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Cooling Load Calculations of Heat Gain for Buildings

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COOLING LOAD CALCULATIONS OF HEAT GAIN FOR BUILDINGS

BY

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COOLING LOAD CALCULATIONS OF HEAT GAIN FOR BUILDINGS

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Adviser   Date

Head, Mechanical Engineering Department   Date

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H. C. C.

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NOMENCLATURE

A	Area of convecting surface
A_n	Area normal to radiation
C_p	Heat capacity
E_b	Emissive power of source
$E_\alpha, E_\beta, E_\gamma$	Emissive power in wavelength interval α, β, γ
h	Convective coefficient
L	Thickness of solid or depth of fluid
$q_{os}(t)$	Solar heat flux on hourly basis
$q_x(t)$	Heat removed or added
r	Coefficient of reflectivity
T	Temperature
$T_w(t)$	Temperature of surface w
t	Time
V	Volume
v	Wind velocity
ϵ	Emissivity
λ	Wavelength
μ	Coefficient of absorption
ρ	Density
σ	Stefan-Boltzmann constant
K	Conductivity
α	Diffusivity
U	Thermal conductance

$q_d(t)$ Difference of heat flux between the case of all energy absorbed and no energy absorbed

$q_1(t)$ Heat flux, released from the wall

$q_e(t)$ $q_d(t) - q_1(t)$

SUBSCRIPT

a,b,c,d,w Convecting surfaces

s Ambient surrounding

g Outside solid

f Airspace

p Inside solid

x Room air

1 Convecting surface of structure adjacent to the ambient air

2 Convecting surface of structure adjacent to the room air

CHAPTER I

INTRODUCTION

Two recent papers [1,2] obtain formulas for temperature variations caused by the varying flow of solar heat through windows. These two works are used in the present study to develop a computer program for calculating the transient cooling load required to maintain the interior of a building at a given temperature. We assume that the building is standing by itself without surrounding radiating buildings or pavements.

The heat gain through semi-transparent single or composite barriers is combined with the accompanying heat gain through the opaque walls; see Figure 1.

The analytical solutions given in [1,2] cannot be represented directly as numerical terms in the cooling load. This is because the useful solutions in [1,2] exist as explicit solutions for the air temperature of the room; the solutions also exist in the form of a formal integral representation with the terms of the cooling load involved in the integrals. To obtain a solution of this part of the problem in numberical terms, two steps are involved:

i. Numerical evaluations of the formal integral representation

ii. Inversion of the indirect solutions to obtain the numerical results for the required cooling load.

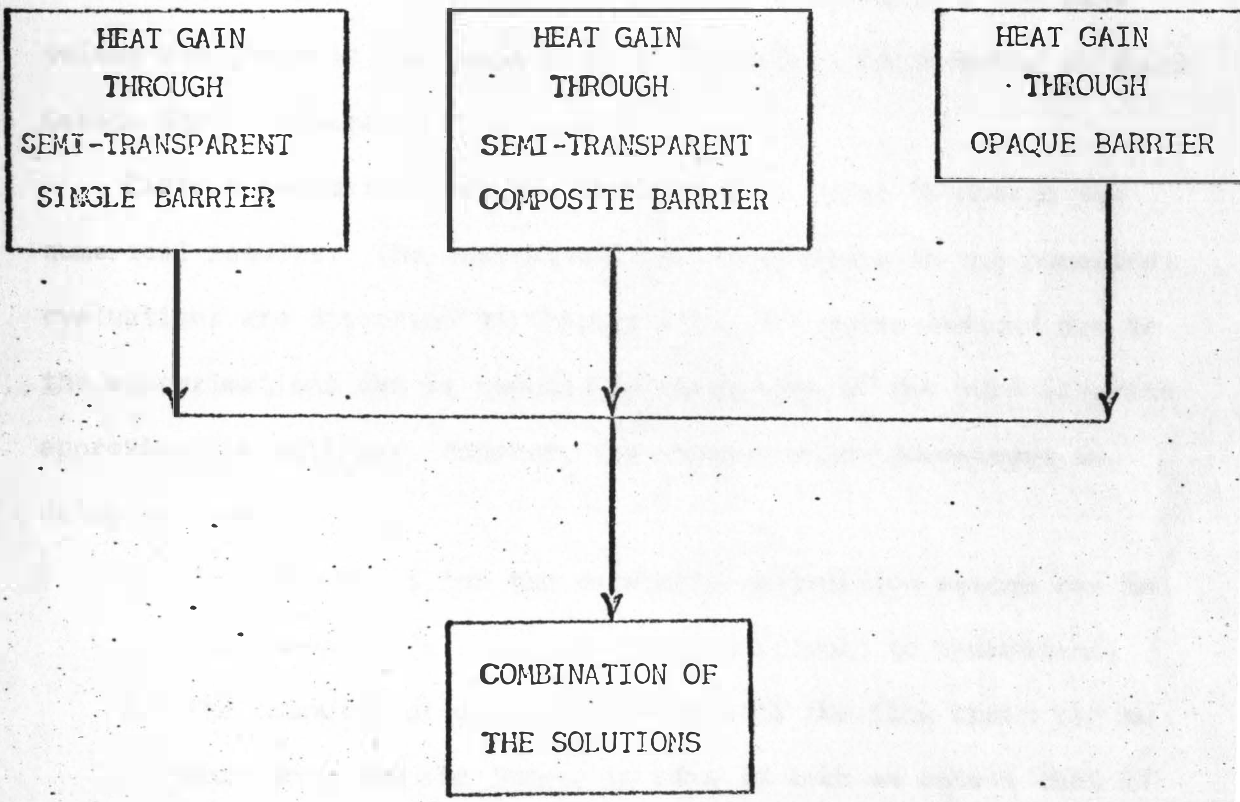

Figure 1. Contributions to the Cooling Load.

Two procedures can be followed to arrive at a solution: One, we use the analytical solutions given in [1,2]; and two, we solve our problem directly from the differential formulation given in these references by numerical method. The mathematical representation of our problem was formulated into a computer program to perform the automatic calculations. This computer program is given in a separate volume available in the Department of Mechanical Engineering at South Dakota State University.

Certain approximations are introduced in order to obtain the numerical results. The approximations associated with the numerical evaluations are discussed in Chapter III. The error induced due to the approximations can be reduced by using some of the more accurate approximation methods. However, the associated disadvantages in doing so are:

1. The flow chart for the automatic calculation system can be more complex in logic, and more difficult to understand.
2. The computer program associated with the flow chart can be more involved and larger in size to such an extent that it is not easily handled.

Also high accuracy can be attained by using smaller increments for each step of the calculations. But the required number of calculations will be greatly increased, and so will be the required computer time.

For engineering applications, we prefer to have our methods simple and practical, so that they can be applied without difficulty by those who are interested.

CHAPTER II

ANALYSIS

A. Assumptions

For the physical interpretation of our problem refer to Figures 2, 3, and [1,2]. Before our problem can be formulated into mathematical terms, certain assumptions are fundamental and have to be introduced into our problem. Those assumptions are:

1. Convective heat transfer is governed by Newton's law of cooling.

2. Room air temperature is uniform throughout.

3. Each solid surface taken into consideration is an isothermal surface.

4. Over each isothermal surface, the convective heat transfer coefficient is constant.

5. Temperature gradients are assumed to be zero in the normal direction through the semi-transparent solid and airspace.

6. Radiation heat transfer is proportional to that calculated from Stefan-Boltzmann's law of thermal radiation.

The Stefan-Boltzmann law of thermal radiation states that

$$q_r = \sigma A(T_1^4 - T_2^4)$$

between solids of temperature T_1 and T_2, where q_r is the amount of thermal radiation energy per unit time, σ is the Stefan-Boltzmann constant and A is the area of the radiating solid body. The Stefan-Boltzmann law

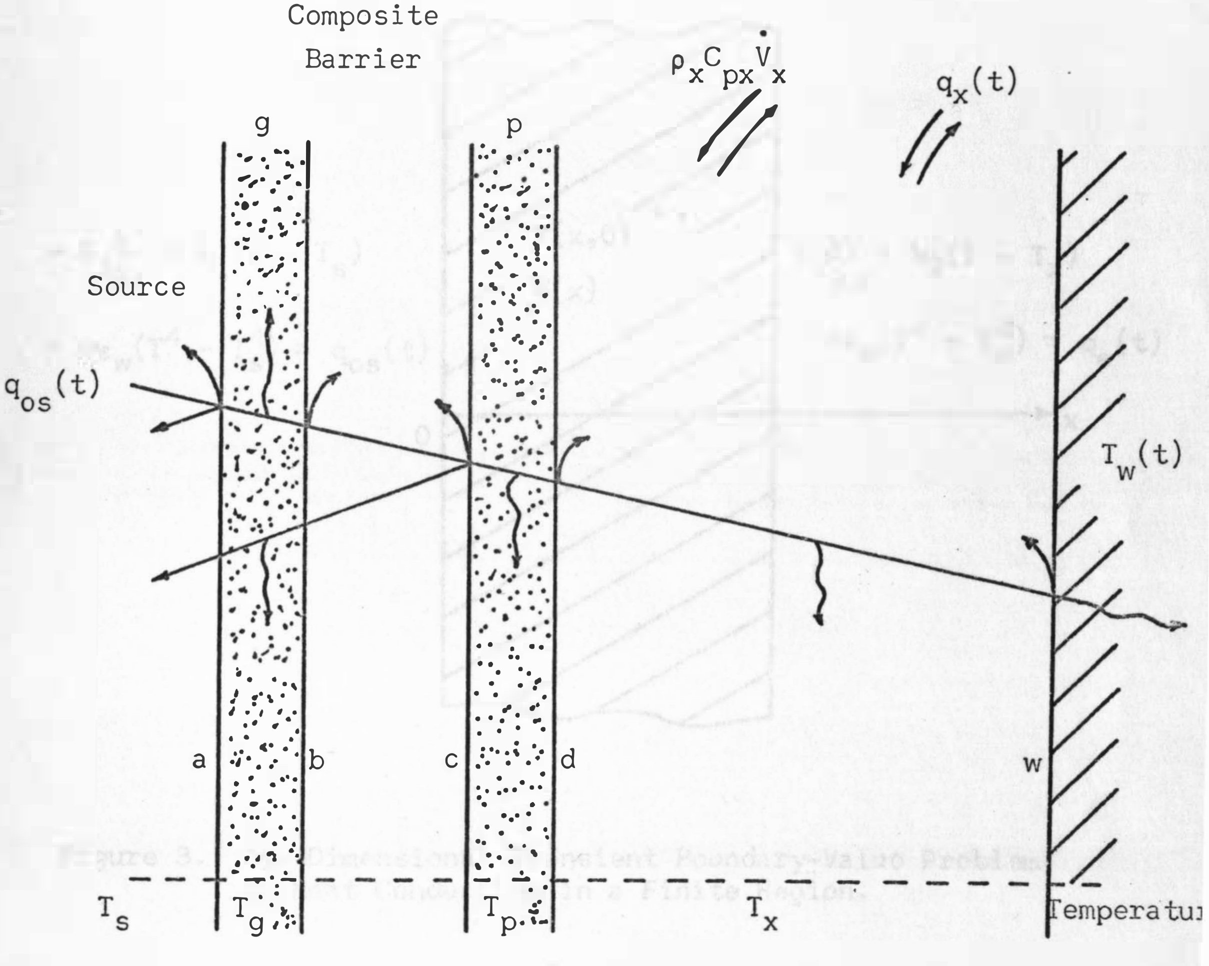

Figure 2. Composite Barrier.

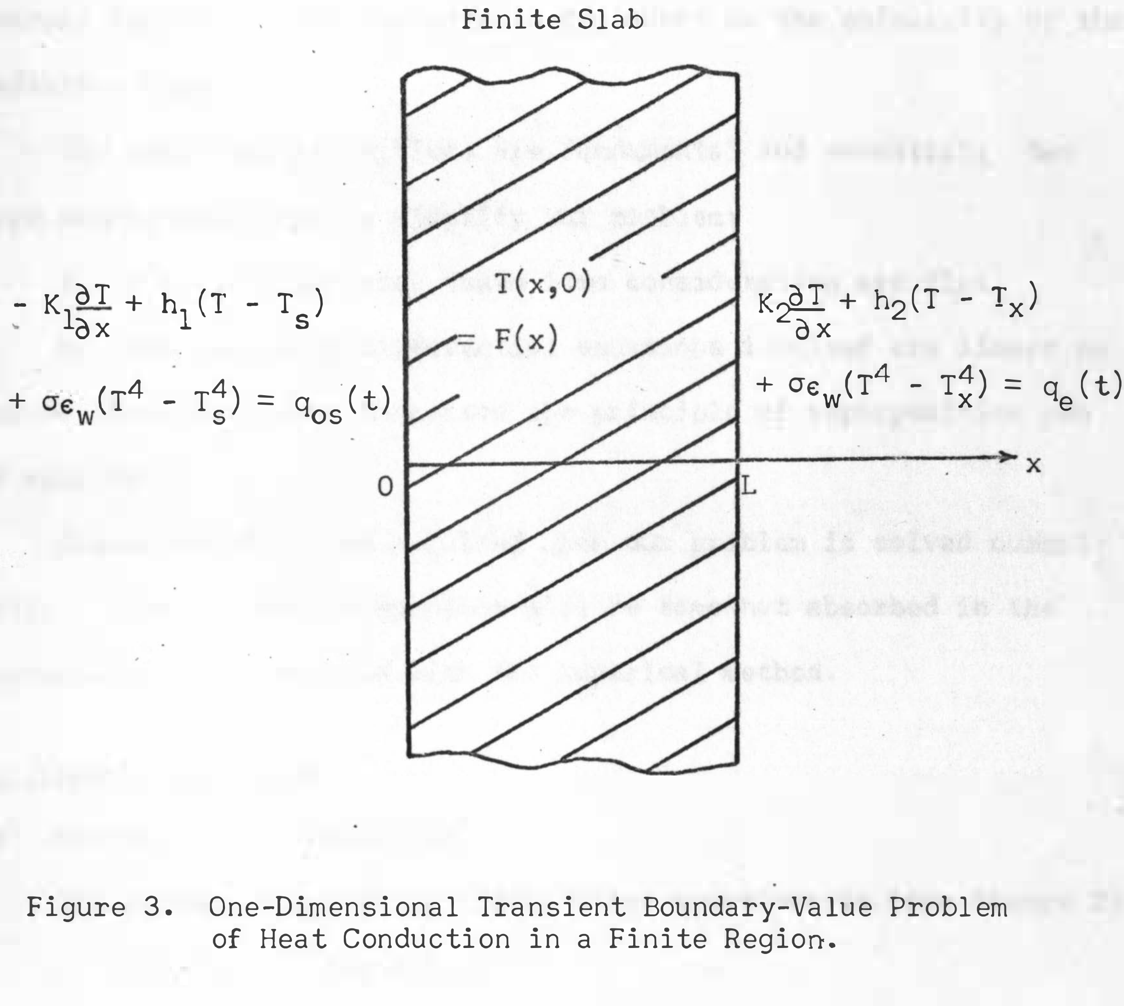

Figure 3. One-Dimensional Transient Boundary-Value Problem of Heat Conduction in a Finite Region.

is only applied to a black body. In the present case, we have

$$q_r(\epsilon) = \sigma\epsilon A(T_1^4 - T_2^4)$$

where ϵ is the emissivity of the radiating solid body. The amount of thermal radiation heat transfer is dependent on the emissivity of the radiating body.

