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Millimeter-wave Hybrid Precoder Design with a Fast Iterative Beam Split and Detection Algorithm

Yifang Nie, Fangwei Li, Allan Wainaina Mbugua, and Wei Fan

***Abstract*—The hybrid analog and digital structure of large-scale antenna array offers a good trade-off between cost and flexibility, which is a promising solution for millimeter-wave (mm-wave) system. This letter proposes a novel hybrid precoder (HP) design with an iterative beam split and detection algorithm, which works well for sparse mm-wave channels. The proposed strategy can extract the dominant propagation paths by iterative trainings, where the number of dominant paths up to that of radio frequency (RF) chains can be detected. The effectiveness and time overhead parameterized by the number of iterations are investigated. The experimental results demonstrated that we can approach optimal HP design with the proposed algorithm with only 8 iterations for measured channels, which significantly reduced the complexity compared to exhaustive search method.**

***Index Terms*—Hybrid precoding, millimeter-wave communications, multipath propagation, beamforming.**

I. Introduction

MILLIMETER-WAVE (mm-wave) communication is seen as a key technology to enable high data rate in future communication systems due to the large available bandwidth. However, the higher free space loss and larger system bandwidth at mm-wave bands compared to sub-6 GHz bands would result in a lower signal-to-noise ratio (SNR) [1]. This has motivated the study of large-scale antenna arrays for achieving highly directional beamforming. The full-digital beamforming structure which requires one radio frequency (RF) chain per antenna element is infeasible for large-scale arrays due to the high cost and power consumption. To reduce the system complexity, the full-analog architecture where only one RF chain is employed is another alternative. However, this system can only support a single stream, therefore providing limited flexibility. The hybrid structure [2], which has less RF chains than antennas, is seen as the most promising solution at mm-wave bands, since it offers a good trade-off between flexibility and cost to support multi-stream transmission [3].

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Many researchers have made efforts to design precoders for hybrid structure in recent years. In [3], [4], [5], the hybrid precoder (HP) designs which exploited the sparse nature of the mm-wave channel [6] were proposed to approach the optimal unconstrained precoder. In [7], [8], beam-training solutions were investigated to design HP by tracking the multipath component (MPC)s. Although these strategies have achieved good results, the original channel state information (CSI) between the transmitter (Tx) and receiver (Rx) antenna radiating elements was assumed to be perfectly known. However, it is intractable for the initial HP design to obtain the original CSI, since the control links have not been completely established. For the initial access, the expected HP design should be simple, fast, and effective. Different from above mentioned schemes, some iterative search (IS) approaches [9], [10] are presented to satisfy the expected needs. In [9], a ping-pong beamforming (PPB) method was fast and suboptimal, since it only converges to the most dominant path while other paths are not involved. The work in [10] which exploited different frequencies to distinguish different angle of arrival (AoA)s was fast and effective, yet the complicated modulations of multiple frequencies are required.

In this letter, a simple and fast iterative training method is proposed for the initial HP design of mm-wave communication systems. The basic idea is to approach the optimal HP design based on the beam-split and multipath detection algorithm. This design, unlike most state-of-the-art methods which necessitate a prior knowledge of the original CSI, is only based on the effective channel between the Tx and Rx antenna ports. Furthermore, the complexity and time overhead are improved, since multiple beams with the same frequency are formed to scan the multipath simultaneously. Finally, the proposed algorithm is shown to be simpler and faster than existing state-of-art algorithms via simulations and experiments.

Notation: $(\cdot)^*$, $(\cdot)^T$, $\mathbb{E}[\cdot]$, $(\cdot)^H$, $\angle(\cdot)$, $||\cdot||_F$, $\mathbf{0}_{M\times N}$ and $\mathbf{I}_K$ denote the complex conjugate, transpose, expectation, Hermitian, phase extraction operator, Frobenius norm, $M\times N$ zero matrix and $K\times K$ identity matrix, respectively.

