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A Neoclassical Growth Model with an Environment Sector

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A NEOCLASSICAL GROWTH MODEL WITH AN ENVIRONMENTAL SECTOR

BY

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AN ENVIRONMENTAL SECTOR

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree. Acceptance of this thesis does not imply that the conclusions reached by the candidate are necessarily the conclusion of the major department.

Thesis Advisor Date

Head, Economics Department Date

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Chapter 1

INTRODUCTION

Pollution, in its various forms is rapidly becoming a major social and economic problem. It is an economic problem because it affects society's common property resources such as air and water. The topic of pollution arises in economic literature as it reaches dangerous levels in many parts of the world and its relation to production and consumption is recognized.

Of the many terms economists use in discussing quantities and types of pollution, the term residuals will be used most often in this paper. A residual can be defined as a by-product waste material that is not productively used but which is somehow disposed. To this definition can be added comparable tangible and intangible side effects. The effect of a residual in society is to lessen physical well-being and/or the psychic satisfaction of living.[1]

The effects of pollution in the form of residuals on physical and mental well-being relate directly to welfare theory. It has been suggested that social welfare will increase with growth in the economy (and the accompanied growth in pollution) until a point is reached beyond which social

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[1]Joe S. Bain, Environmental Decay (Boston: Little, Brown and Company, 1973), pp. 9-10.

welfare declines.[2] This point of view regards some mix of growth and pollution as optimal as related to social welfare. This optimal mix can be determined by an analysis of the trade-off between pollution and per capita consumption.

An optimum point can also be determined in welfare theory by equating the private cost of production with the social cost of production. Pollution is normally not considered a cost by the producer since he is only interested in the relevant direct costs. The social cost of production on the other hand includes all direct and indirect costs. The indirect costs include any costs imposed on others such as noise, heat, smoke, etc.

A flow of residuals in the production process then can be seen to cause a non-optimal welfare position since social cost of production will be greater than private cost of production. In this case marginal social benefit will be less than marginal social cost and welfare will not be maximized.

Because of the relationship between welfare and pollution there is a natural desire to limit the flow of residuals. Two often mentioned solutions to the problem of environmental degradation are a reduction in the growth of real output, and the use of pollution abatement equipment.

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[2] D. Donaldson and P. Victor, "On the Dynamics of Air Pollution", Canadian Journal of Economics, August, 1970, pp. 422-431.

Current discussions often link economic growth as a causative factor in the increase of pollution. Growth in the economy has a number of causes. These include technological innovation, resource discovery and population growth along with the most widely discussed, investment. Economic growth in its advanced stages can cause severe depletion of raw materials, environmental pollution from energy use, and overtaxing of the environment's capacity to absorb and recycle waste products.[3]

A simple solution to growth related pollution would be to reduce the rate of growth and therefore reduce pollution. Herein lies a dilemma. Edwin L. Dole has written that reducing production would cause massive unemployment. His basis for this statement is that the United States labor force grows at approximately one percent a year and potential per worker productivity increases by as much as three percent per year. Maintaining full employment then, requires the economy to grow at four percent a year. Reduction in population growth and improvement in pollution reducing technology are possible alternatives he offers to reducing pollution.[4]

Use of pollution abatement equipment is another alternative in the attempt to return society to an optimum welfare position. Abatement equipment is used to eliminate or reduce

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[3]Matthew Edel, Economics and the Environment (Englewood Cliffs: Prentice Hall, 1973), p. 58.

[4]Ibid., p. 66.

residuals in the environment. Residuals may be recycled, assimilated into the environment, or reduced by means of technologically better production techniques. All of these methods require producers to recognize indirect costs incurred due to residual flows and take positive action to create abatement processes.

Formation of a stock of pollution abatement equipment will require either increased saving (and therefore investment) and/or substitution of other (productive) capital to pollution abatement use. Since capital is now required for productive and non-productive purposes, the formation of abatement capital will have a definite influence on the process of capital accumulation. The most obvious result will be smaller increases in productive capital than would be possible with no requirement for investment in abatement equipment. Through this mechanism there will be an impact on the growth of income and consumption.

