# Laboratory Study of Trail-tube Irrigation 

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## LABORATORY STUDY OF TRAIL-TUBE IRRIGATION

BY
MOHAMED O. SHANNA

A thesis submitted
in partial fulfillment of the requirement for the degree Master of Science, Major in Engineering South Dakota State University

1982

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable for meeting the thesis requirements for this degree. Acceptance of this thesis does not imply that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Shu Tung Chu Thesis Adviser

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## PARTIAL LIST OF SYMBOLS

| Symbol | Explanation | Units Used |
| :---: | :---: | :---: |
| A | Area | $f t^{2}$ |
| a | Area | in ${ }^{2}$ |
| C | Hazen-Williams friction factor | - - |
| $\mathrm{C}_{\text {C }}$ | Coefficient of contraction | - - |
| cfs | Cubic feet per second | $\mathrm{dfs}\left(\mathrm{ft}^{3} / \mathrm{sec}\right)$ |
| $\mathrm{C}_{\mathrm{Q}}$ | Coefficient of discharge | - - |
| $\mathrm{C}_{\mathrm{V}}$ | Coefficient of velocity | - - |
| CV | Coefficient of variation | - - |
| D | Pipe diameter | ft, in |
| $\mathrm{D}_{\mathrm{a}}$ | Depth of application | ft , in |
| d | Perforation diameter | in |
| $\mathrm{dQ}_{\mathrm{x}}$ | Flow rate of an infitesmal line element, $d x$, on tube or flow rate of an infinitesmal perforation | gpm |
| F | Cumulative intake or the depth of infiltrated water | nm, in |
| f | Darcy-Weisbach's friction factor | - - |
| FF | F-factor of trail tubes after Christiansen (1942) | - - |
| g | Acceleration due to gravity $=32.16$ | $\mathrm{ft} / \mathrm{sec}^{2}$ |
| gpm | Gallons per minute | gpm |
| H | Dimensionless total energy loss distribution or the distribution factor | - - |
| $\mathrm{H}_{\mathrm{f}}$ | Head loss due to friction | ft |
| ID | Inside diameter | ft , in |

PARTIAL LIST OF SYMBOLS CONTINUFD

| Symbol | Explanation | Units Used |
| :---: | :---: | :---: |
| $\mathrm{K}_{\mathrm{s}}$ | Scobey's friction factor | - - |
| L | Length of pipe or tube | $f t$ |
| OD | Outside diameter | ft , in |
| P | Pressure head | Psi, ft, in |
| $\mathrm{P}_{\mathrm{X}}$ | Pressure head at point $x$ along the tube | Psi, ft, in |
| Psi | Pounds per square inch | $1 \mathrm{~b} / \mathrm{in}^{2}$ |
| Q | Tube flow rate | gpm, cfs |
| $\mathrm{Q}_{0}$ | Total flow rate of a trail tube | gpm |
| q | Perforation flow rate | gpm, cfs |
| $\mathrm{q}_{\mathrm{x}}$ | ```Perforation flow rate at a distance x along the tube``` | gpm |
| $Q_{x}$ | Flow rate of trail tube at a distance $x$ along the tube | gpm |
| R | Reynold's number | - - |
| $\mathrm{R}_{\mathrm{H}}$ | Hydraulic radius $=\mathrm{D} / 4$ for round pipe flowing full | $f t$ |
| S | Energy loss per foot of pipe $=$ $\mathrm{H}_{\mathrm{f}} / \mathrm{L}$ | - - |
| SD | Standare deviation | - - |
| t | Time | min |
| TR | Time per revolution of a center pivot system | min |
| V | Average flow velocity | $\mathrm{ft} / \mathrm{sec}$ |
| $\mathrm{V}_{\mathbf{s}}$ | Traveling speed of a center pivot system | $\mathrm{ft} / \mathrm{min}$ |
| $v$ | Kinematic viscosity | $\mathrm{ft}^{2} / \mathrm{sec}$ |

PARTIAL LIST GF SYMBOLS CONTINUED

| Symbol | Explanation <br> $\mathbf{x}$ | Units Used <br> at the upstream side of the tube to <br> a point along the tube, where $x$ is <br> less than or equal $L$ |
| :--- | :--- | :--- |
| Z | Potential head or elevation | ft , in |

## INTRODUCTION

Increasing energy costs, continuing interest in more efficient use of water, and the availability of low cost plastic tubes initiated the idea of trail tube irrigation. Trail tubes are perforated poly-flex hoses similar to the laterals of a trickle irrigation system. These tubes connected to the main line of a center pivot system can be used to replace sprinklers. The main line of a center pivot system provides the water supply and the mobility. The arrangement of a trail tube irrigation system is similar to a traveling trickle system (Rawlins, et al., 1979).

Advantages of trail tube irrigation are its low energy consumption and its high water use efficiency. Trail tube operating pressures can be much lower than the pressure used in the conventional center pivot irrigation system. Such reduction in pressure represents a saving in energy consumption. Trail tubes also distribute water near the ground surface, which minimize water losses due to evaporation and wind effects. Decreasing water losses results in an improvement in water use efficiency.

A theoretical analysis of trail-tubes was presented in a paper, "Analysis of Irrigation by Trail Tubes", (Chu, 1982). The purpose of this study is to evaluate the theory by laboratory measurements.

The objectives of the study were:

1. To determine the roughness coefficient of the poly-flex hoses.
2. To determine the discharge coefficient of the perforations in the tubes.
3. To measure the average jet distance of the perforations.
4. To measure the distributions of flow rate and pressure along the tube.
5. To compare the measured distributions with the theoretical results.

## LITERATURE REVIEW

## Friction Loss Formulas

Water flow in pipes is accompanied by a loss of pressure due to friction. This loss depends on the roughness of the inside walls of the pipe, the diameter of the pipe, the viscosity of the water, and the velocity of the flowing water in the pipe. Many formulas for friction loss in pipes express the relationships between the factors involved. These empirical formulas have been developed from test data. The formulas most commonly used are:
A. Darcy-Weisbach's Formula (Pair, 1975)

$$
\begin{equation*}
H_{f}=f \frac{L}{D} \frac{v^{2}}{2 g} \tag{1}
\end{equation*}
$$

Where $H_{f}=$ the loss of pressure in pipeline, $f t$,
$\mathrm{f}=$ the friction factor,
$L=$ the length of line, ft,
D = the pipe diameter, ft,
$\mathrm{V}=$ the average velocity, $\mathrm{ft} / \mathrm{sec}$, and $g=$ the acceleration due to gravity $=32.16 \mathrm{ft} / \mathrm{sec}^{2}$.

In the above formula, the friction factor (f) depends primarily on the roughness of the pipe material, but also on velocity and pipe diameter. The friction factor values range from 0.015 for large smooth pipe to about 0.050 for very rough pipe, Christiansen (1942).
B. Hazen-Williams Formula (Brater and King, 1976)

$$
\begin{equation*}
\mathrm{V}=1.318 \mathrm{C}_{\mathrm{H}}^{.63} \mathrm{~S} .54 \tag{2}
\end{equation*}
$$

Where $V=$ average velocity, ft/sec,

$$
\begin{aligned}
C= & f r i c t i o n-l o s s \text { coefficient, } \\
R_{H}= & D / 4 \text { for round pipe flowing full, } \\
& D \text { is the diameter of pipe in } f t, \text { and } \\
S= & H_{f} / L=\text { energy loss per foot of pipe. }
\end{aligned}
$$

The typical values of the Hazen-Williams friction-loss coefficient (C), are given in Table 1.

Table 1: Hazen-Williams Friction Coefficient, C.
$\qquad$
Extremely smooth and straight 140
Very smooth130

New riveted steel 110
O1d riveted steel 100
O1d cast iron 95
O1d pipes in poor condition $\quad 60$ to 80
C. Scobey's Formula (Schwab, et al. 1966)

$$
\begin{aligned}
& \mathrm{H}_{\mathrm{f}}=\mathrm{K}_{\mathrm{s}} \frac{\mathrm{~L} \mathrm{Q}^{1.9}}{\mathrm{D}^{4.9}}\left(1.45 \times 10^{-8}\right) \\
& \text { Where } \mathrm{H}_{\mathrm{f}}=\text { total friction loss, } \mathrm{ft}, \\
& \mathrm{~K}_{\mathrm{s}}=\text { Scobey's coefficient of retardation, } \\
& \mathrm{L}=\text { length of pipe, } \mathrm{ft}, \\
& \mathrm{Q}=\text { total discharge, } \mathrm{gPm}, \text { and } \\
& \mathrm{D}
\end{aligned}
$$

Valuis of $K_{s}$ range from about 0.3 for smooth pipe to 1.0 or higher for very rough pipe (Christiansen, 1942).

It is confusing that $f, C$, and $K_{s}$ have different names. However, they are similar in nature because they represent empirical constants in different friction formulas. To avoid confusion, these constants will be referred to as friction coefficients in the following study.

## Flow Formula and Orifice Coefficient

The flow rate in pipes and orifices can be described by the continuity equation and the orifice flow formula.

Pipe Flow: Based on the conservation of mass principle, the flow rate of water in a pipe is represented by the product of average velocity of the water and cross-sectional area of the pipe (Pair, et al. 1975).

$$
\begin{aligned}
Q & =\mathrm{A} V \\
\text { Where } Q & =\text { flow rate, cfs, } \\
A & =\text { cross-sectional area of flow, } \mathrm{ft}^{2}, \text { and } \\
V & =\text { average velocity of flow, } \mathrm{ft} / \mathrm{sec} .
\end{aligned}
$$

Orifice Flow: An orifice is an opening with closed circumference through which water flows (Brater and King, 1976). The flow rate of an orifice is described by the orifice flow equation

$$
\begin{equation*}
Q=A C_{Q}(2 g h)^{0.5} \tag{5}
\end{equation*}
$$

Where $Q=$ discharge, cfs,
$A=$ cross-sectional area of the orifice, $f t^{2}$
$C_{Q}=$ the coefficient of discharge which is the product of the coefficient of contraction $\left(C_{C}\right)$ and the coefficient of velocity ( $\mathrm{C}_{\mathrm{V}}$ ),

```
g = acceleration of gravity, ft/sec, and
h = the pressure head, ft
```

For practical convenience the orifice flow equation can be written in the following way (Christiansen and Davis, 1967).

$$
\begin{equation*}
q=38 C_{Q} a(P)^{0.5} \tag{6}
\end{equation*}
$$

Where $q$ = discharge, gpm,
$a=$ orifice area, $\mathrm{in}^{2}$,
$\mathrm{P}=$ pressure head as presented in psi.
The discharge coefficient of an orifice has been the topic of many studies. Hamilton Smith, 1886 (in Brater and King, 1976), reported that for circular orifices with a diameter of 0.02 feet, the discharge coefficients ( $C_{Q}$ ) were found to be 0.632 and 0.611 at a pressure head of 2 and 10 feet, respectively. The $C_{Q}$ was 0.595 for a diameter of 1 foot at heads of 2 and 10 feet.

Values of the discharge coefficient for 1 inch orifices were determined by various investigations and were found to be different. The differences are not entirely due to experimental error. Other factors may contribute, for example the ratio of the orifice diameter to the tank wall, the sharpness of the orifice edge, the smoothness of the inner surface, the orifice plate, and the temperature of the water. When the tank wall thickness is close to the size of the orifice diameter, the contraction will be suppressed and $C_{Q}$ approaches the value of $C_{V}$ (Brater and King, 1976).

The relation between Reynold's number ( $R$ ) and $C_{Q}$ was presented by Brater and King (Figure 12). The dotted line (A-B) in this Figure was
the range of R covered by the tests of Medaugh and Johnson (1940). In this range, the $C_{Q}$ was approximately a constant, equal to 0.60 .

A discharge coefficient of 0.6 for a sharp-edged orifice, a standard orifice, can be found in many text books, for example Pederson (1971).

## Trail-Tube Irrigation

The development of economical plastic tubes during the 1950's helped initiate the practice of trickle irrigation. Such a system applies water at a low rate through mechanical devices, called emitters, located at selected points along plastic tubes. In the $1970^{\prime}$ s, the research in trickle irrigation entered its well developed stage. The two major problems associated with this type of irrigation rere the clogging of the emitters and the high cost of the total quantity of material (Howell, et al., 1980). These problems prompted the introduction of traveling trickle systems. On a trail tube, many small orifices can be substituted by a small number of large orifices to diminish the hazard of clogging. Furthermore, a traveling tube can replace many stationary tubes to reduce the total material cost (Rawlins, et al. 1979).

The traveling trickle system did not receive widespread acceptance because there was no adequate carrier to provide mobility. A traveling sprinkler system was suggested to be used as a carrier for the trickle tubes (Rawlins, et al., 1979), but the replacement of sprinklers with traveling tubes on a center pivot system did not seem to be attractive at that time.

Increasing energy costs and the desire for increased water-use efficiency gradually shifted research interest toward trail tube irrigation. Since 1978, extensive research on trail-tube systems has been conducted in the states of California, Texas, Arizona, and Idaho (Howell, Phene, and Sanders, 1980). These studies utilized the mainframe of a linear-travel sprinkler system as a carrier and water source for trail tubes. A practical difficulty associated with using a lineartravel sprinkler system as a carrier is that the direction of trail tubes has to be reversed at the end of the field so that irrigation can be continued on its returning trip. In this study, the frame of a center pivot irrigation system is to be used as a carrier for the trail tubes. The traveling path of a center pivot system is in concentric circles, and the direction of travel normally does not reverse during operations. The existing difficulty of a linear carrier is avoided by a center pivot tube carrier.

A trail tube is a specific type of irrigation lateral because of its unique application pattern. The fluid mechanics of an irrigation lateral consist of two parts: the total friction loss, and the dimensionless pressure distribution of an irrigation lateral (Chu, 1982).

The friction loss of a lateral in a hand move irrigation system was studied by Christiansen (1942). He introduced an F factor to represent the friction loss of a lateral as a fraction of the friction loss of an associated supply pipe which has the same characteristics. These characteristics include size, length, surface roughness, and total flow rate similar to those of a lateral (Christiansen, 1942). Merrian (1968) described the pressure distribution of a lateral in a hand move
irrigation system. He mentioned that a little over $50 \%$ of the pressure loss occurs in the first $1 / 5$ of the length, and about $87 \%$ in the first 1/2. Chu and Moe (1972) investigated the fluid mechanics of a center pivot irrigation system. They obtained the F factor of a center pivot irrigation system and introduced a distribution factor to describe the pressure distribution of a center pivot system. Chu (1982) defined the distribution factor as a normalized version of the energy grade line. The energy grade line as defined by Brater and King (1976) is the line representing the total energy at any point. Wu, Howell, and Hiler (1979) studied the hydraulics of the lateral in a trickle irrigation system in detail. They prepared design charts for trickle laterals under various operating conditions including conditions of non-uniform slopes and conditions of varying pipe sizes.

## Soil Intake Families

The water intake by soils, Soil Conservation Service (SCS, 1964) is the movement of irrigation water from the surface into and through the soil. Water intake is the expression of several factors including infiltration and percolation.

