POLYTECHNIQUE MONTRÉAL
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POLYTECHNIQUE DE MILAN

Game-theoretic frameworks for the techno-economic aspects of infrastructure sharing in current and future mobile networks

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Cette thèse intitulée :

Game-theoretic frameworks for the techno-economic aspects of infrastructure sharing in current and future mobile networks

présentée par **Lorela CANO**
en vue de l'obtention du diplôme de *Philosophiæ Doctor*
a été dûment acceptée par le jury d'examen constitué de :

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DEDICATION

To my parents for all their love and support,
to my bratty but lovely little sister who has grown into my role model,
to my exceptional aunt Ana and to the memory of her beloved Ru.

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The word *grateful* is the first that comes to mind when writing this section, because I am grateful to so many people that have made my PhD experience unique.

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My PhD experience has been a "tale of two cities" (although completely different from the Dickens' novel). I moved to Milan in 2012 to pursue my master degree at Politecnico di Milano followed then by the PhD. Milan specifically and Italy in general have become my second home: Italian has almost become my second mother tongue and the Italian culture has somehow moulded my personality. In turn, the two years I have spent in Montréal have been so eventful and multi-colored, just like its seasons. I had worshipped many artistic bits of Montréal long before I knew I would go there as a huge fan of the Arcade Fire, Xavier Dolan, Cœur de pirate etc. However, this is incomparable to experiencing Montréal in person: the kindness and outdoorsy nature of its people, the sense of community, the melting-pot welcoming and respecting every single culture and the all year-long festival culture. While in Montréal I tried to experience every single local aspects of it. What I have enjoyed the most was studying French and the culture of Québec and being part of volleyball team of Côtes des Neiges, where I met people of very different ages and nationalities. I was also lucky to be part of the large Italian community of Montréal. In these lines, I would like to thank Filippo, Eleonora, Luca and Giulia with whom I have shared many memorable moments. In particular, I would like to thank my best friend Marta, which experienced this "tale of two cities" very similarly to me. I randomly met Marta in Montréal and suggested to watch the Italian movie "La pazza gioia", without knowing this to be one of her favorite movies. She has been a true friend with a big heart ever since. Luckily we were back to Milan at about the same time and she has always been there for me.

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RÉSUMÉ

Le phénomène de partage d'infrastructure dans les réseaux mobiles a prévalu au cours des deux dernières décennies. Il a pris de l'ampleur en particulier pendant les deux dernières migrations technologiques, à savoir de la 2G à la 3G et de la 3G à la 4G et il sera encore plus crucial à très court terme avec l'avènement de la 5G. En permettant aux Opérateurs de Réseaux Mobiles (ORM) de faire face à la demande croissante des utilisateurs et à la baisse des revenus. Il n'est pas rare non plus que le partage d'infrastructure s'accompagne du partage du spectre, une ressource essentielle et de plus en plus rare pour les réseaux mobiles. Dans ce milieu, la communauté des chercheurs, parmis d'autres, a étudié les multiples aspects techniques du partage d'infrastructure parfois associés au partage du spectre. Entre autres, ces aspects techniques comprennent l'évaluation des performances en termes de métriques de réseau, de gestion de ressources et d'habilitateurs et d'architectures adaptées. Les aspects économiques ont également été abordés, mais généralement en se concentrant étroitement sur l'estimation des économies de coûts des différentes alternatives de partage d'infrastructure.

Cependant, lorsqu'on considère le problème du partage d'infrastructure, et le cas échéant aussi du partage du spectre du point de vue d'un ORM, qui est une entité intéressée à maximiser le profit, il est important d'évaluer non seulement la réduction des coûts de cette infrastructure, et le cas échéant aussi le partage du spectre, mais aussi leur impact sur les performances du réseau et par conséquent sur les revenus de l'ORM. De ce point de vue, la viabilité du partage d'infrastructure ne doit pas être prise pour acquise ; afin d'étudier le problème stratégique d'un ORM concluant un accord de partage avec un ou plusieurs autres ORM, les aspects techniques et économiques doivent être pris en compte. Cette étude constitue le premier objectif de ce projet de recherche doctorale.

Plus précisément, nous avons considéré plusieurs variantes résultant de deux cas où chaque variante a été abordée par un modèle mathématique approprié. Ces variantes répondent à un scénario 4G commun dans lequel il existe un ensemble de ORM avec des parts de marché données qui coexistent dans une zone géographique urbaine dense ; chaque ORM doit décider s'il faut déployer une couche de petites cellules dans la zone et, le cas échéant, s'il doit le faire lui-même ou en concluant un accord de partage en créant un réseau partagé avec certains, ou la totalité, des autres ORM, auquel cas une coalition est créée. Une caractéristique commune importante de ces variantes est le modèle de tarification de l'utilisateur défini comme une fonction linéaire du taux moyen perçu par l'utilisateur en fonction de la coalition dont fait partie l'ORM de l'utilisateur. Un tel modèle de tarification permet de saisir l'impact du

partage d'infrastructure et, le cas échéant, le partage du spectre sur les revenus de l'ORM grâce à une mesure de performance du réseau. À leur tour, les deux résultats clés des modèles abordant ces variantes sont l'ensemble des coalitions qui se forment et le nombre de petites cellules qu'elles déploient.

Les deux cas que nous avons examinés sont : (i) le partage d'infrastructure sans mise en commun du spectre avec le partage des coûts d'infrastructure entre ses membres *a priori*, en fonction de leurs parts de marché relatives et (ii) le partage d'infrastructures avec la mise en commun des fréquences, en supposant que chaque ORM peut posséder une quantité donnée de spectre sous licence et que les membres d'une coalition sont tous prêts à partager leur spectre individuel. Contrairement au cas (i), la repartition des coûts est un résultat du modèle. Ceci permet de rendre compte due fait que les ORM peuvent avoir des mélanges très distincts de parts de marché et de spectre. Une règle de répartition des coûts *a priori* ne peut pas garantir la stabilité de la coalition.

En outre, nous considérons deux variantes pour le cas (i): une variante axée sur le marché, dans laquelle des coalitions sont créées volontairement par les ORM, chacune agissant comme une entité qui maximise son profit – modélisée comme un jeu d'Utilité Non Transférable (UNT) et une deuxième variante dans laquelle un régulateur décide des coalitions à créer de manière à maximiser un objectif global (taux d'utilisation total ou minimal) tout en garantissant un gain non négatif pour chaque ORM – modélisé comme des problèmes de programmation linéaire en nombres entiers mixtes. Deux variantes sont également envisagées pour le cas (ii): une première dans laquelle le problème est formulé comme un jeu coopératif d'UNT, en supposant que les membres d'une coalition sont uniquement disposés à partager le coût de l'infrastructure et à conserver chacun leurs revenus individuels et une deuxième, où les ORM sont également prêts à donner une partie de leurs revenus, formulé comme un jeu coopératif d'Utilité Transférable (UT).

Plusieurs cas ont été examinés en faisant varier les parts de marché des ORM et, pour le cas (ii), également leurs parts de spectre ; nous analysons également l'impact du prix de l'utilisateur par unité de taux sur le résultat. Les principales conclusions pour le cas (i) sont qu'il y a généralement plus d'incitation à coopérer dans la première variante par rapport à la seconde et que pour les deux variantes l'augmentation du prix unitaire de l'utilisateur diminue l'incitation à la coopération, ce qui indique que pour le partage d'infrastructure sans spectre il y a un compromis entre la réduction des coûts et la dégradation du niveau de service et lequel des deux prévaut dépend du prix unitaire de l'utilisateur. Concernant le cas (ii), on peut conclure que la grande coalition est stable dans presque tout les cas indépendamment du prix unitaire de l'utilisateur, ce qui signifie que le gain de mise en commun du spectre dépasse la dégradation du niveau de service du partage d'infrastructure sans mise en commun du spectre ; en outre, la règle de répartition stable des coûts tient compte de la part de marché et de la part de spectre caractérisant chaque ORM. Concernant l'originalité et la contribution de ces travaux, au mieux de nos connaissances, les variantes et leurs modèles respectifs étudiés dans le cadre du premier objectif de ce projet de recherche n'ont pas de précédents dans la littérature.

Pour les réseaux 5G, le découplage d'infrastructure des services dont le partage d'infrastructure devient un pilier important se traduit par l'émergence de nouveaux acteurs tels que les Fournisseurs d'Infrastructure (FI) et les Fournisseurs de Services (FS) où les FI sont responsables du déploiement et gestion de l'infrastructure tandis que les FS pour la fourniture de services aux utilisateurs via des ressources louées ou acquises auprès des FI. Le deuxième objectif de ce projet de recherche est d'étudier les interactions techno-économiques entre plusieurs FI et plusieurs FS qui n'ont pas encore été abordées dans la littérature et pour lesquelles nous proposons un nouveau cadre basé sur un Jeu de Multiple Leaders et Satellites (JMLS).

Plus précisément, nous considérons plusieurs FI avec des infrastructures de réseau individuelles mais qui se chevauchent sur une zone urbaine dense. Les FI sont également supposés fonctionner dans des bandes de spectre disjointes. En outre, dans ce domaine, il existe également un ensemble de FS qui fournissent chacun un type de service donné à un segment de marché donné. Les stations de base (SB) des FI sont supposées être colocalisées, c'est pourquoi nous nous concentrons sur la zone d'une seule cellule de SB où la capacité des cellules de SB de chaque FI dépend de sa technologie de réseau et de la bande passante du spectre disponible. Alors qu'un FI peut desservir plusieurs FS simultanément, dans notre cadre, nous supposons qu'un SP dessert ses utilisateurs finaux grâce à une capacité achetée auprès d'un seul FI. Tous les FI et FS sont des entités rationnelles qui maximisent leur profits individuels. Ansi, le prix unitaire de capacité offert par un InP maximise le profit du FI qui est donné par le produit du prix unitaire et la capacité totale vendue. À son tour, un FS, par la sélection du FI, maximise son profit donné par la différence entre les revenus obtenus de ses utilisateurs (qui sont fonction de la quantité de capacité achetée auprès du FI sélectionné) et le coût de la capacité achetée. Comme prévu, ce problème a été modélisé comme un JMLS à travers lequel nous avons étudié un grand nombre de cas réalistes où le modèle de coût du réseau est basé sur la littérature récente de la 5G et les services fournis par les FS sont caractérisés par des scénarios d'utilisation des Télécommunications Mobiles Internationales pour 2020 et au-delà. Les exemples sont choisis de façon que l'ensemble de FS est le même pour tous, tandis que nous faisons varier les caractéristiques des FI, c'est-à-dire la technologie de réseau et la bande passante du spectre disponible. À partir des résultats numériques obtenus, une conclusion clé est que les caractéristiques technologiques des FI influencent de manière significative la compétitivité et donc la configuration du marché qui en résulte.

ABSTRACT

The phenomenon of infrastructure sharing in mobile networks has been prevalent over the last two decades. It has gathered momentum especially during the last two technology migrations, i.e., from 2G to 3G and from 3G to 4G and it will be even more crucial with the advent of 5G. The key rationale behind such phenomenon is cost reduction as a means for Mobile Network Operators (MNOs) to deal with an increasing user demand but declining revenues. It is also not unusual for infrastructure sharing to go hand in hand with sharing of spectrum, an essential and increasingly scarce resource for mobile networks. In this milieu, the research community (but not only) has addressed multiple technical aspects of infrastructure sharing sometimes combined with spectrum sharing. Among others, such technical aspects include performance evaluation in terms of network metrics, resource management and enablers and adapted architectures. Economic aspects have been addressed as well, but usually with a narrow focus on estimating the cost savings of the different infrastructure sharing alternatives.

However, from the perspective of an MNO, which is a self-interested, profit-maximizing entity, it is important to assess not only the cost reduction that infrastructure sharing, and when applicable, also spectrum sharing bring about, but also their impact on the network performance and consequently on the MNO's revenues. From this perspective, the viability of infrastructure sharing should not be taken for granted; in order to study the strategic problem of an MNO entering a sharing agreement with one or multiple other MNOs, both technical and economic aspects should be taken into account – such study has been the first objective of this doctoral research project.

We have specifically considered multiple variants arising from two cases where each variant has been tackled by an appropriate mathematical model. These variants address a common 4G scenario in which there is a set of MNOs with given market shares that coexist in a given dense urban geographical area; each MNO has to decide whether to deploy a layer of small cells in the area and if so, whether to do that by itself or by entering a sharing agreement, i.e., building a shared network with a subset or all other MNOs (in which case a coalition is created). One key common feature of these variants is the user pricing model which is defined as a linear function of the average rate perceived by the user depending on the coalition joined by the user's MNO; such pricing model allows us to capture the impact that infrastructure sharing, and, when applicable, also spectrum sharing have on the MNO's revenues through a network performance metric. In turn, the two key outcomes of the models tackling these variants are the set of coalitions and the number of small cells they deploy.

The two cases we have considered are: (i) infrastructure sharing without spectrum pooling with the shared infrastructure cost split among its members *a priori*, according to their relative market shares and (ii) infrastructure sharing with spectrum pooling, assuming each MNO may own a given amount of licensed spectrum and that members of a coalition are all willing to pull together their individual spectrum, and conversely to case (i), how the coalition cost are split among its member MNOs is an outcome of the model – this is to account for the fact that MNOs may have very distinct mixtures of market and spectrum shares, hence an *a priori* cost division rule may not guarantee the coalition stability.

Further, we consider two variants for case (i): a market-driven variant, in which coalitions are created voluntarily by the MNOs, each acting as a profit-maximizing entity – modeled as a Non Transferable Utility (NTU) game and a second variant in which a regulator decides the coalitions to be created so as to maximize a global objective (total or minimum user rate) while guaranteeing a non-negative payoff for each MNO – modeled as Mixed Integer Linear Programming problems. Two variants are also considered for case (ii): a first one in which the problem is formulated as an NTU cooperative game, assuming members of a coalition are only willing to share the infrastructure cost and each keep its individual revenues and a second variant in which the MNOs are also willing to give away part of their revenues, thus formulated as a Transferable Utility (TU) cooperative game.

Several problem instances have been considered in which we vary the MNOs' market shares and for case (ii) also their spectrum shares; in addition we also analyze the impact of the user price per unit of rate on the outcome. The main findings for case (i) are that there is generally more incentive to cooperate in the first variant compared to the second and that for both variants the increase of the user unit price decreases the incentive for cooperation which indicates that for infrastructure sharing without spectrum pooling there is a trade-off between reduced cost and service level degradation and which of the two prevails depends on the user unit price. As for case (ii), it can be concluded that the grand coalition is stable for almost all the instances independently of the user unit price meaning that the spectrum pooling gain exceeds the service level degradation of infrastructure sharing without spectrum pooling; additionally, the stable cost division rule accounts for the market and spectrum share characterizing each MNO. Concerning the originality and the contribution of these works, to the best of our knowledge, the variants and their respective models studied under the first objective of this research project have no precedents in the literature.

As for 5G networks, the decoupling of infrastructure from services – for which infrastructure sharing becomes an important pillar – results in the emergence of new stakeholders such as Infrastructure Providers (InPs) and Service Providers (SPs) where the InPs are responsible for the deployment and management of the infrastructure whereas SPs for provisioning services to end users through resources rented/acquired from the InPs. The second objective of this research project is to study the techno-economic interactions among multiple InPs and multiple SPs that have not yet been addressed in the literature and for which we propose a novel framework based on a Multi-Leader-Follower Game (MLFG).

In detail, we consider multiple InPs with individual but overlapping network infrastructures over a dense urban area. InPs are also assumed to operate in disjoint spectrum bands. Further, in this area there is also a set of SPs which provide each a given type of service to a given market segment of end users. The InPs' base stations (BSs) are assumed to be collocated, hence we focus on the area of a single BS cell where the BS cell capacity of each InP depends on its network technology and available spectrum bandwidth. While an InP can serve multiple SPs simultaneously, in our framework we assume that an SP serves its end users through capacity purchased from a single InP. All InPs and SPs are assumed rational entities whose aim is to maximize their individual profits. In these lines, the capacity unit price offered by an InP maximizes the InP's profit which is given by the product of the unit price and the total sold capacity. In turn, an SP, through the InP selection, maximizes its profit given by the difference between the revenues obtained from its end users (which are a function of the amount of capacity purchased from the selected InP) and cost of the purchased capacity. As anticipated, this problem has been modeled as a MLFG through which we have investigated a large number of realistic problem instances which contributes significantly to the novelty of this work. What makes the instances realistic in the context of 5G is that the network cost model is based on recent 5G literature and the services provisioned by SPs are characterized through usage scenarios of International Mobile Telecommunications for 2020 and beyond. The instances are such that the set of SPs is the same across all of them while we vary the InPs' characteristics, i.e., network technology and available spectrum bandwidth. From the numerical results, one key finding is that the technological characteristics of the InPs significantly influence the competition among them and hence the resulting market setting.

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2G	2^{nd} Generation (wireless mobile telecommunications technology)
3G	3^{rd} Generation (wireless mobile telecommunications technology)
3GPP	3^{rd} Generation Partnership Project
4G	4^{th} Generation (wireless mobile telecommunications technology)
5G	5^{th} Generation (wireless mobile telecommunications technology)
5GPPP	5G Infrastructure Public Private Partnership
AMPL	A Mathematical Programming Language
ARPU	Average Revenue Per User
BS	Base Station
BSC	Base Station Controller
BTS	Base Transceiver Station
C-RAN	Cloud RAN
CA	Carrier Aggregation
CAPEX	CAPital EXpenditures
CN	Core Network
CPU	Central Processing Unit
DCS 1800	Digital Cellular System at 1800 MHz
DL	Downlink
DSA	Dynamic Spectrum Access
EAD	Ethernet Access Direct
EDGE	Enhanced Data rates for GSM Evolution
eMBB	enhanced Mobile Broadband
eNB	evolved Node B
EUR	Euro(s)
GBP	Great British Pound(s)
GERAN	GSM EDGE RAN
GSM	Global System for Mobile communications
GSM 900	Global System for Mobile communications at 900 MHz
GSMA	GSM Association
HetNets	Heterogeneous Networks
HSPA	High Speed Packet Access
IaaS	Infrastructure as a Service
IMT	International Mobile Telecommunications

InP	Infrastructure Provider
IoT	Internet Of Things
ISP	Infrastructure Sharing Problem
ITU-R	International Telecommunication Union Radiocommunication
JV	Joint Venture
KPI	Key Performance Indicator
LSA	Licensed Shared Access
LTE	Long-Term Evolution
LTE-A	LTE Advanced
LTE-U	LTE in Unlicensed bands
Mbps	Mega bit per second
MC	Macro Cell
MHz	Mega Hertz
MILP	Mixed Integer Linear Programming
MIMO	Multiple-Input Multiple-Output
MIP	Mixed Integer Programming
MLFG	Multi-Leader-Follower Game
mMTC	massive MTC
mmWave	millimeter Wave
MNO	Mobile Network Operator
MOCN	Multi-Operator Core Network
MTC	Machine Type Communications
MVNO	Mobile Virtual Network Operator
NaaS	Network as a Service
NE	Nash Equilibrium
NFV	Network Functions Virtualization
NP	Nondeterministic Polynomial time
NSP	Network Service Provider
NTU	Non Transferable Utility
O&M	Operations and Maintenance
OLMFSG	One-Leader Multiple-Follower Stackelberg Game
OPEX	OPerational EXpenditures
OTT	Over The Top
PRB	Physical Resource Block
QoS	Quality of Service
RAN	Radio Access Network

RAT	Radio Access Technology
RF	Radio Frequency
RNC	Radio Network Controller
RRH	Remote Radio Head
RSE	RAN Sharing Enhancements
SA1	System Architecture working group 1 (3GPP)
SaaS	Software as a Service
SC	Small Cell
SCP	Small Cell Provider
SDN	Software-defined Networking
SINR	Signal-to-Interference-plus-Noise Ratio
SLA	Service Level Agreement
SMS	Short Message Service
SP	Service Provider
SPE	Subgame Perfect Equilibrium
SPP	Set Partitioning Problem
TO	Telco Operator
TowerCo	Tower Company
TU	Transferable Utility
UL	Uplink
UMTS	Universal Mobile Telecommunications Systems
URLLC	Ultra Reliable Low Latency Communications
VNO	Virtual Network Operator
VO	Virtual Operator
W-CDMA	Wideband Code Division Multiple Access
WiMAX	Worldwide interoperability for Microwave Access
WLAN	Wireless Local Area Network
WMAN	Wireless Metropolitan Area Network
WNV	Wireless Network Virtualization

LIST OF APPENDICES

CHAPTER 1 INTRODUCTION

Up to now, the prevailing business model for a facility-based[1] Mobile Network Operator (MNO) entails (among others): (i) purchasing a spectrum license, (ii) deploying and managing the network infrastructure, (iii) tailoring services for the subscribers (such as voice, text and data etc.) and (iv) handling their billing and accounting. This business model intrinsically implies high upfront costs for the spectrum licenses acquisition and the infrastructure roll-out, but also operational costs to maintain the network during its lifetime. However, with the shift of the MNO core service from voice to data, such approach is failing to be profitable, especially due to the remarkable escalation of the mobile data traffic. To sustain such a demand, MNOs have to densify their networks, increase their spectrum holdings and migrate to new (more spectrally–efficient) technologies which altogether result in substantial capital investments while revenues from users are decreasing. When instead of deploying a dedicated network infrastructure, an MNO decides to share part/all of the infrastructure with other MNOs, it can be able to reduce its capital and operational cost, which in turn makes its business more profitable.

Further, as the mobile ecosystem moves towards the 5G, the network infrastructure and the services it provides are expected to decouple[2], essentially through the Network Functions Virtualization (NFV) and Software-defined Networking (SDN) paradigms. As a result, infrastructure sharing becomes a building block of the 5G framework whereas from a business point-of-view, 5G becomes an enabler to infrastructure sharing, as new stakeholders such as Infrastructure Providers (InPs) and Service Providers (SPs) will emerge.

1.1 Definitions and basic concepts

As a broad definition, infrastructure sharing is the shared use (of existing or jointly deployed) network infrastructure among multiple MNOs. Unsurprisingly, the common driver across the several infrastructure sharing instances accompanying the recent technology migrations is cost reduction – especially at roll-out. However, the degree by which infrastructure sharing contributes in lowering the capital and operational cost depends significantly on the type of

[1] "Facility-based" is a term used to identify a Mobile Network Operator (MNO) which deploys its own network infrastructure as opposed to a Mobile Virtual Network Operator (MVNO) which does not own neither network infrastructure nor a spectrum license but operates by leasing capacity from the former.

[2] It is worth noting that concept of decoupling the network infrastructure from its services has been suggested much earlier than the conception of 5G (see e.g., [2–4]). However, in 5G it is integral to how its architecture is being defined.

network sharing agreement (which network elements are shared, the geographical footprint, whether spectrum is shared etc.). Nevertheless, the degree of sharing is also subject to national and international regulation (which, at the limit, can inhibit such process) [5–8]; further, it is also affected by standardization [9–12] and the industry (vendors) [13, 14].

1.1.1 Evolution of mobile networks and infrastructure sharing

Infrastructure sharing has become instrumental since the roll-out of 3G networks[3], while an even larger number of sharing agreements have been recorded so far for 4G [15]. Moreover, infrastructure sharing is also considered an important enabler for 5G, given its performance targets and in particular the very dense network deployments [16] which will require at least site sharing due to limited site availability. Additionally, in the envisioned 5G architecture [17] there is innate support for infrastructure sharing; two important terms related to infrastructure sharing in 5G are multi-tenancy and network slicing [18]. Multi-tenancy means that multiple tenants (mobile broadband providers, industry verticals etc.) with very different service requirement will coexist in the same network [19]. In turn, network slicing is the means to support multi-tenancy, that is, a means to set up end-to-end logically independent networks from the common pool of network resources which can accommodate the specific requirements of each tenant. Several new stakeholders emerge in this context, e.g., InPs which deploy and manage resources and lease them to SPs[4], which in turn focus only on provisioning services for end users.

1.1.2 Infrastructure sharing alternatives

In the literature, but also among vendors, regulators, the 3rd Generation Partnership Project (3GPP) and other actors, there is not a (strictly) unique classification of the infrastructure sharing alternatives available to MNOs (compare e.g., the classifications provided in [2–4, 13, 15, 20, 21]). However, based on the *architectural scope* [15, 21] or *technical model* [4, 20], i.e., based on which network elements (nodes) MNOs agree to/can share, there are two main types of sharing: *passive* and *active*, the latter comprising the former.

[3] Notice however that nowadays commercial solutions are available also for 2G networks. For instance, Nokia offers a Multi-Operator Core Network (MOCN) solution for the Global System for Mobile Communications (GSM)/Enhanced Data rates for GSM Evolution (EDGE) [13]: in MOCN, MNOs share the Radio Access Network (RAN) and pool together their frequency carriers, that is, they also share their spectrum, while they keep dedicated Core Networks (CN)s. The goal of MOCN is to efficiently operate legacy 2G networks. MNOs are refarming 2G frequency carriers for 3G and 4G since the latter can exploit them more efficiently. Therefore, to maintain suitable levels of the capacity for 2G networks, Nokia's MOCN solution allows pooling spectrum of up to four MNOs.

[4] We have opted for the (more generic) term, i.e., SP instead of tenant since our focus is on techno-economic aspects and not on the implementation of network slicing within the 5G architecture.

Passive sharing (also referred to as site sharing or co-location [4]) implies the sharing of the site physical space and of the non-active elements on the site (such as shelter, cabinet, mast, power supplies, batteries, generators, air-conditioning alarms and access protection [4, 20]. Passive sharing became common around 2000 [4], nowadays it is usually encouraged and – in some countries – even mandated by regulators [6]; it has also given rise to the TowerCompanies (TowerCos) business[5].

Active sharing, on the other hand, extends to active elements of the RAN (such as antennas, Base Transceiver Stations (BTS)/Base Station Controller (BSC) for 2G, Node B/Radio Network Controller (RNC) for 3G and Evolved Node B (eNB) for 4G) and part of the core nodes[6].

A third sharing alternative – the geographical-split network sharing [9] – applies to the case when multiple MNOs with individual spectrum licenses agree to cover disjoint geographical areas of a country; by allowing users of one another to roam in their respective deployed networks, all involved MNOs can provide services nation-wide. Implementation-wise, there are two options: the first is based on national (domestic) roaming, which means that only one CN is connected to each RAN (the one of the MNO deploying the RAN) whereas the second option is based on RAN sharing, where either the (individual) core of each MNO is connected to the each shared RAN covering a specific area or when also part of the core network is common to all MNOs and the latter are connected to the several RANs (covering the entire geographical area) through the common core.

Apart from these alternatives, infrastructure sharing for mobile networks can also apply to the back-haul segment only. Moreover, spectrum sharing alone comes in plenty of different flavors. Conversely, infrastructure sharing among mobile networks and other Radio Access Technologies (RATs) has been studied as well. Further, the consolidated business model of MVNOs represents a particular type of infrastructure sharing: an MVNO is an operator which does not own either a spectrum license or a network infrastructure but leases network capacity from a facility-based MNO to provide services to its subscribers (some references concerning these additional alternatives are provided in Section 2.2.5).

[5]Ollen and Avery [22] discusses different alternatives for passive sharing varying from cash neutral agreements between operators to Joint Ventures (JVs) and TowerCos which pool together sites of several MNOs and lease them to the same MNOs (sale-and-leaseback) and to new entrants.

[6]CN elements related to user billing and accounting are not shared. These elements are individual also for Mobile Virtual Network Operators (MVNOs).

1.1.3 Drivers and barriers of infrastructure sharing

As anticipated, the main driver for infrastructure sharing is cost reduction. The following is a brief summary of the drivers identified by (some) authors in the literature over the last two decades: Village *et al.* in [2] (2002) identify the high roll-out cost and the "need for speed to market" as drivers for network sharing in 3G. The authors in [3] (2005) argue that high license cost and demanding coverage requirement associated with a spectrum license, have forced licensees to turn to network sharing (or at the limit return their licenses when unable to meet such requirements). According to a study carried out by the GSM Association (GSMA) in 2008 [6], infrastructure sharing in 2G/3G was mainly propelled by declining Average Revenue Per User (ARPU), the need for substantial investment for the 3G roll-out both by new entrants and incumbents [7] and the limited number of sites in urban areas (where network densification is particularly needed to deal with congestion). In [15] (2015), it is pointed out that infrastructure sharing is even more crucial nowadays, for the deployment of the Long-Term Evolution (LTE) technology compared to the deployment of 3G (back in 2000) since the amount of data traffic generated by an LTE user is two to three times larger compared to a 3G one, while the data unit price is decreasing. As drivers, the work in [15] lists the followings: spectrum being a scarce resource, the aim to reduce the digital divide (provide broadband services to rural areas as well), the price reduction due to increased competition from a large number of operators in the market, regulator caps for wholesale access prices and, most importantly, the continuous proliferation of the mobile data traffic.

Moreover, it is worth noting that although cost reduction is the main driver for infrastructure sharing (MNOs are, after all, profit-maximizers), there are also several societal and environmental benefits to it; among others, infrastructure sharing contributes in reducing: the digital divide – by making investment affordable in unattractive (rural/low density) areas and emerging markets, the infrastructure redundancy and consequently, the energy consumption and citizens' health and aesthetic concerns regarding electromagnetic emission/site locations but also the barriers for new entrants.

However, there are also barriers to infrastructure sharing. Frisanco *et al.* in [4] identify the MNOs' loss of control over their networks leading to the inability to differentiate themselves in the market. According to [2], a shared network provides the same coverage for all involved MNOs which poses a threat to competitiveness if the MNOs were to compete solely on coverage. Beckman *et al.* in [3] argue along similar lines as [2].

[7] A new entrant which obtained a 3G spectrum license had to provide national coverage within deadlines whereas incumbent MNOs had to deploy additional sites to those already deployed for 2G; although MNOs with 2G infrastructure can generally reuse 2G sites for 3G, a larger number of sites are nevertheless needed to provide coverage for 3G since it operates at higher frequencies than 2G.

1.1.4 Cost savings of infrastructure sharing

The cost saving of the different infrastructure sharing alternatives have been estimated by several actors [2, 4, 6, 15, 20, 21]. For instance, the work in [4] applies a spreadsheet-based financial model to calculate the cost savings of several sharing options ranging from passive to full sharing. In their estimates, Meddour et al. [20] also account for the fact that the Capital Expenditures (CAPEX) and the Operational Expenditures (OPEX) incurred from the different network parts, e.g., the site, the RAN and the core, vary on the type of area under consideration, i.e., whether rural, sub-urban or urban. All estimates confirm that sharing agreements involving more network elements provide larger savings; for instance, in [15], a report of the consultancy firm Coleago, it is revealed that the overall savings can vary in the range 10%-40%, depending of the scope of sharing. Estimates of Vodafone [21] show that in addition to the sharing option, the amount of savings also depends on the number of involved MNOs: larger savings are reported for a network shared by three MNOs as opposed to two.

1.2 Scope and motivation

Infrastructure sharing for mobile (cellular) networks is a multifaceted subject which interests several entities such as MNOs, regulators, standardization bodies, the industry etc. However, this research project will deal with the MNO perspective and it will first tackle problems arising when self-interested, profit-maximizing, conventional (i.e., facility-based) MNOs voluntarily enter long-term infrastructure (and possibly also spectrum) sharing agreements as a means to reduce their CAPEX and OPEX. Then, this research project will explore new stakeholders that are emerging in the context of 5G, such as InPs and SPs and it will address problems arising from their techno-economic interactions.

Notice that the spectrum sharing research field is closely related to infrastructure sharing. Spectrum for mobile networks is inherently a scarce resource, and, nowadays more than ever, vital for MNOs to support the current traffic volume and the envisioned throughput for 5G. In this context, several spectrum sharing frameworks have stemmed, e.g., Dynamic Spectrum Access (DSA), Licensed Shared Access (LSA), cognitive radio and spectrum sharing in the context of LTE in Unlicensed bands (LTE-U). However, this research project addresses spectrum sharing mainly as an additional dimension to infrastructure sharing, with the purpose of evaluating its implications in the incentive for infrastructure sharing; in other words, we do not deal with particular problems, such as, interference management or resource allocation, arising in the context of specific spectrum sharing frameworks.

As discussed in details in Section 2, most of the literature on infrastructure sharing concerning conventional MNOs deals with technical aspects, such as enablers and architectural enhancements, resource management algorithms, performance evaluation in terms of network-related metrics, etc. As far as economic aspects are concerned, the profitability of infrastructure sharing has been usually defined in terms of cost savings estimates, merely accounting for the investment cost to provide coverage under different sharing options. Few works that adopt mathematical approaches (mainly game theory) have modeled the strategic problem of self-interested conventional MNOs deciding to share the network infrastructure. However, also in these works, the impact of infrastructure sharing in the MNO payoffs is perceived only through the cost. In these lines, the viability of infrastructure sharing becomes almost always trivial and the profitability defined in this way does not address the impact that sharing has on network performance metrics, which in turn impact MNO revenues and consequently their overall profit. In this context, there is a need to address both technical and economic aspects when dealing with the strategic problem of self-interested MNOs entering sharing agreements: this is the first objective of this research project. In turn, in the 5G literature, the main focus is on the architecture and implementation of network slicing. There are also works that address the economic viability of 5G networks through techno-economic approaches but, up to date, only from the point of view of a single MNO. However, the presence of new stakeholders in 5G (such as InPs and SPs) sets the ground for a new and competitive mobile market setting that needs to be studied with appropriate models, which is the second objective of this research project.

1.3 Research project objectives

As anticipated, this research project has two broad objectives: the first concerning *infrastructure sharing among conventional MNOs* and the second concerning *infrastructure sharing for decoupled infrastructure from services in the context of 5G.*

1.3.1 Infrastructure sharing among conventional MNOs

The first objective of this research project is to propose techno-economic frameworks to evaluate the viability and profitability of infrastructure sharing among self-interested conventional MNOs under different technical, economic and regulatory settings. Methodology–wise, we resort to mathematical programming and game theory to model scenarios arising when varying technical and economic settings both when sharing is assisted (constrained) by a regulator and when the latter does not intervene.

Specifically, in this first type of problems, we consider the current business model of conventional MNOs, namely, MNOs with individual market shares and spectrum licenses which decide whether to enter greenfield infrastructure sharing agreements with other MNOs, if profitable to do so. In this context, depending on whether spectrum sharing is allowed or possible (from a technical point of view) two cases can arise:

- infrastructure sharing without spectrum pooling and
- infrastructure sharing with spectrum pooling.

1.3.2 Infrastructure sharing for decoupled infrastructure from services in the context of 5G

The second research objective is to study the techno-economic interactions among InPs and SPs in the context of 5G. Specifically, we will consider scenarios involving multiple self-interested InPs and multiple self-interested SPs where the former own network resources (infrastructure and spectrum) but do not provision services for end users whereas the latter provision services for end users through resources rented/acquired from the former. Our aim is to propose a framework based on game theory that addresses the problems of resource pricing from the InPs' point of view and of InP selection and resource demand from the SPs' point of view for several cases which differ from the technological characteristics of the InPs (and the resulting network cost) while the SPs provision 5G services for end users.

1.4 Document outline

This document is organized as follows. In Chapter 2 we review the literature related to our research project. Then, in Chapter 3 we explain how the objectives of the research project have been realized through three journal articles ([23], [24] and [25]). Each of these journal articles has been included in this document as a standalone chapter, i.e., in Chapters 4, 5 and 6, respectively. A general discussion follows in Chapter 7. Conclusions, limitations of our work and some directions for future research are presented in Chapter 8. Finally, Appendices A and B belong to the article presented in Chapter 4 ([23]) whereas Appendices C, D and E, to the article presented in Chapter 6 ([25]).

CHAPTER 2 LITERATURE REVIEW

Infrastructure sharing has been a very prolific research topic over the last three decades. In this literature review we first make a broad chronological classification due to the change in the nature of problems studied over time. We start by providing an overview of the early works on the topic and then we focus on the more recent and up-to-date works. For the latter, we further identify several categories.

2.1 Early works

The works in [2–4, 26–29] are among the earliest on infrastructure sharing (combined at times also with spectrum sharing). With the exception of [28][1], these works have tended to address technical issues of different sharing alternatives, assess the financial profitability through techno-economic approaches, state regulatory standpoints and provide guidelines for the latter and conceive new paradigms for the mobile market.

In [26], which dates back to 1994, Ramsdale states that national roaming[2] is part of the specifications of the Digital Cellular System at 1800 MHz (DCS 1800), unlike the Global System for Mobile Communications at 900 MHz (GSM 900), which supported international roaming only. National roaming was introduced in the DCS 1800 to improve coverage due to smaller cell sizes at 1800 MHz (as opposed to 900 MHz).

Instead, the work in [27] shows the positive impact of infrastructure sharing in financial terms for the Universal Mobile Telecommunications Systems (UMTS), especially for lowly populated areas in which network deployment is dictated by coverage instead of capacity. In turn, MVNOs are suggested as a means to monetize spare resources of an MNO.

Park *et al.* in [29] discuss issues faced by MNOs worldwide when deploying Wideband Code Division Multiple Access (W-CDMA) and propose spectrum trading and infrastructure sharing as means to accelerate the deployment of W-CDMA. However, they emphasize that such means should be cautiously treated by regulators.

The study in [4] proposes a spreadsheet-based financial model to estimate the economic

[1]The study in [28] is an early work on the problem of scheduling users of multiple operators arising from the case when a 3G, facility-based MNO hosts several MVNOs: the authors propose a non-preemptive priority queuing model for circuit-switched traffic applied through an admission control scheme.

[2]National roaming is an infrastructure sharing alternative that allows users of an operator which does not provide coverage in certain areas of a country to be served by the network of another operator of that country covering such areas.

profitability of multiple sharing alternatives and shows that cost can be further reduced if the network operations are outsourced or a joint venture is created.

The authors in [2] discuss technical aspects concerning the infrastructure sharing alternatives at the time; they also anticipate two crucial paradigms: (i) dynamic spectrum trading and (ii) the decoupling of the network infrastructure from services, enabled by infrastructure sharing. It is worth noticing that both these paradigms are ongoing research topics even nowadays. Similarly, according to [3], the advantages of network sharing go beyond cost reduction: based on the product life cycle model, the authors suggest that, under an appropriate regulatory framework, network sharing can steer the monolithic mobile networks industry toward the decoupling of the network infrastructure from services for end users. In other words, based on [2] and [3] infrastructure sharing would lead to new stakeholders such as network/infrastructure providers and service providers which were expected to emerge in the mobile market, the former being responsible for network planning, deployment and management while the latter for dealing only with the development of novel services (possibly specialized and targeting specific market segments [3]).

2.2 More recent and up-to-date works

In this section, in addition to works on infrastructure sharing among conventional MNOs we will also address works on Wireless Network Virtualization (WNV) [30] and network slicing (enabling multi-tenancy) in the context of 5G [31], both based on infrastructure and spectrum sharing. Conversely, WNV and network slicing can be seen as enablers for infrastructure and spectrum sharing. Specifically, from these two research areas, we review works concerning the InP business model given that our second research objective addresses infrastructure sharing for decoupled infrastructure from services. It is interesting to bring up the fact that the decoupling of infrastructure from services was envisioned by some of the early works on infrastructure sharing, namely, [2] and [3]. The concept has been further carried out in the context of WNV and then in the context of network slicing. In fact, the different research efforts on introducing SDN, virtualization in general and NFV in particular into mobile networks seem to have converged into the 5G architecture as enablers for network slicing. In these lines, Samdanis *et al.* in [32] provide a compelling analysis of the path from infrastructure sharing to multi-tenancy.

For the more recent literature, there is a tendency to address *specific problems*, e.g., the problem of resource management, for *specific sharing scenarios*, e.g., infrastructure and spectrum sharing at the RAN. There are at least two ways to go about the classification of this literature, one being problem-centric and the other being methodology-centric. We have opted for the first one in order to highlight the fact that there are many aspects to infrastructure sharing and hence provide the reader with the bigger picture on the topic. Methodology details are discussed only for works that are the most similar to ours. We have identified the following categories/branches for the revised works under the problem-centric classification:

- *performance evaluation*,
- *resource management*,
- *enablers* and *architectures*,
- *energy efficiency* and
- *strategic modeling.*

It is worth pointing out that some of the works may fit in more than one category, but for each such work, we have opted for a single category, the one we believe is the most salient.

Our work fits in the last branch (*strategic modeling*) on which we will dwell on more thoroughly in Section 2.2.6, especially for works that are very similar to ours both in context and methodology. For the latter in particular, we will provide a detailed comparison in order to highlight the contribution of our work in the field. As for the rest of the branches – discussed in Sections 2.2.1–2.2.4 – we will only point to a non-exhaustive related literature for the interested reader and provide some illustrative examples. To complete the picture of infrastructure sharing, in Section 2.2.5 we also mention a few publications concerning sharing scenarios that are different from the ones addressed in this research project.

2.2.1 Performance evaluation

Several authors have addressed the gains of particular infrastructure and/or spectrum sharing scenarios in terms of *network performance metrics*, such as throughput, coverage probability etc. (see e.g., [33–36]) and/or *economic* ones such as CAPEX/OPEX reduction (see e.g., [37–40]). The common approach is to benchmark such scenarios against the baseline case when no sharing takes place and the involved MNOs build individual networks instead. Methodology-wise, both theoretical, mainly stochastic geometry analysis (see e.g., [34, 36, 41, 42]), and simulation approaches (see e.g., [35, 43, 44]) have been adopted. For instance, the work in [35] proposes a virtualized architecture to enable two types of spectrum sharing other than the classical one and capacity sharing (national roaming) and compares the different sharing alternatives with no sharing case. The performance metrics considered in [35] are the sector load and packet drop probability. The authors in [43] analyse how the time and space correlation of the MNO individual traffic loads impacts the gains of infrastructure sharing in the case when MNOs decide to pool together their respective networks. Kibilda *et al.* [41] resort to stochastic geometry to calculate the gains of sharing for the cases of infrastructure and/or spectrum pooling. Their key finding is that the infrastructure and spectrum sharing gains do not sum up when combined since full sharing (infrastructure+spectrum) introduces a tradeoff between data rate and coverage. As 5G is expected to make use of the millimeter wave (mmWave) frequencies [16], the gains of infrastructure and/or spectrum in these frequencies have become the object of several recent works. For instance, Gupta *et al.* in [42] provide a stochastic geometry-based theoretical analysis on the gains of spectrum sharing using a simplified antenna and channel model for the mmWave frequency range. In particular, in [42] it is shown how narrow beams are key for spectrum sharing in the mmWaves. A very similar investigation to [41] is carried by Rebato *et al.* in [44] for mmWaves; the authors highlight the impact of the channel model accuracy when carrying out a quantitative analysis of the sharing gains. The recent work in [36] also addresses infrastructure and spectrum sharing at mmWaves and it resorts to stochastic geometry to derive the probability of Signal-to-Interference-plus-Noise Ratio (SINR) coverage as a performance metric.

2.2.2 Resource management

Problems of resource management arise whenever infrastructure sharing is combined with spectrum sharing, as users of multiple MNOs have to be assigned resources from a shared pool.

Several studies ([45–48]) have proposed algorithms for a multi-operator scheduler, namely when users of multiple MNOs have to be scheduled in the finite resources available in a shared BS. Assuming MNOs agree *a priori* on the resource shares, i.e., how to split the available BS resources among them, the work in [45] adopts the concept of Generalized Processor Sharing for a multi-operator scheduler. For the same setting, Malanchini *et al.* [46] explore the tradeoff between satisfying the resource shares and improving the overall (system) spectral efficiency when the agreed resource shares are violated in a controlled fashion. The work in [47] considers a global scheduler taking decisions for clusters of BSs and therefore scheduling users of multiple MNOs over a 3D time-frequency-space resource grid. In [47] scheduling is performed with the objective of maximizing the overall system utility. The authors in [48] propose a BS virtualization scheme which performs scheduling in two levels, namely, among MNOs, and for each MNO, among its user flows. Hew *et al.* in [49] consider a network shared by multiple MNOs, each of them serving both a set of end users and a set of MVNOs. In this context, the problem of resource allocation is tackled in two steps: first, the resource sharing among MNOs, and then the resource sharing among the users and the MVNOs of each MNO, where the resource sharing at each step is modeled as a bargaining problem. The study in [50] suggests an algorithm that fairly allocates the shared radio resources among MNOs. In [51] the authors propose Remote Radio Head (RRH) assignment algorithms for a SDN-based Cloud Radio Access Network (C-RAN) shared by multiple MNOs.

Concerning WNV, the problem of resource management is crucial in the interaction among an InP and its SPs. We recall that in the context of our research project, an InP is an entity which is responsible for the infrastructure deployment, management and operation and does not serve end users directly whereas an SP is an entity which does not have any resources of its own but purchases or rents resources from an InP to provision services for its end users. It is worth noticing that the terminology concerning the SP varies across different works: such entity is also referred to as a Virtual Operator (VO), Virtual Network Operator (VNO) or MVNO. Also notice that the conventional MVNO obtains resources from an MNO which serves end users of its own, unlike the InP. The key difference lies in the fact that a conventional MVNO competes with its MNO, while there is no such competition between and InP and its SPs/VOs/VNOs/MVNOs. In these lines, some works tend to "misuse" the term InP when they consider the InP to provide services also to end users. Additionally an InP is also referred to as a Network Service Provider (NSP). Moreover, the work in [30] envisions three different types of stakeholders in line with the ones in the cloud computing domain, i.e., the InP providing Infrastructure as a Service (IaaS), the MVNO providing Network as a Service (NaaS) and the SP providing Software as a Service (SaaS). For instance, in [52] the authors address a scenario in which there are multiple InPs, a single MVNO and multiple SPs where the MVNO acts as a reseller of resources from InPs to SPs. However, we will stick to our two-types-of-stakeholders business model, i.e., InPs and SPs which is common among the vast majority of works related to WNV addressed in this research project. Finally, it should be noted that the terms *slicing* and *slice* are also used in some works in non-5G contexts, in the sense that, such works do not consider problem instances that account for 5G service requirements.

There is a large body of literature on resource managements concerning InPs and SPs in the context of WNV. The vast majority considers a single InP and multiple SPs (see e.g., [53–77]). However, there are exceptions: e.g., the work in [78] considers a single InP and a single VNO which serves multiple users through an SDN-based virtualized network provided by the InP. The VNO faces the problem of scheduling its users, each characterized by a maximum delay over a finite time period, through resources rented by the InP with the objective of minimizing the payments made to the InP for the rented resources. There are also works which consider both multiple InPs and multiple SPs (and few other variations with multiple InPs) which will be discussed in Section 2.2.6 as they are generally similar to our work in [25]. As for the literature on a single InP and multiple SPs, it can be broadly classified into two groups based on whether the resource management is driven by pricing ([66–77]) or not ([53–65]). For instance, Ho *et al.* in [68] consider the case when there is a single InP serving multiple MVNOs, each characterized by a fixed number of users and a Service Level Agreement (SLA) given in terms of a minimum resource requirement and a maximum aggregate rate (over all its users). The InP has to decide how to price and allocate its available BS resources among all users of all MVNOs so as to maximize its profit while guaranteeing the SLA of each MVNO. In this work MVNOs are also self-interested as the goal of each MVNO is to maximize its own profit given by the difference between the total rate obtained from resources allocated by the InP and their cost. The problem is then modeled as a one-leader multi-follower variant of the Stackelberg game with the InP being the leader and each MVNO being a follower. Instead, Kamel *et al.* in [63] address a scheduling problem over one time frame which is modeled through mathematical programming. In details, there is a single InP and a set of VOs, each having a fixed number of users and a minimum resource requirement (total Physical Resource Blocks (PRBs) over the time frame). The InP has to decide to which user to assign each PRB and the amount of power to allocate to each PRB so as to maximize the total rate over the time frame while satisfying the maximum power constraint, the minimum resource requirement of each VO and a VO-specific proportional fairness constraint for cell-center and cell-edge users.

In 5G, the problem of resource management reemerges in the context of multi-tenancy and its enabler, network slicing ([17,31]). Tenants (such as MVNOs, Over The Top (OTT) providers and vertical industries) have distinct requirements to support their services which have to be translated into appropriate network resources. It is worth noting that network slicing does not involve only the RAN segment but it can be end-to-end. However, the problem of resource management at the RAN segment has brought about a significant amount of attention from the research community due to the intrinsically complex nature of the radio (wireless) access. For instance, the authors in [32] propose the "5G Network Slice Broker", a centralized scheduler based on the 3GPP specifications for network sharing. The proposed scheduler has a global view of the shared network and applies admission control and resource allocation, translating the tenants' request, with given SLAs, into available network resources. Other examples on resource management at the RAN in the context of multi-tenancy/network slicing are given in: [79–85].

2.2.3 Enablers and architectures

Although the different alternatives for infrastructure and spectrum sharing can be financially attractive for MNOs, they where not always supported by the 3GPP specifications; in fact, while a basic type of network sharing was supported as of Release 5, there was no support for more involved network sharing scenarios for the 3GPP GSM EDGE RAN (GERAN) prior to Release 10 ([10]).

Standardization apart, the research community has largely contributed on the topics of enabling network sharing, e.g., through novel architectures. While passive sharing (i.e., site/tower sharing) is the simplest network sharing alternative to implement, the different types of active sharing demand architectural changes in mobile networks e.g., to guarantee the isolation of the involved MNOs in terms of their private information in order to avoid harming competition, or they demand changes at protocol stack level to implement the novel resource management algorithms etc. According to [86], radio resource management should be delegated to a third party provider to ensure isolation and therefore not to interfere with competition. In [87] the authors introduce AppRAN which relies on a centralized scheduler to perform application-level resource allocation for a shared RAN. In particular, different flavors of virtualization have been widely considered by the research community as candidate enablers for network sharing. For instance, the virtualized network architecture proposed in [88] can support network sharing.

Other works that resort to virtualization are e.g., [48, 89–92]. In particular, the authors of [93] and of [94] propose the “Network without Borders”, namely the virtualized pool of (heterogeneous) wireless resources for which infrastructure and spectrum pooling are essential.

Costanzo *et al.* in [95] suggest an architecture for 4G RAN sharing based on SDN and NFV.

In the context of enabling network slicing in 5G networks, there is a myriad of works that propose architectures or test prototypes based on (i) NFV and/or SDN (see e.g, [96–101]), (ii) changes to the RAN protocol stack (see e.g., [102–104]), or (iii) using features of the new 5G radio ([84]) etc. In particular, the work in [105] proposes an architecture to support network slicing in ultra-dense networks, the one in [106] presents an architecture that supports Internet Of Things (IoT) slices whereas the one in [107] dwells on combining 3GPP specifications for 5G with NFV.

2.2.4 Energy efficiency

Infrastructure and spectrum sharing allow to reduce the energy-consumption OPEX cost particularly in cases when the aggregated network resources (infrastructure and/or spectrum) are redundant. For instance, in rural areas where capacity is not an issue, MNOs can decommission a subset of the aggregated BSs and/or operate at a subset of the aggregated frequency carriers [4], which reduces the energy consumption and (indirectly) the environmental impact. In these lines, since MNOs dimension their networks based on the peak-load traffic predictions, there is intrinsically resource redundancy during the off-peak periods in their individual networks. Consequently, MNOs can agree to roam each other users during the off-peak periods, e.g., overnight, and switch off a subset of their BSs (see e.g., [108, 109]). While the vast majority of infrastructure (and spectrum) sharing problems revolve around economic and technical aspects, some works (see e.g., [108–123]) have taken an energy-efficiency/green networking perspective.