Presently there are few economic growth models which formally take into account residual flows and investment in pollution abatement capital. One such model was developed by Ralph C. d'Arge.[5] This is a Harrod-Domar type model which includes variables for waste flow, abatement capital and changes in pollution.

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[5]Ralph C. d'Arge, "Essay on Economic Growth and Environmental Quality", The Swedish Journal of Economics, March, 1971, pp. 25-41.

The purpose of this paper is to develop a growth model that incorporates residual flows and pollution abatement capital using a neoclassical growth model which is less restrictive in nature than that set forth by d'Arge. The following chapters will include a discussion of the d'Arge model, presentation of the neoclassical model, and finally a comparison of the results obtained in the two models.

Chapter 2

POLLUTION AND THE HARROD-DOMAR GROWTH MODEL[1]

D'Arge bases his analysis on a simple Harrod-Domar type of economic growth model. In this type of model it is assumed that saving is a fixed proportion of income, $S = sY$, where S is saving, Y is income, and s is the average and marginal propensity to save. The change in income is dependent upon the marginal productivity of capital, or $\frac{\Delta Y}{\Delta K}$, the change in income divided by the change in capital. If $\frac{\Delta Y}{\Delta K}$ is denoted as σ and ΔK is equal to investment, I, then $\Delta Y = \sigma I$. It is also assumed that planned saving is equal to planned investment, or

$$sY = S = I \tag{1}$$

We can now obtain:

$$sY = \frac{\Delta Y}{\sigma} \tag{2}$$

since $I = \frac{\Delta Y}{\sigma}$, and

$$\frac{\Delta Y}{Y} = s\sigma \tag{3}$$

This equation shows that the growth rate of income, is a function of the marginal propensity to save and the productivity of capital.

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[1]The analysis presented in this chapter is based on the model discussed in "Essay on Economic Growth and Environmental Quality", by Ralph C. d'Arge, in the Swedish Journal of Economics, March, 1971, pp. 25-41.

D'Arge introduces to the basic model the assumptions that pollution can be reduced by investment in abatement capital and the greater that stock of abatement capital, the lower the level of pollution. He begins with the relationship between residual flows, R, and income, Y.

$$R = r_c(Y-S) + r_yY \tag{4}$$

Residual flow is related to consumption (income minus saving) and total income. r_c is the residual flow per dollar of consumption and r_y is residual flow per dollar of income.

It is then postulated that environmental quality is determined by changes in the level of pollution, P. Changes in the level of pollution are given in equation (5), and are determined by the flow of residuals, investment in pollution abatement equipment, and the natural assimilative capacity of the environment.

$$\dot{P} = R - hIr - \lambda \tag{5}$$

$\dot{P}$ is the change in the level of pollution through time, $\frac{dP}{dt}$, Ir is investment in pollution abatement equipment, h is the rate at which pollution is abated per dollar of investment, and λ is the ability of the environment to assimilate residual flows, or a natural rate of decay of residuals per year. By combining equation (3), (4), and (5), a relation is shown between changes in pollution, investment in abatement equipment, saving, and output. This is shown in equation (6).

$$\dot{P} = (r_c + r_y)Y - r_cS - hIr - \lambda \tag{6}$$

It can be noted in equation (6) that even if saving and investment associated with pollution abatement are both zero, there is a rate of output which could cause no change in density of pollution. D'Arge equates this level of production with a biological equilibrium in which man's production of residuals is in balance with nature's ability to absorb them.

