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Lyapunov Stability Analysis of Power Systems

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OF POWER SYSTEMS

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OF POWER SYSTEMS

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Adviser   Date

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CHAPTER I

INTRODUCTION

Over the past ten years, a vast outflow of research and publications has resulted from the use of Lyapunov's "second or direct method" of stability analysis. This research trend stems from the appearance of the original work of Lyapunov in 1892, more than three quarters of a century ago; but only recently has this concept been appreciated to the point where researchers in the area of stability of dynamic systems are aware of its potentialities.

La Salle and Lefschetz wrote the first comprehensive book in English about Lyapunov stability theory (1). This was immediately followed by a series of papers, both original treatments and translations from Russian and German. Gurel and Lapidus have given an exhaustive bibliography of all this literature (2).

The difficulty in the application of Lyapunov's direct method is that, in general, there is no obvious choice for a function suitable for use as a Lyapunov function. In most cases describing a physical system, the energy stored in the system appears to be a natural candidate. This is neither a necessary nor a sufficient criterion for choosing a function and many examples of stable systems are known where the energy is not a suitable Lyapunov function. Though not completely definitive, five major categories of construction methods can be suggested (2). They are:

1. Chetaev integral methods.
2. Krasovskii quadratic forms.

3. Zubov-type partial differential equations.

4. Lur'e canonical forms.

5. Miscellaneous methods.

A common objective in these varied techniques is the development of a Lyapunov function which need not be related to the system energy.

The extension of this abstract theory to the analysis of real-world problems has posed yet another problem; the conflict between the theorist and the practitioner. Those who are mathematically inclined have seen others as in retreat from rigor while the others have thought those who manipulated symbols impractical.

However, direct methods like Lyapunov's second theorem yield stability conditions without recourse to a complete solution of large system differential equations commonly encountered in practice. The minimization of computer time without sacrificing accuracy is a very important consideration and in this respect the promising applications of direct methods are obvious.

In recent years a number of papers have been written indicating the possibility of using Lyapunov's direct method to analyze the power system transient stability problem. Gless (3) developed Lyapunov functions for single-machine systems and extended these functions to a hypothetical three-machine problem. El-Abiad and Nagappan (4) considered the general multimachine power system and proposed an energy-based Lyapunov function. Fallside and Patel (5) also considered a single-machine system. Yu and Vongsuriya (6) applied Zubov's method of generating a suitable Lyapunov function, but this scheme proved to be quite unwieldy when handling multimachine systems. Pai, Mohan and Gopal Rao (7) used the Popov criterion and applied Kalman's construction procedure to generate Lur'e-type Lyapunov functions. However, this approach was restricted to systems with a single nonlinearity.

It was Willems (8) who applied Popov's criterion to analyze the multimachine transient stability problem. Anderson analyzed the stability of control systems with multiple nonlinearities and, later with Moore, (9) generalized the Popov criterion. These techniques were used in Willems' paper which also reduced the power system to the Aizerman-Popov model. Willems and Willems (10) extended these ideas to a multimachine system.

This treatise envisages the study of large power system stability with the aid of Lyapunov functions. The primary purpose of the investigation is an extensive study of Willems' conjecture (discussed in Chapter III) for the derivation of Lyapunov functions for multimachine power systems. The study extends the Lyapunov method to analyze operations of practical significance such as automatic circuit reclosing. An investigation is made into the effects of parameter variation on the extent of stability (11). New methods of generating Lyapunov functions have also been attempted. Wall's energy metric algorithm (12) is shown to yield a suitable function for the analysis of a second-order system. Two systems have been studied using Lyapunov stability analysis. The advantages and disadvantages of Lyapunov's method are discussed and some suggestions are offered for further research.

CHAPTER II

PART A

LYAPUNOV STABILITY CRITERION

Lyapunov's stability study is based on the formulation of the system state differential equation of the form

$$\dot{X} = A X,$$

where X is an n dimensional state vector, $\dot{X}$ is the state vector derivative and A is an nxn coefficient matrix. The objective is to investigate the stability of the equilibrium point in state space, E^n, by constructing a suitable function V(X), or by devising a sufficient condition for predicting the qualitative behavior of the system in the neighborhood of the origin without actually solving the system differential equation (13).

The point of equilibrium X = 0 is stable in the Lyapunov sense if for $e > o$ there can be chosen $\delta(e) > o$ such that from the following relation $\|X(t_o)\| < \delta(e)$, for all $t > o$. It follows that, $\|X(t)\| < e$

A graphical interpretation is shown in Fig. 2.1. The $x_1 - x_2$ plane is for $t = t_o$. If $\|X_o\|$ is in the interior of the circle of radius δ, the curve X(t) in the motion space remains for all future times inside the cylinder of radius e.

If the equilibrium point is not only stable in the Lyapunov sense, but also

$$\lim_{t \to \infty} \|X(t)\| = 0,$$

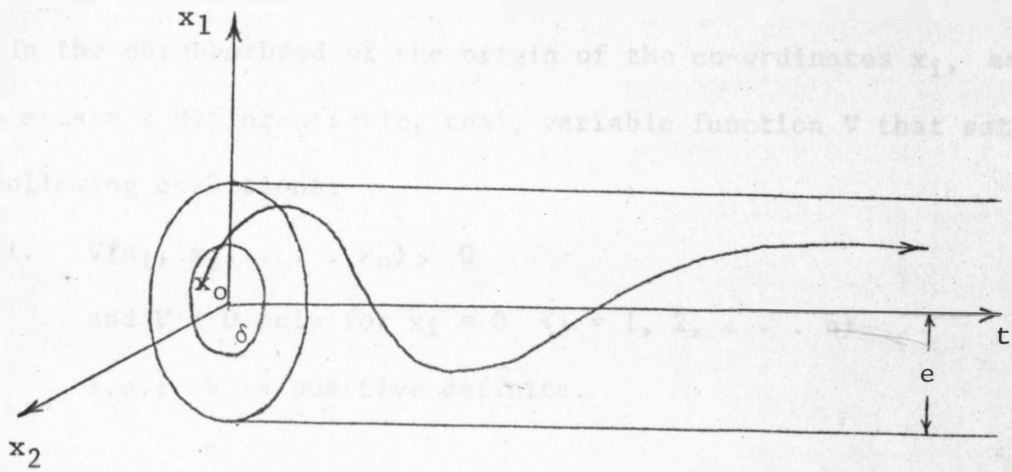


Fig. 2.1. Definition of stability in the Lyapunov sense.

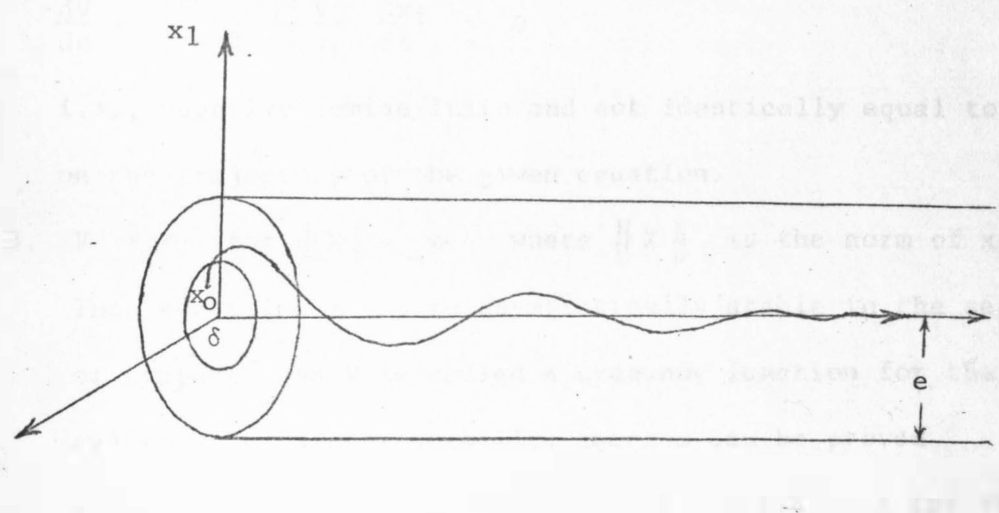


Fig. 2.2 Definition of asymptotic stability in the Lyapunov sense.

the equilibrium point is said to be asymptotically stable. An asymptotically stable system response is shown in Fig. 2.2.

Statement of the theorem

In the neighborhood of the origin of the co-ordinates x_i, assume there exists a differentiable, real, variable function V that satisfies the following conditions:

1. $V(x_1, x_2, \ldots x_n) > 0$

   and V = 0 only for $x_i = 0$ $(i = 1, 2, \ldots n)$

   i.e.; V is positive definite.

2. $$\frac{dV}{dt} = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \frac{dx_i}{dt} < 0 \quad \text{for all } x_i \neq 0 \text{ and } t > 0$$

   i.e., $\frac{dV}{dt}$ is negative definite.

   Or

   $$\frac{dV}{dt} = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \frac{dx_i}{dt} \leq 0$$

   i.e., negative semidefinite and not identically equal to zero on the trajectory of the given equation.

3. $V \rightarrow \infty$ for $\|X\| \rightarrow \infty$ where $\|X\|$ is the norm of x_i.

   Then the point X = 0 is asymptotically stable in the sense of Lyapunov and V is called a Lyapunov function for the system. Lyapunov's stability theorem can be proved geometrically (13), but the proof is omitted here for the sake of brevity.

PART B

TRANSIENT STABILITY PROBLEM OF A POWER SYSTEM

Transient stability refers to the amount of power that can be transmitted with stability when the system is subjected to an "aperiodic disturbance." By aperiodic disturbance is meant one that does not come with regularity and only after intervals such that the system reaches a condition of equilibrium between disturbances. The three principal types of transient disturbances that receive consideration in transient stability studies are:

1. Load increases.
2. Switching operations.
3. Faults with subsequent circuit isolation.

Focussing attention to the severest and perhaps the most frequent transient disturbance, the fault, it is easily seen that the stability of the system is completely dependent on:

(i) the initial condition immediately prior to the fault

(ii) condition during the fault and

(iii) condition subsequent to the isolation of the fault.

An important practical quantity in this context is the critical clearing time. This may be briefly defined as the maximum time the system can sustain a disturbance before being isolated from the fault if stability is to be retained. From a Lyapunov stability standpoint a better concept is obtained. If a maximum bound for stability is established, the critical clearing time is the time required for the V function to reach this boundary, starting from the stable equilibrium point at the origin. A very clear idea of qualitative as well as quantitative factors concerning power system stability can be had from the equal area criterion. However, this is somewhat restricted to a second-order case. Lyapunov stability analysis can be applied to higher-order systems.

CHAPTER III

THE SYSTEM MODEL

Consider a multimachine power system. In order to facilitate the analysis, the following assumptions are made in the study (4):

1. The input power to all the machines in the system remains constant during the entire transient period. In other words time constants of the governors are considered large with respect to the period under study.
2. Each machine is represented by a constant voltage behind the transient reactance. Stator resistances of the machines are neglected.
3. Damping in the system is directly proportional to the rate of change of rotor angles.
4. Loads other than synchronous machines are represented as constant impedances.
5. Saliency of the rotating machines is neglected.

Under these conditions, the dynamic equation describing the i^{th} machine is given by:

$$M_i \frac{d^2 \delta_i}{dt^2} + a_i \frac{d \delta_i}{dt} + P_{ei} - P_{mi} = 0; \; i = 1, 2, \ldots n \qquad (3.1)$$

where

δ_i = Angular divergence of the i^{th} machine.

M_i = Inertia constant of the i^{th} machine.

a_i = Damping coefficient of the i^{th} machine.

P_{ei} = Electrical power output of the i^{th} machine

P_{mi} = Mechanical power input to the i^{th} machine.

The electrical power output for round-rotor machines is given by:

$$P_{ei} = E_i^2 G_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i E_j Y_{ij} \left\{ \cos \theta_{ij} - (\delta_i - \delta_j) \right\} \qquad (3.2)$$

$$i = 1, 2, \ldots n$$

Here, E_i is the internal voltage and G_i is the short circuit conductance of the i^{th} machine. Y_{ij} and θ_{ij} are respectively the modulus and phase angle of the short circuit transfer admittance between the i^{th} and j^{th} machines.