II. System Model

Assume that a base station (BS) and a mobile station (MS) are both equipped with a uniform linear array (ULA) with an inter-element spacing ℓ in a full-connected hybrid beamformer structure [5], as illustrated in Fig. 1. The BS composed of M antennas and M_{RF} RF chains communicates K streams to the MS with N antennas and N_{RF} RF chains. Note that we have $M_{RF}\leq M$, $N_{RF}\leq N$ and $K\leq \min(N_{RF}, M_{RF})$. In the

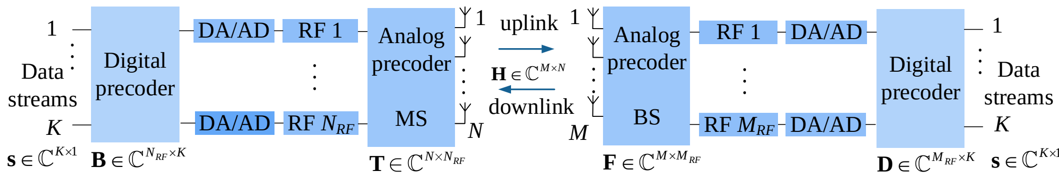

Fig. 1. Simplified block diagram of mm-wave hybrid precoding system.

uplink system, the transmitted signal vector of the MS $\mathbf{s} \in \mathbb{C}^{K\times 1}$ with $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_K$ is precoded by its digital precoder (DP) $\mathbf{B} \in \mathbb{C}^{N_{RF}\times K}$ and analog precoder (AP) $\mathbf{T} \in \mathbb{C}^{N\times N_{RF}}$, propagated through the channel $\mathbf{H} \in \mathbb{C}^{M\times N}$, and decoded by the DP $\mathbf{D} \in \mathbb{C}^{M_{RF}\times K}$ and AP $\mathbf{F} \in \mathbb{C}^{M\times M_{RF}}$ of the BS. Furthermore, we assume that there are Q discrete planar propagation paths between the BS and MS. The original time-invariant narrowband channel transfer function is written as

$$\mathbf{H} = \sqrt{\frac{MN}{Q}} \sum_{q=1}^{Q} \alpha_q \mathbf{a}_T(\phi_q^t) \mathbf{a}_R^T(\phi_q^r) \tag{1}$$

where α_q, ϕ_q^t, ϕ_q^r, $\mathbf{a}_T(\phi_q^t)$ and $\mathbf{a}_R(\phi_q^r)$ are the complex gain, azimuth angle of departure (AoD), azimuth AoA and array response vectors of Tx and Rx for the q^{th} path, respectively. The singular value decomposition (SVD) of the channel can be $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$, where $\mathbf{\Sigma}$ is a diagonal matrix with positive ordered entries, i.e. $\mu_1 \geq ... \geq \mu_Q \geq 0$, and $\mathbf{U} \in \mathbb{C}^{M\times M}$ and $\mathbf{V} \in \mathbb{C}^{N\times N}$ are the left and right singular matrices, respectively. Further, the channel can be denoted as [4]

$$\mathbf{H} = \sum_{q=1}^{Q} \mu_q \mathbf{u}_q \mathbf{v}_q^T \tag{2}$$

where $\mathbf{v}_q \in \mathbb{C}^{N\times 1}$ and $\mathbf{u}_q \in \mathbb{C}^{M\times 1}$ are the columns of $\mathbf{V}$ and $\mathbf{U}$, respectively. With $\beta_k = \mu_k / \sum_{k=1}^{K} \mu_k$, the first K columns of $\mathbf{U}$ and $\mathbf{V}$ are defined as $\overline{\mathbf{U}} = [\beta_1 \mathbf{u}_1, ..., \beta_K \mathbf{u}_K]$ and $\overline{\mathbf{V}} = [\beta_1 \mathbf{v}_1, ..., \beta_K \mathbf{v}_K]$, respectively.

A. Digital Precoding

For the full-digital structure, the transmitted signal for K streams at the MS is weighted by a set of precoding weights $\mathbf{G} \in \mathbb{C}^{N\times K}$, propagated through the channel $\mathbf{H}$, and then recovered at the BS by a set of combining weights $\mathbf{W} \in \mathbb{C}^{M\times K}$. The basic idea is that the precoding and combining weights should be designed to make the product of $\mathbf{W}^T\mathbf{H}\mathbf{G}$ a diagonal matrix so each data stream can be recovered independently. This can be simply achieved according to the SVD of channel, i.e. by setting $\mathbf{G} = \overline{\mathbf{V}}^*$ and $\mathbf{W} = \overline{\mathbf{U}}^*$.

B. Hybrid Precoding

For the hybrid structure shown in Fig 1, the HP is composed of an AP and a DP. The received signal vector after decoding at the BS is indicated as

$$\mathbf{y} = \mathbf{D}^T\mathbf{F}^T(\mathbf{HTBPs} + \mathbf{n}) \tag{3}$$

where $\mathbf{n} \in \mathbb{C}^{M\times 1}$ is a complex Gaussian noise vector with 0 mean and σ^2 variance for each vector element, and $\mathbf{P} = \sqrt{P/K}\mathbf{I}_K$ denotes the transmitted power, respectively.