2.2.5 Miscellaneous

Infrastructure sharing for mobile network segments other than the access

Infrastructure sharing and multi-tenancy can also be applied to specific segments of a mobile network other than the access. For instance, the studies in [124–128] address sharing of the back-haul network whereas the one in [129] deals with the sharing of the core network.

Infrastructure sharing among different types of networks

In the following paragraph we provide some examples of heterogeneous infrastructure sharing. The work in [130] studies sharing among different RATs, the one in [131] addresses sharing between LTE femtocells and Wi–Fi hotspots whereas the one in [132] investigates 3G offloading over Wi–Fi. Kibilda *et al.* [133] deal with sharing among MNOs and OTTs. In [134] the authors propose a RAN architecture for both infrastructure and spectrum sharing between the MNOs and safety services. Instead the study in [135] concerns infrastructure sharing between mobile services and smart grid utilities or intelligent transportation services. Lin *et al.* in [136] address back-haul sharing among mobile networks and fixed networks whereas Simo-Reigadas *et al.* in [137] suggest exploiting the community infrastructure as back-haul for 3G.

Infrastructure sharing for networks other than mobile

The concept of infrastructure sharing is not exclusive to mobile networks. In fact, it has been applied to fixed access networks and problems related to the latter have been recently addressed in the literature (see e.g., [138] and [139]). Apart from fixed access networks, infrastructure sharing has also been proposed for Wi–Fi networks, e.g., in [140].

Spectrum sharing

The overall literature on the different types of spectrum sharing alone (i.e., not combined with infrastructure sharing) is *per se* very vast. Unsurprisingly, as spectrum is a scarce resource for the MNOs, many works within this literature resort to different game theory models (see e.g., [141–144]).

MVNO business model

The relation among the MNO, its MVNO(s) and the end users has been largely addressed through game theory as well (see e.g., [49, 145–148]).

2.2.6 Strategic modeling

This branch consists of works that deal with decision-making problems such as MNOs deciding whether to enter a sharing agreement or not, SPs selecting InPs from which to obtain resources etc. Such works naturally resort to mathematical programming and to game theory in particular when the involved actors are assumed rational, self-interested and payoff-maximizing entities.

Infrastructure sharing among conventional MNOs

The following works concern either greenfield deployment of shared networks [149–152] or the case when shared networks are created by pooling together the existing network infrastructure of at least two MNOs [144, 153–155].

The works in [149–151] address the problem of infrastructure and spectrum sharing arising when a set of MNOs, each with a given number of users (market share) and own spectrum license, plan a greenfield LTE deployment. The strategic problem of coalition formation, namely, which subsets of MNOs voluntarily sign long-term infrastructure and spectrum sharing agreements, is modeled by means of non-cooperative game theory.

Blogowski *et al.* in [152] deal with the particular scenario when two MNOs have to deploy BSs over a given set of candidate sites. For each site, each MNO has to decide whether to install a BS or not; in the former case, if both MNOs decide to install a BSs, it is assumed that is is profitable for both to install a single shared BS. The problem is formulated as a non-cooperative game where the payoff of each player (MNO) is given by its total profit (revenues - cost), calculated over all BSs. It is assumed that each site can serve a given (arbitrary) number of users, e.g., those under its coverage area, which means there are no capacity constraints associated with the sites. Instead, coverage constraints are present and they are expressed as a minimum percentage of users to be served by each MNO (a common constraint associated for spectrum licensees). When the coverage constraint is absent, MNOs can decide independently for each site. Otherwise, the game is no longer separable. The authors describe the propriety of the Nash equilibria of the game for different relationships of the payoff matrix (i.e., by establishing relations between the payoffs obtained under different strategy profiles) and also suggest a centralized solution which Pareto dominates all Nash Equilibria.

The authors in [153] consider the case when a set of MNOs agrees to pool together their current individual RAN networks but make joint decisions for future decommissions, network expansion and upgrades of their shared network; a greedy procedure is proposed to solve the multi-period network planning.

Similarly to the “sale-leaseback” approach of TowerCos (see e.g., [22]), the work in [154] assumes a set of self-interested MNOs decide to pool together their respective network infrastructures and create a JV, responsible for managing their shared network. In turn, MNOs will leaseback network capacity from the JV. The authors propose a Stackelberg game to determine the shares MNOs obtain from the JV and the prices set by the JV to the MNOs and by the MNOs to their respective users.

Notably, the user perspective is considered in [144], which investigates the problem of user-to-BS association when multiple MNOs decide to pool together their respective network infrastructures. The authors propose a non-cooperative game to model the problem of each user selecting its serving BS from the shared pool, independently, so that its individual data rate is maximized.

The work in [155] represents a fresh take on infrastructure sharing. Its authors consider a set of MNOs with individual but overlapping infrastructures (BSs) and individual spectrum licenses; in this setting one of the MNOs (the buyer) can purchase the use of BSs of the other MNOs (the sellers) for serving its own users at its own licensed spectrum. The buyer MNO evaluates whether it can provide a given QoS to its own users through its own infrastructure by increasing the transmission power of its BSs or by purchasing BSs from the seller MNOs. In the latter case, the buyer MNO has to decide from which seller MNOs to buy from and what fraction of their BSs to purchase so as to minimize its expenditures while satisfying the QoS of its users. In turn, the seller MNOs have to decide the fraction of their own BSs to sell so as to maximize their profit (payment from the buyer MNO minus cost of sold BSs) where the competition in quantity among the seller MNOs is modeled as a Cournot market.

From the works on strategic modeling of infrastructure sharing among conventional MNOs, only the works in [149–151] have significant common features with one of our papers, namely, [24]. Indeed, similarly to [24], the works in [149–151] address the coalition formation problem among MNOs in a greenfield deployment scenario where both infrastructure and spectrum can be shared and each MNO is characterized by a given number of users and amount of individual spectrum bandwidth. However, there are some essential differences:

- a non-cooperative game is proposed in [149–151] as opposed to the cooperative games we propose in [24],
- the players' (MNOs') payoff functions are defined only in terms of network costs in [149–151] while in [24] the payoff of an MNO is given in terms of its profit, i.e., the difference between its revenues and network cost, both affected by the coalition the MNO becomes part of; such difference is crucial since in practice MNOs are more likely to be driven by gains in profits in such strategic decision making than by cost reduction alone, and
- coalition cost are split *a priori* in [149–151] – uniformly among coalition members in [149, 151] and based on the Shapley value in [150] – whereas in [24] the cost division among coalition members is an outcome of the game, which is an important aspect, as from our numerical analysis we conclude that an intuitive *a priori* cost division based on the market shares does not always guarantee the coalition stability.

Infrastructure sharing for decoupled infrastructure from services

We remind the reader that we have discussed the varying terminology used across different works related to the InP business model in Section 2.2.2 and that we have maintained the authors' terminology for the considered stakeholders when describing their work and, when necessary, we provide clarifications on how they compare to our terminology. In the following, we will focus on works that tend to exhibit more affinities with our work in [25]. It is worth pointing out that, across the different works very distinct mathematical approaches have been used to study the interaction among InPs and SPs.

Rather exceptionally, the study in [156] tackles the interaction among InPs and MVNOs (analogous to SPs in our model) from the MVNO perspective. In fact, the authors in [156] consider multiple InPs but a single MVNO and propose a model based on contract theory in which the MVNO acts as the employer whereas the InPs as employees.

Instead, Wei *et al.* in [157] take a centralized approach. Specifically, the work in [157] considers multiple InPs and multiple VNOs (analogous to SPs in our model) in the context of WNV. Here, each InP has a given set of users of its own; resources allocated to its own users are referred to as local slices and the total rate across the local slices should be above a given minimum for each InP. Instead, resources allocated to users of an MVNO are referred to as a foreign slices. Each InP is characterized by a given bandwidth (number of subchannels) and power budget for the downlink of a BS. The problem of determining the number of subchannels and amount of power to allocate to each slice by each InP with the objective of maximizing the total rate across all slices while satisfying the bandwidth and power constraints and the minimum rate requirement for the local slices of each InP is formulated by means of an Integer Programming (IP) model. In this model an MVNO can be simultaneously served by multiple InPs, likewise an InP can simultaneously serve multiple MVNOs.

The authors in [158] propose a hierarchical (two layer) combinatorial auction to model the interactions among multiple InPs, multiple MVNOs (analogous to SPs in our model), and multiple end users concerning the resource allocation at BS level (the resources here being transmission power, number of channels and number of antennas).

Maille *et al* in [159] propose an interesting three-stage game concerning the investment of wireless telecommunications providers in different technologies. In essence, at stage 1 the providers decide a subset of technologies to invest on, from 3G, Wi-Fi and Worldwide interoperability for Microwave Access (WiMAX), at stage 2, given their choices at stage 1, the providers then decide the price to offer to the end users, whereas at stage 3, end users select the provider-technology tuple providing the best price *vs.* Quality of Service (QoS) tradeoff.

The work in [160] also models competition among multiple wireless service providers, where each provider has deployed a single and distinct wireless access technology, owns a given amount of spectrum bandwidth and competes with the other providers over end users in bandwidth unit prices. Among the different formulations given by the authors, there is a one-leader, two-follower Stackelberg game in which one of the providers moves first, i.e., announces its bandwidth unit price before the others.

Rose *et al.* [161] study a scenario in which there are multiple Network Service Providers (NSPs), each being able to provide multiple service types, and multiple users with different assessments for the QoS. The authors propose a Multi-Leader-Follower Game (MLFG) in which NSPs first declare the prices for the services they offer and then, based on these prices, the users choose an NSP and a service. Both NSPs and users are profit maximizers; the former aim to maximize their profit from services selected by users and the latter aim to maximize the difference between the QoS assessment of the chosen service and its price.

Although the works in [159–161] do not address the InP business model, they bear methodological similarities with our work in [25]. Nevertheless, among [156–161], the work in [161] is the most comparable to [25]. Specifically, the MLFG that models the interaction among NSPs and users in [161] is, at some extent, similar to the MLFG of our framework (with NSPs and end users in [161] being similar to the InPs and the SPs, respectively in [25]) in both the strategies and the payoff definition of the respectively similar stakeholders. Nevertheless, there are some crucial differences between the two:

- while in [25] we consider a discrete and finite set of SPs, in [161] the authors account for a continuum of end users therefore each subgame of stage two of the MLFG in [161] is a non-atomic game, and
- unlike in [161], we use our proposed framework in [25] to study realistic scenarios concerning *the network technologies and their costs* (based on the elaborate cost models for 5G proposed in [1]), *the services provided to end users and their requirements* (based on usage scenarios for International Mobile Telecommunications (IMT) for 2020 and beyond [162]) and *the user fees.*

Additionally, our work in [25] also bears similarities with the works [1, 163, 164] which study the economic feasibility of 5G networks against the requirements of 5G services; however, in [25] we model the competition among multiple InPs with their own 4G/5G networks whereas in [1, 163, 164] the problem of dimensioning a single 5G network has been addressed.

CHAPTER 3 REALIZATION OF THE RESEARCH PROJECT

This doctoral thesis is organized as a thesis by articles; its core consists of the following three published journal articles:

1. Lorela Cano, Antonio Capone, Giuliana Carello, Matteo Cesana and Mauro Passacantando, "On optimal infrastructure sharing strategies in mobile radio networks", *IEEE Transactions on Wireless Communications*, May 2017 ([23]),

2. Lorela Cano, Antonio Capone, Giuliana Carello, Matteo Cesana and Mauro Passacantando, "Cooperative infrastructure and spectrum sharing in heterogeneous mobile networks", *IEEE Journal on Selected Areas in Communications*, October 2016 ([24]), and

3. Lorela Cano, Giuliana Carello, Matteo Cesana, Mauro Passacantando and Brunilde Sansò, "Modeling the techno-economic interactions among infrastructure and service providers in 5G networks with a multi-leader-follower game", *IEEE Access*, December 2019 ([25]).

These articles are presented in the thesis in order, in Chapters 4, 5 and 6, respectively.

Our work in [165], a proceeding of the 2016 European Wireless conference, was also carried out in the context of this research project. In [165] we address the same problem as in [24] but through a non-cooperative game approach. As the key findings of [24] and [165] are similar, our work in [165] has not been incorporated in this thesis for the sake of conciseness.

As stated in Section 1.3, this research project has two main objectives, the first concerning *infrastructure sharing among conventional MNOs*, whereas the second, *infrastructure sharing for decoupled infrastructure from services in the context of 5G*. Essentially, through the models developed in [23] and [24] we have covered the first objective, whereas through the one in [25], the second objective.

In detail, the papers [23] and [24] study a similar scenario in which there are multiple conventional MNOs with fixed market shares that coexist in one or multiple geographical areas where they aim to deploy a layer of LTE small cells. The problem consists in whether each MNO deploys the new infrastructure or not and if it does, whether to deploy it alone or by entering a sharing agreement (i.e., building a shared network) with some or all other MNOs. Several variants deriving from this scenario have been studied in [23] and [24] through appropriate mathematical models (i.e., either game theory or mathematical programming). These variants differ among them depending on whether (i) spectrum pooling is allowed or not,

(ii) the sharing agreement is market-driven or imposed by a regulator and (iii) the shared infrastructure cost is split *a priori* among the MNOs or it is an outcome of the model. A common feature of all these variants is the user pricing model which is defined as a linear function of the average rate perceived by the user where this rate depends on the user's MNO and the coalition joined by the latter. Through this user pricing model we are then able to capture the impact of infrastructure and spectrum sharing not only on the cost but also on the MNOs' revenues.

In turn, in [25] we study a 5G scenario in which the network infrastructure and services are decoupled. Specifically, we consider a dense urban area in which there are multiple InPs with individual, overlapping network infrastructures but disjoint spectrum bands, and multiple SPs, each providing a single type of service to a specific market segment. Assuming the InPs' BSs are collocated, we concentrate on the area of a single BS cell. The BS cell capacity of a given InP depends on its network technology and available spectrum bandwidth. SPs provision services for their end users in the cell area by purchasing capacity from one of the InPs. Conversely, an InP can serve multiple SPs. Both InPs and SPs are assumed self-interested and profit-maximizers. Consequently, each InP offers its cell capacity at a unit price which maximizes its profit from the overall sold capacity and each SP selects an InP from which to purchase capacity so as to maximize its own profit, i.e., revenues from end users depending on the the amount of purchased capacity minus the cost of the latter. In this setting, SPs compete among them in choosing an InP whereas InPs compete among them in their unit prices. This scenario has been modeled through a MLFG and has been tested for a large number of realistic instances in the context of 5G.

CHAPTER 4 ARTICLE 1 : ON OPTIMAL INFRASTRUCTURE SHARING STRATEGIES IN MOBILE RADIO NETWORKS

Lorela Cano, Antonio Capone, Giuliana Carello, Matteo Cesana and Mauro Passacantando
Published on *IEEE Transactions on Wireless Communications*, May 2017

Abstract

The rapid evolution of mobile radio network technologies poses severe technical and economical challenges to Mobile Network Operators (MNOs); on the economical side, the continuous roll-out of technology updates is highly expensive, which may lead to the extreme where offering advanced mobile services becomes no longer affordable for MNOs which thus are not incentivized to innovate. Mobile infrastructure sharing among MNOs becomes then an important building block to lower the required per-MNO investment cost involved in the technology roll-out and management phases.

We focus on a Radio Access Network (RAN) sharing situation where multiple MNOs with a consolidated network infrastructure coexist in a given set of geographical areas; the MNOs have then to decide if it is profitable to upgrade their RAN technology by deploying additional small-cell base stations and whether to share the investment (and the deployed infrastructure) of the new small-cells with other operators. We address such strategic problem by giving a mathematical framework for the RAN infrastructure sharing problem which returns the "best" infrastructure sharing strategies for operators (coalitions and network configuration) when varying techno-economic parameters such as the achievable throughput in different sharing configurations and the pricing models for the service offered to the users. The proposed formulation is then leveraged to analyze the impact of the aforementioned parameters/input in a realistic mobile network environment based on LTE technology.

4.1 Introduction

Mobile telecommunication networks and services have been characterized by a dramatic uptake in the past two decades which is still to be over. According to [166], the penetration of mobile subscriptions has reached the amazing level of 96% worldwide in 2014, and the traffic delivered through mobile radio networks is expected to reach 11.2 Exabytes/month by 2017 [167] with a considerable share taken by bandwidth-eager services provided by aggressive

Over The Top service providers.

To cope with such fast growing rate, the mobile networks have undergone, and are still undergoing, several technology migration phases cruising from the introduction of third generation (3G) and 3.5G wireless technologies on top of 2G networks to the standardization and deployment of the Long Term Evolution (LTE) with the recent launch of 5G initiatives [168]. The effect of such rapid evolution in the mobile networks technologies poses several technical and economical challenges to Mobile Network Operators (MNOs). On the technical side, the coexistence of multiple technologies in the Radio Access Network (RAN) calls for advanced radio resource orchestration procedures to cope with such heterogeneity. On the economical side, the combined effect of revenues of MNOs that tend to flatten and the network technology updates that are highly expensive may lead to the extreme where offering advanced mobile services becomes no longer affordable for MNOs which are not incentivized to innovate and migrate to new technologies [169].

In this context, the conventional model according to which each MNO retains complete control and ownership of its network is at odds with the large and frequent investments requested on the network infrastructure, and with the increased complexity in the management of the network components. Mobile infrastructure sharing among MNOs thus becomes an important building block to “break" such vertical and inflexible approach, by lowering the required per-MNO investment cost to cope in the technology roll-out and management phases.

Different forms of infrastructure sharing are already in place, ranging from basic unbundling and roaming, to site and spectrum sharing [170]. In these “classical" forms of sharing generally one MNO still retains ownership of the mobile network. On the other hand, we focus here on a RAN sharing scheme in which MNOs share a single radio infrastructure while maintaining separation and full control over the back hauling and respective core networks. In this work, we consider a scenario where multiple MNOs with a consolidated macro cells network infrastructure and consolidated market shares coexist in a given set of geographical areas; the MNOs have to decide if it is profitable to upgrade their RAN technology by deploying additional small-cell base stations and whether to share the investment (and the deployed infrastructure) of the new small-cells with other operators.

We address such strategic problem by providing a mathematical framework for the analysis of the RAN infrastructure sharing problem that takes into account both technical and economical aspects and provides the optimal sharing strategies for MNOs, that include coalitions with other MNOs and network configuration. The proposed infrastructure sharing problem is first tackled from the perspective of a regulatory entity that can impose sharing configurations maximizing the quality of service perceived by all users and then from a single

MNO perspective, in order to account for MNOs as profit-maximizing selfish entities. A Mixed Integer Linear Programming (MILP) formulation is proposed to determine sharing configurations maximizing the quality of service; this formulation includes techno-economic parameters such as the achievable throughput and the pricing models for the service offered to the users. For representing the MNO perspective, we propose a Non Transferable Utility (NTU) coalitional game model. The proposed mathematical framework is then leveraged to analyze the impact of the aforementioned parameters in a realistic mobile network environment based on LTE technology for which numerical values for technical and economic parameters are available. Note however that the proposed approach is general and can be easily applied to other scenarios with different small cell technologies.

The manuscript is organized as follows: Sec. 4.2 reviews the mainstream literature in the field of infrastructure sharing highlighting the main novelties of the proposed approach. In Sec. 4.3, we introduce the reference scenario describing the techno-economic parameters involved in the infrastructure sharing problem and the proposed mathematical framework that allows to represent the problem from the two considered perspectives. Sec. 4.4 describes the considered scenarios and cases while results and insights are reported in Sec. 4.5, where the strategic behavior of MNOs in several different realistic scenarios is analyzed. Our concluding remarks are given in Sec. 4.6.

4.2 Related Work

The literature on infrastructure/resource sharing can be grouped in two main research tracks: (i) works dealing with techno-economic modeling of network sharing and (ii) works on practical algorithms for management and allocation of shared network resources. The first track mostly includes qualitative and quantitative studies of different sharing scenarios and models for estimating capital and operational expenditures. Particular attention is dedicated to the identification of drivers and barriers to network sharing or possible new organization of the mobile network value chain for sharing to be viable.

Meddour et al. [20] suggest guidelines for MNO involved in the sharing process and emphasize the need for subsidization and assistance from regulatory entities. Similarly, Beckman et. al [3] show that the role of regulatory entities is crucial to avoid the decline of market competition.

A recent work by Di Francesco *et al.* [153] introduces a competition-aware network sharing framework in the context of cellular network planning which allows to balance the cost benefit of sharing and the push toward next-generation technologies.

The authors of [4] model the capital and operational expenditures for different levels of sharing and suggest outsourcing as the solution to the challenges posed by network sharing. In [171], the authors propose a benchmark-based model that provides high-quality cost estimates for alternative delivery options of the MNO processes such as "regionalization", "centralization" and "outsourcing". Vaz *et al.* propose a framework to evaluate the performance of heterogeneous network deployment patterns in terms of net present value, capacity, coverage, and carbon footprint [33]. By means of a techno-economic analysis, the work in [37] addresses the cost/revenue viability of different WLAN value network configurations in the presence of MNOs and Service Application Providers and the use cases for which there is incentive to share.

In the field of strategic modeling of resource/infrastructure sharing, it is worth mentioning the works resorting to game theory. Malanchini *et al.* [172] resort to non-cooperative games to model the problems of network selection, when users can choose among multiple heterogeneous wireless access, and of resource allocation in which mobile network operators compete to capture users by properly allocating their radio resources. In [141], spectrum sharing among selfish MNOs in unlicensed bands is modeled as a non-cooperative game. The work in [151] and more extensively in [150] also use a non-cooperative game to model the strategic decision of a MNO regarding sharing its LTE infrastructure in a non–monopolistic telecom market. Another example of 4G infrastructure sharing is given in [131] which considers sharing LTE access network femtocells with other access technologies such as Wi–Fi. Cooperative game theory is used in [49] and [130]; in [49], the resource allocation problem in a shared network is formalized in a two step problem: resource sharing among the operators and resource bargaining among the users and Mobile Virtual Network Operators of each operator; the work in [130] considers not only sharing among MNOs but also among operators of different wireless access technologies.

The research track on practical aspect of resource/infrastructure sharing focuses on algorithms and architectures for managing shared resources. The work in [86] suggests that radio resource management is handled by a third-party service provider or an inter-connection provider to preserve competition and reduce exposure. Anchora *et al.* ([90]) introduce a ns-3 implementation to assess the performance of spectrum sharing in a LTE multi-node/multi-MNO scenario, where a virtual central entity is responsible for applying the sharing policies to the common frequency pool. In [89], virtualization of the wireless medium (spectrum sharing) is proposed to exploit spectrum multiplexing and multi-user diversity while allowing MNOs to remain isolated. Instead, the authors in [40] introduce the Network without Borders concept as a pool of virtualized wireless resources with a shared radio resource manager. Along the same lines, Rahman *et al.* ([91]) introduce a novel architecture based on wireless access network virtualization, where the key tenet is to offload the baseband process from physical base station to backend devices; in this way, the physical base stations can be *sliced* into virtual base stations. In [48], instead, a 2-level radio resource scheduling (among MNOs and for each MNO among its user flows) BS virtualization scheme satisfying the 3GPP SA1 RSE ([173]) requirements has been proposed. The work in [174] proposes the necessary LTE architectural enhancements to adopt capacity, spectrum and hardware sharing, and provides a simulation-based comparative performance analysis of the proposed sharing scenarios and of no sharing case. Johansson [50] provides an algorithm for fair allocation of the shared radio resource among multiple operators.

The aforementioned literature work either abstracts away technical aspects related to the mobile network performance to focus on more economic-oriented analysis and modeling, or the other way around. In our previous work [175], we focus on infrastructure sharing in a single and homogeneous geographical area. To the best of our knowledge, ours is one of the first attempts to strike a better balance between these two aspects of the sharing problem, by quantitatively modeling the relation between technical issues related to the radio communication at the access interface (area coverage, transmission rate, user density and quality observed by users) with economic issues (deployment costs and revenues) in mobile network infrastructure sharing. In this work we provide a more general framework which captures large-scale sharing scenarios featuring multiple geographical areas. Further, we consider two different perspectives: the single decision maker one, where the decision maker is a regulatory authority, and the multiple decision makers perspective, that accounts for the single MNO point of view.

4.3 Modeling the Problem of Mobile Network Infrastructure Sharing

We decided to explore two alternative infrastructure sharing configurations: *socially optimal* configurations providing the best service level for the users, which can be imposed by a regulatory authority[1] and *stable* configurations representing a setting where MNOs act as selfish entities aiming to maximize their profits from upgrading their network. While a centralized approach allows to model the problem of determining *socially optimal* configurations, cooperative game theory is more suitable to determine *stable* configurations. In Section 4.3.1, we introduce the techno-economic parameters representing the considered scenario and provide an MILP formulation for the centralized approach. In Section 4.3.2, we discuss how

[1] It is usually the case that infrastructure sharing agreements are analyzed on a per-case basis by a regulatory authority aiming to assess the impact of such sharing agreements on the users; at the limit, regulators could impose configurations that provide the best service level for the users.

an NTU cooperative game is adopted to determine *stable* configurations. We remark that in Sections 4.3.1 and 4.3.2, we use the term coalition with a slight abuse of terminology to represent a set of MNOs which build a unique shared network, both when they decide to join the coalition based on their profit and when the coalition is suggested as a socially optimal choice. In 4.3.1, the *socially optimal* coalitional structure (partition of the set of MNOs) is selected according to the regulator point of view and each MNO is assigned to its corresponding coalition. Instead, in 4.3.2, each MNO joins the coalition that maximizes its individual profit; in other words, a coalition is *stable* when none of its members has an incentive to leave the coalition.

4.3.1 Socially optimal coalitional structures - an MILP formulation

We consider a set $\mathcal{O}$ of MNOs who have up and running 3G/4G networks over a set $\mathcal{A}$ of dense urban areas: each area $a \in \mathcal{A}$ is populated by N_a users and has a size A_a. Parameter σ_i gives the share of users of MNO $i \in \mathcal{O}$ which is assumed to be equal in each area. The MNOs may consider investing to deploy additional LTE small-cells (HetNets) in some or all the areas. A MNO can either invest by itself or share the investment (and the deployed infrastructure) with a subset (or all) of the other MNOs. Let $\mathcal{S}$ denote the set of all possible coalitions that can be activated for the given set of MNOs (here we consider all possible non-empty subsets, thus $|\mathcal{S}|$ is equal to $2^{|\mathcal{O}|} - 1$). If a MNO invests by itself, the coalition is referred to as *singleton*. $\mathcal{S}_i$ is the set of coalitions MNO i can be part of. Each MNO inherits the customer base from its current network, assuming that users do not change their MNO but may subscribe to a new (LTE) data plan.

We consider the problem of determining the *socially optimal* sharing configurations, that is, how to partition MNOs in coalitions and how many small-cell base stations (BSs) each coalition of MNOs should activate in order to maximize the global service level provided to the users.

In each area a maximum number U_{max} of BSs can be activated by all coalitions.

Users are characterized by parameter δ that represents their willingness to pay for 1 Mbps of LTE rate on a monthly basis and therefore the monthly price of 1 Mbps.

We consider an investment lifetime D (in months). The investment costs are then calculated over the whole D period. Both capital (*e.g.*, site and BS acquisition) and operational (*e.g.*, hardware and software maintenance, land renting and power supply) expenditures contribute to the overall costs of the infrastructure [20].

Let g_{capex} and g^{α}_{opex} denote the fixed CAPEX and annual OPEX components, respectively.

g_{opex}^{α} is calculated as a fixed percentage (ξ) of g_{capex}, i.e., $g_{opex}^{\alpha} = \xi g_{capex}$. We denote by g the cost of a single BS for the investment lifetime D which is determined as the sum of the fixed initial CAPEX and the OPEX accumulated during D, i.e.,

$$g = g_{capex} + \frac{1}{12} D g_{opex}^{\alpha}. \tag{4.1}$$

The BSs installation cost of a coalition is then divided among the coalition members based on their market shares.

We assume that the same coalitional structure will apply to all areas, that is, MNOs will be assigned to the same coalition in all areas, as it can be easier for MNOs to coordinate with the same set of MNOs in all the areas[2]. Table 4.1 recaps the problem's parameters notation.

The partitioning of the set of MNOs $\mathcal{O}$ into a *socially optimal* coalitional structure is modeled as follows. Binary variables y_s represent the coalition activation: y_s equals one if coalition s is activated in all the areas $a \in \mathcal{A}$ and it invests (deploys BSs) in at least one of them. The binary variable x_{is} is equal to one if MNO i is assigned to coalition $s \in \mathcal{S}_i$ and s invests, it equals zero if i is assigned to any other coalition in $\mathcal{S}_i$ but s or s does not invest. Constraints (4.2) guarantee that each MNO i is assigned to at most one coalition from $\mathcal{S}_i$. Constraints (4.3) make sure that if s is activated ($y_s = 1$), all MNOs $i \in s$ are assigned to s.

$$\sum_{s \in \mathcal{S}_i} x_{is} \leq 1, \qquad \forall\, i \in \mathcal{O}, \tag{4.2}$$

$$x_{is} = y_s, \qquad \forall\, s \in \mathcal{S},\ \forall\, i \in s. \tag{4.3}$$

If coalition s is activated, it will deploy a certain number of BSs for each area $a \in \mathcal{A}$, represented by a non-negative integer variable u_s^a. If s is not activated or there is no investment ($y_s = 0$), the corresponding variables u_s^a, for each $a \in \mathcal{A}$, are forced to zero by means of Constraints (4.4). Conversely, a coalition is not active ($y_s = 0$) if it does not deploy any BS in any of the areas (Constraint (4.5)); Constraint (4.6) limits the overall number of BSs

[2]In the case of *stable* sharing configurations, as MNOs decide by themselves which coalition to join, selecting the same coalition (set of collaborating MNOs) in all the areas might also require less time for the sharing agreements to be approved by regulators. Nevertheless, we have also investigated the case in which MNOs are assigned/select a different coalition in each area, which overall does not provide significant gains with respect to forcing the same one over all areas.

deployed by all coalitions in each area.

$$u_s^a \leq U_{max} y_s, \qquad \forall\ s \in \mathcal{S},\ \forall\ a \in \mathcal{A}, \tag{4.4}$$

$$y_s \leq \sum_{a \in \mathcal{A}} u_s^a, \qquad \forall\ s \in \mathcal{S}, \tag{4.5}$$

$$\sum_{s \in \mathcal{S}} u_s^a \leq U_{max}, \qquad \forall\ a \in \mathcal{A}. \tag{4.6}$$

We assess the quality of service provided by MNOs through the average rate perceived by the users, which is an important indicator of the users' level of satisfaction. This rate is different for each area $a \in \mathcal{A}$: firstly because we consider areas with different number of users (N_a) and size (A_a) and secondly because a different number of BSs (u_s^a) may be deployed in different areas. In the proposed model, we define two types of LTE user rate, namely nominal and average, for each coalition $s \in \mathcal{S}$. The nominal user rate is the maximum achievable LTE rate for a certain level of Signal to Interference and Noise Ratio (SINR) and a given system bandwidth[3] that a user perceives when assigned all downlink LTE resource blocks from its serving BS. The downlink SINR depends on the number of BSs activated by the coalition the user belongs to since a larger number of BSs results in the user being on the average closer to its serving BS, and thus receiving a stronger signal, but also closer to the interfering BSs[4]. Thus, the nominal user rate of coalition s in area a, represented by a non-negative continuous variable $\rho_s^{a,nom}$, is a function of the number of deployed BSs u_s^a. The behavior of $\rho_s^{a,nom}$ as a function of u_s^a is investigated by simulating the deployment of the small cell BSs (see Subsec. 4.4.1).

Instead, the average user rate perceived by a user of coalition s in area a is represented by the continuous non-negative variables ρ_s^a and defined in terms of the nominal user rate ($\rho_s^{a,nom}$) and of the load of its serving BS as follows[5]:

$$\rho_s^a = \rho_s^{a,nom}(1-\eta)^{\frac{\sum_{i \in s} \sigma_i N_a}{u_s^a}}, \qquad \forall\ s \in \mathcal{S},\ \forall\ a \in \mathcal{A},$$

where parameter η is the user activity factor, that is, the probability that a user is actually active in his/her serving BS, $\sum_{i \in s} \sigma_i N_a$ is the total number of users that are served by members of coalition s in area a, and the ratio $\frac{\sum_{i \in s} \sigma_i N_a}{u_s^a}$ is the average number of users served by one BS in area a. As a result, $\rho_s^{a,nom}$ is scaled down by the factor $(1-\eta)^{\frac{\sum_{i \in s} \sigma_i N_a}{u_s^a}}$

[3]We consider a 10 Mhz bandwidth in our simulations whether the BS is shared or not.

[4]Since we are considering a nominal rate, any other BS transmission will use a subset or all the resource blocks and therefore unavoidably interfere.

[5]We note that this equation is defined for $u_s^a > 0$, while we set $\rho_s^a = 0$ when $u_s^a = 0$.

which accounts for the average congestion level at a serving BS in a.

In the MILP formulation, the nonlinearity of ρ_s^a in terms of u_s^a is handled by approximating ρ_s^a with a piecewise linear function described by the following constraints:

$$z_s^a \leq u_s^a, \quad \forall\ s \in \mathcal{S},\ \forall\ a \in \mathcal{A}, \tag{4.7}$$

$$\rho_s^a \leq R_s^{a,l} + \alpha_s^{a,l+1}(u_s^a - U_s^{a,l}) + M(1 - z_s^a), \tag{4.8}$$

$$\forall\ s \in \mathcal{S},\ \forall\ a \in \mathcal{A},\ \forall\ l \in \{0, \dots, L-1\},$$

$$\rho_s^a \leq R_s^{a,L} z_s^a, \quad \forall\ s \in \mathcal{S},\ \forall\ a \in \mathcal{A}, \tag{4.9}$$

where z_s^a are binary variables that equal 0 if $u_s^a = 0$ (Constraints (4.7)) and therefore set to zero ρ_s^a when $u_s^a = 0$ (Constraints (4.9)). Constraints (4.8) model the piecewise linear functions approximating ρ_s^a, for any $s \in \mathcal{S}, a \in \mathcal{A}$, where L denotes the number of the linear pieces, $\alpha_s^{a,l}$ denotes the slope of the l-th linear piece, $U_s^{a,l}$ and $R_s^{a,l}$ are the coordinates (number of BSs and user rate, respectively) of the $(l+1)$ breakpoint whereas M is a big positive constant (see Appendix A for the details of the approximation).

Assuming that, in each area a, users of any member of coalition s can be served by any of the BSs activated by s in a, the average user rate provided by MNO i in area a, represented by continuous non-negative variable q_i^a, is equal to the average user rate of the coalition to which i is assigned, that is,

$$q_i^a = \sum_{s \in \mathcal{S}_i} \rho_s^a, \qquad \forall\ i \in \mathcal{O},\ \forall\ a \in \mathcal{A}. \tag{4.10}$$

As for the investment cost and revenues for the MNOs, it is reasonable to model the revenue[6] per MNO i in area a as a continuous non-negative variable r_i^a which is linearly dependent on the MNO's user rate q_i^a in that area as shown in (4.11): δq_i^a is the monthly revenue obtained from one user, which is then multiplied by the investment lifetime D and the number of users $\sigma_i N_a$ of MNO i in area a:

$$r_i^a = \delta D \sigma_i N_a q_i^a, \qquad \forall\ i \in \mathcal{O},\ \forall\ a \in \mathcal{A}. \tag{4.11}$$

[6]The price per unit of service (δ) represents the highest price all current users of each MNO are willing to pay for the new service. Therefore the number of users N is assumed independent of δ. Moreover, the proposed pricing model aims at translating the MNOs level of investment, which affects the service level perceived by users, into revenues. It is outside of the scope of the analysis we propose here to account for pricing models in line with those currently applied by MNOs which involve bundles of services, data caps etc. In the same lines, we do not account for the user migration among MNOs since it is generally determined by "non-technical" parameters such as special tariffs, bundle offers, brand fidelity and more in general marketing strategies.

The cost incurred by MNO i in area a, represented by non-negative continuous variable c_i^a, is a linear function of the number of BSs activated in a by the coalition to which i is assigned, divided among the coalition's members proportionally to their number of users:

$$c_i^a = \sum_{s \in \mathcal{S}_i} g \frac{\sigma_i}{\sum_{j \in s} \sigma_j} u_s^a, \qquad \forall\, i \in \mathcal{O},\ \forall\, a \in \mathcal{A}. \tag{4.12}$$

Although the *socially optimal* infrastructure sharing configurations provide the optimal service level for users, MNOs cannot be forced to undertake lossy investments. Therefore, Constraints (4.13) make sure that each MNO obtains a non-negative profit:

$$\sum_{a \in \mathcal{A}} (r_i^a - c_i^a) \geq 0, \qquad \forall\, i \in \mathcal{O}. \tag{4.13}$$

We consider two candidate objective functions to be maximized to determine the *socially optimal* sharing configurations:

$$\sum_{i \in \mathcal{O}, a \in \mathcal{A}} q_i^a, \tag{4.14a}$$

$$\min_{i \in \mathcal{O}, a \in \mathcal{A}} q_i^a. \tag{4.14b}$$

Objective (4.14a) favors efficiency by maximizing the sum of user rate over all MNOs and areas, whereas (4.14b) maximizes the smallest user rate (over all areas and MNOs), so as to privilege users' fairness. We denote Objectives (4.14a) and (4.14b) by TOT_Q and MIN_Q, respectively and use this notation throughout Section 4.5. Sets and parameters describing the instances are recapped in Table 4.1 whereas variables in Table 4.2. In Appendix B, we prove that the decision version of the problem with objective MIN_Q is NP-complete.

4.3.2 Stable coalitional structures - A non transferable utility cooperative game model

We now describe the problem of determining *stable* infrastructure sharing configurations. We assume that MNOs in a coalition will share their cost while each MNO will keep its individual revenue since the latter is incurred from its own share of users. As a result, the coalition worth, that is, the difference between the coalition global revenues and cost, cannot be redistributed among its members: therefore we adopt solution concepts of NTU cooperative games [178].

The game is formalized as a pair $(\mathcal{O}, V)$, where the player set $\mathcal{O}$ coincides with the set of

Table 4.1 Sets, parameters, and corresponding values

Symbol	Description	Value
$\mathcal{O}$	Set of MNOs	{A,B,C}, $\lvert\mathcal{O}\rvert$=3
$\mathcal{A}$	Set of Areas	{Z1,Z2,Z3}
$\mathcal{S}$	Set of coalitions	{A,B,C,AB,AC,BC,ABC}
$\mathcal{S}_i$	Set of coalitions MNO $i \in \mathcal{O}$ can join	$\{s \in \mathcal{S} \mid i \in s\}$
N_a	Number of users of area $a \in \mathcal{A}$	See Table 4.3
A_a	Size of area $a \in \mathcal{A}$	See Table 4.3
σ_i	Market share of MNO $i \in \mathcal{O}$	M_1:{1/3,1/3,1/3}, M_2:{0.1,0.3,0.6}
U_{max}	Max. number of BSs in the area	4000
δ	Monthly price of 1 Mbps	Equidistant values in [0.02,2]€/Mbps
D	Investment lifetime [months]	120 ([176],[50])
η	User activity factor	0.001
ξ	OPEX annual %	15% [177]
g_{capex}	CAPEX of BS cost	3000€
g	BS cost normalized for D	7500€

Table 4.2 Variable domains and description

Variable	Description
$x_{is} \in \{0,1\}$	1 if MNO $i \in \mathcal{O}$ joins coalition $s \in \mathcal{S}_i$ in all areas, 0 otherwise
$y_s \in \{0,1\}$	1 if coalition $s \in \mathcal{S}$ is created in all areas, 0 otherwise
$u_s^a \in \mathbb{Z}_+$	Number of BSs activated by coalition $s \in \mathcal{S}$ in area $a \in \mathcal{A}$
$z_s^a \in \{0,1\}$	1 if coalition $s \in \mathcal{S}$ activates at least one BS in area $a \in \mathcal{A}$, 0 otherwise
$\rho_s^{a,nom} \geq 0$	Nominal user rate for coalition $s \in \mathcal{S}$ in area $a \in \mathcal{A}$
$\rho_s^a \geq 0$	User rate for coalition $s \in \mathcal{S}$ in area $a \in \mathcal{A}$
$q_i^a \geq 0$	User rate for MNO $i \in \mathcal{O}$ in area $a \in \mathcal{A}$
$c_i^a \geq 0$	Costs of MNO $i \in \mathcal{O}$ in area $a \in \mathcal{A}$
$r_i^a \geq 0$	Revenues of MNO $i \in \mathcal{O}$ in area $a \in \mathcal{A}$

MNOs and V is a function that associates to each non-empty coalition $s \in \mathcal{S}$ a subset of payoff allocation vectors $(\pi_i)_{i \in \mathcal{O}}$, i.e.,

$$V(s) = \{(\pi_i)_{i \in \mathcal{O}} : \ \pi_i \leq p_s^i \quad \forall \ i \in s\},$$

where p_s^i is the optimal payoff of player i in coalition s.

Since each MNO is a self-interested entity that aims to maximize its individual profits from the investment, we define its optimal payoff p_s^i from a given coalition as the largest profit (difference between total revenues and total cost) it can achieve if it becomes part of that coalition. Such payoffs are calculated in the following fashion: given a coalition $s \in \mathcal{S}$, we determine the optimal number of BSs ($\widetilde{u}_s^a$) activated in each area $a \in \mathcal{A}$, calculate each member's revenues and costs for each area and therefore calculate the MNO total profit.

The optimal number $\widetilde{u}_s^a$ of BSs coalition s can deploy in area a is obtained solving the following problem[7]:

$$\max \sum_{i \in s} r_i^a - c_i^a \tag{4.15}$$

$$r_i^a = \delta D \sigma_i N_a \rho_s^a, \qquad \forall\, i \in s, \tag{4.16}$$

$$c_i^a = \frac{\sigma_i}{\sum\limits_{j \in s} \sigma_j} g u_s^a, \qquad \forall\, i \in s, \tag{4.17}$$

$$u_s^a \leq U_{max}, \tag{4.18}$$

$$z_s^a \leq u_s^a, \tag{4.19}$$

$$\rho_s^a \leq R_s^{a,l} + \alpha_s^{a,l+1}(u_s^a - U_s^{a,l}) + M(1 - z_s^a), \tag{4.20}$$

$$\forall\, l \in \{0, \ldots, L-1\},$$

$$\rho_s^a \leq R_s^{a,L} z_s^a, \tag{4.21}$$

$$u_s^a \in \mathbb{Z}_+,\ \rho_s^a \geq 0,\ z_s^a \in \{0,1\}. \tag{4.22}$$

The objective function (4.15) can be rewritten as

$$\sum_{i \in s} \left(\delta D \sigma_i N_a \rho_s^a - \frac{\sigma_i}{\sum\limits_{j \in s} \sigma_j} g u_s^a \right) = \tag{4.23}$$

$$\left(\sum_{i \in s} \sigma_i \right) \left(\delta D N_a \rho_s^a - \frac{1}{\sum\limits_{j \in s} \sigma_j} g u_s^a \right),$$

where ρ_s^a depends on u_s^a. As $\delta D N_a \rho_s^a - \frac{1}{\sum_{j \in s} \sigma_j} g u_s^a$ is independent of the MNOs, the optimal number $\widetilde{u}_s^a$ of BSs is the same for all the players and can be easily computed solving the above problem.

[7]We remark that, in the problem we upper bound the number of BSs activated by each coalition in the area to U_{max} (Constraint (4.18)) since, for the considered instances (see Section 4.5), the total number of BSs activated by any partition of MNOs in the set $\mathcal{O}$ does not exceed U_{max}, that is, the more stringent Constraint (4.6) which limits the number of BSs activated by all coalitions in the area to U_{max} is never tight.

Therefore, the optimal payoff p_s^i of each MNO $i \in s$ is

$$p_s^i = \sum_{a\in\mathcal{A}} \left(\delta D \sigma_i N_a \rho_s^a(\widetilde{u}_s^a) - \frac{\sigma_i}{\sum\limits_{j\in s}\sigma_j} g\widetilde{u}_s^a \right) = \frac{\sigma_i}{\sum_{j\in s}\sigma_j} \sum_{a\in\mathcal{A}} \left(\delta D N_a \rho_s^a(\widetilde{u}_s^a) \sum_{j\in s}\sigma_j - g\widetilde{u}_s^a \right). \tag{4.24}$$

In other words, the optimal payoff allocations p_s^i correspond to dividing the optimal worth of coalition s, i.e., $\sum_{a\in\mathcal{A}} \left(\delta D N_a \rho_s^a(\widetilde{u}_s^a) \sum_{j\in s}\sigma_j - g\widetilde{u}_s^a \right)$, among its members according to their relative market shares, i.e., $\sigma_i / \sum_{j\in s}\sigma_j$.

In the following we look for *stable* infrastructure sharing configurations. We define a sharing configuration as a partition $(s_1, \ldots, s_p)$ of the MNOs set $\mathcal{O}$, where coalitions $s_1, \ldots, s_p \in \mathcal{S}$. A configuration $(s_1, \ldots, s_p)$ is said *stable* if for any $j = 1, \ldots, p$ there is no nonempty subset $s'_j \subset s_j$ such that

$$p^i_{s'_j} > p^i_{s_j}, \qquad \forall\ i \in s'_j,$$

that is, for any coalition s_j no subset of MNOs has incentive to leave it.

4.4 Experimental settings

We run several tests to evaluate how the coalitional structure, the level of investment, and therefore the performance indicators of both the *socially optimal* and *stable* configurations are affected by the user economic standpoint.

The MILP model (Section 4.3.1) and problem (4.15)–(4.22) for any $s \in \mathcal{S}$ and $a \in \mathcal{A}$ (Section 4.3.2) have been implemented in AMPL [179]. We have used Gurobi 6.0 [180] as a MILP solver. All tests were run on an Intel(R) Core(TM) i5-3230M CPU @2.6 Ghz. To keep the computational time limited, for some of the instances the acceptable relative MIP gap of Gurobi was set equal to 1e-6. When optimizing MIN_Q, several equivalent optimal solutions may be found, which may not provide consistent values for the user rate of the non-bottleneck areas and MNOs. When needed, they have been computed in post-processing.

4.4.1 BS deployment simulation

A simulation environment was set up to derive the coalition user rate per area ρ_s^a as a function of each possible number u_s^a of BSs that coalition s can activate in area a, *i.e.*, from 1 up to U_{max}. In details, the entire set of U_{max} BSs is uniformly distributed in a pseudo-random fashion on the considered square areas; 10 sample users are also randomly distributed over each area a. The downlink SINR of each sample user in a for each coalition s ($SINR_s^a$) is calculated for each possible value of u_s^a as a function of: the signal power P_k the sample user receives from its serving BS k (*i.e.*, the BS from which receives the strongest signal), the signal power $\sum_{j\neq k} P_j$ received from the interfering (non-serving) BSs and the white Gaussian noise signal power[8] P_{noise}. Since users are characterized by an activity factor η, the captured interference is scaled down by the load of coalition s in area a, i.e., $l_s^a = 1-(1-\eta)^{\frac{\sum_{i\in\mathcal{O}_s}\sigma_i^a N_a}{u_s^a}}$. $SINR_s^a$ is therefore calculated as follows:

$$SINR_s^a = \frac{P_k}{l_s^a\left(\sum\limits_{j\neq k} P_j\right) + P_{noise}}, \ \forall\ s\in\mathcal{S},\ \forall\ a\in\mathcal{A}. \tag{4.25}$$

The received signal power $P_{rx}[dBm]$ has been calculated according to a three-parameter path loss model (transmitted signal power P_{tx}, fixed path loss C_{pl} and path loss exponent Γ) defined within the GreenTouch Consortium [181]:

$$P_{rx}[dBm] = P_{tx}[dBm] - C_{pl}[dB] - 10\Gamma log(d[km]), \tag{4.26}$$

where d is the sample user–BS distance. The calculated SINR is finally mapped to LTE nominal rate ($\rho_s^{a,nom}$) according to a multilevel SINR–to–rate scheme [181]. A single value for $\rho_s^{a,nom}$ is obtained by averaging over the 10 sample users. An additional averaging is obtained by applying 100 iterations for each value of u_s^a; ρ_s^a is then calculated analytically as the product $\rho_s^{a,nom}(1-\eta)^{\frac{\sum_{i\in s}\sigma_i N_a}{u_s^a}}$, according to the definition in Section 4.3.1.

4.4.2 Instances

We consider three square dense areas (their size and number of users are provided in Table 4.3) and three MNOs (A, B and C) which is quite reasonable for the Italian (also European) telecom playground [150]. Assuming the dense urban areas belong to the same city, we consider the same distribution of users among MNOs in all of them. We report the results obtained for two such user distributions: M_1, MNOs have equal market shares ($\sigma_A = \sigma_B = \sigma_C = 1/3$) and M_2, for which the market shares of A, B and C are 10%, 30% and 60%, respectively ($\sigma_A = 0.1$, $\sigma_B = 0.3$, $\sigma_C = 0.6$).

The values of the user's willingness to pay for 1 Mbps of service on a monthly basis δ were deduced from current data tariff-plans applied by different Italian MNOs. We have considered

[8]The white Gaussian noise signal power accounts for the considered system bandwidth.

100 values in the range $[0.02, 2]$ €/Mbps which were obtained discretizing the range uniformly with a 0.02 step.

The number of available sites for installing small cell BSs in a given geographical area is finite and most likely different for each area. We set U_{max} to 4000 for all the considered areas; such number of BSs it at least one order of magnitude larger than the minimum needed for coverage[9] whereas deploying more BSs would result in only a marginal increase of the average user rate ρ_s for the considered instances (see Figure A.1).

The investment lifetime period D is set to 120 months (see, e.g., [50, 176]) for all instances.

For the two user distributions we generate a scenario for each value of δ, while the rest of parameters ($\mathcal{O}$, $\mathcal{A}$, N_a, A_a, g, U_{max}, η, D) are fixed to the values provided in Table 4.1.

Table 4.3 Characteristics of the set of areas

Area	**Number of users**	**Size**
Z1	$N_1 = 20000$	$A_1 = 4\ km^2$
Z2	$N_2 = 20000$	$A_2 = 0.5\ km^2$
Z3	$N_3 = 10000$	$A_3 = 1\ km^2$

4.5 Results

In this section, we examine the impact of the user economic standpoint (different values of δ) and of the user distribution among MNOs (σ_i) on the coalitional structures and the level of investment first of the *socially optimal* configurations (Subsection 4.5.1) and then of the *stable* configurations (Subsection 4.5.2). The two configurations are then compared in Subsection 4.5.3.