In most practical cases, the transfer conductances $G_{ij} = Y_{ij} \cos \theta_{ij}$ where, $i \neq j$, are negligible and only the transfer susceptance terms need be taken into consideration. Then the electrical power output is given by

$$P_{ei} = E_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i E_j B_{ij} \operatorname{Sin} (\delta_i - \delta_j), \qquad (3.3)$$

$$i = 1, 2, \ldots n$$

Let $(\delta_1^o, \delta_2^o, \ldots \delta_n^o)$ be the equilibrium angles for which stability has to be determined. The dynamic equations describing the motion of the multimachine power system can be written in the state form as (10)

$$\frac{dX}{dt}(t) = AX(t) - Bf\left\{ CX(t) \right\} \qquad (3.4)$$

Here $X = \begin{bmatrix} Y \\ Z \end{bmatrix}$ is a 2n dimensional vector which is the state of the system, the components of the i^{th} elements being:

$$Y_i = d\delta_i/dt$$

$$Z_i = \delta_i - \delta_i^o$$

$$A = \begin{bmatrix} M^{-1}R & 0_n \\ I_n & 0_n \end{bmatrix}, \quad \text{a(2nx2n) matrix}$$

M = diagonal (M_i), R = diagonal ($-a_i$) and 0_n and I_n are respectively zero and identity matrices

$$C = \begin{bmatrix} 0_{mn} & D \end{bmatrix}$$

where 0_{mn} is a zero (m x n) matrix and D is a (m x n) matrix such that

$$\sigma = D\,Z$$

with components $\sigma_1 = Z_1 - Z_2$, $\sigma_2 = Z_1 - Z_3$ . . .

$\sigma_{n-1} = Z_1 - Z_n$, $\sigma_n = \sigma_2 - \sigma_3$, $\sigma_{n+1} = Z_2 - Z_4$ . . .

$\sigma_m = Z_{n-1} - Z_n$

The vector-valued function $f(\sigma)$ has m elements and is of the diagonal type which means that the i^{th} component σ_i of σ depends on the i^{th} component of $f(\sigma)$.

$$f_i(\sigma_i) = \mathrm{Sin}(\sigma_i - \sigma_i^{o}) - \mathrm{Sin}\,\sigma_i^{o}, \qquad i = 1, 2, \ldots n$$

where σ_i^{o} is the i^{th} component of

$$\sigma^{o} \triangleq D \begin{bmatrix} \delta_1^{o} \\ \delta_2^{o} \\ \vdots \\ \delta_n^{o} \end{bmatrix}$$

Also

$$B = \begin{bmatrix} M^{-1} & D^T & E \\ & 0_{nm} & \end{bmatrix}$$

where E = diagonal (e_k) with $e_k = E_i\,E_j\,B_{ij}$ for which i and j are the indices of the components of Z on which σ_k is dependent, i.e.

$\sigma_k = Z_i - Z_j$.

For a typical three-machine system, the values of the above matrices are given in Appendix A.

By means of the technique explained in Appendix B-a, Lur'e-type Lyapunov functions are generated for the set of equations 3.4. According to the definitions of the matrices B and C in the above discussion we have,

$$CB = B^T C^T = 0_{mm}$$

Hence, from the equations in Appendix B-a, it follows that

$$W = 0_{mm}.$$

(W is defined in Appendix B-a.) A set of matrices should be now determined so that $(N + Qs)\,C\,(Is - A)^{-1}\,B$ is a positive real matrix. One solution is $N = 0_{mm}$ and $Q = I_m$, since $sC(Is - A)^{-1}\,B$ is positive real if all damping coefficients are non-negative. This choice leads to Lyapunov functions with a negative semidefinite derivative (8).

With the solution above, the equations in Appendix B are simplified to obtain

$$PA + A^T P = -LL^T \tag{3.5}$$

$$PB = A^T C^T \tag{3.6}$$

L is an auxiliary square matrix. Let P be partitioned into four n x n matrices as

$$P = \begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix}$$

Then, equation (3.6) is equivalent to

$$P_1 M^{-1} D^T = D^T \tag{3.7}$$

$$P_2 M^{-1} D^T = 0_{nm} \tag{3.8}$$

The negative semidefiniteness of

$$PA + A^T P = \begin{bmatrix} P_1 M^{-1} R + R M^{-1} P_1 + P_2 + P_2^T & R M^{-1} P_2^T + P_3 \\ P_2 M^{-1} R + P_3 & 0_{nn} \end{bmatrix} \tag{3.9}$$

is required by equation (3.5) and it follows that

$$P_3 = -P_2 M^{-1} R = -R M^{-1} P_2^T \tag{3.10}$$

The solution for P_1 can be obtained by means of the results of Appendix B-b.

Equation (3.7) yields,

$$(P_1 M^{-1} - I_n) D^T = 0_{nm} \quad \text{and,}$$

$$(M^{-1} P_1 M^{-1} - M^{-1}) D^T = 0_{nm}.$$

Since $(M^{-1} P_1 M^{-1} - M^{-1})$ is symmetric, Appendix B-b gives

$$P_1 = M + \mu M1M \tag{3.11}$$

where 1 is a square matrix of order n whose elements are all equal to 1. μ is a scalar number. P_1 is positive definite if $\mu > \mu_o$ where μ_o is the solution of

$$\det (M + \mu_o M1M) = 0 = (1 + \mu \sum_{i=1}^{n} M_i) \quad (\prod_{i=1}^{n} M_i)$$

Hence, $\mu_o = -1/(\sum_{i=1}^{n} M_i)$

From equation (3.8) and (3.10),

$$R^{-1} P_3 R^{-1} D^T = 0_{nm}$$

and hence,

$$P_3 = \gamma R1R \quad (3.12)$$

$$P_2 = -\gamma R1M \quad (3.13)$$

where γ is a scalar constant.

SYSTEMS WITHOUT DAMPING TORQUES (8)

In a transmission system the damping torques due to asynchronous torques are usually negligibly small. At least from the point of view of a stability study, neglecting damping torques results in a more conservative result thus justifying the assumption.

When the damping torques are neglected, R = 0 and hence,

$P_2 = P_3 = 0_{nm}$.

By Appendix B-a, the Lyapunov function

$$V(X) = X^T PX + 2\int_0^{CX} f(\sigma)^T Q\, d\sigma \quad (3.14)$$

is obtained. Here,

$$X^T PX = \omega^T M \omega + \mu \omega^T M1M\omega$$

where μ is an arbitrary constant satisfying $\mu > \mu_o$ where $\mu_o = -1/(\sum_i M_i)$. The derivative of this Lyapunov function along the solution of the system equation (3.4) is

$$\dot{V}(X) = 0$$

Two conditions are of special interest: (a) If $\mu = 0$, the Lyapunov function becomes,

$$V(X) = \omega^T M \omega + 2\int_0^{CX} f(\sigma)^T \, d\sigma \tag{3.15}$$

Note that $Q = I_m$.

(b) If $\mu = \mu_o$,

$$V(X) = \frac{1}{\sum_i M_i} \left[\sum_i \sum_{j=i+1}^{n} (\omega_i - \omega_j)^2 M_i M_j \right] + 2\int_0^{CX} f(\sigma)^T \, d\sigma \tag{3.16}$$

When one of the machines in the system becomes an infinite bus, equations (3.15) and (3.16) become identical. Equation (3.16) developed through Willems' conjecture is a systematic way of generating a Lyapunov function for a multimachine power system. Equation (3.15) is equal to twice the energy function of the system and so is different from those developed by Gless (3) and El-Abiad and Nagappan (4). However, in computing critical clearing time this does not cause any difference at all.

STABILITY DOMAINS (10)

Let $V(X)$ be as given and $\dot{V}(X)$ along solutions nonpositive for all x. Consider the surfaces $V(X) = k \geq 0$. For small values of k the surfaces are bounded since by assumption $\sigma_i f(\sigma_i) \geq 0$ (Refer to Appendix B-a) for σ_i sufficiently small. Let $k = k_{max}$ be the smallest non-zero value of k for which

$$\frac{\partial V}{\partial x_1} = \frac{\partial V}{\partial x_2} = \frac{\partial V}{\partial x_3} = \ldots = \frac{\partial V}{\partial x_{2n}} = 0$$

has a solution for some X such that $V(X) = k$. Then the region containing the origin and enclosed by the surface $V(X) = k_{max}$ belongs to the domain of attraction of the origin.

The function V(X) vanishes at the origin and is cup-shaped near the origin for $\|X\|$ sufficiently small. Thus the surfaces $V(X) = k$ are bounded for small values of positive k. When k increases, the surface remains bounded until the surface $V(X) = k$ passes through a point where V(X) has a relative minimum, $\frac{\partial V}{\partial x_1} = \frac{\partial V}{\partial x_2} = \cdots = \frac{\partial V}{\partial x_{2n}} = 0$.

In other words, $V(X) = k$ remains bounded until the next equilibrium point closest to the origin is reached. This equilibrium point is unstable and the value of k_{max} is obtained by substituting the co-ordinates of this point into the V(X) function. Thus, the test for stability will be to check whether V(X) has reached k_{max} or not as the system trajectory travels from the origin.

CHAPTER IV

STUDY OF A SECOND-ORDER SYSTEM

PART 1.

The Lyapunov stability concepts discussed in Chapter III will be applied in this chapter to the analysis of a second-order system. Consider the system shown in Fig. 4.1 (15). A 25 MVA, 60 Hertz water-wheel generator is delivering 20 MW over a double circuit

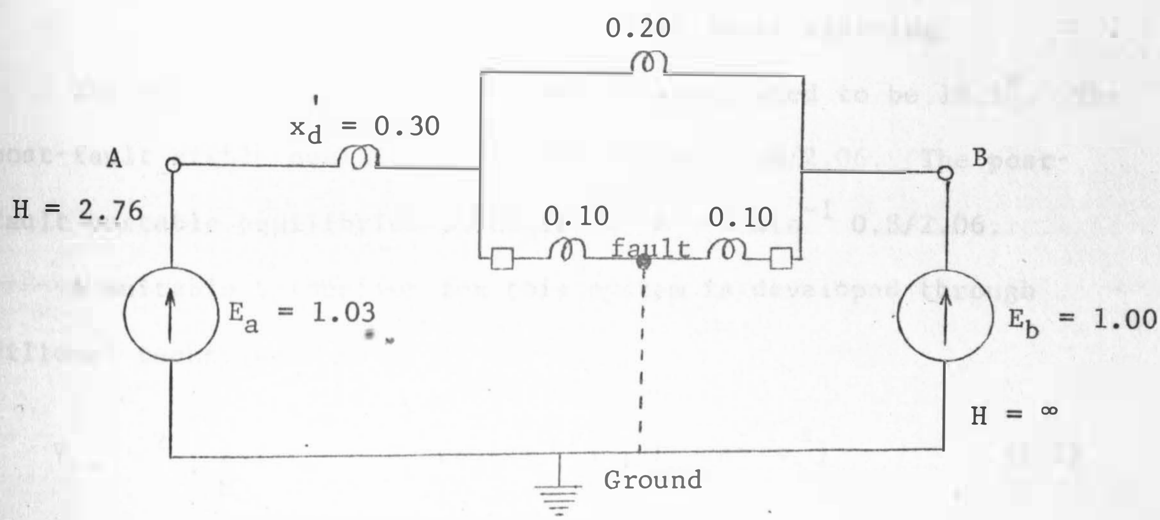


Fig. 4.1. Second-order power system.

transmission line to a large metropolitan system which may be regarded as an infinite bus. The generating unit has a kinetic energy of 2.76 M. joules/MVA at rated speed. The direct-axis reactance of the generator is 0.3 per unit. The transmission circuits have negligible resistance and each has a reactance of 0.2 p.u. on a 25 MVA base. The voltage behind the transient reactance of the generator is

1.03 p.u. and the voltage of the metropolitan system is 1.00 p.u. A three-phase short circuit occurs at the middle of one transmission circuit. The fault is cleared by the simultaneous opening of the circuit breakers at both ends of the line.

The power angle equations giving the output of generator A as a function of the angle δ between voltages E_A and E_B can be easily derived as

$$\begin{aligned} P_{UA} &= 2.58 \sin\delta && \text{before fault} \\ &= 0.936 \sin\delta && \text{during fault} \\ &= 2.06 \sin\delta && \text{after fault clearing.} \end{aligned} \qquad (4.1)$$

The pre-fault equilibrium state is calculated to be 18.1^o. The post-fault stable equilibrium is $\delta^s = \sin^{-1} 0.8/2.06$. The post-fault unstable equilibrium point is $\delta^u = \pi - \sin^{-1} 0.8/2.06$.

A suitable V function for this system is developed through Willems' technique.

$$V = \omega^2 + \frac{2}{M}(-P_m x - P_e \cos(x+\delta^s) + P_e \cos \delta^s) \qquad (4.2)$$

V_{max} is obtained by substituting the co-ordinates of the unstable equilibrium state into the above equation.