If Iy is used to designate investment in productive capital, total investment can be shown as the sum of productive and non-productive (abatement) capital.

$$I = Iy + Ir \tag{7}$$

And according to equations (1) and (2):

$$sY = Iy + Ir \tag{8}$$

$$\Delta Y = \sigma Iy \tag{9}$$

given that $\Delta Y = \frac{dY}{dt} = \dot{Y}$

$$\dot{Y} = \sigma Iy \tag{10}$$

where $\dot{Y}$ equals $\frac{dY}{dt}$, and Iy now equals $\frac{dKy}{dt}$, where Ky represents productive capital. From equations (8) and (10) can be obtained an equation similar to the Harrod-Domar growth equation in equation (3).

$$\frac{\dot{Y}}{Y} = s\sigma - \sigma\left(\frac{Ir}{Y}\right) \tag{11}$$

Without investment in pollution abatement, the results are the same as the Harrod-Domar solution in equation (3), which is the warranted rate of growth, $\frac{\Delta Y}{Y} = s\sigma$. With investment in abatement equipment the growth of income is reduced by this investment.

To examine the relationship between growth and pollution, an equation for change in pollution is obtained by using equations (6), (8), and (11). Solving equation (11) for Ir, substituting into equation (6), and substituting sY for saving from equation (8) into equation (6) yields:

$$\dot{P} = [r_c(1-s) + r_y - hs]Y + \frac{h}{\sigma}Y - \lambda. \qquad (12)$$

To obtain a rate of growth, set $\dot{P}$ equal to zero, multiply through the equation by σ, and divide each term by h, which yields:

$$\dot{Y} + \sigma[\frac{r_c}{h}(1-s) + \frac{r_y}{h} - s]\,Y - \frac{\lambda\sigma}{h} = 0. \qquad (13)$$

Equation (13) is a first order differential equation of the form:

$$\dot{Y} + aY - n = 0 \qquad (14)$$

where $a = \sigma[\frac{r_c}{h}(1-s) + \frac{r_y}{h} - s]$, and $n = \frac{\lambda\sigma}{h}$. The solution to equation (14) is:

$$Y(t) = Ze^{-at} + \frac{n}{a}^{2} \qquad (15)$$

where Z is a constant determined by initial conditions. Equation (15) determines the growth path of income when the change in pollution is assumed to be zero. In this case income will grow at a positive rate if the bracketed term in equation (13) (a in equations (14) and (15)) is negative. This term will be negative if:

$$r_c(1-s) + r_y < hs. \qquad (16)$$

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[2]Ralph C. d'Arge, "Essay on Economic Growth and Environmental Quality", The Swedish Journal of Economics, March, 1971, p. 33.

To clarify the meaning of the model, the h term in equation (16) can be defined as the reduction in pollution per dollar of abatement investment. Using current estimates for the United States, r_c plus r_y is approximately equal to six pounds per dollar, and s is equal to .20. The criterion for a positive rate of growth can then be determined by changing equation (16) to:

$$r_c + r_y - r_c s < hs \qquad (17)$$

$$\frac{r_c + r_y}{s} - r_c < h \qquad (18)$$

if $r = r_c + r_y$, then:

$$\frac{r}{s} - r_c < h. \qquad (19)$$

Using the estimates previously stated, $\frac{r}{s} = 30$. Therefore, for positive growth, h must be greater than $30 - r_c$. Assuming r_c is two pounds per dollar, a dollar of investment in pollution abatement must reduce pollution a minimum of 28 pounds. It can also be seen that if the propensity to save, and therefore the rate of investment, is decreased, the productivity of investment must increase substantially to maintain positive growth.

D'Arge's major conclusions concerning this model are:

1. In the long run the propensity to save influences whether a positive rate of growth is warranted when a constraint is imposed on utilization of the environment.

2. Shifts in the propensity to save not only increase growth potential, but provide necessary investment for pollution abatement.

3. There is overutilization of the natural environment due to the common property character of almost all environmental resources. If there is a failure to define common property rights, there is no incentive to invest in pollution abatement equipment.

4. A high efficiency of abatement investment (i.e., high productivity of abatement capital) is necessary to maintain growth with a minimum of non-productive investment.