C. Reference HP Design

1) Optimal Precoder Design: The goal of HP design is to eliminate the inter-stream interference, under a normalized power constraint of $||\mathbf{TB}||_F^2 = ||\mathbf{FD}||_F^2 = K$. Basically, the HP should be designed to make the product $\mathbf{D}^T\mathbf{F}^T\mathbf{HTB}$ approximate a diagonal matrix so that each stream can be recovered at each channel singular mode. Theoretically, to attain an optimal HP as in [3]-[5], the design should be done to ensure $\mathbf{FD} = \overline{\mathbf{U}}^*$ and $\mathbf{TB} = \overline{\mathbf{V}}^*$.

2) Sparse Precoder Design: The work in [3] presented a near-optimal HP design, which minimizes the gap between the optimal HP weights and the combined weights of the APs and DPs. The optimization at the BS is performed as [3]

$$\begin{aligned} &\arg\min ||\overline{\mathbf{U}}^* - \mathbf{FD}||_F \\ &\text{s.t. } \mathbf{F} \in \mathscr{F}, ||\mathbf{FD}||_F^2 = K \end{aligned} \tag{4}$$

The basic idea is to match the channel by selecting the N_{RF} best steering vectors from a predefined codebook $\mathscr{F}$ [9] and form the linear combination of the selected vectors using the precoder $\mathbf{D}$. However, a prior knowledge of the original CSI and the optimal HP weights are required.

3) Beam-steering Precoder Design: In fact, without a prior knowledge of the CSI, it is intractable to select the expected beam vectors and DP weights. In this case, the traditional beam steering [11] is commonly used to steer the beams with predefined codewords, and the codeword corresponding to the strongest beam direction is recorded. The HP of beam steering can be obtained as $\mathbf{FD} = \mathbf{u}_1^*$, which results in sub-optimal performance, since only one path corresponding to the dominant singular mode is captured.

III. HP Design with Beam Split

In this section, a novel precoder design based on iterative training is described. For the sake of simplicity, we limit our discussion for a single-input multiple-output (SIMO) system i.e. with precoding only performed at the BS. The HP design at the MS can be realized by an analogous design.

A. AP Design

First, an iterative training based on beam split and detection strategy is discussed to extract the dominant propagation paths and then utilized to determine the AP of the BS. The beam training follows a coarse and refined search procedure with a multi-resolution codebook, which is executed in 2 steps explained as follows. In step 1, the MS sends a signal to the BS. The BS then receives the signal and records the normalized gain per RF chain. Basically, the BS first forms M_{RF} training beams with a codebook resolution of $2\pi/M_{RF}$, so that the space can be fully covered by steering the M_{RF} beams initiated by first M_{RF} elements out of M antennas to the angles $\arccos(\frac{2(m_{RF}-1)}{M_{RF}} - 1)$, $m_{RF} \in [1, M_{RF}]$. The full space is decomposed into M_{RF} wide angle regions formed by

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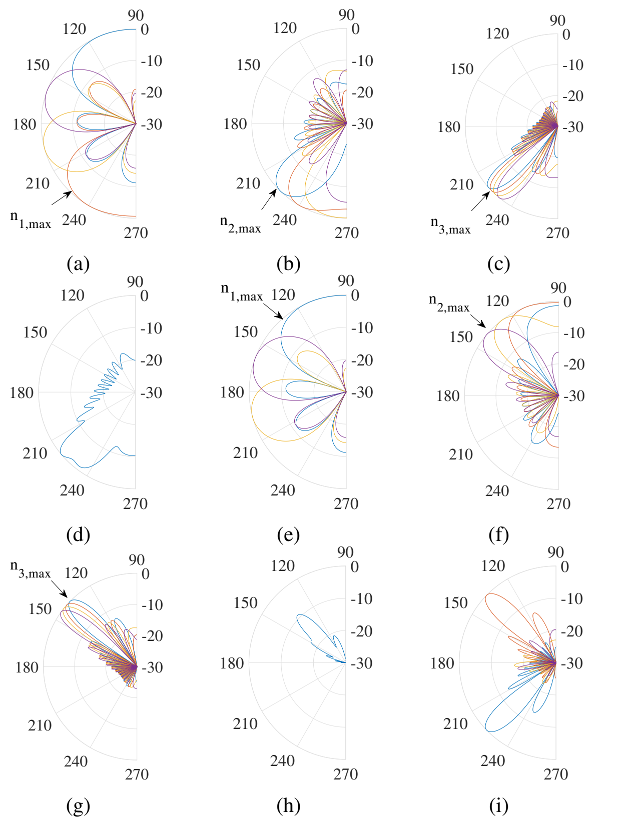