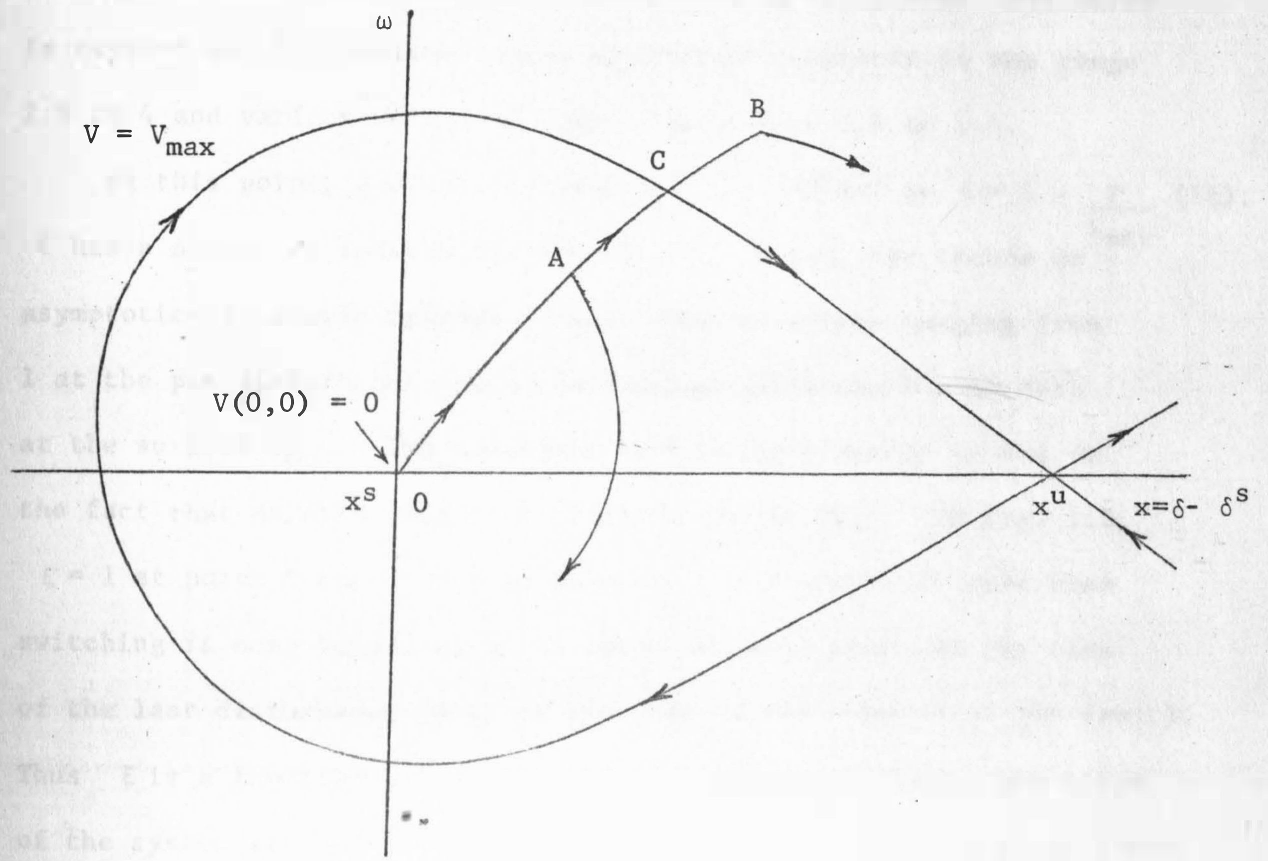
$$V_{max} = \frac{2}{M}(-P_m Y - P_e \cos(Y+\delta^s) + P_e \cos \delta^s) \qquad (4.3)$$

Note that $\frac{d\delta}{dt} = \omega = 0$ at equilibrium conditions. In these equations $x = \delta - \delta^s$ and $Y = \delta^u - \delta^s$. This means that the co-ordinates have been shifted so that the post-fault stable equilibrium state forms the new origin.

For determining the critical clearing time for the system, the system differential equation is numerically integrated from the pre-fault stable equilibrium state along the fault trajectory. The integration is carried out until the V function equals the maximum value, V_{max}. The time at which V equals V_{max} is the critical clearing time.

The concepts involved in this analysis can be explained with the aid of a phase-plane portrait of ω, the velocity, vs x, the displacement, for the system. Referring to Fig. 4.2, V is evaluated at a point A on the disturbance trajectory OACB where A is the point at which the disturbance is removed. As $OA < OC$, the fault clearing at A results in a stable post-fault system. For a clearing at B, i.e. $OB > OC$, Lyapunov theory as applied here fails as an instability criterion. The time corresponding to C, where $V = V_{max}$ gives the critical clearing time.

Summarizing, the Lyapunov function V, as a scalar function of the system state variables, corresponds to a hypersurface in the state-space co-ordinates $(\delta_1, \delta_2, \ldots \delta_n, \omega_1, \omega_2, \ldots \omega_n)$. According to the second method of Lyapunov, if the initial state of the post-disturbance power system is inside the surface $V = V_{max}$, the system trajectory will tend to the equilibrium state as time approaches infinity and the system is asymptotically stable. The initial state external to $V = V_{max}$ could be either stable or unstable since a valid Lyapunov function is only a strong, conservative and sufficient but not necessary condition for some type of stability.



Stable equilibrium: $x^s = \delta^s - \delta^s = 0$

Unstable equilibrium: $x^u = \delta^u - \delta^s$

$\omega^u = \omega^s = 0$

Fig. 4.2. Phase-plane portrait for a second-order power system.

Appendix C gives the computer program for the study of the previous system. The results are tabulated in Table 4.1. The study is carried out for various values of inertia constants in the range 2.5 to 4 and various values of power inputs from 0.8 to 1.4.

At this point, a stability measure ξ is defined as $\xi = 1 - \frac{V}{V_{max}}$ (11). ξ has a number of interesting properties. First, for stable or asymptotically stable systems, ξ will take on values ranging from 1 at the pre-disturbance state, decreasing monotonically to zero at the surface V_{max}. The monotonic decreasing property is due to the fact that dV/dt is negative definite inside V_{max}. In Fig. 4.2, $\xi = 1$ at point 0 and $\xi = 0$ at point C. In a practical case when switching is done before C, ξ is evaluated only once, at the time of the last disturbance (e.g. at the time of the removal of the fault). Thus ξ is a function of the time of the last disturbance, the state of the system at that time, and the parameters of the post-disturbance system. Further examination of ξ reveals that it is dimensionless and normalized and that for a stable system, ξ has the same range of values $(0 < \xi \leq 1)$, regardless of the number of machines or complexity of the power system.

Tables 4.2 and 4.3 give the results of the study of the given system for different values of inertia and mechanical power input respectively in the range mentioned. The values of ξ also are calculated. It is easily seen that ξ always has the same limits. Graphs are plotted to show the variation of ξ and switching time. The x(time) axis crossover point corresponds to critical clearing time. These curves are given in Fig. 4.3 for various values of inertia

TABLE 4.1. POWER INPUT = 0.800 P.U.

V_{max} = 131.23001

TIME	VEL	DELTA	PT	V	ZETA
0.020000	377.685303	18.461945	0.296421	0.6248	0.995239
0.040000	378.361816	19.646469	0.314686	1.1427	0.991292
0.060000	379.005615	21.588821	0.344358	2.0590	0.984310
0.080000	379.601563	24.242935	0.384270	3.4432	0.973762
0.100000	380.136963	27.546295	0.432793	5.3756	0.959037
0.120000	380.601318	31.423401	0.487905	7.9315	0.939560
0.140000	380.987061	35.788177	0.547273	11.1665	0.914909
0.160000	381.290283	40.547943	0.608388	15.1056	0.884892
0.180000	381.510254	45.607269	0.668743	19.7328	0.849632
0.200000	381.649658	50.872192	0.726019	24.9920	0.809556
0.220000	381.714111	56.253265	0.778225	30.7891	0.765380
0.240000	381.711670	61.668930	0.823839	37.0031	0.718029
0.260000	381.652100	67.048096	0.861867	43.4976	0.668539
0.280000	381.546143	72.331207	0.891826	50.1333	0.617974
0.300000	381.404785	77.471710	0.913702	56.7802	0.567323
0.320000	381.238770	82.435074	0.927849	63.3252	0.517449
0.340000	381.058350	87.199142	0.934881	69.6792	0.469030
0.360000	380.872803	91.752609	0.935562	75.7773	0.422561
0.380000	380.690186	96.094315	0.930709	81.5786	0.378354
0.400000	380.517578	100.232025	0.921110	87.0644	0.336551
0.420000	380.360840	104.180496	0.907471	92.2332	0.297164
0.440000	380.225342	107.960968	0.890373	97.0995	0.260081
0.460000	380.115234	111.600388	0.870252	101.6869	0.225125
0.480000	380.034424	115.130081	0.847382	106.0269	0.192053
0.500000	379.986572	118.585785	0.821875	110.1562	0.160587
0.520000	379.975586	122.007217	0.793676	114.1135	0.130431
0.540000	380.004883	125.438904	0.762552	117.9364	0.101300
0.560000	380.079102	128.929352	0.728086	121.6615	0.072914
0.580000	380.202881	132.532791	0.689675	125.3190	0.045043
0.600000	380.382324	136.309372	0.646489	128.9319	0.017512
0.620000	380.624512	140.326782	0.597472	132.5100	-.009753
0.640000	380.938477	144.661804	0.541290	136.0453	-.036693
0.660000	381.334961	149.402710	0.476314	139.5006	-.063024

TABLE 4.2. PERFORMANCE WITH VARIOUS INPUT POWERS

TIME	VEL	DELTA	PT	V	ZETA
0.020000	377.858887	23.302597	0.370284	1.0036	0.990031
0.040000	378.705322	24.783844	0.392353	1.8119	0.982003
0.060000	379.512207	27.214157	0.428010	3.2388	0.967830
0.080000	380.261963	30.537888	0.475525	5.3883	0.946480
0.100000	380.940186	34.680908	0.532508	8.3789	0.916775
0.120000	381.535645	39.554672	0.595968	12.3174	0.877654
0.140000	382.041992	45.060089	0.662457	17.2798	0.828363
0.160000	382.457520	51.093521	0.728287	23.2908	0.768658
0.180000	382.785645	57.552567	0.789809	30.3156	0.698882
0.200000	383.034668	64.341263	0.843652	38.2608	0.619964
0.220000	383.217041	71.375916	0.886956	46.9840	0.533319
0.240000	383.348877	78.588745	0.917484	56.3106	0.440679
0.260000	383.448730	85.932434	0.933640	66.0528	0.343913
0.280000	383.536865	93.382004	0.934370	76.0250	0.244861
0.300000	383.635010	100.936020	0.918996	86.0588	0.145197
0.320000	383.765381	108.618958	0.886992	96.0030	0.046423
0.340000	383.951172	116.480789	0.837755	105.7222	-.050115

VMAX =100.67679

MECHANICAL POWER INPUT = 1.000 P.U.

MECHANICAL POWER INPUT = 1.200 P.U.

TIME	VEL	DELTA	PT	V	ZETA
0.020000	378.032227	28.314926	0.443979	1.4998	0.979501
0.040000	379.049316	30.093002	0.469302	2.6619	0.963616
0.060000	380.020996	33.012665	0.509910	4.7080	0.935651
0.080000	380.928223	37.010391	0.563367	7.7803	0.893658
0.100000	381.755371	42.003098	0.626263	12.0334	0.835526
0.120000	382.492676	47.892822	0.694328	17.6053	0.759370
0.140000	383.136230	54.573761	0.762634	24.5799	0.664040
0.160000	383.689209	61.939667	0.825916	32.9651	0.549430
0.180000	384.162354	69.892120	0.878910	42.6838	0.416595
0.200000	384.572998	78.348907	0.916697	53.5712	0.267785
0.220000	384.944824	87.251526	0.934921	65.3911	0.106229
0.240000	385.307373	96.571701	0.929850	77.8563	-.064145
0.260000	385.694580	106.318130	0.898279	90.6381	-.238847

VMAX = 73.16322

TABLE 4.2 (continued)

MECHANICAL POWER INPUT = 1.400 P.U.

TIME	VEL	DELTA	PT	V	ZETA
0.020000	378.206055	33.559937	0.517447	2.1471	0.956113
0.040000	379.394287	35.635605	0.545325	3.7258	0.923844
0.060000	380.532959	39.046631	0.589592	6.4969	0.867201
0.080000	381.601807	43.724091	0.646884	10.6384	0.782547
0.100000	382.586182	49.579071	0.712505	16.3403	0.665999
0.120000	383.478516	56.509613	0.780535	23.7555	0.514429
0.140000	384.280029	64.410110	0.844133	32.9561	0.326366
0.160000	385.001953	73.181915	0.895934	43.9012	0.102644
0.180000	385.665527	82.744919	0.928497	56.4171	-.153185

VMAX = 48.92285

TABLE 4.2 (continued)

MECHANICAL POWER INPUT = 1.600 P.U.

