotus's argument is formally valid (step by step), unless 'formally valid' is assigned, in an issue-begging way, some sort of strict construal. On the holding account indicated it is not valid at the last stage. Rejecting the step involved, as the Cologne school did, and as relevant theorists now do, is not 'rejecting something essential to primary logic', unless 'primary' is given a partisan construal. Rather it is to reject the extrapolation of a principle, good no doubt for negation-consistent situations, beyond its legitimate range of application.

The Cologne commentators, analysizing the paradox argument in the form given, leave no doubt that the last (junky syllogistic) step cannot be accepted. The reason given is that the initial premiss, of the form P & ~P, can be taken in two different ways: either absolutely, as an asserted contradiction, or hypothetically, for the sake of argument. 'In this proof, it is accepted for the sake of argument, and since both *P* and ~*P* have thus been conceded, one cannot use part of the formal contradiction to deny the other part. That is, ~P & (P ∨ Q) → Q has to be rejected!' (Ashworth 74 p.135, symbolism adjusted). The same grounds for rejection of Disjunctive Syllogism - as distinct from the rule γ of Material Detachment which is admissible, granted as always in those days that the actual world is consistent - are equally explicit in the subsequent work of de Soto (of 1529):

> ...A pair of contradictories can be taken in two ways, one way absolutely and without any supposition, and then the one is destructive of the other, that is, the truth of the one takes away the truth of the other. In the other way, they are taken as conceded by someone from some assumption for the sake of argument to see what follows, and then neither destroys the other, for conceding that one is true, it does not follow that the other is false, since both are conceded to be true. So the inference
>
> Peter disputes or a man is a stone
> and Peter does not dispute,
> hence a man is a stone,
>
> if those contradictories are taken absolutely, is valid. But if they are taken as conceded in some initial premise, it is not valid, for from the fact that a negative is true it does not follow by the assumption given that the affirmative is false, since in the cause of disputation both were conceded (f.74va).

The contrast, between obtaining absolutely (or truth) and holding by assumption, is tantamount to that between actual world (world T) semantical assessment and hypothetical (off-T) assessment. A formally valid principle must be more than merely admissible; it must

hold for suppositional situations also. Thus Disjunctive Syllogism is *not* formally valid. Observe however that de Soto assumes a cancellation account of negation for absolute cases; indeed he goes further, rejecting implications from the impossible (e.g. "God does not exist") to the necessary ("God exists"), apparently on the basis of Abaelard's principle $\sim(\sim A \rightarrow A)$. If so, de Soto is committed to (what is viable enough, though incompatible with relevance logic) a *connexive* relevant framework. But it may be that de Soto intended only to emphasize his repudiation of the paradoxes and of modal construals of the Chrysippean definition of consequence (a consequence is valid iff the negation of the consequent is incompatible with the antecedent) which lead to them. De Soto, like others such as Blanchellus Faventinus who adhered to that, was not convinced by the modal reduction of incompatibility. He sarcastically remarked that 'although "the moderns" accept "If God does not exist, then God exists" on the grounds that the impossible leads to anything he could not bring himself to accept an inference of this form'. He also pointed out he was hardly on his own, appealing 'to common usage to support his doubts about the paradoxes: who would say that if you are a stone, it follows both that you are and that you are not?' (see Ashworth 74, whose account of late medieval activity is heavily exploited, p.127, p.135). Consistently with his stance, he supplied a framework of principles for consequence and conditionality which is appropriatelt relevant, lacking both paradoxes and their source, Disjunctive Syllogism (p.184). The Cologne framework - although apparently based on a different (through reconcilable) definition of consequence: containment rather than Chrysippean - was also bound to be duly relevant; for those commentators espoused a principle of connection of terms for valid consequences.

While the clear emergence of a nondissembling position is the most exciting sociative development in later medieval logic, it was by no means the only event of relevance.[33] The earlier British tradition, set out in textbook form by Strode, gained a following and obtained different elaborations, a significant one of them plainly nontransitive. Cajetan of Thiene not only, like Strode, adopted an information inclusion account of formal consequence and accordingly rejected the paradoxes as formally valid; as well he explained that valid consequence implies connection, that the terms involved are *linked*, and went on to discuss several interpretations of Strode's requirement that the consequent be formally understood in the antecedent (for a summary, see Ashworth 74 p.129). Strode and Cajetan were followed by 'the Cologne commentators, Greve, and John of Glogovic [who] all seem to have accepted the claim that the consequent should be understood in the antecedent without comment; while Major added that a relation of pertinence between the terms was required' (p.130). Here is our first sighting of the connectional requirement explicitly identified as one of relevance.

The importance of relevance, and its failure for material and strict consequence, was also clearly recognised by others who adopted a containment story, notably Ferebrich and the unknown author of *Libellus Sophistarum*. According to the latter, in rejecting paradoxes of formal validity,

> There are ... three kinds of materially valid consequence, that whose validity depends on the terms employed, that where the antecedent is both impossible and irrelevant

> to the consequent, as in "A man is an ass, therefore the stick is in the corner", and that where the consequent is both necessary and irrelevant to the antecedent, as in "You ran, therefore God exists". ... of course ... there were some formally valid consequences whose antecedents were impossible, such as "A man is an ass, therefore a man is an animal", but these held by virtue of some other rule (Ashworth p.135).

Subsequently some went further; Javellus, himself averse to the paradoxes and committed to a formal containment story, reported that 'realists would not even allow the paradoxes to be materially valid, because they exhibited no inferential link, no dependence and no incompatibility between the antecedent and negated consequent' (p.135). They were not alone; the Cologne commentators required connection of terms for both usual types of material consequence.

Important, and importantly different, elaborations of the formal containment account were also offered, along with (different) suggestions as to what went wrong in the paradox arguments and which consequences were formally invalid. The author of *Libellus Sophistarum* explained formal containment as holding just when in making the antecedent true the consequent is made true or verified, i.e. effectively in truth preservational terms. That text also delivers an elegant if weak logical system, which (by contrast with the disappointing formal theory of Major, p.179) is duly relevant (pp.182-3, pp.275-6). Given that the system supplies Simplification, Addition, Composition principles *and* Disjunctive Syllogism, but not presumably the paradoxes (which are *not* cited), Transitivity (also not given, though it was a standard principle) has to fail, both syntactically, and less plausibly, semantically for verification. For *understanding in* - as distinct from formal or intelligible *inclusion* - eventual failure of transitivity is no doubt to be expected; it is just surprising that it is bound to happen in sequences as short as the paradox arguments. That Strode's formal containment account *did* lead to failure of Transitivity was explicitly stated by Sermonete in his commentary. He 'said that a consequence of Strode's view was that something could follow formally from the consequent of a valid consequent without so following from the antecedent, since in the case of the paradox "c is understood in b, b in a, but c is not understood in a"' (p.135). The British nontransitivity bent reaches a long way back; but it was then, as now, a minor affair. The moderns of later medievaldom were heavily committed, like the modern moderns of later capitalism, to modal-style theory (as well as to the reference theory). By contrast with present complacency, however, the modal position did not enjoy an easy or stable dominance. While a few logicians did accept the received strict account of consequence validity, 'many felt that it expressed neither the necessary nor the sufficient conditions for validity' (p.126). Even within the ranks of the faithful then, all the details were not merely up for reassessment, but regularly being reassessed.

Even in those medieval centres where a modal position of some sort became dominant after the thirteenth century and the logical canon, and other approaches declined to minority concerns or were abandoned, certain key elements underlying relevant positions and more satisfactory theories of conditionals, such as containment accounts, did not disappear entirely. Rather they continued to play a part, sometimes a significant part, in the ancillary theories that helped to prop up the dominant position; most notably, in the theory of what ensues

from, and what commitments are made, with false and even certain impossible assumptions. A contemporary parallel lies in the way the dominant logical position is hedged about by (competing) theory-saving supplements, notably theories of counterfactual and counterlogical implication and recently modal theories of conditionals. But whereas contemporary supplementation of the dominant paradigm takes no due account of connection and relevance, and really has as yet no approved theory for accommodating impossible assumptions, medieval supplementation gave sociative considerations an important place. Moreover, the damaging paradoxes of implication were only incorporated into the later medieval logical canon as regards certain types of consequence and inference, such as material consequence. Not only for many situations, most conspicuously those where impossible assumptions operate, but also for significant types of consequence, including that of valid (*bona*) consequences according to several authors, the strict canon was suspended or ditched. Even so, such a "high" medieval approach, and onus of organisation, resembled that of liberated contemporary positions rather than a deep approach. It was, after all, variation *from* the strict canons. By contrast, what deep sociative approaches do is to *reverse* all this. A more general theory which can accommodate impossible assumptions is sought from the outset; then special relaxed or shortcut procedures are introduced where consistency or other prized properties (e.g. truth) can be assumed.