Fig. 2. An illustration of beam split and extraction procedure. (a) first iteration for path 1, (b) second iteration for path 1, (c) third iteration for path 1, (d) pattern for path 1, (e) first iteration of path 2, (f) second iteration for path 2, (g) third iteration of path 2, (h) pattern for path 2, and (i) analog pattern.

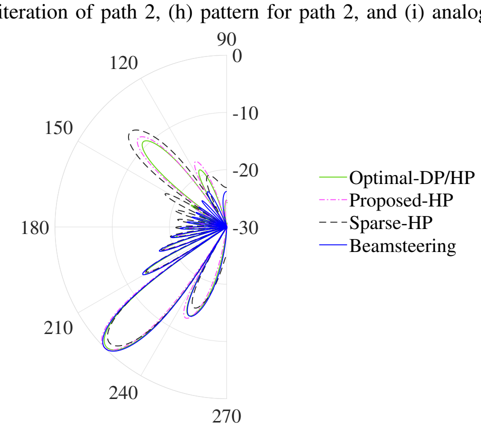

Fig. 3. Combined pattern with the same simulation scenario of Fig. 2.

M_{RF} training beams. The setting is chosen to keep a balance between beam-steering capability and coverage of full space.

Due to the one-to-one relationship between per RF chain and training beam, the $n_{1,max}$ beam with the maximal gain is determined and split further. In step 2, beam split is achieved by steering another M_{RF} beams formed by first $2M_{RF}$ elements with a codebook resolution of $2\pi/M_{RF}^2$ to the angles $\arccos(\frac{2(m_{RF}-1)}{M_{RF}^2}-1)$, $m_{RF}\in[(n_{1,max}-1)M_{RF}, n_{1,max}M_{RF}]$. Further, as more elements are used, the codebook resolution becomes higher and the formed beams become thinner. The basic idea is that we can approach the actual angle of the most dominant path during the iterative training. Once the most dominant path is found, we continue the same search by beam split for the other $M_{RF}-1$ wide angle regions covering the angles $\arccos(\frac{2(m_{RF}-1)}{M_{RF}}-1)$, $m_{RF}\neq n_{1,max}\in[1,M_{RF}]$, (i.e. the wide angle region covering the most dominant path is excluded). Note that each reduction of the trained angle region is caused by deleting the wide beam with the maximal gain indicated by $n_{1,max}$.The whole process stops until all wide angle regions are deleted.

An example is given in Fig. 2 to illustrate the procedure. The propagation channel consists of two paths with impinging angle 135^o and 225^o, and power $-10\,$dB and $0\,$dB, respectively. The BS is modeled with $M_{RF}=4$ RF chains and $M=16$ antennas with an inter-element spacing of $\ell=0.414\lambda$ at 30 GHz. The parameters are set to mimic the measurement settings as explained in Section IV. In the first iteration, the dominant wide beam indicated by $n_{1,max}$ in Fig. 2 (a) is determined. Following the same procedure, $n_{2,max}$ and $n_{3,max}$ can also be determined in the second and third iterations. It can be observed that the angle detected (224.4^o) in Fig. 2 (c) matches well with the target angle (225^o). For the second path, the same procedure can also be followed, except that the main path angle region covered is not detected again. As it can be seen, the detected angle of the second path (135.1^o) also matches the target angle (135^o) well. To gain further insights into the training time of the proposed scheme, we briefly review the number of iterations of existing algorithms, when the propagation paths are determined by the same codebook resolution. More specifically, the beam steering, PPB [9], IS [10] and the proposed method require 32, 8, 3, and 6 iterations, respectively. The proposed scheme outperforms beam steering and PPB, but underperforms IS. The advantage of proposed method compared to IS [10] is that it simply detects the MPCs by only the same frequency not multiple frequencies as in [10].