TIME	VEL	DELTA	PT	V	ZETA
0.020000	378.379883	39.123871	0.590633	3.0073	0.893801
0.040000	379.740479	41.497620	0.620168	5.0647	0.821146
0.060000	381.048828	45.402695	0.666445	8.6622	0.694104
0.080000	382.284912	50.767349	0.724952	14.0083	0.505311
0.100000	383.436279	57.500549	0.789358	21.3146	0.247297
0.120000	384.500000	65.502716	0.851693	30.7281	-.085132
0.140000	385.485352	74.677399	0.902701	42.2730	-.492831

VMAX = 28.31735

TABLE 4.2 (continued)

TABLE 4.3. PERFORMANCE WITH VARIOUS INERTIA CONSTANTS

THE INERTIA CONSTANT H = 2.500

TIME	VEL	DELTA	PT	V	ZETA
0.020000	377.756348	18.538834	0.297614	0.7018	0.995156
0.040000	378.500244	19.843826	0.317717	1.3362	0.990777
0.060000	379.204346	21.980240	0.350289	2.4640	0.982993
0.080000	379.850586	24.891963	0.393902	4.1762	0.971174
0.100000	380.424072	28.503784	0.446587	6.5760	0.954610
0.120000	380.912598	32.725128	0.505908	9.7543	0.932672
0.140000	381.308350	37.453644	0.569093	13.7729	0.904935
0.160000	381.607910	42.580643	0.633219	18.6473	0.871290
0.180000	381.811768	47.995850	0.695444	24.3366	0.832020
0.200000	381.924805	53.591965	0.753223	30.7469	0.787774
0.220000	381.955566	59.269516	0.804505	37.7409	0.739499
0.240000	381.914795	64.940140	0.847846	45.1500	0.688358
0.260000	381.815186	70.529236	0.882444	52.7967	0.635578
0.280000	381.670166	75.976929	0.908090	60.5089	0.582345
0.300000	381.493164	81.238739	0.925072	68.1351	0.529707
0.320000	381.296631	86.285324	0.934032	75.5522	0.478511
0.340000	381.092285	91.101227	0.935827	82.6713	0.429372
0.360000	380.889893	95.683289	0.931398	89.4344	0.382691
0.380000	380.698486	100.038757	0.921666	95.8147	0.338652
0.400000	380.525146	104.184616	0.907454	101.8089	0.297278
0.420000	380.376221	108.145309	0.889439	107.4344	0.258448
0.440000	380.256836	111.951874	0.868118	112.7220	0.221951
0.460000	380.171875	115.640961	0.843799	117.7133	0.187500
0.480000	380.125732	119.254410	0.816588	122.4550	0.154771
0.500000	380.123047	122.839508	0.786380	126.9974	0.123417
0.520000	380.168213	126.448395	0.752861	131.3878	0.093113
0.540000	380.265846	130.139175	0.715496	135.6716	0.063545
0.560000	380.425049	133.976456	0.673500	139.8844	0.034467
0.580000	380.650635	138.032867	0.625827	144.0504	0.005711
0.600000	380.953369	142.390732	0.571117	148.1742	-.022753
0.620000	381.344971	147.145325	0.507673	152.2300	-.050747

VMAX =144.87784

TABLE 4.3 (continued)

THE INERTIA CONSTANT H = 2.760

TIME	VEL	DELTA	PT	V	ZETA
0.020000	377.684570	18.497559	0.296972	0.6181	0.995290
0.040000	378.360352	19.680908	0.315216	1.1364	0.991341
0.060000	379.003418	21.621262	0.344851	2.0530	0.984355
0.080000	379.598877	24.272537	0.384712	3.4378	0.973803
0.100000	380.133789	27.572525	0.433173	5.3704	0.959077
0.120000	380.597656	31.445633	0.488216	7.9256	0.939605
0.140000	380.983154	35.805969	0.547509	11.1595	0.914962
0.160000	381.286133	40.561066	0.608550	15.0962	0.884964
0.180000	381.506104	45.615601	0.668839	19.7208	0.849723
0.200000	381.645508	50.875626	0.726053	24.9760	0.809678
0.220000	381.709961	56.251846	0.778210	30.7680	0.765542
0.240000	381.707520	61.662750	0.823791	36.9760	0.718235
0.260000	381.647949	67.037216	0.861798	43.4640	0.668795
0.280000	381.541992	72.315796	0.891750	50.0930	0.618281
0.300000	381.400635	77.451599	0.913631	56.7329	0.567684
0.320000	381.234863	82.410278	0.927796	63.2719	0.517855
0.340000	381.054688	87.169922	0.934858	69.6205	0.469477
0.360000	380.869141	91.719193	0.935579	75.7130	0.423051
0.380000	380.686523	96.056702	0.930774	81.5091	0.378883
0.400000	380.513916	100.190216	0.921231	86.9903	0.337116
0.420000	380.357178	104.134262	0.907656	92.1550	0.297760
0.440000	380.221191	107.910294	0.890629	97.0160	0.260718
0.460000	380.110596	111.544662	0.870586	101.5983	0.225800
0.480000	380.029297	115.068680	0.847807	105.9334	0.192765
0.500000	379.980957	118.518250	0.822402	110.0581	0.161335
0.520000	379.968994	121.932846	0.794321	114.0098	0.131221
0.540000	379.997314	125.356461	0.763330	117.8272	0.102132
0.560000	380.070313	128.837494	0.729030	121.5465	0.073790
0.580000	380.192627	132.430054	0.690808	125.1981	0.045965
0.600000	380.370361	136.193893	0.647853	128.8055	0.018475
0.620000	380.610596	140.196457	0.599109	132.3793	-.008757
0.640000	380.922119	144.514069	0.543258	135.9115	-.035674
0.660000	381.315674	149.234543	0.478677	139.3668	-.062003

VMAX =131.23001

TABLE 4.3 (continued)

THE INERTIA CONSTANT H = 3.000

TIME	VEL	DELTA	PT	V	ZETA
0.020000	377.629395	18.465805	0.296478	0.5563	0.995392
0.040000	378.252686	19.555313	0.313285	0.9937	0.991769
0.060000	378.848145	21.344391	0.340650	1.7642	0.985387
0.080000	379.402832	23.793716	0.377575	2.9223	0.975795
0.100000	379.906006	26.850037	0.422688	4.5334	0.962451
0.120000	380.348145	30.448685	0.474259	6.6595	0.944840
0.140000	380.722412	34.515762	0.530288	9.3507	0.922549
0.160000	381.024658	38.970566	0.588588	12.6346	0.895350
0.180000	381.253418	43.729630	0.646938	16.5088	0.863260
0.200000	381.409912	48.709244	0.703214	20.9387	0.826568
0.220000	381.498047	53.828491	0.755530	25.8593	0.785812
0.240000	381.523682	59.012360	0.802366	31.1804	0.741738
0.260000	381.494629	64.193359	0.842616	36.7961	0.695224
0.280000	381.419434	69.313904	0.875632	42.5930	0.647209
0.300000	381.307373	74.326218	0.901181	48.4603	0.598611
0.320000	381.167969	79.193512	0.919394	54.2983	0.550256
0.340000	381.010010	83.889542	0.930680	60.0215	0.502852
0.360000	380.842041	88.398163	0.935634	65.5647	0.456938
0.380000	380.671387	92.712051	0.934951	70.8803	0.412910
0.400000	380.504639	96.832123	0.929352	75.9393	0.371007
0.420000	380.347656	100.766296	0.919520	80.7298	0.331328
0.440000	380.205566	104.528427	0.906063	85.2538	0.293857
0.460000	380.082275	108.138306	0.889477	89.5227	0.258498
0.480000	379.981689	111.619507	0.870137	93.5573	0.225080
0.500000	379.906982	115.000061	0.848283	97.3833	0.193389
0.520000	379.861084	118.311142	0.824015	101.0298	0.163186
0.540000	379.847412	121.587723	0.797293	104.5289	0.134204
0.560000	379.868896	124.868500	0.767922	107.9109	0.106191
0.580000	379.928955	128.195679	0.735566	111.2053	0.078905
0.600000	380.031982	131.615784	0.699719	114.4386	0.052124
0.620000	380.182373	135.180756	0.659704	117.6301	0.025689
0.640000	380.386230	138.948013	0.614645	120.7923	-.000503
0.660000	380.650391	142.982666	0.563446	123.9232	-.026435
0.680000	380.983398	147.358566	0.504768	127.0024	-.051940

VMAX =120.73155

TABLE 4.3 (continued)

THE INERTIA CONSTANT H = 3.500

TIME	VEL	DELTA	PT	V	ZETA
0.020000	377.538574	18.413635	0.295668	0.4593	0.995562
0.040000	378.074707	19.348648	0.310103	0.7789	0.992473
0.060000	378.590332	20.887161	0.333689	1.3378	0.987072
0.080000	379.075928	23.000214	0.365691	2.1719	0.979013
0.100000	379.522949	25.648361	0.405098	3.3239	0.967880
0.120000	379.924072	28.783203	0.450625	4.8381	0.953248
0.140000	380.273438	32.348465	0.500759	6.7526	0.934747
0.160000	380.567139	36.282455	0.553829	9.0948	0.912114
0.180000	380.802979	40.519943	0.608068	11.8749	0.885249
0.200000	380.980469	44.994247	0.661725	15.0828	0.854250
0.220000	381.101318	49.639130	0.713160	18.6891	0.819402
0.240000	381.168457	54.391464	0.760935	22.6444	0.781180
0.260000	381.186768	59.192291	0.803885	26.8865	0.740187
0.280000	381.161865	63.988663	0.841162	31.3421	0.697132
0.300000	381.100098	68.734818	0.872251	35.9341	0.652758
0.320000	381.008301	73.392227	0.896942	40.5868	0.607797
0.340000	380.893311	77.930634	0.915303	45.2307	0.562921
0.360000	380.761963	82.327133	0.927617	49.8057	0.518712
0.380000	380.620361	86.566650	0.934319	54.2627	0.475642
0.400000	380.474365	90.640869	0.935941	58.5649	0.434069
0.420000	380.329102	94.547836	0.933052	62.6877	0.394230
0.440000	380.188965	98.290924	0.926215	66.6161	0.356267
0.460000	380.058105	101.878357	0.915953	70.3467	0.320217
0.480000	379.939941	105.322800	0.902720	73.8829	0.286046
0.500000	379.837402	108.640564	0.886890	77.2343	0.253661
0.520000	379.753174	111.851089	0.868740	80.4157	0.222918
0.540000	379.689697	114.976822	0.848448	83.4453	0.193642
0.560000	379.649170	118.042725	0.826091	86.3432	0.165639
0.580000	379.634033	121.076294	0.801643	89.1307	0.138702
0.600000	379.646240	124.108170	0.774962	91.8288	0.112629
0.620000	379.688477	127.170959	0.745807	94.4579	0.087224
0.640000	379.763672	130.300659	0.713815	97.0365	0.062306
0.660000	379.875000	133.536987	0.678493	99.5799	0.037728
0.680000	380.026367	136.923370	0.639216	102.0986	0.013389
0.700000	380.222656	140.508316	0.595208	104.5970	-.010754
0.720000	380.469238	144.346527	0.545511	107.0687	-.034637
0.740000	380.772949	148.499390	0.488989	109.4944	-.058078

VMAX =103.48418

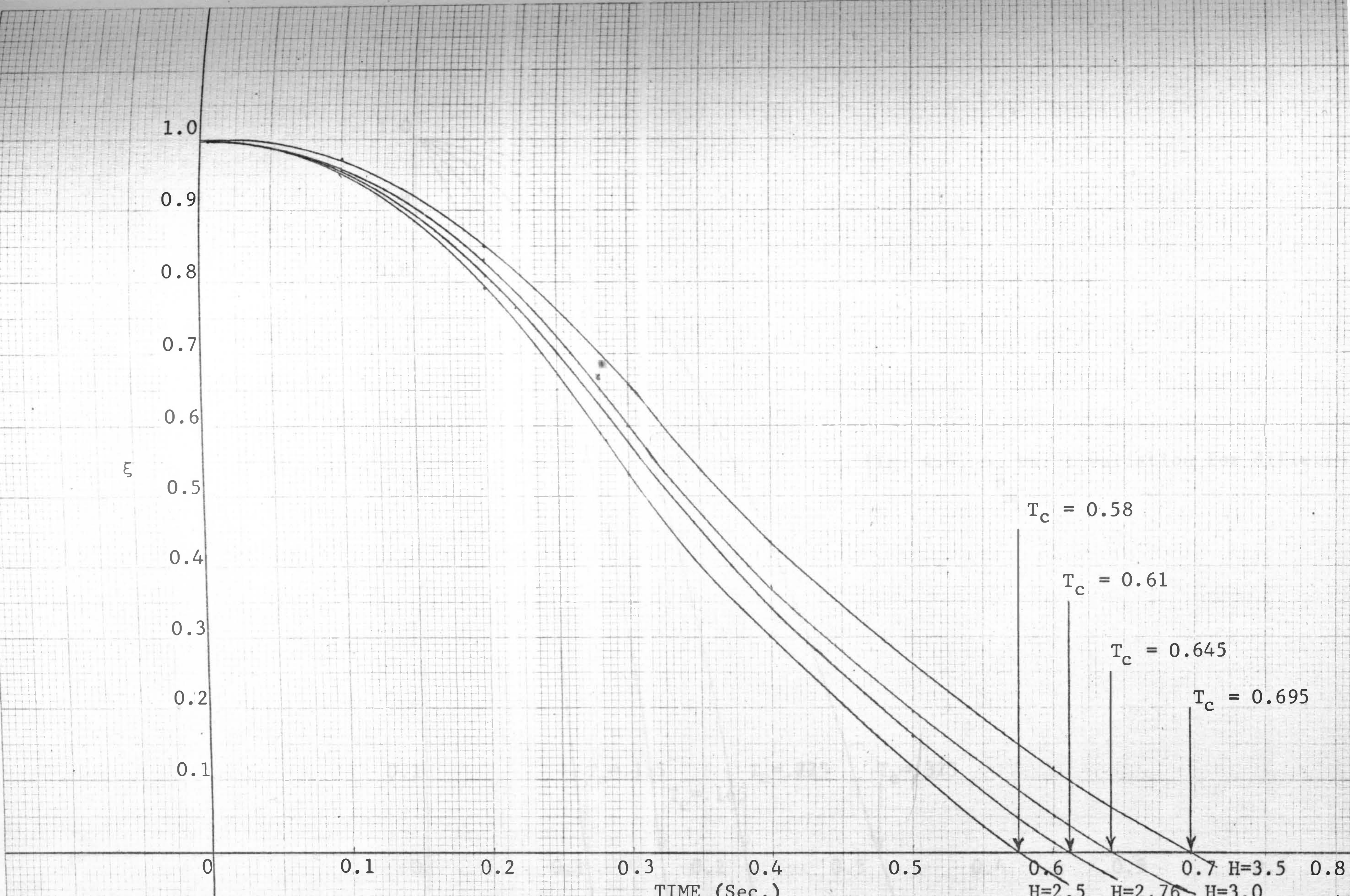


Fig. 4.3. ξ vs. t for various values of inertia constants.

