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# Design of PWM-SMC Controller Using Linearized Model for Grid-Connected Inverter with LCL Filter

Han Li, Weimin Wu, *Member, IEEE,* Min Huang, Henry Shu-hung Chung, *Fellow, IEEE,* Marco Liserre, *Fellow, IEEE,* and Frede Blaabjerg, *Fellow, IEEE* 

Abstract- Nowadays, various Sliding Mode Control (SMC) methods have been successfully applied to digitally controlled Grid-Connected Inverter (GCI) with LCL filter. However, how to design of the Pulse Width Modulation based SMC (PWM-SMC) controller need to be further explored, especially upon the large variation of parameters drift and the delay issue. In this paper, the essence of two classic SMC methods used in power converter area is first analyzed in detail. Thus, a novel design of PWM-SMC controller using linearized model for three-phase GCI with LCL filter is proposed. Based on this, a three-loop step-by-step design of the PWM-SMC controller is developed, by using the closed-loop pole locations. Robust analysis against parameters drift is also studied. In addition, a discrete state observer is adopted to reduce the number of sensors. Further, a discussion between the proposed control strategy with the existing SMC methods and the full-state feedback controller is carried out. Finally, a 3 kW lab device designed on the dSPACE is constructed to verify the feasibility of the proposed strategy and the correctness of theoretical analysis<sup>1</sup>.

*Index Terms-* Grid-connected inverter, *LCL*-filter, Parameters drift, Robust, Sliding Mode Control, Stability analysis, State Observer.

#### I. INTRODUCTION

Due to the environmental concerns and fossil energy crisis problems, in recent years, researches on the renewable energy grid-connected power generation technology have received extensive attention [1]. As the core component of system, the Grid-Connected Inverter (GCI) is charged with the task of injecting high-quality current into power grid in a stable and efficient way. For this purpose, a power filter (L or LCL) is often used to suppress the high-frequency

harmonics generated by switching operations. Compared with the L filter, a third-order LCL filter has been widely adopted, owing to its better ability to attenuate high-frequency harmonics with relatively smaller inductance [2]. However, it has inherent resonant problem that may seriously deteriorate the stability of whole system [3]-[5], especially when grid distortion and parameters drift occur in GCI systems, resulting in a critical challenge in the current regulator design.

Nowadays, to improve the stability, many linear controllers combined with active damping methods have been applied to *LCL*-filtered GCI [6]-[12]. Besides, various nonlinear controllers have been deeply studied, such as predictive controller[13]-[16], Lyapunov-Function based controller [17], passive based controller [18]-[20], adaptive controller [21], [22], deadbeat controller [23]-[25] and SMC controller [26]-[31], etc. As one of nonlinear controllers, the SMC controller has attracted many interests, due to its strong robustness against system parameter uncertainties.

Currently, according to the switching mode, there are two kinds of SMC controllers for *LCL*-filtered GCI. The one is the hysteresis modulation based SMC (HM-SMC) [26]-[27], while the other is the Pulse Width Modulation based SMC (PWM-SMC) [29]-[31].

In HM-SMC methods, Komurcugil et al. [26] had proposed a new sliding surface function with double-band hysteresis scheme, which can reduce the number of the sensors and achieve good dynamic performance. Guzman et al. [27] had integrated kalman filter (KF) with SMC controller for LCL-filtered inverters, using a reduced model to achieve high robustness against LCL parameters drift. Also, Guzman et al. [28] had proposed a control algorithm in natural frame based on SMC together with KF to obtain three decoupled controllers, which can provide the desired dynamics for the grid-injected current. However, the hysteresis modulation will cause the variable switching frequency, resulting in the difficulty of designing the output power filter. Besides, a high sampling frequency has to be adopted for the HM-SMC, increasing the implementation difficulty.

The PWM-SMC can realize the fixed switching frequency operation. Hao et al. [29] adopted the equivalent SMC and inserted the multiple resonant terms into the sliding surface function, which can eliminate steady-state error and suppress the total harmonic distortion (THD) of the grid current effectively. Vieira et al. [30] introduced a dual-loop design strategy and chose the discrete-time SMC controller as the inner loop controller to simplify the current control design. Alali et al. [31] studied two continuous SMC algorithms to prevent very high-frequency switching effects of the SMC. Although all the mentioned PWM-SMC methods can achieve good control performance, the detailed

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Fig. 1. Three-phase LCL-filtered GCI with the proposed design of SMC controller using linearized model.

design guideline of controller parameters is lacking, resulting in the industrial application difficulty.