5. There is a high degree of interdependence between decisions on economic growth and the environment.

6. Economic growth and environmental quality are only compatible in the long run provided that as the growth in output occurs, a significant proportion of investment is directed toward pollution abatement.

Chapter 3

POLLUTION AND THE NEOCLASSICAL GROWTH MODEL

SIMPLE NEOCLASSICAL GROWTH

The simple neoclassical model used in this discussion is based on an analysis of monetary growth by Jerome L. Stein.[1] In this model, full employment of the labor force is assumed, and real output is dependent upon capital and labor. The production function is linear and homogeneous and is given in equation (1).

$$Y = f(K,N) \tag{1}$$

where:

$$N = N_o e^{nt} \tag{2}$$

K equals capital, and N equals the labor force. Equation (2) shows the labor force grows exponentially at rate n. The assumption of linear homogeneity allows:

$$\theta Y = f(\theta K, \theta N) \tag{3}$$

and letting $\theta = \frac{1}{N}$

$$Y = Nf(\frac{K}{N}) \tag{4}$$

or

$$y = f(k) \tag{5}$$

where $k = \frac{K}{N}$, the capital-labor ratio.

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[1]Jerome L. Stein, "Monetary Growth in Perspective", American Economic Review, LX, No. 1, (March, 1970), 85-106.

The equilibrium equation for output is:

$$y = c + i \quad (6)$$

where $y = \frac{Y}{N}$, per capita output, $i = \frac{I}{N}$, per capita investment, and $c = \frac{C}{N}$, per capita consumption where:

$$C = c(K,N) \quad (7)$$

and it can again be shown that with linear homogeneity that

$$c = c(k). \quad (8)$$

Per capita investment, i, can be considered as the sum of two parts: the investment per worker to maintain the current capital-labor ratio, nk, plus the rate of change of the capital-labor ratio through time, $Dk(D \equiv \frac{d}{dt})$. Since

$$i = \frac{DK}{N} \quad (9)$$

we have

$$i = \frac{DK}{N} = nk + Dk \quad (10)$$

By sutstituting equation (10) into equation (6) we obtain

$$y = c + nk + Dk \quad (11)$$

or rearranging

$$Dk = (y - nk) - c \quad (12)$$

Equation (12) is graphed in Figure 1.

The curve (y-nk) represents the amount of per capita output available for per capita consumption plus the change in the capital-labor ratio (i.e., total net production). The shape of this curve is based on the fact that capital is required for production and the law of diminishing returns operates. The c curve represents consumption as a linear function of k, which was also shown in equation (8). Equilibrium is

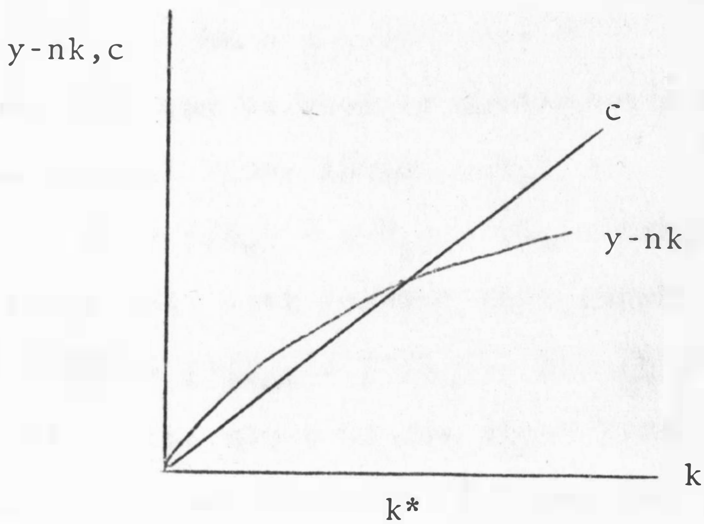

Figure 1

Neoclassical Equilibrium

attained at k^* since, if the capital-labor ratio was below k^*, (y-nk) would exceed c, and per capita output would be available to raise the capital-labor ratio. If the capital-labor ratio was above k^*, it would decline since c would exceed (y-nk).

