1972

The Role of Agriculture in South Dakota's Economy : An Approach to Analysis

Mahlon George Lang

AUTOMATIC CONTROL OF A CLOSED CONDUIT

CUTBACK IRRIGATION SYSTEM

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree. Acceptance of this thesis does not imply that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Advisor Date

Head, Electrical Engineering Department Date

ACKNOWLEDGMENTS

Funds for this research were provided by the Water Resources Institute, South Dakota State University, which is under the direction of Dr. J.L. Wiersma.

Appreciation is extended to the author's advisor, Dr. D.E. Sander, for his suggestions, guidance, and assistance throughout the duration of this study.

The author wishes to express sincere appreciation to his wife, Sharon, for her encouragement and patience while typing this manuscript.

REK

TABLE OF CONTENTS

LIST OF FIGURES AND TABLE

GLOSSARY OF TERMS (11), (12), (13), (24)

ADHESION — The ability of soil-particles to attract and hold water to their surface.

APPLICATION RATE — The equivalent rainfall rate expressed in inches of water depth per hour.

AVAILABLE WATER — Is limited to the capacity of water the root zone can hold between field capacity and the permanent wilting point.

BASIC INTAKE RATE — The nearly constant soil intake rate developed after some time has elapsed from the start of irrigation.

CUT BACK STREAM — Furrow length in 100's of feet times basic intake rate in g.p.m. This corresponds to the cut back flow rate.

DESCRIBING FUNCTION ANALYSIS — A frequency response technique for analyzing nonlinear systems by approximating the nonlinearity with a linear equivalent gain.

DYNAMIC HEAD — This is the sum of:

1. Pressure head required to operate lateral lines in feet.
2. Friction head losses in the main line and submains.
3. Friction head losses in fittings and valves.
4. Total static head including suction lift.

EQUIVALENT GAIN — The linear approximation of a nonlinear transfer function.

FIELD CAPACITY — The moisture percentage, on a dry weight basis, of a soil after a rapid drainage has taken place following an application of water, provided there is no water table within capillary reach of the root zone. This moisture percentage usually is reached within two to four days after an irrigation, the time interval depending on the physical characteristics of the soil.

FURROW INTAKE	Intake rate in gallons per minute per 100 feet of furrow.
FURROW STREAMS	At the beginning of an irrigation the largest stream of water that will not cause erosion in each furrow. This corresponds to the initial flow rate.
GAIN	The ratio of output to input of a component, circuit, or system.
GATED PIPE	Portable metal pipe, usually aluminum, with a number of small gates along one side through which water can be run into corrugations, furrows and borders.
HEAD	Water pressure in a line which is equivalent to that being produced by a body of water stored at this height.
INFILTRATION RATE	The rate at which soil will take water during the irrigation period, expressed in inches of water depth per hour.
IRRIGATION EFFICIENCY	The percentage of applied irrigation water that is stored in the soil and available for consumptive use by the crop.
NET IRRIGATION WATER REQUIREMENT	The amount of water exclusive of precipitation required for crop production or it is the amount of irrigation water that must be stored in the root zone to meet the consumptive use requirement of a crop.
OSMOTIC PRESSURE	Tension caused by salts. Water moves from the solution of lower concentration to the one of higher concentration.
PERCOLATION	The movement of water through the soil profile.
TRANSFER FUNCTION	The mathematical model of a component or system which relates its input to output.

INTRODUCTION

The total supply of water on the earth today is about the same as it was thousands of years ago. Man has manipulated it with respect to time and space but has not changed the total quantity of water. The demand for water has increased because there are more people using it today than ever before. It is estimated that by the year 2020, 1,380 billion gallons of water will be used every day to satisfy the needs of 468 million people in the United States (21). Priorities will therefore have to be established to allocate the limited supply of water among the alternative users. One criterion, based on an economic analysis of the user, for making the allocation decision is presented by Reynolds (21). Unfortunately, agriculture has been awarded a rather low priority level as compared to municipal and domestic users. It is desirable, therefore, to make optimum use of the water allocated for agricultural endeavors.

The largest single use of agricultural allocated water is for irrigation. It is estimated that by the year 2000, 150,000 million gallons of water per day will be used for irrigation purposes in the United States (21). It is estimated that 20 to 40% of the water applied by conventional means is lost due to runoff, evaporation, or deep percolation (3). It is desirable to minimize these losses, thereby optimizing the use of irrigation water. One way to accomplish these goals is through the use of automation.

Automation of surface irrigation systems, especially automation of cutback furrow irrigation systems, can optimize the use of water for irrigation. Even without automation, the cutback system employs two rates of flow which if properly timed can minimize the amount of runoff water. With automation the irrigation efficiency could be further improved. Garton (5) states that not only is irrigation efficiency improved through automation, but the labor savings as a result of automation will pay the installation costs in less than the useful life of the system.

Fishbach (2) states that the three basic components of a well designed automatic system are: (a) The Distribution System, (b) Electric Controls, and (c) The Telemetry System. The distribution system includes the water supply, the interconnecting valves and lines, and some type of outlet system to distribute the water on the land. The electric controls are responsible for adjusting and maintaining the rate of flow for each irrigation station, timing the irrigation of each station, and automatically sequencing the system after a predetermined time interval. The telemetry system measures and records the amount of moisture stored in the root zone of the field.

The scope of this paper is to examine the part of the system which deals with electric controls. This thesis will investigate the use of automated flow control valves to program the application rates of water on the soil during an irrigation.

THE STATE OF THE ART

Until recently the extent of automation in surface irrigation systems was characterized by traveling gun or sprinkler type systems. This type of irrigation system is the predominant form of surface irrigation in the Midwest. This system employs a closed conduit, pressurized supply and sprinkler mechanisms which uniformly distribute water at a rate below the intake rate of the soil. The efficiency and uniformity of the system is determined almost exclusively by the weather and terrain (9). Some of the parameters to be reckoned with are the wind, the amount of direct sunlight, the slope of the land, the pressure in the lines, and the size of the irrigation stream. In the South and West where the land is flat, open channel cutback irrigation is employed. The cutback system has the potential for the greatest efficiency among surface irrigation systems. This system utilizes two rates of flow for furrow irrigation. Initially a high rate of flow, lower than the soil erosion rate and higher than the soil absorption rate, is maintained for a period of time which is long enough to allow the water to reach the end of the irrigation furrow. The flow is then "cutback" to a lower rate equal to the absorption rate of the soil. By dividing the fields into sections or stations, a sequence can be established so that when one station is operating at the low rate of flow, the previous station is off or not irrigating, and the next station is operating at the high rate of flow. This allows the irrigator to maximize the use of available water, thereby improving the efficiency of the supply. The irrigation efficiency of this system is determined by how accurately each of the two flow rates can be maintained. Since flow and pressure head can be assumed linear over a limited pressure range in this system, flow can be regulated by maintaining a constant head. As can be anticipated, to improve efficiency it is more important to maintain a constant flow during the "cutback" flow period than during the initial flow period. Garton states,

> "Since the discharge varies as the square root of the head, the cutback furrow flow varies more widely with changes in supply flow than does the initial furrow flow" (6).

Recent developments include a hybrid type of system implementing the desirable characteristics of the two previously mentioned systems. This system, called a "closed conduit furrow irrigation system", employs the principles of the open channel cutback system with a pressurized closed conduit supply and distribution system. Coupled with a reuse system to recycle the runoff water this system has an irrigation efficiency of 91.9%. This is compared with an efficiency of 64.8% for the same system without the reuse system (3).

The large difference in these two efficiencies is directly related to how closely the two flow rates can be maintained.

Although the advantages of automation more than outweigh the initial and maintenance costs, it is still desirable to minimize the cost of the system. Therefore an attempt should be made to optimize the efficiency of the latter hybrid system without adding the extra expense of a reuse system. The supply and distribution portions of this system have already been designed and tested (23). It was found that with manually adjusted valves, an efficiency of 86% could be maintained. However, it is anticipated that with the utilization of automatic pressure regulating valves this efficiency can be improved. Several attempts have been made at the implementation of these automatic pressure regulating valves by Fishbach and others. One scheme employs a pneumatically actuated valve which has met with limited success. The main drawback to this scheme is the possibility of excessive irrigation should a leak develop in the required air supply line (9). This system would also be more costly since the air compressors must be driven by either a gas or an electric motor.

It is the intent of this paper to present a design for an automatic, electrically actuated, pressure regulating valve and controller. It will include the capability of being programmed for irrigation on a daily or weekly basis. The valve will also be able to maintain a constant programmable output pressure regardless of typical input pressure variations.

THE PROBLEM

The problem is to develop, construct, and test a programmable, electrically actuated, pressure regulating, irrigation valve and controller which is to be incorporated into the existing distribution system (23). The controller consists of two parts, the program control and the valve control. The program control should operate on 120 V., 60 Hz, A.C. power. It should have the capability of programming up to ten valves, two at a time, with adjustable flow rate and duration for each valve. It also should include a twenty-four hour programmable clock and electronic switching logic.

The valve control should have the capability of maintaining a constant output flow corresponding to pressures of 4 to 36 inches of head with an accuracy of ± 2 inches of head. The input to the valve should be allowed to vary between 3 and 6 feet of head. The valve control should include an electro-mechanical positioning system, decision electronics, and a pressure sensing device. It also should operate on 120 V., 60 Hz, A.C. power. Two important constraints on the total system are simplicity and economy.

A typical supply and distribution system, into which the automatic valve will be integrated, consists of 30 foot, quick coupling, eight-inch diameter, aluminum irrigation pipe sections. The distribution system for each station is parallel to the main supply line. It employs two sections of pipe with closed ends, joined together by a tee to form a distribution bank. These two sections have one-inch diameter, equally spaced, orifices corresponding to the furrow spacing. The tee is connected to the main supply line through the automatic pressure regulating valve.

A drawing of the physical system showing the main supply line, automatic valve, and distribution bank is shown in figure 1.

It should be noted that the length "L" of the distribution bank in figure 1 may not always be equal to 60 feet. The length of the distribution bank depends upon the position of the particular station with respect to the supply, the spacing of the holes in the pipe, the initial and cutback flow rates, and the length of the previous distribution bank. Adjusting the lengths of the distribution banks allows the irrigator to use the total potential of the supply. For a series of "k" banks, $k \geq n$, equation 1 determines the length of bank "n" (23):

$$L_n = \frac{Q S_p - (L_{n-1} q_{ch})}{q_i} \qquad \text{(Equation 1)}$$

where: $n = 1, 2, \cdots$, n number of banks,

L_n = length of bank "n" in feet,

THE PHYSICAL SYSTEM

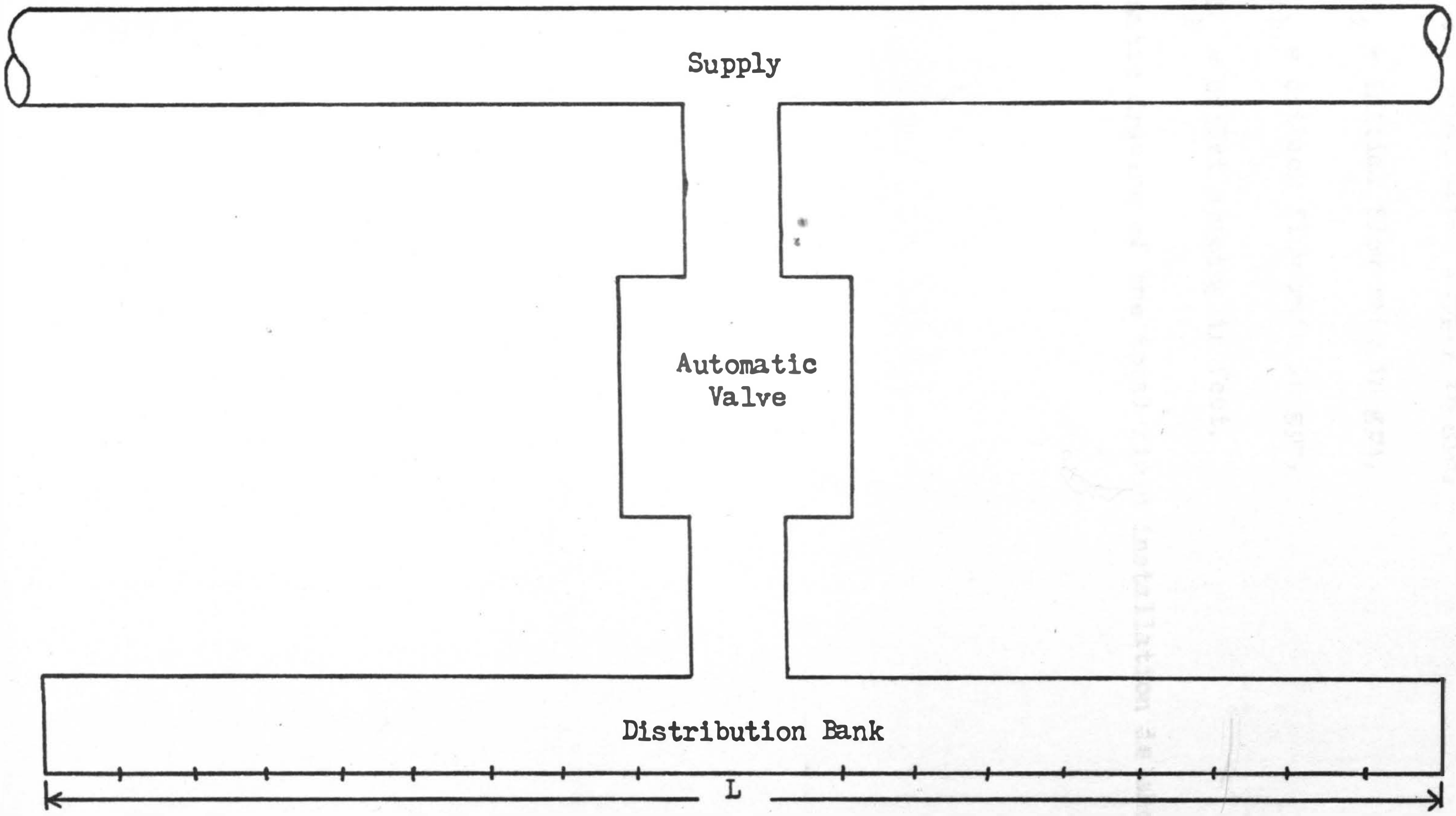


Figure 1

L_{n-1} = length of the preceeding bank in feet,

Q = total water supply in gpm,

q_i = initial flow rate in gpm,

q_{cb} = cutback flow rate in gpm,

and S_p = outlet spacing in feet.