One relevant medieval supplement to the main logical theory, of consequence and of signification (or reference), was that of obligationes (or commitment); another was that of insolubiles. The theory of obligationes, a partly formalised rule-controlled kind of dialogue, guided a method of training in dialectic.[34] The business of obligationes concerned both the theory and practice of dialectical argument; that is, it concerned argument, according to prescribed rules, between disputants. Obligationes discussions - of which there were several types (with rules also varying from place to place) - were closed under restricted (sociative) consequence. A standard introduction to obligationes texts run along these lines: 'Obligation is a certain art whereby some opponent can bind a respondent to reply at the opponent's pleasure to the obligatory sentence posited to him. Alternatively, an obligation is a sentence by virtue of which someone who is obligated is committed to reply affirmatively or negatively to the given sentence' (Billingham's rather full introduction, quoted by Ashworth 83 pp. 309-310). One important kind of obligationes disputation was that called *positio*, in which the opponent presents the respondent with a simple declarative proposition, the positum, which he is obliged to maintain, while the opponent seeks to draw him into explicit contradictions. In *positio* the respondent is then required to concede everything that follows, according to the prescribed inference rules, from the proposition to which he or she is committed and anything else already conceded, to deny everything that is inconsistent with all this, and to respond to what is irrelevant as he or she takes it to be the case. In other kinds of obligationes, the respondent has to maintain a proposition about his own response, or what he knows, or some such. Always certain critical factors are involved: a starting *assumption*, or proposition[s] considered, and an investigation by [two] sides, as to what it *binds* holders and opponents to, where it *leads logically*, according to certain predetermined inferential rules. The business plainly differs from the axiomatic method, with which it has recently been assimilated, in that

it involves *sides* or participants (characteristically two disputing a single proposition). As each obligation thus takes a certain structured dialogue form, a route to contemporary formalization is evident. Elements of such a theory can be neatly formulated, like parts of dialogue theory which the theory anticipates, in two sided tableaux style, with two columns, one for the respondent and one for the opponent, with the rules figuring rather like tableaux rules.[35]

Obligationes theory intrudes upon connected issues of much interest for sociative logic: firstly, the types of assumptions that can legitimately be made, and the types of rules that can be applied in such argument situations. It will quickly emerge that because false, and certain impossible, assumptions may be granted, the standard principles of material and formal consequence cannot be applied, but have to be qualified or suspended. It is this adjustment of consequence rules, made in many handbooks on obligationes, that gives the theory its main sociative interest: not what the theory variously says on assumption or on relevance. For there is now - since the Hilbert school, for all its stress on consistency - nothing very striking in the idea of arbitrary assumptions, including those of any grade of inconsistency. The theory of "natural deduction", for instance, admits *any* well-formed assumptions whatsoever. The worst problems with contemporary logical theory are bound up with where such assumptions lead or are supposed to lead, what they commit adherents to, and so on. On relevance obligationes theory is also a trifle disappointing. For relevance, or "pertinence" as it is better called, is defined in such a restrictive way that it is certain irrelevant statements, including relevant impertinent statements, that become of absorbing interest in the development of obligationes dialogues. In that setting B is pertinent to A, B Pt A, iff B follows from A or is repugnant to A, i.e. ~B follows from A.[36] The relation Pt is reflexive and symmetric, but not transitive. But what discussants, especially those with a sociative bent, want to determine is: what happens when irrelevant statements are infiltrated into dialogues?

The favoured material rules would, if applied to obligationes, have led, like those for mainstream tableaux, to paradox. For instance, in any disputation in which a contingent falsehood is posited, any irrelevant proposition must also be granted! This material paradox derives from the rule for irrelevant propositions, namely, that if a proposition is suitably independent of the positum (i.e. neither follows from, nor is incompatible with the positum, a proposition already granted, or the opposite of any correctly denied proposition), then it must be granted, denied or doubted solely insofar as we know, from outside the dialogue, its truth. The reasoning is like that of familiar paradoxes. Let the positum A be contingent and known to be false, and let B be any irrelevant proposition. Then not-A or B, though irrelevant, is known to be true, and so must be granted, by the rule. As it follows from the two propositions granted, the irrelevant B must also be granted. The upshot is that in any disputation in which a recognised contingent falsehood is posited, arbitrary irrelevant propositions must also be granted.[37] Medieval logicians, such as Burley, were much occupied with such paradoxes. As elsewhere in medieval logic, different rival theories emerged, several of them, like Burley's, still unclear. All these theories satisfied, through their qualification or suspension of the standard material rule, *ex falso quodlibet*, a basic requirement for paraconsistency. Not

surprisingly some of the theories that were devised anticipated features of contemporary paraconsistent and sociative logics. For example, Swineshed appears to have anticipated the recent non-adjunctive approach. In this theory, while one is sometimes required to concede two contradictory statements in a disputation that begins with a noncontradictory hypothesis, nonetheless, one must always deny the conjunction of these contradictories.

Thus certain standard material principles were suspended in obligationes. For example. Ockham commenting on rules of consequence observes

> That though, in a simple consequence, from something possible something impossible does not follow, nevertheless it sometimes happens that if something possible is affirmed, something impossible has to be conceded and something necessary has to be denied; but this can be done only in the art of *Obligatio* [that is, in the art of purely logical disputation] and only for the course of a given disputation (57 p.88).

While this does not upset the principle under discussion, namely $\Diamond A, \sim\Diamond B \ / \sim(A \rightarrow B)$, it does overthrow the paradoxes of implication (to which Ockham is reluctantly coming in this passage) given normal logical connections of implication with what is asserted and denied.

What was required of initial assumptions varied, like the rules determining commitment, from university to university and even logician to logician. Although consistency of assumption was *not* in general required, *blatant* inconsistency tended to be excluded, since explicit inconsistency signalled the termination of a game or dialogue. Albert of Saxony, for instance, contended that propositions from which the contradictory opposite immediately followed should not be admitted (Boehner p.15). Even so *positio* were generally allowed to start from propositions where consistency was in very considerable doubt or had lapsed altogether, such as some important theological propositions concerning the Incarnation and the Trinity. Consider, for instance, the issue of whether the Holy Spirit proceeded from the Son, which involved both their identity and their difference.

As the theory "developed" it appeared to become progressively more restrictive as to what was admitted in the way of impossible assumptions. In earlier days, before the high, more reactionary, stages of medieval logic were reached in the fourteenth century, assumption was much freer. A most significant text recently unearthed[38] is the *Tractatus Emmeranus de impossibili positione* (TE), apparently produced by the Nominalists, followers of Abaelard, active between 1140 and 1200. There several arguments are advanced to show that an impossible positio should be admitted. The first group moves from an appealing and significant analogy through to appeal to high authority. 'Just as we assert that what is possible should be conceded in order to see what follows thereupon; similarly we have it from Aristotle that what is impossible should be conceded in order to see what would happen then.' So, as we noticed, Boethius also correctly argued. Next the point is made that we can quite correctly assert things which involve impossibility, for instance that God is a man. Such assertions as these are certainly significant. Furthermore, we can often enough understand what is thus asserted in such impossible assertions; the point is neatly illustrated. But what is asserted or understood can be investigated as a posit, indeed what is understood can in no

way be excluded.

Even some highly implicit impossibility is destructive of modal theories of consequence, as the authors of TE proceeded to stress. They contended, invoking the analogy with false positio again, that

> everything that follows from the *positum* is to be conceded, where "follows" is construed in terms of a proper [*recta*] consequence. And a consequence is proper when the understanding of the consequence is contained in the understanding of the antecedent.... [Thus] from an impossible obligation not everything follows. Whence the consequence of the followers of Adam [no doubt the Parvipontani] must not be conceded in this [form of] question in which the understanding of the consequent is contained in the understanding of the antecedent (TE).

Thus proper consequence is supplied through content containment. So far, fine. But they immediately, and too quickly, proceeded to impose heavy connexive requirements on proper consequence. 'Since only such consequences are to be admitted, ... a consequence in which a negation follows from an affirmation is not to be admitted... .' The requirement not only ensures $\sim(A \rightarrow \sim A)$, but takes out *all* such implications of the form $A \rightarrow \sim B$, as 'if it is a man, then it is not an ass'. Much of the rest of the tract (TE) is devoted to meeting a series of objections to, and softening, such a problematic outcome of heavy connexivism. The crucial point for present purposes is however that obligationes theory, virtually from its inception, became a stage where a *genuine* consequence could operate, and where the sociative outcomes of false and impossible assumptions could, to a considerable extent, be decently and freely investigated. Unfortunately, later theory became progressively more restrictive, and appears to have degenerated in some centres towards modalism.

But, medieval logic was never, it seems, dialethic; it could not have tolerated some proponent resting comfortably, or however uncomfortably, with an explicit contradiction. Consistency remained a fundamental, and substantially unquestioned, good. But some of the theories, especially parts of obligationes, were paraconsistent; they permitted consequential reasoning from and through inconsistency, even if explicit contradiction could not be a resting stage, but was an endpoint, when the game was up.[39] It is worth observing, however, that obligationes practice could be redesigned so that explicit contradictories are admitted. The game would stop when everything had to be conceded (and so some acknowledged severe falsehood admitted), or differently when some statement was both asserted and rejected.