In equilibrium there will be no change in the capital-labor ratio, so

$$Dk = y - nk - c = 0 \tag{13}$$

and equation (13) can be used to establish a function of the equilibrium capital-labor ratio, $\phi(k_o)$.

$$Dk = \phi(k_o) = f(k_o) - nk_o - c(k_o) \tag{14}$$

Differentiating (14) with respect to k gives

$$\frac{dDk}{dk} = \phi'(k_o) = f'(k_o) - n - \frac{\partial c}{\partial k} < 0 \tag{15}$$

Equation (15) is the slope of the phase line acquired by plotting the values of the distances between the two lines in Figure 1. This phase line is shown in Figure 2.

It can be seen that to have a stable equilibrium at k*, equation (15) must be less than zero (the slope of the phase line being negative). With any positive level of k, it can be seen that the movement in the model will cause a convergence on k*, the equilibrium capital-labor ratio. Maintenance of the equilibrium capital-labor ratio will require that capital, and therefore investment, increase at the same rate as the labor force so as to maintain k*. This implies that capital must grow at rate n.

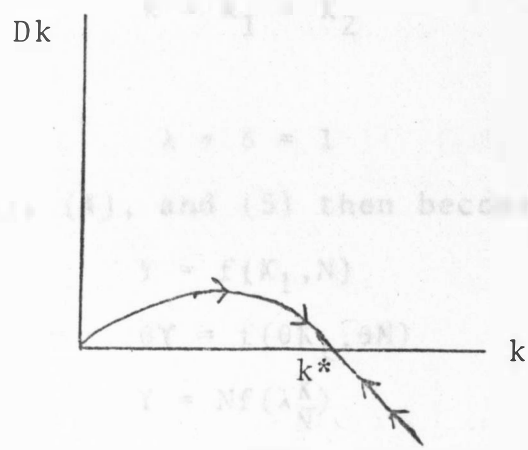

Figure 2

Phase Line

NEOCLASSICAL GROWTH WITH ABATEMENT CAPITAL

Pollution abatement capital is introduced into the model by means of splitting capital, K, into productive capital, and abatement capital which is also considered non-productive in the output sense. Productive capital is denoted as K_1, and

$$K_1 = \lambda K. \tag{16}$$

Abatement capital is denoted as K_2, and

$$K_2 = \delta K \tag{17}$$

$$K = K_1 + K_2 \tag{18}$$

and

$$\lambda + \delta = 1 \tag{18a}$$

Equations (1), (3), (4), and (5) then become

$$Y = f(K_1, N) \tag{19}$$

$$\theta Y = f(\theta K_1, \theta N) \tag{20}$$

$$Y = Nf(\lambda \frac{K}{N}) \tag{21}$$

$$y = f(\lambda k) \tag{22}$$

Substituting equation (22) into equation (14), we have

$$Dk = \phi(k) = f(\lambda k) - nk - c(k) \tag{23}$$

This substitution can be made since a new production function is used but the same overall capital-labor ratio is still being used. Again, this capital-labor ratio influences consumption, and investment $(\frac{DK}{N})$ is still made up of the two parts, nk and Dk, as discussed in equation (10). Differentiation with respect to k yields

$$\phi'(k) = f'(\lambda k) - n - \frac{\partial c}{\partial k} < 0 \tag{24}$$

Equation (24), as equation (15), must be less than zero. This will again cause an equilibrium position as did the mechanism which caused k*, in Figure 2, to be an equilibrium capital-labor ratio. If we call the equilibrium capital-labor ratio k_o, the equilibrium equation becomes

$$Dk = f(\lambda k_o) - nk_o - c(k_o) = 0. \tag{25}$$

To show the relationship of λ to the equilibrium k, we now differentiate totally for k_o and λ. This yields