A schematic drawing of the total field installation is shown in figure 2.

FIELD INSTALLATION SCHEMATIC

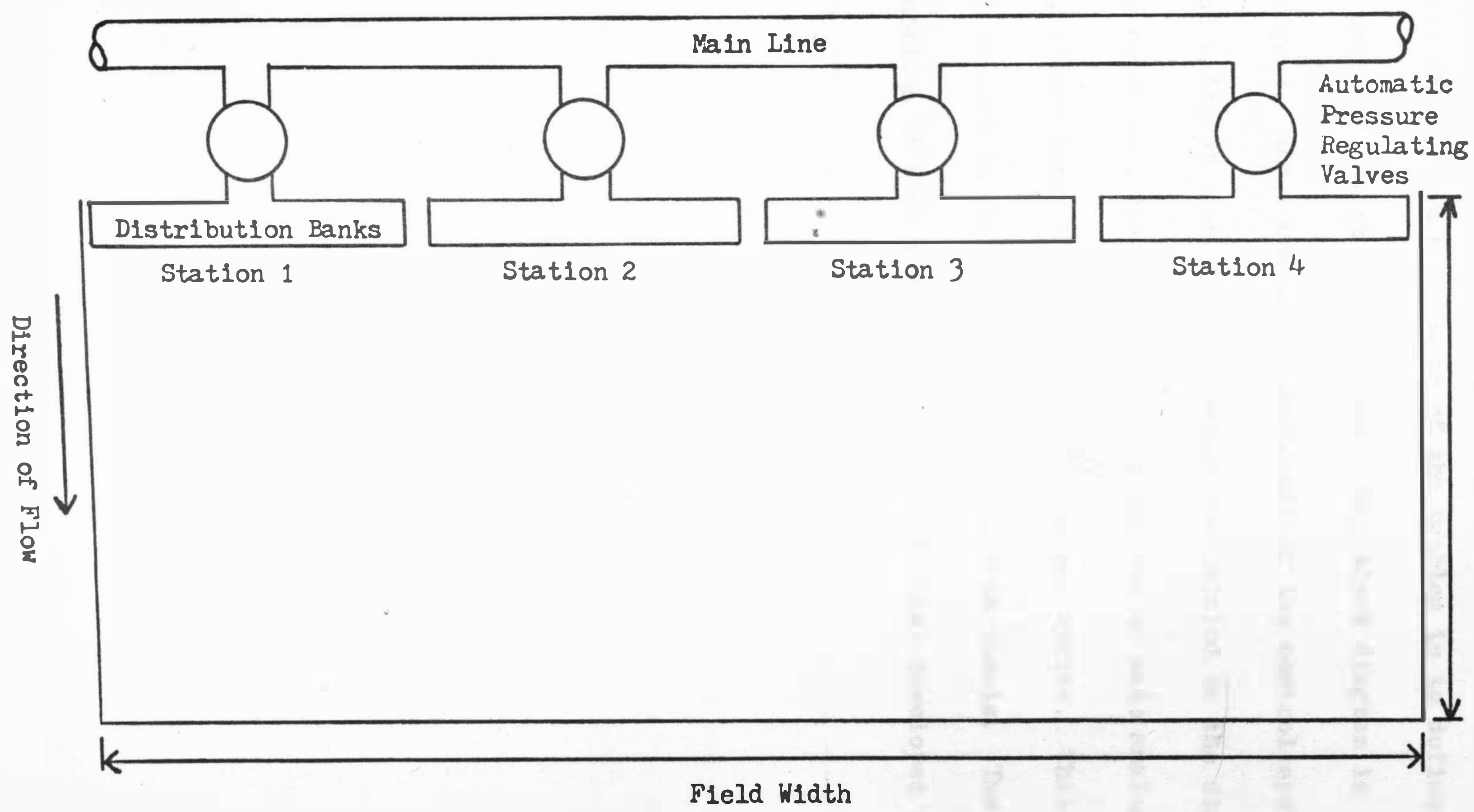


Figure 2

THE SOLUTION

The first step toward a solution of the problem is to define the control system in block diagram form. This block diagram is shown in figure 3. The fundamental components of the control system with physical units of each input or output are labeled on the diagram. It should be noted that the valve control block can be made analogous to a single, closed loop, negative feedback control system. This analogy will be used in the analysis portion of this thesis. The transfer functions of each basic block will be further developed in the following sections.

BLOCK DIAGRAM OF THE TOTAL SYSTEM

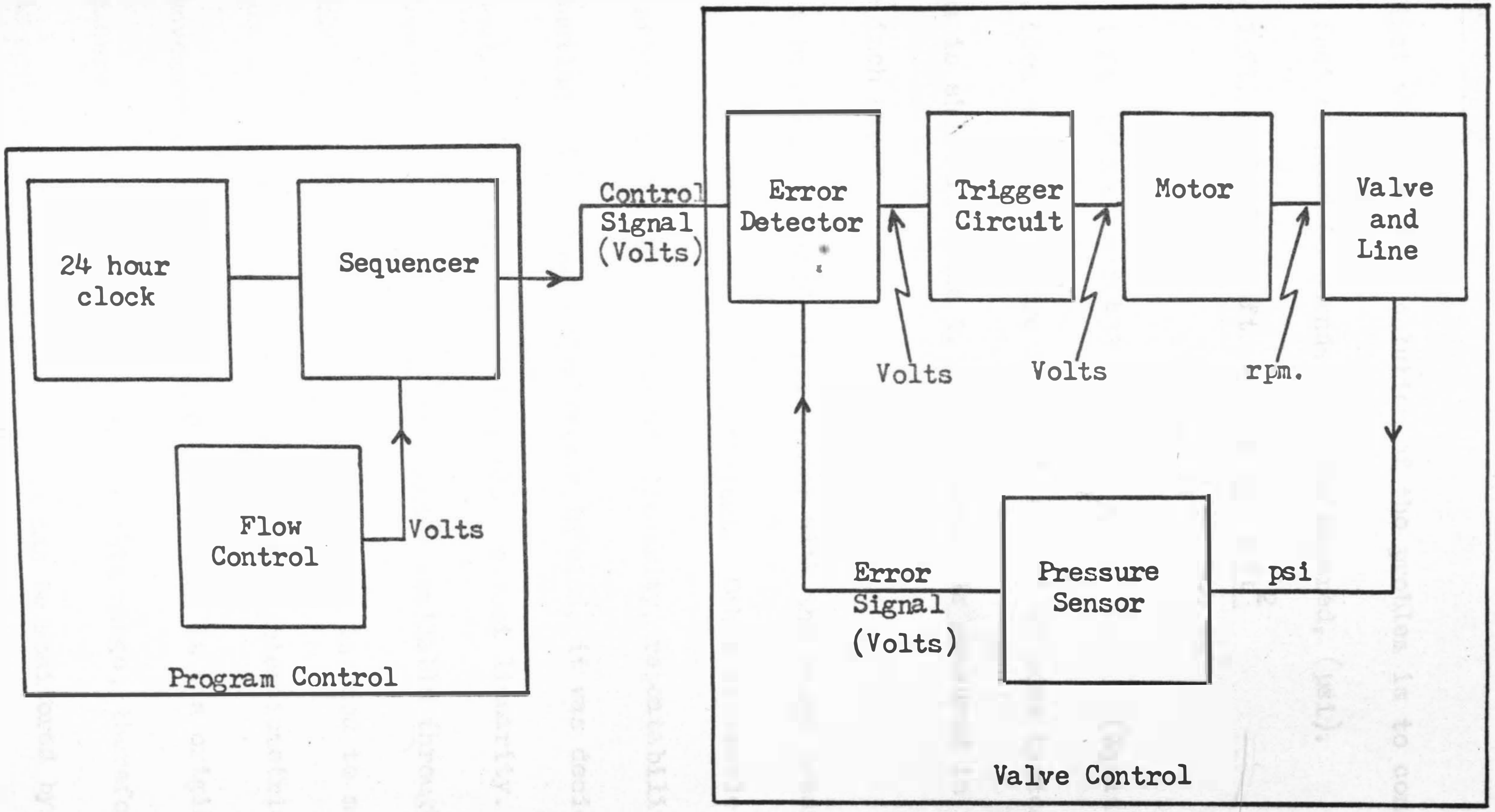


Figure 3

The Valve Control

The Pressure Sensor

The first step toward a solution of the problem is to convert inches and feet of head to pounds per inch squared, (psi).

$$1 \text{ ft. of head} = 1 \text{ ft.} \times 62.4 \frac{\text{lb}}{\text{ft}^3} \times \frac{\text{ft}^2}{144 \text{ in}^2}$$

$$1 \text{ ft. of head} = 0.433 \text{ psi} \qquad \text{(Equation 2)}$$

To give an idea of the pressure involved, a list of some typical conversions is shown in table 1. The pressure is measured in pounds per square inch gauge.

As can be seen from table 1, a sensor with the gauge pressure range of 0.0 to 2.5 psi would be sufficient. Other desirable characteristics for this device include linearity, repeatability, and ease of mounting. With these constraints in mind, it was decided that a potentiometric type movement would yield the best linearity.

A "Giannini 45176-G" pressure transducer available through <u>A.S.T. Servo Systems Incorporated</u> of Newark, New Jersey, is used to measure the pressure in the line. This transducer has a potentiometric, 5000 ohm movement over the 0.0 to 5.0 psi range. It was originally used to measure a pressure differential in this range, therefore, two input ports are available. Gauge pressure can be monitored by

TYPICAL CONVERSIONS

HEAD (in.)	P.S.I.
4	0.144
8	0.288
12	0.433
18	0.649
24	0.866
30	1.082
36	1.299
48	1.732
60	2.165

Table 1

venting one port to the atmosphere while the other is connected to the distribution bank. The desirable characteristics of this device include a resolution and repeatability of 0.3% of full scale. This corresponds to a pressure of 0.015 psi.

The transfer function for this device when biased at 5.0 volts is shown in figure 4. As can be seen from this figure, a slope of unity is a good approximation for determining the transfer function of input pressure to output voltage over the desired operating range of 0.130 to 1.30 psi. Therefore, the assumption will be made that one volt at the output of the sensor is equal to one psi at the input.

The Valve

The valve, furnished by the Water Resources Institute at S.D.S.U., is an Ames "Series S" model. This is an eight-inch, manually operated, gate valve with worm gear drive. Modifications necessary to more easily adapt the valve to motor operation include installation of a sprocket for chain drive and a mounting platform for the motor.

The static transfer function, found experimentally, relating output pressure, input pressure, and valve position is shown in figure 5. The true relationship between valve position and output pressure is constantly changing. This is due to the relationship between discharge or velocity and the input head. As the valve opens