We recall that the user rate as a function of the number of deployed BSs for the different sharing configurations was obtained by means of simulation (Subsection 4.4.1) and that it behaves nonlinearly in the number of BSs; to obtain a MILP formulation of the problem, we have approximated the user rate functions with piecewise linear ones (see Subsection 4.3.1, Appendix A). In order to account for the error introduced by the approximation, we investigate multiple configurations which perform very similarly. This allows us to identify general trends concerning the size and composition of the selected coalitional structures as we vary δ and the user distribution. For each value of δ, we consider as *socially optimal* sharing configurations the ones selected by the optimal solution of problem (4.2)-(4.13), solved ei-

[9] If we consider small cells of 50 m range, the minimum number of small cell BSs for coverage would be roughly 500 for the largest area (Z1).

ther under objective TOT_Q (4.14a) or MIN_Q (4.14b), and all configurations for which the objective function value is at most 0.5% smaller with respect to the optimal one. Similarly, for *stable* sharing configurations, we relax the stability condition as follows: we consider a configuration $(s_1, \ldots, s_p)$ to be *stable* if for any $j = 1, \ldots, p$ there is no nonempty subset $s'_j \subset s_j$ such that

$$\frac{p^i_{s'_j} - p^i_{s_j}}{p^i_{s_j}} > 0.5\%, \qquad \forall\, i \in s'_j.$$

The different outcomes are denoted by the following notation: ABC represents the grand coalition, coalitional structures that consist of a singleton (i.e., a MNO investing alone) and a coalition of two MNOs are denoted by A/BC, B/AC and C/AB[10], whereas the case when no sharing takes place, that is, when each MNO invests by itself, is denoted by A/B/C.

For each possible outcome, we report the values of δ for which the outcome is *socially optimal* under objectives TOT_Q and MIN_Q in Tables 4.4 and 4.5 for user distributions M_1 and M_2, respectively. The results concerning the *stable* configurations are reported in Tables 4.7a and 4.7b for user distributions M_1 and M_2, respectively.

Concerning the level of investment, we report the number of BSs deployed by the sharing configurations only for a subset of the considered values of δ (i.e., {0.02, 0.04, 0.2, 0.4, 1, 2}) due to space limitations. For all values of δ for which we have identified multiple configurations (as illustrated in Tables 4.4, 4.5, 4.7a and 4.7b), we report the results of the configuration selected by the optimal solution of the MILP model for the *socially optimal* configurations in Tables 4.6 and 4.7, for user distributions M_1 and M_2, respectively. Similarly, when multiple configurations are *stable*, only one of them is reported in Tables 4.8a and 4.8b, for user distributions M_1 and M_2 respectively. The notation concerning the number of deployed BSs in Tables 4.6, 4.7, 4.9 is the following: for outcome ABC, the reported number represents the number of BSs deployed by the grand coalition, for outcomes A/BC, C/AB and B/AC, the first number represents the number of BSs deployed by the singleton whereas the second represents the number of BSs deployed by the coalition of two, whereas for outcome A/B/C, the number of BSs deployed by each MNO are reported in order (i.e., the first number corresponds to A, the second to B and third to C).

4.5.1 Socially optimal configurations

As a general rule, results show that as users are willing to pay more (i.e., for higher values of δ) and, as a result, MNOs can afford a larger network cost, the *socially optimal* configu-

[10]We remark that outcomes A/BC, B/AC and C/AB are equivalent for user distribution M_1 since MNOs have equal market shares.

rations consist of smaller and less congested coalitions in order to provide the best service level. Regarding the level of investment, the higher the value of δ, the denser the network deployment as larger revenues make up for increasing network cost.

Table 4.4 Values of δ for which a coalitional structure is *socially optimal* – user distribution M_1

(a) TOT_Q

Coalitional structure	δ
ABC	[0.02, 0.04]
A/BC, B/AC, C/AB	[0.04, 0.1], [0.14, 2]
A/B/C	[0.06, 2]

(b) MIN_Q

Coalitional structure	δ
ABC	[0.02, 0.06]
A/BC, B/AC, C/AB	0.06, 0.1, [0.14, 0.22]
A/B/C	[0.06, 2]

Table 4.5 Values of δ for which a coalitional structure is *socially optimal* – user distribution M_2

(a) TOT_Q

Coalitional structure	δ
ABC	[0.02, 0.06]
A/BC	—
B/AC	0.04, 0.08, [0.12, 0.26]
C/AB	[0.04, 2]
A/B/C	[0.28, 2]

(b) MIN_Q

Coalitional structure	δ
ABC	[0.02, 0.06], 0.1
A/BC	—
B/AC	[0.1, 0.16], [0.2, 2]
C/AB	[0.06, 2]
A/B/C	[0.26, 2]

For very low and high values of δ, results are very similar for both user distribution scenarios (M_1 and M_2). The grand coalition (ABC) outperforms the other configurations for $\delta = 0.02$

for TOT_Q and for $\delta \leq 0.04$ for MIN_Q for both M_1 and M_2. Although ABC is selected also for few other low values of δ for both objectives and user distributions, it performs similarly to other outcomes (Tables 4.4 and 4.5): e.g., for M_2, ABC is selected by TOT_Q also for $\delta = 0.06$ but performs similarly to C/AB. Instead, A/B/C, which represents the case when no sharing takes place, is always among the selected outcomes for $\delta \geq 0.06$ for both TOT_Q and MIN_Q for M_1 (Table 4.4) and for $\delta \geq 0.28$ for TOT_Q and for $\delta \geq 0.26$ for MIN_Q for M_2 (Table 4.5).

However, for intermediate values of δ, results seem more sensitive to the user distribution. For M_1, the equivalent outcomes A/BC, B/AC and C/AB are selected for almost all values of δ in [0.06, 2] for TOT_Q and for some values of δ in [0.06, 0.22] for MIN_Q (Table 4.4). However, since they are always selected alongside A/B/C, that is, they perform very similarly to the case when there is no sharing, there is practically no incentive for sharing also for intermediate values of δ for M_1. Instead for M_2, for δ in [0.08, 0.26], the only *socially optimal* configurations selected by TOT_Q are C/AB and, for a subset of the values of δ in this range, also B/AC (Table 4.4a); similarly for MIN_Q for δ in [0.12, 0.24] (Table 4.4b). In C/AB and B/AC, both coalitions of two MNOs, AB and AC, involve A which has the smallest market share (10%) and therefore introduces the minimum level of interference to a coalition. Moreover, for low values of δ, A benefits from being in a coalition since it cannot afford to invest sufficiently by itself given its small market share[11]. For these values, C/AB is more persistent than B/AC (i.e., it is selected for all δ in [0.04, 2] by TOT_Q and all δ in [0.06, 2] by MIN_Q) since C and AB are smaller (less congested) than AC. In turn A/BC, which involves the largest coalition of two MNOs (BC) and the smallest MNO (A) investing alone, is never selected.

Concerning the level of investment, in Tables 4.5a and 4.5b, we report the number of small cell BSs deployed in each area for the *socially optimal* sharing configuration selected by the optimal solution under TOT_Q and MIN_Q, respectively, for a subset of the considered values of δ ({0.02, 0.04, 0.2, 0.4, 1, 2}) and user distribution M_1. Results concerning user distribution M_2 are reported in Tables 4.6a and 4.6b.

For most instances, both objectives TOT_Q and MIN_Q provide the same coalitional structures but slightly different number of deployed BSs. For instance, for user distribution M_2 and $\delta = 0.02$ (see Tables 4.6a and 4.6b), the grand coalition deploys 5 more BSs under MIN_Q compared to TOT_Q in the largest area (Z1), 16 more in the most congested/dense area (Z2), and 22 BSs less in area Z3 (smaller than Z1 and less congested than Z2). Since the overall profit of each MNO has to be non-negative, objective MIN_Q achieves fairness by “redistributing" BSs across the areas so that the user rate of the worst served ones (Z1 & Z2)

[11]For instance, if all MNOs were to invest by themselves, for $\delta \leq 0.26$ users of MNO A would perceive the worst service level (user rate) due to A’s low level of investment. Instead, for $\delta \geq 0.28$, as A is able to densify its network, users of C perceive the lowest user rate since C is the largest/most congested MNO.

is increased at the expense of sacrificing the user rate of the better served one (Z3) (see also Figure 4.2 and observation (iv) in Section 4.5.4).

Similar observations can be made for both user distribution scenarios concerning the impact of δ on the number of deployed BSs (Tables 4.6–4.7). A little incentive from users (small δ) forces MNOs to deploy only a small number of BSs in order to limit their cost and therefore guarantee an overall positive profit. For example, for user distribution M_2, $\delta = 0.02$, under objective TOT_Q the grand coalition deploys 169 BSs in area Z1, 156 BSs in area Z2 and 110 BSs in area Z3 (Table 4.6a). However, as users are willing to pay more (larger values of δ), more BSs are deployed since higher revenues compensate the costs of deploying more BSs. In particular, all available sites per area (U_{max}) are used up in all the areas for user distribution M_1 under objective TOT_Q when $\delta \geq 0.4$ (Tables 4.5a); instead, for M_2, the U_{max} BSs are exhausted only in areas Z1 and Z2 when $\delta \geq 0.4$ whereas in Z3 the rate saturation is achieved by deploying less than U_{max} BSs when $\delta \geq 0.46$ (Table 4.6a).

Table 4.6 *Socially optimal* coalitional structures and corresponding number of activated BSs – user distribution M_1

(a) TOT_Q

$\mathcal{A}/\delta$	0.02	0.04	0.2	0.4	1	2
Z1	ABC 157	ABC 443	A/BC 1007/1928	A/B/C 1500/1500/1000	A/B/C 1500/1500/1000	A/B/C 1500/1500/1000
Z2	ABC 161	ABC 448	A/BC 1000/2000	A/B/C 1500/1500/1000	A/B/C 1500/1500/1000	A/B/C 1500/1500/1000
Z3	ABC 115	ABC 274	A/BC 706/1496	A/B/C 1500/1500/1000	A/B/C 1500/1500/1000	A/B/C 1500/1500/1000

(b) MIN_Q

$\mathcal{A}/\delta$	0.02	0.04	0.2	0.4	1	2
Z1	ABC 177	ABC 467	A/B/C 1091/1091/1091	A/B/C 1257/1486/1257	A/B/C 1257/1257/1257	A/B/C 1257/1257/1257
Z2	ABC 164	ABC 462	A/B/C 1141/1141/1141	A/B/C 1333/1334/1333	A/B/C 1334/1333/1333	A/B/C 1334/1333/1333
Z3	ABC 91	ABC 232	A/B/C 488/488/488	A/B/C 1080/2288/632	A/B/C 2288/633/633	A/B/C 2288/633/633

4.5.2 Stable configurations

Also for *stable* configurations, the higher the value of δ, the smaller and less congested are the selected coalitions. For low values of δ, MNOs prefer to collaborate with a larger number of MNOs so as to minimize the network cost. Instead, for higher δ, i.e., higher revenues per unit of service provided, MNOs prefer to increase the service level, which in turn requires

Table 4.7 *Socially optimal* coalitional structures and corresponding number of activated BSs – user distribution M_2

(a) TOT_Q

$\mathcal{A}/\delta$	0.02	0.04	0.2	0.4	1	2
Z1	ABC 169	ABC 398	C/AB 2000/1364	A/B/C 700/1300/2000	A/B/C 700/1300/2000	A/B/C 700/1300/2000
Z2	ABC 156	ABC 472	C/AB 1700/1200	A/B/C 617/1383/2000	A/B/C 700/1300/2000	A/B/C 700/1300/2000
Z3	ABC 110	ABC 287	C/AB 1200/716	A/B/C 395/1200/2000	A/B/C 554/1200/2000	A/B/C 554/1200/2000

(b) MIN_Q

$\mathcal{A}/\delta$	0.02	0.04	0.2	0.4	1	2
Z1	ABC 174	ABC 469	C/AB 1776/1191	A/B/C 401/1031/2568	A/B/C 401/1031/2568	A/B/C 401/1031/2568
Z2	ABC 172	ABC 459	C/AB 2185/1472	A/B/C 381/1055/2564	A/B/C 381/1055/2564	A/B/C 381/1055/2564
Z3	ABC 88	ABC 230	C/AB 925/595	A/B/C 309/480/2662	A/B/C 858/480/2662	A/B/C 309/1029/2662

building less congested networks, i.e., either shared networks with fewer and smaller MNOs or individual ones.

Table 4.8 Values of δ for which a coalitional structure is *stable*

(a) User distribution M_1

Coalitional structure	δ
ABC	[0.02, 0.1], [0.16, 0.22], 0.28
A/BC, B/AC, C/AB	[0.02, 0.52], 0.6, [0.98, 2]

(b) User distribution M_2

Coalitional structure	δ
ABC	[0.02, 0.04], [0.1, 0.12], [0.18, 0.30]
A/BC	0.02, 0.06, [0.1, 0.14], [0.18, 0.36], [0.52, 0.54]
B/AC	[0.02, 0.08], [0.12, 0.16], [0.22, 0.52], [0.6, 2]
C/AB	[0.04, 0.06], [0.1, 2]

For user distribution M_1 (see Table 4.7a), when $\delta \leq 0.52$ there is always incentive for sharing, i.e., each MNO is better off building a shared network with at least one other MNO than investing alone. The grand coalition (ABC) is *stable* for all values of δ in [0.02, 0.1] and a subset of values in [0.16, 0.28] but it ceases to be the *stable* when $\delta \geq 0.3$. The equivalent outcomes A/BC, B/AC and C/AB are *stable* for all δ in [0.02, 0.52] but they become unstable for a subset of values of δ in [0.54, 2] which in turn means that in such cases no sharing will take place and MNOs will build individual networks. However, for $\delta \geq 0.3$, A/BC, B/AC and C/AB perform very similarly to A/B/C.

For user distribution M_2 (see Table 4.7b), as δ increases only configurations containing the least congested coalitions of two MNOs remain *stable*. The grand coalition (ABC) and outcome A/BC (which involves the largest coalition of two MNOs) are never *stable* for $\delta \geq 0.32$ and $\delta \geq 0.56$, respectively. For $\delta \geq 0.56$, C/AB and, for a subset of values of δ, also B/AC are *stable*. In particular, outcome C/AB, in which the largest MNO C invests by itself whereas the smaller MNOs A and B collaborate, is always *stable* for $\delta \geq 0.1$.

Table 4.9 *Stable* coalitional structures and corresponding number of activated BSs

(a) User distribution M_1

$\mathcal{A}/\delta$	0.02	0.04	0.2	0.4	1	2
Z1	ABC 67	ABC 157	ABC 443	A/BC 349/686	A/BC 606/1500	A/BC 1000/2000
Z2	ABC 65	ABC 163	ABC 471	A/BC 357/628	A/BC 558/1500	A/BC 1000/2000
Z3	ABC 54	ABC 54	ABC 272	A/BC 178/274	A/BC 298/678	A/BC 490/1000

(b) User distribution M_2

$\mathcal{A}/\delta$	0.02	0.04	0.2	0.4	1	2
Z1	ABC 74	ABC 169	ABC 700	C/AB 700/491	C/AB 1200/700	C/AB 2000/1200
Z2	ABC 69	ABC 156	ABC 472	C/AB 700/465	C/AB 1200/700	C/AB 1700/1200
Z3	ABC 12	ABC 66	ABC 287	C/AB 273/237	C/AB 700/476	C/AB 700/700

Concerning the number of BSs deployed by the *stable* configurations (Tables 4.9), a little incentive from users (small δ) forces MNOs to activate only a small number of BSs in order to limit their cost and therefore guarantee an overall positive profit. For example, for user distribution M_2 and $\delta = 0.02$, the grand coalition is *stable* and it activates 74 BSs in area Z1, 69 BSs in area Z2 and 12 BSs in area Z3 (Table 4.8b). However, as users are willing to pay more (larger values of δ), more BSs are activated since higher revenues compensate the costs of activating more BSs.

4.5.3 Comparison

We now compare the behavior of the *socially optimal* and *stable* configurations. The impact of δ on the two configurations is overall very similar. However, there is incentive for sharing for a larger range of the values of δ in order to maximize the MNOs profits (i.e., for *stable* configurations) compared to maximizing the global/minimum user rate (i.e. for the *socially optimal* configurations). In other words, shared networks can be more beneficial from the MNOs perspective as sharing the network cost allows for larger profits but less beneficial from the user perspective due to the service level degradation experienced in more congested networks. Consider for instance user distribution M_1. The grand coalition ABC is socially optimal for $\delta \in [0.02, 0.04]$ for TOT_Q and for $\delta \in [0.02, 0.06]$ for MIN_Q, but it is stable for a larger number of values of δ between 0.02 and 0.28. In general, under MIN_Q sharing is selected as optimal strategy only for $\delta \leq 0.22$, while sharing configurations are *stable* for a wider range of values (up to $\delta = 2$), which means that for higher values of δ no sharing should takes place in order to provide the best service level, while there is incentive to share in order to maximize the MNOs' profit.

Regarding the level of investment, the higher the value of δ, the denser the network deployment for both configurations as larger revenues make up for increasing network cost. Nevertheless, for the same value of δ more BSs are deployed by the *socially optimal* configurations compared to the *stable* ones, as the former focus on the user rate whereas the latter, focusing on the profit, reflect the trade-off between increased revenues and cost. For instance, for M_1 and $\delta = 0.04$, the grand coalition is selected by TOT_Q and it is *stable*; however, it deploys 443 BSs in area Z1, 448 in Z2 and 274 in Z3 under objective TOT_Q (Tables 4.5a) whereas in order to maximize the MNOs profit, 157 BSs are deployed in area Z1, 163 in Z2 and 54 in Z3 (Table 4.8a).

4.5.4 Performance indicators analysis

We now analyze how different values of δ impact two key performance indicators for the users and the MNOs: the average user rate, $Q_{avg} = \frac{\sum_{i \in \mathcal{O}, a \in \mathcal{A}} q_i^a}{|\mathcal{O}| \times |\mathcal{A}|}$, and the average global profit, $P_{avg} = \frac{\sum_{i \in \mathcal{O}} \sum_{a \in \mathcal{A}} (r_i^a - c_i^a)}{|\mathcal{O}|}$; when multiple configurations are selected for the same value of δ (as reported in Tables 4.4, 4.5, 4.8), we average also over the different configurations. In particular, we analyze the "price" of imposing a fair coalitional structure (objective MIN_Q).

Results show that the *socially optimal* infrastructure sharing configurations outperform *stable* ones in terms Q_{avg} and vice versa for P_{avg}. However, as users are willing to pay more, the two configuration types tend to provide very similar values of Q_{avg} and P_{avg}.

As similar observations regarding the behavior of Q_{avg} and P_{avg} as a function of δ can be drawn for both user distributions M_1 and M_2, we report results concerning only M_2 in Figure 4.1.

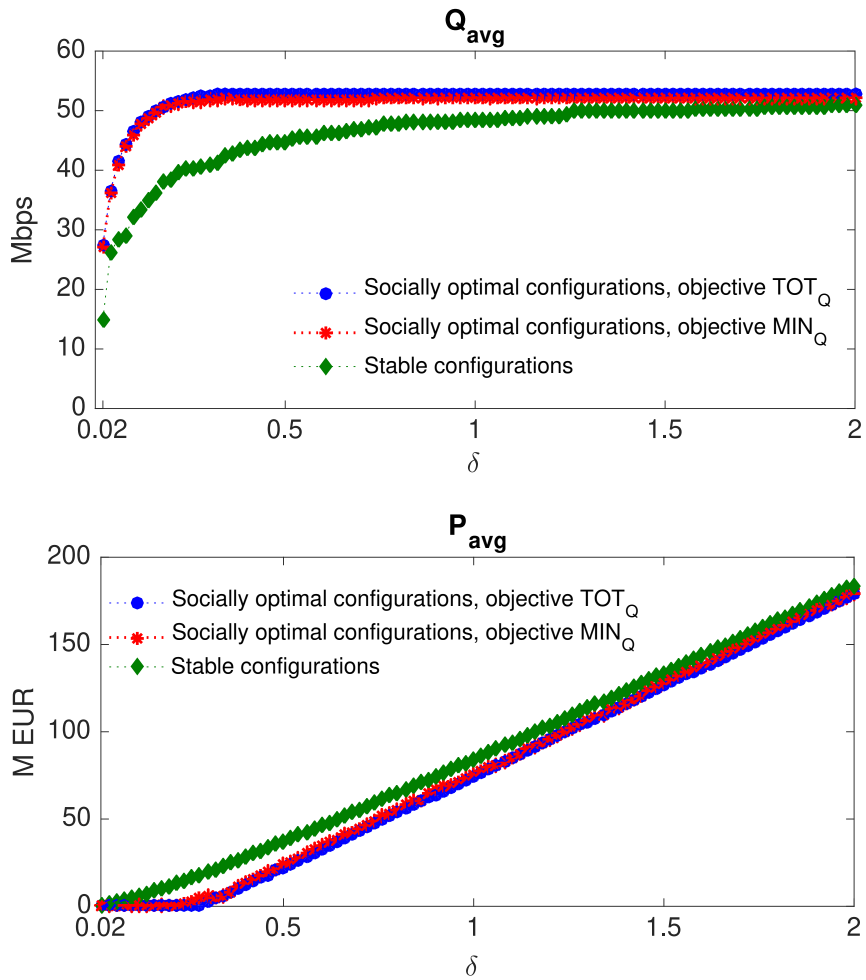

Figure 4.1 Average user rate (Q_{avg}) and average profit (P_{avg}) *vs.* δ – user distribution M_2

As pointed out in Section 4.5.1, the *socially optimal* configurations obtained applying objectives TOT_Q and MIN_Q are the same for most instances and they also provide very similar Q_{avg} (the largest difference across all values of δ is approximately 1.1 Mbps) which can be observed by the overlap of their corresponding plots (see Figure 4.1). Therefore, solutions that are fair to all users in all the areas are also efficient.

More BSs are activated by the *socially optimal* configurations than by *stable* ones (see Subsection 4.5.3) which is reflected in their corresponding Q_{avg} and P_{avg}. The difference in the Q_{avg} provided by the *socially optimal* configurations and *stable* ones for $\delta = 0.02$ is nearly 12.6 Mbps (45.8% gap); it goes down to 4.3 Mbps (8.1%) for $\delta = 1$ and eventually becomes nearly 1.8 Mbps (3.3%) for $\delta = 2$. Thus, for high δ, the two types of configurations provide roughly the same quality of service to the users if they are very interested in the new service.

As far as P_{avg} is concerned, for low values of δ, the difference in the P_{avg} provided by the two types of configurations is significantly different (see Figure 4.1). For $\delta = 0.02$, the configuration selected by TOT_Q provides on the average only 55.2 € per MNO, whereas the stable configurations provide 262306.3 €. This suggests that solutions obtained from objectives TOT_Q and MIN_Q merely satisfy the constraint on having a positive profit while providing, on the average, a 12.6 Mbps higher user rate. However, with the increase of δ, the difference in rate between the two types of configurations becomes negligible, and so does the difference in profit (only 2.8% for $\delta = 2$).

So far we have investigated the average performance indicators (Q_{avg} and P_{avg}). We now analyze how the user rate per area and MNO (Q) and profit per area and MNO (P) are affected by the characteristics of MNOs (market share) and by the characteristics of the areas (size and population, reported in Table 4.3) for both configurations.

Figure 4.2 illustrates the behavior of Q with respect to P in each area, for each MNO for the user distribution M_2 when $\delta = 0.02$. We recall that, when $\delta = 0.02$, the grand coalition (ABC) is *socially optimal* (for both TOT_Q and MIN_Q objectives) and *stable*. For this scenario we can observe that: (i) the *socially optimal* configurations provide in every area higher user rates than the *stable* one, which in turn guarantee higher revenues, (ii) the grand coalition results in all MNOs providing the same user rate to users of the same area, while their profit follows their market shares (see Equations (4.12), (4.24)), (iii) in area Z3, MNOs obtain a negative profit under objective TOT_Q, while the global profit for each MNO is positive, which indicates that a negative balance between costs and revenues can be accepted in some areas by the *socially optimal* configurations, (iv) the objective that favors fairness (MIN_Q) improves the quality of service of the users of the largest area (Z1) and most congested area (Z2) at the cost of lowering the user rate of area Z3 and (v) since the user rate provided by a given coalitional structure in an area depends on the user density, on the size of the area and on the number of BSs activated in that area, a slightly higher user rate is achieved for the small, low user density area (Z3) by the *socially optimal* configurations as the LTE nominal rate is divided among less users and on the average users are closer to their serving BSs.

4.6 Conclusions

This work analyzes the strategic situation in which MNOs have to decide whether to invest in LTE small cells in dense urban areas and whether to share the investment with other MNOs. A mathematical framework is proposed to address the problem of infrastructure sharing for

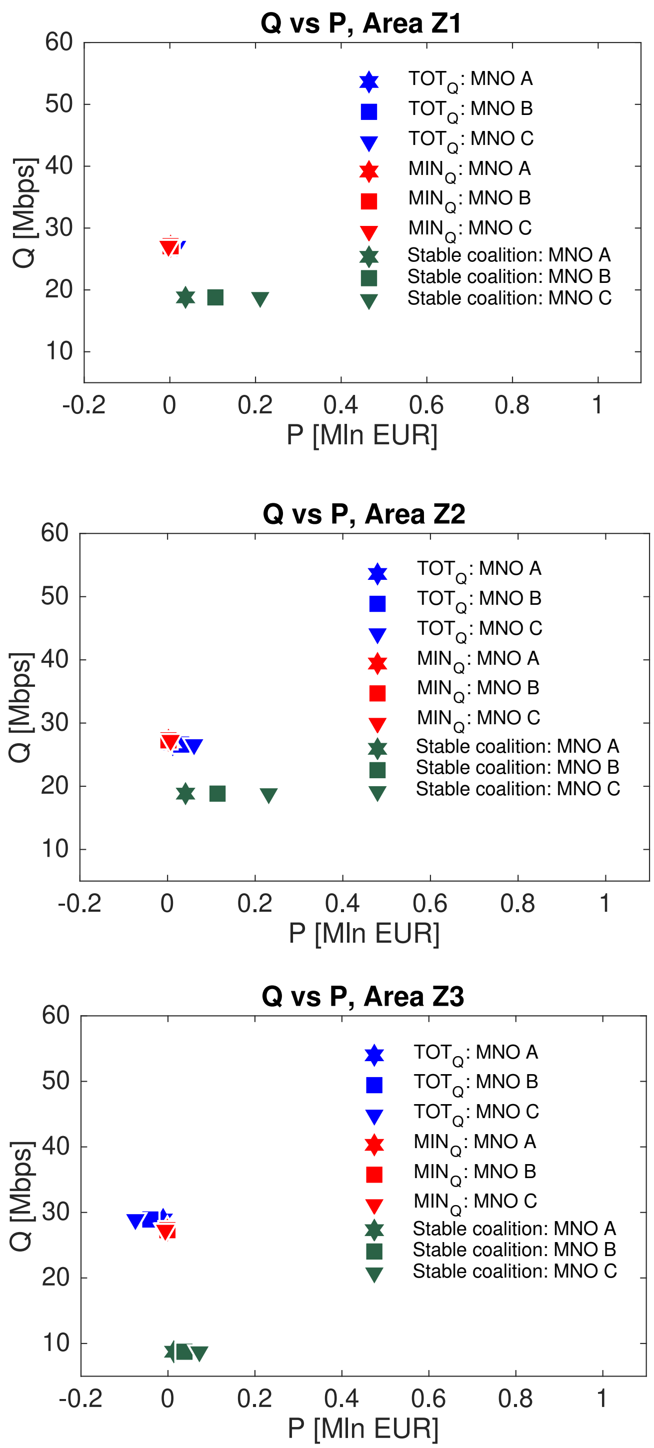

Figure 4.2 User rate (Q) *vs.* profit (P) for each area and MNO – user distribution M_2, $\delta = 0.02$.

the considered scenario. This framework accounts for techno-economic parameters such as the achievable throughput and a general pricing model for the LTE service. The problem has been tackled from two perspectives: the one of a regulatory entity which imposes infrastructure sharing configurations that optimize the quality of service perceived by all users and the MNOs perspective, which captures their competitive and profit-maximizing nature. We propose an MILP formulation to determine socially optimal configurations (regulator perspective) and adopt concepts of cooperative game theory to determine stable configurations (MNOs perspective).

Results show that sharing configurations obtained under both perspectives are strongly affected by how much users are willing to pay for the new services but they also depend on the user distribution (MNOs market shares). Sharing is appealing from both perspectives when users are willing to pay little, regardless of the MNOs market shares as they all struggle with high infrastructure cost. Instead, if users were willing to pay more, there is generally more incentive to share from the MNO perspective and in particular when MNOs have significantly different market shares. For both perspectives, the selected configurations involve less congested coalitions, that is, coalitions of fewer and smaller MNOs, when the market shares are significantly different. When the focus is on the quality of service, such configurations behave very similarly to the case when no sharing takes place, that is, users are best served either by less congested coalitions or when all MNOs build individual networks.

The proposed mathematical framework has proved to be a flexible instrument of limited complexity to analyze in detail the possible strategies for different infrastructure sharing configurations under different techno-economic conditions. It can be further extended to incorporate spectrum management issues and therefore more elaborated game theory models, as well as different classes of users and heterogeneous technologies.

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CHAPTER 5 ARTICLE 2 : COOPERATIVE INFRASTRUCTURE AND SPECTRUM SHARING IN HETEROGENEOUS MOBILE NETWORKS

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Abstract

To accommodate the ever-growing traffic load and bandwidth demand generated by mobile users, Mobile Network Operators (MNOs) need to frequently invest in high spectral efficiency technologies and increase their hold of spectrum resources; MNOs have then to weigh between building individual networks or entering into network and spectrum sharing agreements. We address here the problem of Radio Access Network (RAN) and spectrum sharing in 4G mobile networks by focusing on a case when multiple MNOs plan to deploy small cell Base Stations in a geographical area in order to upgrade their existing network infrastructure. We propose two cooperative game models (with and without transferable utility) to address the proposed problem: for given network (user throughput, MNO market and spectrum shares) and economic (coalition cost, mobile data pricing model) settings, the proposed models output a cost division policy that guarantees coalition (sharing agreement) stability.

5.1 Introduction

The mobile network ecosystem is intrinsically competitive as Mobile Network Operators (MNOs) are self-interested entities. However, expensive technology upgrades to support user demand [167], revenues decline [182], regulators intervention [183] and communities health/environmental concerns are pushing competing operators to cooperate and share their networks. Nevertheless, cost reduction remains the main driver for network sharing: The mobile market is characterized by high upfront cost for acquiring spectrum licenses and deploying and operating the network infrastructure, which is particularly heavy on new-entrants [183]. Nowadays, MNOs need to invest in high spectral efficiency mobile technologies such as LTE-A and 5G and, in particular, to increase their hold of spectrum resources in order to accommodate the exponentially growing demand for mobile data services [167]. To the high upgrade upfront cost amounts the decrease in revenues, strongly due to Over The Top (OTT) applications replacing MNOs main revenue resources such as voice and SMS [184]:

but as investment in new technologies becomes little profitable, innovation is held back.

Network sharing agreements for greenfield network roll-outs have become a means to reduce the high upfront infrastructure cost and, when spectrum sharing is allowed[1], to boost the network capacity by aggregating spectrum resources.

Network sharing can encompass different parts of the network architecture in addition to having different geographical footprints [20, 21]. Passive sharing (site and mast sharing), the most commonplace sharing alternative, is either mandated (or strongly encouraged) by regulators or voluntary adopted given the limited site availability, urban planning constraints and communities aesthetics and health concerns [8, 21, 170]. Up to date, several 50:50 joint ventures for 3G/4G greenfield network deployments have been created; while most concern only sharing of the Radio Access Network (RAN) infrastructure, in some cases, also spectrum is shared [170, 185].

We address here the *common spectrum network sharing* scenario, as defined in the Third Generation Partnership Project (3GPP) specifications for network sharing [173]. Namely, we assume multiple MNOs offer their services through a single (shared) RAN while running individual core networks; "MNOs share the total spectrum obtained from pooling together their respective allocated spectrum portions while it is also possible for MNOs with no allocated spectrum to use the pooled spectrum" [173]. One viable option for implementing such scenario in 4G networks is Carrier Aggregation (CA), an LTE-A standardized feature [187] that enables pooling together the spectrum allocated in different bands[2].

In this work, we consider a set of MNOs with fixed market shares and individual spectrum licenses, which plan to upgrade their RAN by deploying small cell Base Stations (BSs) in order to improve the service provided to their users and thus increase their revenues. MNOs decide whether to upgrade their RAN by themselves or enter into a sharing agreement with other MNOs. If a set (all) of MNOs enters into a sharing agreement, that is, a coalition is created, we assume it will make use of all the aggregated spectrum resources of its members. We propose two cooperative game theory models, with and without transferable utility, to determine stable cost divisions for coalitions of MNOs entering sharing agreements. The proposed models are then leveraged to investigate several network (user throughput, market and spectrum shares) and economic configurations (coalition cost, mobile data pricing model)

[1]In practice, both infrastructure and spectrum sharing viability are subject to national and regional/international regulation. Sharing and/or transferring of licensed spectrum is prohibited in most countries [6]. There are however examples of spectrum sharing: for instance, in Sweden, operator Tele2 is into a 3G license and network sharing agreement with TeliaSonora and it has entered a similar agreement for deploying a 4G network with Telenor [185], [186].

[2]116 operators have commercially launched LTE-A with CA [188]. CA will most likely be an enabler also for future generation networks given the 5G throughput targets.

which aim to represent realistic scenarios.

The main findings of this work are the following:

- If all MNOs contribute with spectrum resources, they prefer building a unique shared RAN due to the combined gain of spectrum aggregation and the cost reduction from sharing the network infrastructure; formally, this means that the reference cooperative game has a nonempty core, which makes the grand coalition preferable to any subcoalition.

- The stable division (among MNOs) of the shared network infrastructure cost depends on both network and economic settings: MNOs with a larger customer base should be accounted for a larger fraction of the network cost; instead, MNOs which contribute with a larger spectrum portion are "rewarded" by a lower cost fraction and, in some cases, not only they are exempted from such cost but also receive part of the other MNO individual revenues, which suggests a way to compensate them from most likely higher cost incurred when acquiring the spectrum license[3].

- A trivial cost division based on the market share does not always guarantee stability; instead, the stable cost division selected by the nucleolus, which accounts also for the MNOs spectrum contribute, makes a better candidate for a cost division policy.

The manuscript is organized as follows: The literature review is presented in Section 5.2. In Section 5.3, we state the problem and define the coalitions cost and the proposed pricing model. The Transferable Utility (TU) and Non-Transferable Utility (NTU) cooperative models are introduced in Section 5.4. The simulation environment and the problem instances are described in Section 5.5. Results obtained with the two cooperative models are analyzed in Section 5.6. In Section 5.7, we discuss some of the assumptions made and the applicability of the proposed models. Our concluding remarks are drawn in Section 5.8.

5.2 Related work

Recent works on resource sharing deal mainly with operational aspects, such as scheduling of shared resources among multiple operators. [45] adopts the Generalized Processor Sharing principle to a multi-operator scheduler when operators agree a priori on their respective resource shares. In the same lines, [189] investigates the trade-off between fairness, that

[3] However, we do not account for the spectrum license cost here. Moreover, the spectrum license cost does not depend only on the amount of bandwidth associated with the license but also on the spectrum band and and the time and place of the spectrum auction.

is, satisfying operators predefined resource shares and the achievable spectral efficiency by deviating from predefined resource shares. [190] introduces the SoftRAN architecture, which extends the concept of Software Defined Networks (SDN) to the RAN. A centralized scheduler for SoftRAN is proposed in [47]: the traffic of multiple operators is allocated over the 3D (time-frequency-space) resource grid with the objective of maximizing the total network utility. [48] proposes a 2-level radio resource scheduling (among MNOs, and for each MNO among its user flows) BS virtualization scheme.

Given the competitive and cooperative nature of resource sharing problems, many works resort to game theory: In [49], the problem of resource allocation in a shared network is formalized in two steps: the resource sharing among the operators, and the resource bargaining among the users and Mobile Virtual Network Operators of each operator. [130] investigates the sharing of different wireless access technologies. In particular, [141–144] tackle spectrum sharing problems. [141] models spectrum sharing among strategic operators in unlicensed bands as a noncooperative game. Instead, [142–144] deal with licensed spectrum. [144] also proposes a noncooperative game but takes the user perspective: assuming MNOs with individual spectrum resources aggregate their RANs, each user then independently selects its serving BS from the shared pool in order to maximize its individual data rate. [142] extends the concept of CA for limited-time sharing of excess spectrum among MNOs that own exclusive spectrum resources. Spectrum scheduling is carried out based on the Nash Bargaining Solution concept while a distributed algorithm is proposed for Bayesian coalition formation when the MNO decisions are made based on incomplete information. In [143], the inter-operator CA does not apply only to the MNOs' unutilized spectrum but, if profitable, MNOs can agree to share a portion of their individual spectrum between their own users and users of another MNO; the level of interference caused by the latter is controlled by means of a pricing mechanism. Such sharing scheme is limited only to two MNOs and the pairing of a set of MNOs for mutually sharing part of their spectrum is modeled as a stable roommate market.

As virtualization and SDN are expected to extend to wireless networks ([191], [192]), new architectures are anticipated ([47], [93], [94]). [93] and [94] envisage a "Network without Borders", as the pool of virtualized wireless resources which defies the current vertically-integrated mobile networks value-chain by introducing new players such as service/infrastructure providers and virtual operators. Inter-operator sharing is argued to be one of the key ingredients of such architecture. The idea is further elaborated in [94], where the focus is on novel spectrum management aspects.

Our work instead belongs to a complementary research branch, whose focus is on the strategic modeling of infrastructure and spectrum sharing. In particular, we consider the sharing of exclusive (licensed) MNO spectrum[4].

On mid-to-long term joint decision making in the context of cellular network planning, [153] introduces a competition-aware network sharing framework which offers a trade-off between the cost benefit of sharing and the incentive for investing in next-generation technologies. While we address a greenfield-deployment of small cell BSs, [153] assumes operators will pool together their existing macro-cell RAN networks and make joint decisions on future changes to their aggregated RAN such as decommissioning, upgrading or adding new sites. Also in [194], a recent work by Kibilda et al., the shared network is created by pooling together the operators' individual network infrastructure and/or their respective licensed spectrum. This work compares the gains from infrastructure and spectrum sharing when adopted separately and combined (full sharing) on the basis of classical performance indicators such as throughput and coverage probability obtained by means of stochastic geometry models; such gains are shown to strongly depend on the spatial correlation of the individual network deployments and densities while infrastructure and spectrum sharing gains do not sum up as full sharing introduces a trade-off between data rate and coverage.

However, [149–151] are the only works which bear explicit similarities with ours: they also tackle the strategic problem of coalition formation in the context of infrastructure and spectrum sharing and consider MNOs with fixed market shares and pre-allocated spectrum; nevertheless, these works resort to non-cooperative game theory. Moreover, players (MNOs) payoffs are expressed only in terms of network cost estimates and the coalition cost are split either uniformly among its members [149, 151] or according to the Shapley value [150]. Instead, we propose more refined payoff models for the MNOs which accounts for both the MNO revenue (as a function of the average user rate perceived by users) and cost. Moreover, in the cooperative games proposed here the coalition cost is not divided a priori among member operators; albeit the way such cost is split determines the coalition stability.

In our previous work [175], we propose a Mixed Integer Linear Programming model to address an infrastructure sharing problem from a centralized/regulatory entity perspective. Instead, in this work and in [165], we take the perspective of the MNOs, which are self-interested entities, and thus resort to game theory models. Further, in [165] we tackle the problem of spectrum and infrastructure sharing addressed here by a non-cooperative approach and formalize it as a generalized Nash equilibrium problem, where the operators strategies consist of the choice of coalition and the fraction of coalition cost to pay. However, the non-cooperative

[4] The literature on dynamic spectrum access, cognitive/software radio etc. dealing with sharing of licensed and unlicensed spectrum [193] is not addressed here.

approach limits the stability analysis to the action of the single player, while the cooperative approach allows to determine whether a coalition is stable or not also in terms of joint actions of its members.

5.3 The Problem

5.3.1 Problem definition

We consider a set $\mathcal{O}$ of MNOs which provide data services to users of a dense urban area through pre-4G macrocell networks but have plans to upgrade their RAN technology by deploying 4G small cells. We assume MNOs inherit the user share from their individual current networks: being N the number of users that populates the given area, each MNO $i \in \mathcal{O}$ has a fixed market share σ_i, that is, user churning is assumed to be null. We also assume that at least one MNO owns a spectrum license of b_i units of bandwidth[5] which it plans to put to use for the network of small cells. Each MNO may decide to deploy its individual network of small cells or collaborate with other MNOs to deploy a shared one. When a set of MNOs decides to deploy a shared network, we assume they will agree on aggregating their individual spectrum. Let $\mathcal{S}$ be the set of all possible coalitions that can be created, that is, the set of all the possible subsets of MNOs agreeing to deploy a shared network. If coalition $s \in \mathcal{S}$ is created, it will deploy a shared network infrastructure of total cost $\widetilde{c}_s$ which has to be divided among its member MNOs. Applying a simple data pricing model, each MNO i incurs revenues $\widetilde{r}_s^i$ from its user subscriptions when in s. The case when MNOs in s agree to share the coalition cost $\widetilde{c}_s$ but keep their individual revenues $\widetilde{r}_s^i$ is formalized as a cooperative game without transferable payoffs. Alternatively, the case when MNOs would be willing to give away also part of their revenues is modeled as a cooperative game with transferable payoffs. The core and nucleolus solution concepts are then leveraged to determine stable cost divisions.

5.3.2 Cost and revenues definition

Since this work addresses sharing at the RAN, the adopted cost model accounts only for radio equipments cost and a simplified leased line pricing model for the backhaul transmission cost as in [50]. Moreover, in [195], it is argued that the cost associated with the RAN dominates the remaining cellular network cost.

An investment period of duration D (months) has been considered. Let g_s be the total cost incurred in D by coalition s from activating and operating one small cell BS: g_s accounts

[5] MNOs with no spectrum license are represented by $b_i = 0$.

for the capital (CAPEX) and operational (OPEX) expenditures of the radio equipment, the backhaul transmission cost and the site build-out cost. We denote by $\widetilde{b}_s$ the aggregated spectrum of coalition s, that is, $\widetilde{b}_s = \sum_{i \in s} b_i$, whereas by β_s the number of MNOs in s which own a spectrum license, that is, $\beta_s = |\{i \in s: \ b_i > 0\}|$. Let $g^{\mathrm{c,r}}_{\mathrm{small}}$ be the radio equipment CAPEX of a typical small cell BS supporting a single carrier. Given that a small cell BS activated by coalition s aggregates β_s carriers, it has to support $\beta_s - 1$ additional carriers. As in [196], we consider a fixed cost for each additional carrier, calculated as a percentage ϕ of the cost $g^{\mathrm{c,r}}_{\mathrm{macro}}$ of a single-carrier macrocell BS. The total radio equipment CAPEX of a small cell BS activated by coalition s, $g^{\mathrm{c,r}}_s$, is then given as follows:

$$g^{\mathrm{c,r}}_s = g^{\mathrm{c,r}}_{\mathrm{small}} + (\beta_s - 1)\phi g^{\mathrm{c,r}}_{\mathrm{macro}}. \tag{5.1}$$

The Operations and Maintenance (O&M) annual cost of the radio equipment is calculated as a percentage ξ of the corresponding total radio CAPEX $g^{c,r}_s$ [50, 177]. The considered backhaul leased line pricing model consists of an upfront fee $g^{\mathrm{c,b}}$ and the annual leasing cost $g^{\mathrm{o,b}}_s$ which are incurred for each BS activated by coalition s. We assume that, in the worst case, $g^{\mathrm{o,b}}_s$ is proportional to the total amount of spectrum (bandwidth) aggregated by any of the BSs of coalition s ($\widetilde{b}_s$). Let $g^{\mathrm{o,b}}_0$ be the annual leased line cost for a reference carrier of b_0 units of bandwidth. We then set $g^{\mathrm{o,b}}_s$ equal to $\widetilde{b}_s\, g^{\mathrm{o,b}}_0 / b_0$. Let $g^{c,s}$ denote the site build-out cost. Finally, the total cost g_s incurred by coalition s from a single small cell BS in D is given by:

$$g_s = g^{\mathrm{c,r}}_s + g^{\mathrm{c,b}} + g^{\mathrm{c,s}} + \frac{D}{12}\left(\xi g^{\mathrm{c,r}}_s + g^{\mathrm{o,b}}_s\right). \tag{5.2}$$

The considered cost parameter values (Table 5.1) refer to HSPA technology as in [50, 196], given that, to the best of our knowledge, CA-enabled equipment cost are not made publicly available by any vendor. Such cost should nevertheless represent a good estimate, at least in orders of magnitude, since as argued by Johansson et al. [196], the physical infrastructure cost of new radio access technologies tend to be similar to the previous ones.

As in [175] and [165], the revenues $\widetilde{r}^i_s$ incurred by MNO i in coalition s are calculated according to a simple data service pricing model, where the latter is defined in terms of the average data rate perceived by users of s. Let $\rho^{nom}_s(u_s)$ be the nominal rate coalition s can provide to its users by activating u_s BSs. For a given level of Signal to Interference and Noise Ratio (SINR) and a given system bandwidth, the nominal user rate in LTE is the maximum rate perceived by a single user when assigned all downlink LTE resource blocks from its serving BS. The downlink SINR is a function of the number of BSs activated by the coalition the

Table 5.1 BS cost model parameters

Symbol	Description	Value
$g^{\mathrm{c,r}}_{\mathrm{small}}$	Single-carrier small cell BS radio equipment cost	3000€ [50]
$g^{\mathrm{c,r}}_{\mathrm{macro}}$	Single-carrier macro cell BS radio equipment cost	20000€ [50]
ϕ	Cost coefficient per additional carrier	0.017 [196]
$g^{c,b}$	Upfront fee for backhaul	2000€ [50]
b_0	Bandwidth of the reference carrier	5 MHz [50]
$g^{\mathrm{o,b}}_0$	Annual leased line cost of the reference carrier	2000€ [50]
$g^{\mathrm{c,s}}$	Site buildout cost	2000€ [50]
ξ	O&M annual percentage	15% [177]

user belongs to: a larger number of BSs results in the user being on the average closer to its serving BS, and therefore receiving a stronger signal, but also closer to the interfering ones[6]. The average rate $\rho_s(u_s)$ perceived by a user in coalition s can be defined in terms of $\rho_s^{nom}(u_s)$ and of the load of its serving BS:

$$\rho_s(u_s) = \rho_s^{nom}(u_s)(1-\eta)^{\frac{\sum_{i\in s}\sigma_i N}{u_s}}, \qquad \forall s \in \mathcal{S}, \tag{5.3}$$

where parameter η is the user activity factor representing the probability that a user is actually active in his/her serving BS, $\sum_{i\in s}\sigma_i N$ is the total number of users of coalition s whereas $(\sum_{i\in s}\sigma_i N)/u_s$ gives the average number of users served by one BS. To obtain $\rho_s(u_s)$, the nominal rate is then scaled down by the factor $(1-\eta)^{(\sum_{i\in s}\sigma_i N)/u_s}$ representing the average congestion level at a serving BS.

Let δ denote the monthly price per user and per unit (Mbps) of data service. As $\rho_s(u_s)$ represents the average user rate provided by coalition $s \in \mathcal{S}$ when it activates u_s BSs [7], the revenues r_s^i each member MNO $i \in s$ can incur when in s at the end of the investment lifetime D, are then modeled linearly in $\rho_s(u_s)$:

$$r_s^i = \delta D \sigma_i N \rho_s(u_s), \qquad \forall i \in s. \tag{5.4}$$

Let $\tilde{u}_s$ be the number of BSs that maximizes the global return on investment of coalition s

[6]When calculating the nominal user rate, any other BS transmission will use at least a subset of the available resource blocks and therefore unavoidably interfere.

[7]The simulation set up to obtain $\rho_s(u_s)$ as a function of u_s for each $s \in \mathcal{S}$ is explained in details in Section 5.5.1.

calculated as:

$$\widetilde{u}_s = \underset{\substack{u_s \in \mathbb{Z}_+ \\ u_s \leq U_{\max}}}{\operatorname{argmax}} \left(\sum_{i \in s} \delta D \sigma_i N \rho_s(u_s) - g_s u_s \right), \qquad \forall s \in \mathcal{S}, \tag{5.5}$$

where $U_{\max}$ is the maximum number of small cell sites coalition s can activate in the area.

Finally, the revenues $\widetilde{r}_s^i$ of MNO i from coalition s and the total cost $\widetilde{c}_s$ of coalition s are the following:

$$\widetilde{r}_s^i = \delta D \sigma_i N \rho_s(\widetilde{u}_s), \qquad \forall s \in \mathcal{S}, \forall i \in s, \tag{5.6}$$

$$\widetilde{c}_s = g_s \widetilde{u}_s, \qquad \forall s \in \mathcal{S}. \tag{5.7}$$

5.4 Cooperative game models

In this section we describe the two cooperative game theory models we developed for the problem. The first one is a Non Transferable Utility (NTU) game, namely we assume that players share the network infrastructure cost, but each keeps its own revenue (Section 5.4.1), while the second one is a Transferable Utility (TU) game, namely, beside sharing the cost, we allow players to partially transfer their revenue to others (Section 5.4.2).

The NTU model represents indeed a more intuitive scenario as MNOs incur revenues from their individual share of users. However, we define the player payoffs in terms of their profits, i.e., as revenues minus cost; therefore, if an MNO benefits from being in a coalition, e.g., due its aggregated spectrum resources, and has no incentive to leave the coalition, even when giving away part of its revenue to the others, then this is worth being investigated by means of the TU model. In other words, the TU model allows to analyze at what extent a coalition is valuable to the MNOs.[8]

For both games, we want to determine whether the grand coalition is selected and, if so, how to make it stable. Thus, we look for the elements of the core, namely the payoff allocations which guarantee that there is no incentive neither for an MNO to leave the grand coalition and build a network by itself nor for any subset of MNOs to create their own coalition/shared network. In other words, whenever the core is nonempty, the grand coalition is preferred by all MNOs.

[8]See Section 5.6 for numerical examples.

5.4.1 A Non Transferable Utility game model

We model the problem as a NTU cooperative game $(\mathcal{O}, V)$, where the set of players coincides with the set $\mathcal{O}$ of MNOs. The set-valued mapping V assigns a set of feasible payoff vectors $V(s)$ to each coalition $s \in \mathcal{S}$ and is defined as follows:

$$V(s) = \left\{ (p_i)_{i\in\mathcal{O}} : \sum_{i\in s} p_i \leq \sum_{i\in s} \widetilde{r}_s^i - \widetilde{c}_s, \quad p_i \leq \widetilde{r}_s^i, \quad \forall\, i \in s \right\}.$$

The value of the payoffs p_i is bounded by the inequalities described above. Inequality $\sum_{i\in s} p_i \leq \sum_{i\in s} \widetilde{r}_s^i - \widetilde{c}_s$ guarantees that the sum of the payoffs of the players does not exceed the overall payoff, which is given by the difference between the sum of the revenues and the coalitional cost. Inequalities $p_i \leq \widetilde{r}_s^i$ make sure that the revenues are not transferred among players by limiting each player's payoff to its respective revenue.

We aim at determining whether the grand coalition is selected by the players or not, and how they decide to share the network cost among them. Thus, we study the core of the game, namely the set of payoff vectors that make the grand coalition preferable to any sub-coalition. To formally define the core, the Pareto efficient frontier F of the set $V(\mathcal{O})$ must be defined as follows:

$$F = \left\{ (p_i)_{i\in\mathcal{O}} : \sum_{i\in\mathcal{O}} p_i = \sum_{i\in\mathcal{O}} \widetilde{r}_{\mathcal{O}}^i - \widetilde{c}_{\mathcal{O}}, \quad p_i \leq \widetilde{r}_{\mathcal{O}}^i, \quad \forall\, i \in \mathcal{O} \right\}.$$

The core of the game is then defined as

$$C = F \setminus \bigcup_{s\in\mathcal{S}} \text{int } V(s),$$

where $\setminus$ denotes the difference between two sets and int denotes the interior of a set.