The classification of soils into intake families or groups is based upon analyzing cylinder-infiltrometer data from large numbers of sites. Soil of minor differences are considered together as a group. The SCS (1976) classified soils into eight intake families or groups. These groups have been assigned numbers such as 0.1, 0.3, . . . 4.0. These numbers approximate the basic-rate values for soils in those families. The basic intake rate is the nearly constant rate developed
after some time has elapsed from the start of irrigation. The time required to infiltrate a certain depth of water cah be calculated by using the equation of the intake families used by SCS, (Hart, 1980). This equation can be written as follows:

$$
F=a t^{b}+c
$$

Where $F=$ the cumulative intake or the depth of infiltrated water, mm , $t=$ the time required to infiltrate a certain depth of water, min, and
$\mathrm{a}, \mathrm{b}$, and $\mathrm{c}=$ constants associated to each intake family.
Values of the constants are shown in Table 2.

Table 2: Constants for Different Intake Families. (Adapted from ASAE, Monograph No. 3, 1980.

| Intake family | a | b | c |
| :---: | :---: | :---: | :---: |
| 0.05 | 0.5334 | 0.618 | 7.0 |
| 0.10 | 0.6198 | 0.661 | 7.0 |
| 0.15 | 0.7110 | 0.683 | 7.0 |
| 0.20 | 0.7772 | 0.699 | 7.0 |
| 0.25 | 0.8534 | 0.711 | 7.0 |
| 0.30 | 0.9246 | 0.720 | 7.0 |
| 0.35 | 0.9957 | 0.729 | 7.0 |
| 0.40 | 1.0640 | 0.736 | 7.0 |
| 0.45 | 1.1300 | 0.742 | 7.0 |
| 0.50 | 1.1960 | 0.748 | 7.0 |
| 0.60 | 1.3210 | 0.757 | 7.0 |
| 0.70 | 1.4430 | 0.766 | 7.0 |
| 0.80 | 1.5600 | 0.773 | 7.0 |
| 0.90 | 1.6740 | 0.779 | 7.0 |
| 1.00 | 1.7860 | 0.785 | 7.0 |
| 1.50 | 2.2840 | 0.799 | 7.0 |
| 2.00 | 2.7530 | 0.808 | 7.0 |

## Methods and Procedures

## Theory

The theory of trail-tube hydraulics (Chu, 1982) can be divided into three parts: tube flow rate distribution, total friction loss, and pressure distribution.

The tube flow rate is described by the following equation.

$$
\begin{equation*}
\frac{Q_{x}}{Q_{0}}=1-B\left(\frac{x}{L}\right)^{0.5}-(1-B) \frac{x}{L} \tag{8}
\end{equation*}
$$

Where $Q_{0}=$ total flow rate of trail tube,

$$
\begin{aligned}
Q_{x}= & \text { the flow rate at a distance } x \text { from the upstream } \\
& \text { end of the tube, }
\end{aligned}
$$

$\mathrm{L}=$ the length of the trail tube,
$B=A_{0}(V L)^{0.5} /\left[A_{0}(V L)^{0.5}+A_{1} L\right]=a$ constant,
$V=$ the constant traveling speed of the trail tube, and $A_{0}$ and $A_{1}$ are the infiltration parameters of Philip's model (1957).

The total friction loss is given by the following equation.

$$
\begin{equation*}
\frac{H_{0}}{H_{f}}=F=\int_{1}^{0}\left[1-B\left(\frac{x}{L}\right)^{0.5}-(1-B)^{\frac{x}{L}}\right]^{1.85} d\left(\frac{x}{L}\right) \tag{9}
\end{equation*}
$$

Where $H_{0}=$ the total friction loss of a perforated tube,

$$
\begin{aligned}
\mathrm{H}_{\mathrm{f}}= & \text { the total friction loss of the trail tube without } \\
& \text { perforations, and } \\
\mathrm{F}= & \text { the } \mathrm{F} \text { factor of trail tube. }
\end{aligned}
$$

The pressure distribution is represented by the following equation.

$$
\begin{equation*}
H=\frac{H_{x}}{H_{0}}=\frac{1}{F} \int_{1}^{x / L}\left[1-B\left(\frac{x}{L}\right)^{0.5}-(1-B) \frac{x}{L}\right]^{1.85} d\left(\frac{x}{L}\right) \tag{10}
\end{equation*}
$$

Where $H=$ the distribution factor of the trail tube after Chu (1982), and

$$
\begin{aligned}
H_{x}= & \text { the friction loss from the point } x \text { to the downstream } \\
& \text { end of tube. }
\end{aligned}
$$

A graphical representation of equations 8,9 , and 10 is provided in Figures 1, 2, and 3, respectively, to facilitate their application in practice. These Figures are the same as the Figures provided by Chu (1982). The numerical procedure to obtain these curves is provided in Appendix C .

The theory was based upon the assumptions that the energy associated with the flow velocity, the kinetic energy, is usually small, and can be geglected, and that the land is level. It was observed in this study that the kinetic energy in the trail tubes was substantial, which contradicts the assumptions made in the previous study (Chu, 1982). However, based upon conservation of energy principle, the theory can be extended to the condition where the land has uniform slopes and where the kinetic energy is not neglible by modifying equation 10. This modified equation represents the distribution of total energy loss rather than the pressure distribution. The modified version of the trail tube theory is to be evaluated in this study.

In the derivation of theoretical results, Equations 8 to 10, the perforation flow rate was represented by the orifice flow formula

$$
Q_{2}-Q_{1}=d Q_{x}=38 C_{Q} a\left(P_{x}\right) 0.5
$$



Figure 1: Dimensionless Tube Flow Rate Distribution; Symbols Defined in Equation 8 .


Figure 2: F factor of Trail Tubes; Symbols Defined in Equations 10 and 8.


Figure 3: Dimensionless Total Energy Loss Distribution (H) of Trail Tubes; Symbols Defined in Equations 8 and 10.

$$
\begin{aligned}
\text { Where } Q_{2}= & \text { the tube flow rate upstream of a perforation, } \\
Q_{1}= & \text { the tube flow rate downstream of a perforation, } \\
d_{x}= & \text { the change of the tube flow rate at a distance } x \\
& \text { away from the first perforation at the upstream } \\
& \text { side, } \\
a= & \text { perforation area in square inches, and } \\
P_{x}= & \text { pressure head at point } x .
\end{aligned}
$$

The perforation flow rate can be obtained using the orifice flow formula, if the pressure and the discharge coefficient are given. The pressure at any point along a tube can be determined with the help of the theoretical pressure distribution, but the discharge coefficient must be determined experimentally.

The total friction loss ( $H_{f}$ ) can be calculated by using one of the friction loss formulas, Equations 1, 2, and 3. But the friction coefficient which represents the smoothness of tube material must be determined experimentally.

Trail-tubes distribute water through perforations along the tubes. Water flowing out of these perforations is in the form of a jet. Jet distance is the maximum horizontal distance from the perforation to the jet fall, Figure 7. This distance is controlled by the operating pressure. A relationship between the pressure and the jet distance is needed in practice to select an appropriate tube spacing. Jet distance will be investigated by laboratory measurements in this study.

## Friction Coefficient Determination

Friction loss was measured by the difference of water level in two
manometers established at both ends of a poly-flex hose, Figure 5. Two pressure regulators connected in series were used to maintain constant pressure during measurement, Figure 4. The tube flow rate was controlled by a gate valve and was measured by using a water-meter and a stop watch. A pressure differential range from 1 to 10 feet of water was used in the tests. Three tube sizes, including tube diameters of $1 / 2,3 / 4$, and 1 inch, were investigated. Tube length was 25 feet for testing the $1 / 2$ inch tube and was 50 feet for other sizes.

The measured friction loss under different flow rates was analyzed to evaluate the friction coefficient in a pipe friction model. Three models were evaluated in this study, including the Darcy-Weisbach formula, the Hazen-Williams formula, and Scoby's formula, Equations 1, 2, and 3. The model which provided the least variation in thr friction coefficient was to be selected to test the theoretical results.

## Discharge Coefficient Determination

Ten equally spaced holes were drilled in a five foot tube. A manometer was established at each end of the tube, Figure 6. This tube was connected to a water-meter, two pressure regulators, and a flow control valve at the water source, Figure 4. Pressure regulators were used for controlling the pressure at the upstream side of the tube. The pressure was measured by the height of the water in the manometer. Average values of the pressure readings from the two manometers were used to represent the operating pressure.

Flow rates were measured from each of the ten perforations by a graduated cylinder and a stop watch. The value of each perforation


Figure 4: Pressure Regulations, Flow Control Value, Water Meter, and Surveying Instrument.


Figure 5: Poly-Flex Tube and Manometers for Friction Loss Measurements.
flow rate was used to determine the discharge coefficient in the orifice flow formula, Equation 6.

The discharge coefficients were obtained for three sizes of perforations $1 / 16,5 / 64$, and $1 / 8$ inch with three sizes of tubes $1 / 2$, $3 / 4$, and 1 inch in diameter under a pressure head ranging from 2 to 10 feet of water.

Jet Distance
Jet distance is the maximum horizontal distance from the perforations to the jet fall, Figure 7, as defined earlier. Jet distance was measured after determining the discharge coefficient, using the same perforated tubes, in the following manner: The perforated tubes were oriented at an angle of $45^{\circ}$ with respect to the ground surface. Under such an angle the jet distance reaches its maximum. Average maximum jet distances of ten perforations were measured in this study. These measurements were determined for each tube size, for three perforation sizes, and at five pressures.

## Distribution of Tube Flow Rate and Pressure

The distribution of tube flow rate determines the application pattern of a trail tube. Philip's infiltration model was used for matching the application pattern of trail tubes to the soil infiltration characteristics (Chu, 1982). There were two parameters included in this model, $A_{0}$ and $A_{1}$, Equation 8. A following numerical example is presented to illustrate the procedures to determine these parameters for a typical soil in the 1.0 intake family. Philip's


Figure 5: Perforated Poly-Flex Tube and Manometers
For Flow Rate Measurement

Figure 7: Perforated Poly-Flex Tube for Jet Distance Measurement
infiltration model (1957) is represented by

$$
\begin{equation*}
F=A_{0} t^{0.5}+A_{1} t \tag{11}
\end{equation*}
$$

Where $F=$ the cumulative infiltration, in, and

$$
t=\text { the infiltration time, min. }
$$

From Table 2, the constants for the 1.0 intake family are, $a=1.786$, $b=0.785$, and $c=7$. Substituting these constants in equation (7) results in

$$
F=1.786 t^{0.785}+7
$$

Rearrangement of this equation gives

$$
\begin{equation*}
t=\left(\frac{F-7}{1.786}\right)^{1.2739} \tag{12}
\end{equation*}
$$

Where $F=$ the accumulative intake or the depth of infiltrated water, mm.

Substituting in equation (12) for

$$
\begin{equation*}
\mathrm{F}=1 \text { inch }(25.4 \mathrm{~mm}), \mathrm{t}=19.5 \text { minutes } \tag{13}
\end{equation*}
$$

and for $\mathrm{F}=2$ inch ( 50.8 mm ), $\mathrm{t}=58.9$ minutes.
Substituting in equation number (11)

$$
\begin{align*}
\text { for } F & =1, \text { and } t=19.5, \text { results in } \\
1 & =A_{0}(19.5)^{0.5}+(19.5) A_{1}  \tag{14}\\
\text { for } F & =2 \text {, and } t=58.9, \text { results in } \\
2 & =A_{0}(58.9)^{0.5}+(58.9) A_{1} \tag{15}
\end{align*}
$$

By solving equation (14) and (15) results in

$$
\begin{align*}
& A_{1}=0.0009 \mathrm{ft} / \mathrm{min}(0.0105 \mathrm{in} / \mathrm{min}), \text { and } \\
& A_{0}=0.0150 \mathrm{ft} / \mathrm{min}^{0.5}\left(0.1801 \mathrm{in} / \mathrm{min}^{0.5}\right) \tag{16}
\end{align*}
$$

Perforation size has a direct effect on the tube flow rate. Selection of perforation size depends on two factors: Hole size and clogging. Small holes are desirable because more holes along the tube provide more precision for tube flow adjustments. However, too small a hole should not be used because of clogging.

The United States Department of Agriculture classifies soils into different groups according to texture (Schwab, et al., 1966). These groups are gravel, sand, silt, and clay. The particle size range of these groups are: more than 2,2 to $0.05,0.05$ to 0.002 , and less than 0.002 mm , respectively. For this study, the smallest recommended perforation size that will allow the passage of large particles (sand $2 \mathrm{~mm})$ is 5/64 inch (2 mm).

The theoretical distribution of tube flow rate and pressure were tested for the conditions of the outermost trail tube on a center pivot system. The radial distance from this tube to the pivot is 1317.5 feet, Figure 9. The center pivot system is designed to apply one inch of water (depth of application) in a period of 3 days (time per revolution). The system is assumed to irrigate on a 1.0 intake family soil, Table 2. The infiltration parameters of 1.0 intake family soil are $A_{0}=0.015 \mathrm{ft} / \mathrm{min}^{0.5}$, $A_{1}=0.0009 \mathrm{ft} / \mathrm{min}(E q u a t i o n 16)$ and the time required to infiltrate 1.0 inch of water was 19.5 minutes, Equation 13.

The following procedure is used to determine the length of tube. Let $\mathrm{V}_{\mathrm{s}}$ be the traveling speed of the center pivot system at a distance equal to $r$ from the pivot, Figure 9. Therefore,

$$
\begin{equation*}
V_{s}=\left(\frac{2 \pi r}{T R}\right) \tag{17}
\end{equation*}
$$

$$
\begin{aligned}
\text { Where } \mathrm{V}_{\mathrm{s}}= & \text { system traveling speed, ft/min, } \\
2 \pi r= & \text { the circumference of a circle with a radius } \mathrm{r}, \\
& \mathrm{ft}, \text { and } \\
\mathrm{TR}= & \text { the time per revolution of a center pivot system, } \\
& \text { min. }
\end{aligned}
$$

The center pivot system is designed to travel a distance equivalent to the tube length, $L$, within a period of time, $t$, which is the time of application described in equation 12. So

$$
\begin{equation*}
V_{s}=\frac{L}{t} \tag{18}
\end{equation*}
$$

Where $L=$ tube length, ft, and
$t=$ the time required to infiltrate a depth of water needs to be applied, min.

Combine equations 17 and 18 to obtain

$$
\begin{align*}
& \mathrm{L}=\left(\frac{2 \pi r}{\mathrm{TR}}\right) \mathrm{t}  \tag{19}\\
& \mathrm{~L}=\left(\frac{2 \pi(1317.5)}{3 \times 24 \times 60}\right) 19.5=(1.9162) 19.5=37.4 \mathrm{feet}
\end{align*}
$$

The tube discharge $\left(Q_{0}\right)$ can be calculated as follows. Let the area to be irrigated by the tube equal A , and the depth of the application equal $D_{a}$. Therefore, the volume of water to be applied equals the product of $A$ and $D_{a}$. This volume to be applied in a time equal to $T R$. So the tube flow rate is

$$
\begin{equation*}
Q_{0}=\left(\frac{A\left(D_{a}\right)}{T R}\right) 7.48 \tag{20}
\end{equation*}
$$

Where $A=$ tube irrigated area, $f t^{2}$

$$
\begin{aligned}
D_{a} & =\text { depth of application, ft. } \\
7.48 & =\text { conversion factor }
\end{aligned}
$$

Tube spacing is selected to be 5 feet in this study. This spacing is equivalent to having one tube in every other crop row. The irrigated area covered by the tube is

$$
A=\pi\left(r^{2}-r_{1}^{2}\right), \text { Figure } 9
$$

Where $r$ = radial distance from the pivot center, $f t$, and

$$
\begin{aligned}
r_{1} & =r-5 \text { feet } \\
A & =\pi\left[(1320)^{2}-(1315)^{2}\right]=41390.5 \mathrm{ft}^{2} \\
D_{a} & =1 \text { inch }=1 / 2 \mathrm{ft}(0.0833 \mathrm{ft}) \\
\mathrm{TR} & =(3)(24)(60)=4320 \mathrm{~min} \\
\mathrm{Q}_{0} & =\left(\frac{41,390.48(0.0833)}{4320}\right) 7.48=5.97 \mathrm{GPM}
\end{aligned}
$$

The B - value for use in equation 8 was calculated

$$
\begin{aligned}
B= & {\left[A_{0}(V L)^{0.5}\right] /\left[A_{0}(V L)^{0.5}+A_{1} L\right] } \\
B= & {\left[0.015(1.92 \times 28)^{0.5}\right] /\left[0.015(1.92 \times 38)^{0.5}+\right.} \\
& 0.0009(38)] \\
B= & 0.79
\end{aligned}
$$

Perforation spacing was calculated starting from the downstream end and working upstream. Minimum operating pressure of 2 pounds per square inch (psi) was selected for the downstream end of the tube, Table 6. The calculations follow the procedure described by Chu (1982). A numerical example is shown below for a tube $3 / 4$ inches in diameter.