PRESSURE SENSOR TRANSFER FUNCTION

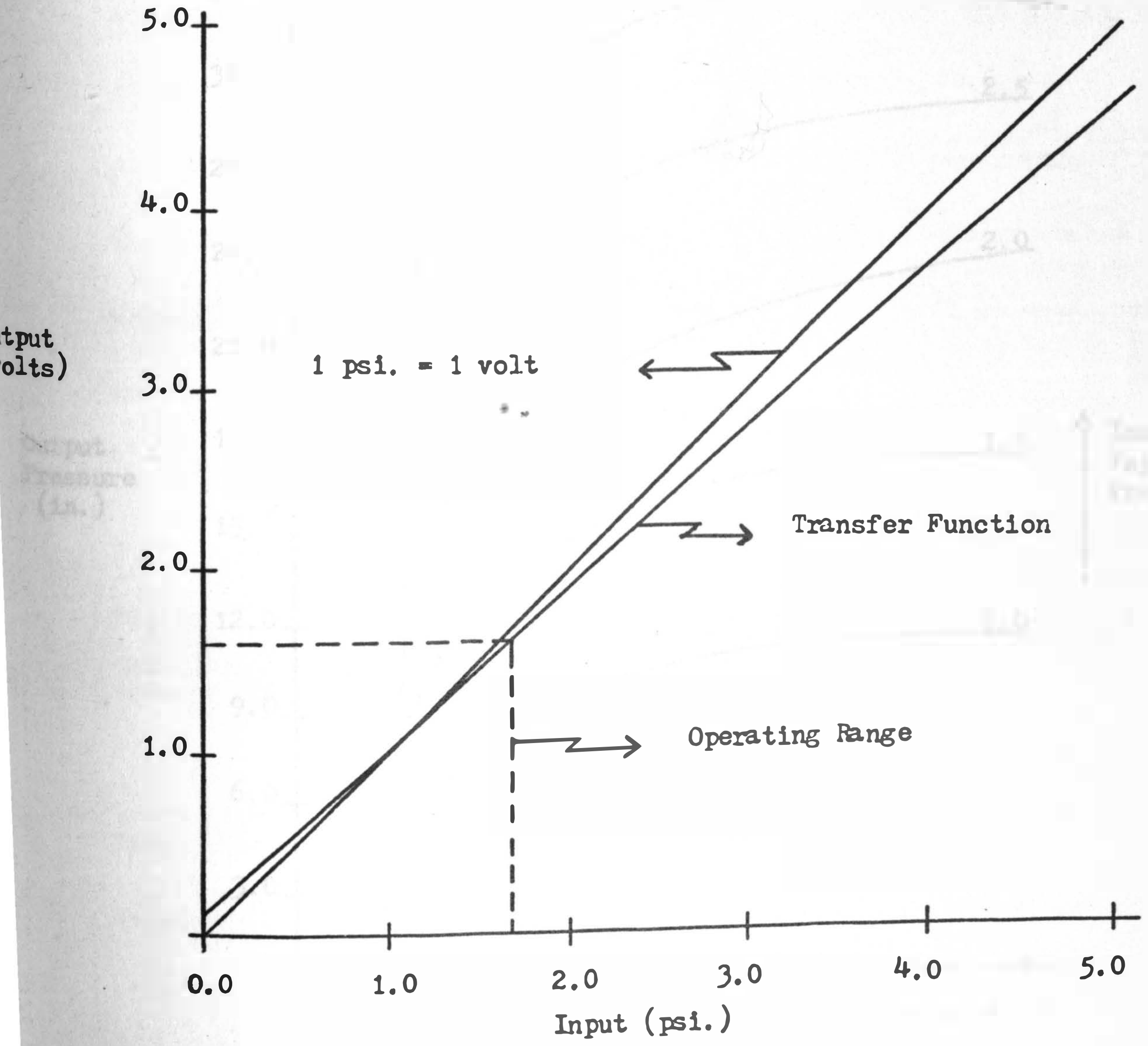


Figure 4

VALVE STATIC TRANSFER FUNCTION

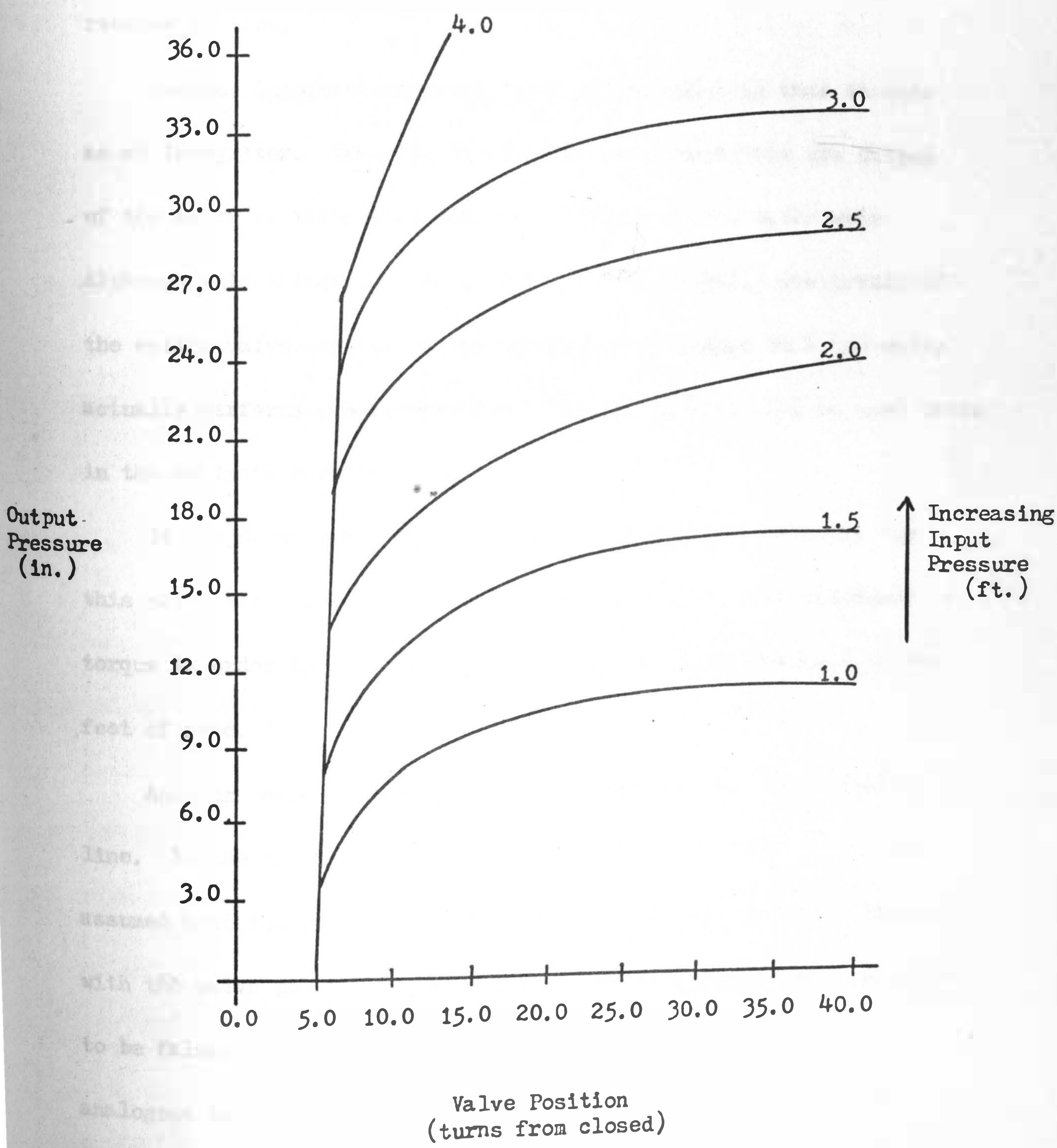


Figure 5

the discharge and velocity increase which cause a decrease in the input pressure due to friction losses. When the valve closes the reverse is true.

Another important characteristic of the valve is that it acts as an integrator. That is, revolutions per minute from the output of the motor is integrated into the position of the valve gate. Although this integration is implicit and is actually the result of the entire valve control, it is convenient to assume that the valve actually performs the integration. This assumption will be used later in the analysis portion of this paper.

It was found experimentally that the torque requirements for this valve were quite high. The motor must supply 15 foot-pounds of torque in order to close the valve against an input pressure of ten feet of head.

Another integral part of the valve block is the distribution line. In the original analysis it was neglected. That is, it was assumed that the pressure in the line responded almost instantaneously with the valve position. Experimental evidence proved this assumption to be false. It was found, however, that the line could be made analogous to a low pass filter for the purpose of mathematically modeling the system. This also will be further expanded in the analysis portion of this paper.

The Motor

Some of the desirable characteristics that the motor must have are high torque, low rpm., low current, and it must be reversible. A motor possessing these characteristics is available through B&B Motor and Control Company in New York. It is a permanent split capacitor, induction, fractional horse power, gear motor. Some of the characteristics of this motor are listed below:

Horse Power	1/6 H.P.
Gear Ratio	36 : 1
R.P.M.	40
Power Requirements	115 V. AC, 2 Amp.
Torque	21 ft. lbs.

The 21 foot-pounds of torque is well above the 15 foot-pounds required to close the valve. The 40 rpm of the motor is geared down such that the valve worm gear turns at 26 rpm. The motor is capable of instantaneous reversal by interchanging two leads. Other desirable qualities are low starting current (120% of running current), and high starting torque (100% of running torque).

The Error Detector

The error detector should have the capability of comparing two D.C. signals, the control signal and the error signal. It should have a double ended input and single ended output. Stability and low offset are two important characteristics this device should possess.

An R.C.A. CA-3001 integrated circuit differential amplifier fulfills the requirements quite well. This device is a monolithic integrated circuit in a TO-5, 12 terminal package. When the circuit is operated in operating mode "C" the following characteristics apply (20), (25):

Power Supplies	± 5 V.
Input Resistance	150 k ohms
Output Resistance	45 ohms
Voltage Gain	10
Input Offset Voltage	1.5 mV.

The circuit diagram for this device is shown in figure 6.

The inputs to the circuit are pins 1 and 6. The output is taken between pins 8 and 11. Note that both the input and output are emitter - follower coupled. This provides a high input impedance and low output impedance. Q3 and Q4 make up the actual differential circuit. Resistors R5 and R6 provide local feedback for the differential circuit which decreases the overall gain of the amplifier but adds stability. Q7 provides the constant current source for biasing the circuit. The connection diagram and transfer function of this circuit are shown in figures 7 and 8 respectively.

Input resistors R1 and R2 were selected to be 3 k ohms in order

CA-3001 CIRCUIT DIAGRAM

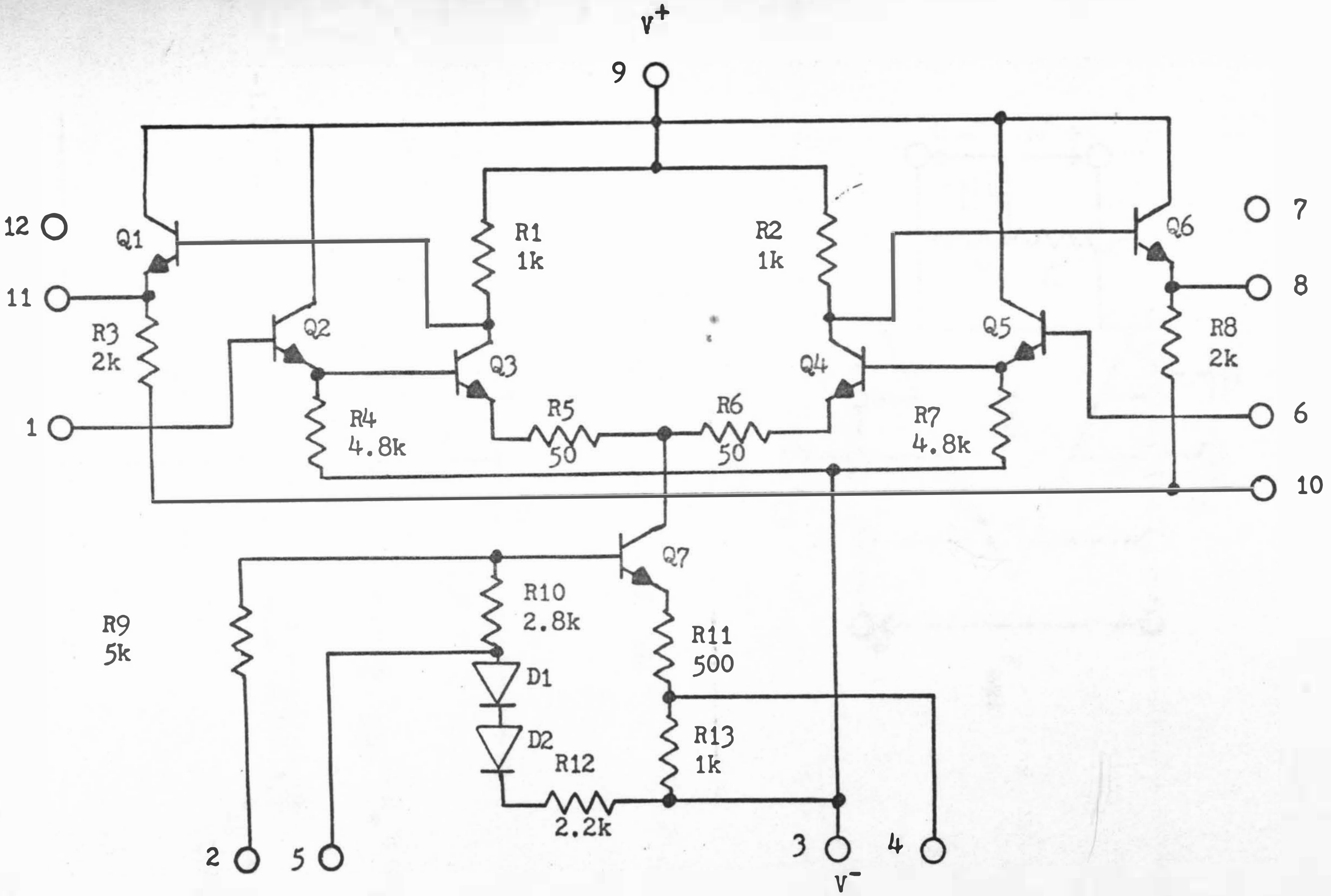


Figure 6

DIFFERENTIAL AMPLIFIER CONNECTION DIAGRAM

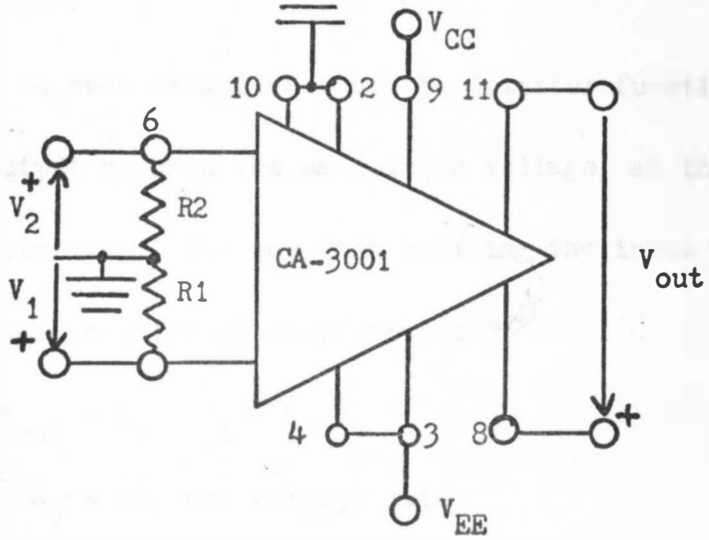


Figure 7

ERROR DETECTOR TRANSFER FUNCTION

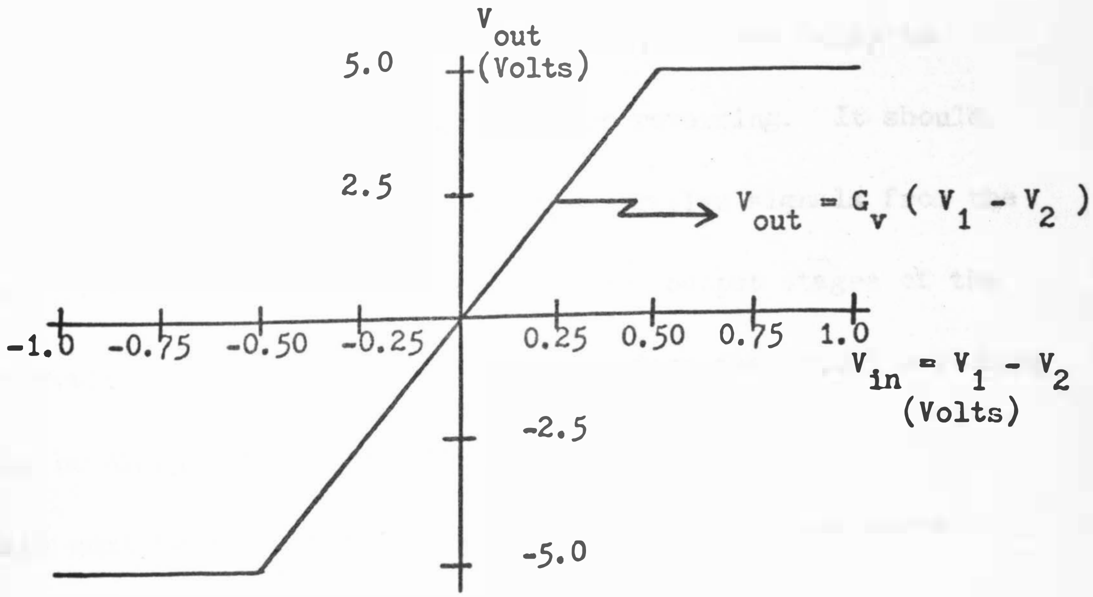


Figure 8

to maximize the voltage gain. Note that the output is taken between pins 8 and 11 in order for the transfer function to be symmetrical about the input.