5.4.2 A transferable utility game model

If we assume that players may partially transfer their revenue to others, then the corresponding model is a Transferable Utility game $(\mathcal{O}, v)$, where $\mathcal{O}$ is the set of players and v is the characteristic function, i.e., a real-valued function which assigns to each coalition $s \in \mathcal{S}$ its overall payoff defined as

$$v(s) = \sum_{i\in s} \widetilde{r}_s^i - \widetilde{c}_s.$$

Notice that this TU game is equivalent to the NTU game described in Section 5.4.1 where the constraints $p_i \leq \widetilde{r}_s^i$ for any $i \in s$ are removed from the definition of $V(s)$.

Similarly to the NTU model, we are interested in finding the set of payoff vectors that make the grand coalition preferable to any sub-coalition, that is the core of the TU game, which is defined as

$$C = \left\{ (p_i)_{i\in\mathcal{O}} : \sum_{i\in\mathcal{O}} p_i = v(\mathcal{O}), \quad \sum_{i\in s} p_i \geq v(s), \quad \forall\ s \subset \mathcal{O} \right\}.$$

We remark that we have defined the characteristic function v assuming the joint strategy space of coalition s is the number of BSs it activates ($0 \leq u_s \leq U_{max}$). Further, the utility of s from activating u_s BSs is given by the corresponding global return on investment, $\sum_{i\in s} r_s^i(u_s) - c_s(u_s)$, where such utility depends only on u_s and it is not affected by the actions of $i \notin s$. Therefore, $\widetilde{u}_s$ represents the strategy of coalition s whereas $v(s) = \sum_{i\in s} \widetilde{r}_s^i - \widetilde{c}_s$ its overall payoff. Although the overall payoff of a coalition is determined maximizing its total return on investment, which does not necessarily maximize the individual return on investment of each of its member MNOs, when such payoff is distributed among them according to a solution in the core, no MNO has an incentive to deviate.

5.4.3 A two MNOs example

Assume that there are only two MNOs, i.e., $\mathcal{O} = \{A, B\}$. For the NTU game (see Figure 5.1a), the set of feasible payoff vectors corresponding to the grand coalition $\{A, B\}$ is

$$V_{NTU}(\{A,B\}) = \left\{ (p_A, p_B) \colon \begin{array}{l} p_A + p_B \leq \widetilde{r}^A_{\{A,B\}} + \widetilde{r}^B_{\{A,B\}} - \widetilde{c}_{\{A,B\}} \\ p_A \leq \widetilde{r}^A_{\{A,B\}} \\ p_B \leq \widetilde{r}^B_{\{A,B\}} \end{array} \right\}.$$

The Pareto efficient frontier F_{NTU} of the set $V_{NTU}(\{A, B\})$ is the line segment with extreme points

$$\Pi_A = \left(\widetilde{r}^A_{\{A,B\}}, \quad \widetilde{r}^B_{\{A,B\}} - \widetilde{c}_{\{A,B\}} \right),$$
$$\Pi_B = \left(\widetilde{r}^A_{\{A,B\}} - \widetilde{c}_{\{A,B\}}, \quad \widetilde{r}^B_{\{A,B\}} \right),$$

where in Π_A the coalitional cost $\widetilde{c}_{\{A,B\}}$ is entirely paid by MNO B, while in Π_B it is entirely paid by A. Therefore, the core C_{NTU} (the bold segment in the figure) is obtained as the difference between F_{NTU} and the union of the interiors of the two halfplanes representing the feasible payoffs for the single player coalitions:

$$\text{int } V(\{A\}) = \{(p_A, p_B) :\ p_A < v(\{A\})\},$$
$$\text{int } V(\{B\}) = \{(p_A, p_B) :\ p_B < v(\{B\})\},$$

where $v(\{A\}) = \widetilde{r}^A_{\{A\}} - \widetilde{c}_{\{A\}}$ and $v(\{B\}) = \widetilde{r}^B_{\{B\}} - \widetilde{c}_{\{B\}}$.

In fact, such halfplanes represents the sets of vectors of payoff such that the single player would earn more alone than joining the grand coalition. Therefore, we can write the core C_{NTU} as follows:

$$C_{NTU} = \left\{ (p_A, p_B) : \begin{array}{l} p_A + p_B = \widetilde{r}^A_{\{A,B\}} + \widetilde{r}^B_{\{A,B\}} - \widetilde{c}_{\{A,B\}} \\ v(\{A\}) \leq p_A \leq \widetilde{r}^A_{\{A,B\}} \\ v(\{B\}) \leq p_B \leq \widetilde{r}^B_{\{A,B\}} \end{array} \right\}.$$

Instead, for the TU game (see Figure 5.1b), the set of feasible payoff vector is the halfplane

$$V_{TU}(\{A,B\}) = \left\{ (p_A, p_B) \colon p_A + p_B \leq \widetilde{r}^A_{\{A,B\}} + \widetilde{r}^B_{\{A,B\}} - \widetilde{c}_{\{A,B\}} \right\},$$

the Pareto efficient frontier is the line

$$F_{TU} = \left\{ (p_A, p_B) : \ p_A + p_B = \widetilde{r}^A_{\{A,B\}} + \widetilde{r}^B_{\{A,B\}} - \widetilde{c}_{\{A,B\}} \right\},$$

and the core

$$C_{TU} = \left\{ (p_A, p_B) : \begin{array}{l} p_A + p_B = \widetilde{r}^A_{\{A,B\}} + \widetilde{r}^B_{\{A,B\}} - \widetilde{c}_{\{A,B\}} \\ p_A \geq v(\{A\}) \\ p_B \geq v(\{B\}) \end{array} \right\}.$$

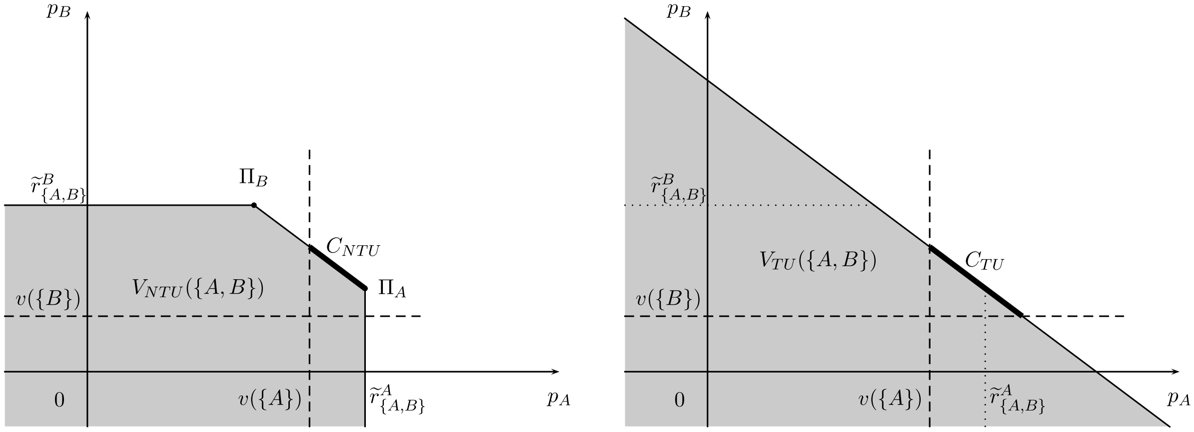
Notice that the core of the NTU game is a subset of the TU one. In the example depicted in Figure 5.1, the grand coalition provides MNO B with a strictly positive margin with respect to investing alone, even if it pays the entire $\widetilde{c}_{\{A,B\}}$ cost, that is, $\widetilde{r}^B_{\{A,B\}} - \widetilde{c}_{\{A,B\}} - v(\{B\}) > 0$. Therefore, it is still profitable for B to be in the grand coalition even it transfers part of its revenues to A (represented by payoff vectors in $C_{TU} \setminus C_{NTU}$).

The following relations between the parameter values determine whether the core of each two players game is empty or not:

1) If $v(\{A,B\}) < v(\{A\}) + v(\{B\})$, that is,

$$\widetilde{c}_{\{A,B\}} - \left(\widetilde{c}_{\{A\}} + \widetilde{c}_{\{B\}} \right) > \left(\widetilde{r}^A_{\{A,B\}} + \widetilde{r}^B_{\{A,B\}} \right) - \left(\widetilde{r}^A_{\{A\}} + \widetilde{r}^B_{\{B\}} \right),$$

then the core of both games is empty as both MNOs are better off investing alone. Roughly speaking, if the additional revenues generated from the grand coalition do not cover its additional cost, than the grand coalition is not stable.



(a) NTU game (b) TU game

Figure 5.1 A two-players example: Pareto frontier and core

2) Otherwise, if $v(\{A,B\}) \geq (v\{A\})+v(\{B\})$, then the core of the TU game is nonempty. In particular, when $v(\{A,B\}) = v(\{A\}) + v(\{B\})$, it consists of a single payoff vector, $\left(p_A = v_{\{A\}},\, p_B = v_{\{B\}}\right)$, which corresponds to the case when MNOs are indifferent between cooperating or not. As for the NTU game:

   2a) If either $v(\{A\}) > \widetilde{r}^A_{\{A,B\}}$ or $v(\{B\}) > \widetilde{r}^B_{\{A,B\}}$, then the core of the NTU game is empty. Notice that $v(\{A\}) > \widetilde{r}^A_{AB}$ means MNO A is better off alone, even if B could pay for the entire $\widetilde{c}_{\{A,B\}}$ cost.

   2b) Otherwise, if $v(\{A\}) \leq \widetilde{r}^A_{\{A,B\}}$ and $v(\{B\}) \leq \widetilde{r}^B_{\{A,B\}}$, then also the core of the NTU game is nonempty.

5.5 Computational tests

5.5.1 Simulation environment

As in [175] and [165], a simulation environment has been set up in Matlab to obtain the average user rate $\rho_s(u_s)$ for each coalition s as function of the number u_s of activated small cell BSs varying from 1 to $U_{\max}$. The u_s BSs and 10 sample users are uniformly distributed in a pseudo-random fashion on the considered square area. The downlink SINR for a reference system bandwidth of a sample user of coalition s, when s activates u_s BSs, is given by:

$$SINR_s = \frac{P_i}{l_s\left(\sum\limits_{\substack{1\leq j\leq u_s\\ j\neq i}} P_j\right) + P_{noise}}, \qquad \forall s \in \mathcal{S}, \tag{5.8}$$

where P_i is the signal power the sample user receives from its serving BS, whereas $\sum_{1\leq j\leq u_s,\ j\neq i} P_j$ is the power received from interfering (non-serving) ones. The received signal power is calculated according to the following three-parameter path loss model (transmitted signal power P_{tx}, fixed path loss C_{pl} and path loss exponent Γ), defined within the Green-Touch Consortium [181]:

$$P_{rx}[dBm] = P_{tx}[dBm] - C_{pl}[dB] - 10\Gamma log(d[km]), \tag{5.9}$$

where d is the sample user–BS distance. The captured interference is then scaled down by the load of coalition s, $l_s = 1 - (1-\eta)^{\frac{\sum_{i\in s}\sigma_i N}{u_s}}$, as users are characterized by an activity factor η. P_{noise} is the white gaussian noise power for the reference system bandwidth.

The resulting SINR is then mapped to LTE spectral efficiency according to a multilevel SINR–to–spectral efficiency scheme [181]. Multiplying the obtained spectral efficiency by the coalition aggregated bandwidth $\widetilde{b}_s$, we obtain the nominal user rate $\rho_s^{nom}(u_s)$. 100 simulation iterations are run for each value of u_s so that an average value for $\rho_s^{nom}(u_s)$ is obtained across all sample users and iterations. Finally, $\rho_s(u_s)$ is obtained from $\rho_s^{nom}(u_s)$ as defined in (5.3).

5.5.2 Instances

We consider instances with 3 MNOs [9], namely A, B and C and a 4 km^2 area populated by 20000 users. $U_{\max}$ is set to 10000, which is an arbitrarily large number of small cells for the considered area size; however, the number of activated small cells by any coalition does not exceed 1500 for all the considered instances. Parameter δ, which represents the monthly price per unit of service and per user, is set equal to equidistant values obtained discretizing the range [0.5,3] with a 0.01 step. We set up 5 scenarios (**S1**–**S5**) with different mixtures of market shares and "spectrum shares"[10] as shown in Table 5.2. The values of the bandwidth associated with the spectrum license of each MNO b_i, are set to standardized bandwidths for LTE/LTE-A ({1.4, 3, 5, 10, 15, 20} MHz) [187]. In particular, scenarios **S4** and **S5** aim at representing cases that may arise under traditional and recent design of spectrum auctions. The extreme case in which only one MNO in the area has succeeded to obtain a spectrum license from the latest auction has been considered; we assume such MNO is either the *smallest MNO* (**S4**), for instance, a new entrant which has benefited from a set-aside

[9] The considered number of MNOs is common for most countries, as far as facility-based operators are concerned [197]. [144] and [151] also consider 3 MNOs. Nevertheless, the proposed approach can be easily extended to more MNOs.

[10] The term "spectrum share" is used analogously with market share to represent the weight of the spectrum of an MNO w.r.t. to the total obtained aggregating the spectrum of all MNOs ($b_i/\sum_{j\in\mathcal{O}} b_j$).

spectrum policy[11] [183], or *the incumbent* (**S5**), which is the most likely to be the highest bidder in a traditional auction.

Table 5.2 Scenarios

	S1			**S2**			**S3**			**S4**			**S5**		
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
σ_i	1/3	1/3	1/3	1/3	1/3	1/3	0.1	0.3	0.6	0.1	0.3	0.6	0.1	0.3	0.6
b_i	5	5	5	1.4	5	10	5	5	5	15	0	0	0	0	15

Parameters notation and their corresponding values are summarized in Table 5.3.

Table 5.3 Sets, parameters and corresponding values

Symbol	**Description**	**Value**
$\mathcal{O}$	Set of MNOs	$\{A,B,C\}$
$\mathcal{S}$	Set of nonempty coalitions	$2^{\mathcal{O}} \setminus \emptyset$
N	Total number of users in the area	20000
A	Area size	4 km^2
$U_{\max}$	Max. number of BSs in the area	10000
δ	Monthly price of 1 Mbps	[0.5,3] €/Mbps
D	Investment lifetime	120 months [176]
η	User activity factor	0.01

5.6 Numerical results analysis

In this section we discuss the results obtained applying first the NTU game model and then the TU game model. Our goal is to highlight the impact of the three main parameters of the problem, namely δ, market share and spectrum share, on the existence of the core and on its features and on the nucleolus. The nucleolus is a well known solution of NTU and TU games (see, e.g., [199, 200]). We use the nucleolus as a suggested solution, as it always belongs to the core, if the core is nonempty, and therefore represents a stable way of assigning payoffs to players. Roughly speaking, the nucleolus minimizes the largest dissatisfaction of the coalitions, thus reducing the inequity among the coalitions, where the dissatisfaction is related to the difference between the coalition value and what its members receive according to the nucleolus.

[11]Despite some countries regulator efforts to encourage competition in mobile networks, by introducing spectrum set-asides during auctions and relaxing their coverage requirements, new entrants do not always succeed in deploying a network which may lead to inefficient spectrum allocations or eventually with the set-aside spectrum ending up in the hands of incumbent MNOs [198].

Across all considered scenarios, when $\delta \in [0.5, 0.53]$, neither the grand coalition nor sub-coalitions find it profitable to invest, thus the core is trivial as it collapses to only one point corresponding to zero investment and thus zero revenues. Instead, for $\delta \in [0.58, 3]$, the core of both games is nonempty (see Table 5.4).

Therefore in Sections 5.6.1 and 5.6.2, we focus our analysis on instances with a nonempty core. For such instances, as payoff allocations in the core make the grand coalition preferable to any subcoalition, only the grand coalition is analyzed. Instead, for the few particular instances with an empty core, we investigate subcoalitions (Section 5.6.3).

In Section 5.6.4, we assess the MNOs' gain from sharing with respect to building individual networks.

Figures 5.2 and 5.3 report the core of the NTU and the TU game, respectively, for $\delta \in$\{0.75, 1.5, 3\} for each considered scenario (**S1**-**S5**).

As we are interested in how the players share the network infrastructure cost, we introduce three values α_A, α_B and α_C which represent the fraction of the overall cost paid by player A, B and C, respectively. The payoff of a player i in the grand coalition $\mathcal{O}$ can be therefore written as

$$p_i = \tilde{r}^i_{\mathcal{O}} - \alpha_i \tilde{c}^i_{\mathcal{O}}.$$

The core is represented in the (α_A, α_B) plane, as $\alpha_C = 1 - \alpha_A - \alpha_B$.

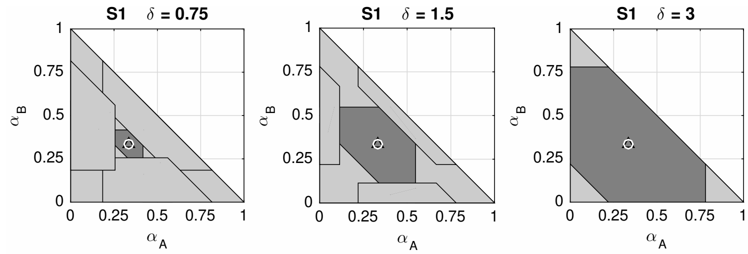
As for the NTU game, in each sub-figure of Figure 5.2 the Pareto efficient frontier F is represented by the triangle with vertices $(0, 0)(0, 1)(1, 0)$, where the diagonal line connecting $(0, 1)$ and $(1, 0)$ represents the payoff values such that $\alpha_C = 0$. The light grey areas represent the sets $V(s)$ for all the sub-coalitions. The core is thus represented by the dark grey area. Beside the core, the nucleolus is reported with a white circle and the market share with a black triangle.

As for the TU game, in each sub-figure of Figure 5.3 the core is represented by the dark grey area, while dashed lines represent lines $\alpha_A = 0$, $\alpha_B = 0$ and $\alpha_C = 0$. The color and symbolic code of Figure 5.3 are the same as the one of Figure 5.2.

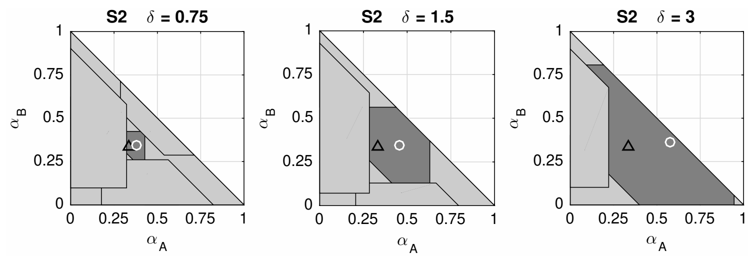
5.6.1 NTU game results

Impact of δ

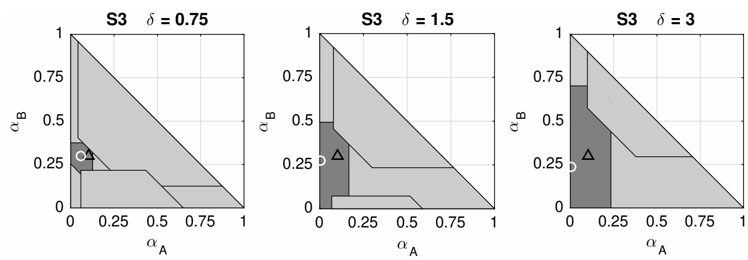
For all the considered scenarios, the core size enlarges with the increasing value of δ, namely the range of the acceptable values of α_A, α_B and α_C increases. Let us consider for instance scenario **S1** (Figure 5.2a): for $\delta = 0.75$, α_A ranges from about 0.25 to about 0.42, while for



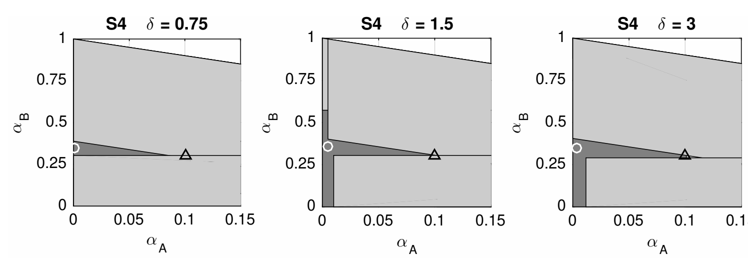
(a) Scenario S1



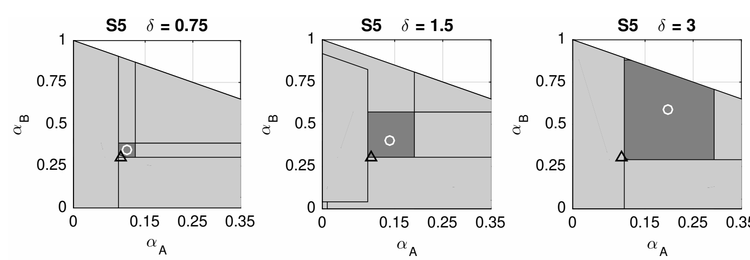
(b) Scenario S2



(c) Scenario S3



(d) Scenario S4



(e) Scenario S5

Figure 5.2 NTU game results: core, nucleolus and market shares

$\delta = 3$ it may rise up to about 0.8. Roughly speaking, with the increasing value of δ and therefore the increasing revenues, players accept more ways of dividing the costs and accept to bear a higher fraction of costs. Further, they may also accept to free one of the players of its fraction of costs. In scenario **S1** each player can be freed from the network cost, as α_A, α_B and α_C can all be equal to 0 for $\delta = 3$, due to the symmetry of the core. However, only one player at a time can be freed from the cost, the other two agreeing to share the overall amount. In fact, in scenario **S1** the market shares as well as the spectrum shares are equal: this results in a symmetric core and makes the market share coincide with the nucleolus. Market share and spectrum share have an impact of the shape of the core, as it will be discussed in the following paragraphs.

Impact of the spectrum share

To highlight the impact of the spectrum share let us compare scenarios **S1** (Figure 5.2a), where all the players have the same market and spectrum share, and **S2** (Figure 5.2b), where all the players have the same market share, but different spectrum shares (1.4:5:10 MHz, respectively). For scenario **S2** the core is not symmetric, differently from scenario **S1**. The acceptable fraction is somehow inversely dependent from the spectrum share: the highest the spectrum share the smallest the minimum fraction allowed. Thus, for $\delta \geq 1.5$, α_C can be equal to 0, as A and B are willing to share the overall cost in order to exploit the spectrum provided by C. For the highest value of δ, α_A can reach 0.95, while α_B is at most 0.8. The market share belongs to the core but it never coincides with the nucleolus, which gets closer to the line representing $\alpha_C = 0$ as δ increases.

Impact of the market share

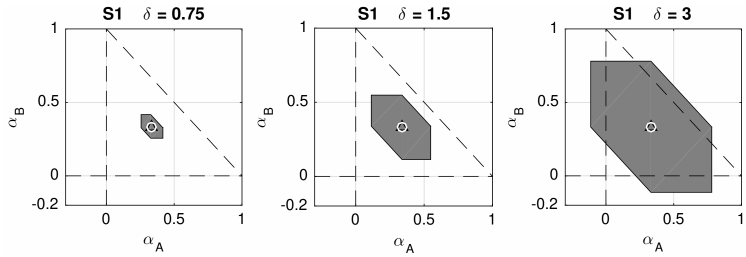
To highlight the impact of the market shares, let us compare scenarios **S1** (Figure 5.2a), where all the players have the same market share, and **S3** (Figure 5.2c), where all the players have the same spectrum share, but different market shares (0.1:0.3:0.6, respectively). In scenario **S3** the core is not symmetric, although as for scenario **S1** it enlarges with the increasing values of δ. The range of acceptable values of α_B is greater than the range of acceptable values of α_A. It is somehow proportional to the market share: in fact B has three times the users of A and for $\delta = 3$ $\alpha_A \in [0, 0.25]$ while $\alpha_B \in [0, 0.75]$. Player C, which has the highest number of users, may accept to pay the overall BSs cost so as to profit of the other two's spectrum: in fact the point $\alpha_A = \alpha_B = 0$ is in the core. Although the market share is in the core, it does not coincide with the nucleolus, not even for $\delta = 0.75$. For higher values of δ the nucleolus suggests to keep α_A and α_B smaller than the corresponding market shares, and to have α_A almost equal to 0.

Combined effect of market and spectrum share

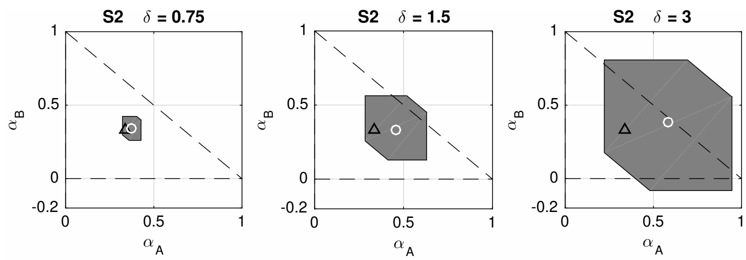
The combined effect of market and spectrum shares is shown in Figures 5.2d and 5.2e, reporting the core for scenarios **S4** and **S5**, respecively. In scenario **S4** the player with the minimum number of users is the only one owning a spectrum, while in scenario **S5** the incumbent player is the only one owning a spectrum. The core of scenario **S4** is very small and is not very sensitive to the value of δ. The acceptable values of α_A are very small, never above 0.15 and $\alpha_A = 0$ is acceptable. For $\delta = 3$, α_B cannot rise above 0.5: this means that the incumbent C should pay most of the cost with a little help from B so as they can both profit from the spectrum of A. Instead, in scenario **S5**, where the incumbent is the one providing the spectrum, α_A and α_B cannot be equal to 0. For smaller values of δ, neither α_C can be equal to 0, as A and B do not find it profitable to cover for the whole expenses due to limited revenues. Instead, for $\delta = 3$, α_C can be null, showing that A and B find it profitable to cover the whole expenses in order to profit of the spectrum of C.

5.6.2 TU game results

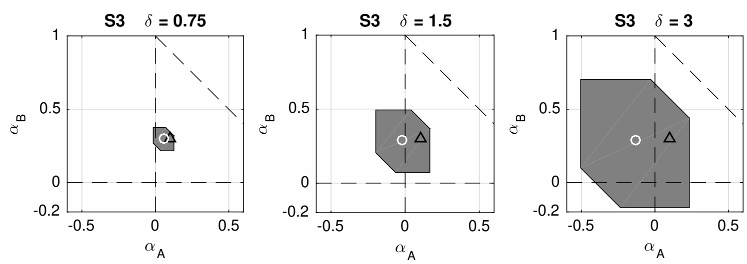
Many remarks can be extended to the TU case whose results are reported in Figure 5.3: for instance, the effect of δ is similar as for the NTU case, as the core enlarges with the increasing value of δ. However, as revenues are assumed to be transferable, the value of α can also be negative, meaning that not only the player does not share the cost but it also receives part of the revenues of the other players. This of course depends on the spectrum and market shares. For scenario **S1** with $\delta = 3$, all the players can receive from others, although not simultaneously. Instead, in scenario **S2** only players B and C, that provide most of the spectrum, can receive revenues from the others, although not simultaneously: they are rewarded for providing spectrum by receiving more than their own revenue. In scenario **S3** with $\delta \geq 1.5$, the players with the smallest number of users can be rewarded: for $\delta = 3$ they can both and simultaneously receive utility from the incumbent C, which finds it profitable to give part of its revenues despite having to bear the whole expenses, as the increased available spectrum provides it with higher revenues. In scenarios **S4** and **S5** the only player providing the spectrum, A in **S4** and C in **S5**, can be rewarded for high values of δ. This is more accentuated in **S4** where A not only provides the overall spectrum but has also the smallest market share.



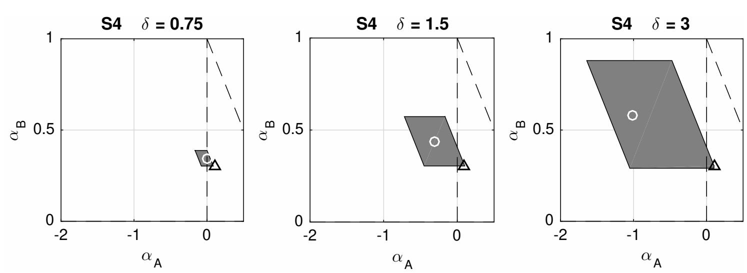
(a) Scenario S1



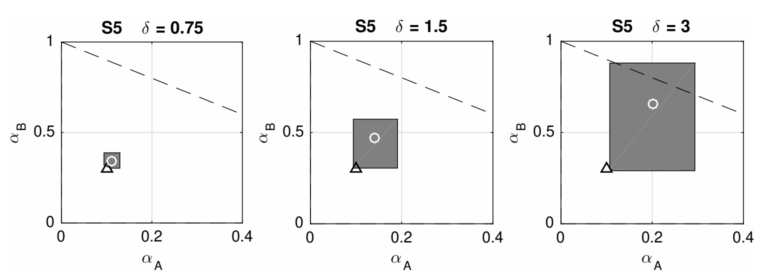
(b) Scenario S2



(c) Scenario S3



(d) Scenario S4



(e) Scenario S5

Figure 5.3 TU game results: core, nucleolus and market shares

5.6.3 Subcoalition analysis

Table 5.4 reports the stable coalitions for different ranges of the value of δ for all scenarios. When $\delta \leq 0.53$, no coalition finds it profitable to invest (denoted by the symbol $-$), whereas for $\delta \geq 0.58$ the core of the grand coalition is nonempty for all scenarios. Instead for instances with an empty core, stable subcoalitions are reported [12].

Table 5.4 Stable coalitions for each scenario and value of δ

	S1	**S2**	**S3**	**S4**	**S5**
$\delta \in [0.5, 0.53]$	$-$	$-$	$-$	$-$	$-$
$\delta = 0.54$	$-$	$-$	$-$	$\{A, C\}$	$\{C\}$
$\delta = 0.55$	$-$	$\{B, C\}$	$-$	$\{A, C\}$	$\{B, C\}$
$\delta = 0.56$	$\{A, B, C\}$	$\{A, B, C\}$	$\{A, B, C\}$	$\{A, C\}$	$\{B, C\}$
$\delta = 0.57$	$\{A, B, C\}$	$\{A, B, C\}$	$\{A, B, C\}$	$\{A, B, C\}$	$\{B, C\}$
$\delta \in [0.58, 3]$	$\{A, B, C\}$	$\{A, B, C\}$	$\{A, B, C\}$	$\{A, B, C\}$	$\{A, B, C\}$

We explore scenarios **S4** and **S5** when δ=0.55, for which we study the core of each subcoalition of 2 MNOs (i.e., $\{A, B\}$, $\{A, C\}$ and $\{B, C\}$)[13]. We recall that in scenario **S4** only A has a spectrum license whereas in **S5**, only C. In case it is feasible for a coalition to invest (i.e., at least one of its members has a spectrum license), Table 5.5 reports whether the core is empty, otherwise, if nonempty, it indicates the range of stable cost fractions (α_i) and obtainable payoffs (p_i) by each member MNO i.

In scenario **S4**, the core of the grand coalition is empty since $v(\{A, B, C\}) < v(\{A, C\})$, i.e., A and C can be both better off in $\{A, C\}$. However, both $\{A, B\}$ and $\{A, C\}$ have a nonempty core. Since B and C cannot invest neither alone nor together, due to the lack of spectrum, both prefer collaborating with A. Instead, A prefers $\{A, C\}$ to $\{A, B\}$: if C were to pay at least 86.26% of the cost of $\{A, C\}$ (which lies inside C's stable range of cost fractions and thus it is profitable (see Table 5.5)), the payoff of A from $\{A, C\}$, would be at least as large as the maximum payoff it can secure from $\{A, B\}$ (123153 €), that is, if B were to pay for all the $\{A, B\}$ cost. Consequently, $\{A, C\}$ will be created whereas B will not invest at all. Such behavior is due to the very low value of δ (i.e., price per unit of service), which limits revenues and in turn the level of investment (i.e., number of activated BS) in order to be profitable. But since $\{A, B, C\}$ is more congested than $\{A, B\}$ and $\{A, C\}$ (no spectrum pooling gain since B and C do not contribute with spectrum) and requires more investment to lower the level of congestion, it is then less profitable. In turn, A can better

[12] Notice that for all entries of the table in which the stable coalition consist of either one or two MNOs, the remaining MNOs do not invest at all.

[13] Similar observations can be drawn also for the other instances for which the grand coalition is not stable.

exploit its spectrum by collaborating with C instead of B, since C has the largest market share and thus can take up a larger fraction of their shared network cost.

For **S5**, the core is empty since the overall payoff of the grand coalition ($v(\{A, B, C\})$) is strictly smaller than the overall payoff of any other subcoalition for which it is profitable to invest. In other words, C is better off in any other subcoalition it can be part of than in $\{A, B, C\}$. Further, also $\{A, C\}$ has an empty core since $v(\{A, C\}) < v(\{C\})$, that is, C is better off by itself than collaborating with A, as A can only cover a small portion of the $\{A, C\}$ cost, given its small market share. Instead, $\{B, C\}$ has a nonempty core, thus B and C will build a shared network while A will not invest at all. Although C could be building its own network ($v(\{C\}) > 0$), it prefers collaborating with B which can pay up to 1/3 of their shared network cost.

It can be observed that, in conditions of very low revenues, and in particular when there is no spectrum pooling gain, smaller coalitions and cooperation with bigger MNOs are preferred.

Table 5.5 Core of subcoalitions for $\delta = 0.55$ (same for the NTU and the TU games)

S4		
$\{A, B\}$	$\alpha_{\mathbf{A}}$: [23.76, 25.41]%	$\mathbf{p_A}$: [0, 123153]€
	$\alpha_{\mathbf{B}}$: [74.59, 76.24]%	$\mathbf{p_B}$: [0, 123153]€
$\{A, C\}$	$\alpha_{\mathbf{A}}$: [12.46, 14.59]%	$\mathbf{p_A}$: [0, 307490]€
	$\alpha_{\mathbf{C}}$: [85.41, 87.54]%	$\mathbf{p_C}$: [0, 307490]€
$\{B, C\}$	*no spectrum license*	

S5		
$\{A, B\}$	*no spectrum license*	
$\{A, C\}$	*empty core (v({A,C})>0)*	
$\{B, C\}$	$\alpha_{\mathbf{B}}$: [34.12, 34.13]%	$\mathbf{p_B}$: [0, 1352]€
	$\alpha_{\mathbf{C}}$: [65.87, 65.88]%	$\mathbf{p_C}$: [344749, 346101]€

5.6.4 Sharing gain

Tables 5.6 and 5.7 summarize the gain of each MNO from joining the grand coalition relative to not sharing, that is, if they were to build individual networks. The values are calculated as $\frac{p_i - \overline{p}_i}{\overline{p}_i} 100\%, \forall i \in \mathcal{O}$, where p_i is the payoff of MNO i from the grand coalition according to the Nucleolus solution whereas $\overline{p}_i$ is its payoff when investing by itself. Notice that when it is either not profitable for an MNO to build its own network (i.e., its revenues do not cover its cost: e.g. MNO A for $\delta = 0.75$) or not feasible (the MNO has no spectrum license: e.g.

Table 5.6 NTU game: sharing gain

		A	B	C
	$\delta = 0.75$	2052.67%	2052.67%	2052.67%
S1	$\delta = 1.5$	290.99%	290.99%	290.99%
	$\delta = 3$	254.94%	254.94%	254.94%
	$\delta = 0.75$	∞	2179.71%	206.64%
S2	$\delta = 1.5$	9374.79%	317.69%	151.37%
	$\delta = 3$	1459.48%	275.62%	146.01%
	$\delta = 0.75$	61995.21%	1998.13%	901.87%
S3	$\delta = 1.5$	833.41%	329.98%	216.03%
	$\delta = 3$	433.20%	298.55%	205.77%
	$\delta = 0.75$	431.54%	∞	∞
S4	$\delta = 1.5$	109.35%	∞	∞
	$\delta = 3$	50.82%	∞	∞
	$\delta = 0.75$	∞	∞	23.22%
S5	$\delta = 1.5$	∞	∞	26.74%
	$\delta = 3$	∞	∞	32.58%

Table 5.7 TU game: sharing gain

		A	B	C
	$\delta = 0.75$	2052.67%	2052.67%	2052.67%
S1	$\delta = 1.5$	290.99%	290.99%	290.99%
	$\delta = 3$	254.94%	254.94%	254.94%
	$\delta = 0.75$	∞	2176.61%	207.15%
S2	$\delta = 1.5$	9378.69%	342.27%	141.57%
	$\delta = 3$	1428.44%	262.00%	154.14%
	$\delta = 0.75$	61817.36%	2010.21%	899.36%
S3	$\delta = 1.5$	918.79%	314.99%	210.30%
	$\delta = 3$	703.23%	265.31%	177.26%
	$\delta = 0.75$	448.76%	∞	∞
S4	$\delta = 1.5$	456.01%	∞	∞
	$\delta = 3$	574.99%	∞	∞
	$\delta = 0.75$	∞	∞	23.10%
S5	$\delta = 1.5$	∞	∞	39.06%
	$\delta = 3$	∞	∞	38.45%

MNOs B and C in **S4** or A and C in **S5**), then $\overline{p}_i = 0$. Such cases are represented by the ∞ symbol (the absolute gain is nevertheless finite).

As we calculate the sharing gain for the Nucleolus solution, which, by definition, tends to select a "fair" solution from the core, the NTU and TU models provide similar gains across all considered scenarios and cases [14].

While the increase of δ increases the number of stable divisions of the grand coalition cost among the MNOs (illustrated by the increase of core size in Figures 5.2 and 5.3), Tables 5.6 and 5.7 indicate decreasing values of the relative gain as δ increases for all scenarios but **S5**. This shows how sharing is more beneficial when low revenues significantly limit the level of investment in network infrastructure an MNO can undertake by itself. Nevertheless, sharing remains profitable even for higher values of δ, as MNOs still benefit from a larger pool of spectrum resources and cost sharing.

As expected, identical MNOs obtain equal gains (scenario **S1**). Scenario **S2** shows the benefit of spectrum pooling: the smaller the MNO spectrum share, the more it benefits from the grand coalition, despite having to pay for a larger fraction of its infrastructure cost. Instead, scenario **S3** shows how MNOs with smaller market shares, which find it more difficult to face the network upfront cost by themselves, incur larger gains from cooperation. In particular, for scenarios **S4** and **S5**, since there is no spectrum pooling gain (only one MNO has a spectrum license), the relative gain is much smaller compared the other scenarios, especially for MNO C (scenario **S5**), which has less difficulties covering its network cost given its large market share (as opposed to A in scenario **S4**, that, despite owning a spectrum license, has limited revenues given its small share of users). Contrarily to the other scenarios, in **S5**, sharing becomes more beneficial for C as δ increases, as A and B can afford to cover a larger fraction of the grand coalition cost.

5.7 Discussion

This work has targeted sharing of 4G small cells and proposed a particular pricing and cost model. However, the proposed game models are useful tools to study other technologies as well (e.g., 3G/4G macro cells) and/or different pricing and cost models. In the following, we discuss the impact of some of the assumptions made and the applicability of the models to alternative settings.

- As investment in network infrastructure and spectrum availability are both key to improving the service level (i.e, data rate here), the proposed pricing model aims at

[14] For scenario **S4**, $\delta = 3$, which represents an extreme case, the nucleolus solutions of the two games are however significantly different: the relative gain of MNO A under the TU model is one order of magnitude larger compared to the NTU one. Such behavior was also reflected in the core size being significantly larger in case of the TU game w.r.t. to the NTU one (Figures 5.2 and 5.3).

translating the two into revenues. Since nowdays MNOs struggle with monetizing their investments (either in infrastructure of spectrum licenses), roughly speaking, the considered revenues would represent an overestimation. Nevertheless, by considering a wide range of such revenues, we were able to see their impact on the stable coalitions and their corresponding cost divisions. We also note that it is outside the scope of this work to investigate pricing models in line with those in the market, such as bundles of different types of services and data usage caps. In these lines, as the churn rate is determined by marketing strategies rather than technical factors, we do not address the user migration among MNOs.

- The considered cost model accounts for the main upfront and operational cost terms related to the radio equipments and for the backhauling cost. Although a more realistic backhauling cost model could be used instead, our goal was to overestimate its cost, in order to have a more significant sharing tradeoff, that is, between benefiting from larger spectrum resources when in a larger coalition but incurring a higher cost to which amounts also a higher level of congestion. The backhaul optimization is also outside the scope of this work.

- We do not account for the spectrum license cost since we assume MNOs have purchased the spectrum license individually and prior to entering a sharing agreement. The amount of spectrum available to a coalition depends then on its members contribution, and thus is not part of its strategy, unlike the investment in network infrastructure. The models can nevertheless take into account such cost as follows: Let $\hat{c}_i$ be the spectrum license cost of MNO $i \in \mathcal{O}$, representing an upfront cost. It is then only profitable for an MNO to be in the grand coalition if its allocated payoff can cover $\hat{c}_i$, that is, $p_i \geq \hat{c}_i, \forall i \in \mathcal{O}$. Such constraint translates into an upper bound on the fraction of cost the MNO would be willing to pay to be in the grand coalition:

$$p_i \geq \hat{c}_i \implies \tilde{r}^i_{\mathcal{O}} - \alpha^i_{\mathcal{O}} \tilde{c}_{\mathcal{O}} \geq \hat{c}_i \implies \alpha^i_{\mathcal{O}} \leq \frac{\tilde{r}^i_{\mathcal{O}} - \hat{c}_i}{\tilde{c}_{\mathcal{O}}}, \forall i \in \mathcal{O}.$$

  If $\left(\tilde{r}^i_{\mathcal{O}} - \hat{c}_i\right) / \tilde{c}_{\mathcal{O}} < 1$, the constraints would reduce the set of feasible payoffs of the proposed models. Further, if $\sum_{i \in \mathcal{O}} \left(\tilde{r}^i_{\mathcal{O}} - \hat{c}_i\right) / \tilde{c}_{\mathcal{O}} < 1$, then the grand coalition would not be created. If the core was empty, the constraints would similarly be extended to subcoalitions.

  We did not carry out such analysis since a spectrum license cost depends on several factors such as the spectrum auction time and place and the spectrum band. However, assuming the spectrum license cost is proportional to its amount of spectrum, the larger the spectrum provided by an MNO, the smaller the cost fraction it would pay to be in a coalition. This behavior is indirectly observed even without explicitly taking into account the spectrum license cost, as MNOs contributing with a larger spectrum share tend to pay less than the others.

5.8 Conclusions

This work investigates the problem of RAN and spectrum sharing in 4G networks for a scenario in which MNOs with fixed market and spectrum shares plan to upgrade their existing RAN by deploying small cell BSs. Each MNO weighs between deploying an individual network or enter a sharing agreement with other MNOs and thus build a shared network. We assume that when MNOs build a shared network, they will aggregate their spectrum resources. A generic mobile data pricing model is introduced to determine revenues incurred by an MNO from each possible coalition (sharing agreement). We propose two cooperative game models to address the problem: if MNOs in a coalition agree to share its cost but keep their individual revenues, the problem is formalized as a non-transferable utility cooperative game; if MNOs would be willing to give away also part of their individual revenues to be in a coalition, a transferable utility game is proposed instead. The core and nucleolus solution concepts are leveraged to determine stable cost divisions.

The proposed models are investigated for several instances with different network and economic settings aiming to represent realistic scenarios. For the vast majority of the considered instances, MNOs are better off building a unique shared RAN than creating sub-coalitions or building individual RANs due to the combined gain from spectrum aggregation and cost reduction from sharing the network infrastructure. The cost division of the shared network infrastructure that guarantees stability depends both on network and economic inputs: MNOs with a larger customer base should be accounted for a larger fraction of the cost; instead, MNOs contributing with a larger spectrum portion are “rewarded” by a lower cost fraction. In particular, MNOs which provide the largest spectrum portion are not only exempted from the network infrastructure cost but can also receive part of the other MNOs revenues. Dividing the cost based on the market share does not always guarantee stability whereas the stable cost division selected by the nucleolus, which in turn accounts also for the MNOs spectrum contribute, makes a better candidate for a cost division policy.

The models we propose here are generic instruments for addressing the problem of network sharing from a strategic perspective as they can accommodate for alternative technologies and/or pricing models and cost functions.

Acknowledgment

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CHAPTER 6 ARTICLE 3 : MODELING THE TECHNO-ECONOMIC INTERACTIONS OF INFRASTRUCTURE AND SERVICE PROVIDERS IN 5G NETWORKS WITH A MULTI-LEADER-FOLLOWER GAME

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Abstract

The decoupling of infrastructure from services, which has been so far a mainstream paradigm in the computational and storage domain, is now becoming a paradigm also for mobile networks. Indeed, 5G must provide a variety of services with very diverse requirements, such as throughput, latency, or reliability, and decoupling infrastructure from service provisioning allows to deal with such heterogeneity. In this context, a new business model, involving two different stakeholders, Infrastructure Providers and Service Providers, has emerged. Besides addressing the technical issues, it is also important to study the economic feasibility and behavior of such new paradigm and the techno-economic interactions among the different stakeholders that play different roles in the mobile network market. In this paper, we propose a multi-leader multi-follower variant of the Stackelberg game to model the considered environment. The proposed model is then fed with realistic data and used to analyze the system behavior and the impact of the technological features of the stakeholders on their competitiveness.

6.1 Introduction

The decoupling of infrastructure from services, a mainstream paradigm in the computational and storage domain, is now being materialized also for mobile networks with the advent of 5G. Up to date, a typical pre-5G Mobile Network Operator (MNO) owns and manages by itself the network resources (infrastructure and spectrum) and provisions services for its end users. However, over time, there has been a progressive deviation from this typical MNO business model which can be witnessed through, e.g., the Mobile Virtual Network Operator (MVNO) business model and infrastructure and/or spectrum sharing agreements among MNOs [6, 201, 202]. The emergence of such new business models, even prior to 5G, has been mainly driven by the need to cut down on infrastructure cost (so as to improve the return on investment) and to increase resource utilization (e.g., when scarce such as spectrum).

As for 5G networks, in addition to delivering higher throughput mobile broadband services, they are also expected to provide support for the Internet of Things and for vertical industries: the International Telecommunication Union Radiocommunication Sector has identified three usage scenarios for the International Mobile Telecommunications (IMT) for 2020 and beyond [162], namely enhanced Mobile BroadBand (eMBB), Ultra-Reliable and Low Latency Communications (URLLCs) and massive Machine Type Communications (mMTCs). In these lines, unlike the previous generations, 5G networks will have to provision heterogeneous services with very distinct requirements in terms of throughput, latency, reliability, connection density, type of end user devices, etc. As a means to deal with such heterogeneity and open up the mobile network to verticals, the decoupling of network infrastructure and resources from service provisioning is considered a design principle by several entities, initiatives and research projects involved/contributing in the 5G architecture definition and standardization [31, 203, 204] with Network Function Virtualization and Software Defined Networking being two key technical enablers. In this context, a key 5G concept is that of network slicing [31], which allows to create logically separated networks (slices) over the set of shared physical network resources where each such slice will be tailored to the service requirements of a specific tenant (i.e., a business entity which provides eMBB/URLLC/mMTC services to end users).

Apart from the architectural aspects of 5G, there is a need to address its economic viability [203] which, by far, has been studied from the point of view of a single MNO [1, 163, 164]. However, one of the implications of the 5G architecture is the emergence of new stakeholders that play different roles in the mobile network market such as, e.g., infrastructure providers and mobile service providers, in addition to the so-called tenants (see, e.g., [1, 31]). The techno-economic interactions among these new stakeholders (such as resource demand and pricing, provider selection, etc.) give rise to new competitive scenarios for the mobile network market requiring suitable models to be studied, which is the object of this work.

In this paper, we devise a mathematical model to capture the technological and economic features of the considered scenarios and the techno-economic interactions among stakeholders. We feed the model with realistic technological and economic parameters describing different network configurations (either 4G or 5G) and end user services (5G usage scenarios). Then, the developed model and data are used to deeply analyze the interactions among stakeholders of the same type and those playing different roles and how their features, both technological and economic, influence their behavior and the resulting mobile market setting.

In this work, we consider two types of stakeholders: Infrastructure Providers (InPs) and

Service Providers (SPs), while end users are represented implicitly. An InP is an entity which owns spectrum licenses, deploys and manages the infrastructure of the mobile network and rents/sells its resources to SPs, but does not provision services for end users. In turn, an SP[1] is an entity which does not own network resources but provisions services to end users through leased/acquired resources. Here the resource sold by InPs (acquired by SPs) is the cell capacity at base station (BS) level and the problem we address is the pricing of the cell capacity from the InPs' perspective and the selection of an InP from which to acquire cell capacity from the SPs' perspective.

Specifically, we consider a dense urban area where there are multiple InPs that own mobile networks and multiple SPs, each provisioning a single type of service to a given number of end users in the area. The SPs provision services for their own end users in the cell area by acquiring cell capacity from only one of the InPs (i.e., from the BS of one of the InPs) while each InP can host multiple SPs. All InPs and SPs are considered profit-maximizers, i.e., each InP offers a cell capacity unit price that maximizes its profit from the amount of cell capacity sold to SPs that select the InP, while each SP selects an InP from which to acquire cell capacity so as to maximize its profit (revenue from own users given the acquired cell capacity minus cost of the latter). As the cell capacity of each InP is fixed and finite, SPs compete among them in selecting an InP from which to acquire cell capacity, whereas InPs compete among them over the cell capacity unit prices to be selected by SPs. In this setting, we formulate the problem of cell capacity pricing from the InP perspective and InP selection from the SP perspective as a multi-leader multi-follower extension of the basic (one-leader one-follower) Stackelberg game [205]; we will refer to the proposed model as the multi-leader-follower game (MLFG) as in [206].

We have applied the proposed MLFG to several realistic scenarios in which services provisioned by SPs are inspired from usage scenarios for IMT for 2020 and beyond [162] and characterized by their respective performance requirements (such as user target rates, connection densities, etc.) as in [207], while we vary the InPs' network technology (whether 4G or 5G) and their spectrum bandwidth availability. To devise meaningful pricing strategies for the InPs across the different scenarios, we propose an InP cost model that accounts for the InP's network technology type and available spectrum bandwidth based on [1], whereas the SP revenue function is based on a noted function in literature [208] that allows to represent how the end user responds to the fee offered by its SP based on the utility achieved from resources assigned by the latter [208]. The proposed MLFG has been instrumental to derive

[1] An SP is equivalent to a tenant in the 5G literature terminology. For instance, in [32], a tenant is either an MVNO, a vertical industry or an Over The Top provider (OTT). In this paper, we have opted for the term SP since the focus of our work is not on the 5G architecture.

insights concerning these scenarios. Indeed, for all the considered instances, it is possible to compute either an equilibrium or an approximation of the equilibrium. Results show that the technological features of the InPs have a significant impact on their competitiveness.

The layout of this paper is the following. In Section 6.2, we identify and review works in the mobile networks literature which are related to ours in terms of methodology and/or in application. The proposed framework and the mathematical models behind it are presented in Section 6.3. Then, in Section 6.4 we explain how the framework has been applied in the context of migrating from 4G to 5G through the characterization of InPs and services provisioned by the SPs and how we set up several scenarios/problem instances for our computational tests. Numerical results concerning these problem instances are presented and analyzed in Section 6.5, whereas conclusions are drawn in Section 6.6.