The above six assumptions are fundamental and essential; two more assumptions greatly simplify our problem:

7. The solid surfaces taken into consideration are flat.

8. The governing differential equations involved are linear or can be linearized, and therefore the principle of superposition can be applied.

Assumption 8 is not required when our problem is solved numerically. However, this assumption will be somewhat absorbed in the approximations associated with the numerical method.

B. Composite Barrier

(a) Mathematical Formulation

The system of governing differential equations is (see Figure 2)

1. For the outside glass:

$$\frac{dT_g}{dt} = \frac{1}{A_o}\Big\{A_{ng}q_{os}(t)(1 - r_g)(1 - \text{SUMA})\left[1 + \frac{A_{np}}{A_{ng}} r_p(1 - \text{SUMA})\right]$$

$$- h_a A_a(T_g - T_s) - \sigma\epsilon_g A_a(T_g^4 - T_s^4) - h_b A_b(T_g - T_f) - \sigma\epsilon_g A_b(T_g^4 - T_f^4)\Big\} \quad (1)$$

2. For the airspace:

$$\frac{dT_f}{dt} = \frac{1}{B_o}\left[h_b A_b (T_g - T_f) + h_c A_c (T_p - T_f)\right] \tag{2}$$

3. For the solid:

$$\frac{dT_p}{dt} = \frac{1}{C_o}\Big\{A_{np} q_{os}(t)(1 - r_g)(1 - r_p)(\text{SUMA})(1 - \text{SUMB}) - h_c A_c (T_p - T_f)$$

$$- \sigma\epsilon_p A_d (T_p^4 - T_x^4) - h_d A_d (T_p - T_x) - \sigma\epsilon_p A_c (T_p^4 - T_g^4)\Big\} \tag{3}$$

4. For the room air:

$$\frac{dT_x}{dt} = \frac{1}{D_o}\Big\{A_{np} q_{os}(t)(1 - r_g)(1 - r_p)(\text{SUMA})(\text{SUMB})(1 - \text{SUMC})$$

$$+ h_d A_d (T_p - T_x) + A_a q_x(t) + h_w A_w\left[T_w(t) - T_x\right]$$

$$+ \sigma\epsilon_p A_d (T_p^4 - T_x^4) + \rho_x C_{px} \dot{V}_x (T_s - T_x)\Big\} \tag{4}$$

where

$A_o = \rho_g C_{pg} V_g$

$B_o = \rho_f C_{pf} V_f$

$C_o = \rho_p C_{pp} V_p$

$D_o = \rho_x C_{px} V_x$

$$SUMA = \sum_{\alpha=1}^{m} \frac{\Delta E_\alpha}{E_b} \exp(-\mu_\alpha L_g)$$

$$SUMB = \sum_{\beta=1}^{n} \frac{\Delta E_\beta}{E_b} \exp(-\mu_\beta L_p)$$

$$SUMC = \sum_{\gamma=1}^{k} \frac{\Delta E_\gamma}{E_b} \exp(-\mu_\gamma L_x)$$

Compared with [1], two more terms are added into the system of governing differential equations. Those terms are:

Thermal radiation heat transfer from the inside solid to the room:

$$\sigma\epsilon_p A_d (T_p^4 - T_x^4)$$

and heat transfer associated with mass transfer:

$$\rho_x C_{px} \dot{V}_x (T_s - T_x)$$

where $\dot{V}_x$ is the volume rate of mass transfer in ft^3/hour.

Heat transfer associated with the mass transfer is considered only when the original system of governing differential equations is solved directly by numerical method. Here the thermal radiation term is added because, for a high emissivity solid exposed to slow moving air, the reradiation factor can be important. For a high transparent solid the term may be neglected. The second term is also added to increase flexibility.

(b) Solutions in Cooling Load

The solutions given in [1] are explicit solutions for the air temperature of the room, and those solutions exist in the form of formal integral representation with the term of the cooling load involved in the integrals. In order to calculate the solutions for cooling load, we proceed as follows:

1. We replace the formal integral by their equivalent sums. Numerically, this is equivalent to evaluating the integrals by the left hand rectangular rule.

2. We evaluate the required cooling load numerically by method of trial and error iteration.

A computer program is available for performing the above mentioned two operations. This computer program is listed in the separate volume.

(c) Numerical Solution

When equations (1 to 4) are simplified, they become

1. $$\frac{dT_g}{dt} = A_1 q_{os}(t) - A_2(T_g - T_s) - A_3(T_g^4 - T_s^4) - A_4(T_g - T_f) - A_5(T_g^4 - T_p^4) \quad (5)$$

2. $$\frac{dT_f}{dt} = B_1(T_g - T_f) + B_2(T_p - T_f) \quad (6)$$

3. $$\frac{dT_p}{dt} = C_1 q_{os}(t) - C_2(T_p^4 - T_x^4) - C_3(T_p - T_f) - C_4(T_p - T_x) - C_5(T_p^4 - T_g^4) \quad (7)$$

4. $$\frac{dT_x}{dt} = D_1 q_{os}(t) + D_2(T_p - T_x) + D_3\left[T_w(t) - T_x\right] + D_4 q_x(t)$$

$$+ D_6(T_p^4 - T_x^4) + D_7(T_s - T_x) \qquad (8)$$

with the initial conditions

$$T_g(0) = T_s(0)$$

$$T_x(0) = 530^oR$$

$$T_p(0) = T_x(0)$$

$$T_f(0) = \tfrac{1}{2}\left[T_g(0) + T_p(0)\right]$$

We wish to have T_x kept constant; therefore we set

$$\frac{dT_x}{dt} = 0; \text{ it follows that}$$

$$T_x = T_x(t) = T_x(0) = 530^oR$$

With these conditions we can numerically integrate equations (5 to 7) simultaneously by the fourth-order Runge-Kutta method. Equation (8) is transposed to solve for the required cooling load:

$$q_x(t) = \frac{1}{D_4}\left\{D_1 q_{os}(t) + D_2(T_p - T_x) + D_3\left[T_w(t) - T_x\right] + D_6(T_p^4 - T_x^4) + D_7(T_s - T_x)\right\} \qquad (9)$$

where the constants in equations (5 to 8) are:

$$A_1 = \frac{1}{A_o} A_{ng}(1 - r_g)(1 - SUMA)\left[1 + \frac{A_{np}}{A_{ng}} r_p(1 - SUMA)\right]$$

$$A_2 = \frac{h_a A_a}{A_o} \qquad A_3 = \frac{\sigma\varepsilon_g A_a}{A_o}$$

$$A_4 = \frac{h_b A_b}{A_o} \qquad A_5 = \frac{\sigma\varepsilon_g A_b}{A_o}$$

$$B_1 = \frac{h_b A_b}{B_o} \qquad B_2 = \frac{h_c A_c}{B_o}$$

$$C_1 = \frac{1}{C_o} A_{np}(1 - r_g)(1 - r_p)\ (SUMA)\ (1 - SUMB)$$

$$C_2 = \frac{\sigma \epsilon_p A_d}{C_o} \qquad C_3 = \frac{h_c A_c}{C_o}$$

$$C_4 = \frac{h_d A_d}{C_o} \qquad C_5 = \frac{\sigma \epsilon_p A_c}{C_o}$$

$$D_1 = \frac{1}{D_o} A_{np}(1 - r_g)(1 - r_p)\ (SUMA)(SUMB)(1 - SUMC)$$

$$D_2 = \frac{h_d A_d}{D_o} \qquad D_3 = \frac{h_w A_w}{D_o}$$

$$D_4 = \frac{A_a}{D_o} \qquad D_6 = \frac{\sigma \epsilon_p A_d}{D_o}$$

$$D_7 = \frac{\rho_x C_{px} \dot{V}_x}{D_o} = \frac{\dot{V}_x}{V_x}$$

C. Single Barrier

(a) Mathematical Formulation

The mathematical formulation can be obtained from [2] with some extensions. The solutions given in [2] were used for solar heating only. In order to apply the solutions of [2] to the present study, a cooling load term has to be added to the system of governing differential equations. Also the thermal radiation heat transfer from the solid to the room is not neglected. A term of heat transfer associated with mass transfer is included only when the system of governing differential equations is solved numerically.

With reference to [2], the system of governing differential equations is:

1. For the solid:

$$\frac{dT_g}{dt} = \frac{1}{B_o}\Big[A_{ng}q_{os}(t)(1 - r_g)(1 - \text{SUMA}) - h_a A_a(T_g - T_s) - \sigma\epsilon_g A_a(T_g^4 - T_s^4)$$

$$- h_b A_b(T_g - T_x) - \sigma\epsilon_g A_b(T_g^4 - T_x^4)\Big] \qquad (10)$$

2. For the room air:

$$\frac{dT_x}{dt} = \frac{1}{B_x}\Big[h_b A_b(T_g - T_x) + A_{ng}q_{os}(t)(1 - r_g)(\text{SUMA}) + \sigma\epsilon_g A_b(T_g^4 - T_x^4)$$

$$+ A_a q_x(t) + \rho_x C_{px}\dot{V}_x(T_s - T_x)\Big] \qquad (11)$$

where

$$B_o = \rho_g C_{pg} V_g$$

$$B_x = \rho_x C_{px} V_x$$

$$\text{SUMA} = \sum_{\alpha=1}^{m} \frac{\Delta E_\alpha}{E_b}\exp(-\mu_\alpha L_g)$$

Somewhat differing from [2], we use here the concept of equivalent thermal radiation coefficients based on the linear temperature difference (this approximation concept will be discussed in the next chapter).

Let

$$h_{ar} = \sigma\epsilon_g(T_g^2 + T_s^2)(T_g + T_s)$$

$$h_{br} = \sigma\epsilon_g(T_g^2 + T_x^2)(T_g + T_x)$$

The method and solutions are given in [2]. The method of equivalent thermal coefficient is used because of convenience in numerical computation.

(b) Solutions in Cooling Load

Applying the solutions given in [2], the method is the same as that for the composite barrier.

(c) Numerical Solution

The system of the governing differential equations (10,11) is rewritten in the form

$$\frac{dT_g}{dt} = B_1 q_{os}(t) - B_2(T_g - T_s) - B_4(T_g - T_x) \qquad (12)$$

$$\frac{dT_x}{dt} = B_6 q_{os}(t) + B_5(T_g - T_x) + B_9 q_x(t) + B_{10}(T_s - T_x) \qquad (13)$$

where

$$B_1 = \frac{1}{B_o} A_{ng}(1 - V_g)(1 - SUMA) \qquad B_2 = \frac{1}{B_o} A_a(h_a + h_{ar})$$

$$B_4 = \frac{1}{B_o} A_b(h_b + h_{br}) \qquad B_5 = \frac{1}{B_x} A_b(h_b + h_{br})$$

$$B_6 = \frac{1}{B_x} A_{ng}(1 - r_g)(SUMA) \qquad B_9 = \frac{1}{B_x} A_a \qquad B_{10} = \frac{1}{B_x} \rho_x C_{px} \dot{V}_x$$

with the initial conditions

$$T_g(0) = T_s(0) \qquad\qquad T_x(0) = 530^{\circ}R$$

Since the air temperature of the room is assumed to be constant, we set

$$\frac{dT_x}{dt} = 0\,; \quad \text{therefore}$$

$$T_x = T_x(t) = T_x(0) = 530^{\circ}R$$

Equation (12) can be integrated numerically. In the present study, we use the fourth-order Runge-Kutta method to integrate equation (12). Equation (13) is transposed to evaluate the required cooling load:

$$q_x(t) = \frac{-1}{B_9}\left[B_6 q_{os}(t) + B_5(T_g - T_x) + B_{10}(T_s - T_x)\right] \qquad (14)$$

D. Opaque Barrier

(a) Mathematical Formulation

The mathematical formulation of the second part of the problem (see Figure 3) can be obtained by a simple energy balance. With our assumptions, this part reduces to a one-dimensional transient boundary-value problem of heat conduction in a finite region. The governing differential equation is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{where} \quad 0 \le x \le L\,, \quad t > 0 \qquad (15)$$

with the initial condition

$$T = F(x) \qquad \text{where} \quad 0 \le x \le L\,, \quad t > 0$$

and boundary conditions

$$- K_1\frac{\partial T}{\partial x} + h_1(T - T_s) + \sigma\epsilon_w(T^4 - T_s^4) = q_{os}(t)$$

at $x = 0$, $t > 0$

$$K_2\frac{\partial T}{\partial x} + h_2(T - T_x) + \sigma\epsilon_w(T^4 - T_x^4) = q_x(t)$$

at $x = L$, $t > 0$ (17)

where

$T = T(x,t)$

$T_s = T_s(t)$

$T_x = T_x(t) = T_x(0) = 530^oR$

(b) Numerical Solution

These equations are solved by the finite difference method. Applying the finite difference approximations (see Discussion) and refering to Figure 3a, we reduce our problem to:

$$T(x_i,t_j + k) = \frac{1}{6}\left[T(x_i + h,t_j) + 4T(x_i,t_j) + T(x_i - h,t_j)\right]$$