$$f_{ko}dk_o + f_\lambda d\lambda - ndk_o - c_{ko}dk_o = 0 \tag{26}$$

where $f_{ko} = \frac{\partial f}{\partial k_o}$, $f\lambda = \frac{\partial f}{\partial \lambda}$, and $c_{ko} = \frac{\partial c}{\partial k_o}$. By rearranging terms we can obtain

$$dk_o(f_{ko} - n - c_{ko}) = -f_\lambda d\lambda. \tag{27}$$

$$\frac{dk_o}{d\lambda} = \frac{-f_\lambda}{f_{ko}-n-c_{ko}} \tag{28}$$

Examining equation (28), we see that the divisor $(f_{ko}-n-c_{ko})$, must be less than zero by equation (15), and since $-f_\lambda$ is negative, the entire term is positive. Therefore

$$\frac{dk_o}{d\lambda} > 0 \tag{29}$$

This term is the change in the capital-labor ratio related to a change in the proportion of capital devoted to production. It indicates that to maintain equilibrium, if the proportion of capital going to productive purposes is increased, there is an increase in the capital labor ratio.

Concentrating on contributions to pollution abatement then, we can see that since $\lambda = 1 - \delta$, $d\lambda = -d\delta$ and from equation (28):

$$\frac{dk_o}{-d\delta} = \frac{-f\lambda}{f_{k_o}-n-c_{k_o}} \tag{30}$$

In this case $\frac{dk_o}{d\delta}$ must be negative, indicating an increase in the proportion of capital going to pollution abatement will cause a decrease in the capital-labor ratio. In Figure 3 we can see that the original k* will necessarily decrease due to the mechanism discussed above, when increases are made in the proportion of capital going to pollution abatement.

Such an increase could cause a decrease in the equilibrium capital-labor ratio along with the indicated decreases in both (y-nk), or net production, and c, per capita consumption.

This position can be compared to d'Arge's situation in equation (11) of Chapter 2. D'Arge shows that capital investment will reduce growth of income. In the case of Figure 3 a mechanism including capital and labor reduces production and consumption and therefore reduces growth.

RESIDUAL FLOWS

To introduce residual flows to the analysis a general pollution function is added.

$$P = p(Y,K_2) \tag{31}$$

Pollution is a function of both output, Y, and pollution abatement capital, K_2. There are two means of minimizing pollution. Investment in K_2 can be increased, or Y can be decreased. A form for per capita investment which includes residual flows must now be found. First, the total differential of equation (31) is:

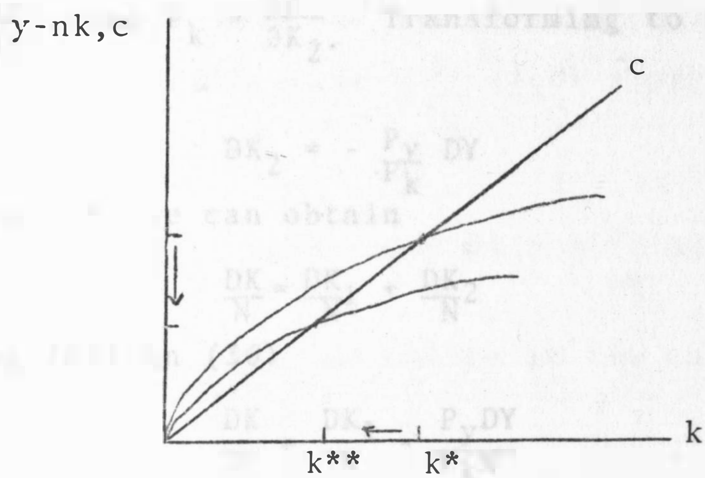

Figure 3

Equilibrium with Abatement
Investment

$$dP = \frac{\partial P}{\partial Y}\, dY + \frac{\partial P}{\partial K_2}\, dK_2 \tag{32}$$