Tube cross-sectional area $(A)=\frac{\pi D^{2}}{4}=\frac{\pi(0.0628)^{2}}{4}=0.0031 \mathrm{ft}^{2}$
$Q_{0}=(5.79)(0.002228)=0.129 \mathrm{cfs}$
Hydraulic radius of tube $\left(R_{H}\right)=\frac{D}{4}=\frac{0.0628}{4}=0.0157 \mathrm{ft}$.

Substituting $H_{f} / L$ for $S$ and solving for $H_{f}$ equation 2 becomes:

$$
\begin{align*}
& \mathrm{H}_{\mathrm{f}}=\left[\frac{\mathrm{V}}{1.318 \mathrm{C}\left(\frac{\mathrm{D}}{4}\right)^{0.63}}\right]^{1.85}(\mathrm{~L}), \text { thus }  \tag{2A}\\
& \mathrm{H}_{\mathrm{f}}=\left[\frac{4.1613}{1.318(135)(0.0157)^{0.63}}\right]^{1.85} \\
& \mathrm{H}_{\mathrm{f}}=4.62 \mathrm{ft}
\end{align*}
$$

From Figure 2, $F=0.214$ for $B=0.79$.
From equation (9), the total friction loss of the perforated tube

$$
\mathrm{H}_{0}=(\mathrm{F})\left(\mathrm{H}_{\mathrm{f}}\right)=(0.214)(4.62)=0.99 \mathrm{ft}
$$

From Figure 3, $H=0.0$ for $x / L=1.0$ (tube end)
From equation (10)

$$
H_{x}=(H)\left(H_{0}\right)=(0.0)(0.99)=0.0 \mathrm{ft} \text { of water }
$$

The pressure at the tube end

$$
\begin{aligned}
P_{x} & =H_{x}+P_{1} \\
& =0.0+4.62=4.62 \mathrm{ft} \text { of water }(2 \mathrm{Psi})
\end{aligned}
$$

From the orifice flow formula, Equation (6)

$$
q=38 C_{Q} a\left(P_{x}\right)^{0.5}
$$

For the perforation diameter (d) $=5 / 64$ inch, area $(a)=\left(\pi d^{2}\right) / 4=\frac{\pi(0.0781)^{2}}{4}=0.0048 \mathrm{in}^{2}$,

$$
\begin{align*}
& \mathrm{q}=38(0.65)(0.0048)\left(\mathrm{P}_{\mathrm{x}}\right)^{0.5},\left(\mathrm{C}_{\mathrm{Q}}=0.65\right. \text { from Table 5) } \\
& \mathrm{q}=0.1184\left(\mathrm{P}_{\mathrm{x}}\right)^{0.5} \tag{21}
\end{align*}
$$

The flow rate from the first perforation on the downstream side is
determined using equation (21).

$$
\mathrm{q}=0.1184(2)^{0.5}=0.1674 \mathrm{gpm}
$$

From Figure 1, $Q_{x} / Q_{0}=0.0$ for $B=0.79$ and $x / L=1.0$

$$
Q_{x}=\left(Q_{0}\right)(0.0)=0.0, \text { where } Q_{x}=\text { tube discharge just beyond } q_{1} \text {. }
$$

The tube flow rate at the next perforation upstream (Between perforation 1 and 2), is

$$
\begin{aligned}
& Q=Q_{x}+q_{x}=0.0+0.1674=0.1674 \mathrm{gpm}, \text { and } \\
& Q / Q_{0}=\frac{0.1674}{5.79}=0.0289
\end{aligned}
$$

From Figure 1, the values of $x / L$ are

$$
\begin{aligned}
& x / L=1 \text { when } Q_{x} / Q_{0}=0.0 \\
& x / L=0.95 \text { when } Q / Q_{0}=0.0289
\end{aligned}
$$

Therefore, the spacing between the adjacent perforations (1 and 2) is

$$
(1-0.95) \mathrm{L}=0.05(38)=1.9 \mathrm{ft}
$$

From Figure 3, for $B=0.79$ (Equation 8)

$$
\mathrm{H}=0.0 \text { for } \mathrm{x} / \mathrm{L}=0.95
$$

From equation (10)

$$
H_{x}=0.0(0.99)=0.0 \mathrm{ft} \text { of water }
$$

The pressure at $x / L=0.95=P_{x}$

$$
P_{x}=H_{x}+P_{1}=0.0+.4 .62 \mathrm{ft} \text { of water (2 Psi) }
$$

From equation (21)

$$
\mathrm{q}=0.1184(2)^{0.5}=0.1674 \mathrm{gpm}
$$

Tube flow rate at the next perforation upstream (between perforation
2 and 3) is

$$
\begin{aligned}
& Q=Q_{x}+q_{x}=0.01674+0.1674=0.3348 \mathrm{gpm} \\
& Q / Q_{0}=\frac{0.3348}{5.79}=0.0578
\end{aligned}
$$

From Figure 1, the values of $x / L$ are

$$
\begin{aligned}
& x / L=0.95, \text { when } Q_{x} / Q_{0}=0.0289 \\
& x / L=0.905, \text { when } Q / Q_{0}=0.0578
\end{aligned}
$$

So, the spacing between perforation 2 and 3 equals

$$
(0.95-0.905) 38=1.71 \mathrm{ft}
$$

The procedures were continued until the discharge and the tube length were close to 5.79 gpm and 38 feet, Table 3.

Based upon the information calculated in Table 3, thirty-four 5/64 inch perforations were drilled in a $3 / 4$ inch poly-flex tube. Six manometer tubes were installed at $0.0,3.8,11.4,19.0,28.5$, and 38.0 feet to obtain the distribution of total energy loss, Figure 8. Pressure regulators were used for controlling the pressure required at the upstream side of the tube. The elevations along the tube at the six manometers taps were measured by using a surveying level. A gate valve and two pressure regulators were used to control water flow.

Flow rate through each perforation was determined by a graduated cylinder and a stop watch. Perforation flow rates were accumulated to obtain the tube flow rate distribution.


Figure 8: Perforated Poly-Flex and Six Manometers to Measure the Flow Rate and Pressure Distribution


Figure 9: Schematic Diagram of Trail-Tube and Center Pivot System Showing the Area to be Irrigated by the Outer Tube, and the Symbols Used For Calculation.

Table 3. Perforation Spacing Along the Trail Tube

| HOLE |  | $\mathrm{H}_{\mathrm{x}}$ | $\mathrm{P}_{\mathrm{x}}$ | $\mathrm{q}_{\mathrm{x}}$ | $\mathrm{Q}_{\mathrm{x}}$ |  |  |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NUMBER | H | IN FEET | IN PSI | IN GPM | IN GPM | $\mathrm{Q}_{\mathrm{x}} \mathrm{Q}_{0}$ | x/L | SPACING |
| 1 | 0.000 | 0.0000 | 2.0000 | 0.1674 | 0.0000 | 0.0000 | 1.000 | 0.000 |
| 2 | 0.000 | 0.0000 | 2.0000 | 0.1674 | 0.1674 | 0.0289 | 0.950 | 1.900 |
| 3 | 0.000 | 0.0000 | 2.0000 | 0.1674 | 0.3348 | 0.0578 | 0.905 | 1.710 |
| 4 | 0.000 | 0.0000 | 2.0000 | 0.1674 | 0.5022 | 0.0867 | 0.860 | 1.710 |
| 5 | 0.000 | 0.0000 | 2.0000 | 0.1674 | 0.6692 | 0.1156 | 0.815 | 1.710 |
|  |  |  |  |  |  |  |  |  |
| 7 | 0.005 | 0.0049 | 2.0021 | 0.1675 | 0.8371 | 0.1446 | 0.775 | 1.710 |
| 7 | 0.010 | 0.0099 | 2.0043 | 0.1676 | 1.0047 | 0.1735 | 0.735 | 1.710 |
| 8 | 0.017 | 0.0168 | 2.0073 | 0.1677 | 1.724 | 0.2025 | 0.690 | 1.710 |
| 9 | 0.027 | 0.0267 | 2.0116 | 0.1679 | 1.3403 | 0.2315 | 0.645 | 1.710 |
| 10 | 0.039 | 0.0386 | 2.0167 | 0.1681 | 1.5084 | 0.2605 | 0.605 | 1.520 |
| 11 | 0.050 | 0.0495 | 2.0214 | 0.1683 | 1.6767 | 0.2896 | 0.565 | 1.520 |
| 12 | 0.068 | 0.0673 | 2.0291 | 0.1687 | 1.8454 | 0.3187 | 0.525 | 1.520 |
| 13 | 0.088 | 0.0871 | 2.0377 | 0.1690 | 2.0144 | 0.3479 | 0.485 | 1.520 |
| 14 | 0.110 | 0.1089 | 2.0471 | 0.1694 | 2.1838 | 0.3772 | 0.445 | 1.520 |
| 15 | 0.138 | 0.1366 | 2.0591 | 0.1699 | 2.3537 | 0.4063 | 0.410 | 1.330 |
| 16 | 0.169 | 0.1673 | 2.0724 | 0.1704 | 2.5241 | 0.4359 | 0.375 | 1.330 |
| 17 | 0.200 | 0.1979 | 2.0857 | 0.1710 | 2.6951 | 0.4655 | 0.340 | 1.330 |
| 18 | 0.240 | 0.2375 | 2.1028 | 0.1717 | 2.8668 | 0.4951 | 0.310 | 1.140 |
| 19 | 0.275 | 0.2722 | 2.1178 | 0.1723 | 3.0391 | 0.5249 | 0.280 | 1.140 |
| 20 | 0.316 | 0.3127 | 2.1354 | 0.1730 | 3.2121 | 0.5548 | 0.250 | 1.140 |
| 21 | 0.364 | 0.3603 | 2.1560 | 0.1738 | 3.3859 | 0.5848 | 0.220 | 1.140 |
| 22 | 0.412 | 0.4078 | 2.1765 | 0.1747 | 3.5606 | 0.6150 | 0.190 | 1.140 |
| 23 | 0.462 | 0.4572 | 2.1979 | 0.1755 | 3.7361 | 0.6453 | 0.165 | 0.950 |
| 24 | 0.510 | 0.5047 | 2.2185 | 0.1764 | 3.9125 | 0.6757 | 0.140 | 0.950 |
| 25 | 0.564 | 0.5582 | 2.2416 | 0.1773 | 4.0898 | 0.7064 | 0.115 | 0.950 |
| 26 | 0.625 | 0.6186 | 2.2678 | 0.1783 | 4.2681 | 0.7372 | 0.095 | 0.760 |
| 27 | 0.680 | 0.6730 | 2.2913 | 0.1792 | 4.4473 | 0.7681 | 0.075 | 0.760 |
| 28 | 0.735 | 0.7274 | 2.3149 | 0.1801 | 4.6274 | 0.7992 | 0.055 | 0.760 |
| 29 | 0.800 | 0.7918 | 2.3428 | 0.1812 | 4.8086 | 0.8305 | 0.042 | 0.475 |
| 30 | 0.840 | 0.8313 | 2.3599 | 0.1819 | 4.9905 | 0.8619 | 0.030 | 0.475 |
| 31 | 0.885 | 0.8759 | 2.3792 | 0.1826 | 5.1731 | 0.8935 | 0.022 | 0.304 |
| 32 | 0.920 | 0.9105 | 2.3942 | 0.1832 | 5.3563 | 0.9251 | 0.015 | 0.266 |
| 33 | 0.940 | 0.9303 | 2.4027 | 0.1835 | 5.5398 | 0.9568 | 0.008 | 0.266 |
| 34 | 0.968 | 0.9580 | 2.4147 | 0.1840 | 5.7238 | 0.9886 | 0.001 | 0.266 |
|  |  |  |  |  |  |  |  |  |

## Results and Discussion

## Friction Coefficient

The measured friction losses for different flow rates of the polyflex tubes are listed in Table A (Appendix A). The measured friction loss ( $H_{f}$ ) as influenced by the velocity for three poly-flex tube sizes is plotted in Figure 10 for practical application. The following three pipe friction formulas were used to obtain the friction coefficients: Darcy-Weisbach's formula, Hazen Williams formula, and Scobey's formula. Variation in friction coefficient was evident for different sizes of tubes, Table 4.

A coefficient of variation (CV) was calculated for each pipe friction model to represent the variation of friction coefficient. Hazen-Williams friction coefficient (C) gives the lease CV, Table 4; 2 percent as compared with 12 percent for the Darcy-Weisbach and 16 percent for the Scobey. Thus, Hazen Williams formula was selected to represent the friction loss of poly-flex hoses.

The average friction coefficient (C) values were 135 for $3 / 4$ inch (ID $=0.7536$ ) and 1 inch ( $I D=1.0363$ ), and 130 for 0.5 inch ( $I D=$ 0.552 ) tubes, Table 4.

To investigate the type of flow in the trail tube, the DarcyWeisbach friction coefficient (f), and Reynold's number are plotted on Moody's diagram, Figure 11. Almost all the measured data points fall in the transition zone and close to the smooth pipe region. This result shows that poly-flex tubes were hydraulically smooth pipes and the type of flow was generally turbulent.


Figure 10: Relation Between Friction Loss in ft Per 1000 ft of Pipe, and the Flow Velocity in ft/sec.

Table 4. Friction Coefficient of Poly-Flex Tubes

| Tube size <br> in inches | Friction Coefficients |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| OD | ID | f | C | $\mathrm{K}_{\mathrm{s}}$ |
| 0.50 | 0.552 | 0.0352 | 130.35 | 0.4330 |
| 0.75 | 0.7536 | 0.0294 | 135.13 | 0.3888 |
| 1.00 | 1.0363 | 0.0279 | 134.29 | 0.3144 |
|  | SD |  | 0.0038 | 2.552 |

Where $f, C$, and $K_{s}$ - friction coefficient calculated from DarcyWeisbach, Hazen-Williams and Scobey model, respectively, Equations 1, 2 and 3.

SD - standare deviation
$\overline{\mathrm{X}}$ - the mean value
OD - outside diameter
ID - inside diameter
CV - coefficient of variation


Figure 11: Measured Darcy-Weisbach Friction Coefficient on Moody's Diagram. (Adapted from, Trans. ASME, 1944).

For smooth pipes the friction loss for turbulent flow depends on flow velocity. Vennard (1961) reported that friction loss is proportional to $v^{1.75}$. It will be noted on page 25 that $H_{f}$ in the equation 2A is proportional to $\mathrm{V}^{1.85}$. This explained why HazenWilliams formula was a suitable model to describe friction loss of poly-flex tubes.