As can be seen from figure 8, the transfer function is linear until the output reaches the saturation voltage, at this time the amplifier saturates. The equation relating the input to the output over the linear portion of operation is:

$$V_{out} = G_v (V_1 - V_2)$$

where: $G_v = 10$, the voltage gain. (Equation 3)

The Trigger Circuit

The main constraints on the trigger circuit are single ended input with double ended output, sufficient dead zone such that the motor will not be running continually, and a short time delay to allow the motor to come to a full stop before reversing. It should be able to respond to both the positive and negative signals from the output of the differential amplifier. Also the output stages of the trigger circuit must incorporate relays to perform the actual switching which will be discussed in the next paragraph.

Relays must be used to do the actual switching because three wires must be switched, two for the direction of rotation and one power line must be broken. Two three - contact relays with 1.4 k ohm coils were used for the contactor devices. The coils could be actuated by the application of ± 17 V. DC. Since the differential amplifier only supplies a maximum of ± 5 volts, a power buffer stage must also be implemented. This stage must supply the needed voltage amplification to actuate the relays.

A time delay on the order of one second is sufficient to allow the motor to come to a complete halt before reversing. This coasting effect is due to the inertial load provided by the valve and motor rotor.

A complementary pair of switching transistors would satisfy the needed power buffer requirements. This stage is operated in the class "B" mode with the n-p-n transistor responding to the positive signals from the differential amplifier and the p-n-p to the negative signals. This stage is not designed to be a linear amplifier, rather the transistors are driven from cut-off to saturation by the output signals of the error detector. Germanium bipolar transistors were chosen in order to minimize the dead zone. The n-p-n transistor is a 2N1304 and the p-n-p transistor is a 2N1305. The circuit diagram for the entire trigger circuit is shown in figure 9.

Note the addition of 1N810 diodes in shunt, and limit switches in series with the relay coils. The diodes, D1 and D2, allow the

TRIGGER CIRCUIT DIAGRAM

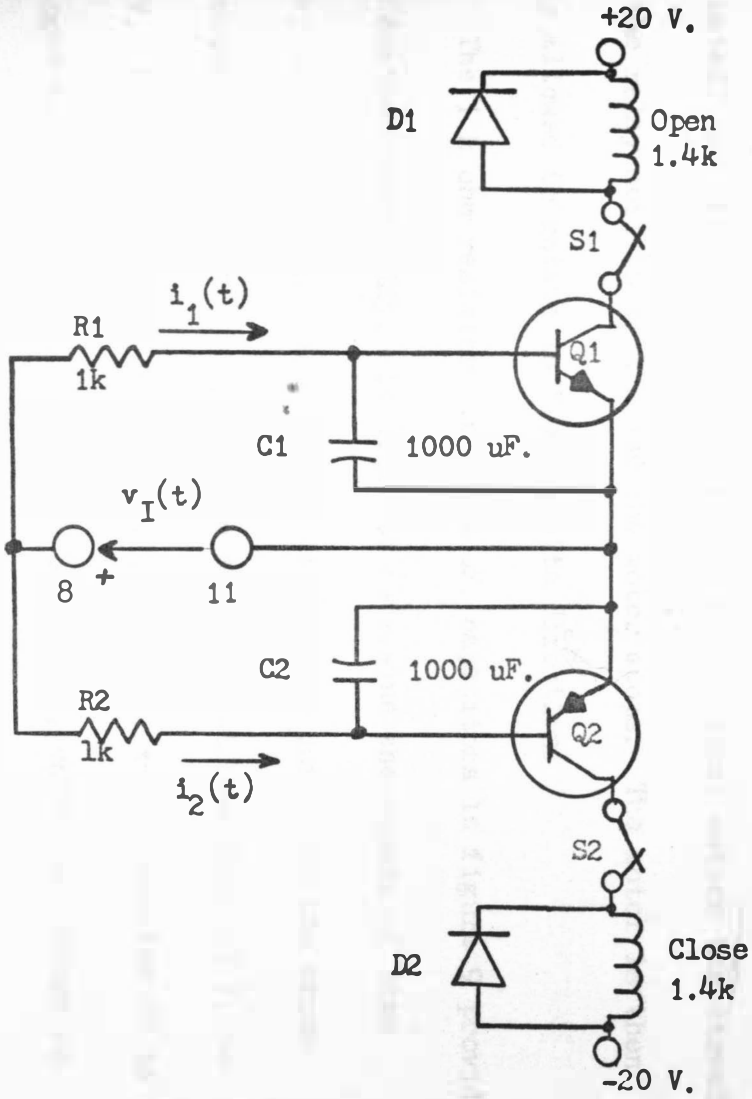


Figure 9

transistors to switch from saturation to cut-off without being held in saturation by the inductive coils (18). The limit switches, S1 and S2, are mounted on the valve shaft guide and allow the valve to be opened or closed only a certain distance before they will be actuated. At the time of actuation of either limit switch the circuit to the relay coil is broken and the motor stops. The motor is then only allowed to rotate in the opposite direction.

The 1 k ohm resistors and 1000 uF. capacitors in figure 9 provide the desired time delay. In order to determine the amount of time delay obtained by this scheme, assume that the output of the error detector is -5 V. At t=0 the error detector switches from -5 V. to +5 V. From figure 9 it can be reasoned that at $t=0^-$ capacitor C1 is charged to -5 V., with minus on the base of Q1, while the charge on C2 is essentially zero because Q2 is conducting. If $v_0(t)$ represents the total voltage across C1, $v_I(t)$ represents the total input voltage (between pins 8 and 11 of the differential amplifier), and V_1 represents the initial voltage on capacitor C1, three equations can be written to describe the situation.

$$v_0(t) = \frac{1}{C1} \int_0^t i_1(t)\, dt + v_0(0^+) \qquad \text{(Equation 4)}$$

$$v_I(t) = i_2(t)\,R2 + \frac{1}{C2}\int_o^t i_2(t)\,dt - \left[\frac{1}{C2}\int_o^t i_1(t)\,dt + i_1(t)\,R1\right] \qquad \text{(Equation 5)}$$

$$V_1 = \frac{1}{C2}\int_o^t i_1(t)\,dt + i_1(t)\,R1 + i_1(t)\,R2 + \frac{1}{C1}\int_o^t i_1(t)\,dt - i_2(t)\,R2 - \frac{1}{C2}\int_o^t i_2(t)\,dt \qquad \text{(Equation 6)}$$

Solving these three equations simultaneously equation 7 is obtained.

$$v_O(t) = V_S\left(1 - e^{\frac{-t}{R1\,C1}}\right) - V_1 \qquad \text{(Equation 7)}$$

where: V_S = the total input swing.

It is known that when $v_O(t)$, in equation 7, reaches +0.4 V. Q1 will start conducting and C1 will charge no further. Setting $v_O(t)$ = +0.4 V., V_S = 10 V., V_1 = 5 V., and solving equation 7 for "t" it is found that:

$$t = 0.776 \text{ seconds} \qquad \text{(Equation 8)}$$

This is the time delay provided by the circuit when the motor is required to reverse. Note that from equation 7 this time is dependent upon the initial charge on C1 and the magnitude of the output voltage swing from the error detector. As is shown in equation 8, the time delay is in the desired one second range. It should also be noted that equation 8 represents the shortest possible time delay since the differential amplifier is switching from saturation in one direction to saturation in the opposite direction. Therefore, the closer the valve comes to the desired position the longer the time delay. This is desirable since it tends to damp out any oscillation of the valve control.

If the motor is assumed to come up to speed instantaneously, compared to any other time constants in the loop, and the gear reduction between the motor and valve is included in the motor output, the transfer function for the trigger circuit and motor is shown in figure 10. Note that some hysteresis is involved. This transfer function is similar to any transfer function for a relay with hysteresis. In figure 10 the output voltage from the error detector, which is the input voltage for the trigger circuit, is converted directly to rpm. Figure 10 also indicates that the dead zone provided by the trigger circuit is 0.6 V. Transferring this voltage to the input of the error detector it is found that $V_1 - V_2$ must be larger than 0.06 V. in order for the motor to be actuated. This corresponds to a pressure difference, from figure 4 and equation 2, of $\pm$ 1.67 inches of head. This is well within the desired $\pm$ 2 inch head accuracy.

The Power Supply

The two DC power supplies required by the circuitry are $\pm$ 5 V. for the error detector and $\pm$ 20 V. for the trigger circuit. As was

TRIGGER CIRCUIT AND MOTOR TRANSFER FUNCTION

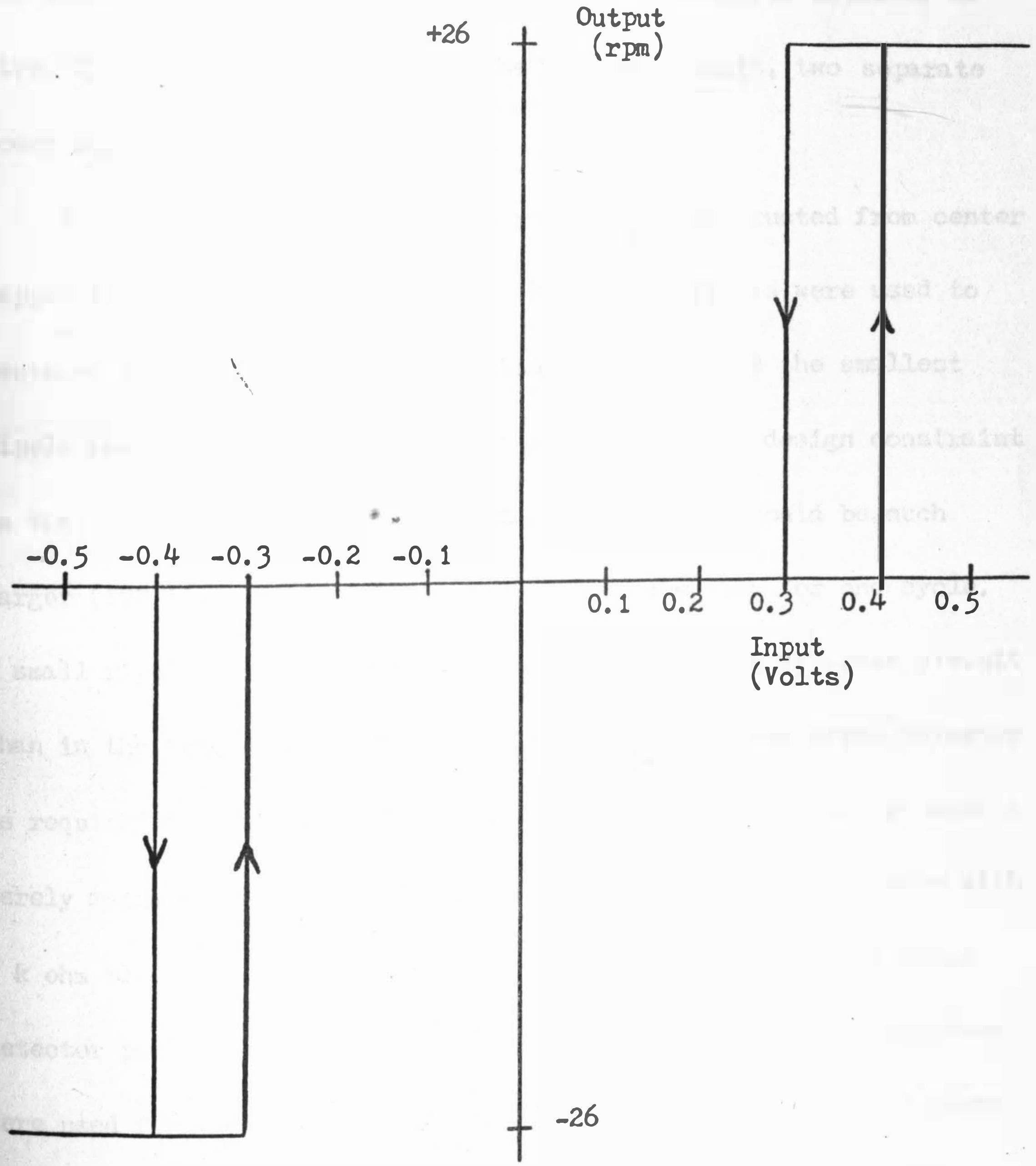


Figure 10

mentioned previously the output of the differential amplifier must be floating with respect to its power supply ground in order to have the output symmetrical. Since the output of the error detector is directly coupled to the input of the trigger circuit, two separate power supplies must be designed.