$i = 2,3,\ldots\ldots,m-1$ $j = 1,2,\ldots\ldots,n-1$ (18)

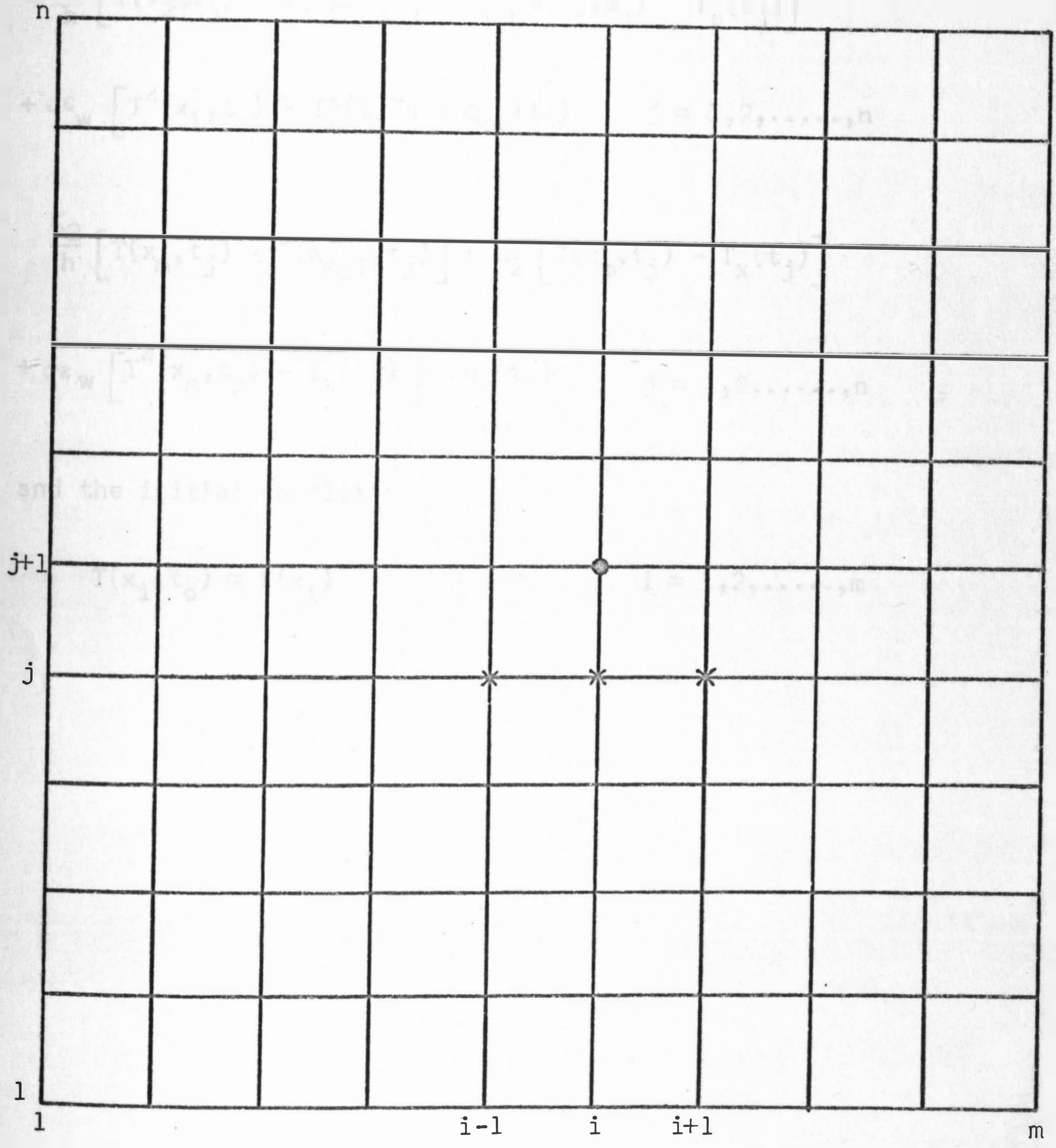

Figure 3a. Finite Difference Stencil Diagram.

The boundary conditions in difference form are

$$- \frac{K_1}{h}\left[T(x_2,t_j) - T(x_1,t_j)\right] + h_1\left[T(x_1,t_j) - T_s(t_j)\right]$$

$$+ \sigma\epsilon_w\left[T^4(x_1,t_j) - T_s^4(t_j)\right] = q_{os}(t_j) \qquad j = 1,2,\ldots\ldots,n$$

$$\frac{K_2}{h}\left[T(x_m,t_j) - T(x_{m-1},t_j)\right] + h_2\left[T(x_m,t_j) - T_x(t_j)\right]$$

$$+ \sigma\epsilon_w\left[T^4(x_m,t_j) - T_x^4(t_j)\right] = q_e(t_j) \qquad j = 1,2,\ldots\ldots,n \tag{19}$$

and the initial condition is

$$T(x_i,t_o) = F(x_i) \qquad i = 1,2,\ldots\ldots,m$$

CHAPTER III

APPROXIMATIONS AND RESULTS

The method of solution discussed in the preceding chapter is associated with the approximations methods discussed in this chapter.

A. Mathematical Simplification

The thermal radiation terms involved in equations (1 to 4), (10 to 11), (15), and (17) are linearized by applying Newton's forward difference method [1]. Consider the thermal radiation heat transfer from a solid next to the ambient surrounding. We have, after applying Newton's forward difference method,

$$q_r = \sigma\epsilon_g \frac{(T_1^4 - T_o^4)}{(T_1 - T_o)} (T_g - T_s)$$

where

$$T_o \leq T_g \leq T_1$$

However, it follows by Stefan Boltzmann's law of thermal radiation, that

$$q_r^t = \sigma\epsilon_g(T_g^4 - T_s^4) = \sigma\epsilon_g\left[(T_g^2 + T_s^2)(T_g + T_s)\right](T_g - T_s)$$

Here q_r^t denotes the true amount of thermal radiation heat transfer. The relative error can be stated as

$$Err(T_g,T_s,T_o,T_1) = \left|\frac{q_r - q_r^t}{q_r^t}\right| = \left|\frac{q_r}{q_r^t} - 1\right|$$

$$= \left|\frac{(T_1^2 + T_o^2)(T_1 + T_o)}{(T_g^2 + T_s^2)(T_g + T_s)} - 1\right|$$

Suppose $T = 500^oR$, $0^oR \le \epsilon \le 50^oR$

$T_g = T + \epsilon$

and $T = T_o \le T_g \le T_1 = T + 50^oR$

If $T_s = 500^oR$

then $\underset{\epsilon \to o}{Err(T_g,T_s,T_o,T_1)} = \left|\frac{580125}{500000} - 1\right| = 0.16025$

If $T_s = 550^oR$

then $\underset{\epsilon \to 50}{Err(T_g,T_s,T_o,T_1)} = \left|\frac{580125}{665500} - 1\right| = 0.12829$

The above examples show that the relative error due to the application of the Newton forward difference method to the equations of thermal radiation is quite significant under certain conditions as far as the energy transmission due to the thermal radiation is concerned. Fortunately, in practically all numerical evaluations, Newton's forward difference method can be replaced by using the concept of equivalent thermal radiation heat transfer coefficients based on linear temperature difference, i.e., let

$q_r = h_r(T_g - T_s)$

where

$$h_r = \sigma\epsilon_g(T_g^2 + T_s^2)(T_g + T_s)$$

During each step of the calculations, h_r is assumed to be constant: It follows that when the increment for each step of the calculations becomes smaller and smaller, the thermal radiation heat transfer we obtain approaches close to the true value. This concept of using h_r is already applied to the finite difference solutions of part 2. A computer subprogram is designed for this purpose. This is also applied to the numerical solutions of part 1. A computer subprogram is available for these calculations.

B. Input Data Approximations

In [3], the normal solar heat flux is given for an hourly basis for several latitudes in the year of 1967.

Over a finite time interval of solar heating, say, from t_o to t_n, the total amount of normal solar radiation energy per unit area can be represented graphically as the shaded area in Figure 4a.

Suppose the normal solar heat flux QON(t) on the finite closed time interval $[t_o, t_n]$ can be represented as a continuous function of time. Then QON(t) is integrable on $[t_o, t_n]$, since a continuous function is integrable over a finite closed interval. Thus the graphical representation of QON(t) versus t is a continuous curve. The total amount of normal solar radiation energy per unit area on $[t_o, t_n]$ can be represented as the integral $IQON = \int_{t_o}^{t_n} QON(t)dt$. On the course of numerical evaluation of this integral IQON, we can approximate it by applying the trapezoidal rule on an hourly basis. This is graphically represented in Figure 4b and is equivalent to replacing the integrand QON(t) by QONT(t),

where

$$QONT(t) = QON(t_i) + (t - t_i)\left[QON(t_{i+1}) - QON(t_i)\right]$$

for

$$i = 1,2,\ldots\ldots,n-1\ , \qquad t_i \leq t \leq t_{i+1}$$

However, if the function QONT(t) is combined with some other time-dependent functions to form the integrand, then this integrand may still not be convenient to be integrated. We thus further approximate IQON = $\int_{t_o}^{t_n}$ QONT(t)dt by an equivalent sum using the left-hand trapezoidal rule, based on a certain increment $\Delta\tau$, which is chosen depending on the required accuracy and economy of computer time. Graphically, this approximation is represented in Figure 4c, while in the course of numerical evaluations, QONT(t) is actually approximated by a step function QONS(τ). Here the closed interval $[t_o, t_n]$ is divided into

$$\left\{\tau_1, \tau_2, \ldots\ldots, \tau_m\right\},$$

where

$$m = \frac{t_n - t_o}{\Delta\tau}$$, and $\Delta\tau$ is chosen suitably so that m is an integer.

Let

$$\tau_i - \tau_{i-1} = \Delta\tau, \quad i = 1,2,\ldots\ldots,m$$

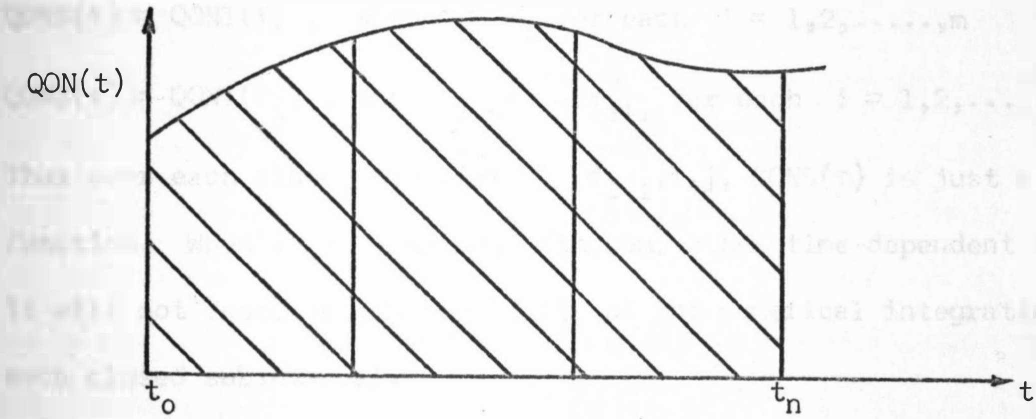

Figure 4a. Graphical Representation of an Integral.

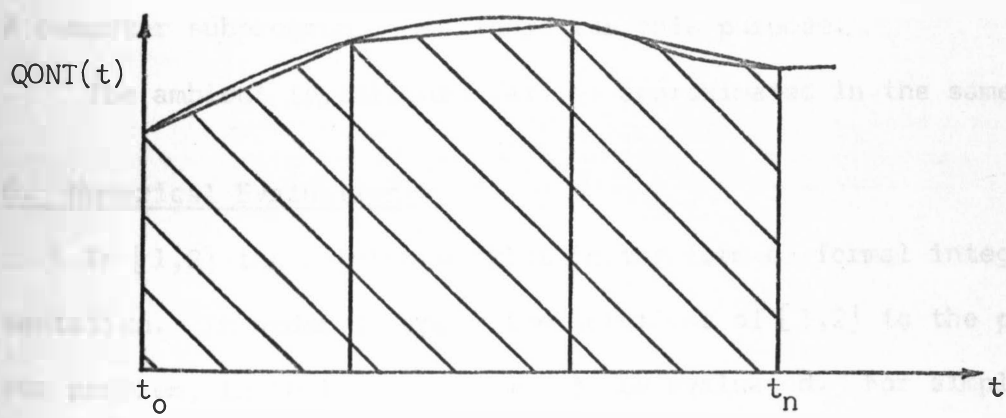

Figure 4b. Trapezoidal Rule.

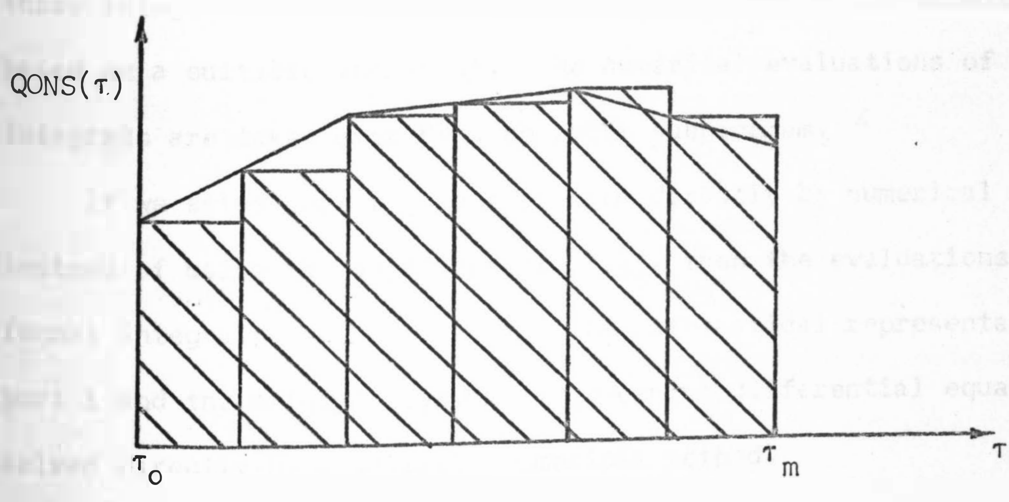

Figure 4c. Rectangular Rule

$QONS(\tau) = QONT(\tau)$, when $\tau = \tau_i$ for each $i = 1,2,.....,m$

$QONS(\tau) = QONT(\tau_i)$, when $\tau_i \leq \tau < \tau_{i+1}$ for each $i = 1,2,.....,m-1$

Thus over each closed subinterval $[\tau_{i-1},\tau_i]$, $QONS(\tau)$ is just a constant function. When it is combined with some other time-dependent functions, it will not increase the complexity of the numerical integration on each closed subinterval.