Aiming at the industrial application, different from the traditional PWM-SMC design based on Lyapunov stability, this paper proposes a step-by-step design guideline of PWM-SMC controller using linearized model for three-phase *LCL*-filtered GCIs, which can help engineers to precisely design a highly robust PWM-SMC controller to against the output filter parameters drift.

The rest of this paper is organized as follows: Section II first presents the accurate error dynamics modeling of GCI with *LCL* filter. Then, the essence of HM-SMC and PWM-SMC is explored in detail in Section III. Based on the above, Section IV proposes a novel design of SMC controller using linearized model together with a step-by-step parameter tuning and robustness analysis. Further, Section V briefly introduces how to adopt a discrete state observer to save sensors, while Section VI demonstrates the experimental validation results on a 3 kW/3-phase/110 V experimental lab setup. Section VII discusses the comparison between the proposed control strategy with the existing SMC methods and the full-state feedback controller. Finally, Section VIII draws some conclusions.

# II. ERROR DYNAMICS MODELING OF GCI WITH LCL FILTER

Fig. 1 shows the three-phase *LCL*-filtered GCI and its basic control structure, where the *LCL* filter consists of an inverter-side inductor of  $L_1$ , a capacitor of C, and a grid-side inductor of  $L_2$ . The equivalent grid impedance is modeled with the inductor of  $L_g$ . The Point of Common Coupling (PCC) voltage  $v_{pcc} = \begin{bmatrix} v_{pcca} & v_{pccb} & v_{pccc} \end{bmatrix}^T$ , the inverter-side current  $i_1 = \begin{bmatrix} i_{1a} & i_{1b} & i_{1c} \end{bmatrix}^T$ , the capacitor voltage  $v_c = \begin{bmatrix} v_{ca} & v_{cb} & v_{cc} \end{bmatrix}^T$ , and the grid-injected current

 $i_2 = \begin{bmatrix} i_{2a} & i_{2b} & i_{2c} \end{bmatrix}^T$  are sensed or calculated for the controller. The operation of system in  $\alpha$ - $\beta$  reference frame can be described by the following equations:

$$L_1 \frac{di_{1\alpha}}{dt} = u_{i\alpha} - i_{1\alpha} r_1 - v_{c\alpha}$$
(1),

$$L_{1}\frac{di_{1\beta}}{dt} = u_{i\beta} - i_{1\beta}r_{1} - v_{c\beta}$$
(2),

$$C\frac{dv_{c\alpha}}{dt} = i_{1\alpha} - i_{2\alpha} \tag{3},$$

$$C\frac{dv_{c\beta}}{dt} = i_{1\beta} - i_{2\beta} \tag{4},$$

$$L_2 \frac{di_{2\alpha}}{dt} = v_{c\alpha} - i_{2\alpha}r_2 - v_{pcc\alpha}$$
(5),

$$L_2 \frac{di_{2\beta}}{dt} = v_{c\beta} - i_{2\beta} r_2 - v_{pcc\beta}$$
(6),

where  $u_{ik(k=\alpha,\beta)}$  is the switching function, and  $r_1$ ,  $r_2$  are the equivalent resistances of inductors  $L_1$  and  $L_2$ , respectively.

According to the angle generated by the Phase-Locked Loop (PLL) and the given reference current, the reference of grid-injected current  $i_2^*$  can be obtained. Furthermore, based on (1)-(6), the reference of the inverter-side current  $i_1^*$  and the reference of capacitor voltage  $v_c^*$  can be also derived. Then three references of the state variables are

$$\begin{cases} i_{2k}^{*} = I_{2} \sin(\omega t + \theta) \\ v_{ck}^{*} = L_{2} i_{2k}^{*} + v_{pcc} + i_{2k}^{*} r_{2} \\ i_{1k}^{*} = i_{2k}^{*} + C v_{ck}^{*} \end{cases} \qquad k = \alpha, \beta \quad (7)$$

Note that the  $\alpha$ -axis and the  $\beta$ -axis are independent with each other. In order to facilitate the description, the next

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design focuses on the  $\alpha$ -axis, while the design in the  $\beta$ -axis is the same as the one in the  $\alpha$ -axis.