It was decided that full wave rectifiers constructed from center tapped transformers would be used. Capacitor filters were used to decrease the ripple factor. Capacitor filters yield the smallest ripple factor with the fewest components. The main design constraint is that the discharge time constant of the filter should be much larger (100 times) than "T/2", where "T" is the time for one cycle. A small ripple factor is more important in the error detector circuit than in the trigger circuit. This is true because the error detector is required to perform linear amplification while the trigger circuit merely actuates the relays. For this reason 1000 uF. capacitors with 1 k ohm bleeder resistors were chosen for the filters of the error detector power supply, while 500 uF. capacitors and 500 ohm bleeders were used for the trigger circuit. The complete valve control power supply is shown in figure 11. Note the voltage dividers for the positive and negative supplies of the error detector are not equal valued. This is due to the fact that the positive supply draws more

VALVE CONTROL POWER SUPPLY

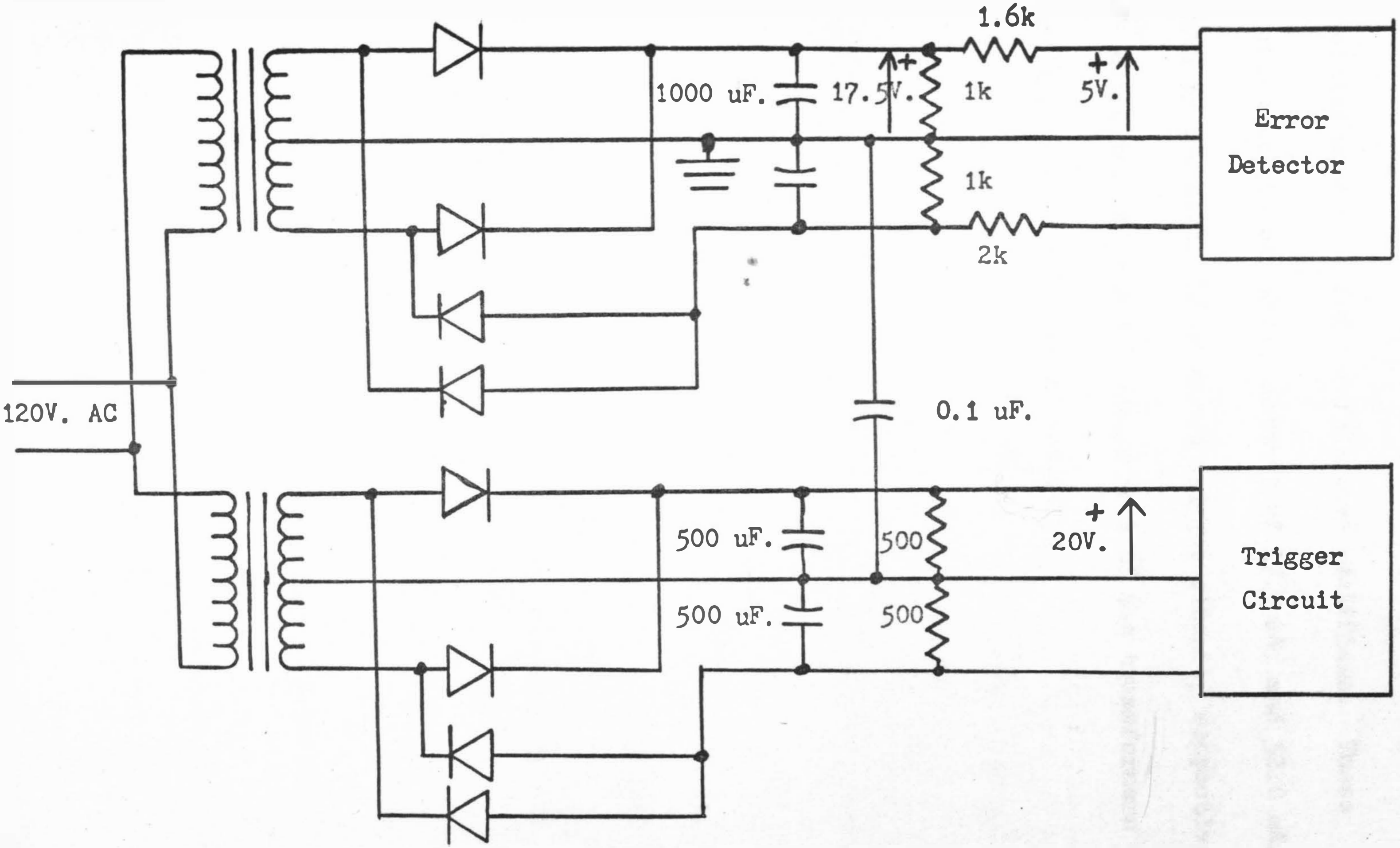


Figure 11

current than the negative supply (20), (25).

1N547 diodes were used for the full wave rectifiers. These diodes are rated at 500 mA. while currents of 27.5 mA. and 52.0 mA. will be drawn by the error detector and trigger circuit, respectively. The 0.1 uF. capacitor between the center taps of the transformers ensures that they are at the same AC potential.

The Program Control

The Sequencer

The heart of the program control is the sequencer. It is responsible for having the right station irrigating at the proper rate of flow. Note from figure 3 that the sequencer transmits the proper DC signal to the error detector of each valve control. Also one program control is responsible for up to ten stations with each station having a valve control.

As is implied by figure 3 the type of logic used by the sequencer is serial in and parallel out since one input signal controls several valves. The inputs are provided by the 24 hour clock and the flow control. Parallel out logic is necessary when at least two stations are irrigating at the same time.

A ten position, three wiper arm stepping switch was found to fill the requirements for the sequencer quite well. The wiper arm is mounted on a ratchet and spring arrangement such that one relay will advance it through the ten positions while a second relay is required to reset the device. The forward relay has a 132 ohm coil which can be actuated by ± 46 V. DC. The reset relay which can be actuated by the same voltage has a 211 ohm coil. The irrigation sequence can be hard-wired along the wiper arm contacts, since once the sequence is established it probably will not be changed for the entire season.

The 24 Hour Clock

The 24 hour clock is the programmable portion of this control. As was pointed out in the previous section, the clock must provide two signals. One signal advances the sequencer through its ten positions, while the other signal resets the sequencer at the end of an irrigation cycle. The clock which was implemented is easily programmable by positioning tabs at the edge of its 24 hour disk. These tabs actuate the advance and reset relays of the sequencer at a minimum of 5 minute intervals. This clock also has the capability of being programmed on a weekly basis. It also has the advantage of skipping any twelve or twenty-four hour period should weather conditions dictate the interruption of the irrigation schedule.

A manual cycle switch is also incorporated into the program control. This switch, which is in parallel with the 24 hour clock, allows the irrigator to manually sequence through the irrigation cycle. Added irrigation may in this manner be supplied to any station or group of stations which may warrant it.

The Flow Control

As has been stated previously, the total range of regulation required by the valve control is from 4 inches to 36 inches of head.

This range can be divided into the high or initial flow rate and the low or cutback flow rate. The low flow rate corresponds to a pressure range of 4 to 20 inches of head, while the high flow rate is in the range of 16 to 36 inches of head. If V_L and V_H represent the voltages corresponding to the low and high flow rates, from figure 4 and equation 2 the voltage ranges are:

$$V_L = 0.144 \text{ V. to } 0.72 \text{ V.} \qquad \text{(Equation 9)}$$

$$V_H = 0.575 \text{ V. to } 1.30 \text{ V.} \qquad \text{(Equation 10)}$$

It is desirable to set V_L and V_H by potentiometers and at nearly the same value for all stations. The design has included the flexibility of changing the high and low flow rates from day to day to compensate for changing weather and soil conditions. Once the irrigation cycle has started, however, these rates need not be changed from station to station.

The complete program control circuit diagram is shown in figure 12. The same power supply design constraints used for the error detector are used here. Note that because of the small resistance of the stepping switch coils the filter capacitor has to be 5000 uF. The 2.7 k ohm bleeder allows discharge of the capacitor when the power is disconnected. WE1265 diodes which are rated at 1 Amp. must be used for the full wave rectifier because of the 370 mA. current

PROGRAM CONTROL CIRCUIT DIAGRAM

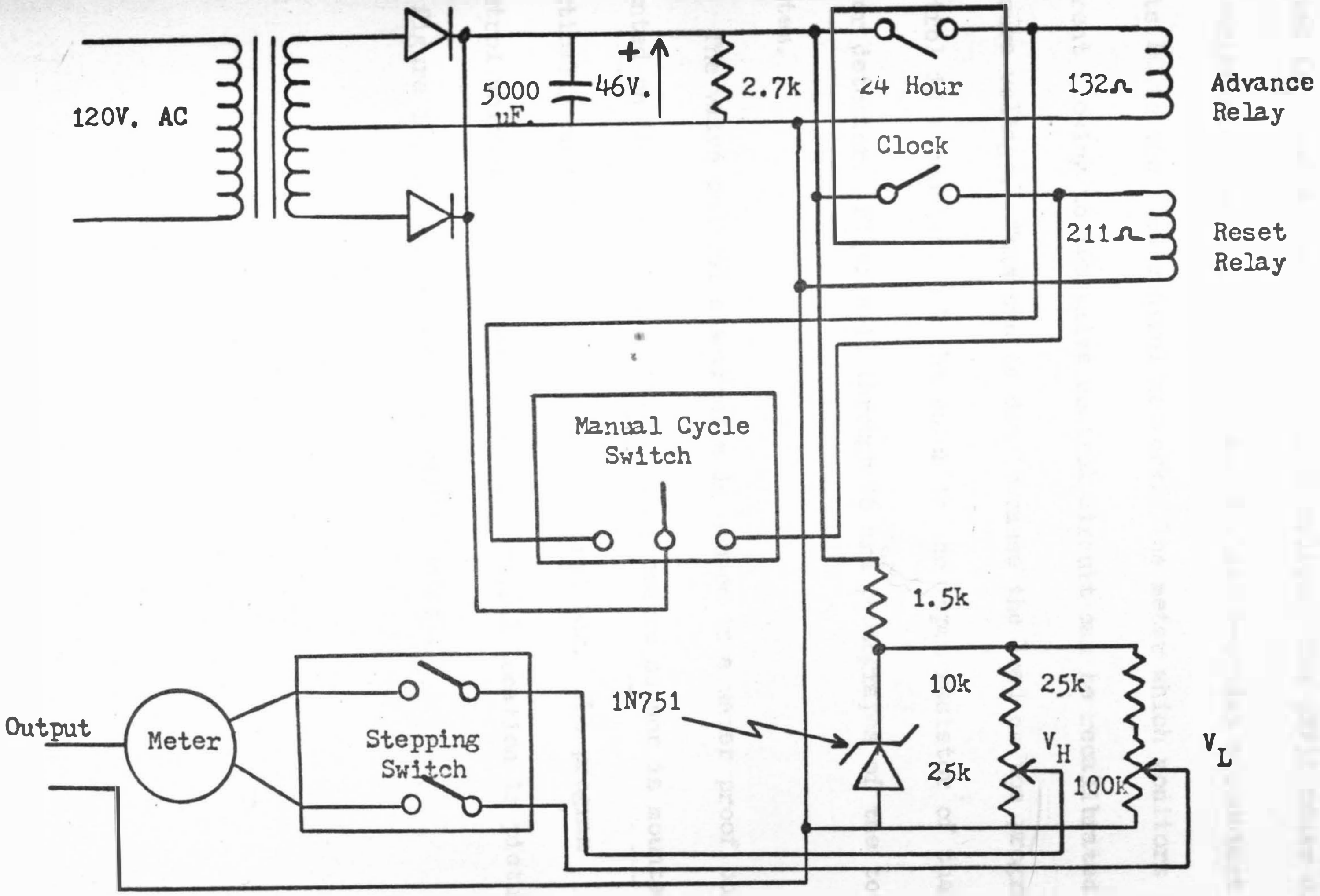
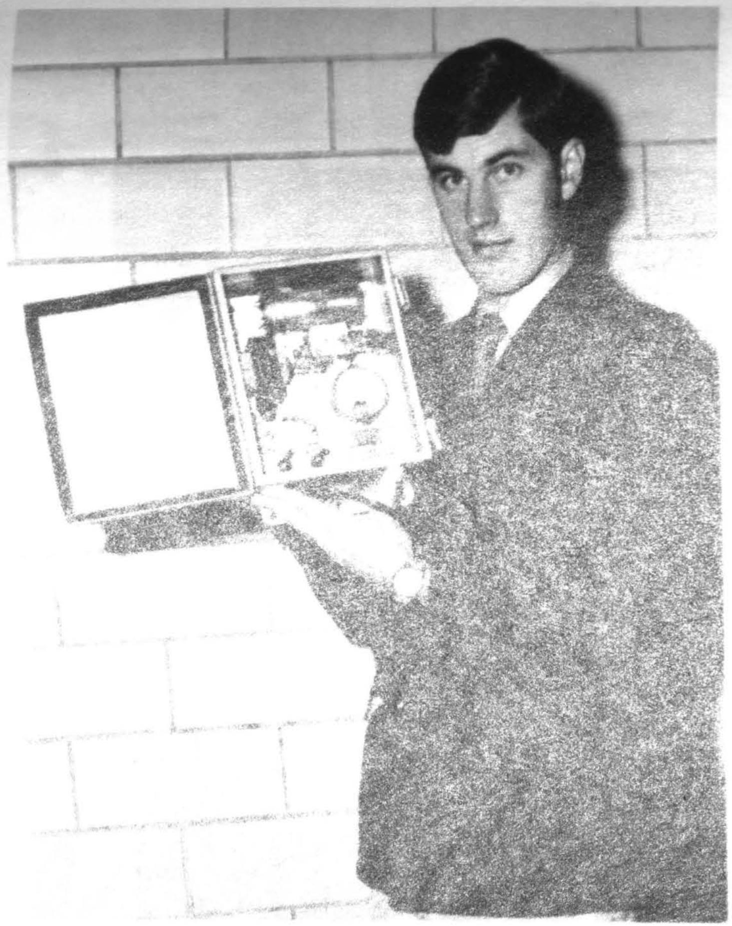


Figure 12

needed to actuate the stepping switch relays. The 1N751 zener diode in series with the 1.5 k ohm voltage divider provides a constant 5.1 volts DC to the flow control network. The meter which monitors the current flowing to the valve control circuit can be recalibrated to measure voltage. This can be done because the load on the program control is a constant 3 k ohm which is the input resistor of the error detector. Figures 13 through 16 are photographs of the total system.

The valve control electronics is housed in a water proof box mounted on the side of the valve. The pressure sensor is mounted vertically on the side of the valve control box. The program control which would be located at some central location is pictured in figure 16, to the left of the valve control.



...ner is pictured holding the Progr... Control which would be mounted at a ...tral location.

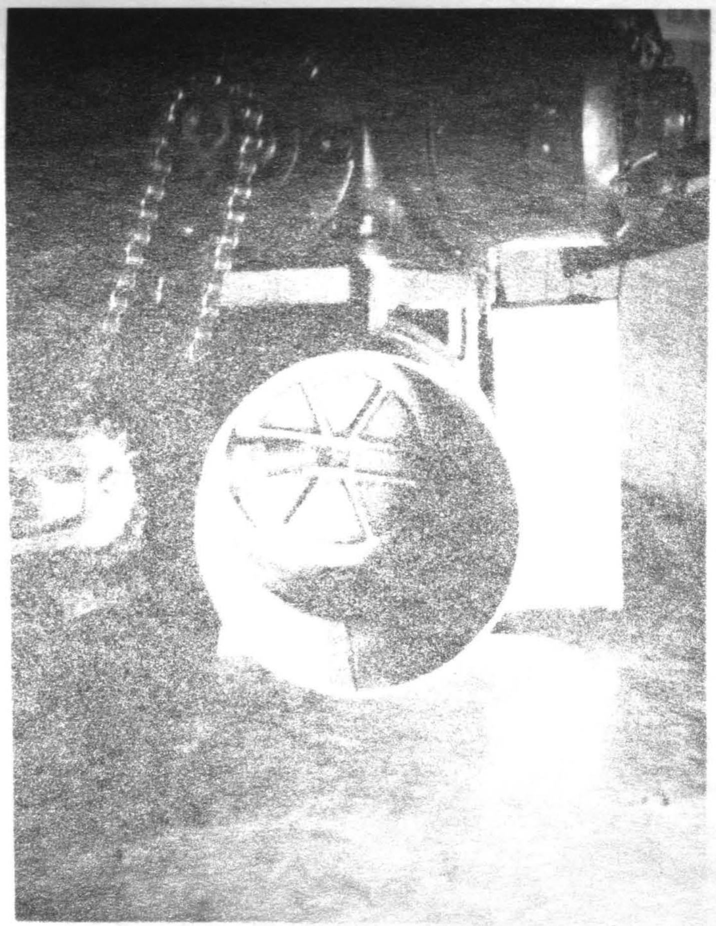


Figure 14 The upstream end of the valve assembly is pictured. The housing for the valve control electronics is mounted on the right side of the valve. The motor and gear mechanism is also shown.