## Discharge Coefficient

The values of measured perforation flow rate are listed in Appendix B. The discharge coefficient was calculated for each measurement using the orifice formula. The average values of perforation discharge, discharge coefficient $\left(C_{Q}\right)$, flow velocity, and Reynold's number for all the perforations were calculated and are summarized in Table 5. The range of average $C_{Q}$ values was from 0.64 to 0.70 as compared with the theoretical value of 0.61 for sharp-edged orifices (Peterson, 1971). The relation between the measured discharge coefficient, and the calculated Reynold's number $R$ was compared with the result published by Brater and King (1976). These measured values as compared with the practical values (Figure 12) show good agreement. The variation is from 2 to 5 percent.

Many factors can cause variability in the discharge coefficient. The most significant factor is probably the tube flow velocity as reported by Fry (1961). The discharge from the perforations is reduced as the velocity in the tube increases. Decreasing a perforation discharge means decreasing $C_{Q}$. This can be seen in Table 5 where the average $C_{Q}$ for 0.5 inch tube was 0.64 , but for the 1 inch tube it was

Table 5. Discharge Coefficient of Perforations

| Tube Diameter in inches | Average Pressure in ft . of water | $\begin{aligned} & \text { Orifice Diameter } \\ & \text { in inches } \\ & 1 / 16 \quad 5 / 64 \quad 1 / 8 \\ & \hline \end{aligned}$ |  |  | $$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | . 6669 | . 6234 | . 6263 |  |  |
|  | 4 | . 6665 | . 6289 | . 6229 |  |  |
| 1/2 | 6 | . 6814 | . 6231 | .6171 |  |  |
|  | 8 | . 6767 | . 6248 | . 6150 |  |  |
|  | 10 | . 6759 | . 6327 | . 6181 |  |  |
|  |  | . 6735 | . 6266 | .6199 | . 64 | 6,594 |
|  | 2 | . 6817 | . 6620 | . 7102 |  |  |
|  | 4 | . 6795 | . 6537 | . 6812 |  |  |
| 3/4 | 6 | . 6664 | . 6520 | . 6730 |  |  |
|  | 8 | . 6956 | . 6515 | . 6687 |  |  |
|  | 10 | .6775 | . 6408 | . 6671 |  |  |
|  |  | . 6801 | . 6520 | . 6800 | .67 | 7,834 |
|  | 2 | . 6594 | . 7052 | . 7417 |  |  |
|  | 4 | . 6557 | . 7087 | . 7471 |  |  |
| 1 | 6 | . 6575 | . 7077 | . 7336 |  |  |
|  | 8 | . 6316 | . 7206 | . 7418 |  |  |
|  | - 10 | . 6319 | . 7143 | . 7333 |  |  |
|  |  | . 6472 | . 7113 | . 7395 | . 70 | 8,195 |



Figure 12: Values of Discharge Coefficient and Reynold's Number, R. (Adapted from Brater and King, 1976).
0.70.

Other factors, such as the ratios of the perforation diameter to the tube wall thickness, the sharpness of the perforations' edge, and the smoothness of the inner surface (Brater and King, 1976) may contribute to the variation as well.

## Jet Distance

The jet distance measurements were made in conjunction with the perforation flow rate measurements. The poly-flex tube was oriented so that the angle between the jet and the ground surface at the perforation is $45^{\circ}$. Average jet distance from the ten perforations was summarized in Table 6. These average values are $2.8,5.4,8.0$ and 9.8 feet for the pressure heads of $2,4,6$, and 8 feet of water, respectively This table gives an estimate of the spacing to be covered with pressures ranging from 2 to 10 feet of water for three perforation sizes (1/16, 5/64, and 1/8 inch).

Tube Flow Rate Distribution
The measured tube flow rates are listed in Table 7. The tube flow rate distribution is needed to compare with the theoretical distribution, Equation 1. In addition it provides the basic information to determine the energy loss distribution, Equation 10. The measured flow rate distribution was compared with the theoretical flow rate distribution, Figure 13. Good agreement was obtained between the theoretical result and the measured data. The maximum variation is about 2.4 percent.

Table 6. Average Measured Jet Distance

| Average Pressure Head Feet | 1 Inch Tube |  |  |  | 3/4 Inch Tube |  |  |  | 1/2 Inch Tube |  |  |  | $\begin{gathered} \text { Average } \\ x \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Orifice Size, Inch |  |  |  | Orifice Size, Inch |  |  |  | Orifice Size, Inch |  |  |  |  |
|  | 1/16 | 5/64 | 1/8 | $\begin{gathered} \text { Average } \\ x \\ \hline \end{gathered}$ | 1/16 | 5/64 | 1/8 | $\begin{gathered} \text { Average } \\ x \end{gathered}$ | 1/16 | 5/64 | 1/8 | $\begin{gathered} \hline \text { Ave rage } \\ x \\ \hline \end{gathered}$ |  |
|  | x | x | x |  | x | x | $x$ |  | x | x | x |  |  |
| 2 | 3.3 | 2.5 | 3.3 | 3 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.8 | 3.3 | 2.8 | 2.8 |
| 4 | 5.8 | 4.5 | 6.6 | 5.6 | 5.0 | 4.2 | 4.8 | 4.6 | 5.4 | 6.0 | 6.6 | 6.0 | 5.4 |
| 6 | 8.3 | 7.1 | 10.8 | 8.8 | 6.6 | 6.6 | 6.6 | 6.6 | 8.0 | 8.3 | 10.0 | 8.7 | 8.0 |
| 8 | 10.8 | 8.7 | 12.5 | 10.6 | 8.3 | 8.3 | 9.0 | 8.5 | 10.0 | 10.8 | - - | 10.4 | 9.8 |
| 10 | - - | -- | 10.5 | - - | 10.0 | 10.0 | 10.0 | 10.0 | - - | - - | - - | - - | - - |

Where $x=j e t$ distance in feet
(angle between jet and ground surface at the perforation is $45^{\circ}$ )

Table 7. Measured Orifices Flow Rate and Tube Flow Rate Distribution

| $\frac{\mathrm{x}}{\mathrm{L}}$ | ORIFICE NUMBER | ORIFICE FLOW RATE GPM | TUBE FLOW RATE GPM | $\frac{Q_{x}}{Q_{0}} v$ | VELOCITY FPS | \% $\frac{\mathrm{v}^{2}}{2 g}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.0 | 1 | 0.1188 | 0.1188 | 0.0 | 0.0 | 0.0 |
|  | 2 | 0.1641 | 0.2829 |  |  |  |
|  | 3 | 0.1710 | 0.4539 |  |  |  |
|  | 4 | 0.1877 | 0.6416 |  |  |  |
|  | 5 | 0.1958 | 0.8374 |  |  |  |
| . 75 | 6 | 0.1640 | 1.0014 | 0.1728 | 0.1728 | 0.00805 |
|  | 7 | 0.1968 | 1.1982 |  |  |  |
|  | 8 | 0.1958 | 1.3940 |  |  |  |
|  | 9 | 0.1500 | 1.5440 |  |  |  |
|  | 10 | 0.1368 | 1.6808 |  |  |  |
|  | 11 | 0.1756 | 1.8564 |  |  |  |
| 0.50 | 12 | 0.1257 | 1.9821 | 0.3419 | 1.42 | 0.0316 |
|  | 13 | 0.1609 | 2.1430 |  |  |  |
|  | 14 | 0.1404 | 2.2834 |  |  |  |
|  | 15 | 0.1474 | 2.4308 |  |  |  |
|  | 16 | 0.2041 | 2.6349 |  |  |  |
|  | 17 | 0.1806 | 2.8155 |  |  |  |
| 0.30 | 18 | 0.1417 | 2.9572 | 0.5102 | 2.13 | 0.0702 |
|  | 19 | 0.1610 | 3.1182 |  |  |  |
|  | 20 | 0.1848 | 3.3030 |  |  |  |
|  | 21 | 0.1831 | 3.4861 |  |  |  |
|  | 22 | 0.1673 | 3.6534 |  |  |  |
|  | 23 | 0.1705 | 3.8239 |  |  |  |
|  | 24 | 0.1382 | 3.9621 |  |  |  |
| 0.10 | 25 | 0.1638 | 4.1259 | 0.7118 | 2.97 | 0.137 |
|  | 26 | 0.1713 | 4.2972 |  |  |  |
|  | 27 | 0.1505 | 4.4477 |  |  |  |
|  | 28 | 0.1706 | 4.6183 |  |  |  |
|  | 28 | 0.1706 | 4.6183 |  |  |  |
|  | 29 | 0.2072 | 4.8255 |  |  |  |
|  | 30 | 0.2052 | 5.0312 |  |  |  |
|  | 31 | 0.1743 | 5.2055 |  |  |  |
|  | 32 | 0.1978 | 5.4033 |  |  |  |
|  | 33 | 0.2126 | 5.6159 |  |  |  |
| 0.0 | 34 | 0.1806 | 5.7965 | 1.0 | 4.17 | 0.27 |



Figure 13: Dimensionless Distribution of Tube Flow Rate; Symbols Defined in Equation 8.

## Energy Loss Distribution

The measured pressure head, elevation, velocity head of water flow at the six manometers on the tube are listed in Table 8. These input data were used to calculate the dimensionless energy loss distribution along the tube. The measured energy loss distribution is compared with the theoretical energy loss distribution, Table 8. A graphical comparison of the two distributions is shown in Figure 14. The discrepancy at the tube length ratio $0.0,0.1,0.3,0.5,0.75$, and 1.0 is $0.0,4.0,17.0,41.0,70.0$, and 0.0 percent, respectively, of the measured data values. The flow rate near the downstream end of the tube is usually small, and the Hazen-Williams formula may become inadequate to describe friction loss under such a condition. The discrepancy observed in the laboratory may be attributed to this cause.

Although the observed deviations between measured and theoretical energy loss distributions appears to be significant on a dimensionless basis, Figure 14 , it was of little practical importance. When the deviation was examined by comparing the measured pressure with the calculated pressure from the theory, the errors were less than 6 percent, which is practically negligible, Table 8.

Table 8. The Measured and the Theoretical Energy Loss Distributions.

| $\frac{x}{L}$ | Measured Values |  | Calculated Values |  |  |  | Theoretical Values |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \mathrm{P} / \mathrm{\gamma} \\ & \mathrm{in} . \end{aligned}$ | $\begin{array}{r} z \\ \text { in. } \end{array}$ | $\begin{gathered} \mathrm{v}^{2} / 2 \mathrm{~g} \\ \mathrm{in} . \end{gathered}$ | $\begin{aligned} & \text { T.H. } \\ & \text { in. } \end{aligned}$ | $\begin{aligned} & \mathrm{H}_{\mathrm{f}} \\ & \mathrm{ft} . \end{aligned}$ | H | H* | $\begin{gathered} \left(\mathrm{H}_{\mathrm{x}}-\mathrm{H}_{\mathrm{L}}\right) \\ \mathrm{ft} . \end{gathered}$ | $\begin{aligned} & H_{x} \\ & \mathrm{ft.}_{\mathrm{x}} \end{aligned}$ | in. | $\begin{aligned} & P / \gamma \\ & \mathrm{in} . \end{aligned}$ |
| 0.00 | 67.3200 | 0.0000 | 3.2400 | 70.5600 | 1.0674 | 1.0000 | 1.000 | 0.99 | 5.61 | 67.44 | 64.20 |
| 0.10 | 56.0000 | 0.0006 | 1.6400 | 66.6406 | 0.7407 | 0.6940 | 0.665 | 0.66 | 5.28 | 63.36 | 61.72 |
| 0.30 | 61.3125 | 0.0319 | 0.8430 | 62.1874 | 0.3697 | 0.3464 | 0.289 | 0.29 | 4.91 | 58.92 | 58.05 |
| 0.50 | 59.6250 | -. 0306 | 0.3790 | 59.9734 | 0.1852 | 0.1735 | 0.102 | 0.10 | 4.72 | 56.64 | 56.29 |
| 0.75 | 58.3125 | -. 0201 | 0.0966 | 58.3890 | 0.0532 | 0.0498 | 0.015 | 0.02 | 4.64 | 55.68 | 55.60 |
| 1.00 | 56.8125 | 0.9381 | 0.0000 | 59.7506 | 0.0000 | 0.0000 | 0.000 | 0.00 | 4.62** | 55.44 | 54.50 |

T.H. - total head along the tube.
$H_{f}$ - energy loss calculated by using Bernolli's equation.
$\mathrm{H}-\mathrm{calculated}$ by using equation $10,\left(\mathrm{H}=\mathrm{H}_{\mathrm{x}} / \mathrm{H}_{0}, \mathrm{H}_{0}=1.0674 \mathrm{ft}\right)$.
$H^{*}$ - values from Figure 3, $B=0.79$.
( $H_{x}-H_{L}$ ) - shows the energy loss distribution along the tube.
$H_{x}$ - the theoretical energy loss distribution along the tube.
** - from page $24,4.62 \mathrm{ft}=2 \mathrm{psi}$.
$P / Y$ - the theoretical pressure distribution $=\left[H_{x}-\left(\frac{V^{2}}{2 g}+2\right)\right], z=$ potential head.


Figure 14: Measured and Theoretical Energy Loss Distribution; Symbols Defined in Equations 10 and 8.

## SUMMARY AND CONCLUSIONS

The friction coefficięnts of poly-flex tubes, the discharge coefficients of perforations along a trail tube, and the jet distances of water distributed by trail tubes were determined by laboratory measurements.

The Hazen-Williams formula was found to be an adequate model to describe friction loss of poly-flex tubes. Friction loss coefficients of measured data based upon Hazen-Williams formula were found to be within a range of 130 to 135.

The discharge coefficients for perforations with sizes of $1 / 16$, 5/64, and $1 / 8$ inch were determined. It was found that the perforation size has little effect on the discharge coefficient but tube size has an effect. The discharge coefficient values were $0.64,0.67$, and 0.7 for tube sizes of $1 / 2,3 / 4$ and 1 inch, respectively.

Jet distances for perforation sizes of $1 / 16,5 / 64$, and $1 / 8$ inch under different pressures were measured. Perforation size has little effect on jet distance, but operating pressure has a major effect on jet distance.

Laboratory data were used to evaluate a theory of tube irrigation developed by Chu (1982). This theory was tested for the conditions of the outermost trail-tube on a center pivot system. The tube was located at 1318 feet away from the pivot. It was to apply 1 inch of water in a period of 3 days on a 1.0 intake family soil. The tube spacing and perforation size were selected to be 5 feet and 5/64 inches. An operating pressure of 2 pounds per square inch was selected
for the downstream end of the tube.

Under the above conditions, perforation spacings were calculated according to the theory. Thirty-four perforations were drilled in a $3 / 4$ inch poly-flex tube based upon the calculated spacings. The flow rate from these perforations and the pressure along the tube of 38 feet in length were measured. Comparisons of these measurements with the theoretical flow rate and the energy loss distribution indicated that

1. The measured discharge distribution was in good agreement with the theory.
2. The measured energy loss distribution was also in good agreement with the theory in general but deviations between the measured and the theoretical distributions were observed near the downstream end of the tube.

Tube discharge near the downstream end is usually small (laminar flow); the Hazen-Williams formula probably is inadequate to represent the friction loss under such a condition. The observed deviation may be attributed to the existence of laminar flow at the downstream end of the tube.