Define three system error dynamics as,

$$\begin{cases} x_{e1} = i_1^* - i_1 \\ x_{e2} = v_c^* - v_c \\ x_{e3} = i_2^* - i_2 \end{cases}$$
(8),

where their derivatives are

$$\dot{x}_{e1} = \dot{i}_{1}^{*} - \dot{i}_{1} = \dot{i}_{1}^{*} - (\frac{1}{L_{1}}(u_{i} - \dot{i}_{1}r_{1} + \dot{i}_{1}^{*}r_{1} - \dot{i}_{1}^{*}r - v_{c} + v_{c}^{*} - v_{c}^{*}))$$

$$= -\frac{1}{L_{1}}x_{e2} - \frac{r_{1}}{L_{1}}x_{e1} - \frac{1}{L_{1}}u_{i} + \dot{i}_{1}^{*} + \frac{v_{c}}{L_{1}} + \frac{r_{1}}{L_{1}}\dot{i}_{1}^{*}$$
(9),

$$\dot{x}_{e2} = \dot{v}_{c}^{*} - \dot{v}_{c} = \frac{1}{C}(\dot{i}_{1}^{*} - \dot{i}_{2}^{*}) - \frac{1}{C}(\dot{i}_{1} - \dot{i}_{2}) = \frac{1}{C}x_{e1} - \frac{1}{C}x_{e3}$$
(10)

$$\dot{x}_{e3} = \dot{i}_{e2}^* - \dot{i}_{e2} = \frac{1}{L_2} (v_c^* - \dot{i}_2^* r_2 - v_{pcc}) - \frac{1}{L_2} (v_c - \dot{i}_2 r_2 - v_{pcc})$$

$$= \frac{1}{L_2} x_{e2} - \frac{r_2}{L_2} x_{e3}$$
(11).

The state-space expression of system error dynamics can be achieved as

$$\begin{aligned} x_{e} &= Ax_{e} + Bu_{i} + D \\ \begin{bmatrix} \vdots \\ x_{e1} \\ \vdots \\ x_{e2} \\ \vdots \\ x_{e3} \end{bmatrix} = \begin{bmatrix} -\frac{r_{1}}{L_{1}} & -\frac{1}{L_{1}} & 0 \\ \frac{1}{C} & 0 & -\frac{1}{C} \\ 0 & \frac{1}{L_{2}} & -\frac{r_{2}}{L_{2}} \end{bmatrix} \begin{bmatrix} x_{e1} \\ x_{e2} \\ x_{e3} \end{bmatrix} + \begin{bmatrix} -\frac{1}{L_{1}} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} u_{i} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} d_{e} \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$
(12),

where  $d_e = i_1^* + \frac{v_c^*}{L_1} + \frac{r_1 i_1^*}{L_1}$ .

#### III. ESSENCE OF HM-SMC AND PWM-SMC

## A. Basic Principle of SMC Theory

For a given control system represented by the state equation

$$x = A(x,t) + B(x,t)u$$
 (13).

In order to construct a SMC system, the following two steps are needed [32]:

1) Design a sliding surface S(x) = 0 to represent a desired system dynamics.

2) Design a variable structure control u(x,t) as

$$u(x,t) = \begin{cases} u^{+}(x,t) & \text{when } S(x) > 0\\ u^{-}(x,t) & \text{when } S(x) < 0 \end{cases}$$
(14)

Any state *x* outside the sliding surface can be driven to reach the sliding surface in finite time. On the sliding surface, the sliding mode takes place, following the desired system dynamics.





Fig. 2 introduces the basic principle of practical SMC system. The middle black line represents the sliding surface S(x) = 0, which is the desired system dynamics. The sliding surface divides the system state into two different parts, where different control of u(x,t) works. Under this variable structure control, the green solid line depicts the system trajectory.

As shown in Fig. 2, when the system state is at point A, the sliding surface function S(x) is less than zero, and then  $u^{-}(x,t)$  controls the system state to move towards the sliding surface. When the system state reaches the sliding surface, it crosses the sliding surface under the control of  $u^{-}(x,t)$ . At this time, the system switches to the control of  $u^{+}(x,t)$  immediately. Due to the inertia (space or time lag), the system state moves from point B towards point C, according to the previous motion mode. The dashed line represents that the trajectory of system state would be unstable without the switching control.