The step function $QONS(\tau)$ is evaluated numerically by the method of linear interpolations and extrapolations: also refer to Figure 4c. A computer subprogram is designed for this purpose.

The ambient temperature data is approximated in the same way.

C. Numerical Evaluation

In [1,2] the solutions exist in the form of formal integral representation. In order to apply the solutions of [1,2] to the part 1 of our problem, those integrals have to be evaluated. For simplification, those integrals involved are approximated by their equivalent sums, based on a suitable increment. The numerical evaluations of those integrals are taken over by a computer subprogram.

If we solve part 1 of our problem directly by numerical methods instead of using the solutions of [1,2], then the evaluations of those formal integrals can be avoided. The mathematical representation of part 1 and the original system of governing differential equations are solved directly by a suitable numerical method.

D. Inversion of Solution

In [1,2] the solutions are given explicitly for the air temperature of the room, but for our purpose, they are considered as indirect solutions, since the actual information we want is the cooling load.

The concept of trial and error is applied. That is, with the other conditions kept unchanged, only the cooling load is varied. Then, we will have the air temperature of the room confined within a certain range of temperature, i.e., from $69^{\circ}F$ to $71^{\circ}F$. If the room air temperature variation is bounded between the given allowance, then the required cooling load is yielded.

When we have a very large building with a comparatively small window, then the variation of the room air temperature due to the amount of solar energy coming in from the window may be small in magnitude. In this case, the inverting of the solutions will be difficult and accuracy will be lost in the final results.

When part 1 of our problem is solved numerically, the solutions are given explicitly as the required cooling load. The troublesome task in inverting the solutions can be avoided.

E. Calculation of Solar Heat Flux

Tabulated in [3] is the normal solar heat flux on an hourly basis. In practical calculations, we want to have the hourly solar heat flux in various directions, for example for a south-facing vertical surface, a horizontal surface, etc. In order to convert the normal data into data for various directions, a certain amount of calculations is required.

The calculations on an hourly basis for solar heat flux for vertical and horizontal surfaces are simple; refer to [4].

For an inclined surface, however, the procedure is not so obvious. Again with reference to [4], the converting formulas follow from projective geometry:

$$\sin H' = \frac{[(\tan H)(1 + \tan^2 B)^{\frac{1}{2}} - (\tan D)](\cos D)}{[(1 + \tan^2 B)(1 + \tan^2 H)]^{\frac{1}{2}}}$$

$$\tan B' = \frac{\tan B}{[\tan H(1 + \tan^2 B)^{\frac{1}{2}} - \tan D]\sin D + \sec D}$$

$$\cos i = \cos H' \cos B'$$

where

H = solar altitude above the true horizon,

B = horizontal angle between the direction of the sun's ray and a normal to the irradiated surface,

D = the angle between a normal to the irradiated surface and the horizon,

i = the incident angle, the angle between a normal of the irradiated surface and the sun's ray.

The above formulas are valid if all the angles involved are within the first quadrant. This follows since, if a vertical surface is irradiated, then the corresponding inclined surfaces are also irradiated. However, the converse is not necessarily true.

In practical calculations, the angle B is in terms of the angle A, the solar azimuth angle. It follows that the angle B lies not necessarily within the first quadrant. In this situation, the angle D will be meaningful only if we know exactly under what conditions a surface is irradiated. Here we consider as follows:

A flat surface is either irradiated or not irradiated, but not both. If a vertical surface is not irradiated, then the corresponding inclined surface will not be irradiated either, providing that the actual solar altitude H is less than the inclined angle taken at the solar azimuth angle (refer to Figure 5); in any other case, the inclined surface is irradiated. The reflected and diffused solar energy is also taken into consideration by the method given in [2]. All the details of the calculations are performed numerically, and a computer subprogram is designed for this purpose.

F. Determination of the Film Coefficients

For a smooth surface exposed to moving air, we use the formulas given in [1] to determine the film coefficients, namely,

$$h = 0.99 + 0.21v, \quad \text{for} \quad v \leq 16 \quad \text{ft/sec}$$

$$h = 0.5(v)^{0.78}, \quad \text{for} \quad v > 16 \quad \text{ft/sec}$$

Thus for $v = 15$ ft/sec, we have $h = 4$.

For very slow moving air, we have $h = 1$.

For the surface adjacent to the airspace, the film coefficients are calculated by using the thermal resistance of the airspace.

$$h = \frac{2}{r}$$

where r is the thermal resistance of the airspace.

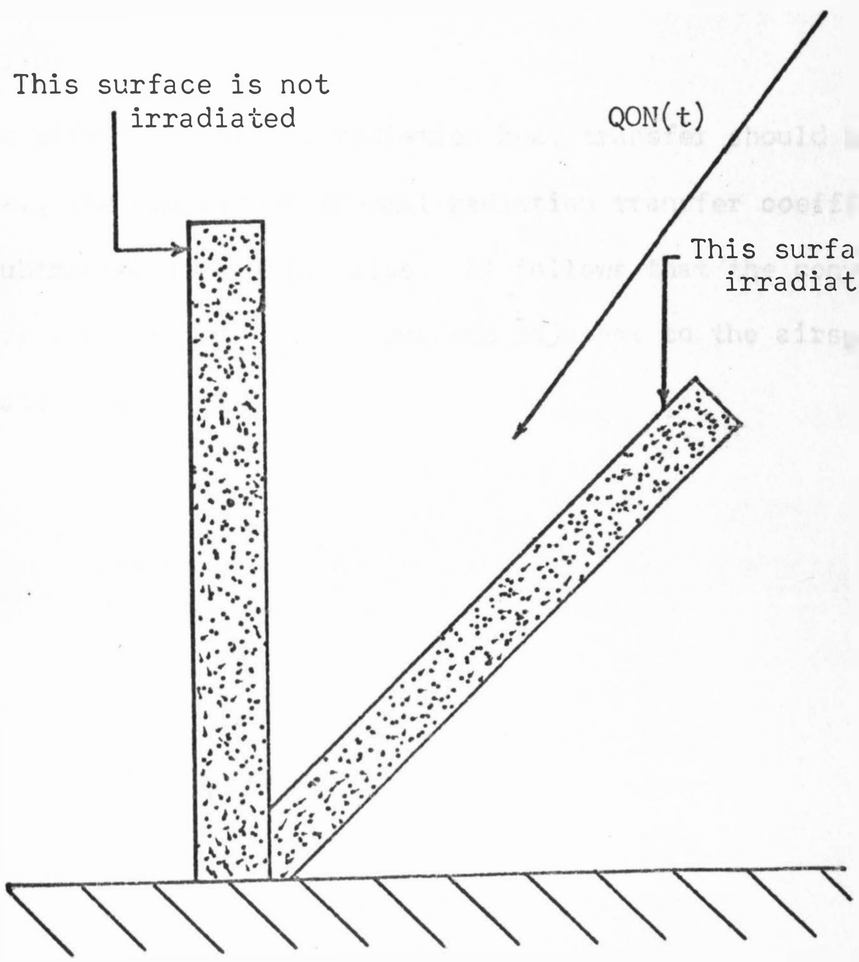

Figure 5. Surface Irradiation.

Given in [4], the thermal resistance for a quarter-inch airspace is r = 0.7107. Thus the equivalent film coefficient is

$$h = \frac{2}{0.7107}$$

However, the effect of thermal radiation heat transfer should be extracted, i.e., the equivalent thermal radiation transfer coefficient should be subtracted from this value. It follows that the convective heat transfer coefficient for the surface adjacent to the airspace has an approximate value of 2.

CHAPTER IV

DISCUSSION OF THE THEORETICAL ERROR ANALYSIS

In Chapter II, we use the finite difference method to solve part 2 and the Runge-Kutta method to solve part 1 of our problem. In this chapter we would like to discuss the finite difference approximation for parabolic partial differential equations and an error estimation for the fourth-order Runge-Kutta method. Also, lag time effect is discussed because this concept is used in solving the example problem.

A. Finite Difference Approximation

Consider a simple case of a function f of one independent variable x; if f is defined on the h_o-neighborhood of x_o, then the difference quotient of f at x_o is

$$\frac{\Delta f}{\Delta x}(x_o) = \frac{f(x_o + h_o) - f(x_o)}{(x_o + h_o) - x_o} ,$$

provided that this difference quotient exists.

If the limit of this difference quotient at x_o exists, or

$$\lim_{h_o \to o} \frac{\Delta f}{\Delta x}(x_o) = \lim_{h_o \to o} \frac{f(x_o + h_o) - f(x_o)}{h_o} ,$$

then this limit is said to be the derivative of f at x_o.

The essential idea of the finite difference approximation is to replace the derivative of a function by its difference quotient. In order to study the deviation of the difference quotient from its corresponding derivatives, we need a more careful and profound consideration. In general, the theory of infinite series and Taylor-series expansions should be taken into consideration.

Suppose f is defined in a h_o-neighborhood of x_o and has a Taylor-series expansion, i.e., f together with its Taylor-series representation are defined on the set

$$N_{h_o}(x_o) = \{x: |x - x_o| < h_o\}.$$

Suppose $0 \leq \Delta x \leq h$, $h < h_o$, and let $f^{(n)}(x_i)$ denote the nth derivative of f at x_i. Then the Taylor-series expansion of f about x_i is

$$f(x) = f(x_i + \Delta x) = \sum_{n=o}^{\infty} \frac{(\Delta x)^n}{n!} f^{(n)}(x_i)$$

where

$$f^{(o)}(x_i) = f(x_i) \ , \quad 0! = 1$$

Let

$\Delta x = h$, then

$$f(x_i + h) = \sum_{n=o}^{\infty} \frac{(h)^n}{n!} f^{(n)}(x_i)$$

Let

$\Delta x = -h$, then

$$f(x_i - h) = \sum_{n=o}^{\infty} \frac{(-h)^n}{n!} f^{(n)}(x_i)$$

From the above expansion, the approximation formula of order h is yielded. For the first derivatives, we have

$$f'(x_i) = \frac{1}{2h}[f(x_i + h) - f(x_i - h)] - \frac{1}{h}\sum_{n=1}^{\infty} \frac{(h)^{2n+1}}{(2n + 1)!} f^{(2n+1)}(x_i)$$

$$= \frac{1}{2h}[f(x_i + h) - f(x_i - h)] + O(h^2)$$

$$= \frac{1}{2h}[f(x_i + h) - f(x_i - h)] + Err$$

It is clear that for h sufficiently small and $n > 2$, the terms of $O(h^n)$ are small in magnitude if compared with the term of $O(h^2)$. Therefore the error term can be considered as proportional to h^2 and $f^{(3)}(x_i)$ in magnitude.

For the second derivative, we have

$$f''(x_i) = \frac{1}{h}[f(x_i + h) - 2f(x_i) + f(x_i - h)] - \frac{1}{h^2}\sum_{n=1}^{\infty} \frac{2h^{2n+2}}{(2n + 2)!} f^{(2n+2)}(x_i)$$

$$= \frac{1}{h}[f(x_i + h) - 2f(x_i) + f(x_i - h)] + O(h^2)$$

$$= \frac{1}{h}[f(x_i + h) - 2f(x_i) + f(x_i - h)] + Err$$

The error term is proportional to h^2 and $f^{(4)}(x_i)$ in magnitude. Also the approximation formula of order h for the first derivative is yielded:

$$f'(x_i) = \frac{1}{h}[f(x_i + h) - f(x_i)] - \frac{1}{h}\sum_{n=1}^{\infty} \frac{h^{n+1}}{(n+1)!} f^{(n+1)}(x_i)$$

$$= \frac{1}{h}[f(x_i + h) - f(x_i)] + O(h)$$

$$= \frac{1}{h}[f(x_i + h) - f(x_i)] + Err$$

The error term is proportional to h and $f''(x_i)$.

B. Parabolic Partial Differential Equation

Consider a special case of the parabolic partial differential equation,

$$u_{xx} = \frac{1}{\alpha} u_t$$

defined on a closed interval $[a,b]$ with initial condition

$$u(x,0) = u_o$$

and boundary conditions of the first kind

$$u(a,t) = u_a$$

$$u(b,t) = u_b$$

Then the applications of the finite difference approximation to this problem yields a straightforward, step by step process to solve for the solutions.

Suppose u has continuous partial derivatives of 2nth order. Let u_{nt} denote the nth partial derivative of u with respect to t, and let

u_{nx} denote the nth partial derivative of u with respect to x. Then

$$u_{2x} = u_{xx} = \frac{1}{\alpha} u_t \ , \quad \text{or}$$

$$u_t = \alpha u_{2x}$$

$$u_{2t} = u_{tt} = \alpha u_{2xt} = \alpha(u_t)_{xx} = \alpha(\alpha u_{xx})_{xx} = \alpha^2 u_{xxxx} = \alpha^2 u_{4x}$$

$$u_{3t} = u_{ttt} = \alpha^2(u_{xxxx})_t = \alpha^2(u_t)_{xxxx} = \alpha^2(\alpha u_{xx})_{xxxx} = \alpha^3 u_{6x}$$

It follows that $u_{nt} = \alpha^n u_{2nx}$

This general relation can be proved by mathematical induction. Extending the finite difference approximation to the partial derivatives, we have

$$u_{xx}(x_i, t_j) = \frac{1}{h^2}[u(x_i + h, t_j) - 2u(x_i, t_j) + u(x_i - h, t_j)]$$

$$- \frac{1}{h^2} \sum_{n=1}^{\infty} \frac{2h^{2n+2}}{(2n + 2)!} u_{(2n+2)x}(x_i, t_j)$$

$$u_t(x_i, t_j) = \frac{1}{k}[u(x_i, t_j + k) - u(x_i, t_j)]$$

$$- \frac{1}{k} \sum_{n=1}^{\infty} \frac{k^{n+1}}{(n + 1)!} u_{(n+1)t}(x_i, t_j)$$

Let

$\lambda = \frac{\alpha k}{h^2}$, then the original partial differential equation can be reduced to

$$u(x_i,t_j + k) - \lambda[u(x_i + h,t_j) + (\frac{1}{\lambda} - 2) u(x_i,t_j) + u(x_i - h,t_j)]$$

$$= \sum_{n=1}^{\infty} h^{2n+2} \frac{\lambda^{n+1}}{(n + 1)!} - \frac{2\lambda}{(2n + 2)!} u_{(2n+2)x}(x ,t)$$

$$= Err$$

$$= \sum_{n=1}^{\infty} (T_n)$$

Suppose u has continuous partial derivatives of order four, then

$Err \leq ERR$,

where

$$ERR = \sum_{n=1}^{\infty} |(T_n)| \geq \sum_{n=1}^{1} |(T_n)|$$

$$= \sum_{n=1}^{1} | h^{2n+2}\left[\frac{\lambda^{n+1}}{(n + 1)!} - \frac{2\lambda}{(2n + 2)!}\right]u_{(2n+2)x}(x_i,t_j)|$$

$$= |h^4\left[\frac{\lambda^2}{2} - \frac{\lambda}{12}\right]u_{xxxx}| = |h^4 \frac{\lambda(6\lambda -1)}{12} u_{xxxx}|$$

Suppose u has a continuous partial derivative of order six, then we can let

$\lambda = \frac{1}{6}$, so that the first term in the summations (T_1) vanishes.