6.2 Related work

Stackelberg games are widely used in the literature to model the interaction among multiple self-interested entities in the field of resource management problems in 5G networks [209]; specific application arena include Heterogenous Networks (HetNets) [210, 211], edge caching [212], edge computing [213], device-to-device communications [214], cognitive networks [215], Cloud Radio Access Networks (C-RANs) [216].

Whilst the aforementioned work is similar to ours only in terms of the adopted methodology, the work in [65, 68, 76, 77, 158, 160, 161, 217, 218] share with ours the same context and application arena targeting the techno-economic interactions arising among multiple stakeholders of mobile radio networks. Among [65, 68, 76, 77, 158, 160, 161, 217, 218], [68, 77, 160, 161, 217] also resort to variants of the Stackelberg game.

[76] resorts to congestion games to address the problem of partitioning the RAN resources of a Telco Operator (TO) (analogous to an InP in our framework) among multiple MVNOs (analogous to SPs in our framework), each with a fixed number of users. In details, the TO's RAN consists of a set of heterogeneous Remote Radio Heads (RRHs) which the TO leases to MVNOs at a fixed RRH-specific price. Then, each MVNO decides how to distribute its own set of users over these RRHs so as to minimize its total cost. Our work differs in the following aspects: (i) in [76] MVNOs compete over a set of RRHs whereas our framework applies to the single BS and (ii) the congestion game proposed by [76] models competition only among MVNOs while the TO is not a player of the game; differently, our MLFG allows to model all involved InPs and SPs as players of the game and in particular thus capturing competition also among multiple InPs (while a single TO is considered in [76]).

In [68], an InP owns a virtualized RAN which hosts multiple MVNOs (analogous to SPs in our framework) each with a fixed number of users. The InP faces the problem of pricing and allocating its available BS resources among the users of all MVNOs so as to maximize its own profit, while satisfying Service Level Agreements signed with the MVNOs which are given in terms of a minimum number of subcarriers per MVNO and a maximum total rate over all MVNO users. The problem is formulated as a one-leader multi-follower Stackelberg game (OLMFSG) with the InP acting as the leader and MVNOs acting as followers. A single InP is considered, whereas we model competition among multiple InPs.

In [160], multiple service providers with distinct wireless access technologies (either a Wireless Metropolitan Area Network (WMAN), a cellular network or a Wireless Local Area Network (WLAN)) and fixed amount of available bandwidth compete among them over prices per unit of bandwidth to be selected by users in a common coverage area. The user sensitivity to changes in price and the user churn among service providers are incorporated in the service providers' payoff functions. The authors propose multiple formulations for the problem, among which a one-leader two-follower Stackelberg game, assuming one of the service providers announces its price before the others. The most significative differences with our approach are: (i) in our framework the selection of an InP by SPs is modeled explicitly as a game (subgame of the proposed MLFG), whereas in [160] the selection of a service provider by users is modeled implicitly (through the service provider payoff function) and (ii) in the MLFG of our framework, InPs announce their prices simultaneously, whereas in the Stackelberg game proposed in [160] one of the service providers moves first.

Rose et al. ([161]) address the problem of service selection from the end user perspective and service pricing from a Network Service Provider (NSP) perspective. They consider multiple NSPs, each providing multiple types of services, and multiple users with different Quality of Service (QoS) evaluation. The NSPs price their offered services so as to maximize their profit, while each user selects a unique service from a single NSP so as to maximize its payoff given by the difference between its evaluation of the QoS of the selected service and its price. The problem is formulated as a MLFG with NSPs acting as leaders (by announcing the prices of their offered services) and users as followers (each selecting a service and an NSP in response to the service prices offered by NSPs). A similar modeling approach is used in [217] which though focuses on the emerging machine type communications (MTC) and introduces in the framework MTC service providers. Differently than our approach, [161] and [217] assume a continuum of end users (which makes each subgame of stage 2 of the MLFG therein a non-atomic game) while the set of SPs in our work is assumed discrete and finite.

Along the same lines, [77] proposes a similar MLFG which however also accounts for a Small

Cell Provider (SCP) (analogous to an InP in our framework), which leases small cell BSs to the NSPs. The interaction among the SCP and the NSPs is modeled through an additional OLMFSG in which the SCP acts as the leader by announcing the spectrum price per small cell BS and NSPs are followers deciding the amount of spectrum to purchase to maximize their individual payoffs. The work in [77] is substantially different from ours: (i) a single SCP is considered in [77], while we have multiple InPs; (ii) since the SCP available spectrum is not bounded in [77], given the spectrum price offered by the SCP, each NSP can derive its optimal amount of spectrum independently, i.e., there is no real competition among the NSPs at stage 2 of the OLMFSG; (iii) while in [77] end users select a service from one of the NSPs, in our framework the user – SP association is given[2].

In [65], the available Physical Resource Blocks (PRBs) of a BS in a C-RAN have to be split among an eMBB, a mMTC slice and an URLLC slice, each requesting a minimum amount. The authors model this problem as bankruptcy game and apply the Shapley value to determine the number of PRBs assigned to each slice. The problem bears similarities with a subproblem of our framework, namely the InP capacity assignment problem (see Section 6.3.3) in which each InP has a fixed amount of capacity per BS cell and SPs that choose to be served by a given InP (each providing either eMBB or mMTC services to a specific market segment of users) request a minimum and maximum of capacity per cell from the latter. While [65] opts for a cooperative game approach for the resource assignment problem, in our framework we propose a two-step lexicographic optimization problem as the assignment is handled in a centralized fashion by the InP, which aims to maximize the total amount of assigned (sold) cell capacity.

Even though our proposed framework is *per se* generic and bears conceptual and formulation similarities with [161] and [77], one of the core contributions of this work is the use of the proposed framework as a means to investigate realistic scenarios in terms of network technologies and related costs, mobile services and related performance requirements, and user tariffing in the context of migrating from 4G to 5G. To this extent, inspired from the usage scenarios for IMT for 2020 and beyond [162], we build up a methodology to evaluate the techno-economic impact of different dimensioning and architectural choices for 5G network. Along these lines, [1, 163, 164] also target a financially sustainable design and development of 5G networks to meet user requirements and envisioned demand for connectivity. However, [1, 163, 164] focus on the dimensioning of a single 5G network, while we address competition

[2]In our work, the interaction between an SP and its set of users is modeled through a noted function in literature [208] which represents the user response to the fee offered by the SP based on the utility perceived by the user from the amount of resources allocated by the SP. Hence, the optimal user fee is affected by the equilibrium of the MLFG or vice versa the user response to the fee offered by the SP affects its strategy in the MLFG.

among multiple InPs with individual 4G/5G mobile networks.

6.3 Framework

To present our framework, we start by describing the problem it addresses in Section 6.3.1. Next, we dwell on the interactions between an SP and its end users in Section 6.3.2 and between an InP and its hosted SPs in 6.3.3. Specifically, in Section 6.3.2 we explain the utility function representing the QoS requirements of the service provisioned by each SP and define the SP revenue function based on a noted function in literature which relates the end user fee to its perceived utility, whereas in 6.3.3 we propose an optimization problem to model how an InP splits its available capacity among its hosted SPs given their requirements. Then, in Section 6.3.4 we formulate the addressed problem as a MLFG.

6.3.1 Problem statement

We consider a mobile ecosystem such that the network infrastructure and its resources are decoupled from service provisioning for end users, which gives rise to two types of actors: InPs and SPs. An InP is the entity that deploys and maintains the cellular network whose resources it then sells/rents to one or multiple SPs. In turn, an SP provisions services for end users through resources acquired/rented from one of the InPs. From a technical point of view, an InP can support multi-tenancy, i.e., it can host multiple SPs over its network infrastructure and resources by relying on the network slicing paradigm [31]. We assume that InPs do not have end users of their own, whereas SPs do not own any network infrastructure.

We consider a geographical area with multiple InPs with individual RANs, and multiple SPs that provision mobile services to end users through RAN resources acquired from InPs. The RAN of each InP consists of a set of BSs and their respective back-hauling links to connect the former with the core network. The specific architecture of the BS is abstracted away to keep the modeling framework as general as possible[3].

The InPs' BSs are assumed to be co-located and their respective cells to overlap, hence we focus on the area of a single BS cell provisioned by all InPs simultaneously through their individual BSs. In turn, this means that an SP can select any of the InPs to serve its user demand within the cell area. The BS cell of a given InP is characterized by an average capacity which depends on the InP's network technology and configuration and its available spectrum resources. The network resources requested by an SP from an InP for a given cell

[3]The proposed model remains valid under different realization of the 5G BSs (e.g., a single Active Antenna Unit, a Radio Unit coupled with a Distributed Unit, etc.).

are expressed in terms of average cell capacity.

Each InP offers its available cell capacity at a certain unit price lower bounded by its unit cost. Based on the InPs' available cell capacities and their offered unit prices, each SP selects an InP from which to acquire cell capacity so as to maximize its profit (difference between revenues from own users and cost incurred from the selected InP, both depending on the amount of acquired cell capacity). The objective of each InP is to maximize the profit from the total amount of cell capacity sold to SPs selecting it. It follows that SPs compete among them for the InPs' cell capacities (as these are finite), while InPs compete among them in cell capacity unit prices to be selected by SPs. Given that InPs and SPs are all self-interested payoff-maximizers, actions taken by any of the actors affect all the others (e.g., by lowering its offered unit price, an InP may be able to attract more SPs or sell more cell capacity to SPs that select it) and we assume that InPs announce their cell capacity unit prices simultaneously and SPs simultaneously select their serving InPs based on these announced prices, then we resort to hierarchical games to model the problem. Specifically, we formulate this problem as a multi-leader-follower game which is an extension of the basic (one-leader one-follower) Stackelberg game. In the proposed model, InPs act as leaders and SPs act as followers. The strategy of each leader is the price per unit of cell capacity which maximizes its profit from the total amount of sold capacity, whereas the strategy of each follower is the choice of an InP which maximizes its profit.

6.3.2 SP service characterization and revenue function

Let $\mathcal{V}$ denote the set of SPs. Each SP v is assumed to provision a single type of service and all end users of v, i.e., users subscribing to the service provisioned by v, are assumed identical. The QoS requirements of the service provisioned by v are given in terms of a minimum and a target user rate (both equal for all users of v). Then, the level of satisfaction of a user of v depends on the rate perceived by the user w.r.t. these minimum and target rates: we represent it by the utility function described in Section 6.3.2. In turn, we adopt the acceptance probability function proposed in [208] to model the user response to a fee offered by its SP depending on its achieved utility, as described in Section 6.3.2. Based on these two functions, in Section 6.3.2 we define the optimal SP revenue in terms of the amount of capacity acquired from its selected InP.

User utility function

Let x_v denote the amount of cell capacity acquired by SP v from its selected InP. Notice that the cell capacity of an InP is intended as its total cell rate (i.e., the product between its spectral efficiency and bandwidth) hence x_v can be a portion of/all the cell rate of the InP selected by v. Let N_v denote the number of users of v and η_v the activity factor of each user of v. We assume that SP v splits x_v uniformly among its identical N_v users. Let $\widetilde{N}_v$ denote the number of simultaneously active users of v which we determine[4] as $\widetilde{N}_v = \max\{1, \eta_v N_v\}$, then each user of v perceives a rate equal to $x_v/\widetilde{N}_v$. The level of satisfaction of a user of v from $x_v/\widetilde{N}_v$ is represented by a variant of the normalized sigmoid utility function [208], defined as

$$u_v(x_v) = \begin{cases} 0, & \text{if } 0 \leq x_v \leq \widetilde{N}_v \underline{\mathcal{X}}_v, \\ \dfrac{\left(\frac{x_v/\widetilde{N}_v - \underline{\mathcal{X}}_v}{\mathcal{X}_v - \underline{\mathcal{X}}_v}\right)^{\xi_v}}{1 + \left(\frac{x_v/\widetilde{N}_v - \underline{\mathcal{X}}_v}{\mathcal{X}_v - \underline{\mathcal{X}}_v}\right)^{\xi_v}}, & \text{if } x_v > \widetilde{N}_v \underline{\mathcal{X}}_v, \end{cases} \tag{6.1}$$

where $\underline{\mathcal{X}}_v$ denotes the minimum user rate characterizing the service provisioned by v, $\mathcal{X}_v$ denotes the user rate which provides a utility value equal to 0.5, i.e., $u_v(\widetilde{N}_v \mathcal{X}_v) = 0.5$, while $\overline{\mathcal{X}}_v$ represents the target user rate of the service provisioned by v, that is the rate value that would make a user of v fully satisfied in practice[5], i.e., $u_v(\widetilde{N}_v \overline{\mathcal{X}}_v) = U$, where $0 < U < 1$ and $U \approx 1$. It follows that

$$\mathcal{X}_v = \underline{\mathcal{X}_v} + \left(\overline{\mathcal{X}}_v - \underline{\mathcal{X}}_v\right) \left(\frac{1-U}{U}\right)^{1/\xi_v},$$

where ξ_v denotes the utility elasticity to x_v (the higher the value of ξ_v, the more step-like the shape of the utility function).

Acceptance probability function

Let p_v denote the fee offered by SP v to each of its users and let $a_v(u_v(x_v), p_v)$ denote the user acceptance probability function proposed in [208] and defined as

$$a_v(u_v(x_v), p_v) = 1 - e^{-A_v u_v(x_v)^{\mu_v} p_v^{-\varepsilon_v}}. \tag{6.2}$$

$a_v(u_v(x_v), p_v)$ relates $u_v(x_v)$, i.e., the level of utility achieved by a user of v when SP v acquires x_v units of capacity (see Equation (6.1)), and p_v, where μ_v and ε_v denote the user sensitivity to changes in utility and to changes in the offered fee, respectively, whereas A_v is a normalizing constant. Assume users of SP v, characterized by μ_v and ε_v, achieve the maximum level of utility $\overline{u}_v$ and are offered the fee $\overline{p}_v$. Let $\overline{q}_v$ denote the probability with

[4]The max operator in $\widetilde{N}_v = \max\{1, \eta_v N_v\}$ makes sure that when $\eta_v N_v < 1$, the rate perceived by a user of v, i.e., $x_v/\widetilde{N}_v$, does not exceed the total available capacity/rate x_v of SP v.

[5]The utility function $u_v(x_v)$ is such that $\lim_{x_v \to \infty} u_v(x_v) = 1$.

which these users reject[6] $\overline{p}_v$, i.e.,

$$\overline{q}_v = 1 - a_v(\overline{u}_v, \overline{p}_v) = e^{-A_v \overline{u}_v^{\mu_v} \overline{p}_v^{-\varepsilon_v}},$$

hence the normalizing constant $A_v = -\overline{u}_v^{-\mu_v} \overline{p}_v^{\varepsilon_v} \log(\overline{q}_v)$ and, as a result, $a_v(u_v(x_v), p_v)$ can be rewritten as

$$a_v(u_v(x_v), p_v) = 1 - \overline{q}_v^{(u_v/\overline{u}_v)^{\mu_v}(p_v/\overline{p}_v)^{-\varepsilon_v}}. \tag{6.3}$$

SP revenue function

Being $a_v(u_v(x_v), p_v)$ the probability that a user of SP v accepts the offered fee p_v when it achieves the level of utility $u_v(x_v)$, then $a_v(u_v(x_v), p_v)p_v$ represents the fee accepted by the user or, in other words, the expected revenue of v from the single user when v acquires x_v units of capacity. Then, as the number of users of SP v is equal to N_v, the total revenue of SP v from x_v units of capacity, when users are offered the fee p_v, can be determined as

$$r_v(x_v, p_v) = N_v a_v(u_v(x_v), p_v)p_v. \tag{6.4}$$

Let $p_v^*(u_v(x_v))$ denote the value of p_v which maximizes $r_v(x_v, p_v)$ for a given x_v and let $r_v^*(x_v)$ be the total optimal revenue of SP v for x_v, i.e. $r_v^*(x_v) = N_v a_v(u_v(x_v), p_v^*(u_v(x_v)))p_v^*(u_v(x_v))$. If $x_v \leq \widetilde{N_v} \underline{\boldsymbol{\mathcal{X}}}_v$, then $r_v(x_v, p_v) = 0$ for any $p_v > 0$ as $u_v(x_v) = 0$ (see Equation 6.1) and $a_v(0, p_v) = 0$ for any $p_v > 0$[7] if $0 < A_v < \infty$, $0 < \mu_v < \infty$ and $0 < \varepsilon_v < \infty$ (see Equation 6.2 and Appendix C for the assumptions on A_v, μ_v and ε_v). This means that $p_v^*(u_v(x_v))$ is indeterminate for $x_v \leq \widetilde{N_v} \underline{\boldsymbol{\mathcal{X}}}_v$, but $r_v^*(x_v) = 0$.

Instead, for $x_v > \widetilde{N_v} \underline{\boldsymbol{\mathcal{X}}}_v$, which implies $u_v(x_v) > 0$ (see Equation 6.1), we show in Appendix C that, when $0 < A_v < \infty$, $0 < u_v(x_v) < \infty$, $0 < \mu_v < \infty$ and $1 < \varepsilon_v < \infty$, we have

$$p_v^*(u_v(x_v)) = \overline{p}_v \left[\frac{\log(\overline{q}_v)}{W_{-1}\left(-\frac{1}{\varepsilon_v} e^{-\frac{1}{\varepsilon_v}}\right) + \frac{1}{\varepsilon_v}} \right]^{\frac{1}{\varepsilon_v}} \left[\frac{u_v(x_v)}{\overline{u}_v} \right]^{\frac{\mu_v}{\varepsilon_v}}, \tag{6.5}$$

where W_{-1} denotes the lower branch of the Lambert W function for the real numbers domain.

[6] The reference rejection probability $\overline{q}_v$ can be determined by polling a large set of users of SP v with known ε_v and μ_v on whether they accept the fee $\overline{p}_v$ when they achieve the maximum level of utility $\overline{u}_v$. Then, $\overline{q}_v$ is set equal to the fraction of users which reject $\overline{p}_v$ [208, 219].

[7] Notice that even for $0 < A_v < \infty$, $0 < \mu_v < \infty$ and $0 < \varepsilon_v < \infty$, $a_v(0, 0)$ is indeterminate (see Equation 6.2). However, in practice, $p_v = 0$ means that SP v obtains zero revenue hence we are interested in $p_v > 0$.

It follows that for $x_v > \widetilde{N}_v \underline{\mathcal{X}}_v$, $0 < A_v < \infty$, $0 < \mu_v < \infty$ and $0 < \varepsilon_v < \infty$, we have

$$a_v(u_v(x_v), p_v^*(u_v(x_v))) = 1 - e^{W_{-1}\left(-\frac{1}{\varepsilon_v} e^{-\frac{1}{\varepsilon_v}}\right) + \frac{1}{\varepsilon_v}}, \tag{6.6}$$

that is, the acceptance probability at $p_v^*(u_v(x_v))$ is a function of only ε_v and independent of $u_v(x_v)$. Let $a_v^* = a_v(u_v(x_v), p_v^*(u_v(x_v))) = 1 - e^{W_{-1}\left(-\frac{1}{\varepsilon_v} e^{-\frac{1}{\varepsilon_v}}\right) + \frac{1}{\varepsilon_v}}$ and $\overline{a}_v = 1 - \overline{q}_v$. Hence, for $0 < A_v < \infty$, $0 < \mu_v < \infty$ and $1 < \varepsilon_v < \infty$, we have

$$r_v^*(x_v) = \begin{cases} 0, & \text{if } 0 \le x_v \le \widetilde{N}_v \underline{\mathcal{X}}_v, \\ N_v a_v^* \overline{p}_v \left(\frac{\log(1-\overline{a}_v)}{\log(1-a_v^*)}\right)^{\frac{1}{\varepsilon_v}} \left(\frac{u_v(x_v)}{\overline{u}_v}\right)^{\frac{\mu_v}{\varepsilon_v}}, & \\ & \text{if } x_v > \widetilde{N}_v \underline{\mathcal{X}}_v. \end{cases} \tag{6.7}$$

6.3.3 InP capacity assignment problem

In the proposed MLFG, the strategy of each SP consists solely in the choice of InP from which to acquire capacity. However, the amount of cell capacity acquired by an SP affects both its revenue (as explained in Section 6.3.2) and its total cost (product of InP unit price with the amount of cell capacity acquired by the SP), hence its payoff (difference between the two). In these lines, given that SPs are rational, none of them can accept an amount of cell capacity which provides a negative payoff. We therefore assume that, given the cell capacity unit price offered by an InP, each SP selecting the InP communicates a minimum and a maximum amount of cell capacity that the SP finds suitable, i.e., the minimum amount of cell capacity that guarantees a non-negative payoff and the payoff-maximizing amount. Based on such cell capacity ranges, the InP determines which of the SPs that select it to serve and how to split its available cell capacity among them so as to maximize its own profit (payoff) while satisfying their cell capacity ranges. We refer to this procedure as the capacity assignment problem and formulate it as an optimization problem detailed in the following paragraphs.

First, we explain how an SP determines its suitable cell capacity range for a given cell capacity unit price. Consider an SP v and a cell capacity unit price $P > 0$ and let:

$$\overline{X}_v(P) = \begin{cases} 0, \quad \text{if } r_v^*(x_v) - Px_v \leq 0, \, \forall\, x_v \geq \widetilde{N}_v \underline{\mathcal{X}}_v, \\ \\ \underset{x_v \geq \widetilde{N}_v \underline{\mathcal{X}}_v}{\text{argmax}} \, \left(r_v^*(x_v) - Px_v\right), \\ \qquad \text{if } \exists\, x_v > \widetilde{N}_v \underline{\mathcal{X}}_v \mid r_v^*(x_v) - Px_v > 0, \end{cases} \tag{6.8}$$

$$\underline{X}_v(P) = \begin{cases} 0 \qquad \text{if } \overline{X}_v(P) = 0, \\ \\ x_v \in [\widetilde{N}_v \underline{\mathcal{X}}_v, \overline{X}_v(P)] \mid r_v^*(x_v) - Px_v = 0, \\ \qquad \text{if } \overline{X}_v(P) > 0, \end{cases} \tag{6.9}$$

where $r_v^*(x_v) - Px_v$ is the profit of SP v when it purchases x_v units of cell capacity at a unit price P. Notice that we consider $\underline{\mathcal{X}}_v > 0$ for each $v \in \mathcal{V}$. For $\underline{\mathcal{X}}_v > 0$, $r_v^*(x_v) = 0$ holds for any $x_v \in (0, \widetilde{N}_v \underline{\mathcal{X}}_v]$ (see Equations (6.1) and (6.7)), which implies $r_v^*(x) - Px < 0$ for any $x \in (0, \widetilde{N}_v \underline{\mathcal{X}}_v]$ as $P > 0$ (being the cell capacity unit price). Therefore, we look for the payoff-maximizing cell capacity of v, denoted as $\overline{X}_v(P)$, for $x \geq \widetilde{N}_v \underline{\mathcal{X}}_v$ (see Equation 6.8). If $r_v^*(x_v) - Px_v \leq 0$ for any $x_v \geq \widetilde{N}_v \underline{\mathcal{X}}_v$, then we impose $\overline{X}_v(P) = 0$ otherwise, if it exists $x_v > \widetilde{N}_v \underline{\mathcal{X}}_v$ such that $r_v^*(x_v) - Px_v > 0$, then $\overline{X}_v(P) > \widetilde{N}_v \underline{\mathcal{X}}_v > 0$.

In turn, $\underline{X}_v(P)$ (see Equation (6.9)) denotes the minimum amount of cell capacity that provides v with a non-negative payoff. If $r_v^*(x_v) - Px_v \leq 0$ for any $x_v \geq \widetilde{N}_v \underline{\mathcal{X}}_v$, for which we imposed $\overline{X}_v(P) = 0$, then we set $\underline{X}_v(P) = 0$ as well. Otherwise, if $\overline{X}_v(P) > \widetilde{N}_v \underline{\mathcal{X}}_v > 0$, then $\underline{X}_v(P)$ is set equal to the unique[8] root of equation $r_v^*(x_v) - Px_v = 0$ in the interval $[\widetilde{N}_v \underline{\mathcal{X}}_v, \overline{X}_v(P)]$. Hence, one has $0 < \widetilde{N}_v \underline{\mathcal{X}}_v < \underline{X}_v(P) < \overline{X}_v(P)$. In summary, for the considered SP payoff function, for $\underline{\mathcal{X}}_v > 0$ and for any unit price $P > 0$, either $\underline{X}_v(P) = \overline{X}_v(P) = 0$ or $\overline{X}_v(P) > \underline{X}_v(P) > 0$.

Let $\mathcal{K}$ denote the set of InPs and C_k the cell capacity of an InP k. C_k is assumed to be a fixed positive quantity. Now consider an InP k which offers a cell capacity unit price $P_k > 0$. Suppose that k is selected by the set of SPs $\mathcal{V}_k \subseteq \mathcal{V}$. Recall that $\underline{X}_v(P_k)$ and $\overline{X}_v(P_k)$ denote the the minimum and maximum amount of capacity requested by SP v at the cell capacity unit price P_k, respectively. Let $\widehat{\mathcal{V}}_k = \{v \in \mathcal{V}_k \mid \overline{X}_v(P_k) > \underline{X}_v(P_k) > 0\}$. The InP assigns a null capacity to any SP $v \in \mathcal{V}_k \setminus \widehat{\mathcal{V}}_k$ as $\underline{X}_v(P_k) = \underline{X}_v(P_k) = 0$. In turn, for $\widehat{\mathcal{V}}_k \neq \emptyset$, the capacity assignment problem is formalized as follows: as C_k is fixed and finite, given the cell capacity ranges of all SPs in $\mathcal{V}_k$, InP k has to decide:

[8] This is always the case for the considered SP payoff function for each $v \in \mathcal{V}$ and for each considered instance. A few examples of the payoff function are provided in Appendix D.

(1) which SPs in $\mathcal{V}_k$ to serve, represented by the binary variables z_{vk}, for any $v \in \mathcal{V}_k$,

(2) how much capacity to allocate to each SP $v \in \mathcal{V}_k$, represented by non-negative variables x_{vk},

so that its profit, $P_k\left(\sum_{v\in\mathcal{V}_k} x_{vk}\right)$, is maximized while the cell capacity ranges of served SPs are satisfied (i.e., if $z_{vk} = 1$, $\underline{X}_v(P_k) \le x_{vk} \le \overline{X}_v(P_k)$, otherwise $x_{vk} = 0$) and its available capacity is not exceeded, i.e., $\sum_{v\in\mathcal{V}_k} x_{vk} \le C_k$. As P_k is fixed in the context of the capacity assignment problem, then the objective function of InP k reduces to $\sum_{v\in\mathcal{V}_k} x_{vk}$.

We opted for a two-step lexicographic approach to formulate the capacity assignment problem. In the first step, InP k solves problem (6.10)–(6.14) to determine the maximum amount of cell capacity it can sell, i.e., $C'_k = \sum_{v\in\mathcal{V}_k} x'_{vk}$ where x'_{vk} denotes the value of variable x_{vk} in the optimal solution of (6.10)–(6.14).

$$\max \sum_{v\in\mathcal{V}_k} x_{vk} \tag{6.10}$$

$$x_{vk} \ge \underline{X}_v(P_k) z_{vk}, \qquad \forall v \in \mathcal{V}_k, \tag{6.11}$$

$$x_{vk} \le \overline{X}_v(P_k) z_{vk}, \qquad \forall v \in \mathcal{V}_k, \tag{6.12}$$

$$\sum_{v\in\mathcal{V}_k} x_{vk} \le C_k, \tag{6.13}$$

$$x_{vk} \ge 0,\ z_{vk} \in \{0,1\}, \qquad \forall v \in \mathcal{V}_k. \tag{6.14}$$

However, there may be multiple equivalent optimal solutions to problem (6.10)–(6.14) such that $\sum_{v\in\mathcal{V}_k} x'_{vk} = C'_k$; these solutions are equivalent from the InP perspective but not necessarily from the SPs' perspective (which may obtain a different amount of capacity in each of these solutions and hence a possibly different payoff value). When the first step of the capacity assignment problem, i.e., problem (6.10)–(6.14), does not have a unique solution, then the InP solves the second step of the capacity assignment problem, represented by problem (6.15)–(6.20):

$$\min\ \zeta_k - \sum_{v\in\mathcal{V}_k} z_{vk} \tag{6.15}$$

$$x_{vk} \geq \underline{X}_v(P_k) z_{vk}, \qquad \forall v \in \mathcal{V}_k, \tag{6.16}$$

$$x_{vk} \leq \overline{X}_v(P_k) z_{vk}, \qquad \forall v \in \mathcal{V}_k, \tag{6.17}$$

$$\sum_{v\in\mathcal{V}_k} x_{vk} = C'_k, \tag{6.18}$$

$$\zeta_k \geq z_{vk} - x_{vk}/\overline{X}_v(P_k), \qquad \forall v \in \mathcal{V}_k \mid \overline{X}_v(P_k) > 0, \tag{6.19}$$

$$x_{vk} \geq 0,\ z_{vk} \in \{0,1\}, \qquad \forall v \in \mathcal{V}_k, \qquad \zeta_k \geq 0. \tag{6.20}$$

The aim of the second step is to select among the multiple optimal solutions of the first step, one which satisfies a fairness criterion from the SPs' perspective while using up C'_k entirely (see Equation (6.18)) as C'_k is the optimal amount of the total assigned cell capacity for InP k determined in the first step. The fairness criterion consists of minimizing the highest among all SPs in $\mathcal{V}_k$ of the relative difference between the maximum amount of capacity requested by an SP (i.e., its payoff-maximizing capacity) and the amount assigned to the SP by the InP. In other words, the InP's capacity assignment accounts for the most "unsatisfied" SP among all. The highest relative difference is represented by the variable $0 \leq \zeta_k \leq 1$ and modeled through constraints (6.19). Consider an SP v with $\underline{X}_v(P_k) = \overline{X}_v(P_k) = 0$ (which means that it is unprofitable for v to purchase capacity from InP k at a unit price P_k): the corresponding optimal value of x_{vk} is equal to zero due to constraints (6.16) and (6.17), and since v is "fully-satisfied", we exclude it from the calculation of ζ_k (see constraints (6.19)). In turn, for an SP v with $\overline{X}_v(P_k) > \underline{X}_v(P_k) > 0$, if $x_{vk} = 0$ (which implies $z_{vk} = 0$ due to constraints (6.16)), the right hand side of constraints (6.19) equals 0, i.e., an SP which is willing to purchase capacity from InP k at a unit price P_k but it is not assigned any capacity is also considered as fully-satisfied to avoid ζ_k being stuck to its upper bound value equal to 1 regardless of the assignment of the other SPs. Therefore, only SPs $v \in \mathcal{V}_k$ such that $\overline{X}_v(P_k) > \underline{X}_v(P_k) > 0$ and $x_{vk} > 0$ (and hence $z_{vk} = 1$ due to constraints (6.17)) influence the value of ζ_k. The second term in the objective function, i.e., $\sum_{v\in\mathcal{V}_k} z_{vk}$, is introduced to deal with equivalent optimal solutions, although uniqueness cannot be guaranteed. Since $\zeta_k \leq 1$, an increase by one of the number of served SPs outweighs the increase of ζ_k from splitting the capacity over a larger set of SPs. Therefore, by minimizing $\zeta_k - \sum_{v\in\mathcal{V}_k} z_{vk}$, we select optimal solutions which are characterized by the largest possible number of served SPs for the fixed capacity C'_k, while the capacity assignment follows the min-max fairness criterion. Notice that for an SP v with $\underline{X}_v(P_k) = \overline{X}_v(P_k) = 0$, although in the optimal solution $x_{vk} = 0$, z_{vk} is set to one by the objective function. However, this does not affect the optimal solution as such v does not use up any capacity given that its respective x_{vk} is equal to zero in optimal solution.

6.3.4 Multi-Leader-Follower Game

As mentioned, all InPs' BSs are co-located hence InPs compete among them to be selected by SPs on a per BS cell basis, which means that the proposed framework applies to each BS cell independently. We assume that each InP k announces its cell capacity unit price P_k to the SPs independently from all other InPs but simultaneously to them. In turn, once the InP unit price profile, $\boldsymbol{P} = \{P_k\}_{k\in\mathcal{K}}$, is known by the SPs, we further assume that also each SP v acts independently but simultaneously to all other SPs in deciding from which InP to acquire capacity in order to serve its users' demand in the area of the considered cell. All involved actors are assumed rational and self-interested, i.e., each of them aims to maximize its individual payoff. Moreover, actions of any actor can affect all other actors, e.g., the InP choice of an SP can affect not only the InPs' payoffs but also the SPs' payoffs given that the cell capacity of each InP is finite and has to be split among SPs selecting the InP. With this setting in mind, we propose a Multi-Leader-Follower game to model the interaction among InPs and SPs. In the proposed model, InPs act as *leaders* (i.e., as the subset of players that move first) by announcing their unit prices to the SPs, whereas SPs act as *followers* as they choose an InP from which to acquire capacity only after the InPs' unit prices have been announced. Formally, this game is a two stage game with observable actions [220]. The game is also of imperfect information since within a stage players move simultaneously, i.e., at stage 1 InPs announce their unit prices simultaneously and at stage 2, for a given InP unit price profile, SPs make their InP choices simultaneously.

As previously argued, since the cell capacity of each InP is fixed and finite and each InP splits its available capacity among SPs that select it, the InP choice of an SP can affect the choices of all other SPs. Hence, for a given InP unit price profile ($\boldsymbol{P} = \{P_k\}_{k\in\mathcal{K}}$) the independent but simultaneous choice of an InP by each SP can be represented by a simultaneous noncooperative game in pure strategies described by the tuple $G^{\mathcal{V}}(\boldsymbol{P}) = \{\mathcal{V}, \{\mathcal{Y}_v\}_{v\in\mathcal{V}}, \{g_v\}_{v\in\mathcal{V}}\}$, where the set of players coincides with the set $\mathcal{V}$ of SPs, $\mathcal{Y}_v$ denotes the strategy set of player v representing its choice of an InP, whereas g_v denotes the payoff of v which is defined for each SP strategy profile and depends on the InP unit price profile (i.e., $g_v = g_v(\boldsymbol{P}, \boldsymbol{y})$). Further, each InP k can anticipate[9] the outcome of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ for any $\boldsymbol{P}$, i.e., k can anticipate the subset

[9]The equilibrium(a) of the MLFG are determined by means of the *sub-game perfect equilibrium* solution

of SPs that will select k at the Nash Equilibrium(a) of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ and consequently determine its payoff for $\boldsymbol{P}$. Therefore, InPs compete among them in cell capacity unit prices to be selected by the SPs: this can be represented by another simultaneous noncooperative game described by the tuple $\mathcal{G}^{\mathcal{K}} = \{\mathcal{K}, \{\mathcal{P}_k\}_{k\in\mathcal{K}}, \{G_k\}_{k\in\mathcal{K}}\}$, where the set of players coincides with the set $\mathcal{K}$ of InPs, $\mathcal{P}_k$ denotes the strategy set of player k representing a unit price range, whereas G_k denotes the payoff of k which is defined for each InP strategy profile (i.e., $G_k : \mathcal{P} \to \mathbb{R}$ where $\mathcal{P} = \prod_{k\in\mathcal{K}} \mathcal{P}_k$). We now detail $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ and $\mathcal{G}^{\mathcal{K}}$ which hereon we will refer to as the SPs' game and InPs' game, respectively.

SPs' game

As detailed in Section 6.3.2, each SP provides a single type of mobile service to a fixed number of end users. As none of SPs owns any network infrastructure/resources, then each SP, to provision the mobile service for its users in the area of a cell, acquires an aggregate amount of cell capacity from one of the InPs which it splits among all of its users in the cell area; the users are then charged by the SP based on the utility achieved from the amount of allocated capacity which results in a total amount of revenue per cell for the SP (see Section 6.3.2). Therefore, the goal of the SP is to select an InP from which to acquire cell capacity in order to maximize its profit (payoff) given by the difference between the cell revenues incurred from the amount of cell capacity assigned by the selected InP and the cost of the latter.

For each InP unit price profile $\boldsymbol{P} \in \mathcal{P}$, when selecting the InP from which to acquire capacity, SPs contend among them for the InPs' fixed and finite capacities; this gives rise to the SPs' game described by $G^{\mathcal{V}}(\boldsymbol{P})$. Formally, the strategy of an SP v is modeled by a set of binary variables $\boldsymbol{y}_v = (y_{vk})_{k\in\mathcal{K}}$ such that $y_{vk} \in \{0, 1\}$ for any $k \in \mathcal{K}$ and $\sum_{k\in\mathcal{K}} y_{vk} = 1$. Let $\boldsymbol{y} = \{\boldsymbol{y}_v\}_{v\in\mathcal{V}}$ denote a strategy profile of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$. Then, let $x_{vk}(P_k, \boldsymbol{y})$ denote the amount of cell capacity obtained by SP v from InP k at unit price P_k given the SP strategy profile $\boldsymbol{y}$: if v does not select k in $\boldsymbol{y}$ (i.e., $y_{vk} = 0$) then clearly $x_{vk}(P_k, \boldsymbol{y}) = 0$, otherwise if v selects k in $\boldsymbol{y}$ (i.e., $y_{vk} = 1$) then $x_{vk}(P_k, \boldsymbol{y})$ is equal to the value of variable x_{vk} in the optimal solution of problem (6.15)–(6.20) when the capacity assignment problem is solved by InP k for the set $\mathcal{V}_k = \{v \in \mathcal{V} : \ y_{vk} = 1\}$, given the cell capacity ranges $[\underline{X}_v(P_k), \overline{X}_v(P_k)]$ for its offered

concept which is an extension of the *backward induction* solution concept for the original one-leader, one-follower Stackelberg game. The idea behind *backward induction* is that the leader assumes that the follower is rational and it anticipates the follower's best response to each action of its own. Therefore, the leader's strategy consists of selecting the action that maximizes its own payoff given the best response of the follower. In case of the MLFG we propose here, leaders anticipate the outcome of the followers' game, i.e., its Nash equilibrium(a), for any action profile of their own, which is, in turn, the main idea behind the *sub-game perfect equilibrium* solution concept. Details concerning the calculation of the equilibrium(a) of the MLFG for the considered problem instances are presented in Section 6.4.5.

unit price P_k (see Section 6.3.3)).

The payoff of v from $\boldsymbol{y}$ is defined as

$$g_v(\boldsymbol{P}, \boldsymbol{y}) = \sum_{k \in \mathcal{K}} \left(r_v^*(x_{vk}(P_k, \boldsymbol{y})) - P_k x_{vk}(P_k, \boldsymbol{y}) \right), \tag{6.21}$$

where $r_v^*(x_{vk}(P_k, \boldsymbol{y}))$ is the total optimal revenue of SP v (see Equation (6.7) and Section 6.3.2) for the amount of cell capacity $x_{vk}(P_k, \boldsymbol{y})$, whereas $P_k x_{vk}(P_k, \boldsymbol{y})$ is the cost incurred by SP v from purchasing the amount of cell capacity $x_{vk}(P_k, \boldsymbol{y})$ at unit price P_k, therefore $g_v(\boldsymbol{P}, \boldsymbol{y})$ is given in terms of the total profit[10] of v.

By definition, a strategy profile $\breve{\boldsymbol{y}} = [\breve{\boldsymbol{y}}_v, \breve{\boldsymbol{y}}_{-v}]$, where $\boldsymbol{y}_{-v}$ denotes the strategies of all SPs but v, is a Nash Equilibrium (NE) of the SPs' game $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ if for each SP $v \in \mathcal{V}$, $\breve{\boldsymbol{y}}_v = \text{argmax}_{\boldsymbol{y}_v \in \mathcal{Y}_v}\ g_v(\boldsymbol{P}, [\boldsymbol{y}_v, \breve{\boldsymbol{y}}_{-v}])$, i.e., if no SP has an incentive to unilaterally deviate from $\breve{\boldsymbol{y}}$. Since there is one SPs' game $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ for each InP unit price profile $\boldsymbol{P} \in \mathcal{P}$, hereon we will use the notation $\breve{\boldsymbol{y}}(\boldsymbol{P})$ to denote the NE strategy profile(s) of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$.

InPs' game

Each InP unit price profile $\boldsymbol{P} \in \mathcal{P}$ may result in a distinct NE of the SPs' game, i.e., in a different partition of the set of SPs over the set of InPs and consequently in different profits for the InPs. To put it differently, InPs compete among them in cell capacity unit prices to be selected by the SPs, which we modeled as the game $\mathcal{G}^{\mathcal{K}} = \{\mathcal{K}, \{\mathcal{P}_k\}_{k \in \mathcal{K}}, \{G_k\}_{k \in \mathcal{K}}\}$, namely the InPs' game. The strategy set of each player k consists of a unit price range, i.e., $\mathcal{P}_k = [\underline{P}_k, \overline{P}]$ where $\underline{P}_k$ denotes the cell capacity unit cost for InP k and $\overline{P}$ denotes the minimum unit price for which no SP is willing to buy capacity. A strategy of InP k is then a cell capacity unit price $P_k \in \mathcal{P}_k$. We impose $P_k \geq \underline{P}_k$ as we assumed InPs to be rational, i.e., they will not accept gains lower than their costs and similarly, as all SPs are also assumed to be rational, they will not purchase cell capacity at a unit price resulting in a negative payoff; in other words, any InP k offering $P_k \geq \overline{P}$, would not sell any cell capacity. The payoff of player k from the strategy (unit price) profile $\boldsymbol{P} = \{P_k\}_{k \in \mathcal{K}}$ is defined as

$$G_k(\boldsymbol{P}) = P_k \left(\sum_{v \in \mathcal{V}} x_{vk}(P_k, \breve{\boldsymbol{y}}(\boldsymbol{P})) \right), \tag{6.22}$$

that is, as the product between the cell capacity unit price of InP k and the total amount of capacity sold to SPs that select k at the NE $\breve{\boldsymbol{y}}(\boldsymbol{P})$ of the SPs' game $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$). Recall that, under the assumption that all SPs are rational, each InP can anticipate $\breve{\boldsymbol{y}}(\boldsymbol{P})$ of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$. If

[10] As for each SP $v \in \mathcal{V}$, $\sum_{k \in \mathcal{K}} y_{vk} = 1$, then $r_v^*(x_{vk}(P_k, \boldsymbol{y})) - P_k x_{vk}(P_k, \boldsymbol{y}) \neq 0$ for at most one InP $k \in \mathcal{K}$.

for some $\boldsymbol{P} \in \mathcal{P}$, the NE of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ is not unique, we assume InPs are pessimistic and each of them independently considers the worst payoff achieved over all the NE of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$. In turn, if $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ has no NE in pure strategies, we would look for its NE in mixed strategies[11].

A strategy profile $\breve{\boldsymbol{P}} = [\breve{P}_k, \breve{\boldsymbol{P}}_{-k}]$, where $\breve{\boldsymbol{P}}_{-k}$ denotes the unit prices offered by all InPs but k, is an NE of the InPs' game $\mathcal{G}^{\mathcal{K}}$ if $\breve{P}_k = \text{argmax}_{P_k \in \mathcal{P}_k} G_k([P_k, \breve{\boldsymbol{P}}_{-k}])$ for any $k \in \mathcal{K}$, i.e., if no InP has an incentive to unilaterally deviate from $\breve{\boldsymbol{P}}$.

6.4 Scenarios and computational tests

In this section, we describe the scenarios that we have addressed by means of the proposed framework. First, we explain how we set up different types of InPs based on their network technology, available spectrum bandwidth, etc., and propose a cost model largely based on [1] to derive a sensible cell capacity unit cost for each InP type (Section 6.4.1). In Sections 6.4.2 and 6.4.3 we dwell on the set of service types that we have set up based on usage scenarios for IMT for 2020 and beyond [162] and on the set of SPs providing such services; in particular, we report how these services have been characterized based on Key Performance Indicators (KPI) requirements from [207] and how SPs set up their user fees based on their service characteristics and user types. The set of scenarios (problem instances) addressed in our computational tests is defined in Section 6.4.4, whereas implementation details concerning these computational tests are reported in Section 6.4.5.

6.4.1 InPs

The considered set $\mathcal{K}$ of InPs consists of InPs which coexist in a dense urban area where each of them has either (i) deployed a legacy (pre-5G) heterogeneous network of macro cells (MCs) and small cells (SCs) prior to the beginning of the studied period and does not upgrade to 5G in the meantime or (ii) just deployed a 5G heterogeneous network of MCs and SCs. We refer to (i) and (ii) as InP types and denote them by $\mathcal{L}$ and $\mathcal{N}$, respectively. The type of each InP k is then represented by a binary parameter λ_k, which equals 1 if k is of type $\mathcal{L}$ and 0 if k is of type $\mathcal{N}$.

[11]If there were no NE in pure strategies for $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ for some $\boldsymbol{P} \in \mathcal{P}$, then we would look for its NE in mixed strategies: formally, we would relax variables $\boldsymbol{y}_v$, $\forall v \in \mathcal{V}$ representing the InP choice of SP v (see Section 6.3.4), i.e., an SP's mixed strategy for the game $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ would be represented by variables $\boldsymbol{\gamma}_v = \{\gamma_{vk}\}_{k\in\mathcal{K}} \mid 0 \le \gamma_{vk} \le 1, \forall k \in \mathcal{K}, \sum_{k\in\mathcal{K}} \gamma_{vk} = 1, \forall v \in \mathcal{V}$ and the expected payoff of v from $\boldsymbol{\gamma}$ given $\boldsymbol{P}$, $\widetilde{g}_v(\boldsymbol{P}, \boldsymbol{\gamma}) = \sum_{\boldsymbol{y}\in\mathcal{P}} P(\boldsymbol{y}|\boldsymbol{\gamma}) g_v(\boldsymbol{P}, \boldsymbol{y})$ where $P(\boldsymbol{y}|\boldsymbol{\gamma})$ is the probability of occurrence of the outcome represented by the pure strategy profile $\boldsymbol{y}$ (i.e., a partitioning of the set of SPs over the set of InPs) given $\boldsymbol{\gamma}$ (the InPs' expected payoff from the equilibrium mixed strategy profile $\breve{\boldsymbol{\gamma}}$ would be calculated in a similar fashion). However, for all considered instances (see Section 6.5.2), there is at least one NE in pure strategies for $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$, $\forall \boldsymbol{P} \in \mathcal{P}$.

We assume that in the considered dense urban area both MCs and SCs of different InPs are colocated. In these lines, at the beginning of the studied period, a site is present in a MC candidate site (and can be used by any of the InPs) if at least one of the InPs has previously deployed a MC BS in it; instead, a site is present in a SC candidate site and can be used by a given InP if the InP itself has previously deployed a SC BS in it. Let $\pi_{MC,k}(\pi_{SC,k})$ denote the probability that InP k has not deployed a MC(SC) BS in a MC(SC) candidate site[12] prior the beginning of the studied period. Then, as MC sites are shared, $\pi_{MC} = \prod_{k \in \mathcal{K}} \pi_{MC,k}$ is the probability that none of the InPs has deployed a MC BS in a MC candidate site prior the beginning of the studied period, i.e., at the beginning of the studied period, a MC site has to be built with probability π_{MC}. In turn, since SC sites are not shared, at the beginning of the studied period, InP k has to build a SC site in a SC candidate site with probability $\pi_{SC,k}$. For an InP k of type $\mathcal{L}$, we consider $\pi_{MC,k} = \pi_{SC,k} = 0$, i.e., we assume k has deployed legacy MCs(SCs) BSs in all available MC(SC) candidate sites. Instead for an InP k of type $\mathcal{N}$, $\pi_{MC,k} = \pi_{SC,k} = 1$ imply that InP k has not previously deployed a legacy network, whereas $0 \leq \pi_{MC,k}, \pi_{SC,k} < 1$ means that InP k has previously deployed a legacy network and can reuse its sites. However, we assume that, at the beginning of the studied period, type $\mathcal{N}$ InPs will deploy 5G MC and SC BSs in all available MC and SC candidate sites and that such InPs will compete with the other InPs solely through their new (5G) network while simply reusing sites of thier previously deployed legacy networks (if any).

For MC sites we have considered 3-sector antennas as in [1, 221, 222], whereas for SC sites, omnidirectional (i.e., 1-sector) ones.

Let B_k denote the total available bandwidth of InP k and $\widehat{B}_k \in [0, B_k]$ the amount of bandwidth associated with a spectrum license whose cost has already been amortized at the beginning of the studied period, while the remaining amount of bandwidth $B_k - \widehat{B}_k$ corresponds to a spectrum license acquired at the beginning of the studied period. In particular, $\widehat{B}_k = B_k$ if InP k is of type $\mathcal{L}$ and $\widehat{B}_k = 0$ if k is of type $\mathcal{N}$ and has no legacy network. We assume that for each InP k, B_k is dynamically shared between the MC and the SC layers and, within the MC/SC layer, the bandwidth is also dynamically shared (and not a priori partitioned) between the Downlink (DL) and Uplink (UL) of each MC sector/SC. Further, as we consider a very dense deployment of SCs, MCs can be assumed idle, hence B_k is then dynamically shared between the DL and the UL of each SC.

We have made the simplifying assumption that the DL and UL spectral efficiencies are equal. For InP k, let $\nu_{MC,k}(\nu_{SC,k})$ denote the MC(SC) average spectral efficiency[13] of both DL and

[12] $\pi_{MC,k}(\pi_{SC,k})$ is intended as an average probability for the considered area, hence it is the same for all MC(SC) candidate sites.

[13] We refer to the average spectral efficiency definition in [223] or equivalently to the cell spectral efficiency

UL. We assume InPs compete among them to be selected by SPs on a per SC basis[14], hence the capacity C_k characterizing InP k is set equal to its total (DL+UL) average capacity of a SC, i.e., $C_k = \nu_{SC,k} B_k$.

The cost per unit of capacity characterizing InP k, i.e., $\underline{P}_k$, is then set equal to the monthly[15] cost per unit of capacity provided in the area of a SC, i.e.,

$$\underline{P}_k = \frac{1}{12\,L\,C_k}\Big[(1-\lambda_k)\left(c^{cpx}_{SC,k} + (A_{SC}/A_{MC})\,c^{cpx}_{MC,k}\right) + \\ + c^{opx}_{SC,k}L + (A_{SC}/A_{MC})\,c^{opx}_{MC,k}L + c^{spec}_{SC,k}\Big], \tag{6.23}$$

where L denotes the duration of the studied period in years, $c^{cpx}_{MC,k}(c^{cpx}_{SC,k})$ and $c^{opx}_{MC,k}$ $(c^{opx}_{SC,k})$ denote the total CAPEX and total annual OPEX incurred by InP k per MC sector (SC site)[16], $A_{MC}(A_{SC})$ denotes the area of a MC sector (SC), respectively, whereas $c^{spec}_{SC,k}$ denotes the spectrum license cost normalized to the area of the SC and to the duration of the studied period. The per sector MC cost terms ($c^{cpx}_{MC,k}$ and $c^{opx}_{MC,k}$) are multiplied by A_{SC}/A_{MC}, i.e., the inverse of the number of SCs per MC sector, to uniformly split the cost of the MC sector among all SCs that overlay the MC sector. The $(1-\lambda_k)$ term sets the CAPEX terms to zero for an InP k of type $\mathcal{L}$ (for which $\lambda_k = 1$); however, k will incur the OPEX of its legacy network. Instead, an InP of type $\mathcal{N}$ (for which $\lambda_k = 0$), incurs both CAPEX and OPEX terms as it deploys its 5G network at the beginning of the study period.