Through the above analysis, we can find that switching plays an important role in SMC system. For example, the reaching law approach is an effective way to design the SMC system [33], which directly specifies the dynamics of the sliding surface function,

$$S = -kS - \varepsilon \operatorname{sgn}(S) \tag{15}$$

The symbolic function could enable the system state to have a certain rate when reaching the sliding surface, so that the system state can cross the sliding surface. Therefore, in a conventional SMC system, the switching of system can be realized by the symbolic function in (15). However, in practical applications, the switching modes are various, especially for power-electronic-based systems with two special modulation modes. Here, a single-phase full-bridge GCI with an L filter is taken as an example to simply elaborate the switching characteristic in HM-SMC and PWM-SMC in following two subsections.

#### B. Switching Characteristic in HM-SMC

Fig. 3 shows the operation of HM-SMC, where this method adopts the hysteresis modulation to achieve a high frequency switching action. For conveniently describing the system, the sliding surface function is chosen as the error of





The switching action can be written as  

$$u = \begin{cases} -1 & S \ge +h \\ 1 & S \le -h \end{cases}$$
(16),

where in the other case (-h < S < +h), the switching action keep the former state. *h* is the width of the hysteresis band, which controls the switching frequency.

As shown in Fig. 3, the hysteresis modulator employs a hysteresis band with the boundary layer to switch the control of the system. In one switching cycle, there are two periods  $t_1$  and  $t_2$ , where the switching actions are u = 1 and u = -1 respectively.

When the sliding surface function S(x) is equal to the value of the lower boundary layer (-h) at point A, the switching occurs and the switching action is u = 1. During the switching action of u = 1, the sliding surface function S(x) moves from -h toward +h.

When the sliding surface function S(x) is equal to the value of the upper the boundary layer (+h) at point B, the switching occurs and the switching action is u=-1. During the switching action of u = -1, the sliding surface function S(x) moves from +h toward -h.

Stated thus, the hysteresis modulator ensures the switching of the HM-SMC system.

#### C. Switching Characteristic in PWM-SMC

As shown in Fig. 4, different from the HM-SMC, the PWM-SMC adopts pulse width modulator to control the system state to move towards the sliding surface indirectly. In one switching cycle, there are two circuit states, where the switching actions are u = -1 and u = 1 respectively.

When the system state is at point A, modulation wave  $u_i$  intersects the carrier, the switching occurs and the switching action is u = 1. During the switching action of u = 1, the sliding surface function S(x) moves from negative toward positive.

When the system state is at point B, modulation wave  $u_i$  intersects the carrier again, the switching occurs and the switching action is u = -1. During the switching action of u = -1, the sliding surface function S(x) moves from positive toward negative.



Fig. 4. Operation of PWM-SMC.

Similar to HM-SMC, pulse width modulator can also ensure the switching of the PWM-SMC system. Although the conclusion is based on a single-phase GCI system, it can be extended to a three-phase GCI system.

Therefore, different from the conventional PWM-SMC design with the nonlinear term in  $u_i$ , the nonlinear term will be eliminated in the proposed design process, where the linear tool can be utilized to analyze the stability. Note that during the design, PWM-SMC systems in [34]-[36] had also ignored the nonlinear term in the control law.

# IV. PROPOSED DESIGN OF PWM-SMC CONTROLLER USING LINEARIZED MODEL

# A. Proposed Reconstructed Control Law with Zero Steady-State Error

The *LCL*-filtered GCI is a typical third-order system. In order to guarantee the global stability and damping effect, we choose the sum of three error dynamics as the sliding surface function, which is defined as [29]

$$S = C_e x_e$$
  
=  $\alpha_1 x_{e1} + \alpha_2 x_{e2} + \alpha_3 x_{e3}$  (17)  
=  $\alpha_1 (i_1^* - i_1) + \alpha_2 (v_c^* - v_c) + \alpha_3 (i_2^* - i_2)$ 

Through the above analysis, the switching function can be realized by PWM, thus the symbolic function is eliminated, the reaching law is rewritten as

$$S = -kS \tag{18}$$

Based on (17)-(18), a control law is deduced as

$$u_{i} = -r_{1}x_{1} - x_{2} + L_{1}\dot{i_{1}^{*}} + v_{c}^{*} + r_{1}\dot{i_{1}^{*}} + L_{1}\frac{\alpha_{2}}{\alpha_{1}}(\frac{1}{C}x_{1} - \frac{1}{C}x_{3}) + L_{1}\frac{\alpha_{3}}{\alpha_{1}}(\frac{1}{L_{2}}x_{2} - \frac{r_{2}}{L_{2}}x_{3}) + k\frac{L_{1}}{\alpha_{1}}S$$
(19)

Through (19), we find that the control law is complex, resulting in the difficulty to design controller parameters.