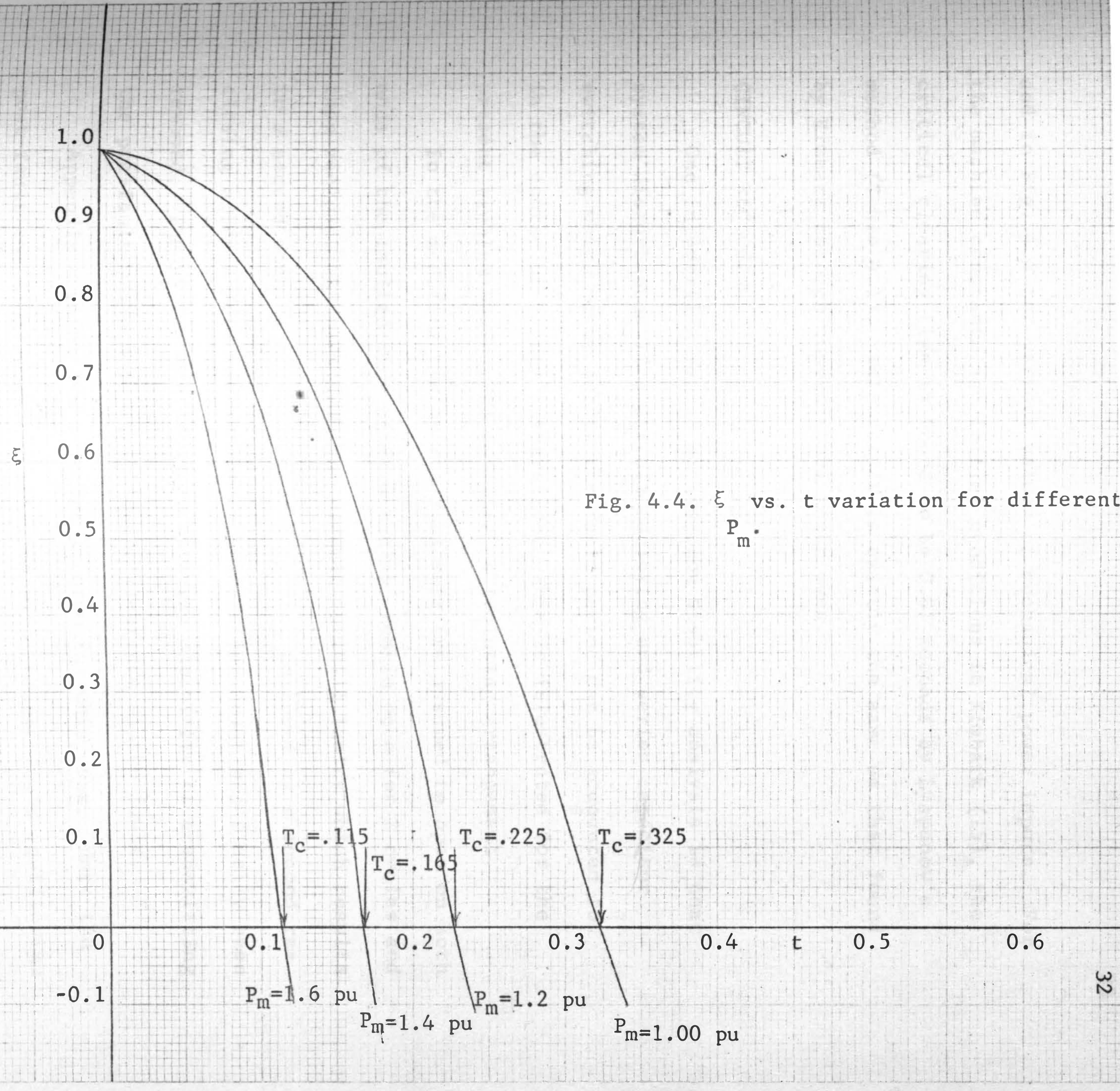


Fig. 4.4. ξ vs. t variation for different P_m.

and in Fig. 4.4 for various values of mechanical power inputs. For the machine constants and initial conditions in Kimbark (15), the critical clearing time is found to be 0.61 seconds by Lyapunov's method (Table 4.1). This result is exactly the same as that found by Kimbark through conventional techniques.

CIRCUIT RECLOSING

One interesting aspect of Lyapunov stability analysis is that system disturbances like circuit reclosing and series capacitor switching which occur during the fault period can be accounted for in the course of system integration. This is illustrated for the present problem having, in addition, a reclosing arrangement.

In the modified system the reclosers are assumed to open on both ends of the faulted line after 4 cycles, remain open for 2 cycles and then reclose. At this time the circuit is still faulty which results in a second opening after another 2 cycles, followed by a second reclosing after 2 more cycles when the transient fault has finally been removed. The system continues to have the capability of transmitting the pre-fault maximum power following clearance of the fault.

Appendix C gives the computer program for this case. Here the post-fault equilibrium states coincide with the pre-fault equilibrium states. Results are tabulated in Table 4.4. The system was found to be stable. A mechanical input of 1.4 p.u. was used. Referring to Table 4.2, without a reclosing arrangement, the critical clearing time for this system was found to be 0.17 second. However, with the reclosing scheme used, at the previous clearing time of 0.17 second,

TABLE 4.4. PERFORMANCE WITH RECLOSING ARRANGEMENT

MECHANICAL POWER INPUT = 1.400 P.U.

TIME	VEL	DELTA	PT	V	ZETA
0.010000	377.600586	33.037537	0.510298	0.1271	0.997215
0.020000	378.205811	33.559891	0.517429	0.5112	0.988802
0.030000	378.804688	34.427292	0.529174	1.1607	0.974574
0.040000	379.394043	35.635300	0.545332	2.0896	0.954227
0.050000	379.970947	37.177490	0.565606	3.3156	0.927371
0.060000	380.532715	39.046036	0.589620	4.8604	0.893531
0.070000	380.583252	41.091003	1.353901	4.8602	0.893537
0.080000	380.595703	43.154007	1.408916	4.8597	0.893547
0.090000	380.571289	45.213470	1.462013	4.8596	0.893549
0.100000	380.511475	47.248749	1.512635	4.8597	0.893547
0.110000	380.990234	49.403152	0.710706	7.0152	0.846331
0.120000	381.452393	51.827209	0.735831	9.5248	0.791357
0.130000	381.896973	54.511032	0.762111	12.4022	0.728328
0.140000	381.688232	57.263367	1.732763	12.4020	0.728331
0.150000	381.443604	59.885590	1.781923	12.4011	0.728352
0.160000	381.167725	62.358490	1.824864	12.4010	0.728354
0.170000	380.547119	64.574646	2.330085	11.1721	0.755274

VMAX = 45.65135

the stability measure ξ was equal to 0.76, thus indicating a large amount of stable region ahead.

Part 2.

WALL'S ENERGY METRIC ALGORITHM

The V function used in the previous analysis as derived by Willems' conjecture can be obtained also by the technique proposed by Wall and Moe (12). Accordingly, from equation (3.1), with $x_1 = \delta - \delta^s$ and $\dot{x}_1 = x_2$,

$$\dot{x}_2 = \frac{-1}{M} (ax_2 + P_e \sin(x_1 + \delta^s) - P_m) \tag{4.4}$$

$$\frac{dx_1}{dx_2} = \frac{x_2}{\frac{-1}{M} (ax_2 + P_e \sin(x_1 + \delta^s) - P_m)}$$

Also, neglecting damping and after cross-multiplying,

$$x_2 dx_2 + \frac{1}{M} (P_e \sin(x_1 + \delta^s) - P_m) dx_1 = 0$$

$$V = \int_o^{x_1} \left(\frac{P_e}{M} \sin(\tau_1 + \delta^s) - \frac{P_m}{M}\right) d\tau_1 + \int_o^{x_2} \tau_2 \, d\tau_2 \tag{4.5}$$

where τ_1 and τ_2 are dummy variables.

Integrating,

$$V = \frac{x_2^2}{2} + \frac{1}{M} (-P_m x_1 - P_e \cos(x_1 + \delta^s) + P_e \cos \delta^s) \tag{4.6}$$

It is readily observed that the Lyapunov functions are equivalent, differing only by a constant factor. When computing the critical clearing time, this constant factor appears both in the V function and $\dot{V}_{max}$ function and hence leads to identical results.

CHAPTER V

ANALYSIS OF AN ACTUAL POWER SYSTEM

In order to establish the effectiveness of Lyapunov's direct method, a multimachine power system is studied in this chapter. The layout of a 60 Hz. three-phase, three-machine system is given in Figure 5.1 (15). The reactances of the lines, expressed in p.u. on a 100 MVA base, are marked on the figure. Line resistances are neglected. Data on the generators and the initial generating station outputs and bus voltages are given in Table 5.1. All loads except those at Murphy and Wieboldt, which are each 200 MW at unity p.f., are neglected. The problem consists of the analysis of the system transient stability when a three-phase short circuit occurs at the point X in Fig. 5.1.

The first step in the analysis is the necessary network reduction of the pre-fault system by a series of star-mesh conversions. The initial operating voltages behind the transient reactances and their angles are calculated. (In the study of a large power system a load flow study prior to the fault will give all these values.) The fault at X in Fig. 5.1 is equivalent to a short circuit on the Patten bus. The application of the short circuit can be represented by connecting this node to the neutral. Similarly, the post-fault conditions correspond to simultaneous opening of the circuit breakers at both ends of the faulty line thus isolating that line from the rest of the system. The fault and post-fault power relations are given in equations 5.1 and 5.2 (15).