There are two potential inaccuracies that are caused by the algorithm. 1) Only one path can be detected within each wide angle region formed by the low codebook resolution of $2\pi/M_{RF}$. However, this might not cause a problem in practice due to the sparse and specular nature of mm-wave channels. 2) The detected path angle is discrete, with resolution determined by the number of RF chains and antennas. Once all dominant path angles are extracted, we can determine the APs of the MS and the BS based on the path directions, as $\mathbf{T}=\frac{1}{\sqrt{N}}[\angle\mathbf{v}_1^*,...,\angle\mathbf{v}_K^*]$ and $\mathbf{F}=\frac{1}{\sqrt{M}}[\angle\mathbf{u}_1^*,...,\angle\mathbf{u}_K^*]$.

B. DP Design

The next step is to determine the DP of the BS, which includes the path gain information. However, the prior knowledge of the original CSI is not available. We resort to construct DP by the knowledge of effective channel between the Tx and Rx antenna ports, in which signals have already experienced linear combination in analog domain. Originally, the MS sends a signal to the BS with an initialized DP of $\mathbf{B}_0=\mathbf{I}_{N_{RF}\times K}$ and the AP $\mathbf{T}$ determined in Section III-A. Subsequently, based on the received signal vector $\tilde{\mathbf{y}}$ before decoding and the AP of the BS $\mathbf{F}$ determined in Section III-A, the BS records its DP as

$$\begin{aligned}\mathbf{D}&=(\mathbf{F}^T\tilde{\mathbf{y}}\mathbf{s}^H)^*=\mathbf{F}^H(\mathbf{H}^*\mathbf{T}^*\mathbf{B}_0^*\mathbf{P}\mathbf{s}^*+\mathbf{n}^*)\mathbf{s}^T\\&\overset{(2)}{=}\mathbf{F}^H\mathbf{U}^*\begin{bmatrix}\mathbf{\Lambda} & \mathbf{0}_{K\times(M-K)}\end{bmatrix}^T+\mathbf{F}^H\mathbf{n}^*\mathbf{s}^T\\&=\begin{bmatrix}\mathbf{\Lambda}_1\mathbf{\Lambda} & \mathbf{0}_{K\times(M_{RF}-K)}\end{bmatrix}^T+\mathbf{F}^H\mathbf{n}^*\mathbf{s}^T\end{aligned} \quad (5)$$

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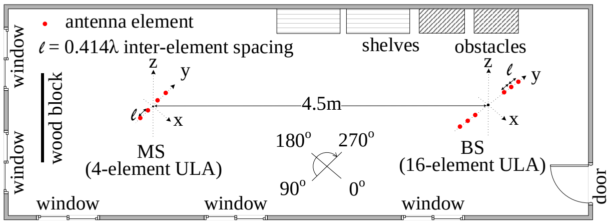

Fig. 4. Measurement scenario.

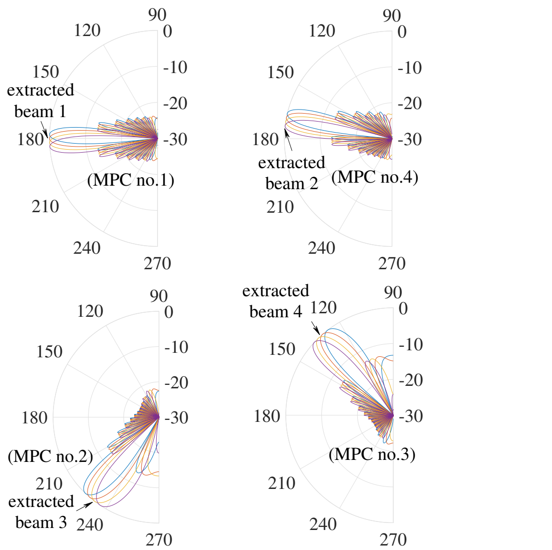

Fig. 5. Beam trainings for MPC no.1 (top-left), no.4 (top-right), no.2 (bottom-left), and no.3 (bottom-right) performed by 16-element ULA with 4 RF chains.

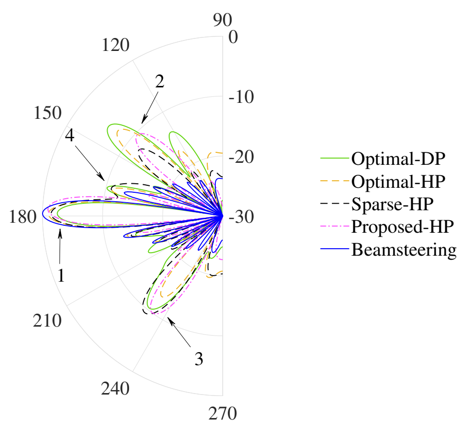

Fig. 6. Combined pattern for measurement scenario.