V. Modernity, enlightenment, and logical stagnation.

Although the traditional logic was expanded, in sociatively relevant ways, in the long period between the late middle ages and the contemporary period, little advance was made in the fundamentals of sociative logic, indeed comparatively little advance was made in logic at all. 'From the 400 years between the middle of the fifteenth and the middle of the nineteenth century we have ... scores of textbooks but very few works that contain anything at once new and good' (Kneales p. 299). So despite the oft-told greatness of the times, for the rise of science and technology, for mathematics and the arts, neither the Renaissance nor the

Enlightenment, nor the early Industrial "Revolution" itself, engendered significant contributions to the growth of logic, sociative or not. For the most part, logic stagnated or even declined, not to revive until well into the nineteenth century.

The situation has not been satisfactorily explained. The attractive (if insufficiently specific) suggestion - that it had an intellectual explanation in the rise and domination, at least during the Renaissance and through the Enlightenment, of ideologies and philosophies unsympathetic or even hostile to logic - is rejected by the Kneales. Instead they are inclined towards a more individualistic and superficial explanation, that 'although the subject survived in the elementary instruction of the universities, it no longer attracted the attention of many of the best minds' (p.298, the latter point is repeated without the qualifying 'many' two pages later). Some pretty good minds were attracted nonetheless, Leibniz for one (as the Kneales themselves observe). Why were other "best minds" no longer attracted, when certainly they mostly encountered logic? Because, presumably, they came across little they were able to get to grips with intellectually. The time was not ripe, for instance, for the algebraic comparison, which occurred to mathematicians in the nineteenth century, once algebra had been much elaborated (though rudiments of the main idea appear in Leibniz). Because, more important, of the depauperate type of logic they came to encounter. With the changeover in education from scholastic to humanistic grammars, all students, including the best, were directed into humanistic studies. The new humanistic texts 'severed the links with scholastic logic and philosophy [and] ... prepared students for the study of Latin poets and orators'. Thus Ashworth (74 p.22) who makes 'humanism ... the culprit' for the decline of logic, 'because it rendered students unfit for the study of logic, rather than because of its more generally seductive properties.' Such an attractive explanation does too much; the changeover, which could hardly have stopped extra-curricula education, would also have blocked the flourishing of scientific and technological investigations. A more likely explanation for the stagnation of logic lies in a combination of factors: the unpreparedness of the times for certain developments, the rich intellectual opportunities elsewhere, the indifference or hostility of humanism and especially humanistic education to scholastic logic, and the rise to ascendancy of empiricism and idealist-rationalism (later reflected in negative influences of Hume and Kant especially), both of which tended to view logic as a closed and empty arena.

Despite the general logical poverty of these intellectually rich human times, things were happening of much interest for the story of sociative logic (as well as for the history of paraconsistent logic; see PL chapter 1). Most of these doings concern attempts to expand the Aristotelian synthesis, traditional formal logic (as it was to be called), to encompass further areas of logic, not so far satisfactorily accommodated under the paradigm. Thus the theory of conditionals, the treatment of singular terms and propositions, the logical analysis of complex terms, the quantification of predicates, and so on. Since the traditional logic elaborated, based on syllogistic, was sociative, in a liberal sense, these extensions may be counted as applications of sociative logic.

Of particular note is a theory of conditionals to be found in modern seventeenth century

"traditional" logic. Though the theory is a reductionistic one, treating conditionals as certain enthymematic syllogistic arguments, it is nonetheless a relevant theory. The reason that relevance results, by complete contrast with contemporary enthymematic accounts of hypothetical reasoning and conditionals, is the tight control on terms and connection which syllogistic theory itself provides.

The traditional theory of conditionals, as it is perhaps appropriately called, is much easier to illustrate than to state in complete generality; and most authors who present the theory content themselves with illustrations, often copious illustrations, of the method. The theory is elegantly illustrated in the Port Royal logic of Arnauld or Arnauld and Nicole (pp. 224-235), which we shall follow, and is repeated in subsequent logic texts such as those of Whately and Mill. Put roughly, the theory is that a conditional holds good if there is an underwriting valid background syllogism, or chain thereof, into which standard form it can be transformed without loose ends. A main contemporary theory of conditionals is but a relaxation of this theory, obtained by extending syllogistic (as its original sense intended) to a wider range of arguments and by giving fuller scope to enthymematic character.

As is the usual way, cruder reductions were progressively refined 'The work of Wallis ... contains an attempt to bring conditional statements within the Aristotelian scheme by treating them as universal. According to Wassis, "*Si universaliter sumendum est, quasi tantumdem valens atque omni casu quo*"' (Kneales p.306). While this may generously be seen as affording a pleasant anticipation of semantical analyses of 'if', as a reduction of conditionals to complete syllogisms, it is too simple, as many examples from Port Royal logic attest. Consider, for instance, negative conclusions drawn from the maxim 'Every feeling of pain is a thought', such as 'if no animal thinks, then no animal feels pain' and 'if some part of man does not think, then some part of man does not feel pain'. The first argument compresses - so it is maintained, a syllogism of the form *Camestres*, the second of the form *Baroco* (Arnauld p.226). More generally, a conditional compresses part of a syllogistic argument. Consider, for instance, the following conditional (similarly 'since' and 'because' statements): 'if every true friend must be ready to give his life for his friend, then there are few true friends since there are few friends so devoted'. The single statement compresses the following syllogism:

> 'All true friends must be ready to give their lives for their friends. There are few people ready to give their lives for their friends. Therefore, there are few true friends' (p.225).

A precise and detailed presentation of the initial theory in traditional fashion would be extremely case-ridden because of the numerous forms involved, and appears not to have been attempted anywhere. As it was only an ancillary theory rounding out and helping to complete the established paradigm, and as everyone competent presumed they could see how the additional theory went and deal with virtually any case that arose, there was probably little pressure to present it in proper contemporary detail. Nor shall we attempt to present the theory in full generality, allowing in an unqualified way for complex and relational terms;

but we can obtain considerable generality and thereby remove much case structure by taking advantage of contemporary formalisations of syllogistic. Let the terms (perhaps complex but with structure unanalysed) be T_1, T_2,...,T_n... , and the term connectors be *a*, *e*, *i*, *o*. Let variable μ range over term connectors. The *well formed* initial *sentences* (*wfs*) of the syllogistic are of the forms $T_i\ a\ T_j$, $T_i\ e\ T_j$, $T_i\ i\ T_j$, $T_i\ o\ T_j$, i.e. $T_i\ \mu\ T_j$ for short, with μ variable. Well formed arguments are built up from sequences of wfs in the standard way. These will comprise both familiar syllogisms, consisting of three linked wfs, and chains of three or more linked wfs (called *sorites*), which are in fact resolvable into sets of familiar syllogisms and their validity so determined. Now the traditional theory of conditionals appears to be as follows: If A then B holds iff there is a well-formed correct (syllogistic) chain $T_1\ \mu\ T_2$, $T_2\ \mu\ T_3$,...,$T_{n-1}\ \mu\ T_n$ underwriting it, with terms T_1 and T_n at least supplied in A and B, and all terms of A and B included in T_1, T_2...T_n (i.e. no loose terms). The theory is perforce a first degree one, but permits higher degree substitution instances.

The traditional theory is bound to be relevant because of its common terms and no loose term requirements. Nothing like intuitionism with its free introduction of irrelevant conjuncts is permitted. Consider, what allows similar irrelevant rubbish, the partial-truth-valued theory of conditionals sketched out by the Kneales in substitution for a proper discussion of the third and fourth positions in the Alexandrian debate on conditionals (p.132). What the Kneales' theory comes to is that "if A then B" behaves like the conjunction, A & B, when A is true, and (what is often wrong) is not assigned a truth value otherwise, when A is false. In particular then, as with intuitionism, A and B, when *any* truths at all, suffice for "if A then B". For example, "all wild lions are confined now to Africa" is a perfectly satisfactory antecedent for "all straightforward conditionals admit of traditional syllogistic analysis". The traditional theory quite correctly does not validate such irrelevant conditionals, since there is no way of syllogistically linking the terms without loose ends. Thus although the traditional theory is unduly roundabout and has some marked weaknesses (to do as usual with the linguistic forms syllogistic can satisfactorily accommodate), in important respects it represents a significant improvement upon the mainstream contemporary theories of conditionals which have displaced it.

Some further features of the emerging theory beyond relevance are also evident, despite evident limitations of the formulation given. For example, as on recent modal theory, Augmentation will fail, i.e. the following principle does not hold in general:

Aug. if A then B / if A & C then B.

For C may introduce loose terms (in special cases of course the term of C may be packed into terms of the syllogistic chain that underwrites "if A then B"). More generally, any principle introducing new unrelated terms will be in trouble.

By contrast however with most recent modal theories, transitivity *will* hold, i.e.

Trans. if A then B, if B then C / if A then C

For the chain underwriting the second premiss can be backed up to that underwriting the

first, with the coupled chains thereby underwriting the conclusion. To be sure, if some of Arnauld's pronouncements were heeded, and only syllogisms proper used to underwrite conditionals, transitivity would break down (sometimes condensation of chains would be possible, but not in general). Moreover, Contraposition too will hold, also in rule form (as inevitable on essentially first degree theory), i.e.