The desired state is no change in pollution, so we postulate

$$dP = 0 \tag{33}$$

and therefore

$$dK_2 = -\frac{P_y}{P_k} dY \tag{34}$$

where $P_y = \frac{\partial P}{\partial Y}$, and $P_k = \frac{\partial P}{\partial K_2}$. Transforming to time derivitives we now have

$$DK_2 = - \frac{P_y}{P_k} DY \tag{35}$$

From equation (18) we can obtain

$$\frac{DK}{N} = \frac{DK_1}{N} + \frac{DK_2}{N} \tag{36}$$

and including (35) in (36)

$$\frac{DK}{N} = \frac{DK_1}{N} - \frac{P_y DY}{P_k N} \tag{37}$$

From equation (10), productive investment can be changed to the form:

$$\frac{DK_1}{N} = n\lambda k + \lambda Dk \tag{38}$$

since $\frac{DK_1}{N} = nk_1 + Dk_1$, and $Dk_1 = \lambda Dk$, $k_1 = \lambda k$. Substituting equation (38) into equation (37), gives total investment as

$$\frac{DK}{N} = n\lambda k + \lambda Dk - \frac{P_y DY}{P_k N} \tag{39}$$

A form must now be found to express $\frac{DY}{N}$ in equation (39) in terms of output. Since

$$Y = Nf(\lambda k) \tag{40}$$

from equations (21) and (22), then

$$DY = Nf_k Dk + yDN \tag{41}$$

where $f_k = \frac{\partial f}{\partial k}$. Therefore, dividing by N

$$\frac{DY}{N} = f_k Dk + yn \tag{42}$$

since $\frac{DN}{N}$ is simply the rate of increase in labor, n. Substituting (42) into (39)

$$\frac{DK}{N} = n\lambda k + \lambda Dk + - \frac{P_y(f_k Dk + yn)}{P_k} \tag{43}$$

and substituting this into the equilibrium equation, y = c + i,

$$y = c + n\lambda k + \lambda Dk - \frac{P_y}{P_k}(f_k Dk + yn) \tag{44}$$

Rearranging terms

$$y(1 + \frac{P_y n}{P_k}) - n\lambda k - c = Dk(\lambda - \frac{P_y f_k}{Pk}) \tag{45}$$

In equilibrium, Dk, the change in the capital-labor ratio, will be zero, so

$$(1 + \frac{P_y}{Pk}n)y - n\lambda k - c = 0 \tag{46}$$

If we set

$$1 + \frac{P_y n}{Pk} = \frac{P_k + P_y n}{Pk} = a \tag{47}$$

and substitute

$$ay - n\lambda k - c = 0 = Dk \tag{48}$$

The term, a, can now be analyzed. P_y, the change in pollution due to change in output is > 0. P_k, the change in pollution due to investment in abatement capital is < 0, and n is a rate > 0. If we then assume that $|P_k| > |P_y n|$, the entire term can be seen to be $0 < a < 1$ in equation (48). This is a valid assumption since sensibly, abatement capital will be more efficient at decreasing levels of pollution than productive capital will be at increasing levels of pollution.

To derive an equation compatible to Figure 1, we can divide equation (48) by λ to get

$$a^{\frac{1}{\lambda}} f(\lambda k) - nk - \frac{1}{\lambda} c = Dk \tag{49}$$

The term $a^{\frac{1}{\lambda}}$ is equal to $\frac{P_k + P_y n}{Pk}$. The vertical axis in Figure 4 is now used to graph $a\frac{1}{\lambda}y$ - nk, and $\frac{1}{\lambda}c$. Given equation (49), and setting the condition $|P_k \lambda| > |P_k + P_y n|$, the dotted lines in Figure 4 show the shift from y-nk to $a\frac{1}{\lambda}y$-nk, and from c to $\frac{1}{\lambda}c$. These shifts create a decrease in the capital-labor ratio from k* to k** when abatement investment and residual flows are included in the model. This capital-labor ratio is indeterminate if $|P_k \lambda| < |P_k + P_y n|$. This may occur since the inequality depends upon the value of λ, the proportion of capital devoted to productive use. A small enough value of λ may cause the inequality to be untrue.