POWER EQUATIONS

i Fault condition

$$P_{ia} = 0.10\cos(86.0 - \delta_b + \delta_a) + 0.1\cos(86.7 - \delta_a + \delta_b) \quad (5.1)$$

$$P_{ib} = 0.24 + 0.10\cos(86.0 - \delta_b + \delta_a) + 0.68\cos(84.7 - \delta_b + \delta_c)$$

$$P_{ic} = 0.09 + 0.10\cos(86.7 - \delta_c + \delta_a) + 0.68\cos(84.7 - \delta_c + \delta_b)$$

ii Post-fault condition

$$P_{ia} = 0.04 + 1.3\cos(79.5 - \delta_a + \delta_b) + 0.59\cos(79.2 - \delta_a + \delta_c)$$

$$P_{ib} = 1.61 + 1.3\cos(79.5 - \delta_b + \delta_a) + 3.09\cos(77.4 - \delta_b + \delta_c) \quad (5.2)$$

$$P_{ic} = 0.33 + 0.59\cos(79.2 - \delta_c + \delta_a) + 3.09\cos(77.4 - \delta_c + \delta_b)$$

The fault and post-fault system admittance matrices needed for the study were calculated and are tabulated (15):

i Faulted System Admittance matrix

	A	B	C
A	0.00 - j 1.84	0.00 + j 0.086	0.00 + j 0.086
B	0.00 + j 0.086	0.24 - j 10.13	0.062 + j 0.668
C	0.00 + j 0.086	0.062 + j 0.68	0.09 - j 4.66

ii Post-fault System Admittance matrix

	A	B	C
A	0.03 - j 1.66	0.205 + j 1.10	0.093 + j 0.493
B	0.205 + j 1.10	1.58 - j 4.54	0.66 + j 2.98
C	0.093 + j 0.493	0.66 + j 2.98	0.33 - j 3.68

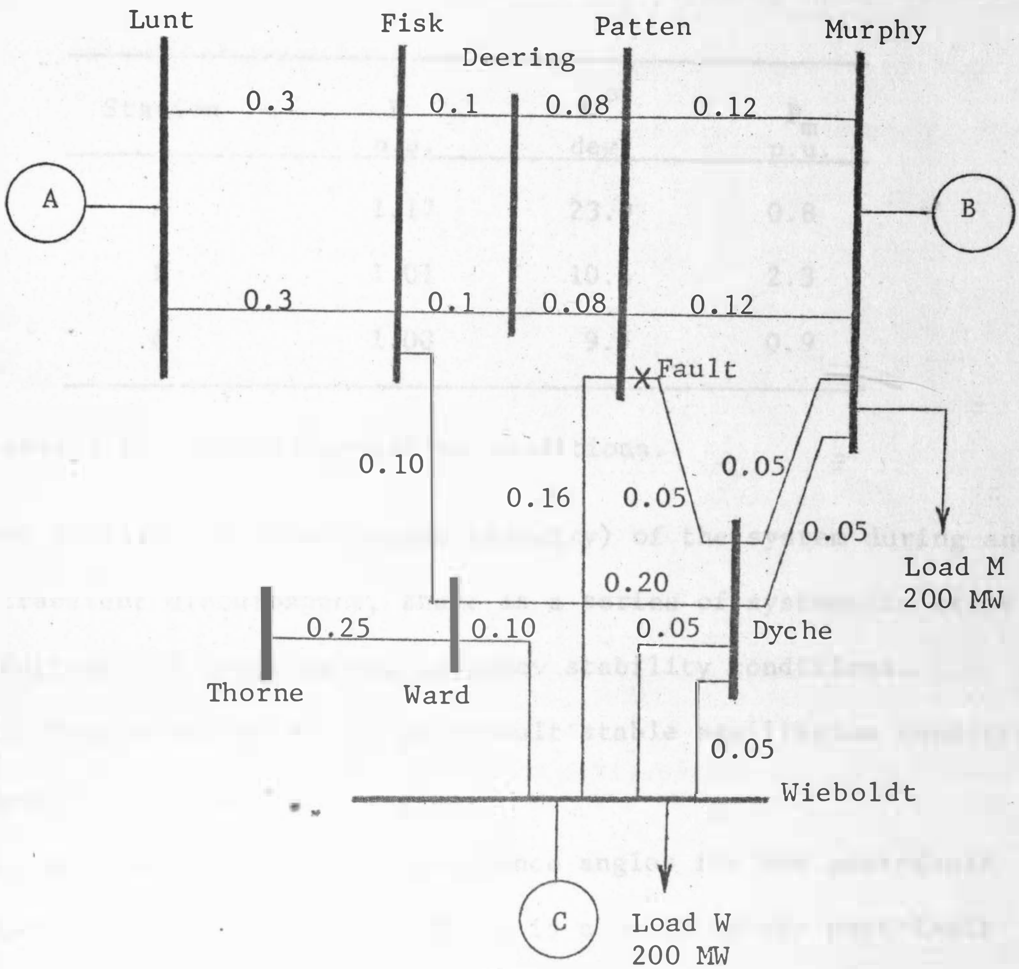


Fig. 5.1. A Three-Machine System. Line reactances are given on a 100 MVA base.

Station	Type	No. of units	Rating/u. M.V.A.	x_d' (p.u.)	H Mj/MVA	Initial Output (MW)	Initial Voltage (p.u.)
Lunt	Hydro	3	35	0.35	3.00	80	1.05
Murphy	Steam	4	75	0.21	7.00	230	1.00
Wieboldt	Steam	2	50	0.18	8.00	90	1.00

Table 5.1. Data for the system shown in Fig. 5.1.

The initial operating conditions are tabulated in Table 5.2 (15). Having thus established the system conditions before the fault and the

Station	E p.u.	δ^{o} deg.	P_m p.u.
A	1.17	23.0	0.8
B	1.01	10.4	2.3
C	1.00	9.5	0.9

Table 5.2. Initial operating conditions.

behavior (related to power output capacity) of the system during and after transient disturbances, there is a series of systematic steps to be followed in applying the Lyapunov stability conditions.

1. Determination of the post-fault stable equilibrium conditions, specifically the rotor angles.

2. Determination of the divergence angles for the post-fault unstable equilibrium state which is closest to the post-fault stable condition.

3. Estimation of the stability domain or, in other words, calculation of the least maximum value of the V function.

4. Forward step-by-step integration of the faulted system to find the critical clearing time.

DETERMINATION OF THE POST-FAULT EQUILIBRIUM STATES.

Determination of the post-fault equilibirium states is the most crucial part in the whole analysis and a proper evaluation of the equilibrium angles is imperative for the success of the method. The equilibrium state of the post-fault system is found by the steepest descent method using Booth's interpolation formula (16).

The post-fault power relations given in equation 5.2 can be written in the form

$$f_i(\delta_1, \delta_2, \ldots \delta_n) - P_{mi} = 0; \quad i = 1, 2 \ldots n \tag{5.3}$$

Define an index of performance

$$\Phi = \sum_{i=1}^{n} (f_i - P_{mi})^2 \tag{5.4}$$

The function Φ has a minimum at the solutions of 5.3 and this minimum value is zero. There are, however, several minima and the method of steepest descent converges only to the local minimum, thus depending heavily on the starting values for iteration. This simply indicates the nonlinear nature of the system equations. As a first guess the stable post-fault equilibrium is chosen to be the pre-fault stable equilibrium, say, $(\delta_1^o, \delta_2^o, \ldots \delta_n^o)$. All co-ordinates are then incremented by a factor ε_r given by

$$\varepsilon_r = -\Phi(o)\, \Phi_r / \sum_{r=1}^{n} (\Phi_r)^2 \tag{5.5}$$

where $\Phi(o)$ is the value of Φ at the initial point and

$$\Phi_r = (\partial\Phi/\partial\delta_r)_o \quad (r = 1, 2, \ldots n) \tag{5.6}$$

i.e., the gradient of the index of performance at the initial conditions.

Increments are calculated as in 5.5. Φ is evaluated at the point given by applying these increments ($\Phi(1)$, say) and also at the point given by applying one-half of these increments say, $\Phi(1/2)$. The final corrections on increments are found by Booth's interpolative formula:

$$\varepsilon_r = -\frac{1}{4} \frac{\Phi(1) - 4\Phi(1/2) + 3\Phi(o)}{\Phi(1) - 2\Phi(1/2) + \Phi(o)} \frac{\Phi(o)\ \Phi_r}{\sum_{r=1}^{n} (\Phi_r)^2} \tag{5.7}$$

There is a danger, however, that the solution might tend to converge to a col instead of a valley, but knowing the minimum value of the index of performance to be zero, a check can easily be incorporated into the program.

El-Abiad and Nagappan (4) discuss some optimum starting values to find the unstable equilibrium state. Unstable equilibrium points are also found by a random search together with the method of steepest descent. Several unstable equilibrium points are found in this way and the one giving the least maximum value of Lyapunov function is chosen. The computer program is given in Appendix E and the results are tabulated in Table 5.3. The method of steepest descent converges rapidly to the local minimum.

SYSTEM STABILITY

The systems of differential equations are numerically integrated along the fault trajectory using Runge-Kutta fourth-order approximation. For each iteration the value of the Lyapunov function also is calculated.

TABLE 5.3. POST-FAULT CONDITIONS

(a) STABLE

$\delta_a = 23.207^o$ $\delta_b = 10.469^o$ $\delta_c = 9.223^o$

(b) UNSTABLE

δ_a^o	δ_b^o	δ_c^o	V_{max}
356.379	-16.165	-17.415	-3.828
174.479	161.952	160.667	-5.330
-81.178	266.217	-95.047	-0.652
83.262	69.294	67.634	-2.0847
-47.960	-60.560	-61.820	2.502
-27.960	-40.560	-41.780	1.798

The Lyapunov function developed by El-Abiad and Nagappan (4) is chosen in order to include the effect of transfer conductances which are specifically given in the problem.

Accordingly,

$$V(\delta_1, \delta_2, \ldots \delta_n, \omega_1, \omega_2, \ldots \omega_n) = \sum_{k=1}^{n} [1/2\, M_k \omega_k^2 + (E_k^2 G_{kk} - P_{ik})$$

$$(\delta_k - \delta_k^s)] + \sum_{k=1}^{n-1} \sum_{j=k+1}^{n} E_k E_j [B_{kj} \{\cos(\delta_k^s - \delta_j^s) - \cos(\delta_k - \delta_j)\}$$

$$+ G_{kj} \{\sin(\delta_k^s - \delta_j^s) - \sin(\delta_k - \delta_i)\}] \qquad (5.8)$$

The maximum value of V is $V(\delta_1^{\,u}, \delta_2^{\,u}, \ldots \delta_n^{\,u})$ obtained by substituting the co-ordinates of the unstable equilibrium state into equation 5.8. The maximum value of V is obtained as 1.798.

The computer program which includes Runge-Kutta fourth-order approximate integration for the three-machine system and calculation of V function at every increment of time is given in Appendix F. Table 5.4 gives the results of this study: the powers and power angles of different machines as time progresses along the fault trajectory. From the tabulated results, the critical clearing time is found to be 0.27 seconds. This result is a conservative figure based on an estimate from Kimbark (15).

TABLE 5.4.

RUNGE-KUTTA FOURTH-ORDER APPROXIMATION

TIME	DELTA-A	VEL-A	POWER-A	DELTA-B	VEL-B	POWER-B	DELTA-C	VEL-C	POWER-C	V	VMAX
0.02000	23.50748	377.87524	0.06047	10.60562	377.34888	0.29680	9.70903	377.35498	0.12305	0.0099	1.7982
0.04000	25.02805	378.75732	0.06542	11.22259	377.70825	0.29401	10.33653	377.72070	0.12072	0.0275	1.7982
0.06000	27.55659	379.63403	0.07223	12.25165	378.06787	0.29014	11.38369	378.08740	0.11752	0.0577	1.7992
0.08000	31.08580	330.50293	0.08079	13.69314	378.42822	0.28520	12.85184	378.45557	0.11353	0.1020	1.7982
0.10000	35.60553	381.36206	0.09091	15.54797	378.78955	0.27927	14.74276	378.82544	0.10881	0.1622	1.7982
0.12000	41.10339	382.20947	0.10239	17.81744	379.15186	0.27242	17.05869	379.19751	0.10347	0.2407	1.7982
0.14000	47.56487	383.04346	0.11492	20.50272	379.51538	0.26475	19.80232	379.57202	0.09767	0.3400	1.7982
0.16000	54.97386	363.86279	0.12817	23.60533	379.88013	0.25641	22.97665	379.94922	0.09159	0.4631	1.7982
0.18000	63.31303	384.66675	0.14173	27.12668	380.24634	0.24756	26.58478	380.32910	0.08541	0.6130	1.7982
0.20000	72.56432	385.45459	0.15511	31.06854	380.61426	0.23842	30.62988	380.71191	0.07940	0.7927	1.7982
0.22000	82.70921	386.22656	0.16778	35.43295	380.98364	0.22922	35.11533	381.09766	0.07381	1.0049	1.7982
0.24000	93.72978	386.98364	0.17917	40.22162	381.35474	0.22027	40.04430	381.48584	0.06893	1.2519	1.7982
0.26000	105.60974	387.72705	0.18866	45.43642	381.72754	0.21185	45.41949	381.87646	0.06506	1.5356	1.7982
0.28000	118.33456	388.45898	0.19563	51.07935	382.10181	0.20432	51.24356	382.26880	0.06253	1.8569	1.7982
0.30000	131.89249	389.18213	0.19945	57.15193	382.47729	0.19801	57.51799	382.66260	0.06163	2.2160	1.7982
0.32000	146.27536	389.90039	0.19951	63.65547	382.85400	0.19327	64.24408	383.05688	0.06268	2.6119	1.7982
0.34000	161.48015	390.61792	0.19526	70.59132	383.23169	0.19045	71.42188	383.45068	0.06596	3.0426	1.7582
0.36000	177.50861	391.33984	0.18621	77.96022	383.60986	0.18989	79.05042	383.84302	0.07170	3.5047	1.7982
0.38000	194.36897	392.07178	0.17194	85.76274	383.98828	0.19190	87.12735	384.23291	0.08012	3.9935	1.7982
0.40000	212.07607	392.81982	0.15217	93.99878	384.36621	0.19672	95.64917	384.61914	0.09138	4.5027	1.7982

Note: Time is in seconds, velocities are in radians/sec., power is in per unit and angles are in degrees.

CHAPTER VI

CONCLUSIONS

Conventional methods of stability analysis involve some form of numerical integration of the system differential equations and plotting of the swing curves for a range of fault clearing times. This technique is advantageous in that any mathematical model can be taken into account without much difficulty, but there are three main disadvantages:

1. To find the boundary of the transient stability region the equations must be integrated for many initial conditions. This is an expensive and time-consuming task.