Contrap. if A then B / if not B then not A.

The justifying argument, a little hairy, involves antilogising the underwriting chain. Strictly more has to be said as regards what underwriting comprises, a matter that becomes crucial in confirming Modus Ponens,

MP. if A then B, A / B,
and in assessing Connexive rules such as
Con. if A then B / not (if A then not B).

For sufficient generality something like the following is required: A is equivalent to (coentails) a conjunction of wfs of an initial segment of the corresponding chain, and B to a disjunction of wfs of a final segment, with initial and final segments not overlapping. (Furthermore, any wfs which are not derived from preceding members are *taken to be* true; this complication is needed to deal with enthymematic examples, best removed however in an initial attempt to capture the traditional theory). While such a specification removes embarrassment concerning MP, it leaves open the admissibility of Con, and also of reflexivity, i.e.

Id. if A then A .

Both Id and Con turn on what types of syllogistic chains are admitted. If syllogistic is always, as was usually thought, progressive and never repetitive or re-entrant, then Id is out, though the theory is easily adjusted to readmit it. The issue of Con is more difficult, and turns on such issues as whether syllogistic can lead to both positive and negative conclusions. While it looks as if the simpler examples traditional theory characteristically worked with would rule this out, and confirm Con, as Aristotle did, problems undoubtedly result. Most notable among these is the undermining of the premisses of Eleatic reduction arguments which take the form

Red. if A then B, if A then not B / not A.

Is this then vacuously admissible? Presumably not, especially as any other irrelevant conclusion would serve as well. But the traditional theory is not clearly enough articulated to give the more decisive answers we might now expect on these issues.

The reduction of all reasoning to syllogistic was pushed further in the eighteenth and nineteenth centuries. The sort of reduction, through compression and underwriting, that the Port Royal logicians applied to conditionals and certain other parts of reasoning, subsequent logicians like Whately, and theoretical hangers-on like Kant, proposed to apply to the *whole* of reasoning, including probabilistic reasoning.[40] The Port Royal logicians themselves seemed prepared to admit, as Leibniz certainly was (cf. Kneales p.322), that not all argument could be brought into syllogistic form. Not so later traditional logicians; the scene was thus set for

the downfall of the traditional paradigm, which proceeded to overextend itself. Thus too, given its (then) limited resources, it became an easy target, open to many counterexamples, for instance, from probability logic and from relational logic (as Leibniz had observed, without observing the damage done to his own metaphysics, and as De Morgan detailed). At least the reduction program pursued by Whately and others had the important merit of making all reasoning relevant - given the unexplicated, but tight and presumably logical, rules of reduction. What a bonus! But the program was never carried through, or even seriously attempted, in a systematic way; it succeeded, while it did, like most other monopolistic paradigms, apart from political strategems, through selective examples, handwaving and bluff. Nor could it have succeeded, without substantial enlargement, as recent developments in syllogistic theory have begun to show (cf. Bacon 85). Indeed, until recently the (relevant) reduction program to syllogistic appeared to be leading nowhere, but simply impeding the progress of logic (whence heavy, and sometimes justified, criticism from proponents of the new mathematical paradigm).

What proved, of course, a much more fruitful line of development was the adaption of algebraic ideas and techniques to logic, by Boole and others. This line of thought in fact stretches back at least to Leibniz, but it took a long time to activate the algebraic rolling stock properly, having to await further maturation of mathematics. Again the movement began in a sociative way, and only later became derailed. For Leibniz's calculus of containment and content is, so far as it goes in main sketches, a sociative logic, fashioned along algebraic lines. This abstract calculus, presented in various partial stages of development, admits of various partial intepretations - among those noticed by Leibniz a propositional interpretation. In one 'fragment on his project for a calculus of inclusion he suggested that the consequent of a conditional proposition might be said to be contained in the antecedent. ... it is clear that we can interpret his formulae as assertions about the relations of propositions' (Kneales p.344, where other interpretations are tabulated). Leibniz's suggestions give some warrant for symbolising not only the containment relation 'is in' by $\leftarrow$, the converse of implication $\rightarrow$, but for representing the conditional in term of the same notation. The result is striking. Proposition 12, for example, of the most developed of Leibniz's many suggestions (conveniently presented in Kneales p.340 ff.) becomes, after notational updating, $C \rightarrow B \rightarrow . A \,\&\, C \rightarrow A \,\&\, B$, i.e. the principle of Factor. Shedding $\rightarrow$-representation of the conditional it is the general first degree principle, if $B \leftarrow C$ then $A \,\&\, B \leftarrow A \,\&\, C$. The example was chosen for two reasons. Firstly, Factorisation of one sort or another plays a significant role in Leibniz's theorizing. Secondly, the most developed calculus *LC*, is an apparently relevant factorisation system (an I system), heavy with factorisation features:- not only Factor itself and complications thereof; but the interconnection of inclusion with identity in principles $C \rightarrow B \rightarrow . C \,\&\, B = C$ and $C \,\&\, B = C \rightarrow . C \rightarrow B$ (Propositions 13 and 14); and the emphasis on identity, $=$, and its substitutivity and other features, as the starting point of a logical calculus (Definitions 1 and 2 and initial Propositions).

As a matter of inspection, this calculus, an $\{\rightarrow, \&\}$ system (without or with a parasitic negation $\sim$), can be represented within an extension of the common first degree system, *FD*,

of entailment. *FD* is extended by substitutional principles, that upon variables and, more important, that for replacement of equals (substitutivity of provable equivalents). Represent propositions of the form '*If* α *and* β, then γ' in rule style as: α, β/γ. It follows from well-known results that *LC*, so represented, is weakly relevant.[41]

In the subsequent algebraic elaboration of logic in the nineteenth century, connection vanished. Though it did not need to vanish at all, it quickly did. The second, algebraic, wave of mathematical logic, called 'the Boolean period' by Bocheński (who presents a neat chronological table, p.279), is said to begin with Boole and De Morgan's publications in 1847. But the logical calculi that Boole and De Morgan somewhat independently devised - though intended, as main applications, to represent both syllogistic forms (by way of familiar dubious extrasystemic translations of A, E, I and O forms) and also intermediate inferences - were essentially first degree algebraic systems. They did not admit nesting of the equality relation (or of it with inequality). They did not contain an inclusion relation or an implication. These enlargements, and the nesting of implication and equivalence were to come rapidly enough in the next thirty years, but they were not present at the beginning. Apart from interesting and damaging curiosities like Boole's symbol ν (for representing particularity) which quickly disappeared, the calculi were narrow algebraic systems, fully formalisable (as in RLR p.118) *without* propositional apparatus. Though it could accordingly be claimed that these systems are (variously) relevant, because the criteria offered do not apply to them, it would be more accurate to say that the question of relevance does not so far arise. However the systems are hardly neutral under intended applications. Much as -(x & -x) = -(y & -y) for any classes or conceptions x and y whatsoever, so, for any universal classes or conceptions u and v whatsoever, u = v, since u = 1 and v = 1. So, under application, any two universally true propositions whatsoever are identified, e.g. "All humans are mortal" says the same as "All kangaroos are herbivores". Content relevance is hardly preserved. Furthermore, Boolean logic, which Boolean algebra straightaway induces (see RLR p.119), is irrelevant.

The trouble arose from the introduction, by algebraic analogy, of 0 (read *Nothing*!) and 1(read (the) *Universe*), conforming, in effect, to the equations 0 & y = 0 and 1 & y = y (e.g. Boole p.47), with y = 1 and y = 0 representing, respectively, truth and falsity of corresponding propositions (representations adopted from Boole through MacColl: see Bocheński p.309). The rather inevitable result, in quite ordinary algebraic settings, is that any designated truth will be assigned to 1 at the top (and designated falsehoods to 0 at the bottom of a Boolean lattice), and so distinct truths will be identified. The fault lay at bottom with the (elective) class model, assumed throughout by Boole, which is entirely inadequate for propositional representation. Mistaken identifications are not the only defective outcome, simply among the worst and the basis of relevance shedding. (Compare the problem with strict implication: because A & ~A = B & ~B, substituting in Simplification gives A & ~A ⥽ B). Another little remarked outcome consists in unsatisfactory interconnections for Nothing and Universe. According to Boole, not-Nothing = Universe, but according to pre-analytic assessments, not-Nothing is something or other; in Boolean symbols, -0 = ν. It took about a century from simplistic beginnings for a satisfactory algebraic treatment of truth and falsity

to eventuate, with scrapping of the dud equations, $x = 1$ and $x = 0$, and replacement of 1 and 0 respectively by truth filters, and falsity ideals (cf. ENT p.193). Really satisfactory treatments of Nothing, Something, and their discredited mates, will have to wait their time.

With the rise of the crude Boole-Schröder algebraic representation of traditional logic and the ensuing development of classical logic in the late nineteenth century, niceties of connection vanished in favour of mathematical analogies and *calculability*. Relevance changed from being something automatic, that was taken largely for granted, to being something that wasn't guaranteed and didn't matter anyway. Its status changed from a 'Not to worry. I'm OK, George' to a 'Don't Care. I'm unnecessary, Will'. With the ensuing adjustments in logical paradigms, former logical necessities like relevance of conditionals and implications, and good connections in logic, became luxuries and matters of nonstandard concern.