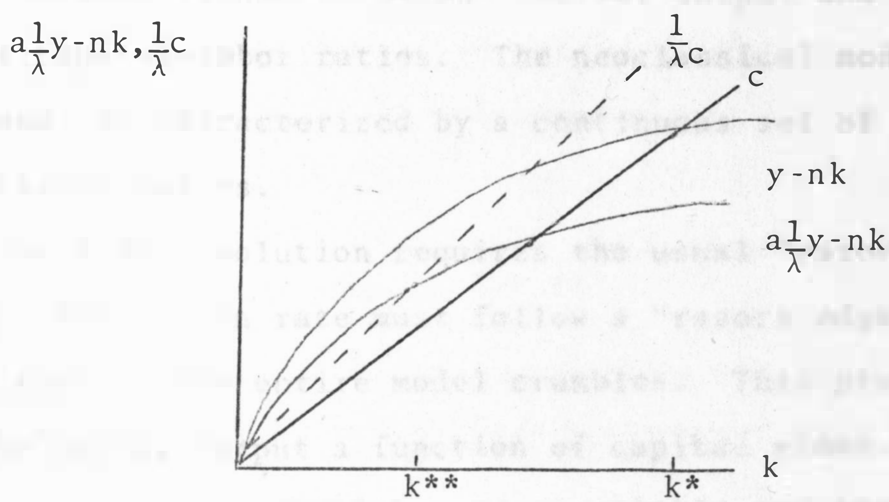

Figure 4

Equilibrium With Residual Flows

Chapter 4

CONCLUSION

A primary difference between the two models discussed is of course the assumptions concerning the capital-labor ratios. D'Arge assumes constant capital output and therefore constant capital-labor ratios. The neoclassical model, on the other hand, is characterized by a continuous set of alternative capital-labor ratios.

The d'Arge solution requires the usual "razors edge" analogy. The growth rate must follow a "razors edge" course through time or the entire model crumbles. This problem is caused by making output a function of capital alone. The neoclassical model includes labor as a variable and therefore the "razors edge" time path does not arise.

Contrary to d'Arge, the neoclassical model's growth is independent of the saving rate. Instead, a large number of other variables play an important role. Figure 4, derived from equation (49), indicates that both curves may shift to any possible degree based on the values of P_y, P_k, n, and λ. To maintain as high an equilibrium capital-labor ratio as we had in Figure 1, the values of the above variables must be delicately balanced.

Both P_y and P_k are highly influenced by the state of the art concerning pollution abatement. In determining the value of $\frac{Pk + P_y n}{Pk}$ I assumed that $|P_k| > |P_y n|$ which determined that $0 < a < 1$. This is actually a necessary condition for any equilibrium at all. As I stated, it is also a valid assumption. We are then concerned with finding production techniques which are as pollution free as possible, and developing pollution abatement processes which are highly efficient. D'Arge was in agreement with this conclusion. This model also requires that the growth rate of labor, n, and the productive proportion of capital, λ, be balanced so as to keep shifts in the curves to a minimum. Both n and λ influence shifts in (y-nk). λ also has an influence on the c cruve by way of its reciprocal.

There is still no incentive to invest in abatement capital present, and it is assumed that this incentive will have to come from an outside source. The major conclusion possible is that a high efficiency of abatement capital, and pollution free production processes are necessary to maintain a level of growth equal to that possible if pollution were not a problem. In addition, an attempt must be made to balance growth of labor and productive investment with these other two variables so as to reach an optimum state.

There are a number of ways the analysis may be expanded which may shed more light on the mechanism, and the balance required. It would be beneficial to determine what happens to k as there is a change in λ. This would give an indication of moves that may be made to increase growth.

It is also possible to add a monetary sector by means of another function. This would highly complicate the model, but may add some significant results and allow more concrete conclusions.

Finally, the model here, employs a non-changing production function. If technological change is allowed to occur, again a significant result may be obtained.

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