In details, $c^{cpx}_{MC,k}$, $c^{cpx}_{SC,k}$, $c^{opx}_{MC,k}$, $c^{opx}_{SC,k}$ and $c^{spec}_{SC,k}$ are determined as follows:

$$c^{cpx}_{MC,k} = \frac{1}{3}\Big[(1/|\mathcal{K}|)\,\pi_{MC}c^{c,s}_{MC} + c^{c,a}_{MC,k} + c^{c,f}_{MC,k} + \\ + \lceil B_k/B_0\rceil c^{c,rf}_{MC,k} + \lceil m_k(B_k/B_0)\rceil c^{c,bp}_{MC,0} + c^{c,bh}_{MC,k}\Big], \tag{6.24}$$

$$c^{opx}_{MC,k} = \frac{1}{3}\Big[(1/|\mathcal{K}|)c^{o,s}_{MC} + c^{o,r\&u}_{MC} + c^{o,v}_{MC} + \\ + \Xi^{l\&m}_{MC}\left(\lceil B_k/B_0\rceil c^{c,rf}_{MC,k} + \lceil m_k(B_k/B_0)\rceil c^{c,bp}_{MC,0}\right) + c^{o,bh}_{MC,k}\Big], \tag{6.25}$$

$$c^{cpx}_{SC,k} = \pi_{SC,k}c^{c,s}_{SC} + c^{c,a}_{SC,k} + c^{c,f}_{SC,k} + c^{c,bh}_{SC,k}, \tag{6.26}$$

$$c^{opx}_{SC,k} = c^{o,s}_{SC} + c^{o,r\&u}_{SC} + c^{o,v}_{SC} + \Xi^{l\&m}_{SC}c^{c,a}_{SC,k} + c^{o,bh}_{SC,k}, \tag{6.27}$$

$$c^{spec}_{SC,k} = c^{spec}_0(B_k - \widehat{B}_k)A_{MC}L, \tag{6.28}$$

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definition in [224].

[14]When the average spectral efficiency is considered and the SPs' users can be assumed uniformly distributed over the considered geographical area, the same solution should apply to all SCs in the area.

[15]$\underline{P}_k$ is defined as a monthly cost to match the timescale of the SPs' user fee (see Section 6.4.3). Consequently, the InP price strategy $P_k \geq \underline{P}_k$ (see Section 6.3.4, is a monthly price per unit of average SC capacity.) In these lines, the payoff of each InP $k \in \mathcal{K}$, i.e., G_k, (as defined by Equation (6.22)) and the payoff of each SP $v \in \mathcal{V}$, i.e., g_v, (as defined by Equation (6.21)) also correspond to a one-month period.

[16]Notice that for SCs, we refer to the site costs since we consider 1-sector SC sites.

where the cost terms that make up $c^{cpx}_{MC,k}$, $c^{cpx}_{SC,k}$, $c^{opx}_{MC,k}$ and $c^{opx}_{SC,k}$ and values given to these cost terms are based on the cost model and respective values in [1]. Notice that in Equations (6.24) and (6.25), the 1/3 multiplier has been introduced to derive the cost per MC sector since each MC cost term therein refers to the total cost of all three sectors of a 3-sector MC site. As for Equations (6.26) and (6.27), the values obtained from [1] for the cost terms $c^{c,s}_{SC}$, $c^{c,f}_{SC,k}$, $c^{o,s}_{SC}$, $c^{o,r\&u}_{SC}$, $c^{o,v}_{SC}$ (which will be defined in the consecutive paragraphs) are costs per site for 2-sector small cell sites but also for 1-sector picocell sites whereas the values for $c^{c,a}_{SC,k}$ are costs per sector for 2-sector small cell sites, hence we deemed all these values to be reasonable also for the 1-sector SC sites considered here without introducing any multipliers.

We now define the individual cost terms involved in Equations (6.24)–(6.28) and explain how their values have been set in order to characterize the InPs considered here. First, $c^{c,s}_{MC}(c^{c,s}_{SC})$ denotes the MC(SC) site civil works and acquisition cost. $c^{c,s}_{MC}$ is weighted by π_{MC} and divided by the number of InPs since we assume that each MC site will be shared by all InPs[17]. In turn, for each InP k, $c^{c,s}_{SC}$ is weighted by the probability that k has to build a SC by itself at the beginning of the studied period ($\pi_{SC,k}$) given that SC sites are not shared. In these lines, also the MC site rental cost $c^{o,s}_{MC}$ is uniformly split among all InPs (see Equation (6.25)), whereas the SC site rental cost $c^{o,s}_{SC}$ is not (see Equation (6.27)).

$c^{c,a}_{MC,k}(c^{c,a}_{SC,k})$ is the MC(SC) antenna cost for InP k. $c^{c,f}_{MC,k}(c^{c,f}_{SC,k})$ are the feeder (cable connecting active antenna to equipment cabinet), install and test and commission cost per MC(SC) site for InP k. $c^{c,rf}_{MC,k}$ is a baseline Radio Frequency (RF) front end cost per MC site for a baseline bandwidth $B_0 = 20$ MHz for InP k which has to be scaled by $\lceil B_k/B_0 \rceil$ (i.e., the ratio between the total bandwidth of InP k and the baseline bandwidth), whereas $c^{c,bp}_{MC,0}$ is a baseline baseband processing cost for 3 sectors of a $B_0 = 20$ MHz 2x2 MIMO channel (see Section 11.5.1.3 in [1]) that needs to be scaled by $\lceil m_k(B_k/B_0) \rceil$, where m_k is a factor[18] that allows to estimate the relative amount of base band processing for InP k for $B_0 = 20$ MHz units of bandwidth given its antenna MIMO order w.r.t. to the baseline (i.e., the $B_0 = 20$ MHz 2x2 MIMO channel). Notice that we have introduced the ceiling operator in the scaling factors of $c^{c,rf}_{MC,k}$ and $c^{c,bp}_{MC,0}$ in order to be conservative as in [1] there is no explicit expression of the cost scaling operation for none of the two.

In the following, we explain how starting from the cost model in [1], we set the values of $c^{c,a}_{MC,k}$, $c^{c,f}_{MC,k}$, $c^{c,rf}_{MC,k}$, m_k, $c^{c,a}_{SC,k}$ and $c^{c,f}_{SC,k}$ for each InP k depending on its type. For an InP

[17] Notice thatif there is at least one InP k with $\lambda_k = 1$, for which $\pi_{MC,k} = 0$, and hence $\pi_{MC} = \min_{k \in \mathcal{K}} \pi_{MC,k} = 0$, a site is already present in each MC candidate site hence $c^{c,s}_{MC}$ is not incurred.

[18] Let $\mathcal{M}_k$ denote the product of the number of MIMO streams with the number of the spatial beams (see Section 11.4.2. in [1]) that correspond to MC antenna MIMO order for InP k and let $\mathcal{M}_0$ denote the value of this product for the baseline 2x2 MIMO channel where $\mathcal{M}_0 = 2$ (see Table 11-9 in [1]). We then set $m_k = \mathcal{M}_k/\mathcal{M}_0$ (see Section 11.5.1.3 in [1]).

k of type $\mathcal{L}$ ($\lambda_k = 1$), we have considered the following values for the MC and SC average spectral efficiency for both DL and UL: $\nu_{k,MC} = 2.2$ bps/Hz and $\nu_{SC,k} = 2.6$ bps/Hz, which are the required DL average spectral efficiency values for IMT-Advanced systems for base urban coverage and microcellular environments, respectively [224]. Instead, for an InP k of type $\mathcal{N}$ ($\lambda_k = 0$), we have set $\nu_{MC,k} = 6.6$ bps/Hz and $\nu_{SC,k} = 7.8$ bps/Hz as ITU-R expects the average spectral efficiency for IMT for 2020 and beyond to be three times higher than for IMT-Advanced [162],[223]. Since the 5G radio interface has not been defined yet, we cannot anticipate the spectral efficiency improvements it will bring about, hence we have assumed that the required spectral efficiency for IMT for 2020 and beyond will be achieved through high order MIMO antennas, although there are several factors that affect the achieved spectral efficiency [225]. We have considered the antenna configurations (MIMO order + frequency band) presented in [1] and, when possible, for each InP k we have selected antenna configurations that would best match its MC(SC) average spectral efficiency $\nu_{MC,k}$ ($\nu_{SC,k}$), otherwise we have associated[19] InPs of type $\mathcal{L}/\mathcal{N}$ with the least/most complex (and hence expensive) antenna configurations while some of the cost terms for InPs of type $\mathcal{N}$ have also been overestimated so as to account for the factor-of-three difference between the average spectral efficiency of type $\mathcal{N}$ and $\mathcal{L}$ InPs. For instance, to choose the MC antenna configurations among those listed in [1], we were mainly driven by their respective average spectral efficiency values: the different 2x2 MIMO operation modes provide average spectral efficiency values in the range 2.23 – 2.88 bps/Hz whereas 64x2 MIMO ones provide average spectral efficiency values in the range 5.53 – 7.14 bps/Hz which makes the former suitable for an InP k of type $\mathcal{L}$ ($\nu_{MC,k} = 2.2$ bps/Hz), whereas the latter suitable for an InP k of type $\mathcal{N}$ ($\nu_{MC,k} = 6.6$ bps/Hz). Instead, for SCs, as the average spectral efficiencies of the two configurations listed in [1] are not provided, we associate InPs of type $\mathcal{L}/\mathcal{N}$ with the lowest/highest MIMO order configuration. Let M denote the MIMO order of an antenna. Specifically, for each InP k of type $\mathcal{L}$, we have assumed that its MCs operate only at sub-1GHz and low frequency bands with $M = 2$ antennas, whereas its SCs operate at low and medium unpaired bands with $M = 2$ antennas. Instead, for each InP of type $\mathcal{N}$, we have assumed that its MCs operate both at sub-1GHz and low frequencies with $M = 4$ antennas and at medium frequencies with $M = 64$ antennas, whereas its SCs operate at low and medium frequency bands with $M = 4$ antennas. Values of $c^{c,a}_{MC,k}$, $c^{c,f}_{MC,k}$, $c^{c,rf}_{MC,k}$, m_k, $c^{c,a}_{SC,k}$ and $c^{c,f}_{SC,k}$ depending on the antenna configuration(s) associated with the type of InP k are then set as reported in Table 6.2 based on [1]. Some details concerning these values follow. According to

[19]It is worth pointing out that the aforementioned association of InPs to antenna configurations based on their types has not been used for a network deployment simulation but it only serves to obtain an estimate of the cost incurred by an InP for providing a certain average cell capacity based on its available bandwidth and average spectral efficiency.

[1], for sub-1GHz and low frequencies multiband MC antennas are available, hence an InP k of type $\mathcal{L}$ ($\lambda_k = 1$), deploys only one $M = 2$ antenna per MC site. Instead, as MCs of an InP k of type $\mathcal{N}$ operate at two frequency band groups (i.e., sub-1GHz and low frequency bands and medium frequency bands) that require individual radio equipment, we set $c^{c,a}_{MC,k}$ equal to the sum of the antenna cost of the two frequency band groups and $c^{c,f}_{MC,k}$ equal to the sum of feeder, install and test and commission cost of the two frequency band groups. In turn, $\lceil B_k/B_0 \rceil c^{c,rf}_{MC,k}$ and $\lceil m_k(B_k/B_0) \rceil c^{c,bp}_{MC,0}$ for k of type $\mathcal{N}$ have been overestimated by setting $c^{c,rf}_{MC,k}$ and m_k equal to the respective values for $M = 64$ antennas at medium frequency bands (which are both higher than the respective values for $M = 4$ antennas at sub-1GHz and low frequency bands).

$c^{o,r\&u}_{MC}(c^{o,r\&u}_{SC})$ denotes the annual rates and utilities for a MC(SC) site, whereas $c^{o,v}_{MC}(c^{o,v}_{SC})$ the annual vendor service fee. The annual licensing and maintenance cost per MC(SC) site are calculated as fraction $\Xi^{l\&m}_{MC}(\Xi^{l\&m}_{SC})$ of the active equipment cost which in case of MC sites corresponds to the sum of the total RF front end cost ($\lceil (B_k/B_0) c^{c,rf}_{MC,k} \rceil$) and the total base band processing cost ($\lceil m_k(B_k/B_0) c^{c,bp}_{MC,0} \rceil$) whereas in case of SC sites it corresponds to the antenna cost ($c^{c,a}_{SC,k}$) as the RF front end and the baseband processing unit are part of the integrated active equipment [1].

In particular, the MC(SC) CAPEX and OPEX backhauling cost of InP k, $c^{c,bh}_{MC,k}(c^{c,bh}_{SC,k})$, $c^{o,bh}_{MC,k}(c^{o,bh}_{SC,k})$ depend on the type of backhauling selected by k. We have considered the set of backhauling options presented in [1] which we denote as $\mathcal{T}$. For each option $t \in \mathcal{T}$, the capacity (C^{bh}_t), CAPEX ($c^{c,bh}_t$) and annual OPEX ($c^{o,bh}_t$) per backhauling link are reported in Table 6.3. We assume that each InP has deployed individual backhauling links for the SCs and MCs, i.e., there is no aggregation of the traffic of the SCs at the underlying MC site. Then, each InP k determines its best (minimum cost) option for the SCs, denoted by $t^*_{SC,k}$, as

$$t^*_{SC,k} = \arg\min_{t \in \mathcal{T}} \left\{ \left\lceil \frac{\nu_{SC,k}\, B_k}{C^{bh}_t} \right\rceil (c^{c,bh}_t + L c^{o,bh}_t) \right\}, \tag{6.29}$$

hence $c^{c,bh}_{SC,k} = \left\lceil (\nu_{SC,k} B_k) / C^{bh}_{t^*_{SC,k}} \right\rceil c^{c,bh}_{t^*_{SC,k}}$ and $c^{o,bh}_{SC,k} = \left\lceil (\nu_{SC,k} B_k) / C^{bh}_{t^*_{SC,k}} \right\rceil c^{o,bh}_{t^*_{SC,k}}$. Similarly, the best backhauling option $t^*_{MC,k}$ for the MC sites (i.e., for all three sectors per site) for InP k is determined as

$$t^*_{MC,k} = \arg\min_{t \in \mathcal{T}} \left\{ \left\lceil \frac{3\nu_{MC,k}\, B_k}{C^{bh}_t} \right\rceil (c^{c,bh}_t + L c^{o,bh}_t) \right\}, \tag{6.30}$$

therefore, $c^{c,bh}_{MC,k} = \left\lceil (3\nu_{MC,k} B_k) / C^{bh}_{t^*_{MC,k}} \right\rceil c^{c,bh}_{t^*_{MC,k}}$ and $c^{o,bh}_{MC,k} = \left\lceil (3\nu_{MC,k} B_k) / C^{bh}_{t^*_{SC,k}} \right\rceil c^{o,bh}_{t^*_{MC,k}}$.

Finally, in Equation (6.28), c_0^{spec} denotes the reference annual spectrum license cost per unit of bandwidth and unit of geographical area which, multiplied by the amount of bandwidth associated with the spectrum license acquired at the beginning of the studied period ($B_k - \widehat{B}_k$), the area of the SC (A_{SC}) and the studied period (L), provides the spectrum license cost $c_{SC,k}^{spec}$ per SC for the studied period. c_0^{spec} was derived from the outcome of the 5G spectrum auction in the UK [226] by first calculating the average cost per MHz of the total auctioned spectrum and then dividing the latter with the area of the UK and the license duration (20 years) [227].

Values given to the cost terms and related parameters throughout Equations (6.24)–(6.28) are summarized in Table 6.2. Notice that we have not considered cost inflation over time and that all values obtained from [1] and [226], originally in GBP currency, have been converted to EUR using a conversion rate 1.11 EUR/GBP.

Table 6.1 InP related parameters

Notation	Definition	Unit
λ_k	binary parameter: 1 if InP k of type $\mathcal{L}$, 0 if of type $\mathcal{N}$	—
$\mathcal{L}$	label to represent an InP that has a legacy network & does not upgrade to 5G	—
$\mathcal{N}$	label to represent an InP that deploys a 5G network at the beginning of the studied period	—
B_k	total available bandwidth of InP k	MHz
$\widehat{B}_k$	amount of bandwidth of InP k associated with an amortized spectrum license	MHz
$\pi_{SC,k}$	probability that InP k has not deployed a legacy SC in a SC candidate site	—
$\pi_{MC,k}$	probability that InP k has not deployed a legacy MC in a MC candidate site	—
$\pi_{MC}=\min_{k\in\mathcal{K}} \pi_{MC,k}$	probability that a site has to be built in a MC candidate site	—
A_{MC}	area of a MC sector	km^2
A_{SC}	area of a SC	km^2
$c_{MC,k}^{cpx}$	per sector total CAPEX of a 3-sector MC site for InP k	EUR
$c_{SC,k}^{cpx}$	total CAPEX of a SC site for InP k	EUR
$c_{MC,k}^{opx}$	per sector total annual OPEX of a 3-sector MC site for InP k	EUR/year
$c_{SC,k}^{opx}$	total annual OPEX of a SC site for InP k	EUR/year
$c_{SC,k}^{spec}$	spectrum license cost normalized to A_{SC} and L for InP k	EUR
$\underline{P}_k$	monthly overall cost incurred by InP k to provide one unit (1 Mbps) of capacity	EUR/Mbps/month

6.4.2 Service types

In this work, we address the provision of two types of 5G services motivated by two usage scenarios identified by ITU-R for IMT for 2020 [162], namely eMBB and mMTC. We have characterized these services using KPIs of the 5GPPP project FANTASTIC-5G ([207]) for use cases defined therein. Specifically, for eMBB we consider the KPIs of use case 7 (dense urban society below 6 GHz) in [207], whereas for mMTC, KPIs of use case 3 (sensor networks) in [207]. Let $d_{eMBB}(d_{mMTC})$ denote the density of devices that request eMBB(mMTC) services. We have set $d_{\text{eMBB}} = 25000$ devices/km^2 and $d_{\text{mMTC}} = 600000$ devices/km^2 according to the

Table 6.2 Cost model parameters

Notation	Definition	Value	Unit
B_0	baseline bandwidth	20	MHz
L	duration of the studied period	10	years
$c_{MC}^{c,s}$	site civil works & acquisition cost for a MC site	51282	EUR
$c_{MC,k}^{c,a}$	total antenna cost per MC site for InP k		
	$\lambda_k = 1$: M=2 antenna at sub-1GHz & low bands	1776	EUR
	$\lambda_k = 0$: M=4 antenna at sub-1GHz & low bands + M=64 antenna at medium band	10656	EUR
$c_{MC,k}^{c,f}$	total feeder, install, test and commission costs for all antennas of a MC site for InP k		
	$\lambda_k = 1$: M=2 antenna at sub-1GHz & low bands	4884	EUR
	$\lambda_k = 0$: M=4 antenna at sub-1GHz & low + M=64 antenna at medium band	9768	EUR
$c_{MC,k}^{c,rf}$	RF front end cost per 20MHz bandwidth per MC site for InP k		
	$\lambda_k = 1$: M=2 antenna at sub-1GHz & low bands	12487.5	EUR
	$\lambda_k = 0$: M=64 antenna at medium band	39960	EUR
$c_{MC,0}^{c,bp}$	baseline baseband processing cost for a 3-sector MC with 2×2 MIMO for each 20 MHz	4162.5	EUR
m_k	scaling factor for baseband processing cost (see Table 11-9 and Section 11.5.1.3 in [1])		
	$\lambda_k = 1$: M=2 antenna configuration	1	–
	$\lambda_k = 0$: M=64 antenna configuration	6	–
$c_{MC}^{o,s}$	annual MC site rental cost	22200	EUR
$c_{MC}^{o,r\&u}$	annual rates and utilities cost for a MC site	11100	EUR
$c_{MC}^{o,v}$	annual vendor services cost for a MC site	3552	EUR
$\Xi_{MC}^{l\&m}$	fraction of active equipment cost (total RF front end + BBU processing cost) to calculate annual licensing & maintenance cost for a MC site	0.1	–
$c_{SC}^{c,s}$	site civil works and acquisition cost for a SC site	5328	EUR
$c_{SC,k}^{c,a}$	antenna cost per SC site for InP k		
	$\lambda_k = 1$: M=2 antenna at low & medium unpaired bands	277.5	EUR
	$\lambda_k = 0$: M=4 antenna at low & medium unpaired bands	555	EUR
$c_{SC,k}^{c,f}$	feeder, install, test and commission costs per SC site for InP k		
	$\lambda_k = 1$: M=2 antenna at low & medium unpaired bands	777	EUR
	$\lambda_k = 0$: M=4 antenna at low & medium unpaired bands	777	EUR
$c_{SC}^{o,s}$	annual SC site rental cost	1110	EUR
$c_{SC}^{o,r\&u}$	annual rates and utilities cost for a SC site	599.4	EUR
$c_{SC}^{o,v}$	annual vendor services cost for a SC site	0	EUR
$\Xi_{SC}^{l\&m}$	fraction of active equipment cost (SC antenna cost) to calculate annual licensing & maintenance cost for a SC site	0.25	–
c_0^{spec}	spectrum license cost per MHz, unit of area and year [226], [227]	1.6331	EUR/MHz/km^2/year

Table 6.3 Capacity and cost of different backhauling options [1]

Backhauling type (t)	Capacity per link (C_t^{bh})	CAPEX ($c_t^{c,bh}$)	annual OPEX ($c_t^{o,bh}$)
dark fiber (1 Gbps)	1 Gbps	35409 EUR	1248.75 EUR
dark fiber (10 Gbps)	10 Gbps	36630 EUR	1248.75 EUR
dark fiber (100 Gbps)	100 Gbps	39405 EUR	1248.75 EUR
Ethernet Access Direct (EAD) Managed	1 Gbps	2331 EUR	3496.5 EUR

device density values considered in [207] for the respective use cases. The average number of devices in the area of one SC that request services of a given type can be determined as the product of the device density with the area of the SC, i.e., $d_{eMBB}A_{SC}$ for eMMB and $d_{mMTC}A_{SC}$ for mMTC. In turn, the area of a MC sector (A_{MC}) and the area of a SC (A_{SC}), have been derived from their respective inter site distances, D_{MC} and D_{SC}. As mentioned, each MC site has three sectors. We also assume that the cells of a MC site (one per sector) are hexagonal and that MC sites are located at the corner of these cells, therefore the MC inter site distance (D_{MC}) is equal to three times the side of a hexagonal cell [1,221,222]. Instead, SC antennas are assumed to be omnidirectional hence the SC inter site distance (D_{SC}) is equal to twice the cell radius. Therefore $A_{MC} = (1/(2\sqrt{3}))D_{MC}^2$ and $A_{SC} = (1/4)\pi D_{SC}^2$. Values $D_{MC} = 0.5$ km and $D_{SC} = 0.05$ km have been used as suggested in [207] for a urban area for both use cases 3 and 7 therein.

6.4.3 SPs

We consider three market segments for eMBB services, each served by a unique SP, while a fourth SP provisions mMTC services. eMBB services are characterized only by their DL demand; the UL demand, being generally much lower, is assumed to be equal to zero. Instead, mMTC services are characterized only by their UL demand, as they are mainly UL biased [207], while their DL demand is set equal to zero. Values given to parameters characterizing the service and the users of each SP are reported in Table 6.4.

Concerning the user utility function (see Section 6.3.2), for SPs providing eMBB services, we set $\underline{\mathcal{X}}_v$ equal to the required value for the user experienced data rate[20] in the DL for use case 7 in [207] (same for all market segments), whereas $\overline{\mathcal{X}}_v$ varies across the eMBB market segments as reported in Table 6.4 assuming users of different market segments have different target rates. For the fourth SP (which provides mMTC services), we set $\underline{\mathcal{X}}_4$ and $\overline{\mathcal{X}}_4$ equal to the minimum and maximum required value for the user experienced data rate in the UL for use case 3 in [207], respectively. Further, the elasticity parameter ξ_v was set to 2 for all eMBB SPs (minimum value considered in [208]) and to 20 for the mMTC SP (maximum value considered in [208]) to account for the fact that the eMBB traffic is more elastic than the mMTC one.

As for the acceptance probability function (see Section 6.3.2), we set the user sensitivity to changes in utility equal to the value considered in [208] for all SPs, i.e., $\mu_v = 2$, but we vary the user sensitivity to changes in the offered fee (ε_v) across SPs as reported in Table 6.4. We assume that mMTC users have a high sensitivity to changes in the offered fee ($\varepsilon_4 = 4$, which is the value considered in [208]), while the eMBB market segments served by SPs 1, 2 and 3 are assumed to have low, medium and high sensitivity to changes in the offered fee, respectively, represented by values $\varepsilon_1 = 2$, $\varepsilon_2 = 3$ and $\varepsilon_3 = 4$. Given that the considered utility function u_v is such that $0 \leq u_v(x_v) \leq 1$ for any x_v (see Equation (6.1)), then, by definition, the maximum utility level is equal to 1 for all SPs, i.e., $\overline{u}_v = 1$. It is reasonable to

[20] In [207], the user experienced data rate is defined as the 5 percentile user rate hence we use its required value as a minimum for the average user rate.

assume that each SP v will tailor its reference offered fee $\overline{p}_v$ to the service requirements of its own users (represented by the utility function here), hence we set $\overline{p}_v = 0.4\mathcal{X}_v\,(1 + 1/\xi_v)$. Recall that $\mathcal{X}_v$ is the rate value that provides a user of SP v with a utility value equal to 0.5 and that $\mathcal{X}_v = \underline{\mathcal{X}_v} + \left(\overline{\mathcal{X}}_v - \underline{\mathcal{X}}_v\right)\left((1-U)/U\right)^{1/\xi_v}$, where $U = 0.999$ has been considered (see Section 6.3.2). We set the values of $\overline{q}_v$ as reported in Table 6.4. We make the following assumptions on the behavior of the rejection probability $\overline{q}_v$ as a function of μ_v, ε_v, $\overline{u}_v$ and $\overline{p}_v$:

(i) for any two SPs $v, w \in \mathcal{V}$, such that $\mu_v = \mu_w$, $\varepsilon_v = \varepsilon_w$, $\overline{u}_v = \overline{u}_w$, we assume $\overline{q}_v = \overline{q}_w$ even if $\overline{p}_v \neq \overline{p}_w$, i.e., when users of v and w are equally sensitive to changes in utility and in the offered fee and they perceive a maximum level of utility, we expect them to reject the considered reference offered fee $\overline{p}_v$ and $\overline{p}_w$, respectively, with the same probability, since $\overline{p}_v$ and $\overline{p}_w$ reflect their respective service requirements;

(ii) for any given μ_v, $\overline{u}_v$ and $\overline{p}_v$, we expect $\overline{q}_v$ to be non-decreasing in ε_v, $\lim_{\varepsilon_v \to 0} \overline{q}_v(\mu_v, \varepsilon_v, \overline{u}_v, \overline{p}_v) = 0$ and $\lim_{\varepsilon_v \to \infty} \overline{q}_v(\mu_v, \varepsilon_v, \overline{u}_v, \overline{p}_v) = 1$ for each $v \in \mathcal{V}$.

In [228–232], the normalizing constant A (see Equation (6.2) and Section 6.3.2) is set equal to 0.1 for $\varepsilon = 4$, $\mu = 2$, $\overline{p} = 1$ and $\overline{u} = 1$, therefore the corresponding reference rejection probability $\overline{q} = e^{-A} \approx 0.9$. We then set $\overline{q}_v = 0.9$ for any SP v with $\varepsilon_v = 4$, $\mu_v = 2$, $\overline{u}_v = 1$ and the considered $\overline{p}_v$, in line with assumption (i), whereas for SPs with ε_v equal to 3 and 2 we set $\overline{q}_v$ equal to 0.6 and 0.3, respectively, as per assumption (ii).

Concerning the number of users or, alternatively, devices[21] subscribing to each SP, first let σ_v denote the market share of SP v for the service offered by v. We assume that the eMBB market segment served by SP 1 makes up 20% of the eMBB market (i.e., $\sigma_1 = 0.2$), whereas the eMBB market segments served by SPs 2 and 3 make up 30% and 50%, respectively (i.e., $\sigma_2 = 0.3$ and $\sigma_3 = 0.5$). SP 4 is assumed to serve the entire mMTC market (i.e., $\sigma_4 = 1$). Then, the number of devices in the area of a SC that have subscribed to each SP are: $N_v = \sigma_v d_{eMBB} A_{SC}$, for any $v \in \{1, 2, 3\}$ (eMBB SPs), and $N_4 = \sigma_4 d_{mMTC} A_{SC}$ (mMTC SP), where d_{eMBB} and d_{mMTC} denote the eMBB and mMTC device density, respectively, whereas A_{SC} the area of a SC (see Section 6.4.2). Further, for eMBB SPs we consider a device activity factor equal to 0.1, i.e., $\eta_v = 0.1$ for any $v \in \{1, 2, 3\}$ as in [203, 207], whereas for the mMTC SP we assume $\eta_4 = 0.01$ (as sensors tend to become active less often).

6.4.4 Instances

In our numerical tests, for InPs of type $\mathcal{N}$, i.e., for InPs which deploy a 5G network, we have considered two particular cases, labeled as $\mathcal{N}^{(1)}$ and $\mathcal{N}^{(2)}$. Specifically, $\mathcal{N}^{(1)}$ refers to an InP

[21] We use the terms device and user interchangeably.

Table 6.4 Parameters characterizing the service and the users of each SP.

v	service type	utility function			acceptance probability			# users & activity factor		
1	eMBB	$\underline{\mathcal{X}}_1$=50 Mbps	$\overline{\mathcal{X}}_1$=5000 Mbps	ξ_1=2	μ_1=2	ε_1=2	$\overline{q}_1$=0.3	σ_1=0.2	N_1=9.82	η_1=0.1
2	eMBB	$\underline{\mathcal{X}}_2$=50 Mbps	$\overline{\mathcal{X}}_2$=2500 Mbps	ξ_2=2	μ_2=2	ε_2=3	$\overline{q}_2$=0.6	σ_2=0.3	N_2=14.73	η_2=0.1
3	eMBB	$\underline{\mathcal{X}}_3$=50 Mbps	$\overline{\mathcal{X}}_3$=500 Mbps	ξ_3=2	μ_3=2	ε_3=4	$\overline{q}_3$=0.9	σ_3=0.5	N_3=24.54	η_3=0.1
4	mMTC	$\underline{\mathcal{X}}_4$=0.00016 Mbps	$\overline{\mathcal{X}}_4$=1 Mpbs	ξ_4=20	μ_4=2	ε_4=4	$\overline{q}_4$=0.9	σ_4=1	N_4=1178.10	η_4=0.01

k for which:

(1) $B_k \geq \widehat{B}_k = 20$ MHz, i.e., k has amortized the spectrum license cost of 20 MHz of bandwidth from its total available (B_k) and it may have acquired a new spectrum license (if $B_k - \widehat{B}_k > 0$);

(2) $\pi_{MC,k} = 0.3$ and $\pi_{SC,k} = 0.5$, i.e., k has not deployed a legacy MC BS in a MC candidate site with probability equal to 0.3 and analogously for SCs for which such probability is assumed equal to 0.5.

In turn, $\mathcal{N}^{(2)}$ refers to an InP k for which:

(1) $B_k > \widehat{B}_k = 0$, i.e., k does not own any spectrum license whose cost has been amortized but has acquired a new spectrum license of B_k units of bandwidth;

(2) $\pi_{MC,k} = \pi_{SC,k} = 1$, i.e., no legacy MC/SC BSs of k are present in any of the MC/SC candidate sites or, in other words, k has not previously deployed a legacy network.

Instead, as mentioned in Section 6.4.1, for an InP k of type $\mathcal{L}$ which does not upgrade to 5G we assume:

(1) $B_k = \widehat{B}_k > 0$, i.e., k has amortized the spectrum license cost of all its available bandwidth, meaning that k does not acquire any new spectrum licenses;

(2) $\pi_{MC,k} = \pi_{SC,k} = 0$, i.e., k has deployed legacy MC/SC BSs in all available MC/SC candidate sites hence it does not deploy additional MCs and SCs during the studied period.

We then set up several instances with two InPs ($|\mathcal{K}| = 2$) and four SPs ($|\mathcal{V}| = 4$). Across these instances, we vary the type and total available bandwidth of the two InPs, but consider the same set of four SPs (as described in Section 6.4.3). The instances are described and labeled in Table 6.5 where, e.g., for the instance labeled as A10, the first InP is of type $\mathcal{N}^{(1)}$ and its total available bandwidth B_1 is equal to 100 MHz, whereas the second InP is of type $\mathcal{L}$ and $B_2 = 100$ MHz.

Table 6.5 Instances and respective labels

	(B_1, B_2)										
	(20,20)	(20,60)	(60,20)	(60,60)	(80,80)	(20,100)	(40,100)	(60,100)	(80,100)	(100,100)	(120,100)
$(\mathcal{N}^{(1)}, \mathcal{L})$	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11
$(\mathcal{N}^{(2)}, \mathcal{N}^{(1)})$	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11

6.4.5 Computational tests

The proposed framework was implemented in Matlab, whose solvers have been used in the implementation to calculate $\overline{X}_{vk}(P_k)$ and $\underline{X}_{vk}(P_k)$ according to Equations (6.8) and (6.9), respectively, and to determine an optimal solution of the capacity assignment problem formulated as a two-step optimization problem (see Section 6.3.3).

The value of $\overline{P}$ (i.e., the minimum monthly price per unit of average SC capacity which is unprofitable for all SPs) has been determined as follows: for each SP v, let $\overline{P}_v$ denote the minimum value of P for which $\overline{X}_v(P) = 0$ (see Equation (6.8)) and let $\overline{P}_v^{\circ}$ denote an upper bound for $\overline{P}_v$ (which we calculate through a heuristic that provides $\overline{P}_v^{\circ} \leq \overline{P}_v + 0.001$); we then set $\overline{P} = \max_{v \in \mathcal{V}} \overline{P}_v^{\circ}$.

To solve the MLFG numerically, we have discretized the continuous InP price strategy sets $\mathcal{P}_k = [\underline{P}_k, \overline{P}]$, (see Section 6.3.4), i.e., hereon, $\mathcal{P}_k = \{\underline{P}_k, \ldots, \overline{P}\}$, for any $k \in \mathcal{K}$. Consequently, the resulting set of InP price profiles $\mathcal{P} = \prod_{k \in \mathcal{K}} \mathcal{P}_k$ is also discrete and finite. We determine the Subgame Perfect Equilibrium(a) (SPE) [220] of the two-stage MLFG as follows:

(1) for each InP price profile $\boldsymbol{P} \in \mathcal{P}$, we look for the NE in pure strategies of the corresponding SPs' game, i.e., for $\breve{\boldsymbol{y}}(\boldsymbol{P})$ of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$, (see Section 6.3.4) from which we can calculate the payoff $G_k(\boldsymbol{P})$ of each InP for the price profile $\boldsymbol{P}$ according to Equation (6.22) – if there are multiple NE in pure strategies for $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$, then $G_k(\boldsymbol{P})$ is set equal to the minimum payoff attained by k among all these NE;

(2) we look for the NE in pure strategies[22] of the InPs' game, i.e., for $\breve{\boldsymbol{P}}$ of $\mathcal{G}^{\mathcal{K}}$ (see Section 6.3.4).

The NE of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ and of $\mathcal{G}^{\mathcal{K}}$ were determined through exhaustive search. In the definition of the NE in pure strategies for $\mathcal{G}^{\mathcal{K}}$ and for $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ we have introduced an absolute margin $\Delta = 10^{-6}$ EUR (recall that the payoffs G_k and g_v are all given in EUR). For instance, the

[22] As mentioned, the price strategy set $\mathcal{P}_k$ of each InP $k \in \mathcal{K}$ is discrete which means that the InPs' game $\mathcal{G}^{\mathcal{K}}$ is formally a non-cooperative game in strategic form, hence we look for its NE in pure strategies.

InP price profile $\breve{\boldsymbol{P}}$ is an NE of the $\mathcal{G}^{\mathcal{K}}$ iff

$$G_k([\breve{P}_k, \breve{\boldsymbol{P}}_{-k}]) \geq G_k([P_k, \breve{\boldsymbol{P}}_{-k})] - \Delta, \qquad \forall\ P_k \in \mathcal{P}_k,\ \forall\ k \in \mathcal{K}, \tag{6.31}$$

where $\breve{\boldsymbol{P}}_{-k}$ denotes the prices of all other InPs but k. Δ was introduced to account for the inaccuracy caused by inherent tolerances of the Matlab solvers.

Concerning the discretization of the originally continuous InP unit price strategy sets $\mathcal{P}_k = [\underline{P}_k, \overline{P}]$, we initially created a unit price strategy set consisting of 30 logarithmically-spaced values in the range $[\underline{P}_k, \overline{P}]$. The MLFG resulting from these discrete InP unit price strategy sets has at least one SPE for all instances but B4 and B5. Instead, for both B4 and B5, although there is at least one NE in pure strategies for each SPs' game, there is no NE in pure strategies for the InPs' game and thus no SPE for the MLFG. Then, for both B4 and B5, for each InP k, we created an alternative unit price strategy set consisting of 60 values in the range $[\underline{P}_k, \overline{P}]$ with the majority of these values in price ranges where we expected the NE of $\mathcal{G}^{\mathcal{K}}$ to be based on the best response mappings of $\mathcal{G}^{\mathcal{K}}$ resulting from the initial discrete InP unit price strategy sets (see Section 6.5.2). As there was no NE for $\mathcal{G}^{\mathcal{K}}$ neither for B4 nor for B5 even for the MLFG resulting from the alternative discrete InP unit price strategy sets, we settled on suggesting as a solution for $\mathcal{G}^{\mathcal{K}}$ an InP unit price profile $\boldsymbol{P}^{\diamond}$ with a small (0.53% for B4 and 3.89% for B5) maximum relative payoff difference from the InPs' best response (see Equation (E.2) and Section 6.5.2 for details).

6.5 Numerical results analysis

In this section, we report and analyze numerical results concerning the equilibrium(a) of the considered problem for the instances defined in Section 6.4.4. To start with, in Section 6.5.1 we explain the notation used in reporting these results. Then, in Section 6.5.2 we discuss the existence and multiplicity of equilibria across these instances. Instead, in Section 6.5.3 we analyze the impact of the InPs' network technology and available spectrum bandwidth on the equilibrium strategies of the players, i.e., on the capacity unit price offered by the each InP and the InP choice of each SP.

6.5.1 Notation summary

For the sake of brevity, hereon we will simplify the terminology as follows:

- the term equilibrium will refer to equilibrium of the overall game, i.e., to the *sub-game* *perfect equilibrium* of the MLFG which consists of the Nash Equilibrium InP capacity unit price profile of the InPs' game, i.e., $\breve{\boldsymbol{P}}$ of $\mathcal{G}^{\mathcal{K}}$ and of the Nash Equilibrium SPs' choice of InP of the SPs' game resulting from $\breve{\boldsymbol{P}}$, i.e., $\breve{\boldsymbol{y}}(\breve{\boldsymbol{P}})$ of $\mathcal{G}^{\mathcal{V}}(\breve{\boldsymbol{P}})$, where both are Nash Equilibria in pure strategies;

- the term capacity will refer to the average SC capacity of an InP;
- the term spectral efficiency will refer to the average SC spectral efficiency of an InP;
- the term unit cost will refer to the total monthly cost per unit of average SC capacity of an InP;
- the term unit price will refer to the monthly price per unit of average SC capacity offered by an InP at the equilibrium.

For all considered instances A1–A11 and B1–B11, the values of the main parameters characterizing the InPs and the SPs and the equilibrium outcomes are reported in Tables 6.7–6.14, where Tables 6.7, 6.9, 6.11 and 6.13 concern the InPs, whereas Tables 6.8, 6.10, 6.12 and 6.14, the SPs. The definitions and unit of measurements of the notation used across Tables 6.7–6.14 are provided in Table 6.6. When reporting numerical values in the text, the respective units of measurement have been omitted. Notice also that:

- in Tables 6.8, 6.10, 6.12 and 6.14, for each SP v (column two), column three reports the InP selected by v at the equilibrium, i.e., k for which $\breve{y}_{vk}(\breve{\boldsymbol{P}}) = 1$; in particular, the symbol – has been reported in all the columns starting from the third one for each SP for which it is not profitable to purchase capacity from any of the InPs at their equilibrium capacity unit prices and hence it cannot provide services to its users;
- in Tables 6.7–6.14 values for $\underline{P}_k$, $\breve{P}_k$, $\breve{G}_k$, $a_v^* p_v^*(\breve{u}_v)$, $\breve{g}_v$ and $\breve{r}_v^*/\breve{x}_{vk}$ are reported rounded to two decimals, whereas values for C_k, $\breve{C}'_k$, $\underline{X}_v(\breve{P}_k)$, $\breve{x}_{vk}$, $\overline{X}_v(\breve{P}_k)$ and $\breve{u}_v$ are reported rounded to three decimals to highlight the differences;
- in Tables 6.8, 6.10, 6.12 and 6.14, when the reported values for $\breve{x}_{4k}$ across different instances are distinct but the respective reported values for $\breve{u}_4$ are equal among them and/or the respective reported values for $a_4^* p_4^*(\breve{u}_4)$) are equal among them, this is due to the aforementioned rounding. Consider e.g., instances A7 and A8 in Table 6.10: for A7, $\breve{x}_{4k} = 10.343$ Mbps, whereas for A8, $\breve{x}_{4k} = 10.355$ Mbps, while for both of them $\breve{u}_4 = 0.987$ and $a_4^* p_4^*(\breve{u}_4) = 0.12$ EUR/month. In fact, distinct values of $\breve{x}_{4k}$ for the two instances imply distinct values of the respective $\breve{u}_4$, but the latter differ from one

another not before the fourth decimal; similarly, the respective values of $a_4^* p_4^*(\breve{u}_4)$ differ not before the third decimal.

Table 6.6 Summary of notation used in Tables 6.7–6.14

Notation	Definition	Unit
$\underline{P}_k$	unit cost of InP k (see Equation (6.23))	EUR/Mbps/month
$\breve{P}_k \geq \underline{P}_k$	unit price offered by InP k (at the equilibrium)	EUR/Mbps/month
$\overline{P}$	minimum unit price unprofitable for all SPs (see Section 6.4.5)	EUR/Mbps/month
C_k	available capacity of InP k (see Section 6.4.1)	Mbps
$\breve{C}'_k \leq C_k$	capacity sold by InP k at the equilibrium	Mbps
$\breve{G}_k$	payoff of InP k at the equilibrium (see Equation (6.22))	EUR/month
$\mathcal{W}_k$	subset of SPs that select InP k at the equilibrium and are assigned non-zero capacity	—
$\underline{X}_v(\breve{P}_k)$	minimum capacity requested by SP v from the selected InP k at $\breve{P}_k$ (see Equation (6.9))	Mbps
$\overline{X}_v(\breve{P}_k)$	maximum capacity requested by SP v from the selected InP k at $\breve{P}_k$ (see Equation (6.8))	Mbps
$\breve{x}_{vk}$	capacity assigned to SP v by the selected InP k at the equilibrium (see Section 6.3.4)	Mbps
$\breve{u}_v$	utility obtained by a single user of SP of v at the equilibrium (see Equation (6.1))	—
$a_v^* p_v^*(\breve{u}_v)$	monthly fee accepted by a user of SP v for $\breve{u}_v > 0$ (see Section 6.3.2)	EUR/month
$\breve{g}_v$	payoff of SP v at the equilibrium (see Equation (6.21))	EUR/month
$\breve{r}_v^*$	total revenue of SP v at the equilibrium (see Equation (6.7))	EUR/month
$\breve{r}_{vk}^* / \breve{x}_{vk}$	revenue per unit of purchased capacity for SP v at the equilibrium	EUR/Mbps/month

6.5.2 Existence and multiplicity of equilibria

For the considered instances, it is always possible to find an equilibrium when the InPs are different, either for the technology or for the available spectrum bandwidth. Instead, if the InPs are very similar, it might be difficult to find an equilibrium, unless the spectrum bandwidth is very low or very high. Concerning the equilibria multiplicity, which results[23] from the equilibria multiplicity of the InPs' game at stage 1 and/or of the SPs' game stage 2, the multiple equilibria are always equivalent for all players (i.e., for all InPs and all SPs) since each player obtains the same payoff in all of them, hence they represent the same system behavior; at stage 1, the equilibria multiplicity occurs because there is an InP which is not selected by any SP for any offered unit price, whereas at stage 2 it occurs because some SPs are not provided with capacity in any of the equilibria, therefore it is not relevant which InP they select.

Specifically, no equilibrium was found for instances B4 and B5 (see Section 6.4.5); however, for both of them, it is possible to determine an approximate equilibrium as explained in Appendix E. In turn, a single equilibrium was found for instances A8–A11 and B7–B11 and multiple equivalent ones for the rest of the instances. As for the equilibria equivalence for instances with multiple equilibria, some illustrative examples follow.

[23]Let $n^{\mathcal{K}}$ denote the number of NE in pure strategies of $\mathcal{G}^{\mathcal{K}}$ where $n^{\mathcal{K}} \geq 1$, and let $\breve{\boldsymbol{P}}_i$ denote the unit price profile of the i-th NE of $\mathcal{G}^{\mathcal{K}}$ where $1 \leq i \leq n^{\mathcal{K}}$. Then let $n_i^{\mathcal{V}}$ denote the number of NE in pure strategies of $\mathcal{G}^{\mathcal{V}}(\breve{\boldsymbol{P}}_i)$ where $n_i^{\mathcal{V}} \geq 1$. The number of *sub-game perfect equilibria* of the MLFG is then equal to $\sum_{i=1}^{n^{\mathcal{K}}} n_i^{\mathcal{V}}$.

Consider instance A7 (see Tables 6.9 and 6.10) for which the equilibria multiplicity derives from stage 2. In details, for A7, the InPs' game at stage 1 has a unique NE $\breve{\boldsymbol{P}} = (\breve{P}_1 = 1.87, \breve{P}_2 = 1.80)$, whereas the SPs' game at stage 2 for $\breve{\boldsymbol{P}}$ has two NE denoted by (i) and (ii) in Table 6.10: in (i) SP 3 selects InP 1, whereas in (ii) it selects InP 2, while in both (i) and (ii) SP 1 selects InP 2 whereas SPs 2 and 4 select InP 1. In (i), SP 3 requests a minimum amount of capacity equal to 173.051 Mbps and a maximum of 175.857 Mbps from InP 1 given $\breve{P}_1 = 1.87$ whereas in (ii) SP 3 requests a minimum of 160.391 Mbps and a maximum of 176.817 Mbps from InP 2 given $\breve{P}_2 = 1.80$. However, SP 3 is allocated a null capacity in both (i) and (ii); in fact, in (i), InP 1 (which serves SPs 2 and 4) does not have enough spare capacity to serve SP 3 ($C_1 - \breve{C}_1' = 97.257 < \underline{X}_3(\breve{P}_1) = 173.051$ Mbps), whereas in (ii) InP 2 has allocated all its available capacity to SP 1 ($\breve{x}_{12} = \breve{C}_2' = C_2 = 260$ Mbps). Formally, the unique NE of of the game at stage 1 and the NE (i) and (ii) of the game at stage 2 imply two equilibria for instance A7. However, it can easily be seen that these two equilibria are equivalent for all SPs: each of the SPs 1, 2 and 4 is served by the same InP, at the same unit price and with the same amount of capacity in both equilibria hence each of them obtains the same payoff in both, while SP 3 is not served in neither equilibria resulting in a null payoff in both. The two equilibria are equivalent also from the InPs' perspective: each InP sells the same amount of capacity at the same unit price in both equilibria thus obtaining the same payoff in both.

For instances A1, A2 and B1 as well, the equilibria multiplicity derives from stage 2. However, for these instances, unlike for A7, the multiplicity of NE for the stage 2 game is due to there being at least one SP for which it is not profitable to buy capacity from any InP , hence each such SP is indifferent to the InP choice. Recall that for such SPs, in Tables 6.8, 6.10 and 6.12 we report the symbol – in all columns starting from the third one. Consider, for instance, instance B1: it is SPs 1, 2 and 3 for which it is not profitable to purchase capacity from any of the InPs, while SP 4 selects and is fully served by InP 2 (see Table 6.12). Formally, there are 8 equilibria for B1 since the stage 1 game has a unique NE (see Table 6.11), whereas the stage 2 game has 8 NE resulting from SPs 1, 2 and 3 selecting either InP 1 or InP 2 but acquiring a null capacity from either, while in all these NE SP 4 is served with the same amount of capacity and at the same unit price by InP 2. Clearly, payoff-wise, these equilibria are equivalent for all SPs and all InPs.

For instances A5 and B6 the equilibria multiplicity derives instead from stage 1. Let us consider instance A5 (similarly then for B6). For A5, $\mathcal{G}^{\mathcal{K}}$ has multiple NE which are all unit price profiles $\breve{\boldsymbol{P}} = (\breve{P}_1, \breve{P}_2)$ such that $\breve{P}_1 = 1.77$, whereas $\breve{P}_2$ can take any value in the considered discrete unit price strategy set of InP 2, i.e., $\mathcal{P}_2 = \{\underline{P}_2 = 2.24, \ldots, \overline{P} = 14.86\}$ (see Table 6.7). Although each such $\breve{\boldsymbol{P}}$ induces a distinct stage 2 game $\mathcal{G}^{\mathcal{V}}(\breve{\boldsymbol{P}})$, all these stage

2 games have the same unique NE reported in Table 6.12 in which all SPs select and are served by InP 1, hence InP 2 sells a null capacity and obtains a null payoff. Thus, formally, there are $|\mathcal{P}_2|$ equilibria for instance A5, but each player obtains the same payoff in all of them.

In turn, instances A3, A4, A6, B2 and B3 have multiple NE at both stages. For the NE multiplicity at stage 1 (stage 2), similar observations to those made for instances A5 and A6 (A1, A2 and B1) apply from which one can easily see the equivalence among the resulting equilibria. Nevertheless, it is worth clarifying that for A3, A4, A6, B2 and B3, each distinct NE unit price profile at stage 1 results in the same set of NE at stage 2, which are *per se* equivalent among them. For example, instance A3 has $2|\mathcal{P}_2|$ equilibria since each NE unit price profile $\breve{\boldsymbol{P}} = (\breve{P}_1, \breve{P}_2)$ with $\breve{P}_1 = 1.99$ and $\breve{P}_2 \in \mathcal{P}_2 = \{\underline{P}_2 = 8.90, \ldots, \overline{P} = 14.86\}$ at stage one (see Table 6.7) results in two NE at stage 2, due to SP 3 not finding it profitable to purchase capacity from any of InPs hence being indifferent to the InP choice (see Table 6.8).

6.5.3 Technology and spectrum availability impact on competition among InPs

Recall that for each InP k, 1) its network technology type and 2) its available spectrum bandwidth (B_k) affect its average SC capacity (C_k) and its total cost per unit of average SC capacity ($\underline{P}_k$) as explained in Section 6.4.1. In the following paragraphs we will then analyze the impact of 1) and 2) on the competition among InPs to be selected by SPs.