Then

$$ERR \geq \sum_{n=1}^{2} |(T_n)|$$

$$= \left| h^6 \left[\frac{\lambda^3}{6} - \frac{2\lambda}{6!} \right] u_{xxxxxx} \right|$$

$$= \left| h^6 \left[\frac{\lambda(60\lambda^2 - 1)}{360} \right] u_{6x} \right|$$

$$= \left| h^6 \left[\frac{\frac{1}{6}(\frac{5}{3} - 1)}{360} \right] u_{6x} \right|$$

$$= \left| \frac{h^6}{3240} u_{xxxxxx} \right|$$

C. Error Estimation for the Fourth-Order Runge-Kutta Method

The following work of an error estimation for the fourth-order Runge-Kutta method suggested by Kutta is given in [5].

A single-step method can be written in the form

$$y_{n+1} = y_n + h\phi(x_n, y_n; h)$$

$$y(x_{n+1}) = y(x_n) + h\phi(x_n, y(x_n); h) - r(x_n, h)$$

where

$r(x_n, h)$ denotes the local truncation error.

Let

$\epsilon_n = y_n - y(x_n)$ be the error at the nth step. Then

$$\epsilon_{n+1} = \epsilon_n + h[\phi(x_n, y_n; h) - \phi(x_n, y(x_n); h)] + r(x_n, h)$$

or

$$\epsilon_{n+1} = (\epsilon_p)_{n+1} + (\epsilon_T)_{n+1}$$

where

$$(\epsilon_p)_{n+1} = \epsilon_n + h[\phi(x_n, y_n; h) - \phi(x_n, y(x_n); h)]$$

denotes the propagation error, and

$$(\epsilon_T)_{n+1} = r(x_n, h)$$

denotes the truncation error.

Suppose $\left| \frac{\partial f(x,y)}{\partial y} \right| \leq K$

For the fourth-order Runge-Kutta Method suggested by Kutta, we have

$$\phi(x,y;h) = \sum_{s=1}^{4} \alpha_s k_s(x,y)$$

$$k_1(x,y) = f(x,y)$$

$$k_s(x,y) = f(x + \mu_s h, y + \lambda_s h k_{s-1}) \qquad (s>1),$$

$$\alpha_1 = \tfrac{1}{2}\alpha_2 = \tfrac{1}{2}\alpha_3 = \alpha_4 = \frac{1}{6}$$

$$\mu_2 = \mu_3 = \tfrac{1}{2}\mu_4 = \tfrac{1}{2} \qquad \lambda_2 = \lambda_3 = \tfrac{1}{2}\lambda_4 = \tfrac{1}{2}$$

After applying the mean value theorem many times, we have

$$\phi(x,y;h) - \phi(x,\bar{y};h) \leq K[\alpha_1 + \alpha_2(1 + \lambda_1 hK) + \alpha_3(1 + \lambda_3 hK(1 + \lambda_2 hK))$$

$$+ \alpha_4(1 + \lambda_4 hK(1 + \lambda_3 hK(1 + \lambda_2 hK)))] \, |y - \bar{y}|$$

$$= K(1 + \tfrac{1}{2}hK + \frac{1}{6}h^2K^2 + \frac{1}{24}h^3K^3)\,|y - \bar{y}|$$

Let

$$x = x_n \ , \quad y = y_n \ , \quad \bar{y} = y(x_n) \ ; \quad \text{then, we have}$$

$$(\epsilon_p)_{n+1} = |\epsilon_n + h\{\phi(x_n,y_n;h) - (x_n,y(x_n);h)\}|$$

$$\leq \left(1 + hK + \frac{(hK)^2}{2!} + \frac{(hK)^3}{3!} + \frac{(hK)^4}{4!}\right)|\epsilon_n|$$

For the local truncation error, we assume

$$\left|\frac{\partial^m f}{\partial x^i \partial y^k}\right| \leq \frac{M^m}{N^{k-1}} \qquad (0 \leq m \leq 4)$$

Lotkin [6] derived the truncation error bound

$$r(x_n,h) = \rho h^5 + 0(h^6) \ ,$$

$$|\rho| = \frac{73}{720} NM^4$$

where

$$\max\left|\frac{\partial^m f}{\partial x^{m-k} dy^k}\right| = K_{mk} \ ,$$

$$\max_{0 \leq k \leq m} N^{k-1} K_{mk} = L_m \ , \qquad \max_{1 \leq m \leq 4} L_m^{\frac{1}{m}} = M$$

Let

$$|\rho h^5 + 0(h^6)| \leq Th^5$$

then

$$|r(x_n,h)| \leq Th^5$$

Let

$$1 + hK + \frac{(hK)^2}{2!} + \frac{(hK)^3}{3!} + \frac{(hK)^4}{4!} = 1 + hP$$

It follows that

$$|\epsilon_{n+1}| \leq (1 + hP)|\epsilon_n| + Th^5$$

since

$$\epsilon_{n+1} = (\epsilon_p)_{n+1} + (\epsilon_T)_{n+1}$$

If

$$\epsilon_o = 0 \ , \quad p > 0 \ , \quad \text{then}$$

$$\epsilon_n = \frac{Th^4}{P}[(1 + hP)^n - 1]$$

A computer program for calculating the error bound for given h, P and $n = \frac{15}{h}$ is available. Some results are listed in Figure 6.

D. Lag Time Effect

The method for calculations of the lag time effect is given by Dr. Edward Lumsdaine, and may be stated as follows:

Assume the specific heat capacity of the structure and film coefficient for the surface of the structure adjacent to the room air are uniform throughout, then the governing differential equation for the lag time effect is

$$\rho C_p V_w \frac{dT_w}{dt} = q_d(t)A_1 - h_2A_2(T_w - T_x)$$

h \ P	0.05	0.1	0.2	0.5	1.0
0.01	0.223×10^{-6}	0.348×10^{-6}	0.951×10^{-6}	0.354×10^{-4}	0.303×10^{-1}
0.02	0.360×10^{-5}	0.556×10^{-5}	0.152×10^{-4}	0.557×10^{-3}	0.451
0.05	0.139×10^{-3}	0.217×10^{-3}	0.587×10^{-3}	0.206×10^{-1}	0.142×10^{2}
0.1	0.223×10^{-2}	0.345×10^{-2}	0.925×10^{-2}	0.301	0.162×10^{3}
0.2	0.355×10^{-1}	0.547×10^{-1}	0.144	0.407×10^{1}	0.139×10^{4}

Figure 6. Error Bound for the Fourth-Order Runge-Kutta Method.

The solution is

$$T_w(t) - T_x = \frac{A_1}{\rho C_p V_w} \exp(-\frac{h_2 A_2}{\rho C_p V_w}) \cdot \int_o^t q_d(t) \exp \frac{h_2 A_2}{\rho C_p V_w} dt$$

$$q_1(t) = h_2 A_2 [T_w(t) - T_x]$$

$$q_1(t) = \frac{h_2 A_2}{\rho C_p V_w} A_1 \exp\left(-\frac{h_2 A_2}{\rho C_p V_w}\right) \cdot \int_o^t q_d(t) \exp\left(\frac{h_2 A_2}{\rho C_p V_w}\right) dt$$

This last equation is subject to numerical integration, and a computer subprogram is available for performing this task.

E. Example Problem

To provide an example for the direct application of our method, we compare with an example problem of cooling load calculations given on page 504 of [3] by using our method.

A building 80 feet deep, 50 feet wide and 10 feet high is located at 40 degree north latitude, with the following data:

Section	Net Area	U-Value	Thickness
Roof	4000	0.12	4
South wall	405	0.39	12
East wall	765	0.48	8
North exposed walls	170	0.48	8
West and North party wall	1065	0.25	13
Door in North wall	35	0.59	1.7
Door in East wall	35	0.59	1.7
Door in South wall	35	0.59	1.7

For each window, based on a unit area:

$\rho_g = 160$ $\rho_f = 0.071$ $\rho_p = 75$ $\rho_x = 0.071$

$C_{pg} = 0.186$ $C_{pf} = 0.24$ $C_{pp} = 0.35$ $C_{px} = 0.24$

$\dot{V}_g = 0.021$ $V_f = 0.083$ $V_p = 0.021$ $V_x = 40000$

$A_a = 1$

$A_a = A_b = A_c = A_d = A_{ng} = A_{np}$

$\epsilon_{pg} = 0.92$ $\epsilon_{pp} = 0.92$

$r_{pp} = 0.04$ $r_p = 0.7$

SUMA = 0.78 SUMB = 0.15

SUMC = 0 , if all energy is absorbed by the room air,

or SUMC = 1 , if no energy is absorbed by the room air.

The air temperature of the room is kept constant at 535°R for this example problem. These data are directly input into the computer program, and the results are given in Figures 7 and 8.

Time	TS	Roof	S. wall	E. wall	N. wall exposed	N. & W. party wall
6	533	385	-2	-1	-1	-5
7	536	4100	-5	203	6	-7
8	540	10939	1	1192	43	1
9	543	18914	13	2876	115	2
10	547	25984	58	4727	219	9
11	551	31891	158	6302	350	31
12	554	36027	328	7354	503	76
13	554	37861	569	7813	668	150
14	554	36962	870	7887	825	256
15	554	33913	1207	7795	958	388
16	554	39177	1546	7624	1058	540
17	554	22787	1856	7405	1128	704
18	554	16007	2110	7132	1172	870

N. door	E. door	S. door	S. glass	N. glass	Total
-1	26	1	-49	-6	348
0	209	22	91	59	4677
11	436	85	361	129	13199
29	585	185	658	198	23576
54	641	304	965	280	33241
82	618	422	1238	361	41453
114	531	523	1434	425	47315
140	455	581	1515	460	50212
156	409	590	1480	468	49903
165	377	555	1381	475	47214
171	351	488	1240	474	42671
174	325	408	1117	477	36382
176	293	336	1038	455	29591

Figure 7. Results for the Example Problem in Btu/Hr.

Time	TS	Roof	S. wall	E. wall	N. wall exposed	N. & W. party wall
6	533	397	0	4	0	2
7	536	4116	1	212	8	4
8	540	10945	3	1195	44	5
9	543	18921	17	2881	116	8
10	547	25990	62	4733	220	17
11	551	31900	163	6310	352	41
12	554	36033	333	7362	505	86
13	554	37861	574	7820	669	158
14	554	36958	874	7892	826	262
15	554	33912	1210	7799	959	393
16	554	29187	1549	7629	1060	546
17	554	22798	1858	7409	1129	710
18	554	16025	2112	7136	1173	876

N. door	E. door	S. door	S. glass	N. glass	Total
-1	26	1	21	31	482
0	209	22	197	100	4868
11	436	86	462	166	13352
30	586	186	757	238	23738
54	642	305	1050	318	33390
83	618	423	1297	393	41578
114	532	524	1460	449	47397
140	455	581	1508	475	50241
156	410	590	1458	479	49904
165	378	555	1357	482	47211
171	352	489	1225	484	42692
174	325	409	1121	484	36416
176	293	337	1055	463	29647

Figure 8. Results for the Example Problem: Solutions by Numerical Method in Btu/Hr.

Component	Btu/Hr
Roof	30840
S. wall	3060
E. wall	8000
N. exposed wall	910
N. & W. party wall	3680
Door N. E.	780
Door S.	920
S. glass	3480
N. glass	900
Total	52570

Figure 9. Results for the Example Problem. Taken from Reference [3] at 4:00 P.M.

CHAPTER V

CONCLUSIONS

The cooling load given in Table 4, page 472 of [3] is compared with Tables III and IV in the Appendix of this thesis. The cooling loads given in [3] are all positive in value; while, seen from Tables III and IV of this thesis, some of the cooling loads are negative in the winter time and when the sun cannot reach the surface. Otherwise the results are close. This is due to the fact that in our analysis, the influence of transient ambient temperature is taken into consideration. The comparison between Tables 9-11 to 9-14 of [4] and Tables V to VIII in the Appendix of this thesis shows the same tendency.

Figure 9 is a retabulation of Table 36, page 506 of [3], a summary of calculations for the example problem based on the designated hour 4:00 P.M., and has a subtotal cooling load of 52570 (Btu/Hr). Figures 7 and 8 in this thesis are the results for the same example problem given in [3] using the present analysis. As seen from Figures 7 and 8, the maximum cooling loads occur at 1:00 P.M. and are about 96 per cent of 52570. At 4:00 P.M. the cooling loads are about 81 per cent of 2570.

To generate the results for the cooling load of a building, e.g. results in Figure 8, only 1 minute and 48 seconds were consumed on the IBM-360-40 computer, at a cost of approximately $2.00.

Complete calculations are given in the Appendix for heat gain through single and double glass for 40° North Latitude.