The deviation between measured and theoretical energy loss distribution appeared to be significant on a dimensionless basis. But when this deviation was examined by comparing the measured pressure with the theoretical pressure the errors were less than 6 percent and would be negligible in practice.

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APPENDIXES

APPENDIX A

Friction Coefficient Data

## SYMBOLS USED IN APPENDIX A

$$
\begin{aligned}
& H_{1}= \text { the pressure at the upstream side of the tube } \\
& \text { in feet of water } \\
& H_{2}= \text { the pressure at the downstream side of the } \\
& \text { tube in feet of water } \\
& G P M= \text { gallons per minute } \\
& F P S= \text { feet per second } \\
& \mathrm{V}^{2} / 2 g= \text { velocity head } \\
& H_{f}, f, C \text {, and } R_{s}, \text { see equations } 1,2 \text {, and } 3 . \\
& R=\text { Reynold's number (at } \left.60^{\circ} F\right), \text { for tubes. }
\end{aligned}
$$

Table A: Measured Friction Loss and Calculated Friction Coefficients

| Tube <br> size <br> inch | $\begin{gathered} \mathrm{H}_{1} \\ \text { feet } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { feet } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{\mathrm{f}} \\ \mathrm{ft} / \mathrm{L} \end{gathered}$ | Discharge GPM | Velocity FPS | $\mathrm{v}^{2} / 2 \mathrm{~g}$ <br> feet | Fricti f | Coef C | ents K s | R |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 1 / 2 \\ & (\mathrm{~L}=25 \\ & \text { feet }) \end{aligned}$ | 4.7377 | 4.6023 | 0.1354 | 0.5111 | 0.6852 | 0.0073 | 0.0341 | 145.00 | 0.3747 | 02,590 |
|  | 0.9043 | 0.4148 | 0.4896 | 0.8829 | 1.1836 | 0.0218 | 0.0413 | 125.00 | 0.4795 | 04.474 |
|  | 1.0710 | 0.4877 | 0.5833 | 0.9847 | 1.3201 | 0.0271 | 0.0396 | 127.00 | 0.4643 | 04,990 |
|  | 1.9043 | 0.9981 | 0.9063 | 1.3533 | 1.8143 | 0.0512 | 0.0326 | 138.00 | 0. 3943 | 06,858 |
|  | 2.0710 | 0.9460 | 1.1250 | 1.3704 | 1.8372 | 0.0525 | 0.0394 | 124.00 | 0.4779 | 06,944 |
|  | 3.9043 | 1.9668 | 1.9375 | 1.9432 | 2.6051 | 0.1055 | 0.0338 | 131.00 | 0.4239 | 09,847 |
|  | 4.0710 | 1.9460 | 2.1250 | 1. 9733 | 2.6455 | 0.1088 | 0.0359 | 127.00 | 0.4515 | 09,999 |
|  | 5.9043 | 2.9252 | 2.9792 | 2.4141 | 3.2364 | 0.1629 | 0.0337 | 129.00 | 0.4316 | 12,233 |
|  | 6.1127 | 3.0293 | 3.0834 | 2.4342 | 3.2634 | 0.1656 | 0.0343 | 128.00 | 0.4397 | 12,335 |
|  | 7.9043 | 4.0085 | 3.8958 | 2,8170 | 3.7766 | 0.2217 | 0.0323 | 130.00 | 0.4209 | 14,275 |
|  | 8.4043 | 4.2795 | 4.1250 | 2.9039 | 3.8931 | 0.2356 | 0.0322 | 130.00 | 0.4207 | 14,715 |
|  | 9.9043 | 5.0814 | 4.8229 | 3.1676 | 4.2466 | 0.2804 | 0.0316 | 131.00 | 0.4170 | 16,051 |
|  |  |  |  |  |  |  | 0.0351 | 130.35 | 0.4330 |  |
| $\begin{aligned} & 3 / 4 \\ & (\mathrm{~L}=50 \\ & \text { feet }) \end{aligned}$ | 0.9949 | 0.2631 | 0.7318 | 1.8010 | 1.2954 | 0.0261 | 0.0350 | 131.75 | 0.4248 | 06,685 |
|  | 1.9949 | 0.3360 | 1.6589 | 2.8472 | 2.0480 | 0.0652 | 0.0320 | 133.89 | 0.4037 | 10,568 |
|  | 3.9949 | 0.5652 | 3.4292 | 4.1950 | 3.0174 | 0.1416 | 0.0300 | 133.26 | 0.3997 | 15,570 |
|  | 5.9949 | 0.3465 | 5.6484 | 5.6724 | 4.0801 | 0.2588 | 0.0272 | 137.64 | 0.3711 | 21,054 |
|  | 8.9949 | 2.2631 | 6.7318 | 6.1853 | 4.4490 | 0.3077 | 0.0274 | 136.52 | 0.3752 | 22,958 |
|  | 7.9949 | 0.5808 | 7.4141 | 6.5422 | 4. 7058 | 0.3443 | 0.0270 | 137.06 | 0.3714 | 24,283 |
|  | 9.9949 | 0.5652 | 9.4297 | 7.3801 | 5.3085 | 0.4381 | 0.0270 | 135.79 | 0.3757 | 27,393 |
|  |  |  |  |  |  |  | 0.0294 | 135.13 | 0.3888 |  |
| $\begin{aligned} & 1.0 \\ & (\mathrm{~L}=50 \\ & \text { feet }) \end{aligned}$ | 2.0287 | 0.7631 | 1.2656 | 5.6242 | 2.1393 | 0.0712 | 0.0307 | 132.42 | 0.4025 | 15,180 |
|  | 4.0287 | 1.6798 | 2.3490 | 7.9815 | 3.0360 | 0.1433 | 0.0283 | 134.58 | 0. 3841 | 21,543 |
|  | 6.0287 | 2.6329 | 3.3958 | 9.8360 | 3.7414 | 0.2176 | 0.0270 | 135.92 | 0.3734 | 26,549 |
|  | 8.0287 | 3.5808 | 4.4479 | 11.4549 | 4.3572 | 0.2952 | 0.0260 | 136.82 | 0.3661 | 30,919 |
|  | 10,0287 | 4.0704 | 5.9583 | 12.9115 | 4.9113 | 0.3750 | 0.0274 | 131.70 | 0.4620 | 34,850 |
|  |  |  |  |  |  |  | 0.0279 | 134.29 | 0.3144 |  |

## APPENDIX B

Measured Perforation Flow Rate and
Calculated Discharge Coefficient

## SYMBOLS USED IN

## APPENDIX B

$\mathrm{H}_{1}=$ the pressure at the upstream side of the tube
$\mathrm{H}_{2}=$ the pressure at the downstream side of the tube H = average pressure
$C_{Q}=$ the discharge coefficient
$V=$ average velocity of flow of orifices in $\mathrm{ft} / \mathrm{sec}$ $R=$ Reynold's number (at $t=60^{\circ} \mathrm{F}$ ), for orifices

Note: - Some values are eliminated from $1 / 2$ and $3 / 4$ inch tubes because of either too large or too small perforations (total perforations were 10).

- Values between parenthesis are averages.

Table Bl: Measured Perforation Flow Rate and Calculated Discharge Coefficient

| Tube <br> size <br> inch | $\begin{aligned} & \text { Orifice } \\ & \text { size } \\ & \text { Inch } \end{aligned}$ | Orifice number | $\begin{gathered} \mathrm{H}_{1} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \text { Average } \\ \text { H } \\ \text { Inch } \end{gathered}$ | Discharge GPM | $\mathrm{C}_{\mathrm{Q}}$ | Aver V FPS | Re |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1/2 | 1/16 | 1 | 24.079 | 23.954 | 24.0165 | 0.0567 | 0.5235 |  |  |
|  |  | 2 |  |  |  | 0.0826 | 0.7626 |  |  |
|  |  | 3 |  |  |  | 0.0863 | 0.7968 |  |  |
|  |  | 4 |  |  |  | 0.0770 | 0.7109 |  |  |
|  |  | 5 |  |  |  | 0.0726 | 0.6703 |  |  |
|  |  | 6 |  |  |  | 0.0704 | 0.6500 |  |  |
|  |  | 7 |  |  |  | 0.0600 | 0.5539 |  |  |
|  |  |  |  |  |  | (0.0776) | (0.6669) | 8.12 | 3,467 |
|  |  | 1 | 48.079 | 47.954 | 48.0165 | 0.0802 | 0.5237 |  |  |
|  |  | 2 |  |  |  | 0.1159 | 0.7568 |  |  |
|  |  | 3 |  |  |  | 0.1191 | 0.7777 |  |  |
|  |  | 4 |  |  |  | 0.0986 | 0.6438 |  |  |
|  |  | 5 |  |  |  | 0.1085 | 0.7084 |  |  |
|  |  | 6 |  |  |  | 0.1033 | 0.6745 |  |  |
|  |  | 7 |  |  |  | 0.0896 | 0.5809 |  |  |
|  |  |  |  |  |  | (0.1021) | (0.6665) | 10.68 | 4,562 |
|  |  | 1 | 72.079 | 71.954 | 72.0165 | 0.0982 | 0.5236 |  |  |
|  |  | 2 |  |  |  | 0.1433 | 0.7640 |  |  |
|  |  | 3 |  |  |  | 0.1451 | 0.7736 |  |  |
|  |  | 4 |  |  |  | 0.1347 | 0.7182 |  |  |
|  |  | 5 |  |  |  | 0.1337 | 0.7128 |  |  |
|  |  | 6 |  |  |  | 0.1286 | 0.6856 |  |  |
|  |  | 7 |  |  |  | 0.1110 | 0.5918 |  |  |
|  |  |  |  |  |  | (0.1278) | (0.6814) | 13.36 | 5,711 |

Table B1: Continued.

| Tube size inch | $\begin{aligned} & \text { Orifice } \\ & \text { size } \\ & \text { inch } \end{aligned}$ | Orifice number | $\begin{gathered} \mathrm{H}_{1} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { inch } \end{gathered}$ | Average H inch | $\begin{aligned} & \text { Discharge } \\ & \text { GPM } \end{aligned}$ | $\mathrm{C}_{\mathrm{Q}}$ | Aver V FPS | age R |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 96.079 | 95.954 | 96.0165 | 0.1127 | 0.5204 |  |  |
|  |  | 2 |  |  |  | 0.1641 | 0.7577 |  |  |
|  |  | 3 |  |  |  | 0.1569 | 0.7245 |  |  |
|  |  | 4 |  |  |  | 0.1562 | 0.7212 |  |  |
|  |  | 5 |  |  |  | 0.1551 | 0.7162 |  |  |
|  |  | 6 |  |  |  | 0.1519 | 0.7014 |  |  |
|  |  | 7 |  |  |  | 0.1289 | 0.5952 |  |  |
|  |  |  |  |  |  | (0.1465) | (0.6767) | 15.32 | 6,546 |
|  |  | 1 | 120.079 | 119.954 | 120.0165 | 0.1254 | 0.5179 |  |  |
|  |  | 2 |  |  |  | 0.1888 | 0.7797 |  |  |
|  |  | 3 |  |  |  | 0.1836 | 0.7583 |  |  |
|  |  | 4 |  |  |  | 0.1752 | 0.7237 |  |  |
|  |  | 5 |  |  |  | 0.1573 | 0.6497 |  |  |
|  |  | 6 |  |  |  | 0.1723 | 0.7116 |  |  |
|  |  | 7 |  |  |  | 0.1429 | 0.5902 |  |  |
|  |  |  |  |  |  | (0.1636) | (0.6759) | 17.11 | 7,310 |
| 1/2 | 5/64 | $1$ | 24.079 | 23.954 | 24.0165 | 0.1032 | 0.6092 |  |  |
|  |  | 2 |  |  |  | 0.1136 | 0.6706 |  |  |
|  |  | 3 |  |  |  | 0.1033 | 0.6098 |  |  |
|  |  | 4 |  |  |  | 0.1134 | 0.6694 |  |  |
|  |  | 5 |  |  |  | 0.1012 | 0.5974 |  |  |
|  |  | 6 |  |  |  | 0.1089 | 0.6428 |  |  |
|  |  | 7 |  |  |  | 0.0957 | 0.5649 |  |  |
|  |  |  |  |  |  | (0.1056) | (0.6234) | 7.07 | 3,775 |

Table B1: Continued.

| Tube size inch | $\begin{aligned} & \text { Orifice } \\ & \text { size } \\ & \text { inch } \end{aligned}$ | Orifice number | $\begin{gathered} \mathrm{H}_{1} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \text { Average } \\ \text { H } \\ \text { inch } \end{gathered}$ | Discharge GPM | $\mathrm{C}_{\mathrm{Q}}$ | Aver V FPS | ge |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| , |  | 1 | 48.079 | 47.954 | 48.0165 | 0.1439 | 0.6008 |  |  |
|  |  | 2 |  |  |  | 0.1618 | 0.6755 |  |  |
|  |  | 3 |  |  |  | 0.1503 | 0.6275 |  |  |
|  |  | 4 |  |  |  | 0.1570 | 0.6555 |  |  |
|  |  | 5 |  |  |  | 0.1438 | 0.6003 |  |  |
|  |  | 6 |  |  |  | 0.1598 | 0.6671 |  |  |
|  |  | 7 |  |  |  | 0.1379 | 0.5757 |  |  |
|  |  | (0.1506) |  |  |  | (0.6289) | 10.08 | 5,383 |
|  |  | 1 | 72.079 | 71.954 | 72.0165 | 0.1769 | 0.6030 |  |  |
|  |  | 2 |  |  |  | 0.1909 | 0.6508 |  |  |
|  |  | 3 |  |  |  | 0.1812 | 0.6177 |  |  |
|  |  | 4 |  |  |  | 0.1937 | 0.6603 |  |  |
|  |  | 5 |  |  |  | 0.1754 | 0.5979 |  |  |
|  |  | 6 |  |  |  | 0.1929 | 0.6576 |  |  |
|  |  | 7 |  |  |  | 0.1684 | 0.5741 |  |  |
|  |  | (0.1828) |  |  |  | (0.6231) | 12.23 | 6,534 |
|  |  | 1 | 96.079 | 95.954 | 96.0165 | 0.2042 | 0.6029 |  |  |
|  |  | 2 |  |  |  | 0.2207 | 0.6518 |  |  |
|  |  | 3 |  |  |  | 0.2100 | 0.6200 |  |  |
|  |  | 4 |  |  |  | 0.2222 | 0.6560 |  |  |
|  |  | 5 |  |  |  | 0.2006 | 0.5922 |  |  |
|  |  | 6 |  |  |  | 0.2258 | 0.6666 |  |  |
|  |  | 7 |  |  |  | 0.1979 | 0.5843 |  |  |
|  |  | (0.2116) |  |  |  | (0.6248) | 14.16 | 7,564 |

Table B1: Continued.