Let us first consider instances A1–A11 for which InP 1 has a new (5G) network (type $\mathcal{N}^{(1)}$) whereas InP 2 has a legacy (4G) network (type $\mathcal{L}$), while their available spectrum bandwidths vary across the instances. As for instances A1–A5 (see Tables 6.7 and 6.8), InP 2 does not sell capacity to any SP, thus obtaining a null payoff, in all the instances but A2, even when SPs are not fully satisfied from InP 1 (e.g., in case of instance A5). InP 1, instead, always serves at least one SP (but in instance A2). Indeed, for A1, A3, A4 and A5, InP 1 is preferred to InP 2 by at least one SP because InP 1 can offer a lower unit price since it is more cost-efficient, i.e., it has a lower unit cost ($\underline{P}_1 < \underline{P}_2$) and because it has sufficient available capacity. InP 1 has a lower unit cost and a higher capacity than InP 2 due to InP 1 having a higher spectral efficiency (resulting in a higher cell capacity for equal amounts of spectrum bandwidth) and due to $B_1 \geq B_2$. However, notice that for equal amounts of bandwidth, InP 1 incurs a higher total cost per cell than InP 2 to attain a higher spectral efficiency and the total cell cost increases with the spectrum bandwidth. Instead for instance A2, $B_2 = 3B_1$ hence $C_1 = C_2$ while $\underline{P}_1 > \underline{P}_2$ (i.e., $\underline{P}_1 C_1 > \underline{P}_2 C_2$), meaning that the legacy (4G) InP 2, which owns the triple of spectrum holdings of the new (5G) InP 1, provides the same amount of capacity as the latter but more cost-efficiently.

As for the SPs, when the spectrum bandwidth is low, and the unit costs and hence the (equilibrium) unit prices are high, only SP 4 is served and provided with the maximum amount of requested capacity from the selected InP (instances A1 and A2). With the increasing spectrum bandwidth and decreasing unit costs and hence unit prices (instances A3 and A4), other SPs are served and provided with the maximum requested capacity. Finally, in instance A5 all the SPs are served. Although the maximum requested capacity is not provided to any of them by InP 1, they all obtain a higher payoff from selecting InP 1 due to its lower unit price.

Instances A6–A11 are such that for all of them $B_2 = 100$ MHz (see Table 6.5) and hence $C_2 = 260$ Mbps (see Table 6.9) given that InP 2 is of type $\mathcal{L}$. Instead, B_1 increases with a step of 20 MHz from A6 to A11 starting from $B_1 = 20$ MHz for A1 (see Table 6.5), therefore C_1 increases accordingly from 156 Mbps for A6 to 936 Mbps for A11 (see Table 6.9) given that InP 1 is of type $\mathcal{N}^{(1)}$. Among A6–A11, only for instance A6, the legacy (4G) InP is more cost-efficient than the new (5G) InP (i.e., $\underline{P}_2 < \underline{P}_1$) and has a higher capacity (i.e., $C_2 > C_1$). Indeed, InP 1 always provides capacity to at least one SP, but in instance A6. On the contrary, for A7–A11 one has $\underline{P}_1 < \underline{P}_2$ and $C_1 > C_2$. With the increasing spectrum bandwidth of InP 1 and its decreasing unit price, InP 1 serves an increasing number of SPs. When the spectrum bandwidths are comparable or InP 1 has a greater amount (A10 and A11), InP 1 serves all the SPs. In general, InP 1 is able to offer a unit price higher than its unit cost, while InP 2 is always selling at a unit price equal to its unit cost but for instance A6. InP 1 does not sell all its available capacity but in instance A9, when it first serves SP 1, which is served by InP 2 as long as the spectrum bandwidth of InP 1 is below 80 Mhz.

As for the SPs, SP 1 and SP 4 are always served. SP 2 and SP 3 are not served in instance A6 as they cannot afford the offered unit prices. Instead, in instance A7, SP 3 can actually afford the unit prices of both InPs but it is not served as neither InP has sufficient available capacity to satisfy its minimum requested capacity. When an SP is served, it is usually provided with the maximum requested capacity. Exceptions are SP 1 in instance A7 and A8, where SP 1 cannot be provided with the maximum requested capacity due to the limited capacity of InP 2, and instance A9, where the available capacity of InP 1 makes it impossible for it to serve completely the three SPs that select it.

Instances B1–B11 (see Tables 6.11 and 6.14) are analogous to the respective A1–A11 in terms of spectrum bandwidth availabilities of the two InPs, but for B1–B11 both InPs have deployed a new (5G) network and InP 1 is of type $\mathcal{N}^{(2)}$, i.e., a sheer new entrant, whereas InP 2 is of type $\mathcal{N}^{(1)}$, i.e., InP 2 reuses available sites and spectrum licenses from its legacy (4G) network when it upgrades to the new (5G) network. In particular, for B1–B11, when the spectrum bandwidths of the two InPs are equal (i.e., $B_1 = B_2$) then also their capacities are equal (i.e., $C_1 = C_2$), which is the case for instances B1, B4, B5 and B10. However, for these latter instances, the unit cost of InP 1 is slightly higher than the one of InP 2 (i.e., $\underline{P}_1 > \underline{P}_2$), reflecting the disadvantage of InP 1 for being a new entrant.

For instances B1–B5, when the capacity is low and the unit cost high, the least cost-efficient InP sells no capacity and hence obtains a null payoff: this is the case of InP 1 in B1 and B2, and of InP 2 in instance B3. Moreover, for B2 and B3, the least cost-efficient InP induces no competition (similarly to A3, A4 and A5), therefore the unit price of the other (most cost-efficient) InP is determined solely by the SPs' demand and its own available capacity. Concerning the SPs, some of them are not served across B1–B3 because neither $\breve{P}_1$ nor $\breve{P}_2$ are profitable for them.

As for instances B4 and B5, we recall that we did not find a NE for the InPs' game, hence we suggested as a solution the unit price profile $\boldsymbol{P}^\diamond = (P_1^\diamond, P_2^\diamond)$ which is calculated according to Equation (E.2) and it can be considered an approximate NE (see Appendix E). In these lines, for both instances, values reported under $\breve{P}_1$ and $\breve{P}_2$ in Table 6.11, which are marked by the symbol $\diamond$, are in fact the values of $P_1^\diamond$ and $P_2^\diamond$, respectively. For B4, the SPs' game for $\boldsymbol{P}^\diamond$ has two distinct NE in pure strategies denoted as (**i**) and (**ii**) when reported in Tables 6.11 and 6.12. This NE multiplicity is due to the fact that the two InPs are very similar ($P_1^\diamond = 1.23$, $P_2^\diamond = 1.22$ and $C_1 = C_2 = 468$). However, neither NE is preferred by all InPs or all SPs. In fact, in both (**i**) and (**ii**) SP 4 is served by InP 2 at the same unit price ($P_2^\diamond = 1.22$) and with the same amount of capacity ($\breve{x}_{42} = 10.558$) hence SP 4 is indifferent between the two NE. Instead, SPs 2 and 3 prefer (**ii**), in the sense that they attain a higher payoff from (**ii**), whereas SP 1 prefers (**i**) which means that SPs 1, 2 and 3 are all better off in the NE in which they are served by the cheapest InP[24]. In turn, InP 1 prefers (**i**), whereas InP 2 prefers (**ii**) since each InP is able to sell more capacity and hence attain a higher payoff when serving both SPs 2 and 3 instead of SP 1. For instance B5 instead, the SPs' game for $\boldsymbol{P}^\diamond$ has a unique NE. In particular, this $\boldsymbol{P}^\diamond$ is such that $P_1^\diamond = 1.09 > P_2^\diamond = 0.94$ despite $\underline{P}_1 = 0.94 > \underline{P}_2 = 0.90$ which shows that InP 1 leverages the fact that C_2 is not sufficiently large for all SPs to be served by InP 2. In fact, even though $P_1^\diamond > P_2^\diamond$, at the unique NE of SPs' game for $\boldsymbol{P}^\diamond$, SP 3 selects and is served by InP 1 from which it obtains $\breve{x}_{31} = 190.370$ at $P_1^\diamond = 1.09$. If SP 3 were to select InP 2 while SPs 1, 2 and 4 still selected and were served by InP 2, then InP 2

[24] In fact, as reported in Table 6.12, SPs 1, 2 and 3 attain a higher payoff when served by InP 2 since in addition to InP 2 offering a lower unit price than InP 1 (i.e., $P_2^\diamond < P_1^\diamond$), the maximum amount of capacity requested by SPs 1, 2 and 3 at $P_2^\diamond$ is higher than the respective one at $P_1^\diamond$ (i.e., $\overline{X}_v(P_2^\diamond) > \overline{X}_v(P_1^\diamond)$, $\forall v \in \{1,2,3\}$) and C_2 is sufficiently large for InP 2 to provide each SP that selects it in each of these NE with its maximum requested capacity (i.e., $\breve{x}_{12} = \overline{X}_1(P_2^\diamond)$ in NE (**i**) and $\breve{x}_{22} = \overline{X}_2(P_2^\diamond)$ and $\breve{x}_{32} = \overline{X}_3(P_2^\diamond)$ in NE (**ii**)).

would split $C_2 = 624$ among all SPs and SP 3 would obtain an amount of capacity equal to 147.825 at $P_2^{\diamond} = 0.94$ which would lower its payoff value by 35.86% w.r.t. the value attained in the NE.

Concerning instances B6–B11, it results that InP 1 becomes more cost-efficient than InP 2 only for instance B11 for which $B_2 > B_1$. Nevertheless, InP 2 is unaffected by the presence of InP 1 only for instance B6 for which InP 1 has only 20 MHz of spectrum bandwidth resulting in a high unit cost ($\underline{P}_1 = 3.55$ as opposed to $\underline{P}_2 = 0.73$). Specifically, for B6, all SPs select and are served by InP 2 and $\breve{P}_2$ is determined solely by the SPs' demand and the available capacity of InP 2. Instead, in instances B7–B8, although all SPs still select and are served by InP 2, the unit price offered by InP 2 at the equilibrium is dictated by the unit cost of InP 1 ($\breve{P}_2$ is the highest discrete unit price value lower than $\underline{P}_1$). Indeed, as the spectrum bandwidth of InP 1 increases, its capacity increases whereas its unit cost decreases making InP 1 more competitive hence forcing InP 2 to lower its offered unit price which in turn increases the amount of capacity requested by the SPs. When the spectrum bandwidths are comparable or InP 1 has a greater amount (B10–B11), the SPs move from InP 2 to InP 1 and InP 2 is forced to sell at its unit cost. As for the SPs, they are always fully served, but in instance B9, where SPs 1, 2 and 3 select InP 2 which is not able to fully serve them whereas SP 4 opts for InP 1, despite its higher unit price, so as to obtain all the requested capacity.

On the overall, we notice that there is more head-to-head competition when InPs are of the same type. Indeed, more recent 5G InPs are preferred w.r.t. older ones (4G ones), but if the latter provide much more spectrum bandwidth, thus resulting more cost-efficient. In this case 5G InP is either less cost-efficient or does not have sufficient capacity for all SPs. Further, there should be sufficient bandwidth even for a 5G InP to be affordable for all 5G services given realistic user fees.

Table 6.7 Key equilibrium outcomes related to the InPs — instances A1–A5

Instance	$\underline{P}_1$	$\underline{P}_2$	$\breve{P}_1$	$\breve{P}_2$	C_1	C_2	$\breve{C}'_1$	$\breve{C}'_2$	$\breve{G}_1$	$\breve{G}_2$	$\mathcal{W}_1$	$\mathcal{W}_2$
A1	3.41	8.90	8.50	8.90	156	52	9.592	0	81.55	0	{4}	∅
A2	3.41	2.98	3.41	3.33	156	156	0	10.055	0	33.52	∅	{4}
A3	1.18	8.90	1.99	{8.90,...,$\overline{P}$=14.86}	468	52	460.326	0	917.48	0	{1,2,4}	∅
A4	1.18	2.98	1.99	{2.98,...,$\overline{P}$=14.86}	468	156	460.326	0	917.48	0	{1,2,4}	∅
A5	0.90	2.24	1.77	{2.24,...,$\overline{P}$=14.86}	624	208	624.000	0	1105.80	0	{1,2,3,4}	∅

Table 6.8 Key equilibrium outcomes related to the SPs — instances A1–A5

Instance	v	k	$\underline{X}_v(\breve{P}_k)$	$\breve{x}_{vk}$	$\overline{X}_v(\breve{P}_k)$	$\breve{u}_v$	$a^*_v p^*_v(\breve{u}_v)$	$\breve{g}_v$	$\breve{r}^*_v/\breve{x}_{vk}$
A1	1	–	–	–	–	–	–	–	–
	2	–	–	–	–	–	–	–	–
	3	–	–	–	–	–	–	–	–
	4	1	7.799	9.592	9.592	0.942	0.12	60.03	14.76
A2	1	–	–	–	–	–	–	–	–
	2	–	–	–	–	–	–	–	–
	3	–	–	–	–	–	–	–	–
	4	2	6.949	10.055	10.055	0.977	0.12	110.61	14.33
A3	1	1	190.156	251.008	251.008	0.622	54.02	30.02	2.11
	2	1	163.424	199.007	199.007	0.547	28.08	16.86	2.08
	3	–	–	–	–	–	–	–	–
	4	1	6.556	10.311	10.311	0.986	0.12	124.24	14.04
A4	1	1	190.157	251.008	251.008	0.622	54.02	30.02	2.11
	2	1	163.424	199.007	199.007	0.547	28.08	16.86	2.08
	3	–	–	–	–	–	–	–	–
	4	1	6.556	10.311	10.311	0.986	0.12	124.24	14.04
A5	1	1	161.214	250.977	266.747	0.622	54.01	85.48	2.11
	2	1	140.492	196.525	208.873	0.537	27.74	60.22	2.08
	3	1	158.099	166.741	177.217	0.613	12.57	12.94	1.85
	4	1	6.471	9.757	10.370	0.958	0.12	125.47	14.63

Table 6.9 Key equilibrium outcomes related to the InPs — instances A6–A11

Instance	$\underline{P}_1$	$\underline{P}_2$	$\check{P}_1$	$\check{P}_2$	C_1	C_2	$\check{C}'_1$	$\check{C}'_2$	$\check{G}_1$	$\check{G}_2$	$\mathcal{W}_1$	$\mathcal{W}_2$
A6	3.41	1.80	$\{3.41,...,\overline{P}=14.86\}$	2.08	156	260	0	255.376	0	531.72	$\emptyset$	{1,4}
A7	1.74	1.80	1.87	1.80	312	260	214.743	260.000	401.40	468.00	{2,4}	{1}
A8	1.18	1.80	1.83	1.80	468	260	393.144	260.000	718.04	468.00	{2,3,4}	{1}
A9	0.90	1.80	1.77	1.80	624	260	624.000	10.362	1105.80	18.65	{1,2,3}	{4}
A10	0.73	1.80	1.68	1.80	780	260	675.940	0	1136.71	0	{1,2,3,4}	$\emptyset$
A11	0.62	1.80	1.66	1.80	936	260	678.429	0	1129.28	0	{1,2,3,4}	$\emptyset$

Table 6.10 Key equilibrium outcomes related to the SPs — instances A6–A11

Instance		v	k	$\underline{X}_v(\check{P}_k)$	$\check{x}_{vk}$	$\overline{X}_v(\check{P}_k)$	$\check{u}_v$	$a^*_v p^*_v(\check{u}_v)$	$\check{g}_v$	$\check{r}^*_v/\check{x}_{vk}$
A6		1	2	213.666	245.087	245.087	0.608	52.79	7.95	2.11
		2	–	–	–	–	–	–	–	–
		3	–	–	–	–	–	–	–	–
		4	2	6.589	10.289	10.289	0.985	0.12	123.33	14.07
A7	**(i)**	1	2	164.024	260.000	264.666	0.643	55.78	79.64	2.11
		2	1	148.471	204.400	204.400	0.568	28.79	41.85	2.07
		3	1	173.051	0	175.857	0	–	0	–
		4	1	6.509	10.343	10.343	0.987	0.12	125.52	14.01
	(ii)	1	2	164.024	260.000	264.666	0.643	55.78	79.64	2.11
		2	1	148.471	204.400	204.400	0.568	28.79	41.85	2.07
		3	2	160.391	0	176.817	0	–	0	–
		4	1	6.509	10.343	10.343	0.987	0.12	125.52	14.01
A8		1	2	164.024	260.000	264.666	0.643	55.78	79.64	2.11
		2	1	144.708	206.343	206.343	0.575	29.03	50.64	2.07
		3	1	163.186	176.446	176.446	0.703	13.45	7.84	1.87
		4	1	6.493	10.355	10.355	0.987	0.12	125.97	13.99
A9		1	1	161.214	254.964	266.747	0.631	54.81	86.25	2.11
		2	1	140.492	199.647	208.873	0.549	28.17	60.98	2.08
		3	1	158.099	169.389	177.217	0.641	12.84	15.04	1.86
		4	2	6.482	10.362	10.362	0.987	0.12	126.24	13.98
A10		1	1	152.987	273.711	273.711	0.671	58.26	111.63	2.09
		2	1	134.438	213.265	213.265	0.599	29.85	81.00	2.06
		3	1	152.816	178.567	178.567	0.719	13.60	33.53	1.87
		4	1	6.433	10.396	10.396	0.988	0.12	127.47	13.94
A11		1	1	151.554	275.071	275.071	0.674	58.49	116.33	2.09
		2	1	133.396	214.124	214.124	0.602	29.95	84.66	2.06
		3	1	152.028	178.832	178.832	0.721	13.62	36.59	1.87
		4	1	6.426	10.402	10.402	0.988	0.12	127.65	13.94

Table 6.11 Key equilibrium outcomes related to the InPs — instances B1–B5

Instance	$\underline{P}_1$	$\underline{P}_2$	$\breve{P}_1$	$\breve{P}_2$	C_1	C_2		$\breve{C}'_1$	$\breve{C}'_2$	$\breve{G}_1$	$\breve{G}_2$	$\mathcal{W}_1$	$\mathcal{W}_2$
B1	3.55	3.41	3.55	3.41	156	156		0	10.043	0	34.27	∅	{4}
B2	3.55	1.18	{3.55,...,$\overline{P}$=14.86}	1.99	156	468		0	460.143	0	917.88	∅	{1,2,4}
B3	1.23	3.41	2.06	{3.41,...,$\overline{P}$=14.86}	468	156		453.265	0	932.84	0	{1,2,4}	∅
B4	1.23	1.18	$1.23^\diamond$	$1.22^\diamond$	468	468	(i)	426.404	326.953	525.89	399.29	{2,3}	{1,4}
							(ii)	315.067	438.082	388.58	535.01	{1}	{2,3,4}
B5	0.94	0.90	$1.09^\diamond$	$0.94^\diamond$	624	624		190.370	624.000	207.35	584.04	{3}	{1,2,4}

Table 6.12 Key equilibrium outcomes related to the SPs — instances B1–B5

Instance		v	k	$\underline{X}_v(\breve{P}_k)$	$\breve{x}_{vk}$	$\overline{X}_v(\breve{P}_k)$	$\breve{u}_v$	$a_v^* p_v^*(\breve{u}_v)$	$\breve{g}_v$	$\breve{r}_v^*/\breve{x}_{vk}$
B1		1	–	–	–	–	–	–	–	–
		2	–	–	–	–	–	–	–	–
		3	–	–	–	–	–	–	–	–
		4	2	6.967	10.043	10.043	0.976	0.12	109.82	14.35
B2		1	2	190.471	250.896	250.896	0.622	53.99	29.60	2.11
		2	2	163.699	198.937	198.937	0.546	28.07	16.53	2.08
		3	–	–	–	–	–	–	–	–
		4	2	6.557	10.311	10.311	0.986	0.12	124.23	14.04
B3		1	1	205.287	246.667	246.667	0.612	53.12	13.86	2.11
		2	1	178.664	196.304	196.304	0.536	27.71	4.03	2.08
		3	–	–	–	–	–	–	–	–
		4	1	6.580	10.295	10.295	0.985	0.12	123.57	14.06
B4	**(i)**	1	2	122.136	316.395	316.395	0.743	64.51	246.93	2.00
		2	1	113.199	239.532	239.532	0.679	32.43	182.18	1.99
		3	1	139.726	186.872	186.872	0.771	14.09	115.34	1.85
		4	2	6.207	10.558	10.558	0.991	0.12	132.29	13.75
	(ii)	1	1	122.811	315.067	315.067	0.741	64.34	243.12	2.00
		2	2	112.740	240.378	240.378	0.681	32.50	185.07	1.99
		3	2	139.485	187.145	187.145	0.773	14.10	117.59	1.85
		4	2	6.207	10.558	10.558	0.991	0.12	132.29	13.75
B5		1	2	107.215	351.059	352.994	0.787	68.32	342.14	1.91
		2	2	102.825	262.305	263.751	0.732	34.11	256.80	1.91
		3	1	137.031	190.370	190.370	0.789	14.25	142.52	1.84
		4	2	6.026	10.636	10.695	0.992	0.12	135.32	13.66

Table 6.13 Key equilibrium outcomes related to the InPs — instances B6–B11

Instance	$\underline{P}_1$	$\underline{P}_2$	$\check{P}_1$	$\check{P}_2$	C_1	C_2	$\check{C}'_1$	$\check{C}'_2$	$\check{G}_1$	$\check{G}_2$	$\mathcal{W}_1$	$\mathcal{W}_2$
B6	3.55	0.73	{3.55,...,$\overline{P}$=14.86}	1.87	156	780	0	650.537	0	1214.45	∅	{1,2,3,4}
B7	1.81	0.73	1.81	1.68	312	780	0	675.761	0	1137.25	∅	{1,2,3,4}
B8	1.23	0.73	1.23	1.11	468	780	0	778.145	0	864.84	∅	{1,2,3,4}
B9	0.94	0.73	0.94	0.90	624	780	10.694	780.000	10.02	704.49	{4}	{1,2,3}
B10	0.76	0.73	0.76	0.73	780	780	201.031	684.881	153.28	502.69	{3}	{1,2,4}
B11	0.65	0.73	0.72	0.73	936	780	892.532	0	642.29	0	{1,2,3,4}	∅

Table 6.14 Key equilibrium outcomes related to the SPs — instances B6–B11

Instance	v	k	$\underline{X}_v(\check{P}_k)$	$\check{x}_{vk}$	$\overline{X}_v(\check{P}_k)$	$\check{u}_v$	$a^*_v p^*_v(\check{u}_v)$	$\check{g}_v$	$\check{r}^*_v/\check{x}_{vk}$
B6	1	2	171.470	259.799	259.799	0.642	55.74	62.26	2.11
	2	2	148.251	204.506	204.506	0.568	28.80	42.33	2.07
	3	2	171.615	175.889	175.889	0.698	13.41	0.72	1.87
	4	2	6.509	10.344	10.344	0.987	0.12	125.55	14.00
B7	1	2	153.092	273.613	273.613	0.671	58.24	111.30	2.09
	2	2	134.515	213.203	213.203	0.599	29.85	80.74	2.06
	3	2	152.875	178.548	178.548	0.719	13.60	33.31	1.87
	4	2	6.433	10.396	10.396	0.988	0.12	127.46	13.94
B8	1	2	116.180	329.198	329.198	0.761	66.03	282.38	1.97
	2	2	108.730	248.547	248.547	0.701	33.15	211.92	1.96
	3	2	137.423	189.794	189.794	0.787	14.23	138.29	1.84
	4	2	6.143	10.606	10.606	0.992	0.12	133.45	13.69
B9	1	2	105.594	340.210	358.017	0.774	67.23	352.74	1.94
	2	2	101.776	253.685	266.963	0.713	33.53	264.58	1.95
	3	2	134.005	186.105	195.846	0.767	14.05	176.76	1.85
	4	1	6.027	10.694	10.694	0.993	0.12	135.31	13.59
B10	1	2	97.388	387.952	387.952	0.823	71.46	416.81	1.81
	2	2	96.566	286.109	286.109	0.776	35.46	312.24	1.83
	3	1	131.953	201.031	201.031	0.834	14.65	206.33	1.79
	4	2	5.866	10.820	10.820	0.995	0.12	137.49	13.44
B11	1	1	96.699	390.870	390.870	0.826	71.68	422.40	1.80
	2	1	96.137	287.976	287.976	0.779	35.56	316.36	1.82
	3	1	131.360	202.855	202.855	0.840	14.71	214.98	1.78
	4	1	5.853	10.830	10.830	0.995	0.12	137.65	13.43

6.6 Conclusion

In this work, we address a mobile ecosystem in which the network infrastructure and resources are decoupled from services provisioned for end users giving rise to two types of stakeholders: InPs and SPs. InPs deploy and manage the mobile network and sell their resources to SPs through which the latter provision services for the end users. We consider a case in which there are multiple InPs and multiple SPs and the resource sold/purchased by InPs/SPs is the amount of capacity per BS cell assuming the cell area is provisioned by each InP through its individual BS. We model the problem of cell capacity pricing from the InP perspective and of the choice of an InP from which to acquire capacity from the SP perspective as a multi-leader-follower game. The proposed model has been applied in the context of migration from 4G to 5G for several scenarios in which InPs are characterized by different network technologies and available spectrum bandwidths, whereas SPs provide different 5G mobile services. To set up realistic scenarios, the InP cost structure and the service characterization are based on recent 5G literature.

The analysis of the obtained equilibria shows that more recent InPs are preferred w.r.t. older ones. Older InPs can be competitive if they provide much more spectrum bandwidth, thus resulting more cost-efficient. When the InPs have the same technology, the new entrant ones are less preferred. Indeed, they incur a slightly higher unit cost thus being less competitive.

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CHAPTER 7 GENERAL DISCUSSION

The business model of a conventional (facility-based) Mobile Network Operator (MNO) is very monolithic, i.e., the MNO alone is responsible for the network deployment, its management over time and the tailoring and provisioning of services for end users. However, the concept of infrastructure sharing has been attractive to conventional MNOs for more than two decades now. In few words, one can define infrastructure sharing as the shared use of existing or jointly-deployed infrastructure among multiple MNOs. The straightforward benefit of sharing infrastructure among MNOs is that of sharing its cost. Due to the simultaneous ever-increasing user demand and declining revenues, infrastructure sharing is particularly appealing to an MNO when the latter deploys a new technology and faces high upfront costs for the infrastructure roll-out of and for the acquisition of licensed spectrum. In other words, infrastructure sharing is one of the means for a conventional MNO business model to remain profitable.

The technical aspects of infrastructure sharing, which at times comes hand in hand with spectrum sharing, have been addressed by the research community, especially in terms of resource management, performance evaluation, enablers and adapted architectures. In turn, the economic viability of the different infrastructure sharing alternatives has been generally studied through cost reduction estimates. Instead, the strategic modeling branch, to which our research project belongs to, is much narrower. Specifically, while there are works that address the problem of MNOs deciding to enter infrastructure and possibly also spectrum sharing agreements, such works have limited their focus on the cost savings that infrastructure and spectrum sharing bring about. However, MNOs are self-interested and profit-maximizing entities thus when evaluating the profitability of infrastructure and possibly also of spectrum sharing it is vital to also take into account the impact on revenues depending usually on the network performance metrics, which means that a proper strategic modeling requires a joint study of both technical and economic aspects – this has been the object of study of our first frameworks.

Moreover, with the advent of 5G, the concept of infrastructure sharing becomes more pervasive in the context of multi-tenancy and its enabler, network slicing. In essence, network slicing consists of creating logically separated networks over a single shared network infrastructure. Apart from infrastructure sharing, the decoupling of the network infrastructure from the services it provides is another salient notion in 5G, giving rise to multiple stakeholders in the mobile market such as InPs and SPs. In such a new mobile market setting, the techno-economic interactions among the different stakeholders which have not yet been addressed by appropriate tools. In this research project we proposed a novel framework based on a multi-leader-follower game to address such problem.

7.1 Contribution

The key contributions of this research project are the following:

– Multiple novel techno-economic frameworks based on mathematical programming and game theory for infrastructure sharing among conventional MNOs

We have considered conventional MNOs with given market shares which have to decide on whether to deploy individual networks or enter sharing agreements with a subset of or all the other MNOs. Two cases have been addressed: (i) infrastructure sharing without spectrum pooling and (ii) infrastructure sharing with spectrum pooling, assuming each MNO may own a given amount of licensed spectrum bandwidth. We have proposed both a centralized model (MILP formulation) and an NTU cooperative game for case (i), whereas for case (ii), both an NTU and a TU cooperative game. Interesting results have been obtained when testing these models over different problem instances (see Section 8.1.1 for the key findings).

– A novel techno-economic framework based on game theory for infrastructure sharing for decoupled infrastructure from services in the context of 5G

The techno-economic interactions among InPs and SPs have been modeled as a MLFG. The model has been tested with multiple realistic problem instances across which we vary the InPs' characterization in terms of their network technology (either 4G or 5G) and the amount of their licensed spectrum bandwidth while the same set of SPs has been considered across all instances. What makes these instances realistic in the 5G context are (i) the considered InP cost model – based on 5G literature and (ii) the characterization of the services provisioned by the SPs – based on usage scenarios of IMT for 2020 and beyond. An important outcome of the numerical analysis is that the technological features of the InPs significantly affect the resulting mobile market setting (see Section 8.1.2 for the key findings).

– Broad literature review on infrastructure sharing

The aim of the literature review presented in Chapter 2 was to provide the reader with an overall complete picture of the research on infrastructure sharing both chronologically-wise and scope-wise, as opposed to simply pointing out the contribution of our work compared to similar ones.

7.2 Impact on the research area

Our work in [23] and [24], presented in Chapters 4 and 5, respectively, have each received several citations from the research community. The work in [25], presented in Chapter 6, has not yet received any citations since it was made available online only in December 2019. All the citations of [23] and [24], with the exception of [122], belong to research branches other than strategic modeling or have studied problems which are significantly different from ours. Instead, in [122] the authors have adopted the NTU cooperative game framework proposed in our work in [24] in the context of a Cloud RAN (C-RAN) with focus on energy efficiency. This shows that our proposed framework is flexible as it can be adapted and used for studying other network sharing scenarios in addition to those addressed in our work.

7.3 Insights and new perspectives

The key findings of [23–25] are presented in detail in Chapter 8. In the following paragraphs we will provide some insights and new perspectives deriving from these findings.

From [23, 24] it results that there is generally incentive for infrastructure sharing among conventional MNOs when infrastructure sharing is accompanied by spectrum sharing in which case the benefit of sharing can be paralleled to the economies of scale. In turn, infrastructure sharing in the context of 5G can be seen from a different perspective, that of using the shared infrastructure among different types of services which have different characteristics and hence allow the InPs to better exploit the network resources and monetize the investments in their assets (infrastructure and spectrum) as opposed to using them for a single type of service.

As for infrastructure sharing with spectrum pooling studied in [24], for which there is almost always incentive for MNOs to collaborate and build a unique shared network (coalition), the fact that an *a priori* cost division rule (such as cost sharing proportionally to the MNOs' market shares) does not always guarantee the coalition stability shows that the proposed model can be a useful tool for MNOs when deciding to enter a sharing agreement and how to split the cost of the shared network.

Concerning [25], one key finding is that the technological features of the InPs have a significant impact on the competition among them in the mobile market. In these lines, in a market consisting of a 4G InP and a 5G InP, it turns out that the 4G InP is preferred to a 5G one only when the former has much more spectrum bandwidth than the latter. This shows that the amount of spectrum bandwidth allocated to 5G networks during auctions is important for 5G to cost-effectively enter a market in which an incumbent 4G network is present, hence regulators may want to intervene in such auctions if they wish to promote 5G entering the market. Moreover, when considering a market with two 5G InPs in which one is a new-entrant and the other owns a legacy network (which it can partially reuse), when both InPs have equal amount of bandwidths, the new entrant is less cost-effective hence less competitive. Therefore, regulators might want to intervene in spectrum auctions and guarantee sufficient spectrum bandwidth for the new entrant in order to induce competition and avoid a monopoly market, e.g., through spectrum set-asides for new entrants.

CHAPTER 8 CONCLUSION AND RECOMMENDATIONS

8.1 Summary of Works

In this chapter we summarize the problems addressed in [23–25] and the key results obtained applying the respective proposed frameworks for the considered problem instances.

8.1.1 Infrastructure sharing among conventional MNOs

The first type of problems that have been tackled by this research project in [23] and [24] share the following reference scenario:

A set of conventional MNOs with given market shares plan to deploy a layer of LTE small cells to their existing macrocell infrastructure in one or multiple given geographical areas. MNOs have to decide whether to deploy their individual network of small cells or cooperate with a subset/all other MNOs and build a shared network in the given area(s).

We have explored several variants that arise from this scenario in [23, 24] which we have modeled by suitable mathematical approaches. The different variants address the following aspects: (i) whether spectrum can be pooled, (ii) whether the regulator intervenes and (iii) whether the cost division rule is an input or an output of the model.

One key common input to these models is the user pricing model which allows us to capture the impact that infrastructure sharing has on the revenues of collaborating MNOs so that their strategic decision of entering a sharing agreement is not simply driven by the impact on the cost, but on its overall profit. The proposed user pricing depends on the average rate perceived by users of an MNO which in turn depends on its sharing agreement, i.e., on the coalition joined by the MNO where each coalition member is characterized by a given market share and a given spectrum availability.

In turn, the key output is the network sharing configuration, i.e., the coalitions of MNOs which are formed up and their respective level of investment (i.e., number of deployed small cells in our case). Such sharing configurations correspond either to the case when coalitions are created by MNOs voluntarily, so that their respective individual profits are maximized, or imposed by the regulator, so that a global objective is optimized.

Infrastructure sharing without spectrum pooling

In [23], we consider the case when there is no spectrum pooling, i.e., the same amount of spectrum resources is available to any type of coalition, regardless of the number of members in the coalition. We consider two variants here:

- A regulator forces MNOs to adopt a network sharing configuration which maximizes the total/minimum user rate, while making sure that the MNO profits cannot be negative. We model this case as a Mixed Integer Linear Programming Problem (MILP).
- Each MNO joins the coalition that maximizes its individual profit. This case is instead modeled as cooperative game with non-transferable utility.

Among others, we analyze the impact that the price per unit of service has on the obtained network configurations (coalitions and number of activated small cells).

Two key findings of such analysis are the followings:

- There is generally more incentive to cooperate so as to maximize the MNOs individual profits (second variant), as opposed to maximizing the user rate (first variant).
- However, for both variants, the increase of user price per unit of service (and therefore of the MNO revenues for the same service of level provided/level of network investment) decreases the incentive to cooperate since MNOs can afford larger investments by themselves and build less congested networks which in turn provides them with higher revenues. This shows that infrastructure sharing without spectrum pooling introduces a trade-off between reduced cost and service level degradation and whether the former outweighs the latter depends on the MNO revenues.

Infrastructure sharing with spectrum pooling

In [24] we address a problem similar to [23], but, in addition, we assume MNOs (or at least one of them) owns a certain amount of spectrum. We also assume that for any type of coalition, its members will agree to pool together their individual spectrum. We model this problem as a cooperative game and derive solutions belonging to the core (that is, we determine payoff allocations that make the grand coalition stable). Unlike in [23], where the cost are divided a priori, based on the MNOs market shares, here ([24]) the cost division is an outcome of the game. In other words, how cost are divided can make the grand coalition stable or not. This further "degree of liberty" is introduced since MNOs can have different mixtures of market share and available spectrum and such mixture should influence their payoff allocations.

In [24], we carry out a similar analysis of the impact of unit price on the game outcome, also. The key findings are the following:

- The grand coalition is stable for the vast majority of considered instances (representing MNOs with different mixture of market share and spectrum holdings) regardless of the unit price: this means that the spectrum pooling gain outweighs the service level degradation introduced by infrastructure sharing alone.
- The stable cost division rule reflects the MNOs individual market share and available spectrum: e.g., MNOs with a small market share and large available spectrum can be at the limit exempted from the network infrastructure cost.

8.1.2 Infrastructure sharing for decoupled infrastructure from services in the context of 5G

Instead in [25], we have addressed a problem concerning a different mobile market setting representing future (5G) networks in which the mobile network infrastructure and the mobile services are decoupled. A brief description of the problem follows:

In a dense urban area, there are multiple InPs with individual mobile networks that overlap. We focus in the area of a single BS cell (served simultaneously by all InPs through their individual BSs, i.e., the latter are assumed to be co-located). However, InPs are assumed to operate in disjoint spectrum bands. None of the InPs provides services to end users. In turn there are multiple SPs with no network infrastructure and resources and each such SP provides only one type of mobile service to a unique market segment. Each SP provisions services for its end users in the cell area by purchasing cell capacity from only one of the InPs (however an InP can host multiple SPs). Each entity (InP/SP) is assumed a profit-maximizer. In these lines, each InP offers its available cell capacity at a unit price that maximizes its profit from the overall amount of sold capacity whereas each SP chooses an InP from which to purchase cell capacity with the objective of maximizing its profit (given by the difference between revenues incurred from own users for the amount of cell capacity purchased from the selected InP and the resulting cost of the this cell capacity. The cell capacity of each InP is fixed (and finite) hence SPs compete among them in selecting an InP from which to purchase cell capacity whereas InPs compete among them in the cell capacity unit prices to be selected by the SPs.

For this setting, to tackle the problem of cell capacity unit pricing by the InPs and InP choice by the SPs, we propose a multi-leader-follower game (an extension of the basic Stackelberg game involving a single leader and a single follower).

We have set up several realistic instances which differ in the characterization of the set of InPs in terms of network technology (whether 4G or 5G) and of their available spectrum bandwidths where the InP cost model is based on recent 5G literature. In turn, for all instances we consider the same set of SPs (whose provisioned services are characterized through usage scenarios of International Mobile Telecommunications (IMT) for 2020 and beyond). Applying the proposed multi-leader-follower game to these instances always allows us to obtain an equilibrium or an approximation of an equilibrium. From the equilibria obtained for the different instances, one key outcome is that the competitiveness among the InPs is considerably affected by their technological features. In particular:

- An InP with 5G technology is generally preferred by SPs to an InP with 4G when their spectrum bandwidths are comparable due to the former's higher cost-efficiency.
- An InP with 4G technology becomes competitive only when its available spectrum bandwidth is significantly larger than the one of a 5G InP.
- When all InPs have 5G networks, the new entrant is generally less competitive due to its slightly higher unit cost.

As for the SPs, in order to maximize their profit, some of the them may opt for a more expensive InP when the cheapest InP has not sufficient available capacity.

8.2 Limitations and future research

In [23–25] we have proposed suitable mathematical models to analyze several infrastructure and spectrum sharing scenarios arising in different technological environments. Although our models are very detailed and realistic, there are still some aspects that can be potential leads for future research in the topic. One such aspect is user churning which is not trivial to deal with. To model user churning, a detailed socio-economic analysis of the user behavior is needed, which goes beyond the scope of our research project. A sensitivity analysis of the infrastructure cost models can be another research lead, given the lack of realistic (MNO-provided) infrastructure cost models in the literature. As for [25], the model proposed therein can be further enriched by considering additional types of SPs, multiple service types provided by the single SP and by allowing the latter to purchase capacity from multiple InPs.

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APPENDIX A PIECE-WISE LINEAR APPROXIMATION OF THE USER RATE

We recall that the nominal user rate ($\rho_s^{a,nom}$) is computed by means of the simulation described in Subsection IV-A whereas the average user rate (ρ_s^a) is derived from $\rho_s^{a,nom}$ according to Equations (A.1). ρ_s^a is then approximated by a concave piecewise linear function in order to formulate the problem as a MILP.

$$\rho_s^a = \rho_s^{a,nom}(1-\eta)^{\frac{\sum_{i\in\mathcal{O}_s}\sigma_i N_a}{u_s^a}}, \quad \forall\, s \in \mathcal{S},\ \forall\, a \in \mathcal{A}. \tag{A.1}$$

Figure A.1 illustrates the simulated nominal user rate $\rho_s^{a,nom}$, the average user rate ρ_s^a and the piece-wise linear function approximating ρ_s^a for coalition ABC in area Z1 (similarly for all the other considered areas and coalitions). In the following, we explain how the approximation was modeled in the MILP formulation.

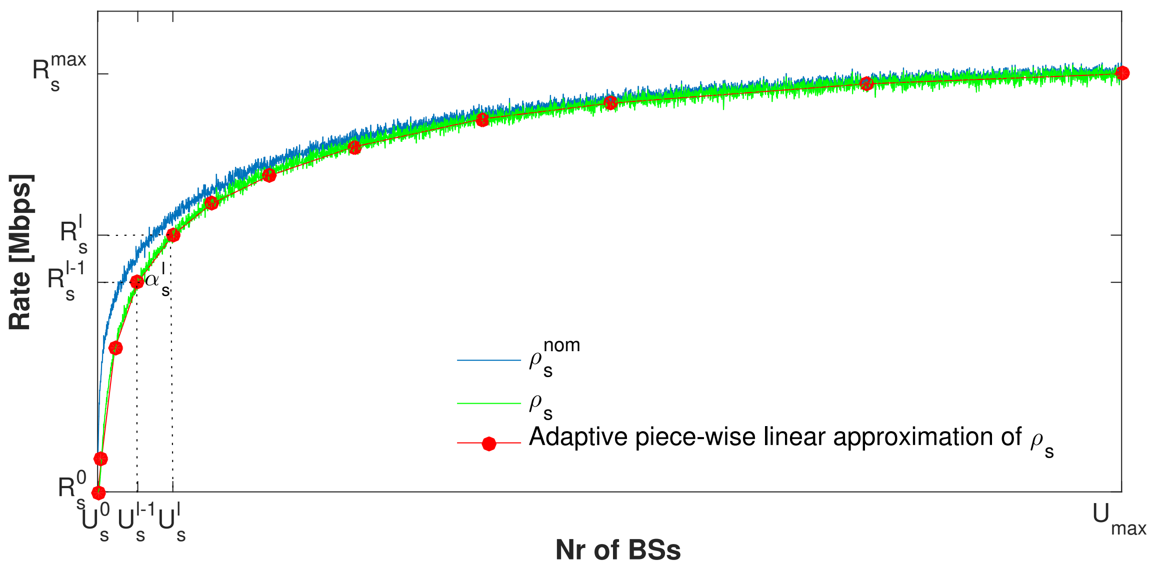


Figure A.1 Simulated nominal user rate ($\rho_s^{a,nom}$), average user rate (ρ_s^a) and adaptive piece-wise linearization for coalition ABC in area Z1 (20000 users, 4 km^2).

As mentioned, L denotes the number of linear pieces (intervals) that approximate ρ_s^a. We have considered equal values of L for all the coalitions $s \in \mathcal{S}$ and all the areas $a \in \mathcal{A}$. L was set to 11 for user distribution M_1 and to 10 for M_2. For each interval $l \in \{1, \ldots, L\}$, coalition s and area a, $[U_s^{a,l-1},\ U_s^{a,l}]$ represents the range of the number of BSs that characterize the l^{th} interval, $R_s^{a,l}$ is the average user rate when s activates $U_s^{a,l}$ BSs in a and $\alpha_s^{a,l}$ is the slope associated with the l^{th} interval. The average user rate ρ_s^a obtained by activating u_s^a BSs, with

$u_s^a \in [U_s^{a,l-1}, U_s^{a,l}]$, is therefore equal to $R_s^{a,l-1} + \alpha_s^{a,l}(u_s^a - U_s^{a,l-1})$. Equations (A.2) show how these parameters are related with one another.

$$\begin{aligned} R_s^{a,0} &= \rho_s^a(1), \quad \forall\ s \in \mathcal{S},\ \forall\ a \in \mathcal{A}, \\ R_s^{a,l} &= R_s^{a,l-1} + \alpha_s^{a,l}(U_s^{a,l} - U_s^{a,l-1}), \\ &\forall s \in \mathcal{S},\ \forall\ a \in \mathcal{A},\ \forall\ l \in \{1, \dots, L\}. \end{aligned} \tag{A.2}$$

In particular, $U_s^{a,0}$ is equal to 1, whereas $U_s^{a,L}$ is equal to U_{max}, $\forall s \in \mathcal{S}$ and $\forall a \in \mathcal{A}$. Thus, the average user rate ρ_s^a obtained by activating u_s^a BSs can be reformulated as:

$$\begin{gathered} \rho_s^a = \min_{l \in \{0,\dots,L-1\}} \{R_s^{a,l} + \alpha_s^{a,l+1}(u_s^a - U_s^{a,l})\}, \\ \forall\ s \in \mathcal{S},\ \forall\ a \in \mathcal{A}. \end{gathered} \tag{A.3}$$

As ρ_s^a is maximized by any of the considered objective functions, Equations (A.3) can be replaced by Constraints (4.8). Notice that, the auxiliary binary variables z_s^a equal zero when either no BSs are activated by s in a ($u_s^a = 0$ and therefore $z_s^a = 0$ due to Constraint (4.7)) or s is not active ($y_s = 0$ and therefore $z_s^a = 0, \forall a \in \mathcal{A}$ due to Constraints (4.4) and (4.7)). In turn, when $z_s^a = 0$, we should also have $\rho_s^a = 0$, which is guaranteed by Constraints (4.9) while Constraints (4.8) are made redundant by the term $M(1 - z_s^a)$, where $M = 1000$.

APPENDIX B PROOF OF NP-COMPLETENESS

The optimization problem with objective MIN_Q and Constraints (4.2)–(4.13) will be denoted by *Infrastructure Sharing Problem* (ISP).

Theorem. The decision version of (ISP) is NP-complete.

Proof. The decision version of (ISP) can be formulated as:

Given a threshold $\bar{Q} > 0$ on the quality, are there variables y_s, z_s^a, u_s^a and ρ_s^a, with $s \in \mathcal{S}$ and $a \in \mathcal{A}$, such that Constraints (4.2)–(4.13) *are satisfied and $MIN_Q \geq \bar{Q}$?*

We will prove that the decision version of (ISP) is NP-complete by reduction from the *Set Partitioning Problem* (SPP) which is a well-known NP-complete problem (see, e.g., [233]). We recall the decision version of (SPP):

Given a universe $\mathcal{U}$, a family $\mathcal{C}$ of subsets of $\mathcal{U}$ and a positive integer K, is there a subset $\mathcal{C}' \subseteq \mathcal{C}$ such that $|\mathcal{C}'| \leq K$ and each element of the universe $\mathcal{U}$ belongs to exactly one member of $\mathcal{C}'$?

The proof is carried out in 3 steps.

1. The decision version of (ISP) is a NP problem because verifying that a given solution is a YES one requires O($|\mathcal{O}| + L|\mathcal{S}|$) number of operations.

2. It is possible to make a polynomial time transformation of any instance $\mathcal{I}_{SPP}$ of the decision version of (SPP) into an instance $\mathcal{I}_{ISP}$ of the decision version of (ISP). Given $\mathcal{I}_{SPP}$=($\mathcal{U}$, $\mathcal{C}$, K), we build $\mathcal{I}_{ISP}$=($\mathcal{O}$, $\mathcal{S}$, $\mathcal{A}$, N_a, $\{\sigma_i\}_{i\in\mathcal{O}}$, U_{max}, L, $\{U_s^l\}_{l=0}^L$, $\{R_s^l\}_{l=0}^L$, $\{\alpha_s^l\}_{l=1}^L$, δ, D, g, $\bar{Q}$) as follows:

   - $\mathcal{O} = \mathcal{U}$, $\mathcal{S} = \mathcal{C}$, $|\mathcal{A}| = 1$, $N_a = |\mathcal{U}|$, $\sigma_i = 1/|\mathcal{U}|$ for any $i \in \mathcal{O}$, $U_{max} = K$, $L = K-1$.
   - For any coalition $s \in \mathcal{S}$, we set $U_s^0 = 1, U_s^1 = 2, \ldots, U_s^{K-1} = K$.
   - Given an arbitrary coalition $\overline{s} \in \mathcal{S}$, we set:

     $R_s^0 = |\overline{s}|/|s|$ for any $s \in \mathcal{S}$,

     $R_s^l = R_s^{l-1} + \alpha_s^l$ for any $s \in \mathcal{S}$ and $l = 1, \ldots, K-1$,
   - For any coalition $s \in \mathcal{S}$, we set $R_s^0 > \alpha_s^1 > \alpha_s^2 > \cdots > \alpha_s^{K-1} > 0$.
   - $\delta = 1$, $D = 1$, $g = |\overline{s}|$, $\bar{Q} = \min_{s\in\mathcal{S}} R_s^0$.

   It is clear that such transformation can be done in polynomial time with respect to size of $\mathcal{I}_{ISP}$.

3. $\mathcal{I}_{ISP}$ is a YES instance if and only if $\mathcal{I}_{SPP}$ is a YES instance.

   First, we prove the *if* part. Since $\mathcal{I}_{SPP}$ is a YES instance, there is a subset $\mathcal{C}' \subseteq \mathcal{C}$ such that $|\mathcal{C}'| \leq K$ and each element of the universe $\mathcal{U}$ belongs to exactly one member of $\mathcal{C}'$. We define the variables

$$y_s = z_s^a = u_s^a = \begin{cases} 1 & \text{if } s \in \mathcal{C}', \\ 0 & \text{otherwise,} \end{cases} \qquad \rho_s^a = \begin{cases} R_s^0 & \text{if } s \in \mathcal{C}', \\ 0 & \text{otherwise.} \end{cases}$$

   It is easy to check that Constraints (4.2)–(4.5) are satisfied. Constraint (4.6) is fulfilled since

$$\sum_{s \in \mathcal{S}} u_s^a = |\mathcal{C}'| \leq K = U_{max}.$$

   The values of variables z_s^a, u_s^a and ρ_s^a guarantee that Constraints (4.7)–(4.9) hold. Furthermore, Constraints (4.13) on the nonnegative profit of MNOs hold because

$$\sum_{a \in \mathcal{A}} (r_i^a - c_i^a) = \delta D \sigma_i N_a q_i^a - \sum_{s \in \mathcal{S}_i} g \frac{\sigma_i}{\sum_{j \in s} \sigma_j} u_s^a = \sum_{s \in \mathcal{S}_i} R_s^0 u_s^a - \sum_{s \in \mathcal{S}_i} \frac{|\overline{s}|}{|s|} u_s^a = 0.$$

   Finally, since $\mathcal{C}'$ is a partition of $\mathcal{O}$, any MNO i belongs to a unique coalition $s_i \in \mathcal{C}'$ and $q_i^a = \rho_{s_i}^a = R_{s_i}^0 \geq \bar{Q}$ for any $i \in \mathcal{O}$, that is $MIN_Q \geq \bar{Q}$. Therefore, $\mathcal{I}_{ISP}$ is a YES instance.

   Now, we prove the *only if* part. Assume that $\mathcal{I}_{ISP}$ is a YES instance, i.e., there are variables y_s, z_s^a, u_s^a and ρ_s^a, with $s \in \mathcal{S}$ and $a \in \mathcal{A}$, such that all the Constraints (4.2)–(4.13) are satisfied and $MIN_Q \geq \bar{Q}$. For any $i \in \mathcal{O}$ we have $q_i^a \geq \bar{Q} > 0$, hence we get from Constraints (4.4), (4.7) and (4.9) that for any $i \in \mathcal{O}$ there exists a unique coalition $s_i \in \mathcal{S}_i$ such that $y_{s_i} = 1$. Thus, $u_{s_i}^a \geq 1$ by Constraint (4.5). On the other hand, $r_i^a = \delta D \sigma_i N_a q_i^a = \rho_{s_i}^a$ and

$$c_i^a = \sum_{s \in \mathcal{S}_i} g \frac{\sigma_i}{\sum_{j \in s} \sigma_j} u_s^a = g \frac{\sigma_i}{\sum_{j \in s_i} \sigma_j} u_{s_i}^a = \frac{|\overline{s}|}{|s_i|} u_{s_i}^a = R_{s_i}^0 u_{s_i}^a.$$

   Since $0 \leq r_i^a - c_i^a = \rho_{s_i}^a - R_{s_i}^0 u_{s_i}^a$, we obtain from Figure B.1 that $u_{s_i}^a \leq 1$. Thus, for any activated coalition s (i.e., $y_s = 1$) the number of deployed BSs is $u_s^a = 1$. If we define

$$\mathcal{C}' = \{s \in \mathcal{S} : \ y_s = 1\},$$

then $\mathcal{C}'$ is a partition of $\mathcal{U}$ and $|\mathcal{C}'| = \sum_{s\in\mathcal{S}} u_s^a \leq U_{max} = K$, therefore $\mathcal{I}_{SPP}$ is a YES instance.