Figure [illegible] [illegible] Sander points out the vertically mounted pressure sensor. The downstream end of the valve assembly is also pictured.

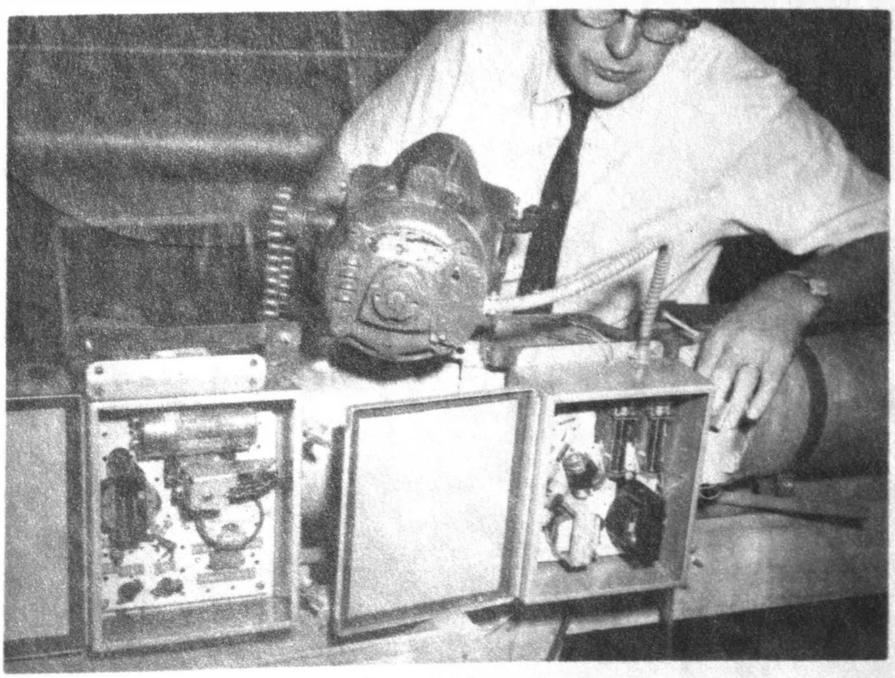


Figure 16. Dr. Wiersma is pictured with the Program Control on the left and the valve control on the right.

ANALYSIS

As was stated in a previous section the valve control block, in figure 3, can be made analogous to a closed loop, negative feedback, control system. From the transfer functions of the components within the loop, as shown in figures 4,5,8 and 10, it is evident that the valve control has some nonlinear components. The problem is to analyze the valve control with these nonlinear components interconnected with linear components.

Several techniques have been presented by Gibson (7), and others for the analysis of this type of nonlinear system. One technique which seems to work quite well is the describing function technique.

The Describing Function Technique (24)

The end result of any solution technique in control system theory is to try to reduce the system's block diagram to two component blocks which constitute the forward and feedback paths of the original system. This can be accomplished by representing each component block by its transfer function in the frequency domain and combining these component blocks by conventional block diagram techniques (16). When nonlinear component blocks are present in the system, more care must be taken in combining the blocks. The extension of conventional techniques requires that the nonlinearity be described by an equation in the frequency domain and that this equation be compatible for use with the transfer functions of the linear components. These requirements yield an "approximate transfer function" or "describing function" which is a linear approximation of the effect of the nonlinearity in the frequency domain.

This describing function hereafter referred to as an "equivalent gain", is derived by assuming a pure sinusoidal signal of constant amplitude and frequency is applied to the input of the nonlinearity. After steady state conditions prevail, the Fourier series of the output waveform is obtained. Assuming that there is no DC component and no subharmonics in the output waveform, the fundamental term in this Fourier series has the same frequency as the input signal but may differ in amplitude and phase. The equivalent gain is then defined as being the ratio of the magnitude of the fundamental term in the Fourier series of the output waveform to the magnitude of the input sinusoid at a phase angle which is the angle between the two waveforms. An equivalent gain may be found for all permissible amplitudes and frequencies of the input waveform.

Mathematically, if the input wave is represented by:

$$\text{Input} = A \sin \omega t \qquad \text{(Equation 11)}$$

and the fundamental term in the Fourier series of the output is:

$$\text{Fundamental term} = F(A,\omega)\left[\sin \omega t + \phi(A,\omega)\right] \quad \text{(Equation 12)}$$

then the equivalent gain for this nonlinearity is defined as:

$$\text{Equivalent gain} = \left|\frac{F(A,\omega)}{A}\right| \;\angle\, \phi(A,\omega) \quad \text{(Equation 13)}$$

The three basic assumptions applied in the derivation of the describing function are:

1. The input to the nonlinearity is a pure sinusoidal wave.
2. The output contains no DC component, no subharmonics, and all higher harmonics may be neglected.
3. The nonlinearity is not time-varying.

In a closed loop control system the describing function approximation is quite accurate if the linear part of the system contains low pass filters. This constraint allows the higher harmonics of the output waveform of the nonlinearity to be filtered out such that the feedback signal to the input of the nonlinearity is nearly a sinusoid. As will be pointed out in the next section, the valve control satisfies this requirement.

Applying the Describing Function Technique

It should be noted that all functions of time, $f(t)$, in this analysis are assumed to be zero for $t<0$. Therefore, the Fourier transform mentioned in the previous section is identical to the Laplace transform with the Fourier operator, $j\omega$, being replaced by the Laplace operator, s. These two transforms will be used interchangeably in this section when it is advantageous.

The equivalent control system block diagram of the valve control is shown in figure 17. The components of this diagram are somewhat different from those shown in figure 3. The $G_1(s)$ block in figure 17 constitutes the error detector and the time constant portion of the trigger circuit in figure 3. It is assumed, for the purpose of analysis, that the error detector is operating in the linear portion of its transfer function. That is, the input to the error detector, v_I, is greater than or equal to 0.04 V. but less than or equal to 0.5 V. The transfer function of figure 8 is replaced by a constant linear gain of ten. The $\frac{a}{s+a}$ portion of the $G_1(s)$ block is the "s" domain transfer function of the R-C input stage of the trigger circuit. This can be obtained by taking the Laplace transform of the time derivative of equation 7 with capacitors C1 and C2 initially discharged. When this is done, it is found that $a = \frac{1}{R1C1} = \frac{1}{R2C2} = 1$.

The $N_1(E_1)$ block in figure 17 presents the first nonlinearity to the system. This block is, of course, the trigger circuit and motor transfer function shown in figure 10. E_1, the input to this block is the output voltage of the error detector. The constant output of this block is 0.434 revolutions per second (rps) which is

EQUIVALENT BLOCK DIAGRAM

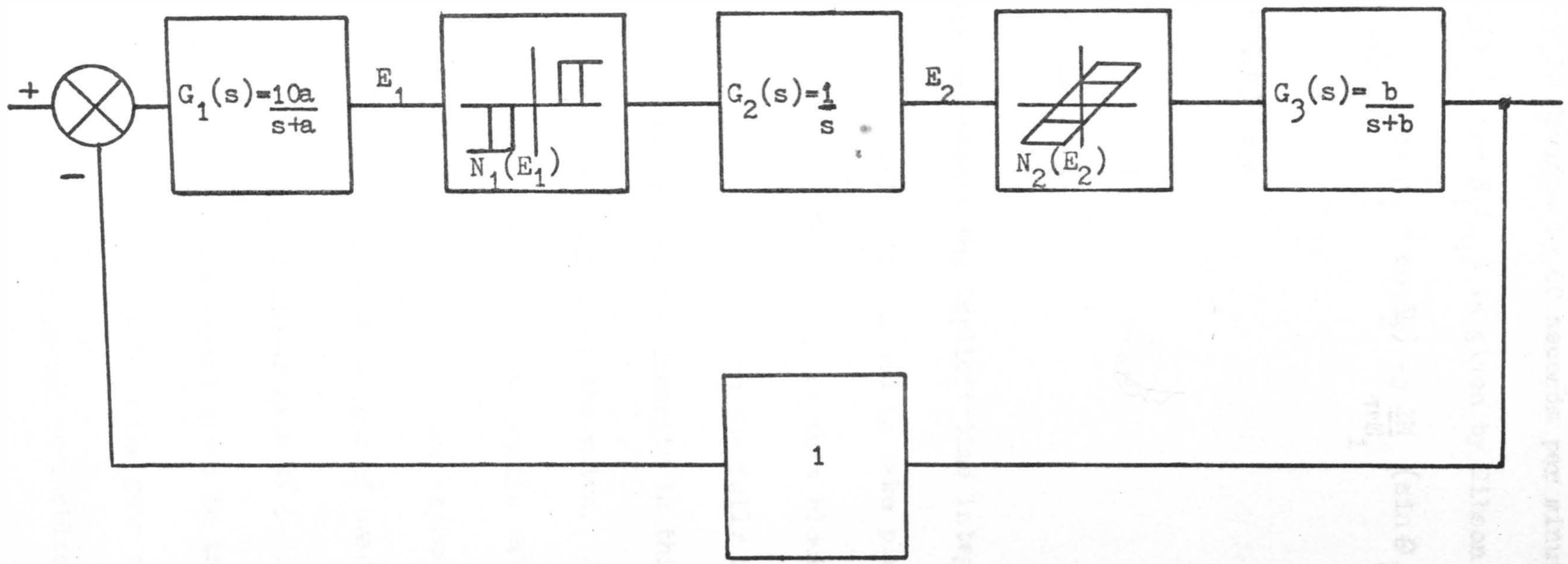


Figure 17

the 26 rpm in figure 10 divided by 60 seconds per minute. An equivalent gain, $K_1(E_1)$, for $N_1(E_1)$ is given by Gibson (7):

$$K_1(E_1) = \frac{2M}{\pi E_1}(\cos\theta_1 + \cos\theta_2) - j\frac{2M}{\pi E_1}(\sin\theta_1 - \sin\theta_2)$$

(Equation 14)

where: $M = 0.434$ rps

$$\theta_1 = \sin^{-1}\frac{0.4}{E_1}$$

$$\theta_2 = \sin^{-1}\frac{0.3}{E_1}$$

The $G_2(s)$ block represents the implicit time integration performed by the system. This integration is assumed to take place between the motor output and valve input. The output of this block, E_2, has the units of valve position in revolutions from the fully closed position.

The $N_2(E_2)$ block is the second nonlinearity in this system. This block represents the transfer function of the valve. As can be noted this block is quite different from the static valve transfer function presented in figure 5. A straight line approximation for these curves for an input pressure of four feet of head is presented here. The end result is a constant linear gain of 5.15 inches of output head per revolution. The horizontal lines in the transfer function of this block represent backlash in the gear mechanism between the motor and the valve. The amount of backlash, "c", which is difficult to measure in the physical system, will be a variable in the derivation of an equivalent gain for this nonlinearity. An equivalent gain, $K_2(E_2)$, for the $N_2(E_2)$ nonlinearity is given by Gibson (7):

$$K_2(E_2) = \frac{2N}{\pi}\left(\frac{\pi}{4} + \frac{\theta_3}{2} + \frac{\sin 2\theta_3}{4}\right) - j\,\frac{2N}{\pi}\cos^2\theta_3 \qquad \text{(Equation 15)}$$

where: $N = 0.186 \frac{\text{psi}}{\text{rev.}}$

$\theta_3 = \sin^{-1} \frac{c}{E_3}$

c = amount of backlash in revolutions.

The last component block in figure 17 which need be considered is the $G_3(s)$ block. This block represents the transfer function for the irrigation line and discharge bank. As was stated in a previous section, the line can be made analogous to a R-C low pass filter for the purpose of analysis. This equivalent circuit is shown in figure 18.

R1 in figure 18 is a resistor which is related to the friction losses in the line from the input of the valve to the pressure sensor input tap. R2 is a resistor which is related to the friction losses in the distribution bank. The capacitance, C, in this circuit is dependent upon the length and diameter of the distribution bank, and the rate of discharge from the bank. The value of these components is very difficult to measure. However, the overall effect of this component can be observed and a transfer function developed.

The transfer function, G(s), of the equivalent circuit in figure 18 is:

$$G(s) = \frac{\frac{1}{R1C}}{s + \frac{R1+R2}{R1R2C}}$$ (Equation 16)

If R2 is much greater than R1, as would normally be the case, since the orifice diameter is much smaller than the line diameter equation 16 reduces to:

$$G(s) = \frac{\frac{1}{R1C}}{s + \frac{1}{R1C}}$$ (Equation 17)

If "b" is equated to $\frac{1}{R1C}$, $G_3(s)$ is obtained:

$$G_3(s) = \frac{b}{s+b}$$ (Equation 18)

A value for "b" must now be found.