| Tube <br> size <br> Inch | $\begin{aligned} & \text { Orifice } \\ & \text { size } \\ & \text { Inch } \end{aligned}$ | Orifice number | $\begin{gathered} \mathrm{H}_{1} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \text { Average } \\ \text { H } \\ \text { inch } \end{gathered}$ | Discharge GPM | $\mathrm{C}_{\mathrm{Q}}$ | Aver V FPS | Re |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 120.079 | 119.954 | 120.0165 | 0.2308 | 0.6095 |  |  |
|  |  | 2 |  |  |  | 0.2518 | 0.6649 |  |  |
|  |  | 3 |  |  |  | 0.2308 | 0.6095 |  |  |
|  |  | 4 |  |  |  | 0.2556 | 0.6750 |  |  |
|  |  | 5 |  |  |  | 0.2312 | 0.6105 |  |  |
|  |  | 6 |  |  |  | 0.2498 | 0.6596 |  |  |
|  |  | 7 |  |  |  | 0.2272 | 0.6000 |  |  |
|  |  |  |  |  |  | (0.2396) | (0.6327) | 16.04 | 8,565 |
| 1/2 | 1/8 | 1 | 23.8915 | 23.7665 | 23.8290 | 0.2594 | 0.6001 |  |  |
|  |  | 2 |  |  |  | 0.2664 | 0.6163 |  |  |
|  |  | 3 |  |  |  | 0.2552 | 0.5904 |  |  |
|  |  | 4 |  |  |  | 0.2705 | 0.6258 |  |  |
|  |  | 5 |  |  |  | 0.2913 | 0.6739 |  |  |
|  |  | 6 |  |  |  | 0.2928 | 0.6774 |  |  |
|  |  | 7 |  |  |  | 0.2594 | 0.6001 |  |  |
|  |  |  |  |  |  | (0.2707) | (0.6263) | 7.08 | 6,048 |
|  |  | 1 | 47.8915 | 47.7665 | 47.8290 | 0.3572 | 0.5833 |  |  |
|  |  | 2 |  |  |  | 0.3677 | 0.6004 |  |  |
|  |  | 3 |  |  |  | 0.3626 | 0.5921 |  |  |
|  |  | 4 |  |  |  | 0.3859 | 0.6302 |  |  |
|  |  | 5 |  |  |  | 0.4091 | 0.6680 |  |  |
|  |  | 6 |  |  |  | 0.4157 | 0.6788 |  |  |
|  |  | 7 |  |  |  | 0.3720 | 0.6075 |  |  |
|  |  |  |  |  |  | (0.3815) | (0.6229) | 9.97 | 8,523 |

Table B1: Continued.

| Tube size Inch | $\begin{aligned} & \text { Orifice } \\ & \text { size } \\ & \text { inch } \end{aligned}$ | Orifice number | $\begin{gathered} \mathrm{H}_{1} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \text { Average } \\ \text { H } \\ \text { inch } \end{gathered}$ | Discharge GPM | $\mathrm{C}_{\mathrm{Q}}$ | Aver V FPS | Re |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 71.8915 | 71.7665 | 71.8290 | 0.4258 | 0.5674 |  |  |
|  |  | 2 |  |  |  | 0.4439 | 0.5915 |  |  |
|  |  | 3 |  |  |  | 0.4403 | 0.5867 |  |  |
|  |  | 4 |  |  |  | 0.4674 | 0.6228 |  |  |
|  |  | 5 |  |  |  | 0.4993 | 0.6653 |  |  |
|  |  | 6 |  |  |  | 0.5084 | 0.6774 |  |  |
|  |  | 7 |  |  |  | 0.4569 | 0.6088 |  |  |
|  |  | (0.4631) |  |  |  | (0.6171) | 12.11 | 10,346 |
|  |  | 1 | 95.8915 | 95.7665 | 95.8290 | 0.4971 | 0.5735 |  |  |
|  |  | 2 |  |  |  | 0.5123 | 0.5910 |  |  |
|  |  | 3 |  |  |  | 0.5067 | 0.5845 |  |  |
|  |  | 4 |  |  |  | 0.5324 | 0.6142 |  |  |
|  |  | 5 |  |  |  | 0.5773 | 0.6660 |  |  |
|  |  | 6 |  |  |  | 0.5807 | 0.6699 |  |  |
|  |  | 7 |  |  |  | $0.5253$ | 0.6060 |  |  |
|  |  | (0.5331) |  |  |  | (0.6150) | 13.94 | 11,910 |
|  |  | 1 | 119.8915 | 119.7665 | 119.8290 | 0.4997 | 0.5155 |  |  |
|  |  | 2 |  |  |  | 0.5824 | 0.6008 |  |  |
|  |  | 3 |  |  |  | 0.6286 | 0.6485 |  |  |
|  |  | 4 |  |  |  | 0.5933 | 0.6121 |  |  |
|  |  | 5 |  |  |  | 0.6516 | 0.6722 |  |  |
|  |  | 6 |  |  |  | 0.6417 | 0.6620 |  |  |
|  |  | 7 |  |  |  | 0.5965 | 0.6154 |  |  |
|  |  | (0.5991) |  |  |  | (0.6181) | 15.66 | 13,385 |

Table B2: Measured Perforation F1ow Rate and Calculated Discharge Coefficient

| Tube <br> size <br> inch | ```Orifice size inch``` | Orifice number | $\begin{gathered} \mathrm{H}_{1} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { inch } \end{gathered}$ | Average <br> H <br> inch | $\begin{aligned} & \text { Discharge } \\ & \text { GPM } \end{aligned}$ | $\mathrm{C}_{\mathrm{Q}}$ | $\begin{gathered} \text { Ave } \\ \mathrm{V} \\ \text { FPS } \end{gathered}$ | rage $\mathbf{R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3/4 | 1/16 | 1 | 24.3775 | 24.2838 | 24.3306 | 0.0712 | 0.6194 |  |  |
|  |  | 2 |  |  |  | 0.0905 | 0.7873 |  |  |
|  |  | 3 |  |  |  | 0.0685 | 0.5959 |  |  |
|  |  | 4 |  |  |  | 0.0798 | 0.6943 |  |  |
|  |  | 5 |  |  |  | 0.0693 | 0.6029 |  |  |
|  |  | 6 |  |  |  | 0.0834 | 0.7256 |  |  |
|  |  | 7 |  |  |  | 0.0858 | 0.7465 |  |  |
|  |  |  |  |  |  | (0.0784) | (0.6817) | 8.20 | 3,503 |
|  |  | 1 | 48.3775 | 48.2838 | 48.3306 | 0.1226 | 0.7568 |  |  |
|  |  | 2 |  |  |  | 0.1241 | 0.7660 |  |  |
|  |  | 3 |  |  |  | 0.1013 | 0.6253 |  |  |
|  |  | 4 |  |  |  | 0.0983 | 0.6068 |  |  |
|  |  | 5 |  |  |  | 0.1016 | 0.6272 |  |  |
|  |  | 6 |  |  |  | 0.1113 | 0.6870 |  |  |
|  |  | 7 |  |  |  | 0.1114 | 0.6871 |  |  |
|  |  |  |  |  |  | (0.1184) | (0.6795) | 12.38 | 5,291 |
|  |  | 1 | 74.3775 | 72.2838 | 76.3306 | 0.1507 | 0.7402 |  |  |
|  |  | 2 |  |  |  | 0.1319 | 0.6479 |  |  |
|  |  | 3 |  |  |  | 0.1169 | 0.5742 |  |  |
|  |  | 4 |  |  |  | 0.1206 | 0.5924 |  |  |
|  |  | 5 |  |  |  | 0.1503 | 0.7383 |  |  |
|  |  | 6 |  |  |  | 0.1370 | 0.6729 |  |  |
|  |  | 7 |  |  |  |  | $0.6990$ |  |  |
|  |  |  |  |  |  | (0.1357) | (0.6664) | 14.19 | 6,064 |

Table B2: Continued.

| Tube <br> size <br> inch | Orifice <br> size <br> inch | Orifice <br> number | $\mathrm{H}_{1}$ <br> inch | $\mathrm{H}_{2}$ <br> inch | Average <br> H <br> inch | Discharge <br> GPM | $\mathrm{C}_{\mathrm{Q}}$ | Average <br> V | RS |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

Table B2: Continued.

| Tube <br> size <br> Inch | $\begin{aligned} & \text { Orifice } \\ & \text { size } \\ & \text { inch } \end{aligned}$ | Orifice number | $\begin{gathered} \mathrm{H}_{1} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \text { Average } \\ \text { H } \\ \text { inch } \end{gathered}$ | Discharge GPM | $\mathrm{C}_{\mathrm{Q}}$ | Ave V FPS | ${ }_{R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 48.1588 | 48.0650 | 48.1119 | 0.1602 | 0.6681 |  |  |
|  |  | 2 |  |  |  | 0.1464 | 0.6106 |  |  |
|  |  | 3 |  |  |  | 0.1564 | 0.6523 |  |  |
|  |  | 4 |  |  |  | 0.1475 | 0.6152 |  |  |
|  |  | 5 |  |  |  | 0.1656 | 0.6907 |  |  |
|  |  | 6 |  |  |  | 0.1733 | 0.7228 |  |  |
|  |  | 7 |  |  |  | 0.1477 | 0.6160 |  |  |
|  |  | (0.1567) |  |  |  | (0.6537) | 10.49 | 5,601 |
|  |  | 1 | 72.1588 | 72.0650 | 72.1119 | 0.1955 | 0.6660 |  |  |
|  |  | 2 |  |  |  | 0.1780 | 0.6064 |  |  |
|  |  | 3 |  |  |  | 0.1908 | 0.6500 |  |  |
|  |  | 4 |  |  |  | 0.1802 | 0.6139 |  |  |
|  |  | 5 |  |  |  | 0.2018 | 0.6875 |  |  |
|  |  | 6 |  |  |  | 0.2128 | 0.7249 |  |  |
|  |  | 7 |  |  |  | 0.1806 | 0.6152 |  |  |
|  |  | (0.1914) |  |  |  | (0.6520) | 12.81 | 6,842 |
|  |  | 1 | 96.1588 | 96.0650 | 96.1119 | 0.2275 | 0.6713 |  |  |
|  |  | 2 |  |  |  | 0.2040 | 0.6020 |  |  |
|  |  | 3 |  |  |  | 0.2200 | 0.6492 |  |  |
|  |  | 4 |  |  |  | 0.2085 | 0.6153 |  |  |
|  |  | 5 |  |  |  | 0.2336 | 0.6893 |  |  |
|  |  | 6 |  |  |  | 0.2476 | 0.7306 |  |  |
|  |  | 7 |  |  |  | 0.2043 | 0.6029 |  |  |
|  |  | (0.2124) |  |  |  | (0.6515) | 14.22 | 7,592 |

Table B2: Continued.

| Tube size <br> inch | $\begin{aligned} & \text { Orifice } \\ & \text { size } \\ & \text { inch } \end{aligned}$ | Orifice number | $\begin{gathered} \mathrm{H}_{1} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \text { Average } \\ \text { H } \\ \text { inch } \end{gathered}$ |  |  | Aver V FPS | age $\mathrm{R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 120.1588 | 120.0650 | 120.1119 | 0.2270 | 0.5992 |  |  |
|  |  | 2 |  |  |  | 0.2273 | 0.6000 |  |  |
|  |  | 3 |  |  |  | 0.2451 | 0.6470 |  |  |
|  |  | 4 |  |  |  | 0.2328 | 0.6145 |  |  |
|  |  | 5 |  |  |  | 0.2600 | 0.6863 |  |  |
|  |  | 6 |  |  |  | 0.2769 | 0.7309 |  |  |
|  |  | 7 |  |  |  | 0.2301 | 0.6074 |  |  |
|  |  |  |  |  |  | (0.2427) | (0.6408) | 16.24 | 8,675 |
| $3 / 4$ | 1/8 | 1 | 24.1275 | 24.0337 | 24.0805 | 0.2889 | 0.6649 |  |  |
|  |  | 2 |  |  |  | 0.3241 | 0.7459 |  |  |
|  |  | 3 |  |  |  | 0.2919 | 0.6718 |  |  |
|  |  | 4 |  |  |  | 0.3223 | 0.7417 |  |  |
|  |  | 5 |  |  |  | 0.3147 | 0.7242 |  |  |
|  |  | 6 |  |  |  | 0.3038 | 0.6992 |  |  |
|  |  | 7 |  |  |  | $0.3144$ | 0.7235 |  |  |
|  |  |  |  |  |  | $(0.3086)$ | (0.7102) | 8.07 | 6,895 |
|  |  | 1 | 48.1275 | 48.0337 | 48.0805 | 0.4002 | 0.6518 |  |  |
|  |  | 2 |  |  |  | 0.4581 | 0.7461 |  |  |
|  |  | 3 |  |  |  | 0.4171 | 0.6934 |  |  |
|  |  | 4 |  |  |  | 0.3873 | 0.6308 |  |  |
|  |  | 5 |  |  |  | 0.4003 | 0.6520 |  |  |
|  |  | 6 |  |  |  | 0.4233 | 0.6894 |  |  |
|  |  | 7 |  |  |  | $0.4290$ | $0.7051$ |  |  |
|  |  |  |  |  |  | (0.4232) | (0.6812) | 11.06 | 9,455 |

Table B2: Continued.

| Tube <br> size <br> Inch | $\begin{aligned} & \text { Orifice } \\ & \text { size } \\ & \text { inch } \end{aligned}$ | Orifice number | $\begin{gathered} \mathrm{H}_{1} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \text { Average } \\ \text { H } \\ \text { inch } \end{gathered}$ | Discharge GPM | $\mathrm{C}_{\mathrm{Q}}$ | Aver V FPS | ${ }_{R}^{\operatorname{age}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\cdots$ |  | 1 | 72.1275 | 72.0337 | 72.0806 | 0.4851 | 0.6453 |  |  |
|  |  | 2 |  |  |  | 0.5586 | 0.7430 |  |  |
|  |  | 3 |  |  |  | 0.5029 | 0.6690 |  |  |
|  |  | 4 |  |  |  | 0.4754 | 0.6324 |  |  |
|  |  | 5 |  |  |  | 0.4810 | 0.6399 |  |  |
|  |  | 6 |  |  |  | 0.5113 | 0.6801 |  |  |
|  |  | 7 |  |  |  | 0.5271 | 0.7011 |  |  |
|  |  | (0.5059) |  |  |  | (0.6730) | 13.23 | 11,303 |
|  |  | 1 | 96.1275 | 96.0337 | 96.0806 | 0.5582 | 0.6431 |  |  |
|  |  | 2 |  |  |  | 0.6350 | 0.7316 |  |  |
|  |  | 3 |  |  |  | 0.5860 | 0.6751 |  |  |
|  |  | 4 |  |  |  | 0.5386 | 0.6206 |  |  |
|  |  | 5 |  |  |  | 0.5562 | 0.6408 |  |  |
|  |  | 6 |  |  |  | 0.5823 | 0.6709 |  |  |
|  |  | 7 |  |  |  | 0.6067 | 0.6990 |  |  |
|  |  | (0.5804) |  |  |  | (0.6687) | 15.17 | 12,967 |
|  |  | 1 | 120.1275 | 120.0337 | 120.0806 | 0.6287 | 0.6479 |  |  |
|  |  | 2 |  |  |  | 0.6957 | 0.7170 |  |  |
|  |  | 3 |  |  |  | 0.6637 | 0.6840 |  |  |
|  |  | 4 |  |  |  | 0.6014 | 0.6198 |  |  |
|  |  | 5 |  |  |  | 0.6245 | 0.6436 |  |  |
|  |  | 6 |  |  |  | 0.6457 | 0.6654 |  |  |
|  |  | 7 |  |  |  | $\begin{gathered} 0.6718 \\ (0.6473) \end{gathered}$ | $\begin{gathered} 0.6923 \\ (0.6671) \end{gathered}$ | 16.92 | 14,462 |