There were other streams of thought in the nineteenth century where connection did matter, or should have, and where sociative logics really were required - not merely in the emerging social sciences, but even to demonstrate the viability of whole ideological movements, most notably German idealism and some of its off-shoots. But although such philosophical enterprise cried out for sustaining logical foundations, of paraconsistent and sociative cast, none of appropriate detail were furnished (despite many texts allegedly about logic). Indeed idealism proved a hostile environment for the growth of sophisticated logic; and most of the development of mathematical logic has occurred in a very different milieu, primarily that of reductionistic mathematics, but with inputs from pragmatism and positivism. Nonetheless parts of idealism were subsequently drawn heavily upon by logical and foundational theory, conspicuously intuitionism, especially elements of the precipitating idealism of Kant (the pivotal figure who also 'began the production of the curious mixture of metaphysics and epistemology which was presented as logic by Hegel and the other Idealists of the nineteenth century': Kneales p.355) Much less momentously according to mainstream history, but more relevant for sociative history, ideas drawn from Kant (and by him from Leibniz and Locke) also played a justificatory role in Parry's elaboration of containment logics.

As the very name 'analytic implication' indicates, the origins of systems of analytic implication or logical containment have been speculatively linked with Kant's celebrated explication of analyticity. As Kant explained it, 'the connection of the predicate with the subject is thought through immediately', or, as Anderson and Belnap try to explain Kant, '"S is P" is to be *analytic* just in case the subject is in some sense partially identical with the predicate' (ENT, p.429). Transcribed as a necessary condition for implication, Kant's dictum then becomes: $A \rightarrow B$ is to be an *analytic* implication only if the consequent B is in some sense partially identical with the antecedent A. So far, if partial identity is taken in an obvious way as symmetrical, this does not swing Kant's dictum in the direction of containment systems, rather than the relevance ones (cf. ENT p.155). If, however, traditional containment accounts of analyticity (and implication) are recalled, accounts appealed to alternatively by Kant, then partial identity is cashed as containment, and the dictum is

swung in the conceptive direction. It remains only, a substantive matter, to replace 'in some sense' by 'as regards sentential variables', a matter that can be argued on the grounds that it is containment of content that counts and that content is carried by variables. The latter claim, though implicit in much logical theory concerning form and content, is highly contestable (cf. RCR); for surely such forms p & q and p ∨ q have different content though they have the same variables? Though the distinctive features of analytic implication systems can be tenuously linked with Kant, there is no investigation of any such systems in Kant (who thought logic was a closed shop) or indeed in propositional form until Parry arrived on the Harvard scene earlier this century.

VI. The contemporary period: the rise of mathematical logic and sociative alternatives to the triumph of irrelevance.

In certain respects logic has flourished in the last century as never before. But it has been a strange monocultural, probably unstable, mathematically complex but philosophically poor, mega-development. By contrast with earlier major periods, the classical, degenerate, problem-making theory appeared first, by a good margin (the system of Frege's *Begriffschrift*, the conventional starting point, is dated as 1879, and Russell's equivalent *theory of implication* 1906: on both see Prior, p.301, who follows and reinforces the received history). Naturally the theory did not spring from nothing; the way had of course been prepared through significant earlier development and supplementation, especially in algebraic elaborations exhibiting similar degeneracy of traditional logic. Moreover, some of the most celebrated, but now questionable, developments, such as the theory of quantifiers, were not only long anticipated, but broke through in several places somewhat independently - reflecting a common pattern of human intellectual discovery.

What the history sketched reveals is that classical logic is not in a privileged historical position. It is a rather recent upstart, a quite minor character pushed to prominence by new but crude mathematical technology. Morover, almost as soon as it was realised what was being claimed as to conditionals and implication under the new "mathematical logic", there were objections, even outrage, particularly (thanks to Russell's publicizing activities) in England and North America, but also in France. Corresponding with Russell on the subject of nested implications, Couturat wrote: 'These sorts of implications ... are very lacking in evidence, or paradoxical, or even unintelligible ... . And the formulas which I consider as *obvious*, such as : pq ⊃ p, p ⊃ q. q ⊃ r: ⊃ : p ⊃ r, are only derivable [in your system] by means of the law of importation, which is again one of these paradoxes ... the equivalence p ⊃ q. =. ~p ∨ q is yet another paradox' (letter of 8 November 1903). 'In general, [Russell's] correspondence shows that the introduction of ⊃ was seen as a radical, and counter-intuitive innovation, and not only by Couturat.'[42] While many informed intellectuals (not exactly a large band) protested, few did anything much about it. The first systematic attempts at repairs and improvements were largely modal in orientation (and due principally to MacColl, an underrated researcher, Johnson and Lewis). These attempts *did* offer improvements to the theory of logical implication or entailment - though not that of conditionals - and they were eventually absorbed in the broader classical paradigm, as extensions of classical logic. But

these improvements were resisted by the earlier narrower paradigm, whose commitment to extensionality such modal explications did upset; and they still are resisted in logical backwaters.

Lewis's investigations of strict implication were particularly influential, they have been important to all subsequent alternative enterprises, and they have largely withstood subsequent extensionalist attempts to discredit or eliminate them. Much subsequent alternative enterprise has in fact consisted in varying techniques and methods, assumptions and systems worked out for expanding galaxy of modal logics (as modal logics themselves mostly advanced through adaption or elaboration of methods first devised for, and in the simplified setting of, many-valued logics). Lewis himself engaged in a good deal of variation of system and much experimentation in arriving at his set of strict systems, in the course of this investigation arriving at some systems which collapsed back into classical logic, and some which were in fact relevant (see RLR p.356). After his rediscovery of the medieval paradoxes of implication, his investigation narrowed, his position rigidified. Lewis became convinced that there was no paradox-free "promised land" (something logicians, of all people, tend to be *much* too easily persuaded of), that some paradox *had* to be lived with. Such a conviction has not proved highly robust; Lewis's conviction did not transfer even to those who helped him investigate the properties of strict implication systems by such new-fangled techniques as matrix methods, Parry in particular.

In part because of Lewis's influence at Harvard, in part owing to Scheffler and his students, a brief golden age of alternative logic, of which Parry has told (e.g. chapter 10 below), followed at Harvard. Among the fruits of this period were Nelson's rediscovery of connexive logic (a framework Parry sets aside too lightly), and Parry's systems of analytic implication, which represent types of containment logics. Most of this alternative work (theses or equivalents bearing on implication by Bronstein, Emch, Henle, Nelson, Resinger, Shen, Weiss, and others) has however vanished from easy public access.

Parry's work on analytic implication, recorded in an unpublished Harvard dissertation in 1932 (and reported briefly and inaccurately in Parry 33), itself disappeared from view for more than thirty years until interest in it was revived by Anderson and his students, and their colleagues and competitors. Since then information concerning such conceptivist systems has increased rapidly, with the result that several of the main open questions (e.g. those raised in ENT pp.430 ff.) concerning Parry's principal system *PA1* are now resolved or can be straightforwardly solved. As usual, the resolution of such problems has generated another set of problems, concerning a wider class of transcending systems. These include relations with other similar sociative logics, arrived at in slightly different way, such as relatedness logics, problems of quantification, and so on. Now that they have been developed to some extent, many applications for containment logics have been noticed. These typically involve areas of systemic limitations, areas where contexts are limited or information available is restricted in some way. Salient examples include: logics of fiction, and surrounding areas such as ideologies and dreams; dialogue logics; and frame and other problems in artificial intelligence (see

further RCR).

Much of the remainder of the more accessible, contemporary sociative story has also been told elsewhere in sufficient detail for present purposes; and the rest is readily pieced together.[43] Most of the contemporary strands involve no very organised movement, but, in the fashion of much contemporary alternative activity, simply comprise a few individuals or small groups of individuals writing largely in isolation. Certainly such a pattern obtained with connexive logics, which have been advanced substantially independently by Nelson, Angell, McCall and others. Connexive principles have also been espoused by many others who have done little or no systemic work; connexive principles just refuse to go away easily, no doubt because there is a viable intuitive picture underpinning them (based on a cancellation view of negation, as speculatively explained in NC). Until very recently, containment logics for instance were represented by a one-man band, Parry, playing, so he argued, technical variations on a theme drawn from Kant. Likewise Ackermann, first prime technical mover of the contemporary nondissembling approach, worked entirely on his own (in a high school, having lost Hilbert's patronage). Again with nontransitive logics, there is no orchestrated movement, no school, though those promoting such systems tend to have been British based (not North American as with connexive theories) and to know one another.