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APPENDIX

A. Constants and Data

Constants for generating the tables of heat gain are: A building of 20 feet deep, 20 feet wide and 10 feet high, with window on each surface of the walls.

For each window on a unit area basis:

$\rho_g = 160$ $\rho_g = 0.071$ $\rho_p = 160$ $\rho_x = 0.071$

$C_{pg} = 0.186$ $C_{pf} = 0.24$ $C_{pp} = 0.186$ $C_{px} = 0.24$

$V_g = 0.021$ $V_f = 0.083$ $V_p = 0.021$ $V_x = 4000$

$A_a = 1$

$A_a = A_b = A_c = A_d = A_{ng} = A_{np}$

$\epsilon_{pg} = 0.92$ $\epsilon_{pp} = 0.92$

$r_g = 0.04$ $r_p = 0.04$

SUMA = 0.78 SUMB = 0.78

SUMC = 0 , all energy is absorbed by the room air.

For a single barrier, data for the air space and inner solid is not needed. For generating the tables to compare with that from Hutchinson's book, only the south facing window is taken into consideration.

The normal solar heat flux for Tables I to IV is taken from [3]. The normal solar heat flux for Tables V to VII is taken from [4].

The ambient temperature data is taken from weather stations for the year of 1967, for the following locations:

Houston, Texas	$29^{o}39'$
Dallas, Texas	$32^{o}51'$
Tulsa, Oklahoma	$36^{o}12'$
Omaha, Nebraska	$41^{o}18'$
Brookings, South Dakota	$44^{o}19'$
Fargo, North Dakota	$46^{o}54'$

SYMBOLS FOR TABLES

Symbol	Meaning
QDN	Direct normal solar heat flux
TS	Ambient temperature
S	South facing
E	East facing
N	North facing
W	West facing
Hor	Horizontal facing
i	Incident angle

TABLE I - 1

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (DOUBLE BARRIER)

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Jan	8	141	481	40	57	-25	-25	-4
	9	238	483	94	78	-24	-24	27
	10	274	485	131	59	-20	-20	56
	11	289	487	152	21	-16	-16	73
	12	293	489	158	-13	-14	-3	80
	13	289	501	155	-8	-8	48	77
	14	274	502	131	-9	-9	83	58
	15	238	503	87	-13	-13	88	27
	16	141	502	24	-19	-19	54	-11
One day's total				972	153	-148	185	383
Feb	7	55	481	-4	37	-28	-28	-16
	8	219	482	51	99	-26	-26	19
	9	271	483	94	101	-21	-21	58
	10	293	486	126	72	-16	-16	89
	11	303	488	146	30	-11	-11	108
	12	306	501	158	-2	-4	9	119
	13	303	503	151	-2	-2	61	113
	14	293	505	130	-3	-4	102	92
	15	271	506	94	-5	-7	121	60
	16	219	505	45	-13	-12	93	-18
	17	55	504	-13	-19	-19	2	-17
One day's total				978	285	-150	284	643
Mar	7	171	496	13	102	-14	-14	18
	8	250	497	51	134	-11	-11	60
	9	281	499	88	124	-5	-5	100
	10	297	503	118	93	1	1	131
	11	304	506	136	48	5	5	151
	12	306	510	143	10	9	21	159
	13	304	513	136	10	10	74	151
	14	297	515	117	9	9	118	131
	15	281	516	87	6	6	145	98
	16	250	516	48	1	1	140	56
	17	171	514	5	-7	-7	99	10
One day's total				942	530	4	573	1065

TABLE I - 2

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (DOUBLE BARRIER)

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Apr	6	89	508	-9	66	-2	-9	9
	7	207	510	5	134	-2	-3	53
	8	253	512	38	148	5	5	99
	9	275	514	71	134	11	11	136
	10	286	516	95	102	15	15	166
	11	292	519	112	60	19	19	183
	12	294	522	118	22	22	34	189
	13	292	524	111	22	24	83	182
	14	286	524	93	21	21	124	161
	15	275	525	67	18	17	153	130
	16	253	524	35	13	13	158	89
	17	207	524	7	5	7	124	45
	18	89	523	-4	-4	8	42	3
One day's total				739	741	158	756	1445
May	5	1	507	-13	9	-7	-13	-7
	6	143	510	-5	95	18	-5	27
	7	216	514	7	140	17	5	73
	8	249	518	25	149	13	13	118
	9	267	521	54	134	20	20	154
	10	277	523	76	104	24	24	182
	11	282	524	90	63	27	27	197
	12	284	526	94	29	28	39	203
	13	282	526	88	28	27	84	195
	14	277	527	72	26	26	122	176
	15	267	527	49	23	23	148	147
	16	249	527	25	18	22	157	108
	17	216	527	13	13	30	138	65
	18	143	526	3	4	23	75	23
	19	1	525	-5	-4	-3	-2	-4
One day's total				573	831	288	832	1657

TABLE I - 3

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (DOUBLE BARRIER)

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
June	5	21	522	-2	29	9	-2	6
	6	154	523	6	111	36	6	44
	7	215	526	15	147	33	15	90
	8	246	529	28	154	25	24	132
	9	262	531	53	140	30	28	166
	10	272	533	75	111	34	33	194
	11	276	535	87	72	38	36	209
	12	278	536	91	38	38	49	214
	13	276	537	85	38	39	91	206
	14	272	538	71	36	36	129	188
	15	262	538	51	32	33	155	159
	16	246	538	29	29	35	164	125
	17	215	538	24	23	48	148	81
	18	154	537	14	14	41	93	39
	19	21	535	5	5	10	14	6
One day's total				632	979	485	983	1859
July	5	2	521	-3	18	3	-3	2
	6	137	525	7	103	30	7	40
	7	208	529	20	147	30	18	86
	8	241	533	36	157	26	26	128
	9	259	526	65	145	32	33	165
	10	269	538	86	116	36	38	192
	11	274	540	100	77	40	41	210
	12	276	542	105	44	43	54	213
	13	274	543	99	42	43	98	207
	14	269	544	84	40	41	134	189
	15	259	544	61	38	38	158	161
	16	241	544	38	34	37	167	123
	17	208	543	27	27	45	148	80
	18	137	542	17	17	36	86	37
	19	2	541	9	9	9	11	9
One day's total				751	1014	489	1016	1842

TABLE I - 4

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (DOUBLE BARRIER)

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Aug	6	80	521	1	69	7	1	18
	7	191	524	16	136	10	10	63
	8	236	528	49	153	20	20	108
	9	259	531	81	143	26	26	147
	10	271	535	106	115	33	33	176
	11	277	539	123	76	38	38	198
	12	279	543	130	42	41	52	201
	13	277	542	122	40	40	98	195
	14	271	543	103	37	37	135	174
	15	259	542	77	34	34	161	142
	16	236	542	46	29	29	164	104
	17	191	542	22	21	24	130	60
	18	180	542	12	12	23	50	18
One day's total				888	907	362	918	1604
Sep	7	149	515	23	103	-1	-1	29
	8	230	518	63	140	5	5	73
	9	263	521	100	134	13	13	113
	10	279	524	129	105	17	17	144
	11	287	526	146	61	22	22	162
	12	290	529	151	25	24	36	168
	13	287	531	145	25	26	85	160
	14	279	532	125	24	24	127	139
	15	263	533	97	21	20	151	107
	16	230	533	59	16	15	144	68
	17	149	532	18	7	7	100	22
One day's total				1056	637	172	699	1185

TABLE I - 5

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (DOUBLE BARRIER)

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Oct	7	48	506	9	38	-13	-13	-2
	8	203	508	64	110	-7	-7	37
	9	257	511	109	116	1	1	77
	10	280	514	142	92	6	6	108
	11	290	517	161	50	9	9	127
	12	293	519	165	13	12	24	133
	13	290	521	158	13	12	72	124
	14	280	521	137	11	12	111	102
	15	257	521	101	6	6	128	70
	16	203	520	52	0	0	99	29
	17	48	519	-1	-8	-8	10	-5
One day's total				1097	441	30	440	800
Nov	8	136	495	46	63	-17	-17	4
	9	232	496	100	85	-15	-15	36
	10	267	498	137	67	-10	-10	65
	11	283	501	158	30	-6	-8	83
	12	287	504	164	-3	-3	8	89
	13	283	507	157	-2	-2	53	82
	14	267	508	131	-4	-4	86	61
	15	232	508	87	-8	-8	91	30
	16	136	507	25	-15	-15	55	-8
One day's total				1005	213	-80	243	442
Dec	8	88	485	26	32	-25	-25	-12
	9	217	486	90	67	-24	-24	17
	10	261	487	130	53	-20	-20	44
	11	279	489	151	19	-16	-16	61
	12	284	492	158	-13	-14	-4	67
	13	279	494	148	-14	-14	39	59
	14	261	496	122	-15	-15	69	40
	15	217	496	72	-19	-19	65	10
	16	88	495	-5	-26	-26	12	-22
One day's total				892	84	-173	96	264

TABLE II - 1

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (DOUBLE BARRIER)

SOLUTIONS BY NUMERICAL METHOD

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Jan	8	141	481	52	63	-26	-26	2
	9	238	483	103	76	-24	-24	33
	10	274	485	136	52	-19	-19	60
	11	289	487	154	12	-16	-16	75
	12	293	489	159	-12	-13	8	80
	13	289	501	152	-8	-8	57	75
	14	274	502	124	-9	-9	87	52
	15	238	503	75	-13	-13	82	20
	16	141	502	11	-20	-20	47	-15
One day's total				966	141	-148	196	382
Feb	7	55	481	8	48	-28	-28	-10
	8	219	482	60	102	-25	-25	27
	9	271	483	102	96	-20	-20	65
	10	293	486	131	64	-15	-15	93
	11	303	488	150	21	-10	-10	111
	12	306	501	158	-2	-3	20	119
	13	303	503	148	-2	-2	70	109
	14	293	505	123	-3	-3	108	86
	15	271	506	86	-7	-7	121	52
	16	219	505	31	-14	-14	75	10
	17	55	504	-18	-20	-20	-10	-19
One day's total				979	283	-147	286	643
Mar	7	171	496	21	111	-13	-13	27
	8	250	497	59	134	-10	-10	69
	9	281	499	95	118	-3	-3	108
	10	297	503	122	84	2	2	137
	11	304	506	138	38	6	6	154
	12	306	510	142	10	9	32	158
	13	304	513	133	10	10	84	148
	14	297	515	112	9	9	125	125
	15	281	516	80	5	5	147	90
	16	250	516	40	-0	0	133	47
	17	171	514	-2	-8	-8	90	1
One day's total				940	511	7	593	1064

TABLE II - 2

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (DOUBLE BARRIER)

SOLUTIONS BY NUMERICAL METHOD

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Apr	6	89	508	-7	83	-2	-7	18
	7	207	510	12	139	0	0	63
	8	253	512	46	147	7	7	107
	9	275	514	76	129	12	12	143
	10	286	516	100	94	17	14	169
	11	292	519	113	50	21	21	185
	12	294	522	117	24	23	45	189
	13	292	524	108	23	23	93	178
	14	286	524	88	21	21	131	155
	15	275	525	61	17	17	155	123
	16	253	524	28	12	12	154	81
	17	207	524	5	4	8	107	36
	18	89	523	-4	-4	8	25	-2
One day's total				743	739	167	760	1445
May	5	1	507	-11	28	0	-11	-1
	6	143	510	-3	107	18	-3	37
	7	216	514	10	143	16	7	84
	8	249	518	32	147	15	15	126
	9	267	521	60	129	21	21	161
	10	277	523	80	97	25	25	185
	11	282	524	92	55	28	28	199
	12	284	526	94	29	29	50	201
	13	282	526	85	28	28	93	191
	14	277	527	68	26	26	129	170
	15	267	527	43	22	22	152	138
	16	249	527	22	18	24	155	100
	17	216	527	11	11	31	128	57
	18	143	526	2	2	17	56	16
	19	1	525	-4	-4	-4	-3	-4
One day's total				581	838	296	842	660

TABLE II - 3

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (DOUBLE BARRIER)

SOLUTIONS BY NUMERICAL METHOD

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
June	5	21	522	0	48	17	0	14
	6	154	523	9	120	37	9	54
	7	215	526	18	150	32	18	99
	8	246	529	34	152	26	25	139
	9	262	531	58	134	31	31	172
	10	272	533	78	103	35	35	196
	11	276	535	89	64	38	38	210
	12	278	536	91	39	38	58	212
	13	276	537	83	38	38	100	202
	14	272	538	67	36	36	135	182
	15	262	538	45	33	34	157	152
	16	246	538	28	28	39	161	115
	17	215	538	22	22	48	138	73
	18	154	537	12	12	34	74	31
	19	21	535	4	4	6	7	4
One day's total				638	983	489	983	1653
July	5	2	521	-1	37	10	-1	9
	6	137	525	10	115	31	10	49
	7	208	529	22	151	29	20	95
	8	241	533	43	155	28	28	137
	9	259	526	70	139	34	34	172
	10	269	538	90	108	38	38	196
	11	274	540	102	68	42	42	210
	12	276	542	104	43	43	63	213
	13	274	543	97	43	43	106	203
	14	269	544	80	41	41	141	182
	15	259	544	56	37	37	162	152
	16	241	544	35	32	39	164	114
	17	208	543	25	25	45	137	70
	18	137	542	15	15	30	67	30
	19	2	541	9	9	9	10	9
One day's total				757	1018	499	1021	1841

TABLE II - 4

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (DOUBLE BARRIER)

SOLUTIONS BY NUMERICAL METHOD

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Aug	6	80	521	3	85	8	3	27
	7	191	524	23	142	13	12	73
	8	236	528	56	152	21	21	117
	9	259	531	87	138	28	28	154
	10	271	535	111	107	34	34	181
	11	277	539	125	67	38	38	198
	12	279	542	128	41	41	61	201
	13	277	543	119	40	40	106	190
	14	271	542	99	37	37	141	167
	15	259	542	72	33	33	162	135
	16	236	542	42	27	27	159	95
	17	191	542	20	20	24	114	50
	18	80	542	12	12	24	37	14
One day's total				897	901	368	916	1602
Sep	7	149	515	32	112	1	1	38
	8	230	518	72	140	8	8	82
	9	263	521	107	129	15	15	121
	10	279	524	133	97	20	20	148
	11	287	526	148	53	23	23	164
	12	290	529	151	26	25	47	168
	13	287	531	142	26	26	95	157
	14	279	532	120	24	24	133	134
	15	263	533	89	20	20	152	100
	16	230	533	51	14	14	135	58
	17	149	532	11	6	6	89	14
One day's total				1056	647	182	718	1184