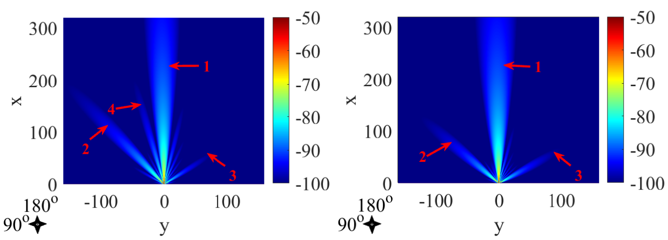

Fig. 7. E-field pattern of optimal HP (left) and proposed HP (right)

where $\mathbf{\Lambda} = \mathrm{diag}(\rho_1, ..., \rho_K)$ and $\mathbf{\Lambda}_1 = \mathrm{diag}(r_1, ..., r_K)$, with $\rho_k = \frac{\mu_k\sqrt{P}}{\sqrt{NK}} = \frac{\mu_k\sqrt{P}}{\sqrt{NK}}\mathbf{v}_k^H\angle\mathbf{v}_k$ and $r_k = \frac{\sqrt{P}}{\sqrt{MK}} = \frac{\sqrt{P}}{\sqrt{MK}}\angle\mathbf{u}_k^T\mathbf{u}_k^*$, respectively. We can have $\mathbf{D} \approx \begin{bmatrix}\mathbf{\Lambda}_1\mathbf{\Lambda} & \mathbf{0}_{K\times(M_{RF}-K)}\end{bmatrix}^T$ since the SNR of large-scale array $\eta = \frac{P}{K\sigma^2}\mathbb{E}[||\mathbf{H}||^2]$ can be assumed high. Thus, after normalized operator, we have $\mathbf{FD} \approx \overline{\mathbf{U}}^*$. Likewise, we have $\mathbf{TB} \approx \overline{\mathbf{V}}^*$. Fig. **??** shows that the proposed design effectively approaches the optimal HP.

IV. Experimental Validation

The proposed method is validated in measured mm-wave channels in an indoor office scenario. The measurement campaign is conducted with a vector network analyzer (VNA) based on channel sounder as outlined in [12], where the measurement scenario is depicted in Fig. 4. The BS and MS are equipped with vertically polarized omni-directional antennas. The BS and MS ULAs are formed by moving the antenna to predefined positions using precision positioning stages with an inter-element spacing of 0.414λ at 30 GHz. For this analysis, we assume that the BS and MS both have 4 RF chains with 16 and 4 radiating elements, respectively. The spatial region considered is in the broadside of the ULA, from 90^o to 270^o.

For the performance analysis, the proposed algorithm is then compared to the optimal HP, conventional beam-steering and sparse HP algorithms as outlined in Section II-C. Fig. 5 and Fig. 6 show the beam patterns during and after the beam training, respectively. Besides, Fig. 7 illustrates the field distribution of the proposed scheme, which shows that the beams are steered to dominant propagation directions. The proposed method has successfully extracted the expected beam vectors, except for the extracted beam 2 with respect to MPC no.4. It can be observed from Fig. 5 that there exists an angle deviation between the extracted beam 2 and the actual direction of MPC no.4. This is due to the fact that the detected angle is discrete, as explained in Section III-A. The BS steers the beams with coarse-refined search during the training with the resolution limit of 4 RF chains. For the coarse search, MPC no.1 and no.4 covered by a same training beam are indistinguishable, which leads to the missing of MPC no.4 as shown in Fig. 7. However, as the number of RF chains or antenna elements increases, the proposed design can approach the optimal HP, since more MPCs can be accurately captured. The proposed method is superior to beam steering but slightly inferior to sparse HP in this measurement result. As discussed, the main advantage of this algorithm compared to the sparse HP is that it works without a prior knowledge of the original channel.

V. Conclusion

In this letter, a HP design method with beam split training was proposed. The HP design was formulated as a problem of tracking the dominant MPCs in a sparse mm-wave channel. Without the knowledge of original CSI, the AP design exploited an iterative beam split strategy to extract beam vectors covering the dominant paths, while the DP was associated with corresponding gains. Under the hybrid constraint, the proposed method outperforms beam steering, attaining transmission at the dominant modes of channel with low estimation latency. This was verified with simulations and measurements.

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