Separating the sliding surface function, the control law can be rewritten as,



Fig. 5. Block diagram of the proposed design of SMC controller using linearized model

$$u_{i} = L_{1}\dot{i}_{1}^{*} + v_{c}^{*} + r_{1}\dot{i}_{1}^{*} + (-r_{1} + L_{1}\frac{\alpha_{2}}{\alpha_{1}}\frac{1}{C} + kL_{1})x_{e1}$$

+ 
$$(-1 + L_1 \frac{\alpha_3}{\alpha_1} \frac{1}{L_2} + k \frac{L_1}{\alpha_1} \alpha_2) x_{e2}$$
 (20).

+ 
$$\left(-L_1\frac{\alpha_2}{\alpha_1}\frac{1}{C}-L_1\frac{\alpha_3}{\alpha_1}\frac{r_2}{L_2}+k\frac{L_1}{\alpha_1}\alpha_3\right)x_{e^3}$$

As shown in (20), the gain of each state feedback variable is determined by three or four sliding mode parameters (sliding surface function parameters and the gain of the reaching law). Based on (20), by simplifying the control law and reintegrating the sliding mode parameters, three new uncoupled controller parameters can be obtained. They are

$$r_{d1} = (-r_1 + L_1 \frac{\alpha_2}{\alpha_1} \frac{1}{C} + kL_1)$$
(21)

$$r_{d2} = (-1 + L_1 \frac{\alpha_3}{\alpha_1} \frac{1}{L_2} + k \frac{L_1}{\alpha_1} \alpha_2)$$
(22)

$$r_{d3} = \left(-L_1 \frac{\alpha_2}{\alpha_1} \frac{1}{C} - L_1 \frac{\alpha_3}{\alpha_1} \frac{r_2}{L_2} + k \frac{L_1}{\alpha_1} \alpha_3\right) \quad (23)$$

Note that, using the determined new three controller parameters to solve the original sliding surface function parameters and the gain of reaching law, we can obtain an infinite number of solutions. Thus, the sliding surface function parameters and the gain of reaching law must exist. As long as the sliding surface passes through the origin, the system would converge to the origin, where the Lyapunov stability of PWM-SMC is not affected.

Therefore, a new control law can be derived as

$$u_i = L_1 d_e + r_{d1} x_{e1} + r_{d2} x_{e2} + r_{d3} x_{e3}$$
(24)

However, in a practical system, LCL filter parameters and parasitic resistances have tolerances.  $L_1, L_2, C, r_1$  and  $r_2$ represent the actual parameters of system, while  $L_1, L_2, C$ ,  $r_1$  and  $r_2$  represent the estimated parameters, respectively. There is a deviation between the actual parameter and the estimated one, and the LCL filter parameters used in the PWM-SMC controller are the estimated parameter. According to [29], this deviation would cause the steadystate error of system, where the multiple resonant terms can effectively eliminate it. Therefore, in our case, a PR term is instead of the gain of  $r_{d3}$ , and the transfer function is,

$$G_{c}(s) = k_{p} + \frac{2k_{r}\omega_{i}s}{s^{2} + 2\omega_{i}s + \omega_{0}^{2}}$$
(25)

where  $k_p$  and  $k_r$  are the proportional and resonant gains, and  $\omega_i$  and  $\omega_o$  are the cutoff frequency and the fundamental frequency, respectively.

Due to the adoption of digital control, it is necessary to consider the delay issue after obtaining the control law.  $T_s$  is the sampling period. The zero-order hold is

$$G_{zoh}(s) = \frac{1 - e^{-I_s \cdot s}}{s}$$
(26).

Thus, when a  $T_s$  computation delay is addressed, the total delay  $G_d(s)$  is

$$G_d(s) = \frac{1}{T_s} e^{-T_s \cdot s} G_{zoh}(s) \approx e^{-1.5T_s \cdot s}$$
(27).

Thus, the whole control block diagram of proposed design of SMC controller using linearized model can be drawn in Fig. 5.