When the total system was assembled and tested in the hydraulics laboratory, it was found to oscillate. The frequency of oscillation was one cycle every nine seconds. The output of the pressure sensor was monitored by a strip-chart recorder. A portion of this data is shown in figure 19. The spikes in this waveform are due to the actuation of the relays which allow the motor to rotate. The spike which occurs at t=0 in figure 19 corresponds to opening of the relay which allows the motor to open the valve. It is shown that the pressure

LINE EQUIVALENT CIRCUIT

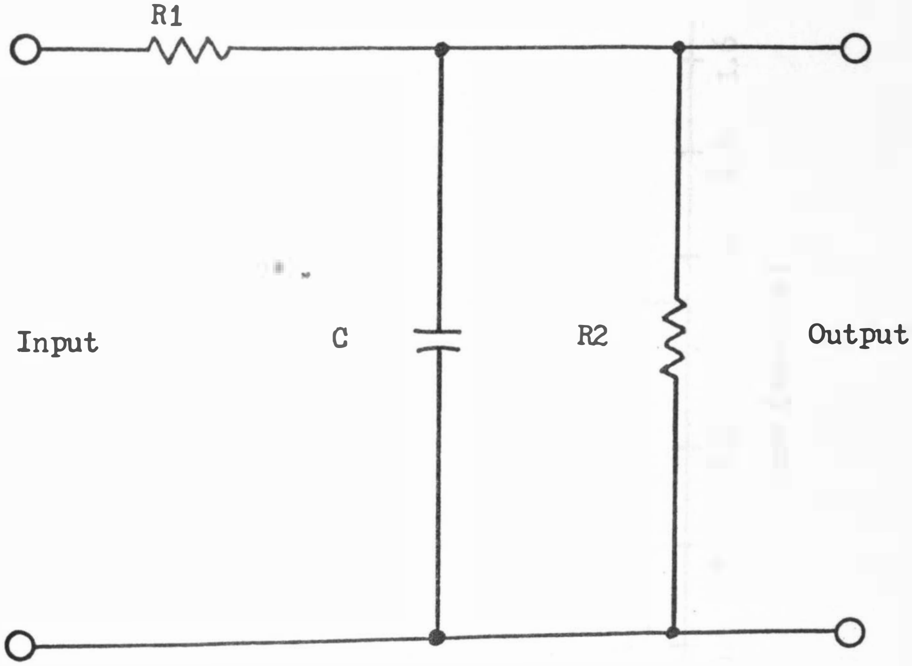


Figure 18

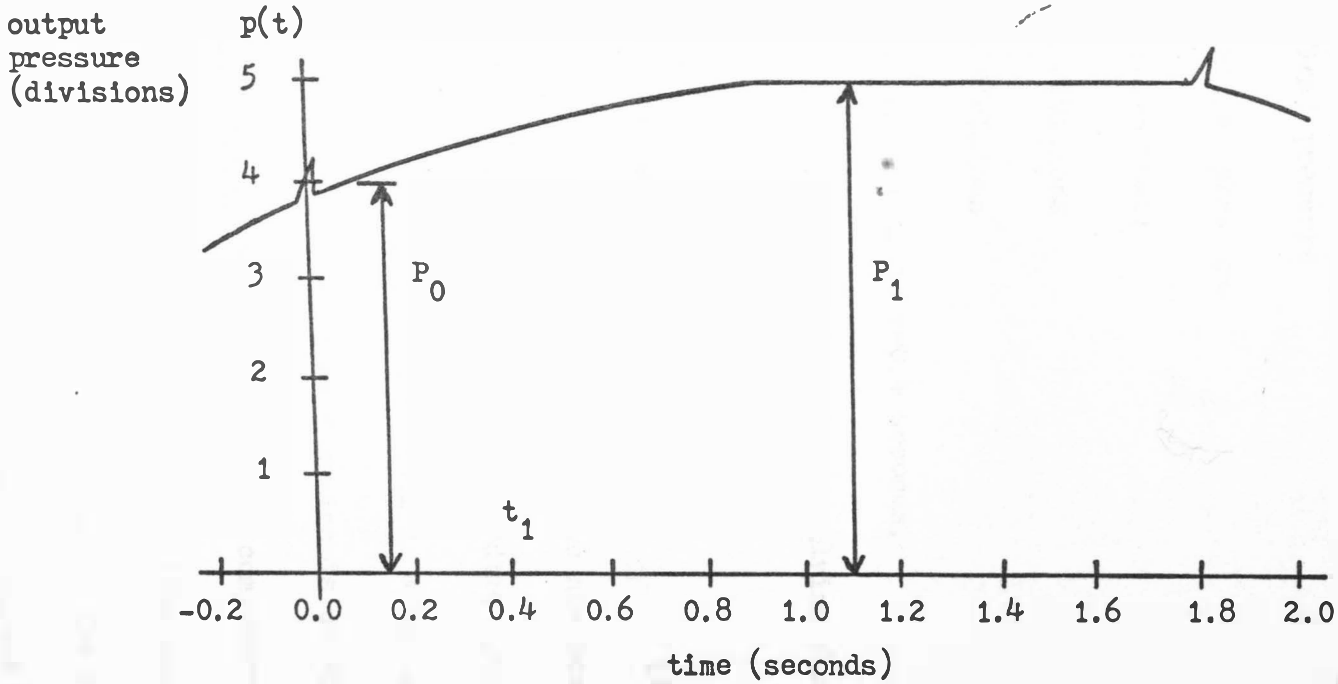

output
pressure
(divisions)
p(t)
5
4
3
2
1
P0
P1
t1
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
time (seconds)

Figure 19

continues to rise for a period of time after the motor has stopped. This implies the low pass filter analogy previously mentioned. The equation for the output pressure, $p(t)$, from $t=0$ to $t=t_1$ is:

$$p(t) = (P_1 - P_0)\,(1 - e^{-bt}) + P_0 \qquad \text{(Equation 19)}$$

where: P_1 = final pressure

P_0 = initial pressure

In figure 19 it is shown that:

P_1 = 5 divisions

P_0 = 4 divisions

$p(0.4)$ = 4.5 divisions at $t=0.4$ seconds.

Substituting these values into equation 19 and solving for "b" it is found that:

$$b = 1.74\ /\text{second} \qquad \text{(Equation 20)}$$

All of the component blocks in figure 17 have now been analyzed with the exception of the single block in the feedback path. This is, of course, the pressure sensor. Since this is a unity gain device which merely changes units, its transfer function is unity.

The previous implication is that all of the component blocks in the forward path of figure 17 can now be combined into one equivalent transfer function. This, however, is not the case. The best that can be done is to combine the linear parts into an equivalent linear transfer function and the nonlinear parts into an equivalent gain. Care must again be taken, for as is shown in figure 17 the nonlinearities are separated by a linear component. This requires a special technique.

A procedure for handling this situation is outlined by Thaler and Pastel (24). The equivalent gain of the combination of the two nonlinearities is desired:

$$K_{eq}(E_1\omega) = K_1(E_1)\ K_2(E_2) \qquad \text{(Equation 21)}$$

E_2 must be redefined in terms of E_1:

$$E_2 = E_1K_1(E_1)\ G_2(\omega) \qquad \text{(Equation 22)}$$

where: $G_2(\omega) = \frac{1}{\omega}$

Note that the 90° phase lag introduced by $G_2(\omega)$ has been neglected. This can be done because the equivalent gain is amplitude and frequency sensitive but not phase sensitive to its input.

A FORTRAN computer program was written and run on the IBM 360 digital computer to evaluate $K_{eq}(E_1\omega)$ as a function of radian frequency ω, the input signal $\frac{E_1}{10}$, and the backlash c. This program is shown in appendix A of this paper.

The linear component blocks of figure 17 can be combined by conventional techniques (16) to yield:

$$G_{eq}(s) = \frac{10ab}{s(s+a)(s+b)} \qquad \text{(Equation 23)}$$

The Fourier transform for the equivalent linear transfer function can be obtained by replacing "s" in equation 23 with "$j\omega$". A computer program, which is shown in appendix B, was written to evaluate this Fourier transform as a function of frequency.

The total valve control has now been reduced to two components, the equivalent gain $K_{eq}(E_1,\omega)$ and the equivalent linear transfer function $G_{eq}(j\omega)$. The transfer function, T.F., for the valve control now becomes:

$$\text{T.F.} = \frac{K_{eq}(E_1,\omega)\, G_{eq}(j\omega)}{1 + K_{eq}(E_1,\omega)\, G_{eq}(j\omega)} \qquad \text{(Equation 24)}$$

The denominator of equation 24, the characteristic equation, is most important. When the denominator of equation 24 takes on the value of zero, system instability could result. In particular, the system could oscillate. A plot of $G_{eq}(j\omega)$ and $\frac{-1}{K_{eq}(E_1,\omega)}$ can be made in the frequency domain. The intersections of these two loci are solutions to the characteristic equation. The two previously mentioned computer programs are used to obtain the data for this plot which is shown in figure 20.

The nonlinearity's family of curves in figure 20 are all calculated at a constant frequency of one cycle every nine seconds, f = 0.111 Hz.

The "X" indicates the point on the equivalent linear transfer function curve where f = 0.111 Hz. As is shown an intersection depicting a limit cycle is indicated for a backlash, "c", of 3/8 revolution. This is a reasonable value.

FREQUENCY DOMAIN PLOT

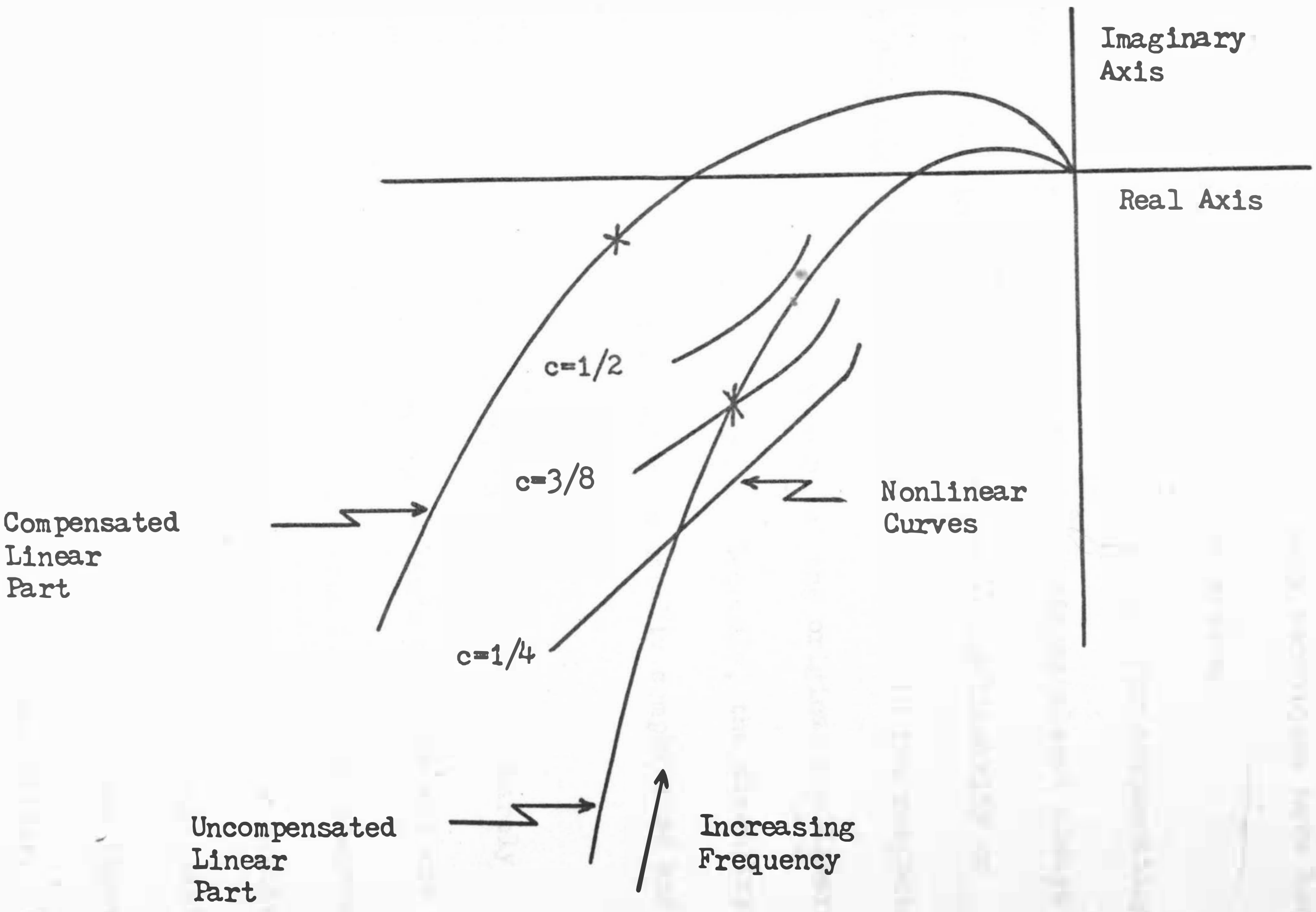


Figure 20

COMPENSATION

It is desirable to manipulate either the linear or nonlinear locus in figure 20 to avoid an intersection of the two and thus avoiding the possibility of oscillation. Many techniques have been proposed to accomplish this for this type of system.

Recent papers (19), (22) describe techniques for compensating the nonlinear part of the system. These techniques almost always employ the addition into the system of an extra nonlinearity or extra high frequency input signal. This is costly in two respects. First, these techniques tend to smooth out the original nonlinearity which would be undesirable in this case. Secondly, the circuitry required to implement these techniques is usually complicated and costly.

Linear compensation techniques, on the other hand, rarely produce any undesirable effects in the system's response and are more easily implemented. The "classical" technique for linear compensation is presented by Kochenberger (14). His method is to add a passive element filter into the system directly preceeding the nonlinearity. In this manner, phase lead or phase lag compensation of the linear locus can be obtained by the use of a high or low pass filter. The only disadvantage in this method is that by using passive elements in the compensation network the steady state error and thus the resolution is increased. This is due to the fact that the gain in any passive element network must be less than unity. The technique which will be used here is to employ an active element filter for the compensator network. In this manner the resolution of the control will not have to be sacrificed since the minimum gain is unity.

The circuit to be used will employ a UA-741 operational amplifier operated in the voltage follower mode. This operating mode ensures that the circuit gain will be a minimum of unity. Thus the resolution of the control is not affected. Unlike the inverting mode, the voltage follower mode provides no added phase shift between input and output.

The approximate voltage gain equation for the voltage follower mode is given by (4):

$$A_v = 1 + Z_F/Z_I \qquad \text{(Equation 25)}$$

where: Z_I = circuit input impedance

Z_F = impedance in the feedback loop.

If Z_F in equation 25 is replaced by a parallel R-C network, $R_F \| C$, and Z_I with a resistor, R_I, the circuit's "s" domain transfer function, $G_4(s)$, becomes:

$$G_4(s) = 1 + \frac{\frac{1}{R_I C}}{s + \frac{1}{R_F C}} \qquad \text{(Equation 26)}$$

This yields the desired low pass filter effect with the advantage of having no attenuation.