Table B3: Measured Perforation Flow Rate and Calculated Discharge Coefficient

| Tube size <br> Inch | $\begin{aligned} & \text { Orifice } \\ & \text { size } \\ & \text { Inch } \end{aligned}$ | Orifice number | $\begin{gathered} \mathrm{H}_{1} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \text { Average } \\ \text { H } \\ \text { inch } \end{gathered}$ | Discharg GPM | e $\mathrm{C}_{\mathrm{Q}}$ |  | $\mathrm{age}_{\mathrm{R}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.0 | 1/16 | 1 | 24.5015 | 24.439 | 24.4703 | 0.0753 | 0.6887 |  |  |
|  |  | 2 |  |  |  | 0.0655 | 0.5991 |  |  |
|  |  | 3 |  |  |  | 0.0827 | 0.7564 |  |  |
|  |  | 4 |  |  |  | 0.0739 | 0.6759 |  |  |
|  |  | 5 |  |  |  | 0.0770 | 0.7043 |  |  |
|  |  | 6 |  |  |  | 0.0752 | 0.6878 |  |  |
|  |  | 7 |  |  |  | 0.0713 | 0.6521 |  |  |
|  |  | 8 |  |  |  | 0.0688 | 0.6293 |  |  |
|  |  | 9 |  |  |  | 0.0741 | 0.6777 |  |  |
|  |  | 10 |  |  |  | 0.0575 | 0.5259 |  |  |
|  |  |  |  |  |  | (0.0721) | (0.6594) | 7.54 | 3,222 |
|  |  | 1 | 48.5015 | 48.4390 | 48.4703 | 0.1036 | 0.6733 |  |  |
|  |  | 2 |  |  |  | 0.0899 | 0.5842 |  |  |
|  |  | 3 |  |  |  | 0.1152 | 0.7487 |  |  |
|  |  | 4 |  |  |  | 0.1037 | 0.6739 |  |  |
|  |  | 5 |  |  |  | 0.1075 | 0.6986 |  |  |
|  |  | 6 |  |  |  | 0.1061 | 0.6895 |  |  |
|  |  | 7 |  |  |  | 0.0959 | 0.6232 |  |  |
|  |  | 8 |  |  |  | 0.1032 | 0.6707 |  |  |
|  |  | 9 |  |  |  | 0.1032 | 0.6707 |  |  |
|  |  | 10 |  |  |  |  |  |  |  |
|  |  |  |  |  |  | $(0.1009)$ | (0.6557) | 10.55 | 4,509 |
|  |  | 1 | 72.4975 | 72.4350 | 74.4663 | 0.1257 | 0.6681 |  |  |
|  |  | 2 |  |  |  | 0.1404 | 0.7462 |  |  |
|  |  | 3 |  |  |  | 0.1398 | 0.7430 |  |  |
|  |  | 4 |  |  |  | 0.1188 | 0.6314 |  |  |
|  |  | 5 |  |  |  | 0.1343 | 0.7138 |  |  |

Table B3: Continued.


Table B3: Continued.

| Tube <br> size <br> inch | ```Orifice size Inch``` | Orifice number | $\begin{gathered} \mathrm{H}_{1} \\ \text { Inch } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { Inch } \end{gathered}$ | Average H inch | Discharge GPM | $C_{Q}$ | Aver <br> V <br> FPS | ge R |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.0 | 5/64 | 1 | 24.5015 | 24.4390 | 24.4703 | 0.1404 | 0.8206 |  |  |
|  |  | 2 |  |  |  | 0.1206 | 0.7047 |  |  |
|  |  | 3 |  |  |  | 0.1173 | 0.6858 |  |  |
|  |  | 4 |  |  |  | 0.1231 | 0.7192 |  |  |
|  |  | 5 |  |  |  | 0.1077 | 0.6292 |  |  |
|  |  | 6 |  |  |  | 0.1280 | 0.7480 |  |  |
|  |  | 7 |  |  |  | 0.1045 | 0.6105 |  |  |
|  |  | 8 |  |  |  | 0.1228 | 0.7179 |  |  |
|  |  | 9 |  |  |  | 0.1252 | 0.7315 |  |  |
|  |  | 10 |  |  |  | 0.1171 | 0.6844 |  |  |
|  |  |  |  |  |  | (0.12067) | (0.7052) | 8.08 | 4,313 |
|  |  | 1 | 48.525 | 48.4390 | 48.4703 | 0.1936 | 0.8038 |  |  |
|  |  | 2 |  |  |  | 0.1684 | 0.6993 |  |  |
|  |  | 3 |  |  |  | 0.1615 | 0.6705 |  |  |
|  |  | 4 |  |  |  | 0.1684 | 0.6995 |  |  |
|  |  | 5 |  |  |  | 0.1595 | 0.6622 |  |  |
|  |  | 6 |  |  |  | 0.1832 | 0.7607 |  |  |
|  |  | 7 |  |  |  | 0.1479 | 0.6142 |  |  |
|  |  | 8 |  |  |  | 0.1812 | 0.7525 |  |  |
|  |  | 9 |  |  |  | 0.1794 | 0.7450 |  |  |
|  |  | 10 |  |  |  | $0.1636$ | $0.6793$ |  |  |
|  |  |  |  |  |  | $(0.17067)$ | $(0.7052)$ | 11.42 | 6,101 |
|  |  | 1 |  |  |  |  |  |  |  |
|  |  | 1 | 72.5015 | 72.4390 | 72.4703 |  |  |  |  |
|  |  | 2 |  |  |  | $0.2045$ | $0.6944$ |  |  |
|  |  | 3 |  |  |  | 0.2045 | 0.6944 |  |  |
|  |  | 4 |  |  |  | 0.2032 | $0.6901$ |  |  |
|  |  | 5 |  |  |  | 0.2003 | 0.6801 |  |  |
|  |  |  |  |  |  |  |  | - |  |

Table B3: Continued.

| Tube <br> size <br> Inch | ```Orifice size Inch``` | Orifice number | $\begin{gathered} \mathrm{H}_{1} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \text { Average } \\ \text { H } \\ \text { inch } \end{gathered}$ | $\begin{gathered} \text { Discharge } \\ \text { GPM } \end{gathered}$ | $\mathrm{C}_{Q}$ |  | $\mathrm{lge}_{\mathrm{p}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 |  | 6 |  |  |  | 0.2223 | 0.7549 |  |  |
|  |  | 7 |  |  |  | 0.1799 | 0.6110 |  |  |
|  |  | 8 |  |  |  | 0.2226 | 0.7559 |  |  |
|  |  | 9 |  |  |  | 0.2150 | 0.7302 |  |  |
|  |  | 10 |  |  |  | 0.1992 | 0.6765 |  |  |
|  |  |  |  |  |  | (0.2084) ( | (0.7077) | 13.95 | 7,449 |
|  |  | 1 | 96.5015 | 96.4390 | 96.4703 | 0.2699 | 0.7943 |  |  |
|  |  | 2 |  |  |  | 0.2383 | 0.7015 |  |  |
|  |  | 3 |  |  |  | 0.2609 | 0.7679 |  |  |
|  |  | 4 |  |  |  | 0.2322 | 0.6834 |  |  |
|  |  | 5 |  |  |  | 0.2305 | 0.6784 |  |  |
|  |  | 6 |  |  |  | 0.2549 | 0.7503 |  |  |
|  |  | 7 |  |  |  | 0.2148 | 0.6322 |  |  |
|  |  | 8 |  |  |  | 0.2579 | 0.7590 |  |  |
|  |  | 9 |  |  |  | 0.2502 | 0.7365 |  |  |
|  |  | 10 |  |  |  |  |  |  |  |
|  |  |  |  |  |  | $(0.2448)$ | (0.7206) | 16.38 | 8,751 |
|  |  | 1 | 120.5015 | 120.4390 | 120.4703 | 0.3094 | 0.8150 |  |  |
|  |  | 2 |  |  |  | 0.2688 | 0.7080 |  |  |
|  |  | 3 |  |  |  | 0.3002 | 0.7907 |  |  |
|  |  | 4 |  |  |  | 0.2615 | 0.6888 |  |  |
|  |  | 5 |  |  |  | 0.2288 | 0.6027 |  |  |
|  |  | 6 |  |  |  | 0.2817 | 0.7420 |  |  |
|  |  | 7 |  |  |  | 0.2159 | 0.5687 |  |  |
|  |  | 8 |  |  |  | 0.3052 | 0.8039 |  |  |
|  |  | 9 |  |  |  | 0.2937 | 0.7736 |  |  |
|  |  | 10 |  |  |  | 0.2468 | 0.6501 |  |  |
|  |  |  |  |  |  | (0.2712) ( | (0.7143) | 18.15 | 9,694 |

Table B3: Continued.

| Tube size inch | $\begin{aligned} & \text { Orifice } \\ & \text { size } \\ & \text { inch } \end{aligned}$ | Orifice number | $\begin{gathered} \mathrm{H}_{1} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \mathrm{H}_{2} \\ \text { inch } \end{gathered}$ | $\begin{gathered} \text { Average } \\ \text { H } \\ \text { inch } \end{gathered}$ | Discharge GPM | e $C_{Q}$ | Aver V FPS | age $\mathrm{R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.0 | 1/8 | 1 | 24.8140 | 24.7515 | 24.7828 | 0.3326 | 0.7545 |  |  |
|  |  | 2 |  |  |  | 0.3278 | 0.7436 |  |  |
|  |  | 3 |  |  |  | 0.3101 | 0.7035 |  |  |
|  |  | 4 |  |  |  | 0.3323 | 0.7538 |  |  |
|  |  | 5 |  |  |  | 0.3430 | 0.7781 |  |  |
|  |  | 6 |  |  |  | 0.3740 | 0.7810 |  |  |
|  |  | 7 |  |  |  | 0.3074 | 0.6973 |  |  |
|  |  | 8 |  |  |  | 0.3330 | 0.7554 |  |  |
|  |  | 9 |  |  |  | 0.2905 | 0.6590 |  |  |
|  |  | 10 |  |  |  |  |  |  |  |
|  |  |  |  |  |  | (0.3270) | $(0.7417)$ | 8.55 | 7,306 |
|  |  |  | 48.8140 | 48.7515 | 48.7828 | $0.4665$ | 0.7573 |  |  |
|  |  | 2 |  |  |  | $0.4631$ | 0.7488 |  |  |
|  |  | 3 |  |  |  | 0.4399 | 0.7113 |  |  |
|  |  | 4 |  |  |  | 0.4760 | 0.7697 |  |  |
|  |  | 5 |  |  |  | 0.4814 | 0.7784 |  |  |
|  |  | 6 |  |  |  | 0.4906 | 0.7933 |  |  |
|  |  | 7 |  |  |  | 0.4548 | 0.7354 |  |  |
|  |  | 8 |  |  |  | 0.4299 | 0.6951 |  |  |
|  |  | 9 |  |  |  | 0.4427 | 0.7158 |  |  |
|  |  | 10 |  |  |  | 0.4757 | 0.7692 |  |  |
|  |  |  |  |  |  | (0.4621) | (0.7471) | 12.08 | 10,324 |
|  |  | 1 | 72.8140 | 72.7575 | 72.7828 | 0.5722 | 0.7574 |  |  |
|  |  | 2 |  |  |  | 0.5530 | 0.7320 |  |  |
|  |  | 3 |  |  |  | 0.5345 | 0.7075 |  |  |
|  |  | 4 |  |  |  | 0.5811 | 0.7692 |  |  |
|  |  | 5 |  |  |  | 0.5733 | 0.7589 |  |  |

Table B3: Continued.


## APPENDIX C

Numerical Solution of Equations 8, 9, and 10.

SYMBOLS USED IN

## APPENDIX C

Where

$$
\begin{aligned}
f= & {\left[\left(1+B \frac{x}{L}\right)-\left(B \frac{x}{L}+\frac{x}{L}\right)\right]^{1.85} d\left(\frac{x}{L}\right) } \\
d a= & \text { area }=\frac{d x}{6}\left(y_{0}+4 y_{1}+y_{2}\right) \\
d_{x}= & \text { the interval of integration divided into an even } \\
& \text { number subinterval of width } d_{x} .
\end{aligned}
$$

$y_{0}, y_{1}, y_{2}=$ the only functional values in the above interval.
$H_{x} / H_{f}=\int_{1}^{x / L}(f)$, Equation 11 (Chu, 1982).
$\mathrm{H}=$ Energy loss distribution
$=\frac{1}{F} \int_{1}^{x / L}(f),($ Equation 10).
$F=F-$ Factor $=$ the sum of da, (Equation 9).
$Q_{X} / Q_{0}=$ Discharge distribution along the tube, (Equation 8).

Table Cl: Numerical Solution of Equations 8, 9, and 10.
$B=0$

| x/L | f | Simpson's factor | Product | $\mathrm{y}_{0}+4 \mathrm{y}_{1}+\mathrm{y}_{2}$ | da | $\frac{\mathrm{H}_{\mathrm{x}}}{\mathrm{H}_{\mathrm{f}}}$ | H | $Q_{x} / Q_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 1.0000 | 1 | 1.0000 |  |  | (0.3504) | 1.0000 | 1.0000 |
| 0.05 | 0.9095 | 4 | 3.6380 |  |  |  |  | 0.9500 |
| 0.10 | 0.8229 | 1 | 0.8229 | 5.4609 | 0.0910 | 0.2594 | 0.7403 | 0.9000 |
| 0.15 | 0.7403 | 4 | 2.9612 |  |  |  |  | 0.8500 |
| 0.20 | 0.6618 | 1 | 0.6618 | 2.4459 | 0.0741 | 0.1853 | 0.5288 | 0.8000 |
| 0.25 | 0.5873 | 4 | 0.3492 |  |  |  |  | 0.7500 |
| 0.30 | 0.5169 | 1 | 0.5169 | 3.5279 | 0.0588 | 0.1265 | 0.3610 | 0.7000 |
| 0.35 | 0.4507 | 4 | 1.8028 |  |  |  |  | 0.6500 |
| 0.40 | 0.3887 | 1 | 0.3887 | 2.7084 | 0.0451 | 0.0814 | 0.2323 | 0.6000 |
| 0.45 | 0.3309 | 4 | 1.3236 |  |  |  |  | 0.5500 |
| 0.50 | 0.2774 | 1 | 0.2774 | 1.9897 | 0.0332 | 0.0482 | 0.1376 | 0.5000 |
| 0.55 | 0.2283 | 4 | 0.9132 |  |  |  |  | 0.4500 |
| 0.60 | 0.1836 | 1 | 0.1836 | 1.3742 | 0.0229 | 0.0253 | 0.0722 | 0.4000 |
| 0.65 | 0.1434 | 4 | 0.5736 |  |  |  |  | 0.3500 |
| 0.70 | 0.1078 | 1 | 0.1078 | 0.8650 | 0.0144 | 0.0109 | 0.0311 | 0.3000 |
| 0.75 | 0.0700 | 4 | 0.2800 |  |  |  |  | 0.2500 |
| 0.80 | 0.0509 | 1 | 0.0509 | 0.4387 | 0.0073 | 0.0036 | 0.0103 | 0.2000 |
| 0.85 | 0.0299 | 4 | 0.1196 |  |  |  |  | 0.1500 |
| 0.90 | 0.0141 | 1 | 0.0141 | 0.1846 | 0.0031 | 0.0005 | 0.0014 | 0.1000 |
| 0.95 | 0.0039 | 4 | 0.0156 |  |  |  |  | 0.0499 |
| 1.00 | 0.0039 | 1 | 0.0000 | 0.0297 | 0.0005 | 0.0000 | 0.0000 | 0.0000 |
|  |  |  |  |  | (0.3504) |  |  |  |