By contrast again, both relevance and relational logical approaches have come to involve very loosely affiliated schools. Relevance logic developed around Anderson and Belnap, at first at Yale from 1958, but chiefly at Pittsburgh to which they both soon moved and where they gathered around them, especially in the 60s, an active and intelligent group of graduate scholars (see ENT p.xx). Anderson and Belnap and others *much* elaborated a very fruitful synthesis of the formal and philosophical work which had preceded them: notably, variations upon Ackermann's technical achievements combined with adaptions of methods of Fitch and of Curry, on the one hand,[44] combined with significant explications of the demands for connection and relevance in a theory of entailment urged by a small chain of opponents of strict implication before them, on the other. Different again, relational logics were explicitly launched (though the idea is an older one) in 1979 in *Philosophical Studies*, mainly by a small group of North Americans, mostly committed at bottom to classical logic, from Wellington, New Zealand, a (declining) centre for modal logics.

In Australia, as in the north, opposition to Russellian logic and insistence upon connection goes back a fair way. But the opposition took a stronger form, which unfortunately helped to delay the development of any sort of contemporary logic in Australia. In the regionally important Sydney sphere of influence, dominated by J. Anderson from the late 20s, Russellian logic was roundly rejected, at least until the 60s. In particular, material-implication was dismissed philosophically because it supplied no requisite connection of components.

> '... one reason why the doctrine of "material implication" is a philosophical blind-alley is just its ignoring of such connection; implication then becomes quite arbitrary or "magical" - divorced from inquiry ... |as| where reference to pressures is lacking ...

> the connections between the barometer and the weather would be magical rather than scientific connections' (Anderson p.145, rearranged).

The relevant logic movement, based in Australia, while applauding constructive features of Andersonian criticism of classical logic, is not an off-shoot or continuation of Andersonianism, but an independent development. The relevant movement differs from Anderson and Belnap's relevance enterprise, though it grew up alongside it intellectually, rather as a slightly younger sibling, and borrowed heavily from it as well as contributing to it. (It has benefitted through contributions of a major defector from the relevance enterprise, Meyer, who is not a relevantist either.) Since the movements are popularly confused, it is worth setting out some of the salient differences (these are presented in more detail in RLR).

•. The relevant movement rejects *most* of the sweeping themes of the relevance enterprise, for instance that E is entailment, that it is the logic of relevance and necessity, that R is relevant implication, that E and R (and perhaps T) are the important sentential relevance-preserving logics. According to the movement, the selection of E (and of R) was premature, and mistaken. If there is a unique logic of entailment - increasingly doubtful as evidence accumulates - it lies somewhere among deep relevant logics.

•. The movement is committed to paraconsistency, and to taking inconsistent theories, situations and worlds seriously. Thus, for example, the relevant movement courteously disowns Ackermann, whose logics were not paraconsistent. By contrast, the main text of relevance enterprise, ENT, is dedicated to Ackermann, 'whose insights' are correctly said to have 'provided the impetus for this enterprise'; and at the very beginning of the text (p. xxx) relevance logic is held up 'as a new branch of mathematical logic, initiated by ... Ackermann'. Ackermann enjoys no place in the history of paraconsistent logic, though some of his technical contributions have, like those of Church, proved valuable in the development of relevant paraconsistent logic. In fact, very much of his activity was directed, like that of other members of the Hilbert school, at establishing consistency, and at exorcising inconsistency - a practice of which Anderson, if not later Belnap, strongly approved (see e.g. his review of Wittgenstein).

To descend again to routine grimy city life and twist the logical scene back into real perspective, it only needs stressing that *all* these alternative logical positions are tiny, with but few genuine proponents, and with at least as many fellow travellers. The overwhelming bulk of contemporary logical theory makes no claim to relevance or connection, the logics involved are not sociative, and most logical practitioners are substantially uninformed about sociative logics of any sort. Moreover logic itself has largely ceased to be an independent disicpline. It does not count for much in the contemporary world, and much of what role and support it has derives from its use as a servile means to other ends. In service roles however lies some new hope for sociative logics. Fortunately there are other purer grounds as well.

The ideas of alternative nonstandard logics are, in main relevant cases, obvious, appealing and exciting. So they keep reappearing or being rediscovered, especially on the fringes. Only in intellectually repressive or inactive societies will these ideas be kept from developing and flourishing, and ultimately from pulling down the present anti-pluralistic,

narrow and authoritarian, logical establishment and structure. Things logical can change; given a little time and a fair run they will change. Many major happenings in sociative logics, helping to make traditional concerns good, are very recent; whereas ideological evolution remains slow, and challenging ideas still take much time to spread. Despite much internal and ideological opposition, but fortunately for its satisfactory survival, logic has increased enormously in diversity in the last thirty years - in ways even very few intellectuals appreciate. Contemporary relevant logics, in particular, only really began as a distinctly technical enterprise in the 50s. Semantics and other analyses of certain modal logics, which were to give enormous stimulus to the investigation of a burgeoning range of modal and intensional logics, were only beginning to be worked out at about the same time. Though the prospect of reasoning successfully with inconsistent information and from inconsistency still went largely unrecognised (so Popper could republish his astonishing, but rapidly outmoded, diatribe against dialectical logics), paraconsistent logics and all that were also beginning. Much more was still to come (for one survey, see PL).

But though logic has come a long way very recently, it has a longer way to go, both in *whom* it involves and *what* it investigates. There are, for instance, virtually no black researchers, and exceedingly few women are engaged; and for all the proclaimed rationality of modern humans and their institutions, logic touches comparatively little human practice. Differently, there remain *many* notions of considerable logical import, *some* of historical significance, of which we lack decent accounts or, sometimes, a clear appreciation. To the satisfactory elucidation of these, sociative logics can make essential contributions.

NOTES

1. See especially PL (and and its extract OP), which should be read in conjunction with this material. Conversely, this material enrichs PL, which is scanty on several topics of relevant interest.

2. The positions of original researchers are mispackaged not only because of lack of boxes, but because of the presuppositions researchers and historians brought and continue to bring with them. For instance, the recent American history of the long debate on universals has been *seriously* distorted by the ontological assumptions that the Americans typically bring with them. The result is that significant positions get eliminated from consideration; for instance, those according to which universals are perfectly good objects of their own sort, not names, not mental constructs, not something else, which furthermore do *not* exist. Each generation needs to rewrite history not merely because of new informational bases (which may represent contractions on former bases), but to get past the presuppositions of predecessors (only of course to impose some of their own).

   It should be evident that the present minority sketch of some logical history - which *is* highly selective, with a view to bringing out some of the neglected sociative tradition - has presuppositions of its own. To some extent, usual Whiggish rankings are reversed. For example, a Pseudo-Scotus (or Buridan or another), who tries to make the paradoxes of implication an inescapable feature of the consequence relation, is not here seen as making a significant advance on Abaelard who is committed to some sort of paradox-free connexive structure; Pseudo-Scotus so far from representing historical

progress takes us a step backwards. Not only pre-moderns get this sort of vulgar (Whiggish) treatment and distortion; more recent examples are Bolzano and MacColl, Peano and Cantor.

3. Though the argument proceeds through an example, the principle involved is stated in a quite general way. Certainly the principles and its derivation were taken to be unrestricted in Boethius and in medieval authors, both those who adopted it and those who later criticised it. But see Montgomery and Routley 68 on the ways of evading generality here. Thus, while Aristotle's work permits of, and mostly encourages, a connexivist interpretation, it is not entirely decisive.

The argument presented for AR1 also makes some defensible assumptions about Aristotle's notion of negation; indeed the issue of negation is critical to the question (see NC). By and large, Aristotle's discussion of negation notions supports a cancellation story and connexive themes. For instance, 'Affirmation is the statement of something of something. Negation, the statement of something [as subtracted] *from* something' (28b 17a25-6). 'In Aristotle's discussion of repugnance ..., an external negation is used to cancel the negation of a consequent and the result is claimed to be equivalent to an affirmative hypothetical' (Martin 87, p.60); i.e. effectively $\sim(A \rightarrow B)$ is equated with $(A \rightarrow \sim B)$ as in hyperconnexivism.

4. For some of the difficulties see E. Martin 83, and Thom, p.243 ff.

5. There was more to it than that. There was also a much discussed rivalry between Peripatetics and Stoics; on this intriguing feud, touched upon below, see e.g. Frede.

6. Thus a rich, and not easily absorbed, new satellite city of literature upon Aristotle's syllogistic, mostly bent upon fitting it within mainstream logical theory, is by-passed - conveniently for energy- and time-expenditure. Both the axiomatic and natural-deduction approaches, now often presented as exhaustively competing to explicate and formalise Aristotle's theory of the "assertoric" syllogism, would foist upon Aristotle principles and procedures there is little or no evidence he accepted, or would have or should have accepted. Aristotle's underlying or operational logic, or "propositional logic", certainly was not what Łukusiewicz ascribes to him, classical two-valued logic or alternatively, if future contingents are duly allowed for, a three-valued logic. Nor was it any many-valued logic neatly contained within classical logic. Nor was it such a logic reformulated in a natural deduction guise. All those include not only implicational paradoxes and irrelevance, and associated bad news such as the deduction theorem (an integral part of any "natural deduction" approach, and a straightforward outcome of logics favoured on axiomatic approaches); but they also include a variety of circular statements derivable in the run of syllogistic systems, such as "All As are As" and "If all As are Bs and all Bs are Bs then all As are Bs". As Corcoran, one of the leading proponents of the ill-named "natural deduction" approach, admits, statements such as that he represents as Axx obtain mention nowhere in Aristotle's writings. (But Corcoran, apparently oblivious of progressive reasoning, uses *this* to suggest that 'Aristotle had no idea of logical axioms' or 'logical truth', 73 p.216!)