TABLE II - 5

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (DOUBLE BARRIER)

SOLUTIONS BY NUMERICAL METHOD

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Oct	7	48	506	21	56	-11	-11	6
	8	203	508	75	115	-5	-5	46
	9	257	511	117	113	2	2	84
	10	280	514	147	83	7	7	113
	11	290	517	163	41	11	11	129
	12	293	519	166	13	13	34	131
	13	290	521	155	13	13	82	121
	14	280	521	130	10	10	116	97
	15	257	521	92	5	5	126	62
	16	203	520	39	-2	-2	80	21
	17	48	519	-6	-9	-9	0	-8
One day's total				1099	438	34	442	802
Nov	8	136	495	58	70	-17	-17	11
	9	232	496	109	83	-14	-14	43
	10	267	498	143	61	-9	-9	69
	11	283	501	161	22	-5	-5	85
	12	287	504	165	-3	-3	17	89
	13	283	507	153	-2	-2	61	79
	14	267	508	125	-5	-5	90	56
	15	232	508	76	-9	-9	84	23
	16	136	507	13	-16	-16	48	-11
One day's total				1003	201	-80	255	444
Dec	8	88	485	40	42	-25	-25	-6
	9	217	486	100	66	-24	-24	23
	10	261	487	135	47	-20	-20	48
	11	279	489	154	10	-16	-16	63
	12	284	492	157	-14	-14	5	66
	13	279	494	144	-14	-14	47	56
	14	261	496	114	-16	-16	72	35
	15	217	496	57	-21	-21	55	3
	16	88	495	-14	-27	-27	0	-25
One day's total				887	73	-177	94	263

TABLE III - 1

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (SINGLE BARRIER)

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Jan	8	141	481	36	50	-57	-57	-23
	9	238	483	105	71	-46	-46	22
	10	274	485	145	44	-39	-39	55
	11	289	587	166	-2	-34	-34	74
	12	293	589	189	-12	-12	12	97
	13	289	501	169	-18	-18	59	79
	14	274	502	134	-21	-21	93	50
	15	238	503	73	-29	-29	83	9
	16	141	502	-3	-38	-38	39	-32
One day's total				1014	45	-294	98	331
Feb	7	55	481	-18	29	-61	-61	-38
	8	219	482	53	102	-49	-49	14
	9	271	483	105	97	-39	-39	62
	10	293	486	139	59	-33	-33	95
	11	303	488	180	28	-8	-8	135
	12	306	501	179	-9	-9	17	134
	13	303	503	165	-10	-10	76	120
	14	293	505	136	-13	-13	119	92
	15	271	506	88	-20	-20	131	48
	16	219	505	23	-29	-29	74	-2
	17	55	504	-34	-37	-37	-25	-49
One day's total				1015	197	-308	202	611
Mar	7	171	496	7	116	-34	-34	14
	8	250	497	59	145	-23	-23	71
	9	281	499	105	131	-11	-11	121
	10	297	503	138	92	-3	-3	156
	11	304	506	160	40	4	4	178
	12	306	510	164	8	8	36	182
	13	304	513	153	8	8	97	170
	14	297	515	126	6	6	143	141
	15	281	516	87	0	0	167	99
	16	250	516	36	-10	-10	146	44
	17	171	514	-14	-21	-21	94	-11
One day's total				1021	515	-76	616	1165

TABLE III - 2

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (SINGLE BARRIER)

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Apr	6	89	508	-19	89	-14	-20	11
	7	207	510	9	159	-6	-6	69
	8	253	512	50	168	4	4	122
	9	275	514	87	147	11	11	165
	10	286	516	117	109	19	19	199
	11	292	519	135	59	25	25	219
	12	294	522	138	28	28	54	222
	13	292	524	124	24	24	108	207
	14	286	524	102	23	23	154	181
	15	275	525	66	16	16	179	139
	16	253	524	29	10	10	177	91
	17	207	524	1	0	5	120	36
	18	89	523	-11	-10	4	22	-8
One day's total				828	822	149	847	1653
May	5	1	507	-25	24	-11	-24	-12
	6	143	510	-6	124	19	-6	42
	7	216	514	12	169	18	8	99
	8	249	518	39	173	18	18	149
	9	267	521	71	151	25	25	190
	10	277	523	94	112	28	28	217
	11	282	524	109	64	33	33	235
	12	284	526	109	32	32	57	235
	13	282	526	99	33	33	110	224
	14	277	527	78	29	29	151	197
	15	267	527	48	25	25	177	160
	16	249	527	23	19	27	180	115
	17	216	527	9	9	33	146	62
	18	143	526	-2	-2	15	59	14
	19	1	525	-9	-9	-9	-9	-9
One day's total				649	953	315	953	1918

TABLE III - 3

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (SINGLE BARRIER)

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
June	5	21	522	-4	55	17	-4	13
	6	154	523	12	144	45	12	66
	7	215	526	25	180	40	25	121
	8	246	529	44	182	34	33	168
	9	262	531	74	162	40	40	207
	10	272	533	98	126	46	46	236
	11	276	535	110	78	48	48	251
	12	278	536	111	50	50	74	253
	13	276	537	103	50	50	124	242
	14	272	538	82	46	46	163	217
	15	262	538	55	42	42	188	181
	16	246	538	36	36	49	192	137
	17	215	538	27	27	58	163	85
	18	154	537	13	13	37	83	33
	19	21	535	2	2	4	4	2
One day's total				824	1193	606	1273	2228
July	5	2	521	-4	43	10	-4	9
	6	137	525	15	140	40	15	63
	7	208	529	33	185	41	30	120
	8	241	533	58	190	40	40	170
	9	259	526	89	169	47	47	209
	10	269	538	114	133	53	53	238
	11	274	540	129	87	57	57	256
	12	276	542	130	58	58	82	257
	13	274	543	121	58	58	133	245
	14	269	544	99	54	54	173	219
	15	259	544	70	49	49	197	183
	16	241	544	44	41	50	197	136
	17	208	543	32	32	56	162	84
	18	137	542	21	21	37	78	36
	19	2	541	13	13	13	13	13
One day's total				964	1273	651	1273	2238

TABLE III - 4

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (SINGLE BARRIER)

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Aug	6	80	521	0	100	6	0	30
	7	191	524	31	170	18	18	91
	8	236	528	71	183	19	19	144
	9	259	531	110	169	41	41	190
	10	271	535	141	135	50	50	224
	11	277	539	157	87	56	56	243
	12	279	542	158	55	55	81	243
	13	277	543	143	50	50	130	227
	14	271	542	120	48	48	172	200
	15	259	542	89	43	43	196	162
	16	236	542	53	37	37	192	115
	17	191	542	28	28	33	137	62
	18	80	542	18	18	33	45	20
One day's total				1119	1123	489	1137	1951
Sep	7	149	515	32	129	-6	-6	40
	8	230	518	84	164	8	8	97
	9	263	521	128	152	18	18	144
	10	279	524	158	114	24	24	176
	11	287	526	178	64	31	30	197
	12	290	529	181	33	33	60	201
	13	287	531	168	32	32	116	187
	14	279	532	143	30	30	160	159
	15	263	533	105	25	25	180	118
	16	230	533	57	15	15	157	66
	17	149	532	11	6	6	102	44
One day's total				1245	764	216	832	1399

TABLE III - 5

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (SINGLE BARRIER)

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Oct	7	48	506	13	56	-26	-26	-5
	8	203	508	84	130	-11	-11	50
	9	257	511	136	129	-1	-1	97
	10	280	514	172	96	7	7	132
	11	290	517	190	44	11	11	150
	12	293	519	194	14	14	40	153
	13	290	521	179	11	11	93	138
	14	280	521	148	7	7	133	109
	15	257	521	101	0	0	142	66
	16	203	520	38	-9	-9	85	17
	17	48	519	-15	-17	-17	-8	-17
One day's total				1240	461	-14	465	890
Nov	8	136	495	51	65	-39	-39	-6
	9	232	496	118	86	-28	-28	39
	10	267	498	159	61	-20	-20	73
	11	283	501	183	18	-13	-13	94
	12	287	504	188	-9	-10	15	99
	13	283	507	171	-11	-11	65	85
	14	267	508	136	-15	-15	96	56
	15	232	508	76	-23	-23	86	15
	16	136	507	2	-31	-31	43	-26
One day's total				1084	142	-190	205	429
Dec	8	88	485	24	26	-55	-55	-32
	9	217	486	101	61	-45	-45	10
	10	261	487	144	39	-38	-38	42
	11	279	489	168	-2	-32	-32	62
	12	284	492	172	-29	-29	-6	65
	13	279	494	157	-28	-28	44	54
	14	261	496	119	-33	-33	71	26
	15	217	496	49	-41	-41	47	-13
	16	88	495	-34	-48	-48	-17	-47
One day's total				900	-55	-349	-31	167

TABLE VI - 1

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (SINGLE BARRIER)

SOLUTIONS BY NUMERICAL METHOD

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Jan	8	141	481	38	51	-57	-57	-23
	9	238	483	103	70	-47	-47	21
	10	274	485	143	41	-41	-41	53
	11	289	487	164	-6	-37	-37	71
	12	293	489	173	-29	-29	-4	80
	13	289	501	166	-21	-21	57	76
	14	274	502	132	-23	-23	92	49
	15	238	503	74	-28	-28	84	10
	16	141	502	-2	-36	-36	41	-31
One day's total				956	38	-319	88	307
Feb	7	55	481	-16	31	-60	-60	-37
	8	219	482	53	101	-50	-50	13
	9	271	483	102	93	-42	-42	58
	10	293	486	137	56	-36	-36	92
	11	303	488	162	9	-25	-25	117
	12	306	501	174	-14	-14	14	129
	13	303	503	162	-13	-13	73	117
	14	293	505	134	-14	-14	118	90
	15	271	506	89	-19	-19	132	49
	16	219	505	23	-28	-28	74	-1
	17	55	504	-33	-36	-36	-25	-35
One day's total				987	207	337.	173	580
Mar	7	171	496	8	117	-34	-34	15
	8	250	497	57	144	-25	-25	69
	9	281	499	101	126	-16	-16	116
	10	297	503	134	87	-8	-8	151
	11	304	506	154	34	-2	-2	172
	12	306	510	159	3	4	32	178
	13	304	513	149	5	5	94	167
	14	297	515	124	4	4	142	140
	15	281	516	86	0	0	167	99
	16	250	516	39	-7	-7	149	47
	17	171	514	-11	-18	-18	97	-8
One day's total				1000	491	-97	596	1146

TABLE VI - 2

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (SINGLE BARRIER)

SOLUTIONS BY NUMERICAL METHOD

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Apr	6	89	508	-19	90	-14	-20	11
	7	207	510	7	157	-8	-8	68
	8	253	512	48	162	1	1	120
	9	275	514	85	144	8	8	163
	10	286	516	113	104	15	15	195
	11	292	519	130	54	21	21	214
	12	294	522	135	24	24	51	219
	13	292	524	124	24	24	108	206
	14	286	524	101	22	22	153	180
	15	275	525	68	18	18	180	141
	16	253	524	30	11	11	177	91
	17	207	524	3	2	7	121	37
	18	89	523	-9	-9	6	23	-7
One day's total				772	659	135	830	1452
May	5	1	507	-25	24	-12	-25	-13
	6	143	510	-10	120	15	-10	38
	7	216	514	7	164	13	3	94
	8	249	518	34	168	14	14	145
	9	267	521	68	148	22	22	187
	10	277	523	92	110	27	27	216
	11	282	524	106	61	31	31	232
	12	284	526	109	32	32	57	235
	13	282	526	98	31	31	109	222
	14	277	527	78	29	29	152	197
	15	267	527	48	25	25	177	160
	16	249	527	24	19	27	181	115
	17	216	527	11	11	34	147	64
	18	143	526	0	0	17	60	16
	19	1	525	-7	-7	-7	-7	-7
One day's total				633	906	298	938	1886

TABLE VI - 3

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (SINGLE BARRIER)

SOLUTIONS BY NUMERICAL METHOD

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
June	5	21	522	-4	55	17	-4	13
	6	154	523	9	141	42	9	63
	7	215	526	21	176	36	21	117
	8	246	529	41	179	31	30	165
	9	262	531	71	158	37	37	205
	10	272	533	94	122	43	43	233
	11	276	535	108	76	47	47	249
	12	278	536	110	48	48	72	252
	13	276	537	101	48	48	123	241
	14	272	538	82	46	46	164	217
	15	262	538	55	42	43	189	181
	16	246	538	36	36	49	193	137
	17	215	538	28	28	60	164	87
	18	154	537	17	16	41	87	37
	19	21	535	6	6	7	8	6
One day's total				775	1139	595	1184	2203
July	5	2	521	-5	42	9	-5	8
	6	137	525	11	136	36	11	59
	7	208	529	27	179	35	25	115
	8	241	533	54	185	36	36	165
	9	259	526	86	166	44	44	206
	10	269	538	111	130	50	50	235
	11	274	540	125	84	54	54	252
	12	276	542	128	56	56	80	255
	13	274	543	119	56	56	132	244
	14	269	544	99	54	54	173	220
	15	259	544	71	50	50	197	183
	16	241	544	47	44	52	199	138
	17	208	543	34	34	58	165	87
	18	137	542	23	23	40	81	39
	19	2	541	15	15	15	15	15
One day's total				945	1254	643	1257	2221

TABLE IV - 4

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (SINGLE BARRIER)