Table C2: Numerical Solution of Equations• 8, 9, and 10.
$\mathrm{B}=0.2$

| x/L | f | $\begin{aligned} & \text { Simpson's } \\ & \text { factor } \end{aligned}$ | Product | $y_{0}+4 y_{1}+y_{2}$ | da | $\frac{\mathrm{H}_{\mathrm{x}}}{\mathrm{H}_{\mathrm{f}}}$ | H | $Q_{x} / Q_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 1.0000 | 1 | 1.0000 |  |  | (0.3128) | 1.0000 | 1.0000 |
| 0.05 | 0.8489 | 4 | 3.3956 |  |  |  |  | 0.9153 |
| 0.10 | 0.7512 | 1 | 0.7512 | 5.1468 | 0.0858 | 0.2270 | 0.7257 | 0.8567 |
| 0.15 | 0.6657 | 4 | 2.6628 |  |  |  |  | 0.8025 |
| 0.20 | 0.5881 | 1 | 0.5881 | 4.0021 | 0.0667 | 0.1603 | 0.5125 | 0.7505 |
| 0.25 | 0.5169 | 4 | 2.0676 |  |  |  | 0.5125 | 0.7000 |
| 0.30 | 0.4513 | 1 | 0.4513 | 3.1070 | 0.0518 | 0.1085 | 0.3469 | 0.6505 |
| 0.35 | 0.3907 | 4 | 1.5628 |  |  | 0.108 | 0.346 | 0.6017 |
| 0.40 | 0.3348 | 1 | 0.3348 | 2.3489 | 0.0391 | 0.0694 | 0.2219 | 0.5535 |
| 0.45 | 0.2834 | 4 | 1.1336 |  |  | 0.069 | 0.221 | 0.5058 |
| 0.50 | 0.2364 | 1 | 0.2364 | 1.7048 | 0.0284 | 0.0410 | 0.1311 | 0.4586 |
| 0.55 | 0.1936 | 4 | 0.7744 |  |  |  | 0.131 | 0.4117 |
| 0.60 | 0.1550 | 1 | 0.1550 | 1.1658 | 0.0194 | 0.0216 | 0.0691 | 0.3650 |
| 0.65 | 0.1206 | 4 | 0.4824 |  |  |  | 0.0691 | 0.3188 |
| 0.70 | 0.0903 | 1 | 0.0903 | 0.7277 | 0.0121 | 0.0095 | 0.0304 | 0.2726 |
| 0.75 | 0.0643 | 4 | 0.2572 |  |  |  |  | 0.2268 |
| 0.80 | 0.0424 | 1 | 0.0424 | 0.3899 | 0.0065 | 0.0030 | 0.0096 | 0.1811 |
| 0.85 | 0.0248 | 4 | 0.0992 |  |  |  |  | 0.1356 |
| 0.90 | 0.0117 | 1 | 0.0117 | 0.1533 | 0.0026 | 0.0004 | 0.0013 | 0.0903 |
| 0.95 | 0.0032 | 4 | 0.0128 |  |  |  |  | 0.0451 |
| 1.00 | 0.0000 | 1 | 0.0000 | 0.0245 | $\begin{gathered} 0.0004 \\ (0.3128) \end{gathered}$ | 0.0000 | 0.0000 | 0.0000 |

Table C3: Numerical Solution of Equations 8, 9, and 10. $B=0.4$

| x/L | f | $\begin{aligned} & \text { Simpson's } \\ & \text { factor } \end{aligned}$ | Product | $y_{0}+4 y_{1}+y_{2}$ | da | $\frac{\mathrm{H}_{\mathrm{x}}}{\mathrm{H}_{\mathrm{f}}}$ | H | $\mathrm{Q}_{\mathrm{x}} / \mathrm{Q}_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 1.0000 | 1 | 1.0000 |  |  | (0.2770) | 1.0000 | 1.0000 |
| 0.05 | 0.7903 | 4 | 3.1612 |  |  |  |  | 0.8806 |
| 0.10 | 0.6826 | 1 | 0.6826 | 4.8432 | 0.0807 | 0.1963 | 0.7087 | 0.8135 |
| 0.15 | 0.5947 | 4 | 2.3788 |  |  |  |  | 0.7551 |
| 0.20 | 0.5185 | 1 | 0.5185 | 3.5799 | 0.0597 | 0.1366 | 0.4931 | 0.7011 |
| 0.25 | 0.4507 | 4 | 1.8028 |  |  |  |  | 0.6500 |
| 0.30 | 0.3898 | 1 | 0.3898 | 2.7111 | 0.0452 | 0.0914 | 0.3300 | 0.6009 |
| 0.35 | 0.3346 | 4 | 1.3384 |  |  |  |  | 0.5534 |
| 0.40 | 0.2846 | 1 | 0.2846 | 2.0128 | 0.0335 | 0.0579 | 0.2090 | 0.5070 |
| 0.45 | 0.2393 | 4 | 0.9572 |  |  |  |  | 0.4617 |
| 0.50 | 0.1984 | 1 | 0.1984 | 1.4402 | 0.0240 | 0.0339 | 0.1224 | 0.4172 |
| 0.55 | 0.1616 | 4 | 0.6464 |  |  |  |  | 0.3734 |
| 0.60 | 0.1287 | 1 | 0.1287 | 0.9735 | 0.0162 | 0.0177 | 0.0639 | 0.3301 |
| 0.65 | 0.0997 | 4 | 0.3988 |  |  |  |  | 0.2875 |
| 0.70 | 0.0743 | 1 | 0.0743 | 0.6018 | 0.0100 | 0.0077 | 0.0278 | 0.2453 |
| 0.75 | 0.0526 | 4 | 0.2104 |  |  |  |  | 0.2036 |
| 0.80 | 0.0346 | 1 | 0.0346 | 0.3193 | 0.0053 | 0.0024 | 0.0087 | 0.1623 |
| 0.85 | 0.0202 | 4 | 0.0808 |  |  |  |  | 0.1212 |
| 0.90 | 0.0095 | 1 | 0.0095 | 0.1249 | 0.0021 | 0.0003 | 0.0011 | 0.0807 |
| 0.95 | 0.0026 | 4 | 0.0104 |  |  |  |  | 0.0401 |
| 1.00 | 0.0000 | 1 | 0.0000 | 0.0199 | 0.0003 | 0.0000 | 0.0000 | 0.0000 |
|  |  |  |  |  | (0.2770) |  |  |  |

Table C4: Numerical Solution of Equations 8, 9, and 10. $B=0.6$

| x/L | f | Simpson's factor | Product | $y_{0}+4 y_{1}+y_{2}$ | da | $\frac{\mathrm{H}_{\mathrm{x}}}{\mathrm{H}_{\mathrm{f}}}$ | H | $Q_{x} / Q_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 1.0000 | 1 | 1.0000 |  |  | (0.2439) | 1.0000 | 1.0000 |
| 0.05 | 0.7336 | 4 | 2.9344 |  |  |  |  | 0.8458 |
| 0.10 | 0.6170 | 1 | 0.6170 | 4.5514 | 0.0759 | 0.1680 | 0.6888 | 0.7703 |
| 0.15 | 0.5274 | 4 | 2.1096 |  |  |  |  | 0.7076 |
| 0.20 | 0.4528 | 1 | 0.4528 | 3.1794 | 0.0530 | 0.1150 | 0.4715 | 0.6516 |
| 0.25 | 0.3887 | 4 | 1.5548 |  |  |  |  | 0.6000 |
| 0.30 | 0.3324 | 1 | 0.3324 | 2.3400 | 0.0390 | 0.0760 | 0.3116 | 0.5514 |
| 0.35 | 0.2820 | 4 | 1.1304 |  |  |  |  | 0.5050 |
| 0.40 | 0.2382 | 1 | 0.2382 | 1.7010 | 0.0284 | 0.0476 | 0.1952 | 0.4605 |
| 0.45 | 0.1987 | 4 | 0.7948 |  |  |  |  | 0.4175 |
| 0.50 | 0.1635 | 1 | 0.1635 | 1.1965 | 0.0199 | 0.0277 | 0.1136 | 0.3757 |
| 0.55 | 0.1323 | 4 | 0.5292 |  |  |  |  | 0.3350 |
| 0.60 | 0.1047 | 1 | 0.1047 | 0.7974 | 0.0133 | 0.0144 | 0.0590 | 0.2953 |
| 0.65 | 0.0806 | 4 | 0.3224 |  |  |  |  | 0.2563 |
| 0.70 | 0.0597 | 1 | 0.0597 | 0.4868 | 0.0081 | 0.0063 | 0.0258 | 0.2180 |
| 0.75 | 0.0421 | 4 | 0.1684 |  |  |  |  | 0.1804 |
| 0.80 | 0.0275 | 1 | 0.0275 | 0.2556 | 0.0043 | 0.0020 | 0.0005 | 0.1433 |
| 0.85 | 0.0160 | 4 | 0.0640 |  |  |  | 0.0005 | 0.1068 |
| 0.90 | 0.0075 | 1 | 0.0075 | 0.0990 | 0.0017 | 0.0003 | 0.0012 | 0.0710 |
| 0.95 | 0.0020 | 4 | 0.0080 |  |  |  | 0.0012 | 0.0352 |
| 1.00 | 0.0000 | 1 | 0.0000 | 0.0155 | $\begin{gathered} 0.0003 \\ (0.2439) \end{gathered}$ | 0.0000 | 0.0000 | 0.0000 |

Table C5: Numerical Solution of Equations 8, 9, and 10. $\mathrm{B}=0.8$

| $x / L$ | f | $\begin{aligned} & \text { Simpson's } \\ & \text { factor } \end{aligned}$ | Product | $y_{0}+4 y_{1}+y_{2}$ | da | $\frac{H_{x}}{H_{f}}$ | H | $\mathrm{Q}_{\mathrm{x}} / \mathrm{Q}_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 1.0000 | 1 | 1.0000 |  |  | (0.2127) | 1.0000 | 1.0000 |
| 0.05 | 0.6789 | 4 | 2.7156 |  |  |  |  | 0.8111 |
| 0.10 | 0.5544 | 1 | 0.5544 | 4.2700 | 0.0712 | 0.1415 | 0.6654 | 0.7270 |
| 0.15 | 0.4638 | 4 | 1.8552 |  |  |  |  | 0.6602 |
| 0.20 | 0.3913 | 1 | 0.3913 | 2.8009 | 0.0467 | 0.0948 | 0.4458 | 0.6022 |
| 0.25 | 0.3309 | 4 | 1.3236 |  |  |  |  | 0.5500 |
| 0.30 | 0.2793 | 1 | 0.2793 | 1.9942 | 0.0332 | 0.0616 | 0.2897 | 0.5019 |
| 0.35 | 0.2346 | 4 | 0.9384 |  |  |  |  | 0.4567 |
| 0.40 | 0.1957 | 1 | 0.1957 | 1.4134 | 0.0236 | 0.0380 | 0.1787 | 0.4141 |
| 0.45 | 0.1616 | 4 | 0.6464 |  |  |  |  | 0.3733 |
| 0.50 | 0.1317 | 1 | 0.1317 | 0.9738 | 0.0162 | 0.0218 | 0.1025 | 0.3343 |
| 0.55 | 0.1056 | 4 | 0.4224 |  |  |  |  | 0.2967 |
| 0.60 | 0.0829 | 1 | 0.0829 | 0.6365 | 0.0106 | 0.0112 | 0.0527 | 0.2603 |
| 0.65 | 0.0633 | 4 | 0.2532 |  |  |  |  | 0.2250 |
| 0.70 | 0.0466 | 1 | 0.0466 | 0.3827 | 0.0064 | 0.0048 | 0.0226 | 0.1906 |
| 0.75 | 0.0326 | 4 | 0.1304 |  |  |  |  | 0.1572 |
| 0.80 | 0.0212 | 1 | 0.0212 | 0.1982 | 0.0033 | 0.0015 | 0.0071 | 0.1245 |
| 0.85 | 0.0122 | 4 | 0.0488 |  |  |  |  | 0.0924 |
| 0.90 | 0.0057 | 1 | 0.0057 | 0.0757 | 0.0013 | 0.0002 | 0.0009 | 0.0612 |
| 0.95 | 0.0015 | 4 | 0.0060 |  |  |  |  | 0.0303 |
| 1.00 | 0.0000 | 1 | 0.0000 | 0.0117 | 0.0002 | 0.0000 | 0.0000 | 0.0000 |
|  |  |  |  |  | (0.2127) |  |  |  |

Table C6: Numerical Solution of Equations 8, 9, and 10.
$B=1.0$

| x/L | f | Simpson's factor | Product | $y_{0}+4 y_{1}+y_{2}$ | da | $\frac{\mathrm{H}_{\mathrm{x}}}{\mathrm{H}_{\mathrm{f}}}$ | H | $Q_{x} / Q_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 1.0000 | 1 | 1.0000 |  |  | (0.1840) | 1.0000 | 1.0000 |
| 0.05 | 0.6261 | 4 | 2.5044 |  |  |  |  | 0.7764 |
| 0.10 | 0.4950 | 1 | 0.4950 | 3.9994 | 0.0667 | 0.1173 | 0.6375 | 0.6838 |
| 0.15 | 0.4040 | 4 | 1.6160 |  |  |  |  | 0.6127 |
| 0.20 | 0.3340 | 1 | 0.3340 | 2.4450 | 0.0408 | 0.0765 | 0.4158 | 0.5528 |
| 0.25 | 0.2774 | 4 | 1.1096 |  |  |  |  | 0.5000 |
| 0.30 | 0.2304 | 1 | 0.2304 | 1.6740 | 0.0279 | 0.0486 | 0.2641 | 0.4523 |
| 0.35 | 0.1908 | 4 | 0.7632 |  |  |  |  | 0.4084 |
| 0.40 | 0.1570 | 1 | 0.1570 | 1.1506 | 0.0192 | 0.0294 | 0.1598 | 0.3676 |
| 0.45 | 0.1280 | 4 | 0.5120 |  |  |  |  | 0.3292 |
| 0.50 | 0.1031 | 1 | 0.1031 | 0.7721 | 0.0129 | 0.0165 | 0.0897 | 0.2928 |
| 0.55 | 0.0818 | 4 | 0.3272 |  |  |  |  | 0.2584 |
| 0.60 | 0.0635 | 1 | 0.0635 | 0.4938 | 0.0082 | 0.0083 | 0.0451 | 0.2253 |
| 0.65 | 0.0480 | 4 | 0.1920 |  |  |  |  | 0.1938 |
| 0.70 | 0.0350 | 1 | 0.0350 | 0.2905 | 0.0048 | 0.0035 | 0.0190 | 0.1633 |
| 0.75 | 0.0243 | 4 | 0.0972 |  |  | 0.0035 | 0.0190 | 0.1340 |
| 0.80 | 0.0156 | 1 | 0.0156 | 0.1478 | 0.0025 | 0.0010 | 0.0054 | 0.1055 |
| 0.85 | 0.0089 | 4 | 0.0356 |  |  |  | 0.0054 | 0.0780 |
| 0.90 | 0.0041 | 1 | 0.0041 | 0.0553 | 0.0009 | 0.0001 | 0.0005 | 0.0512 |
| 0.95 | 0.0011 | 4 | 0.0044 |  |  |  | 0.0005 | 0.0253 |
| 1.00 | 0.0000 | 1 | 0.0000 | 0.0085 | $\begin{gathered} 0.0001 \\ (0.1840) \end{gathered}$ | 0.0000 | 0.0000 | 0.0000 |