2. The procedure must be carried out separately for each numerical example. This is particularly important at the design level where system parameters are frequently changed. For design purposes it would be preferable to have analytical expressions (even if conservative) for the stability boundary in terms of the system parameters.

3. For large interconnected power systems extremely large computer facilities are necessary to accurately represent the high voltage grid. Because of large inertias and long ties in such power systems, swings must be calculated for several seconds, thus involving extensive computer time.

Direct methods like that of Lyapunov stability analysis are the answer to these problems. It should be emphasized, however, that direct methods do not exclude simulation. Lyapunov's method instead of extended simulation, substitutes computation of the transient stability region of the post-fault system. However, extensive post-fault simulation is the most crucial and time-consuming part of stability analysis by conventional computer methods.

Summarizing, the chief advantage of using Lyapunov's direct method in power system stability analysis is that one need not make trials for the values of critical clearing time for a given fault type and location. Alternatively, if the clearing time is already fixed, i.e the circuit breaker has been selected, it is not necessary to calculate the swing curve data beyond the switching time. This advantage is realized because by calculating the value of the V function for the state variables at the clearing time and comparing it with V_{max}, it is possible to draw conclusions about stability. Furthermore, in the conventional technique, prediction of stability or instability depends on studying the behavior of the swing curve over a small initial period, which in some cases may give false results. For example, it may happen that initially the swing curve indicates a tendency toward stability and one may conclude that the respective machine is stable, whereas the later uncomputed part of the swing curve might have indicated instability. Such a situation cannot arise with Lyapunov's method because satisfaction of this criterion predicts asymptotic stability.

The expressions for V functions afford a more quantitative indication of the effects on the system stability for different parameter variations. The stability measure ξ, defined and determined for various conditions in Chapter IV, is an example of this feature. From the results of this study it is shown that the critical clearing time increases as the inertia of the machine is increased and decreases as the mechanical power input to the machine increases. Such a study can be conducted for the variation of any other significant system parameter and is obviously a very useful aid in design. Conventional techniques would have resulted in an excessive amount of computer time for an equivalent study.

Willems' conjecture and the Lyapunov functions derived in Chapter III result in a considerably larger stability domain than what was obtained by using the function suggested by El-Abiad and Nagappan (4). However, the former functions fail to include the effects of transfer conductance.

One interesting but laborious aspect of Lyapunov's method when analyzing multimachine power systems is the evaluation of the post-fault stable and unstable equilibrium states. The method of steepest descent for solution and minimization of simultaneous nonlinear equations(16), though published as early as 1949 and not cited in standard texts on optimization, was found to be an excellent technique for evaluating the local minima. The method is of immense value when seeking local minima in simultaneous trigonometric equations where a large number of solutions exist. The rate of convergence depends somewhat on the choice of initial conditions. In implementing this technique the equilibrium point closest to the pre-fault initial state was taken to be the stable equilibrium state. The unstable equilibrium state was evaluated by a random search.

The critical clearing time obtained for the single-machine system is exact while that for the multimachine system seems rather conservative. One possible reason is that the particular V function chosen relies heavily on the total energy of the system as viewed from a synchronously rotating reference and does not consider another significant aspect, the distribution of this energy in the system. Additional terms describing the effects of the distribution of energy may be used to improve the Lyapunov function to yield a more accurate figure for the critical clearing time.

An alternative to the cut-and-try method would be desirable for evaluating post-fault states. El-Abiad[d] and Nagappan considered an approximate initial guess by taking two machines at a time, finding the stable and unstable states for them and using these values for further iterations using the method of steepest descent. Luders (17) also investigated this problem, but his considerations were very general and conclusions indefinite. Apparently this is a promising area of research and also the most significant in Lyapunov stability analysis of power systems at present.

There are many occurrences in a time sequence of events following a disturbance, such as circuit reclosing, generator dropping or series capacitor switching. Most of these disturbances can be taken care of in a straight-forward manner during the course of numerical integration along the fault trajectory. The case of circuit reclosing is illustrated in Chapter IV and is shown to improve the stability measure at the time considered from a zero value to more than 0.7.

The numerical integration of the system differential equations was carried out by Runge-Kutta fourth-order approximation. The selection was made on the basis of convenience in computation and maximum accuracy of results. The time interval was taken to be 0.02 second when the transient period was not subjected to a series of disturbances as in the case where automatic reclosing was employed. With the introduction of automatic reclosing the time interval was reduced to 0.01 second to improve accuracy. The results of swing curve data agree with corresponding values obtained through conventional techniques by Kimbark (15), for the swing along the fault trajectory.

An absolutely accurate representation of a power system involves voltage regulators, governors, relaying schemes, phase shifters and other control devices. Thus, the system model no longer appears simple and in several cases becomes nonautonomous. Lyapunov stability analysis for nonautonomous nonlinear systems is complex and not well-developed. Generation of optimum Lyapunov functions is a potentially important area of research. However, at the present state of the art, there is no doubt that Lyapunov stability evaluation is a very promising and applicable technique when a somewhat conservative result is acceptable. The Lyapunov technique may be preferred over laborious and expensive digital simulations, especially at the design level, and it may be reasonable to foresee, within the near future, the partial or complete replacement of conventional techniques by this new direct method.

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APPENDICES

APPENDIX A

EXAMPLE OF A THREE-MACHINE MODEL

For a three-machine system described by the equivalence of the state vector differential equation (3.4) and the system differential equation (3.1), the vectors and matrices become:

$$A = \begin{bmatrix} -a_1/M_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -a_2/M_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -a_3/M_3 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$CX(t) = \sigma = \begin{bmatrix} Z_1 - Z_2 \\ Z_1 - Z_3 \\ Z_2 - Z_3 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} E_1E_2B_{12}/M_1 & E_1E_3B_{13}/M_1 & 0 \\ -E_1E_2B_{12}/M_2 & 0 & E_2E_3B_{23}/M_2 \\ 0 & -E_1E_3B_{13}/M_3 & -E_2E_3B_{23}/M_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

APPENDIX B

a. CONSTRUCTION OF LYAPUNOV FUNCTIONS

Suppose N is a positive definite diagonal (mxm) matrix, Q a positive semidefinite diagonal (mxm) matrix, C an (mxn) matrix, B an (nxm) matrix, A an (nxn) matrix and I the (nxn) identity matrix. Then there exists a positive definite symmetric matrix P satisfying the equations,

$$\begin{aligned} PA + A^T P &= -LL^T \\ PB &= C^T N - LW + A^T C^T Q \\ W^T W &= QCB + B^T C^T Q \end{aligned} \tag{B.1}$$

where W and L are auxiliary square matrices. The above theorem has been proven by Anderson (9) and is used to compute Lyapunov functions for the systems defined by equation (3.4). A useful Lyapunov function for this system from Anderson's theorem is

$$V(X) = X^T P X + 2\int_0^{CX} f(\sigma)^T Q \, d\sigma \tag{B.2}$$

which is positive definite if $\sigma_i \; f_i(\sigma_i) \geq 0$ for all σ_i $(i = 1, 2, \ldots m)$.

The derivative of the V function for the solutions of the system equation (3.4) is

$$\dot{V}(X) = \dot{X}^T PX + X^T P\dot{X} + 2\, f(CX)^T C\dot{X} \tag{B.3}$$

Substituting from (3.4) and (B.1),

$$\dot{V}(X) = -\left\{X^T L - f(CX)^T W^T\right\}\left\{L^T X - Wf(CX)\right\} - 2X^T C^T N f(CX) \quad (B.4)$$

and is negative semidefinite if the same conditions on the nonlinearity hold. In the particular case that $N = 0_{nm}$,

$$\dot{V}(X) = -\left\{X^T L - f(CX)^T W^T\right\}\left\{L^T X - Wf(CX)\right\} \quad (B.5)$$

is negative semidefinite for any nonlinearity $f(\sigma)$.

b. SOLUTION OF PARTICULAR MATRIX EQUATION

Consider the equation

$$YD^T = 0_{nm} \quad (B.6)$$

where Y is an unknown symmetric (nxn) matrix, D is an (mxn) matrix, as defined in Chapter III, and 0_{nm} is an (nxm) matrix with all zero elements. The matrix D^T contains $m = n(n-1)/2$ columns and each column contains only two non-zero elements (Appendix A), + 1 on the i^{th} row and -1 on the j^{th} row, so that any (i,j) pair is included. Then, all the elements on the same row of Y are equal. Moreover, since Y is symmetric, it follows that all elements of Y are equal. Hence, a necessary and sufficient condition for a symmetric matrix to be the solution of (B.6) is that the matrix has the form $Y = \mu 1$ where μ is a scalar constant and 1 is an nxn matrix with all elements equal to 1.

APPENDIX C

PERFORMANCE OF THE SECOND ORDER POWER SYSTEM

```
      REAL M,K,L
      DIMENSION K(4),L(4)
      H=2.76
      PM=0.8
      KK=0
   77 KK=KK&1
      IF(KK.EQ.7) GO TO 111
      WRITE(12,177)PM
  177 FORMAT(1H1,12X,26H MECHANICAL POWER INPUT = ,1F5.3,5H P.U.)
      WRITE(12,15)
   15 FORMAT(1H0,12X,4HTIME,10X,3HVEL,10X,5HDELTA,8X,2HPT,12X,1HV,8X,4HZ
     CETA)
      A=2.*3.1416*60.
      T=0.
      W=A
      C1=2.58
      C2=0.936
      C3=2.06
      DEL=ATAN((PM/C1)/(SQRT(1.-(PM/C1)**2)))
      DELO=DEL
C     DT=0.02
   40 J=1
      M=H/(3.1416*60.)
      T=T&DT
      IF(ABS(T-0.58)).LE.0.01) GO TO 72
      T=T&DT
      GO TO 44
   72 T=0.6000
```

```
44 IF(T.GT.1.) GO TO 100
   F=W
   DELZ=DEL
   K(J)=(W-A)*DT
   GO TO 60
61 L(J)=(1./M)*(PM-PT)*DT
   DEL=DELZ&K(J)/2.
   N=J
   J=2
   K(J)=((W&L(N)/2.)-A)*DT
   GO TO 60
63 L(J)=(1./M)*(PM-PT)*DT
   DEL=DELZ&K(J)/2.
   N=J
   J=3
   K(J)=((W&L(N)/2.)-A)*DT
   GO TO 60
65 L(J)=(1./M)*(PM-PT)*DT
   DEL=DELZ&K(J)/1.
   N=J
   J=4
   K(J)=((W&L(N)/1.)-A)*DT
   GO TO 60
67 L(J)=(1./M)*(PM-PT)*DT
   DEL=DELZ&(1./6)*(K(1)&2.*K(2)&2.*K(3)&K(4))
   DEG=DEL*180./3.1416
   W=F&(1./6.)*(L(1)&2.*L(2)&2.*L(3)&L(4))
   D2=ATAN((PM/C3)/SQRT(1.-(PM/C3)**2))
   X=DEL-D2
   Y=3.1416-D2*2.
   U=W-A
   V=(U**2)/2.&1./M*(-PM*X-C3*COS(X&D2)&C3*COS(D2))
   VMAX=(1./M)*(-PM*Y-C3*COS(Y&D2)&C3*COS(D2))
C     A STABILITY MEASURE IS DEFINED AS ZETA=(1-V/VMAX)
C     DIFFERENT VALUES OF ZETA ARE COMPUTED FOR DIFFERENT VALUES OF TIME
C     AND DIFFERENT VALUES OF MECHANICAL POWER INPUT
```

```
C     ZETA EQUALS UNITY AT THE STABLE EQUILIBRIUM POINT AND BECOMES ZERO
C     AT THE UNSTABLE POINT.
      ZETA=1.-(V/VMAX)
      WRITE(12,17)T,W,DEG,PT,V,ZETA
   17 FORMAT(1H ,9X,1F10.6,1F14.6,1F13.6,1F12.6,2X,1F9.4,3X,1F8.6)
      IF(V.GT.VMAX.AND.(V-VMAX).GE.5.) GO TO 100
      GO TO 40
   60 PT = C2*SIN(DEL)
   70 IF(J-1)61,61,62
   62 IF(J-2)64,63,64
   64 IF(J-3)66,65,66
   66 IF(J-4)68,67,68
   68 GO TO 70
  100 WRITE(12,99)VMAX
   99 FORMAT(1H0,50X,6HVMAX =,1F9.5)
      PM=PM&0.2
      GO TO 77
  111 STOP
      END
```