As to rule or conditional formulations of syllogistic, evidently Aristotle and the Peripatetics deployed both (certainly if the conjunctor 'therefore' is taken as signalling a rule). But the rule formulations are not those of "natural deduction" in a contemporary sense; nor are the conditional formulations suggested strictly axiomatic in a modern sense. No doubt it is feasible to develop a theory of syllogistic, much like Aristotle's which furthermore does not presuppose propositional logic (cf. the propositional-free formulation of algebras in RLR p.117). Such a system may or may

not be axiomatic; it could, for instance, be "genetic", in the way mathematics was before the fuller rise of the axiomatic method last century.

The mixed strategy of Thom, which aims to combine and duly restrict the satisfactory elements in the main approaches, both rules and axioms, no doubt leads to a reconstruction of non-intensional syllogistic significantly closer to Aristotle's theory of non-modal syllogisms than the work he builds upon. The reconstruction properly renders the theory both progressive and relevant. But the attractive reconstruction in the first part of his text is not sustained in the later part of the text, where Thom's own propositional theory is elaborated. There (in fine accord with a proclaimed dogmatic procedure, p.11), connexivism is out, progressivism is out, identity is in, etc. Thom loses touch with Aristotle and Peripatetic traditions, but surfaces bearing the implicational part of relevant system *R-W* - no great gift, but at least weakly relevant. More authentic is the approximation to Aristotle's implicit propositional logic which Slater proposes. But Slater's approach remains unduly algebraic and reductionistic, perhaps collapsing altogether. The problem of more satisfactory reconstructions remains open.

7. Some small beginnings only have been made on the task. As well as the items referred to in fn 4, see Martin and Meyer 83 and RLR chapter 11.

8. See Mates p.47. The original much-quoted passage comes from Sextus Empiricus, *Outlines of Pyrrhonism* Bk II, pp.110-112. Much of what we know about the on-going conditionals debate is conveniently collected in the Appendices of Mates' important *Stoic Logic*.

9. Bocheński likewise suggests, wrongly, that 'connexive' implication, the implication of the third position, is an ancient form of *strict* implication (p.119). But he offers no supporting evidence or argument; and it looks as if he is prepared to construe 'strict' implication so widely that it includes connectional implications of a non-Lewis type. Gould (e.g. 67 p.153), De Rijk, and several others also repeat Mates' misconstrual of the third position.

10. For details see, e.g., Prior 57. Gould is, like Mates, confronted by the gratuitous problem of how to distinguish Chrysippus from Diodorus, given that both are committed to strict implication. He fails the test. For not only is what he says (on the last page of 67) implausible, insofar as not obscure; furthermore his modelling is simply defective. Note too, Cicero's argument, supposed to bolster the strict case, where incompatibility is replaced by impossibility of a conjunction (step (4) to (5) in Gould, p.158) does not require anything like a strict account; for it only uses $A \rightarrow B \dashv \sim(A \,\&\, \sim B)$ and $\sim(A \circ B) \dashv \sim(A \,\&\, B)$.

11. The story is told by Cicero. The term omitted from the quote, drawn from Mates p.55, is the bracketed '(material)'. Contemporary, and dubious, assumptions are again being interposed.

12. The resolution of the problem proposed shares significant common ground with the earlier resolution of Frede, who reconstrues the connective used in schemes 6 and 7 as subdisjunction (which amounts truth-functionally in the two argument case to $\sim(A \,\&\, B)$). But taking conjunction as suitably intensional avoids the remoteness of subdisjunction and the implausibility, forced by a classical framework, of representing conjunction as a special sort of *dis*junction. The weakness otherwise of Frede's resolution, and the serious defects of shuffles like that of the Kneales, has been explained by Stump.

13. The quotations are Stump's translations of passages from Boethius's *In Ciceronis Topica*. This whole section on Boethius on Cicero is *heavily* dependent upon Stump 87; nonetheless it does not come down quite where she does.

14. Mill's notoriety-grabbing theme that all argument involves *petitio* lurks in some of the background here. Galen recognised such a problem with truth-functional arguments like the Stoic third indemonstrable, where knowing the conjunctive premiss truth-functionally involves knowing the conclusion; he proposed avoiding the difficulty by intensionalising! Hence too part of the motivation for later amendment of the Stoic "indemonstrables".

15. The theory of topics covers several matters which ought to be of more contemporary interest, but have largely slid off the logical agenda. In Boethius , one is that of relevant enthymematic argument; another (following Cicero) concerns methods of argument discovery. A main method of argument discovery in fact amounts to ascent up a proof tree; for what are surveyed at each node are the sources that would lead from above, by recognised rules, to the sort of proposition under scrutiny at the given node. A similar general procedure can be (noneffectively) applied in filling out enthymemes. For other roles of topical argument, see Martin 87 and Woods and Walton.

16. What is indicated in the text is is the easy part of the story, which was related to us by Miguel de Castro (who referred back on invited inference to Geiss and Zwicky 71 and Kartunnen 71); the hard part is making it work.

17. Martin tries hard to extract a containment account of conditionals from Boethius's obfuscating prose, an account which would reveal a natural historical source for Abaelard's containment theory. At one stage Boethius perhaps appears to extend the association condition to an implicit containment interpretation, when he remarks that it may not occur that something is understood as antecedent unless in the same what is consequent is understood. But as Martin honestly observes, this remark is ambiguous, and the larger context appears to resolve the remark decisively against a containment-in-understanding point, that understanding the antecedent involves understanding the consequent, in favour of a contrast theory point, that 'antecedent' cannot be understood independently of 'consequent' and vice versa.

    Elsewhere Boethius gives what is, at first sight, a different account of "the" conditional: 'The sense of the conditional proposition is this: that it would be the one thing just so long as it were the other...', an account suggestively similar to modern truth-preservation-over-situations analyses. There is no conflict of accounts. For it is now realised, in some places at any rate, that such a situational conjunction or overlap analysis can be seamlessly joined with other analyses. such as association or containment ones.

18. See Martin EA, early version, p.2. Much of this section follows Chris Martin's detailed discussion (unpublished parts of which he kindly made available). Martin suggests that investigations of the sort he has helped initiate will result in a new appreciation of the vacillating fortunes and features of logic, at least up to the end of the Middle Ages. We suspect he is right. Much of what is offered here, supporting the same bold theme, is heavily parasitic on his pioneering work, especially the summary of the twelfth century debate on conditionals. Otherwise unreferenced quotations in this section are drawn from EA.

19. *Dialectica p.271.* This yields, after some standard modal manipulation contested in

later medieval work, Martin's condition I: for the truth of a conditional it is necessary that it be impossible that the antecedent be true and the consequent false.

20. Abaelard argued, in effect, that natures are *positively* characterised; it is no part of the nature of a man that he not be a stone. And he contended that maximal propositions of the theory of topics supplying conditionals, are restricted to those connecting natures - a very narrow theory of conditionals. For requisite details of Abaelard's theory, see again Martin, e.g. EA. One tangential feature, to which Martin does not draw attention, is that Abaelard's treatment of opposites resembles an important move in the theory of objects. Conditionals concerning opposites, which assert that if one of a pair obtains the other does not or is absent, are consistency assumptions, which exclude impossible objects which have both of a pair of opposites.

21. A similar theme is to be encountered as Australian naturalism, a contemporary school that is very "scholastic" in its practice, edging crabwise from ad hoc distinction to special counterexample-defeating distinction. This school has not got around however to putting its restrictive theory of properties to work in the logic of conditionals.

22. It is not clear that the relevance requirement on its own is sufficiently general; e.g. let B_o be a statement duly relevant to A_o, such as an appropriate compound of it. For connexive success, at least all inconsistent conjuncts A_o & $\sim B_o$ need to be excluded from Simplification. But maybe the Porretani intended to exclude all these, and perhaps ban simplification altogether. Then however fallaciousness is no longer apparent in all cases of simplification removed. A revealing discussion of the fallacy of non-cause-as-cause, which helps bring out the relevant use-in-argument requirement it would instate, is given in Hamblin.

23. Parallel themes, not yet taken to be bizarre, are still found nowadays, e.g. "the false is never a cause", "nothing that is false is a reason". The Melidune theme itself appears to have been adopted by Frege, who held that there cannot be an argument with false premisses! (It is thus easy to see, from one angle, why he was happy with material implication; cases with false antecedents could, as Don't Cares, be treated as convenience and simplicity appeared to dictate). The claim destroyed usual hypothetical proofs; thus reductio proofs, in particular, had to be paraphrased as perfect conditional syllogisms: see Dummett 81 pp.308-10. Again we owe these details to Martin.