In figure 20 it is shown that an added 25^{o} of phase lag at the frequency of 0.111 Hz, would move the linear locus out of the range of possible intersections. With this as a design constraint and picking C and R_I to be 10 uF. and 10 k ohms respectively, R_F was found to be 78.4 k ohms. The total compensator circuit is shown in figure 21.

The 8.1 k ohm resistor in series with pin 3 represents the equivalent DC parallel of Z_F and Z_I. This helps to maintain DC stability. The 10 k ohm potentiometer between pins 1 and 5 can also be adjusted to maintain zero offset. The 100 k ohm potentiometer in series and the 8.3 k ohm resistor in shunt with the input represents a voltage divider network which adjusts the voltage gain of the circuit and thus the resolution of the control. This effectively shortens or lengthens the dead zone of the trigger circuit in figure 9. The $\pm$ 18 V. power supply which is required by this circuit is available from the error detector power supply in figure 11.

The computer program in appendix B is revised to include equation 26 in the equivalent transfer function of the linear part of the control. This program is shown in appendix C of this paper.

COMPENSATOR CIRCUIT DIAGRAM

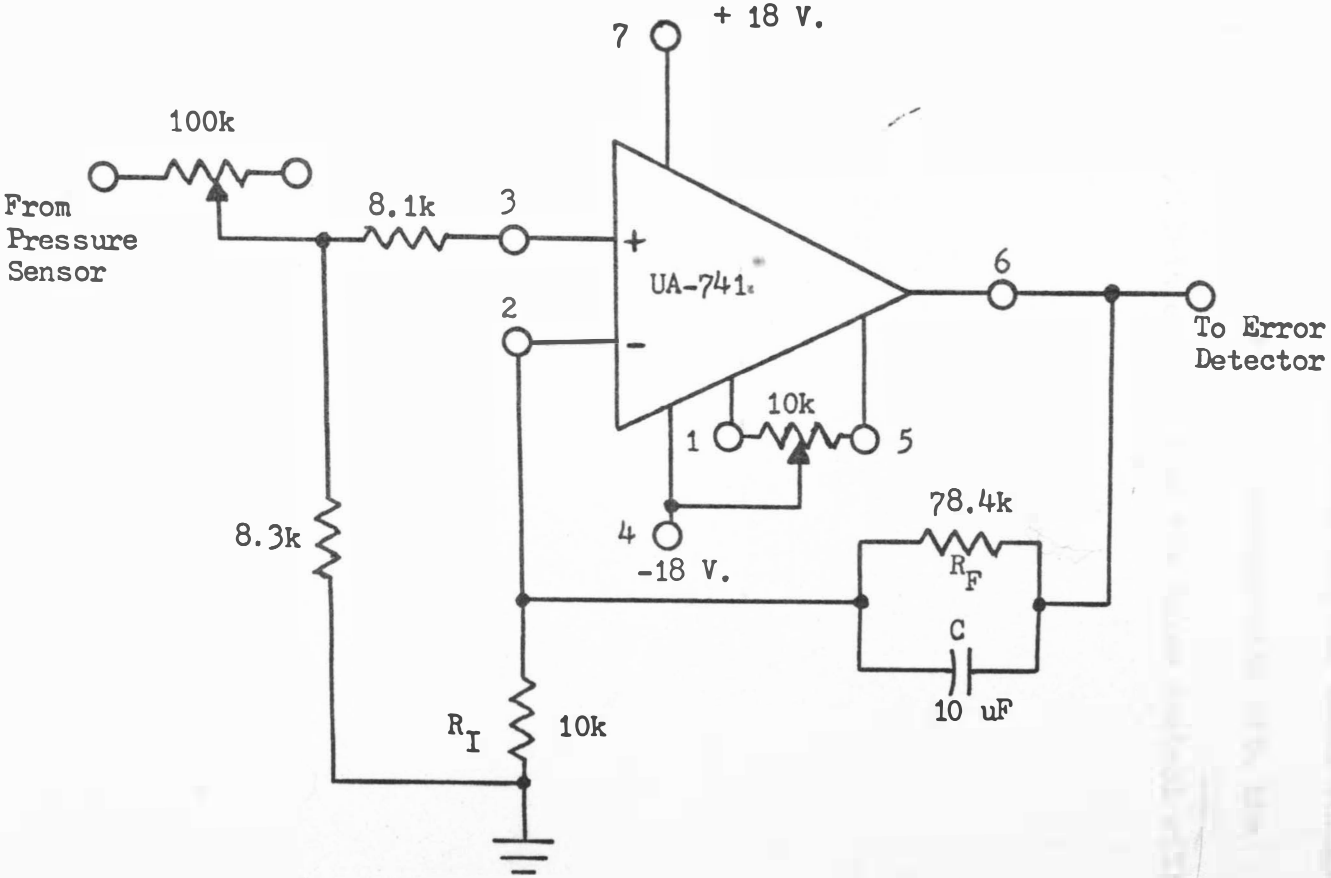


Figure 21

The locus of the compensated linear part in the frequency domain is also shown in figure 20.

As is shown in figure 20 the compensation has added enough phase lag into the control to avoid an intersection with the equivalent gain loci. This ensures that the valve control will not be unstable.

CONCLUSIONS AND RECOMMENDATIONS

It should be pointed out here, that the system is by no means a finished product. At best it represents an experimental model used in a study to determine the feasibility of controlling irrigation by electrical means. The concepts and methods used throughout this paper are all valid and seemed to work quite well. However, there have been several things left for further investigation.

When the compensated system was assembled and tested, it met with limited success. The valve control did not oscillate over the range of the valve characteristics which were considered. It did oscillate at a much lower frequency and larger amplitude when the valve was operating in the flatter portion of its characteristic curves. This could be due to the tremendous difference in the gain of the valve at these two positions. When the gain of the linear part of the system decreases the linear locus in figure 20 moves closer to the origin. This produces an intersection with the nonlinear loci at a lower frequency. Further research should be done and a study made into the area of finding the proper characteristics that a valve should possess to optimize the effect of the valve control.

Another area for further investigation includes a study investigating the feasibility of a digital time sampling system rather than the analog positioning system which was developed. In this system the valve control would be less sensitive to small variations in supply over short periods of time. This implies that some guide lines must be developed for the stability of the supply.

The value of the components in equivalent circuit of figure 18 has been left for further investigators. Modification of this circuit may be necessary, to incorporate the effect of the air which is trapped in the distribution bank during an irrigation. This effect is analogous to an underdamped spring mass system. The addition of an inductor "L" into the equivalent circuit would simulate this effect.

The last area for further investigation involves a study of the reliability, maintainability, and useful life of the system. As was pointed out in the introduction and state of the art, a useful life of at least 15 years is mandatory. This allows the total system to be economical.

BIBLIOGRAPHY

1. Bekey, G.A. Preparing Problems for Solution on an Analog Computer, Richmond, California: Berkeley Division, Beckman Instruments Inc., 1957.

2. Fishbach, P.E. "Design of an Automated Surface Irrigation System with Reuse System," Automation of Irrigation and Drainage Systems, National Irrigation and Drainage Specialty Conference, American Society of Civil Engineers, pp. 219-236, November, 1968.

3. Fishbach, P.E. "Efficiencies of an Automated Surface Irrigation System with and without a Runoff Reuse System," Paper No. 69-716, ASAE Winter Meeting, December, 1969.

4. Fitchen, F.C. Electronic Integrated Circuits and Systems, New York: Van Norstrand, 1970.

5. Garton, J.E. "Automation and Semi-Automation of Surface Irrigation," Automation of Irrigation and Drainage Systems, National Irrigation and Drainage Specialty Conference, American Society of Civil Engineers, pp. 237-252, November, 1968.

6. Garton, J.E., Beasely, R.P., and A.D. Barefoot, "Automating of Cut-Back Furrow Irrigation," Agricultural Engineering, Vol. 45, No. 6, pp. 328-29, June, 1964.

7. Gibson, J.E. Nonlinear Automatic Control, New York: McGraw-Hill Inc., 1963.

8. Golden, W.H., and C.H. Weaver "A Study of Contactor Servo-mechanisms with Positive Hysteresis," Transactions of the AIEE, Vol. 82, pt. 2, pp. 105-111, May, 1963.

9. Haise, H.R. and E.G. Kruse, "Automation of Surface Irrigation Systems: The State of the Art," Automation of Irrigation and Drainage Systems, National Irrigation and Drainage Specialty Conference, American Society of Civil Engineers, pp. 175-200, November 1968.

10. Hanson, R.E. "Water Distribution With Traveling Gun Sprinklers," Preprint No. 1321, paper presented at the ASCE National Water Resources Engineering Meeting in Phoenix, Arizona, January, 1971.

11. "Irrigator's Dictionary," Irrigation Age, Vol. 5, No. 4, pp. 46-47, November, 1970.

12. "Irrigator's Dictionary," Irrigation Age, Vol. 5, No. 5, pp. 38-39, December, 1970.

13. "Irrigator's Dictionary," Irrigation Age, Vol. 5, No. 7, pp. 33-34, February, 1971.

14. Kochenburger, R.J. "A Frequency Response Method for Analyzing and Synthesizing Contactor Servomechanisms," Transactions of the AIEE, Vol. 69, pt. 1, pp. 270-284, 1950.

15. Kuo, B.C. Automatic Control Systems, 2nd ed., New Jersey: Prentice-Hall Inc., 1967.

16. Langill, A.W. Automatic Control System Engineering, Vol. 1, New Jersey: Prentice-Hall Inc., 1965.

17. Meyer, W.A. "Controls: Key to Successful Automation," Automation of Irrigation and Drainage Systems, National Irrigation and Drainage Specialty Conference, American Society of Civil Engineers, pp. 169-174, November, 1968.

18. Millman, Jacob and Herbert Taub, Pulse Digital, and Switching Waveforms, New York: McGraw-Hill Inc., 1965.

19. Oldenburger, R. and Y. Ikebe, "Linearization of Time - Independent Nonlinearities by Use of an Extra Signal and Extra Nonlinearity," Transactions of the ASME, Vol. 89, pp. 241-250, 1967.

20. RCA Linear Integrated Circuits, Technical Series 1C-42, RCA Corporation, 1970.

21. Reynolds, J.E. "Allocating Water Among Alternative Uses," Journal of the Irrigation and Drainage Division, ASCE proceedings, pp. 85-92, March, 1971.

22. Sen, A.K. and R.A. Johnson, "A Nonlinear Compensation Technique for Systems Involving Hysteresis," IEEE Transactions on Automatic Control, Vol. AC-13, No. 6, December, 1968.

23. Snoozy, Robert S. "Design of the Distribution Portion of an Automated Closed Conduit Cut-Back Furrow Irrigation System," M.S. thesis in Agricultural Engineering, S.D.S.U., 1971.

24. Thaler, G.J. and M.P. Pastel, Analysis and Design of Nonlinear Feedback Control Systems, New York: McGraw-Hill Company, Inc., 1962.

25. Theriault, G.E. "Application of the RCA-CA3001 Integrated-Circuit Video Amplifier," RCA Integrated Circuits Application Note, ICAN-5038, September, 1967.

26. Van Valkenburg, M.E. Network Analysis, 2nd ed., New Jersey: Prentice-Hall Inc., 1964.

APPENDIX A

This is an IBM 360 computer program which evaluates the equivalent gain of the nonlinear part of the system. Radian frequency, backlash, and input voltage are variables in this program.

```
C      DESCRIBING FUNCTION ANALYSIS OF NONLINEARITY
       COMPLEX A1,A2,X,AEQ
   11  FORMAT(1H ,11F10.3)
       C=0.250
   10  E1=0.50
   20  Q1=ARSIN(0.4/E1)
       F=0.111
       Q2=ARSIN(0.3/E1)
       B1=(0.276/E1)*(COS(Q1)&COS(Q2))
       B2=(-0.276/E1)*(SIN(Q1)-SIN(Q2))
       A1=CMPLX(B1,B2)
       X=(E1*A1)/(F*6.28)
       E3=CABS(X)
       Q3=ARSIN(C/E3)
       D1=0.118*(0.785&(Q3/2.0)&(SIN(2.0*Q3))/4.0)
       D2=-0.118*(COS(Q3)**2)
       A2=CMPLX(D1,D2)
       AEQ=A1*A2
       Z1=REAL(-AEQ)
       Z2=AIMAG(-AEQ)
       Y=1.0/CABS(AEQ)
       Q4=-57.3*ATAN2(Z2,Z1)
       WRITE(12,11)A1,A2,Y,Q4,E1,C,F,AEQ
       E1=E1&0.10
       IF(E1.LE.1.0)GO TO 20
       C=C&0.125
       IF(C.LE.1.0)GO TO 10
       END
```

APPENDIX B

This is an IBM 360 computer program which evaluates the equivalent transfer function of the linear uncompensated part of the system. Radian frequency is the only variable in this program.

```
C       LINEAR PART OF THE SYSTEM
        COMPLEX D,E,X,R
      1 FORMAT(1H ,3F10.3)
        F=0.05
        A=1.0
        B=1.740
      2 W=6.28*F
        D=CMPLX(0.0,W)
        C=CMPLX(((W**2)&14.4)/((W**2)&1.64),-10.0*W/((W**2)&1.64))
        E=CMPLX(A,W)
        R=CMPLX(B,W)
        X=(10.0*A*B)/(D*E*R)
        G=CABS(X)
        Q=57.3*ATAN2(AIMAG(X),REAL(X))
        WRITE(12,1)G,F,Q
        F=F&0.025
        IF(F.LE.0.750)GO TO 2
        END
```

APPENDIX C

This is an IBM 360 computer program which evaluates the equivalent transfer function of the compensated linear part of the system. Radian frequency is the only variable in this program.

```
C       ACTIVE ELEMENT COMPENSATION OF THE LINEAR PART
        COMPLEX C,D,E,X,R
      1 FORMAT(1H ,3F10.3)
        F=0.05
        A=1.0
        B=1.740
      2 W=6.28*F
        D=CMPLX(0.0,W)
        C=CMPLX(((W**2)&14.4)/((W**2)&1.64),-10.0*W/((W**2)&1.64))
        E=CMPLX(A,W)
        R=CMPLX(B,W)
        X=(1.41*A*B*C)/(D*E*R)
        G=CABS(X)
        Q=57.3*ATAN2(AIMAG(X),REAL(X))
        WRITE(12,1)G,F,Q
        F=F&0.025
        IF(F.LE.0.750)GO TO 2
        END
```