APPENDIX D

PERFORMANCE WITH AUTOMATIC RECLOSING

```
      REAL M,K,L
      DIMENSION K(4),L(4)
      H=2.76
      PM=1.4
      KK=0
   77 KK=KK&1
      IF(KK.EQ.7) GO TO 111
      WRITE(12,177)PM
  177 FORMAT(1H1,12X,26H MECHANICAL POWER INPUT = ,1F5.3,5H P.U.)
      WRITE(12,15)
   15 FORMAT(1H0,12X,4HTIME,10X,3HVEL,10X,5HDELTA,8X,2HPT,12X,1HV,8X,4HZ
     CETA)
      A=2.*3.1416*60.
      T=0.
      W=A
      C1=2.58
      C2=0.936
      C3=2.06
      DEL=ATAN((PM/C1)/(SQRT(1.-(PM/C1)**2)))
      DELO=DEL
      D2=DELO
      DT=0.01
   40 J=1
      M=H/(3.1416*60.)
      T=T&DT
      IF(T.GT.0.5) GO TO 100
      F=W
      DELZ=DEL
      K(J)=(W-A)*DT
      GO TO 60
```

```
 61 L(J)=(1./M)*(PM-PT)*DT
    DEL=DELZ&K(J)/2.
    N=J
    J=2
    K(J)=((W&L(N)/2.)-A)*DT
    GO TO 60
 63 L(J)=(1./M)*(PM-PT)*DT
    DEL=DELZ&K(J)/2.
    N=J
    J=3
    K(J)=((W&L(N)/2.)-A)*DT
    GO TO 60
 65 L(J)=(1./M)*(PM-PT)*DT
    DEL=DELZ&K(J)/1.
    N=J
    J=4
    K(J)=((W&L(N)/1.)-A)*DT
    GO TO 60
 67 L(J)=(1./M)*(PM-PT)*DT
    DEL=DELZ&(1./6)*(K(1)&2.*K(2)&2.*K(3)&K(4))
    DEG=DEL*180./3.1416
    W=F&(1./6.)*(L(1)&2.*L(2)&2.*L(3)&L(4))
    Y=3.1416-D2*2.
    X=DEL-D2
    U=W-A
    V=(U**2)/2.&1./M*(-PM*X-C3*COS(X&D2)&C3*COS(D2))
    VMAX=(1./M)*(-PM*Y-C3*COS(Y&D2)&C3*COS(D2))
    ZETA=1.-(V/VMAX)
    WRITE(12,17)T,W,DEG,PT,V,ZETA
 17 FORMAT(1H ,9X,1F10.6,1F14.6,1F13.6,1F12.6,2X,1F9.4,3X,1F8.6)
    IF(T.GE.(1./6.)) GO TO 100
    GO TO 40
 60 PT = C2*SIN(DEL)
C   AUTOMATIC CIRCUIT RECLOSING EMPLOYED
```

```
      IF(T.GE.(8./60.).AND.T.LT.(1./6.0)) PT=C3*SIN(DEL)
      IF(T.GE.(6./60.).AND.T.LT.(8./60.)) PT=C2*SIN(DEL)
      IF(T.GE.(4./60.).AND.T.LT.(6./60.)) PT=C3*SIN(DEL)
      IF(T.GE.(1./6.0)) PT=C1*SIN(DEL)
   70 IF(J-1)61,61,62
   62 IF(J-2)64,63,64
   64 IF(J-3)66,65,66
   66 IF(J-4)68,67,68
   68 GO TO 70
  100 WRITE(12,99)VMAX
   99 FORMAT(1H0,50X,6HVMAX =,1F9.5)
  111 STOP
      END
```

APPENDIX E

MINIMIZATION BY THE METHOD OF STEEPEST DESCENT

```
C     BOOTHS INTERPOLATION FORMULA IS USED TO COMPUTE THE INCREMENTS IN
C     THE VALUES OF DIFFERENT ROTOR ANGLES.
      RR=3.1416*1./180.
      K=0
      L=0
      KK=0
      WRITE(12,66)
   66 FORMAT(1H0,20X,40HPOST FAULT STABLE EQUILIBRIUM CONDITIONS)
      WRITE(12,44)
   44 FORMAT(1H0,30X,2HDA,13X,2HDB,13X,2HDC,21X,2HSO)
      DAO=23.*3.1416/180.
      DBO=10.4*3.1416/180.
      DCO=9.5*3.1416/180.
C     THE FIRST GUESS FORMS THE ANGLES OF THE PRE-FAULT EQUILIBRIUM
C     CONDITIONS.   THESE VALUES ARE CLOSE TO THE POST-FAULT EQUILIBRIUM
      T1=79.5*3.1416/180.
      T2=79.2*3.1416/180.
      T3=77.4*3.1416/180.
   60 DA=DAO
      DB=DBO
      DC=DCO
      IF(L.GE.12) GO TO 111
      IF(KK.GT.300) GO TO 111
      P=1.3*COS(T1-DA&DB)&0.59*COS(T2-DA&DC)-0.76
      Q=1.3*COS(T1-DB&DA)&3.09*COS(T3-DB&DC)-0.69
      R=0.59*COS(T2-DC&DA)&3.09*COS(T3-DC&DB)-0.57
      SO=P**2&Q**2&R**2
      AD=DA*180./3.1416
```

```
      BD=DB*180./3.1416
      CD=DC*180./3.1416
      WRITE(12,33)AD,BD,CD,SO
   33 FORMAT(1H0,20X,3F15.5,10X,1F15.5)
  151 IF(L.GT.10.AND.K.GT.25) GO TO 111
      IF(L.GE.1.AND.SO.LE..0001) GO TO 144
      IF(L.EQ.1.AND.K.GT.75) GO TO 144
      IF(L.GT.1.AND.K.GT.25) GO TO 144
      IF(SO.LE.0.0001) GO TO 100
      SA=2.*P*(1.3*SIN(T1-DA&DB)&.59*SIN(T2-DA&DC))&2.*Q*(-1.3*SIN(T1-DB
     C&DA))&2.*R*(-0.59*SIN(T2-DC&DA))
      SB=2.*P*(-1.3*SIN(T1-DA&DB))&2.*Q*(1.3*SIN(T1-DB&DA)&3.09*SIN(T3-D
     CB&DC))&2.*R*(-3.09*SIN(T3-DC&DB))
      SC=2.*P*(-0.59*SIN(T2-DA&DC))&2.*Q*(-3.09*SIN(T3-DB&DC))&2.*R*(0.5
     C9*SIN(T2-DC&DA)&3.09*SIN(T3-DC&DB))
      U=-SO/(SA**2&SB**2&SC**2)
      EOA=U*SA
      EOB=U*SB
      EOC=U*SC
      DA=DAO&EOA/2.
      DB=DBO&EOB/2.
      DC=DCO&EOC/2.
      P=1.3*COS(T1-DA&DB)&0.59*COS(T2-DA&DC)-0.76
      Q=1.3*COS(T1-DB&DA)&3.09*COS(T3-DB&DC)-0.69
      R=0.59*COS(T2-DC&DA)&3.09*COS(T3-DC&DB)-0.57
      SH=P**2&Q**2&R**2
      DA=DAO&EOA
      DB=DBO&EOB
      DC=DCO&EOC
      P=1.3*COS(T1-DA&DB)&0.59*COS(T2-DA&DC)-0.76
      Q=1.3*COS(T1-DB&DA)&3.09*COS(T3-DB&DC)-0.69
      R=0.59*COS(T2-DC&DA)&3.09*COS(T3-DC&DB)-0.57
      S1=P**2&Q**2&R**2
```

```
      E=-((S1-4.*SH&3.*SO)/(4.*(S1-2.*SH&SO)))*(SO/(SA**2&SB**2&SC**2))
      EA=E*SA
      EB=E*SB
      EC=E*SC
      DAO=DAO&EA
      DBO=DBO&EB
      DCO=DCO&EC
      K=K&1
      KK=KK&1
      GO TO 60
  100 L=L&1
      K=0
      DAO=147.7*RR
      DBO=5.3*RR
      DCO=9.45*RR
      WRITE(12,77)
   77 FORMAT(1H0,20X,42HPOST FAULT UNSTABLE EQUILIBRIUM CONDITIONS)
      GO TO 60
  144 L=L+1
      WRITE(12,77)
      K=0
      XY=L*5
      DBO=10.4*RR-XY*RR
      DCO=9.5*RR-XY*RR
      XX=L*20
      DAO=70.*RR&XX*RR
      IF(L.LT.3) DAO=70.*RR
      GO TO 60
  111 STOP
      END
```

APPENDIX F

```
      STABILITY ANALYSIS OF A MULTI-MACHINE SYSTEM
C
      INTEGER H,Z
      REAL M,L,K
      DIMENSION D(3),DO(3),DZ(3),F(3),K(3,4),L(3,4),M(3),PM(3),P(3),DEL(
     C3),S(3),U(3),E(3),B(3,3),G(3,3),W(3),O(3),C(3,3),Q(3,3)
      R=3.1416/180.
      DT=0.02
      Q(1,1)=-1.66
      Q(2,2)=-4.54
      Q(3,3)=-3.68
      Q(1,2)=1.10
      Q(1,3)=0.493
      Q(2,3)=2.98
      C(1,1)=0.03
      C(2,2)=1.58
      C(3,3)=0.33
      C(1,2)=.205
      C(2,3)=0.66
      C(1,3)=0.093
      DO 18 J=1,2
      DO 18 I=1,3
      C(I,J)=C(J,I)
      Q(I,J)=Q(J,I)
      PM(I)=P(I)
   18 CONTINUE
      A=376.99
      V=0.
```

```
      T=0.
      DATA W/3*376.99/,L/3*0./
      DO 9 H=1,6
      READ(11,10)I,J,GG,PP,AA,BB,CC,DD,EE,FF
   10 FORMAT(2I1,7F10.4,1F8.6)
      IF(H.GT.3) GO TO 8
      PM(J)=AA
      S(J)=BB*R
      U(J)=CC*R
      D(J)=DD*R
      E(J)=EE
      M(J)=FF
      DO(J)=D(J)
    8 G(I,J)=GG
      G(J,I)=G(I,J)
      B(I,J)=PP
      B(J,I)=B(I,J)
      WRITE(12,7)I,J,GG,PP,AA,BB,CC,DD,EE,FF
    7 FORMAT(1H ,2I2,8F12.4)
    9 CONTINUE
      WRITE(12,3)
    3 FORMAT(1H1,35X,38HRUNGE-KUTTA FOURTH ORDER APPROXIMATION)
   16 WRITE(12,89)
   89 FORMAT(1H0,4X,4HTIME,4X,7HDELTA-A,3X,5HVEL-A,5X,7HPOWER-A,3X,7HDEL
     CTA-B,3X,5HVEL-B,5X,7HPOWER-B,3X,7HDELTA-C,3X,5HVEL-C,5X,7HPOWER-C,
     C6X,1HV,8X,4HVMAX)
      VM=1.7982
   40 N=1
      CC=0.
      T=T&DT
```

```
      IF(T.GT.0.4) GO TO 111
      DO 51 I=1,3
      F(I)=W(I)
      DZ(I)=D(I)
   51 CONTINUE
      DO 59 J=1,4
      DO 52 I=1,3
      K(I,J)=(W(I)&(L(I,N)*CC)-A)*DT
   52 CONTINUE
      IF(J.EQ.1) GO TO 49
      DO 47 Z=1,3
      IF(J.LE.3) D(Z)=D(Z)&K(Z,J)/2.
      IF(J.EQ.4) D(Z)=D(Z)&K(Z,J)
   47 CONTINUE
   49 P(1)=0.1*COS(86.*R-D(1)&D(2))&0.1*COS(86.7*R-D(1)&D(3))
      P(2)=0.24&.1*COS(86.*R-D(2)&D(1))&.68*COS(84.7*R-D(2)&D(3))
      P(3)=.09&.1*COS(86.7*R-D(3)&(D(1)))&.68*COS(84.7*R-D(3)&D(2))
      DO 70 I=1,3
      L(I,J)=(1./M(I))*(PM(I)-P(I))*DT
   70 CONTINUE
      N=J
      IF(J.LT.3) CC=0.5
      IF(J.EQ.3) CC=1.
   59 CONTINUE
      DO 75 I=1,3
      D(I)=DZ(I)&(1./6.)*(K(I,1)&2.*K(I,2)&2.*K(I,3)&K(I,4))
      W(I)=F(I)&(1./6.)*(L(I,1)&2.*L(I,2)&2.*L(I,3)&L(I,4))
      DEL(I)=D(I)/R
      O(I)=W(I)-A
   75 CONTINUE
```

```
      VV=0.
      WW=0.
      DO 80 I=1,3
      WW=WW&0.5*M(I)*(O(I)**2)&((E(I)**2)*C(I,I)-PM(I))*(D(I)-S(I))
   80 CONTINUE
      DO 92 Z=1,2
      H=Z&1
      DO 92 J=H,3
      VV=VV&E(Z)*E(J)*(Q(Z,J)*(COS(S(Z)-S(J))-COS(D(Z)-D(J)))&C(Z,J)*(SI
     CN(S(Z)-S(J))-SIN(D(Z)-D(J))))
   92 CONTINUE
      V=VV+WW
      WRITE(12,88)T,(DEL(I),W(I),P(I),I=1,3),V,VM
   88 FORMAT(1H ,1F9.5,9F10.5,2X,2F10.4)
      GO TO 40
  111 STOP
      END
```