SOLUTIONS BY NUMERICAL METHOD

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Aug	6	80	521	0	99	5	0	30
	7	191	524	26	166	14	14	87
	8	236	528	67	179	25	25	139
	9	259	531	105	163	35	35	184
	10	271	535	134	128	44	44	218
	11	277	539	152	82	50	50	238
	12	279	542	156	54	54	80	242
	13	277	543	145	52	52	132	228
	14	271	542	121	48	48	172	201
	15	259	542	89	44	44	196	162
	16	236	542	53	37	37	192	115
	17	191	542	29	29	33	137	63
	18	80	542	19	19	33	46	21
One day's total				1096	1100	474	1123	1828
Sep	7	149	515	31	129	-7	-7	39
	8	230	518	81	161	5	5	94
	9	263	521	124	148	14	14	140
	10	279	524	155	110	21	21	173
	11	287	526	174	60	27	27	193
	12	290	529	178	30	30	57	197
	13	287	531	167	31	31	114	185
	14	279	532	142	29	29	158	158
	15	263	533	105	25	25	180	118
	16	230	533	59	17	17	158	68
	17	149	532	13	8	8	104	16
One day's total				1229	748	200	831	1381

TABLE IV - 5

HEAT GAIN FOR 40 DEGREE NORTH LATITUDE (SINGLE BARRIER)

SOLUTIONS BY NUMERICAL METHOD

Date	Time A.M.	QDN	TS	S	E	N	W	Hor
Oct	7	48	506	14	57	-26	-26	-5
	8	203	508	82	128	-14	-14	47
	9	257	511	132	125	-5	-5	93
	10	280	514	167	91	3	3	127
	11	290	517	187	41	8	8	147
	12	293	519	191	11	11	38	150
	13	290	521	178	11	11	94	138
	14	280	521	148	8	8	134	109
	15	257	521	103	2	2	144	67
	16	203	520	39	-7	-7	87	18
	17	48	519	-14	-15	-15	-7	-15
One day's total				1227	452	-24	456	876
Nov	8	136	495	53	66	-39	-39	-5
	9	232	496	116	84	-30	-30	37
	10	267	498	156	57	-24	-24	70
	11	283	501	178	13	-18	-18	89
	12	287	504	183	-14	-14	11	94
	13	283	507	170	-12	-12	64	83
	14	267	508	136	-15	-15	97	56
	15	232	508	78	-21	-21	88	16
	16	136	507	3	-29	-29	45	-25
One day's total				1073	129	-202	194	415
Dec	8	88	485	26	27	-54	-54	-31
	9	217	486	100	60	-46	-46	9
	10	261	487	142	37	-41	-41	40
	11	279	489	165	-6	-36	-36	58
	12	284	492	169	-32	-32	-8	62
	13	279	494	154	-31	-31	41	51
	14	261	496	118	-33	-33	71	25
	15	217	496	50	-39	-39	49	-13
	16	88	495	-33	-47	-47	-17	-46
One day's total				891	-64	-359	-41	155

TABLE V - 1

HEAT GAIN FOR SOUTH-FACING 1/4 INCH WINDOW: LATITUDE 30 DEGREE

Date	Time A.M.	QDN	i	ANALYTICAL		NUMERICAL	
				D	S	D	S
Oct	8	253	64	75	99	82	98
	9	286	58	107	136	112	132
	10	303	53	129	161	132	157
	11	312	50	142	176	143	172
	12	314	49	148	179	146	176
	13	312	50	144	170	140	169
	14	303	53	130	150	125	150
	15	286	58	106	122	101	122
	16	253	64	80	89	73	89
Nov	8	220	58	80	102	88	102
	9	276	51	117	147	123	144
	10	295	45	143	175	146	171
	11	305	41	157	192	159	187
	12	308	40	163	194	162	191
	13	305	41	157	185	155	183
	14	295	45	142	164	137	162
	15	276	51	115	125	107	126
	16	220	58	76	77	67	79
Dec	8	193	56	69	87	80	87
	9	265	48	113	135	119	134
	10	286	42	137	162	140	160
	11	297	38	150	177	152	174
	12	300	37	155	178	154	177
	13	297	38	149	169	147	168
	14	286	42	133	144	128	146
	15	265	48	104	102	94	104
	16	193	56	54	47	45	46
Jan	8	220	58	69	87	80	87
	9	276	51	113	135	119	134
	10	295	45	137	162	140	160
	11	305	41	150	177	152	174
	12	308	40	155	178	154	177
	13	305	41	149	169	147	168
	14	295	45	133	144	128	146
	15	276	51	104	102	94	104
	16	220	58	54	47	45	46

TABLE V - 2

HEAT GAIN FOR SOUTH-FACING 1/4 INCH WINDOW: LATITUDE 30 DEGREE

Date	Time A.M.	QDN	i	ANALYTICAL		NUMERICAL	
				D	S	D	S
Feb	8	253	64	65	74	71	75
	9	286	58	92	107	97	107
	10	303	53	113	133	116	129
	11	312	50	125	148	127	144
	12	314	49	130	153	130	148
	13	312	50	127	146	125	143
	14	303	53	114	127	110	126
	15	286	58	93	98	87	98
	16	253	64	65	65	59	65
Mar	7	157	83	12	17	17	17
	8	223	76	36	50	42	48
	9	259	70	60	80	65	76
	10	275	65	80	101	83	98
	11	283	62	92	116	94	111
	12	286	61	96	116	96	115
	13	283	62	92	109	90	108
	14	275	65	79	91	76	91
	15	259	70	60	63	55	66
	16	223	76	35	32	30	35
	17	157	83	10	3	6	7
Apr	8	241	87	12	22	17	21
	9	274	81	33	50	38	48
	10	283	76	51	69	54	66
	11	289	74	61	81	63	78
	12	291	72	66	82	65	81
	13	289	74	60	77	59	74
	14	283	76	51	66	49	63
	15	274	81	36	44	31	42
	16	241	87	16	25	14	23

TABLE VI - 1

HEAT GAIN FOR SOUTH-FACING 1/4 INCH WINDOW: LATITUDE 35 DEGREE

Date	Time A.M.	QDN	i	ANALYTICAL		NUMERICAL	
				D	S	D	S
Oct	8	245	63	69	83	75	83
	9	282	57	99	125	106	121
	10	299	50	127	156	131	152
	11	306	45	144	174	146	170
	12	310	44	148	179	149	174
	13	306	45	145	169	141	166
	14	299	50	128	145	122	143
	15	282	57	101	112	95	112
	16	245	63	72	77	66	78
Nov	8	195	57	62	72	72	73
	9	268	49	105	126	112	123
	10	288	42	132	159	137	154
	11	295	36	150	177	152	172
	12	300	35	155	180	155	176
	13	295	36	150	168	147	167
	14	288	42	132	143	126	143
	15	268	49	100	98	91	100
	16	195	57	52	41	42	42
Dec	8	286	55	57	60	66	61
	9	245	46	96	110	103	109
	10	273	38	126	143	130	140
	11	286	33	142	160	144	158
	12	292	31	147	164	147	161
	13	286	33	140	153	137	150
	14	273	38	122	125	116	125
	15	245	46	88	80	79	81
	16	186	55	44	27	35	29
Jan	8	195	57	56	59	65	60
	9	268	49	98	113	104	109
	10	288	42	125	146	129	141
	11	295	36	142	165	144	159
	12	300	35	148	168	148	164
	13	295	36	143	158	140	155
	14	288	42	125	132	120	132
	15	268	49	93	88	85	89
	16	195	57	46	30	36	31

TABLE VI - 2

HEAT GAIN FOR SOUTH-FACING 1/4 INCH WINDOW: LATITUDE 35 DEGREE

Date	Time A.M.	QDN	i	ANALYTICAL D	ANALYTICAL S	NUMERICAL D	NUMERICAL S
Feb	8	245	63	55	52	60	53
	9	282	57	82	91	88	89
	10	299	50	107	120	112	118
	11	306	45	123	138	125	134
	12	310	44	129	143	128	139
	13	306	45	123	136	121	131
	14	299	50	106	115	102	110
	15	282	57	83	85	77	81
	16	245	63	55	53	50	50
Mar	7	157	82	5	1	11	1
	8	223	74	31	38	36	35
	9	259	68	56	70	62	67
	10	273	62	78	95	81	91
	11	281	59	90	109	92	104
	12	283	58	94	111	95	108
	13	281	59	91	106	89	102
	14	273	62	80	88	75	86
	15	259	68	58	60	53	60
	16	223	74	33	28	27	29
	17	157	82	5	-3	-1	-2
Apr	8	241	84	13	16	19	17
	9	268	77	36	46	41	44
	10	282	72	54	67	57	65
	11	287	69	66	79	67	77
	12	289	68	69	85	69	80
	13	287	69	67	79	65	76
	14	282	72	55	63	52	61
	15	268	77	37	39	32	38
	16	241	84	14	13	10	12

TABLE VII - 1

HEAT GAIN FOR SOUTH-FACING 1/4 INCH WINDOW: LATITUDE 40 DEGREE

Date	Time A.M.	QDN	i	ANALYTICAL D	ANALYTICAL S	NUMERICAL D	NUMERICAL S
Oct	8	230	63	62	73	71	74
	9	273	53	100	121	107	118
	10	292	46	127	152	131	148
	11	300	41	143	170	146	166
	12	303	39	149	173	149	170
	13	300	41	142	163	140	160
	14	292	46	126	138	121	137
	15	273	53	96	99	88	99
	16	230	63	55	50	48	51
Nov	8	180	56	54	56	62	57
	9	240	46	94	110	103	108
	10	276	38	127	147	132	144
	11	288	32	146	167	148	163
	12	289	29	151	172	151	167
	13	288	32	145	159	142	157
	14	276	38	124	128	117	127
	15	240	46	87	79	78	80
	16	180	56	41	24	32	26
Dec	8	180	55	46	41	54	42
	9	221	44	83	90	91	90
	10	260	35	116	127	121	125
	11	273	28	134	148	137	145
	12	277	25	140	152	140	149
	13	273	28	132	140	129	137
	14	260	35	111	107	104	107
	15	221	44	74	60	66	61
	16	180	55	34	13	26	14
Jan	8	180	56	45	38	53	40
	9	240	46	85	93	93	91
	10	276	38	117	128	122	126
	11	288	32	135	146	137	144
	12	289	29	140	169	141	152
	13	288	32	140	152	138	149
	14	276	38	119	121	113	120
	15	240	46	83	72	74	74
	16	180	56	37	18	28	19

TABLE VII - 2

HEAT GAIN FOR SOUTH-FACING 1/4 INCH WINDOW: LATITUDE 40 DEGREE

Date	Time A.M.	QDN	i	ANALYTICAL		NUMERICAL	
				D	S	D	S
Feb	8	230	63	46	38	53	40
	9	273	53	81	83	93	83
	10	292	46	106	114	122	111
	11	300	41	122	130	137	128
	12	303	39	128	155	141	137
	13	300	41	127	140	137	114
	14	292	46	111	117	113	114
	15	273	53	83	79	74	78
	16	230	63	43	32	28	31
Mar	7	134	81	-2	-14	4	-13
	8	217	72	25	27	32	25
	9	247	64	54	66	61	61
	10	265	57	79	93	84	89
	11	275	53	94	112	98	106
	12	277	51	102	117	102	112
	13	275	53	97	107	94	104
	14	265	57	81	85	77	83
	15	247	64	56	53	51	53
	16	217	72	27	14	21	17
	17	134	81	-4	-21	-8	-18
Apr	8	238	81	15	13	20	13
	9	264	74	38	46	44	44
	10	279	68	60	72	65	70
	11	284	63	76	87	76	84
	12	286	63	78	92	78	87
	13	284	63	75	87	73	82
	14	279	68	60	66	57	63
	15	264	74	40	38	35	38
	16	238	81	15	9	11	9

TABLE VIII - 1

HEAT GAIN FOR SOUTH-FACING 1/4 INCH WINDOW: LATITUDE 45 DEGREE

Date	Time A.M.	QDN	i	ANALYTICAL		NUMERICAL	
				D	S	D	S
Oct	8	218	62	57	65	66	66
	9	267	52	97	115	105	113
	10	282	42	127	150	132	146
	11	292	36	145	167	147	165
	12	295	34	151	173	150	169
	13	292	36	144	160	141	158
	14	282	42	124	133	118	131
	15	267	52	91	92	83	90
	16	218	62	49	43	41	41
Nov	8	164	55	44	38	51	40
	9	220	45	81	91	89	89
	10	248	34	115	133	121	128
	11	270	27	139	157	141	153
	12	276	24	146	158	145	157
	13	270	27	135	140	131	140
	14	248	34	109	106	102	106
	15	220	45	73	62	65	61
	16	164	55	31	13	23	12
Dec	8	178	54	41	24	45	27
	9	193	42	64	62	70	61
	10	218	31	95	103	102	100
	11	253	23	123	131	126	129
	12	267	22	131	135	126	129
	13	253	23	118	115	112	112
	14	218	31	89	78	82	77
	15	193	42	59	42	54	44
	16	178	54	33	10	27	11
Jan	8	164	55	35	20	43	22
	9	220	45	72	72	79	71
	10	248	34	105	110	110	108
	11	270	27	126	130	128	130
	12	276	24	132	133	130	132
	13	270	27	121	117	117	115
	14	248	34	96	85	89	83
	15	220	45	60	38	52	39
	16	164	55	18	-12	9	-12

TABLE VIII - 2

HEAT GAIN FOR SOUTH-FACING 1/4 INCH WINDOW: LATITUDE 45 DEGREE

Date	Time A.M.	QDN	i	ANALYTICAL D	ANALYTICAL S	NUMERICAL D	NUMERICAL S
Feb	8	218	62	37	18	44	20
	9	267	52	72	69	79	68
	10	282	42	101	100	105	100
	11	292	36	118	118	119	116
	12	295	34	123	122	122	120
	13	292	36	115	110	112	108
	14	282	42	95	83	89	82
	15	267	52	63	42	54	40
	16	218	62	22	-6	13	-8
Mar	7	117	80	-7	-25	-1	-24
	8	207	70	23	19	29	18
	9	238	61	54	59	60	56
	10	259	53	80	87	84	84
	11	268	49	94	103	97	100
	12	271	46	101	109	100	104
	13	268	49	94	94	91	94
	14	259	53	77	73	72	71
	15	238	61	49	38	44	38
	16	207	70	18	-2	11	-1
	17	117	80	-15	-38	-19	-37
Apr	8	235	79	14	6	20	7
	9	260	70	41	43	46	42
	10	275	63	64	67	67	66
	11	281	59	76	84	78	80
	12	283	58	80	87	80	84
	13	281	59	76	80	74	77
	14	275	63	61	62	58	58
	15	260	70	39	31	33	29
	16	235	79	9	-3	3	-5