24. This containment account runs right through medieval thought, from Abaelard in the twelfth century, through Kilwardby, Ockham and Strode, to Paul of Venice in the fifteenth century. The story continues, through Javellus and others in the sixteenth century, Leibniz in the seventeenth, to the present. Moody, who locates it in Strode and Paul, dismisses it, in good logical positivist fashion, as giving a psychological definition (p.71). But that is to make the crude if common confusion of the more highly intensional with the psychological.

25. Strictly, the coupling is more complex. Even if the logical truth of an implication (as distinct from that of a conditional) is tantamount to the correctness of a consequentia, the latter directly supplies an inference warrant while the former does not. Thus they differ in their relations to the operation of inference, or detachment.

26. Psuedo-Scotus proposed the following definition of *consequence*: 'A consequence is a hypothetical proposition composed of an antecedent and consequent by means of a conditional connective or one expressing a reason (*rationalis*) which signifies that if they, viz. the antecedent and consequent, are formed simultaneously, it is impossible

that the antecedent be true and the consequent false' (Bocheński p.190). For a discussion of the basic definition and the careful qualifications Pseudo-Scotus sometimes added, see Kneales pp.277-288.

27. The rules of *consequentiae* are from Ockham's *Summa Totius Logique*, iii(iii), 37. As well as being repeated in many medieval texts, they are quoted in several contemporary commentaries, e.g. Kneales p.291.

28. The argument, too dependent on omission as opposed to explicit rejection, is hazardous - especially as medieval logicians commonly omitted assumptions they used. And some contrary Burleigh material may, all too easily, come to light!. It should be added that there are also some problems with Boh's reconstruction on which the argument tends to rely. For Burleigh made various qualifications to syllogistic and to nonsyllogistic reasoning, as if they satisfied some different principles, which Boh passes over (see also Moody pp.92-3).

29. Burleigh's modal principles can also be included in an approprate sublogic of $\Box R$. The main principles are the connected group, $A \rightarrow B \therefore \sim\Diamond(A \circ \sim B)$; $\Diamond(A \circ \sim B) \therefore \sim(A \rightarrow B)$; $\Diamond A, \sim\Diamond B / \sim (A \rightarrow B)$.

30. Buridan claimed 'Ad omnem propositionem copulativam ex duabus invicem contradictoris constitutam sequi quamlibet aliam ... consequentia formali' (I8, 7a). See also Moody's discussion of Buridan's emumerations of the types of formal consequence (p.77 especially, also p.88, p.90).

31. But it should be S2, it seems. The evidence for S3 rather than S2 (i.e. $A \rightarrow B \rightarrow . \Box A \rightarrow \Box B$ instead of $A \rightarrow B \therefore \Box A \rightarrow \Box B$) is flimsy at best. The evidence - from Strode and Paul, both to be counted out - suggests the rule form only.

32. See Hughes' commentary on the *Logica Magna*, attached to his edited translation, from which the unpaginated quotations from Paul earlier in the paragraph of the main text are drawn. George Hughes kindly supplied this material before publication; hence the lack of page numbers. A preliminary comparison of the different theories of consequence of *Logica Magna* and *Logica Parva* is given in Perreiah's appendix to the latter edition, pp. 336-7.

33. Much else of sociative interest remains to be investigated in the later medieval period; for instance, more precise details of the logical frameworks various schools and authors were committed to. Further logicians whose work may repay study include Blanchellus Faventinus, Fonseca, Kesler, Maiolus, Juvallus, and Regius.

34. There is now an expanding literature on obligationes, in some of which it is said that the earlier literature on the topic is not very satisfactory, and in much of which it is claimed that the practice is not well understood, a claim given *some* confirmation by the diverse accounts offered of what it is really about. (In this respect it resembles contemporary literature about a main predecessor of obligationes, namely topics.) Thus it has been presented as a set of schoolboy exercises (cf. Stump 85), as the rules for dialectical exchange, as anticipating the modern axiomatic method (Boehner p.15), as a theory of counterfactual reasoning (Spade 82), as a logic of disputation (Stump 85). But the prize in these varying presentations undoubtedly goes to Moody, who - after a decidedly misleading description of '*obligationes*... as forms of agreement concerning things to be assumed for the purposes of argument, and the exposition indicates how to state such agreements without having loop-holes which would afford one's adversary the chance of gaining an advantage in the argument' - asserts that 'this

topic is not very relevant to logic ... and can conveniently be passed over' (65, reissue of a much earlier work, p.294). It seems that obligationes theory can go some way to satisfying several of these varying roles. The theory certainly contained the rudiments of a dialogue logic, and has a good deal in common with modern tableaux theory, tableaux theory with nonstandard rules. And it did investigate what emerged from counterfactual, and sometimes counterlogical, assumptions; and thus insofar as conditionalisation was permitted, it gave a direct guide to which counterfactual conditionals could be sustained. The theory may also have been (as Martin says) the context, otherwise rather mysterious, in which medieval resolutions of semantical paradoxes, naturally arose. Much of the preceding literature on the topic is noted in Spade.

35. Such tableaux representation is of course not uniquely determined, even as to format. Rather better than standard two column (O/R) tableaux - which impose excessive constraints on connectives like negation - may be the following scheme of interactive tableaux. Each side, both O(pponent) and R(espondent), is assigned *two* columns, one of which represents what is asserted (or follows from what is granted) and the other what is denied (cf. a tableaux formulation of the "American plan", RLR chapter 3).

36. This passage borrows from a communication from Stephen Read on Stump 85. To Read, who read and commented constructively on the present essay, a considerable debt is owed.

37. Again by Martin (whose translation is used), building on De Rijk. For various interesting and curious later restrictions on what could be posited, see Ashworth 83 pp.314-5.

38. A notion of pertinence also appears in the medieval literature on insolubilia, where a (self-referring) sentence "falsifies itself" just in case it is pertinent to inferring it is false; see Spade 82, p.250, who remarks that 'this notion of "relevance" is a complex one, and needs further study'. But a notion of pertinence directly relevant to contemporary logical concerns *does* appear in late medieval literature.

39. Even Martin lapses into the commonplace (if convenient) confusion of paraconsistency with dialethism. 'The logic of the Middle Ages were never paraconsistent, but there was an urgent reason for denying that in general everything follows from an impossibility' (Martin 86 p.571).

40. The idea appears to be ancient. Thus '...Alexander's defence of Aristotelian logic consists largely of attempts to show that all interesting arguments can be represented as categorial syllogisms. A belief in the universality of the categorical syllogism was probably common among the Peripatetics. There are also grounds for thinking that some Stoics held an analogous belief about their logic: every deductive argument can be represented as a propositional one' (Mueller p.174: the qualifications slid on, in easy fashion, such as 'interesting', 'deductive', make very significant differences). Whence in part the genesis of the ancient feud between Peripatetics and Stoics (cf. Frede).

41. In a fuller investigation of Leibniz's various fragmentary calculi (perhaps, like contemporary attempts of individual researchers at formalising preanalytic notions, only hazardously and ahistorically lumped together), much turns on how negation is accommodated, no straightforward matter given, if Lenzen is believed, Leibniz's 'somewhat uncertain and partly erroneous theory of negation' (p.3). On the treatment of negation (especially), stands the theme, advanced by a bickering phalanx of Leibniz scholars, that Leibniz's ('explicitly given') logic is a strict implicational system, namely

$S2^{\circ}$ or perhaps $S3^{\circ}$ according to Lenzen's detailed investigations (e.g. p.24, p.1). Such a modal theme deserves - a bit like the parallel findings planted on the Stoics and medieval "mainstream" - to be viewed with some scepticism. The arguments to the theme depend upon, what looks methodologically unsound, pooling virtually all of Leibniz's work - which apparently results an inconsistent corpus (an easy strict route to the sought conclusion) - and then consistencizing in a way that suits sought results. The arguments also depend on strategies such as illegitimately upgrading what are but one-way conditionals (perhaps only rule linkages) to biconditionals or even definitional status (cf. Lenzen's moves to P3 and P1 on p.4 and to P1S, p.13).

Leibniz's calculus *LC* will be examined in more detail elsewhere, along with other Factor systems of historic interest, such as that of MacColl in his anticipation of Frege. MacColl's later (1906) sentential logic amounts to S2.

42. Letter from Alasdair Urquhart (1 Dec 1987), who supplied the quote from Couturat.

43. For instance, in ENT, RLR and PL. But for proper historical purposes, it has not been told in nearly enough detail. That however is a task for someone else, more energetic and further removed from the action.

44. For example, only system *T* (of "ticket entailment") appears to be strictly due to Anderson and Belnap (and to Belnap precisely), if precision is to be insisted upon in these matters. *E* was obtained as postulate-chopping variation of Ackermann's *II* systems; in original intention it was equivalent to them, and it was duly proved theoremwise equivalent to them. Bacon and Prawitz appear to be responsible for the elevation of system *R*, with later help from Meyer, who pushed *NR* as an explication of *E*.

Ticket entailment in general, not as uniquely (but with a certain arbitrariness) encapsulated in system *T*, has a long, interesting history (some of it traced in C.Martin). Both Boethius and Abaelard maintained that a conditional is true or an enthymeme valid if there is a topical connection between antecedent and consequent, a connection, guaranteed by a topical rule, very like that of an "inference ticket".

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