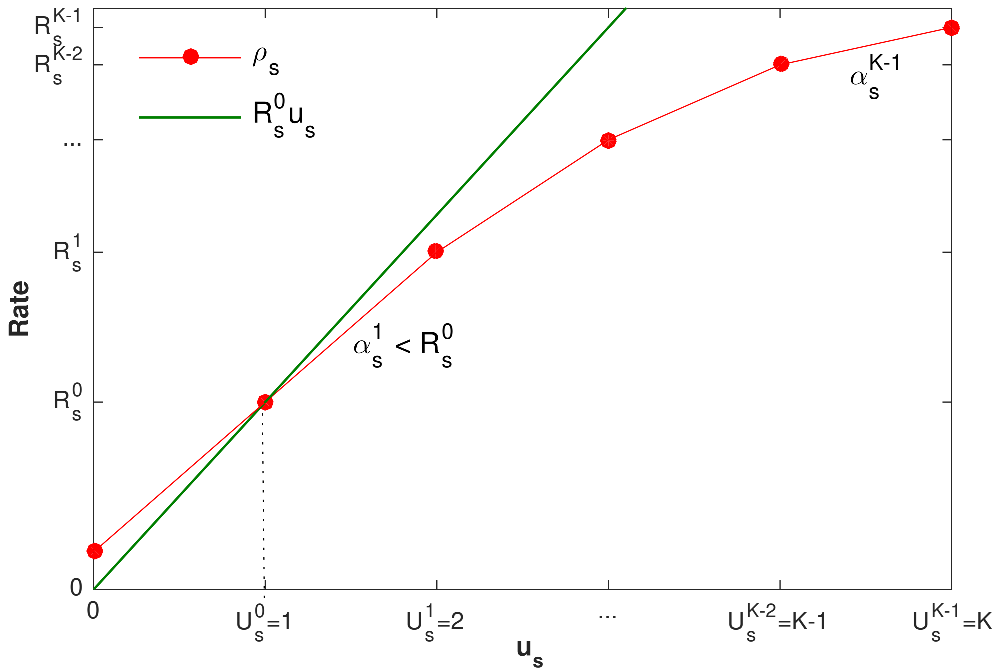


Figure B.1 Graphical illustration of the number of BSs activated by coalitions.

APPENDIX C OPTIMAL USER FEE DERIVATION

In the following, we derive the optimal user fee from the SP perspective, i.e., the fee *offered* to a user by its SP so that the SP revenue is maximized. We show that this fee is a function of the level of utility perceived by the user which is *per se* a function of the amount of capacity allocated to the user by the SP. As explained in Section 6.3.2, each SP $v \in \mathcal{V}$ splits its available cell capacity x_v uniformly among its users and the utility perceived by the single user from the allocated capacity, i.e., $u_v(x_v)$ is represented by Equation (6.1). Further, if a user of SP v perceiving the utility $u_v(x_v)$ is offered a fee p_v, it will accept it with a probability $a_v(p_v, u_v(x_v))$ given by Equation (6.2) (or equivalently (6.3)), therefore, $a_v(p_v, u_v(x_v))p_v$ represents the fee *accepted* by the user. It follows that, for a given amount of cell capacity x_v, which implies a level of utility for the single user equal to $u_v(x_v)$, the optimal user fee for SP v is the value of the p_v which maximizes $a_v(p_v, u_v(x_v))p_v$.

For ease of notation, hereon, we drop the argument x_v of the utility function $u_v(x_v)$ as we derive the optimal offered fee for a fixed level of utility. We also drop the SP subscript v from all parameters and variables since the optimal fee derivation is analogous for all SPs. We assume $0 < \varepsilon < \infty$, $0 < \mu < \infty$, $0 < \overline{p} < \infty$, $0 < \overline{u} < \infty$, $0 < \overline{q} < 1$, $0 < p < \infty$, and $0 < u < \infty$. It can be easily argued that these are all sensible assumptions. First, recall that an SP polls a large set of its own users characterized by ε and μ (i.e., the sensitivities to changes in price and utility, respectively) on whether they accept the fee $\overline{p}$ when they perceive a maximum level of utility ($\overline{u}$) and then it sets $\overline{q}$ equal to the fraction of users that reject it. As explained in Section 6.3.2, the normalizing constant $A = -\overline{p}^{\varepsilon}\overline{u}^{-\mu}\log(\overline{q})$, therefore for the assumed values of $\overline{p}$, ε, $\overline{u}$, μ and $\overline{q}$, we have $0 < A < \infty$. Recall also that $a(p, u) = 1 - e^{-Ap^{-\varepsilon}u^{\mu}} = 1 - \overline{q}^{(\overline{p}/p)^{\varepsilon}(u/\overline{u})^{\mu}}$ given the definition of A (as detailed in Section 6.3.2). Concerning ε and μ, they are both assumed positive constants in [208] and positive bounded values are considered in literature ([208, 219, 228–232]). In fact, ε and μ should be estimated through realistic measurements [208], hence, in practice, it cannot be that $\varepsilon = \infty$ or $\mu = \infty$ as users cannot be infinitely sensitive to changes in the offered fee or the perceived utility. Consider the equivalent definition of $a(p, u)$, i.e., $a(p, u) = 1 - \overline{q}^{(\overline{p}/p)^{\varepsilon}(u/\overline{u})^{\mu}}$ and suppose that $0 < \mu < \infty$, $0 < \overline{p} < \infty$, $0 < \overline{u} < \infty$, $0 < \overline{q} < 1$, $0 < p < \infty$, and $0 < u < \infty$ but $\varepsilon = \infty$. It follows that

$$a(p,u) = \begin{cases} 1 & \text{if } p < \overline{p}, \\ 0 & \text{if } p > \overline{p}, \\ \text{indeterminate} & \text{otherwise,} \end{cases}$$

hence $a(p,u)p$ is maximized by a fee equal to $\overline{p} - \Delta$, where Δ is an infinitely small positive constant. Now, suppose that $0 < \varepsilon < \infty$, $0 < \overline{p} < \infty$, $0 < \overline{u} < \infty$, $0 < \overline{q} < 1$, $0 < p < \infty$, and $0 < u < \infty$ but $\mu = \infty$. It follows that

$$a(p,u) = \begin{cases} 0 & \text{if } u < \overline{u}, \\ 1 & \text{if } u > \overline{u}, \\ \text{indeterminate} & \text{otherwise,} \end{cases}$$

therefore, if $u < \overline{u}$, $a(p,u)p = 0$ for any offered price $p \neq \infty$, i.e., any $p \neq \infty$ generates zero revenue for the SP, whereas for $u > \overline{u}$, $a(p,u)p$ is maximized by any $p \neq \infty$. Further, if $\overline{u} = 0$ (where by definition $\overline{u}$ is the maximum utility perceived by the user), then it would make no sense to look for the optimal fee, as no rational user would be willing to pay for a service which provides no utility. For the considered utility function (see Equation (6.1)), $\overline{u} \leq 1$ hence $\overline{u} < \infty$. Even if we were to consider a different utility function, it would still be reasonable to assume that $\overline{u} < \infty$ since the utility is a function of the allocated capacity which is *per se* physically limited. As for $\overline{p}$, the value $\overline{p} = \infty$ is impractical whereas $\overline{p} = 0$ would result in $\overline{q} = 0$ (as no rational user would reject a service providing a maximum level of utility $\overline{u}$ when offered for free) and therefore $a(p,u) = 1 - \overline{q}^{(\overline{p}/p)^{\varepsilon}(u/\overline{u})^{\mu}}$ would be indeterminate for any value of u and p, which means that the SP cannot make use of the acceptance probability function if it were to poll its users with $\overline{p} = 0$. Next, for $0 < \varepsilon < \infty$, $0 < \mu < \infty$, $0 < \overline{p} < \infty$, and $0 < \overline{u} < \infty$, $\overline{q} = 0$ would result in $a(p,u) = 1 - \overline{q}^{(\overline{p}/p)^{\varepsilon}(u/\overline{u})^{\mu}} = 1$, $\forall u \in (0,\infty)$, $\forall p \in (0,\infty)$ and vice versa, $\overline{q} = 1$ would result in $a(p,u) = 0$, $\forall u \in (0,\infty)$, $\forall p \in (0,\infty)$, which means that in both cases the SP cannot make use of the acceptance probability function. In practice, if an SP estimated $\overline{q} = 0$ ($\overline{q} = 1$), we would expect it to re-poll the users with a higher (lower) value of $\overline{p}$ until it attains[1] a value of $\overline{q}$ in (0,1). As for p, while $p = \infty$ is impractical, for the assumed parameter values, $p = 0$ would instead result in $a(p,u) = 1, \forall u \in (0,\infty)$ and, as a result, in $a(p,u)p = 0, \forall u \in (0,\infty)$ which is the minimal value of $a(p,u)p$ hence we look for $p \in (0,\infty)$. Finally, we have assumed $0 < u < \infty$, where $u < \infty$ can be justified

[1] In practice, it should be unlikely for the SP to attain $\overline{q} = 0$ for $\overline{p} \to \infty$. Instead, if the SP attained $\overline{q} = 1$ for $\overline{p} \to 0$, it means the service it proposes has no market.

in the same fashion as $\overline{u} < \infty$ since by definition $u \leq \overline{u}$, whereas $u = 0$ is not interest: for the assumed parameter values, when $u = 0$, $a(p,u) = 0$, $\forall p \in (0,\infty)$, and, as a result, $a(p,u)p = 0$, $\forall p \in (0,\infty)$, i.e., there is no optimal fee as the SP incurs no revenue when the user achieves no utility.

Now, for a given u, we look for p which maximizes $a(p,u)p$, which we denote as $p^*(u)$. Recall that $a(p,u) = 1 - e^{-Ap^{-\varepsilon}u^{\mu}}$ (see Equation (6.2)), therefore to determine $p^*(u)$ we solve

$$\frac{\partial\left((1 - e^{-Ap^{-\varepsilon}u^{\mu}})p\right)}{\partial p} = 0,$$

that is, the equation

$$1 - e^{-Ap^{-\varepsilon}u^{\mu}} - \varepsilon A p^{-\varepsilon} u^{\mu} e^{-Ap^{-\varepsilon}u^{\mu}} = 0. \tag{C.1}$$

First, let $z = \varepsilon A p^{-\varepsilon} u^{\mu} + 1$. Equation (C.1) becomes equivalent to

$$1 - z e^{(1-z)/\varepsilon} = 0. \tag{C.2}$$

Then let $y = -z/\varepsilon$ which allows to rewrite (C.2) as

$$y e^{y} = (-1/\varepsilon) e^{(-1/\varepsilon)}. \tag{C.3}$$

If we denote a solution of Equation (C.3) by y^*, then the optimal offered price for the givel level of utility u, i.e., $p^*(u)$, corresponding to y^* is

$$p^*(u) = \left[(Au^{\mu}) / (-y^* - 1/\varepsilon)\right]^{1/\varepsilon}. \tag{C.4}$$

It follows that the acceptance probability of $p^*(u)$, given u, is

$$\begin{aligned} a(p^*(u), u) &= 1 - e^{-A(p^*(u))^{-\varepsilon}u^{\mu}} \\ &= 1 - e^{-AA^{-1}(-y^*-1/\varepsilon)u^{-\mu}u^{\mu}} \\ &= 1 - e^{y^* + 1/\varepsilon}. \end{aligned} \tag{C.5}$$

Notice that $a(p^*(u), u)$ is independent of u and it only depends on ε (as from (C.3) y^* only depends on ε) hence, hereon, we refer to $a(p^*(u), u)$ by a^*. Equation (C.3) has one easily identifiable solution, $y^* = -1/\varepsilon$, for which, however, $p^*(u) = \infty$ as $A \neq 0$, $u \neq 0$, $\mu \neq \infty$ and $0 < \varepsilon < \infty$ (see Equation (C.4)), whereas $a^* = 0$ (see Equation (C.5)) and, as a result $a^* p^*(u) = 0 \times \infty$ (i.e., the optimal accepted fee is indeterminate). However, depending on the value of ε, $y^* = -1/\varepsilon$ may not be the only solution of (C.3). To determine all solutions of Equation (C.3), we proceed as follows. Let $\alpha = (-1/\varepsilon)e^{-1/\varepsilon}$. Equation (C.3) becomes equivalent to

$$ye^y = \alpha, \tag{C.6}$$

whose solutions are given by the noted Lambert W function. As here $y = -z/\varepsilon = -Ap^{-\varepsilon}u^{\mu} - 1/\varepsilon \in \mathbb{R}$, we consider the real-valued variant of the Lambert W function which we denote as $W : \alpha \to y$, where $\alpha \in [-1/e, +\infty)$. The lower bound of α is due to the fact that the minimum value of the function $f(y) = ye^y$, attained at $y = -1$, is equal to $-1/e$. Since $\alpha = -(1/\varepsilon)e^{-1/\varepsilon}$ and $0 < \varepsilon < \infty$, here we also have that $\alpha < 0$. W is single valued for $\alpha = -1/e$, whereas for $\alpha \in (-1/e, 0)$, it is double-valued as illustrated by Figure C.1. The upper branch of W (for which $W \geq -1$), is denoted as W_0, whereas the lower branch (for which $W \leq -1$) as W_{-1}, where both W_0 and W_{-1} are *per se* single-valued functions of α. It follows that for $\varepsilon = 1$, which implies a value of $\alpha = (-1/\varepsilon)e^{-1/\varepsilon} = -1/e$, Equation (C.6)

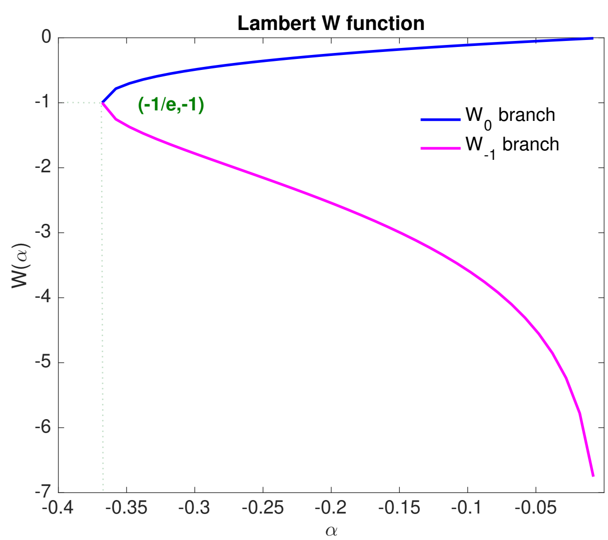


Figure C.1 Lambert W function for $\alpha \in [-1/e, 0)$.

admits a single solution $y^* = W(-1/e) = W_0(-1/e) = W_{-1}(-1/e) = -1$ (see Figure C.1) which coincides with the solution $y^* = -1/\varepsilon = -1$ of the equivalent Equation (C.3) obtained by inspection. Instead, for $\varepsilon \in (0, \infty)$ with $\varepsilon \neq 1$, for which $\alpha \in (-1/e, 0)$, Equation (C.6) admits two solutions: $y_0^* = W_0(\alpha)$ and $y_{-1}^* = W_{-1}(\alpha)$. In summary, based on the value of ε which results in $\alpha = -(1/\varepsilon)e^{-(1/\varepsilon)}$, there are three cases concerning the solution(s) of Equation (C.6):

1. For $\varepsilon = 1$, which implies $\alpha = -\left(1/\varepsilon\right)e^{-(1/\varepsilon)} = -1/e$, Equation (C.6) admits a single

solution $y^* = W(-1/e) = W_0(-1/e) = W_{-1}(-1/e) = -1$ (see Figure C.1)). From (C.4), the optimal fee for the given level of utility u corresponding to $y^* = -1$, i.e.,

$$\begin{aligned} p^*(u) &= \left[(Au^\mu) / (-y^* - 1/\varepsilon)\right]^{1/\varepsilon} \\ &= (Au^\mu) / (1 - 1) \\ &= \infty, \end{aligned}$$

as $A > 0$, $u > 0$, $\mu < \infty$. Then from (C.5), the acceptance probability of $p^*(u)$ for the given level of utility u, i.e.,

$$a^* = 1 - e^{y^* + 1/\varepsilon} = 1 - e^{-1+1} = 0$$

and, as a result, $a^* p^*(u) = 0 \times \infty$, i.e, the optimal accepted fee for the given level of utility u is indeterminate.

2. For $0 < \varepsilon < 1$, which implies $\alpha = -(1/\varepsilon)\, e^{-(1/\varepsilon)} \in (-1/e, 0)$ and hence $W(\alpha)$ being double-valued, Equation (C.6) admits two solutions: $y_0^* = W_0(\alpha) > -1$ and $y_{-1}^* = W_{-1}(\alpha) = -1/\varepsilon < -1$. Let a_0^* denote the acceptance probability of $p_0^*(u)$ for the given level of utility u and, analogously, a_{-1}^*, the acceptance probability of $p_{-1}^*(u)$ for u. From (C.4), we have

$$\begin{aligned} p_0^*(u) &= \left[(Au^\mu) / (-y_0^* - 1/\varepsilon)\right]^{1/\varepsilon} \\ &= \left[(Au^\mu) / (-W_0(\alpha) - 1/\varepsilon)\right]^{1/\varepsilon}. \end{aligned}$$

Due[2] to $-1 < W_0(\alpha) < 0$ and $0 < \varepsilon < 1$, which imply $-\infty < -W_0(\alpha) - 1/\varepsilon < 0$ and given that $0 < A < \infty$, $0 < u < \infty$, $\mu < \infty$ and $0 < \varepsilon < 1$, then

$$p_0^*(u) \in \begin{cases} (0, \infty) & \text{if } 1/\varepsilon \text{ is an even integer,} \\ (-\infty, 0) & \text{if } 1/\varepsilon \text{ is an odd integer,} \\ \mathcal{C} & \text{otherwise,} \end{cases}$$

which means that if $1/\varepsilon$ is not an even integer, then $p_0^*(u)$ is an infeasible solution. From (C.5),

$$a_0^* = 1 - e^{y_0^* + 1/\varepsilon} = 1 - e^{W_0(\alpha) + 1/\varepsilon} \in (-\infty, 0)$$

[2] Recall that here $\alpha < 0$ and since $W_0(\alpha)$ is strictly increasing in α, then $W_0(\alpha) < W_0(0) = 0$.

as $0 < W_0(\alpha) + 1/\varepsilon < \infty$ for which $1 < e^{W_0(\alpha)+1/\varepsilon} < \infty$. As $-\infty < a_0^* < 0$, we can conclude that even when $1/\varepsilon$ is an even integer, $p_0^*(u)$ is infeasible as it cannot be accepted with a negative probability.

As for $y_{-1}^* = W_{-1}(\alpha) = -1/\varepsilon$, we have that

$$\begin{aligned} p_{-1}^*(u) &= \left[(Au^\mu) / \left(-y_{-1}^* - 1/\varepsilon\right)\right]^{1/\varepsilon} \\ &= \left[(Au^\mu) / (1/\varepsilon - 1/\varepsilon)\right]^{1/\varepsilon} = \infty \end{aligned}$$

as $A > 0$, $u > 0$, $\mu < \infty$ and $0 < \varepsilon < 1$ whereas

$$a_{-1}^* = 1 - e^{y_{-1}^* + 1/\varepsilon} = 1 - e^{-1/\varepsilon + 1/\varepsilon} = 0,$$

and, as a result, $a_{-1}^* p_{-1}^* = 0 \times \infty$.

3. For $1 < \varepsilon < \infty$, $\alpha = -(1/\varepsilon)\, e^{-(1/\varepsilon)} \in (-1/e, 0)$, therefore $W(\alpha)$ is double-valued and consequently Equation (C.6) admits two solutions: $y_0^* = W_0(\alpha) = -1/\varepsilon > -1$ and $y_{-1}^* = W_{-1}(\alpha) < -1$. As for case 2, a_0^* and a_{-1}^* denote the acceptance probability of $p_0^*(u)$ and $p_0^*(u)$, respectively, for the given level of utility u. From (C.4) and (C.5), we get

$$\begin{aligned} p_0^*(u) &= \left[(Au^\mu) / (-y_0^* - 1/\varepsilon)\right]^{1/\varepsilon} \\ &= \left[(Au^\mu) / (1/\varepsilon - 1/\varepsilon)\right]^{1/\varepsilon} \\ &= \infty \end{aligned}$$

as $A > 0$, $u > 0$, $\mu < \infty$ and $1 < \varepsilon < \infty$, while

$$a_0^* = 1 - e^{y_0^* + 1/\varepsilon} = 1 - e^{-1/\varepsilon + 1/\varepsilon} = 0,$$

hence $a_0^* p_0^* = 0 \times \infty$.

Concerning the solution $y_{-1}^* = W_{-1}(\alpha)$,

$$\begin{aligned} p_{-1}^*(u) &= \left[(Au^\mu) / \left(-y_{-1}^* - 1/\varepsilon\right)\right]^{1/\varepsilon} \\ &= \left[(Au^\mu) / (-W_{-1}(\alpha) - 1/\varepsilon)\right]^{1/\varepsilon} \in (0, \infty), \end{aligned}$$

as $0 < A < \infty$, $0 < u < \infty$, $\mu < \infty$, $0 < -W_{-1}(\alpha) - 1/\varepsilon < \infty$ (due[3] to $-\infty <$

[3] Recall that here $\alpha < 0$ and since $W_{-1}(\alpha)$ is strictly decreasing in α then $W_{-1}(\alpha) > \lim_{\alpha \to 0^-} W_{-1}(\alpha) = -\infty$.

$W_{-1}(\alpha) < -1$ and $1 < \varepsilon < \infty$) and $\varepsilon > 0$. In turn, as $-\infty < W_{-1}(\alpha) + 1/\varepsilon < 0$ and, as a result, $0 < e^{W_{-1}(\alpha)+1/\varepsilon} < 1$,

$$a^*_{-1} = 1 - e^{y^*_{-1}+1/\varepsilon} = 1 - e^{W_{-1}(\alpha)+1/\varepsilon} \in (0,1).$$

Then, due to $0 < p^*_{-1}(u) < \infty$ and $0 < a^*_{-1} < 1$, $0 < a^*_{-1}p^*_{-1}(u) < \infty$.

$$\begin{aligned} p^*(u) &= \left(\frac{Au^\mu}{-W_{-1}\left(-\frac{1}{\varepsilon}e^{-\frac{1}{\varepsilon}}\right) - \frac{1}{\varepsilon}} \right)^{\frac{1}{\varepsilon}} \\ &= \overline{p} \left(\frac{\log(\overline{q})}{W_{-1}\left(-\frac{1}{\varepsilon}e^{-\frac{1}{\varepsilon}}\right) + \frac{1}{\varepsilon}} \right)^{\frac{1}{\varepsilon}} \left(\frac{u}{\overline{u}}\right)^{\frac{\mu}{\varepsilon}} \in (0,\infty), \end{aligned}$$

which is accepted with a probability

$$a(p^*(u),u) = 1 - e^{W_{-1}\left(-\frac{1}{\varepsilon}e^{-\frac{1}{\varepsilon}}\right)+\frac{1}{\varepsilon}} \in (0,1),$$

hence the optimal accepted fee $a(p^*(u),u)p^*(u) \in (0,\infty)$.

APPENDIX D EXAMPLES OF THE SPS' PAYOFF FUNCTION

The payoff of an SP (defined in Equation (6.21)) is the difference between its revenue (see Equations (6.7) and (6.1)) and its cost for a given amount of acquired cell capacity at a given cell capacity unit price. We drop the SP subscript v from the aforementioned formulas and write in extensive form the SP payoff as function of the amount of acquired cell capacity x at a given cell capacity unit price P as follows:

$$g(x) = \begin{cases} -Px, & \text{if } 0 \le x \le \widetilde{N}\underline{\mathcal{X}}, \\ Na^*\overline{p}\left[\frac{\log(1-\overline{a})}{\log(1-a^*)}\right]^{1/\varepsilon}\left(\frac{1}{\overline{u}}\right)^{\mu/\varepsilon}\left[\frac{\left(\frac{x/\widetilde{N}-\underline{\mathcal{X}}}{\mathcal{X}-\underline{\mathcal{X}}}\right)^{\xi}}{1+\left(\frac{x/\widetilde{N}-\underline{\mathcal{X}}}{\mathcal{X}-\underline{\mathcal{X}}}\right)^{\xi}}\right]^{\mu/\varepsilon} - Px, \\ \qquad \text{if } x > \widetilde{N}\underline{\mathcal{X}}. \end{cases} \tag{D.1}$$

Let $\mu = \varepsilon = 2$, $Na^*\overline{p}\left(\frac{\log(1-\overline{a})}{\log(1-a^*)}\right)^{1/\varepsilon} = 1$, $\overline{u} = 1$, $\widetilde{N} = 1$, $\underline{\mathcal{X}} = 1$ and $\mathcal{X} = 10$. For these values of parameters, in Figures D.1 and D.2 we plot $g(x)$ in terms of x for all combinations of two different values for each of the remaining parameters: i.e., for values 2 and 20 for ξ (the utility elasticity) and for values 0.03 and 0.1 for P (the cell capacity unit price). Values 2 and 20 are the minimum and maximum values considered for ξ in this work (see Section 6.4.3) and also in literature [208]. As for P, given the considered values for all other parameters, values 0.03 and 0.1 are simply two values that allow to illustrate the two different cases concerning the calculation of the minimum and maximum amount of cell capacity requested by the SP for a given cell capacity unit price P, i.e., $\underline{X}(P)$ and $\overline{X}(P)$, where

$$\overline{X}(P) = \begin{cases} 0, & \text{if } g(x) \le 0,\ \forall x \ge \widetilde{N}\underline{\mathcal{X}}, \\ \underset{x \ge \widetilde{N}\underline{\mathcal{X}}}{\operatorname{argmax}}\ g(x), & \text{if } \exists\, x > \widetilde{N}\underline{\mathcal{X}} \mid g(x) > 0, \end{cases}$$

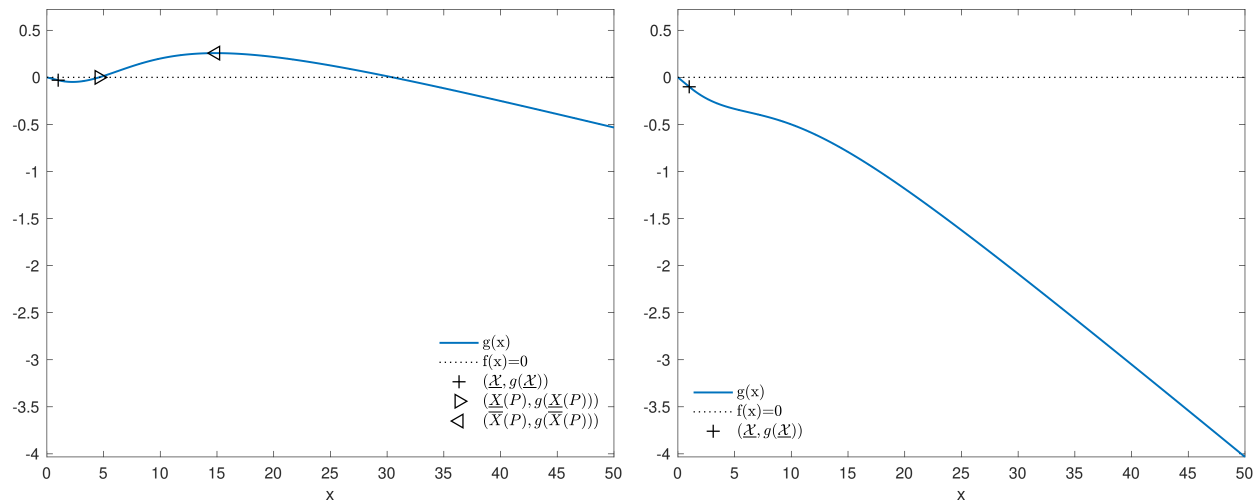
$$\underline{X}(P) = \begin{cases} 0, & \text{if } \overline{X}(P) = 0, \\ x \in [\widetilde{N}\underline{\mathcal{X}}, \overline{X}(P)] \mid g(x) = 0, & \text{if } \overline{X}(P) > 0. \end{cases}$$

Notice that for $0 < x \le \widetilde{N}\underline{\mathcal{X}}$, one has $g(x) < 0$ as in this range $g(x) = -Px$ (see Equation (D.1), Figures D.1 and D.2). Hence, we look for $\underline{X}(P)$ and $\overline{X}(P)$ for $x > \widetilde{N}\underline{\mathcal{X}}$. From Figures D.1b and D.2b, we can see that for both values of ξ, when $P = 0.1$, $g(x) < 0$, $\forall x \ge \widetilde{N}\underline{\mathcal{X}}$, as a result, we force $\underline{X}(P) = \overline{X}(P) = 0$. Instead, from Figures D.1a and D.2a we can see that, for both values of ξ when $P = 0.03$, one has:

(1) there exists $x > \widetilde{N}\underline{\mathcal{X}}$ such that $g(x) > 0$, therefore, $\overline{X}(P) > \widetilde{N}\underline{\mathcal{X}} > 0$ and $g(\overline{X}(P)) > 0$;

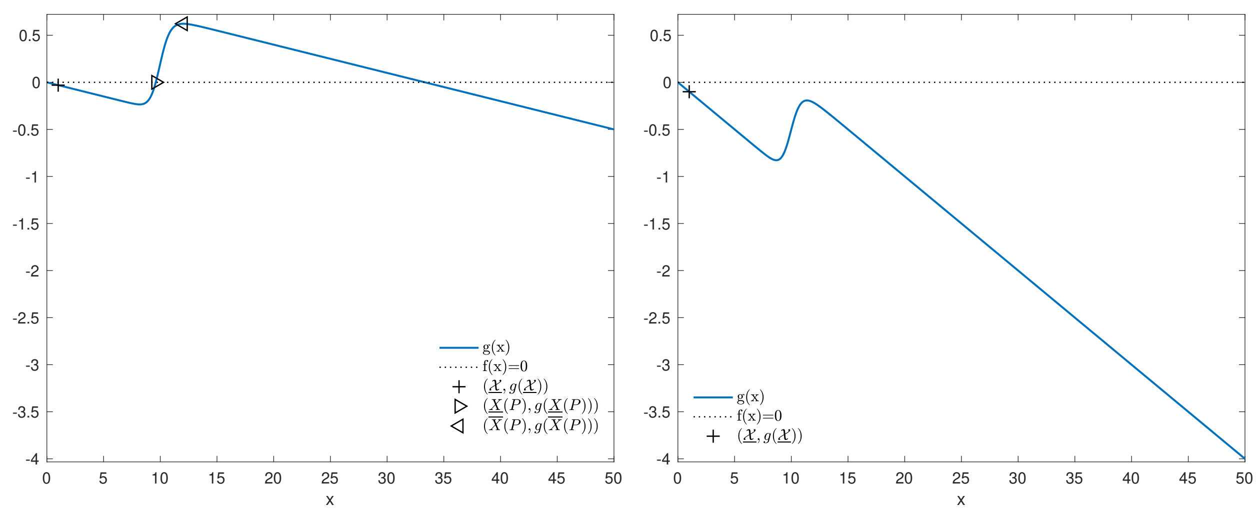
(2) $g(x)$ has a unique zero in $[\widetilde{N}\underline{\mathcal{X}}, \overline{X}(P)]$, hence there is a unique value for $\underline{X}(P)$;

(3) $\overline{X}(P) > \underline{X}(P) > \widetilde{N}\underline{\mathcal{X}} > 0$, as $g(\widetilde{N}\underline{\mathcal{X}}) < 0$ whereas $g(\overline{X}(P)) > 0$.



(a) $P = 0.03$ (b) $P = 0.1$

Figure D.1 SP payoff function examples for utility elasticity $\xi = 2$.



(a) $P = 0.03$ (b) $P = 0.1$

Figure D.2 SP payoff function examples for utility elasticity $\xi = 20$.

APPENDIX E APPROXIMATED EQUILIBRIA

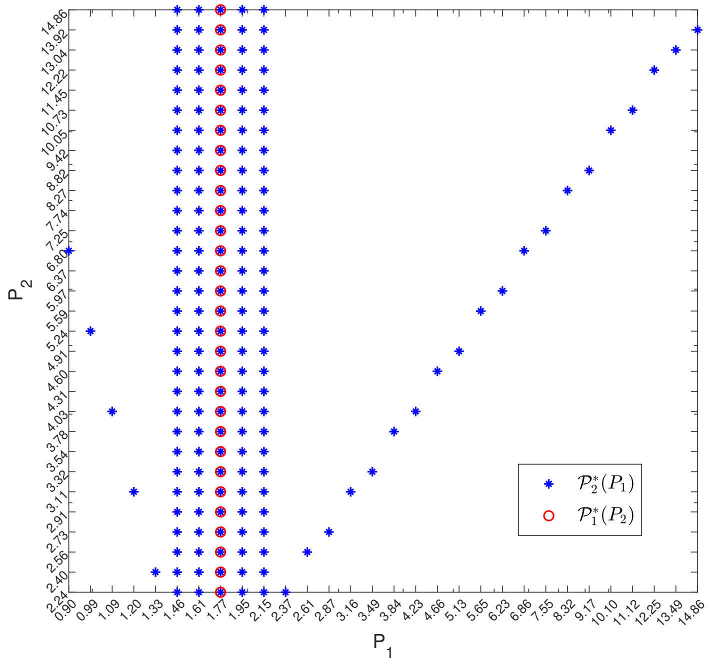
For the InPs' game $\mathcal{G}^{\mathcal{K}}$, since there are two players (InPs) for each considered instance, the existence and multiplicity of its NE in pure strategies can be also depicted graphically through the InPs' best response functions. The best response $\mathcal{P}_k^*$ of any InP k is the set of strategies

$$\begin{aligned}\mathcal{P}_k^*(\boldsymbol{P}_{-k}) = \{P_k \in \mathcal{P}_k \mid \\ G_k([P_k, \boldsymbol{P}_{-k}]) \geq \max_{P'_k \in \mathcal{P}_k} G_k([P'_k, \boldsymbol{P}_{-k}]) - \Delta\}, \\ \forall\, \boldsymbol{P}_{-k} \in \textstyle\prod_{j \in \mathcal{K} \setminus \{k\}} \mathcal{P}_j,\end{aligned} \tag{E.1}$$

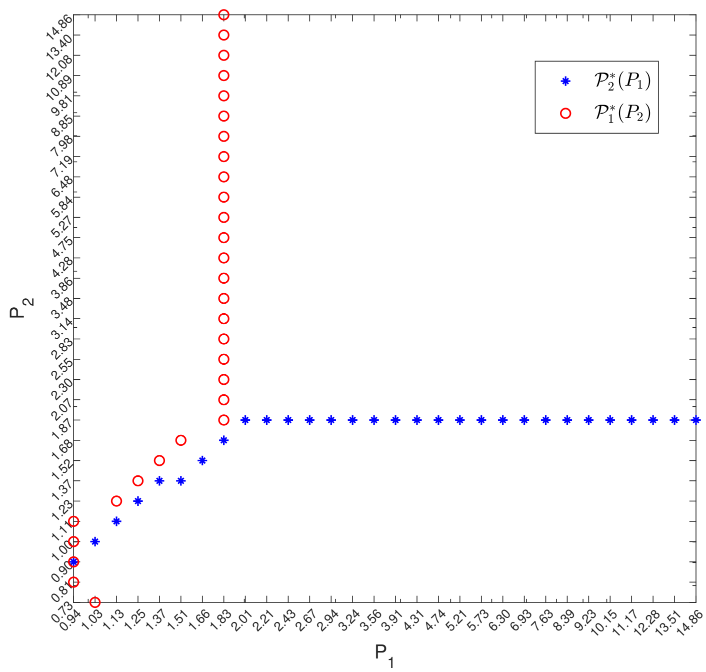
that is, $\mathcal{P}_k^*(\boldsymbol{P}_{-k})$ is the set of all InP k unit prices that maximize its payoff within a margin[1] Δ (see Section 6.4.5) given $\boldsymbol{P}_{-k}$, i.e., given the unit prices offered by all other InPs but k.

For two illustrative instances, A5 and B9, in Figures E.1a and E.1b, we have plotted in blue ($*$ markers) the best response function of InP 2, that is the payoff-maximizing unit price(s) for InP 2 for each possible unit price that InP 1 can offer (i.e., $\mathcal{P}_2^*(P_1)$ for any $P_1 \in \mathcal{P}_1 = \{\underline{P}_1, \ldots, \overline{P}\}$) and in red ($\circ$ markers) the best response function of InP 1, that is the payoff-maximizing unit price(s) for InP 1 for each possible unit price that InP 2 can offer (i.e., $\mathcal{P}_1^*(P_2)$ for any $P_2 \in \mathcal{P}_2 = \{\underline{P}_2, \ldots, \overline{P}\}$). The NE InP unit price profile(s) $\breve{\boldsymbol{P}}$ of the InPs' game for A5 and B9 are then represented by all intersections between $\mathcal{P}_1^*$ and $\mathcal{P}_2^*$ in Figures E.1a and E.1b, respectively. For instance A5 (see Figure E.1a), as also reported in Table 6.7, there are $|\mathcal{P}_2| = 30$ NE such that $\breve{P}_1 = 1.77$ EUR/Mbps/month and $\breve{P}_2 \in \mathcal{P}_2 = \{\underline{P}_2 = 2.24, \ldots, \overline{P} = 14.86\}$ EUR/Mbps/month. In fact, as previously mentioned, all these NE are equivalent for both InPs in terms of achieved payoffs ($\breve{G}_1 = 1105.80$, $\breve{G}_2 = 0$) EUR/month: in each such NE, i.e., for each such $\breve{\boldsymbol{P}}$, InP 1 offers the unit price $\breve{P}_1 = 1.77$ EUR/Mbps/month (which is strictly lower than $\underline{P}_2 = 2.24$ EUR/Mbps/month, i.e., the lowest unit price that InP 2 can offer) and is selected by all four SPs in the unique NE of the respective $\mathcal{G}^{\mathcal{V}}(\breve{\boldsymbol{P}})$ (see Table 6.8), while InP 2, not being selected by any SP and therefore not selling any capacity even when it offers $\breve{P}_2 = \underline{P}_2$, is indifferent between all unit prices it can offer, each proving it with zero payoff (i.e., for InP 2 any unit price $P_2 \in \mathcal{P}_2$ is a best response to $\breve{P}_1$). Instead, for instance B9 (see Figure E.1b) there is a single intersection between $\mathcal{P}_1^*$

[1] The absolute payoff margin $\Delta = 10^{-6}$ EUR introduced in the NE definition (see Equation (6.31)) to deal with numerical issues brought about by solver tolerances (as explained in Section 6.4.5) has been applied to best response definition accordingly.



(a) Instance A5



(b) Instance B9

Figure E.1 InPs' best response functions for $\mathcal{G}^{\mathcal{K}}$ — example of multiple NE (a) and unique NE (b).

and $\mathcal{P}_2^*$, therefore a unique NE for $\mathcal{G}^{\mathcal{K}}$, $\breve{\boldsymbol{P}} = (\breve{P}_1 = 0.94, \breve{P}_2 = 0.90)$ EUR/Mbps/month, as reported in Table 6.13 as well; at the unique NE of the respective $\mathcal{G}^{\mathcal{V}}(\breve{\boldsymbol{P}})$, InP 1 is selected only by SP 4 to which it sells capacity at its minimum unit price $\breve{P}_1 = \underline{P}_1 = 0.94$ EUR/Mbps/month, i.e., at a unit price equal to its unit cost, while all the other SPs (1, 2 and 3) select the more cost-efficient InP (2) which at the equilibrium offers a unit price $\underline{P}_2 < \breve{P}_2 < \underline{P}_1$ (see Tables 6.13 and 6.14).

As anticipated in Section 6.4.5, for instances B4 and B5, there is no NE in pure strategies for the InPs' game $\mathcal{G}^{\mathcal{K}}$ resulting from the initial discrete InP unit price strategy sets $\mathcal{P}_k$ (each made up of 30 logarithmically-spaced discrete values in $[\underline{P}_k, \overline{P}]$), although there is at least one NE in pure strategy for each SPs' game $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ for any $\boldsymbol{P} \in \mathcal{P}$. For this setting, the absence of NE for $\mathcal{G}^{\mathcal{K}}$ for B4 and B5 can be witnessed in Figures E.2a and E.2b, respectively, where the InPs' best response functions, i.e., $\mathcal{P}_1^*$ and $\mathcal{P}_2^*$, do not intersect.

If we were to linearly interpolate $\mathcal{P}_1^*$ and $\mathcal{P}_2^*$ depicted in Figure E.2a and determine the intersection of their interpolations, then, for instance B4, we would expect the NE InP unit prices to be within the following ranges: $\breve{P}_1 \in [\underline{P}_1 = 1.23, 1.34]$ EUR/Mbps/month and $\breve{P}_2 \in [\underline{P}_2 = 1.18, 1.29]$ EUR/Mbps/month. Analogously for B5 (see Figure E.2b), we would expect $\breve{P}_1 \in [1.03, 1.13]$ EUR/Mbps/month and $\breve{P}_2 \in [\underline{P}_2 = 0.90, 0.99]$ EUR/Mbps/month. On this basis, for each InP $k \in \mathcal{K}$, we set up[2] an alternative unit price strategy set $\mathcal{P}_k$ made up of 60 discrete values in $[\underline{P}_k, \overline{P}]$ such that the vast majority of these values lie in the respective aforementioned range where we expect the NE unit price to be for InP k. However, even for the MFSG resulting from these alternative discrete InP unit price strategy sets, for both B4 and B5, there still is no NE in pure strategies for $\mathcal{G}^{\mathcal{K}}$ although, there is at least one NE in pure strategies for $\mathcal{G}^{\mathcal{K}}(\boldsymbol{P})$ for any $\boldsymbol{P} \in \mathcal{P}$. In absence of an NE for $\mathcal{G}^{\mathcal{K}}$, we consider as a solution for $\mathcal{G}^{\mathcal{K}}$ the InP unit price profile(s) denoted by $\boldsymbol{P}^\diamond$ and determined as

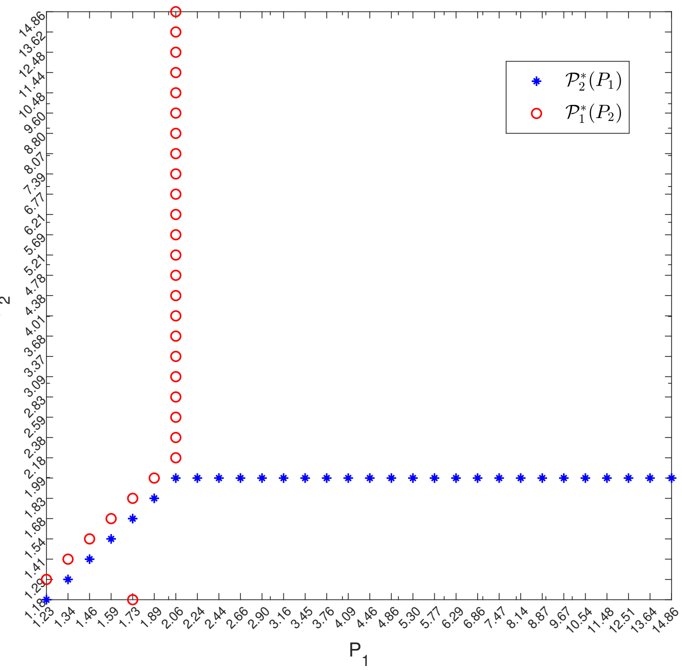
$$\boldsymbol{P}^\diamond = \underset{\boldsymbol{P}=[P_k, \boldsymbol{P}_{-k}] \in \mathcal{P}}{\operatorname{argmin}} \left[\max_{k \in \mathcal{K}} \ \delta_k([P_k, \boldsymbol{P}_{-k}]) \right], \tag{E.2}$$

where

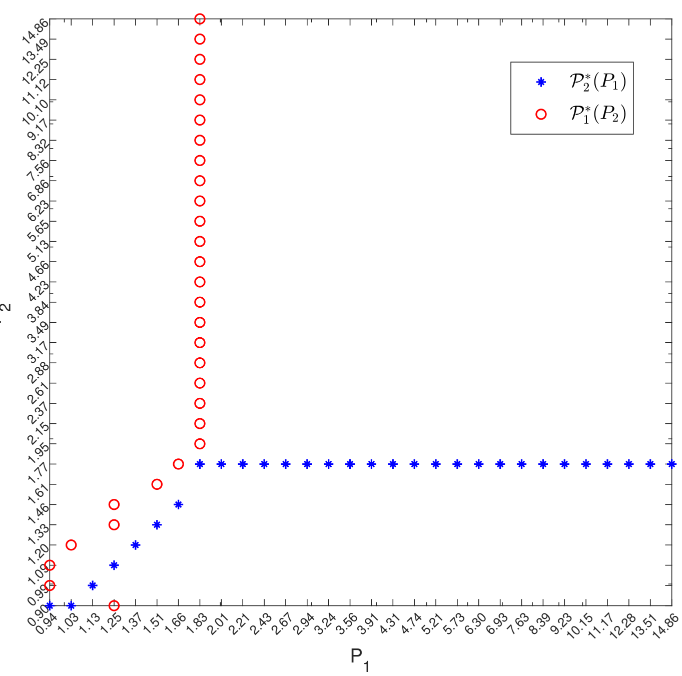
$$\delta_k([P_k, \boldsymbol{P}_{-k}]) = \frac{\max\limits_{P'_k \in \mathcal{P}_k} G_k([P'_k, \boldsymbol{P}_{-k}]) - G_k([P_k, \boldsymbol{P}_{-k}])}{\max\limits_{P'_k \in \mathcal{P}_k} G_k([P'_k, \boldsymbol{P}_{-k}])}$$

is the relative difference between the payoff of InP k from $\boldsymbol{P} = [P_k, \boldsymbol{P}_{-k}]$ and the maximum payoff that k can obtain by unilaterally deviating from $\boldsymbol{P}$. In Equation (E.2), we set $\boldsymbol{P}^\diamond$ equal

[2]The alternative discrete InP unit price strategy sets were set up as follows. For instance B4, $\mathcal{P}_1$ consists of: 50 linearly-spaced values in $[\underline{P}_1 = 1.23, 1.34]$, 5 linearly-spaced values in $[1.35, 2.06]$ and 5 linearly-spaced values in $[2.07, \overline{P} = 14.86]$ whereas $\mathcal{P}_2$ consists of: 50 linearly-spaced in values in $[\underline{P}_2 = 1.18, 1.29]$, 5 linearly-spaced values in $[1.3, 2.18]$ and 5 linearly-spaced values in $[2.19, \overline{P} = 14.86]$. For instance B5, $\mathcal{P}_1$ consists of: $\underline{P}_1 = 0.94$, $(\underline{P}_1 + 1.03)/2$, 50 linearly-spaced values in $[1.03, 1.13]$, 3 linearly-spaced values in $[1.14, 1.83]$ and 5 linearly-spaced values in $[1.84, \overline{P} = 14.86]$ whereas $\mathcal{P}_2$ consists of: 50 linearly-spaced values in $[\underline{P}_2 = 0.90, 0.99]$, 5 linearly-spaced values in $[1, 1.95]$ and 5 linearly-spaced values in $[1.96, \overline{P} = 14.86]$.



(a) Instance B4



(b) Instance B5

Figure E.2 InP best response functions for $\mathcal{G}^{\mathcal{K}}$ — initial, logarithmically-spaced sets $\mathcal{P}_k$ for any $k \in \mathcal{K}$.

to the InP unit price profile(s) which provide the minimum value for $\max_{k \in \mathcal{K}} \delta_k([P_k, \boldsymbol{P}_{-k}])$[3].

[3] If $\mathcal{G}^{\mathcal{K}}$ had a NE $\breve{\boldsymbol{P}}$, then $\boldsymbol{P}^{\diamond} = \breve{\boldsymbol{P}}$ and $\min\limits_{\boldsymbol{P} \in \mathcal{P}} \max\limits_{k \in \mathcal{K}} \delta_k(\boldsymbol{P}) = 0$.

For both B4 and B5, we have calculated $\boldsymbol{P}^{\diamond}$ for the MFSG resulting from the alternative $\mathcal{P}_k$ described above (i.e., for the $\mathcal{P}_k$, $\forall k \in \mathcal{K}$ made up of 60 discrete values in $[\underline{P}_k, \overline{P}]$ with the vast majority of these values where we expect the NE to be by looking at Figures E.2a and E.2b, respectively). It results that for both B4 and B5 there is a unique $\boldsymbol{P}^{\diamond}$. For B4, $\boldsymbol{P}^{\diamond} = (P_1^{\diamond} = 1.23, P_2^{\diamond} = 1.22)$ EUR/Mbps/month with $\max_{k \in \mathcal{K}} \delta_k(\boldsymbol{P}^{\diamond}) = 0.53$, whereas for B5, $\boldsymbol{P}^{\diamond} = (P_1^{\diamond} = 1.09, P_2^{\diamond} = 0.94)$ EUR/Mbps/month with $\max_{k \in \mathcal{K}} \delta_k(\boldsymbol{P}^{\diamond}) = 3.89$, hence we deemed these $\boldsymbol{P}^{\diamond}$ as reasonable solutions for $\mathcal{G}^{\mathcal{K}}$. Notice also that, although these $\boldsymbol{P}^{\diamond}$ are not NE of $\mathcal{G}^{\mathcal{K}}$ for B4 and B5, it turns out that for B4, $P_1^{\diamond} = 1.23 \in [\underline{P}_1 = 1.23, 1.34]$ EUR/Mbps/month and $P_2^{\diamond} = 1.22 \in [\underline{P}_2 = 1.18, 1.29]$ EUR/Mbps/month, and for B5, $P_1^{\diamond} = 1.09 \in [1.03, 1.13]$ EUR/Mbps/month and $P_2^{\diamond} = 0.94 \in [\underline{P}_2 = 0.90, 0.99]$ EUR/Mbps/month, which are the InP unit price ranges where we would expect the NE of $\mathcal{G}^{\mathcal{K}}$ to be by looking at the best response functions of $\mathcal{G}^{\mathcal{K}}$ for the initial MFSG illustrated in Figures E.2a and E.2b, respectively.

For both B4 and B5, the respective values of $P_1^{\diamond}/P_2^{\diamond}$ are reported in Table 6.11 under $\breve{P}_1/\breve{P}_2$, whereas the outcomes of the respective SPs' game $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P}^{\diamond})$ in Table 6.12. In particular, for B4, $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P}^{\diamond})$ turns out to have two distinct NE in pure strategies denoted by **(i)** and **(ii)** in Tables 6.11 and 6.12 and analyzed in Section 6.5.3.