The closed-loop transfer function is

$$G(s) = \frac{I_2(s)}{I_2^*(s)} = \frac{a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{b_5 s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}$$
(28),

where

$$\begin{aligned} a_{5} &= C_{f}L_{1}L_{2} \\ a_{4} &= C_{f}L_{1}r_{2}^{'} + C_{f}L_{2}r_{1}^{'} + C_{f}L_{2}r_{d1}^{'} + 2C_{f}L_{1}L_{2}\omega_{i} \\ a_{3} &= C_{f}L_{1}L_{2}\omega_{o}^{2} + L_{1}^{'} + L_{2}^{'} + L_{2}r_{d2} + C_{f}r_{1}r_{2}^{'} \\ &+ C_{f}r_{2}r_{d1}^{'} + 2C_{f}L_{1}r_{2}\omega_{i} + 2C_{f}L_{2}r_{1}\omega_{i} + 2C_{f}L_{2}r_{d1}\omega_{i} \\ a_{2} &= k_{p} + r_{1}^{'} + r_{2}^{'} + r_{d1} + 2L_{1}\omega_{i} + 2L_{2}\omega_{i} + r_{2}^{'}r_{d2} \\ &+ 2L_{2}r_{d2}\omega_{i} + C_{f}L_{1}r_{2}\omega_{o}^{2} + C_{f}L_{2}r_{1}\omega_{o}^{2} + C_{f}L_{2}r_{d1}\omega_{o}^{2} \\ &+ 2C_{f}r_{1}^{'}r_{2}^{'}\omega_{i} + 2C_{f}r_{2}r_{d1}\omega_{i} \\ a_{1} &= 2k_{p}\omega_{i} + 2k_{r}\omega_{i} + 2r_{1}^{'}\omega_{i} + 2r_{2}^{'}\omega_{o}^{2} + 2L_{1}^{'}\omega_{i} + 2L_{2}^{'}\omega_{i} \\ &+ 2r_{d1}\omega_{i} + L_{1}^{'}\omega_{o}^{2} + L_{2}\omega_{o}^{2} + 2L_{1}^{'}\omega_{o}^{2} + C_{f}r_{2}r_{d1}\omega_{o}^{2} \\ &= 2k_{p}\omega_{i}^{2} + r_{1}^{'}\omega_{o}^{2} + 2r_{1}^{'}\omega_{o}^{2} + 2r_{2}^{'}\omega_{o}^{2} \\ &+ 2r_{2}r_{d2}\omega_{i} + L_{2}r_{d2}\omega_{o}^{2} + C_{f}r_{1}r_{2}^{'}\omega_{o}^{2} + C_{f}r_{2}r_{d1}\omega_{o}^{2} \\ &= 2k_{p}\omega_{o}^{2} + r_{1}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{d1}\omega_{o}^{2} + r_{2}r_{d2}\omega_{o}^{2} \\ &= 2k_{p}\omega_{o}^{2} + r_{1}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{1}^{'}\omega_{o}^{2} + r_{2}^{'}r_{d2}\omega_{o}^{2} \\ &= 2k_{p}\omega_{o}^{2} + r_{1}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{2}r_{d2}\omega_{o}^{2} \\ &= 2k_{p}\omega_{o}^{2} + r_{1}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{2}r_{d2}\omega_{o}^{2} \\ &= 2k_{p}\omega_{o}^{2} + r_{1}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{2}r_{d2}\omega_{o}^{2} \\ &= 2k_{p}\omega_{o}^{2} + r_{1}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{2}r_{d2}\omega_{o}^{2} \\ &= 2k_{p}\omega_{o}^{2} + r_{1}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{2}^{'}r_{d2}\omega_{o}^{2} \\ &= 2k_{p}\omega_{o}^{2} + r_{1}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{2}^{'}r_{d2}\omega_{o}^{2} \\ &= 2k_{p}\omega_{o}^{2} + r_{1}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{2}^{'}r_{d2}\omega_{o}^{2} \\ &= 2k_{p}\omega_{o}^{2} + r_{1}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{2}^{'}r_{d2}\omega_{o}^{2} \\ &= 2k_{p}\omega_{o}^{2} + r_{1}^{'}\omega_{o}^{2} + r_{2}^{'}\omega_{o}^{2} + r_{2$$

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Fig. 6. Proposed design flowchart.

$$\begin{split} b_{3} &= C_{f}L_{1}L_{2}e^{1.5s\cdot T_{s}}\,\omega_{o}^{2} + L_{2}r_{d2} + L_{1}e^{1.5s\cdot T_{s}} \\ &+ L_{2}e^{1.5s\cdot T_{s}} + C_{f}r_{2}r_{d1} + C_{f}r_{1}r_{2}e^{1.5s\cdot T_{s}} + 2C_{f}L_{2}r_{d1}\omega_{i} \\ &+ 2C_{f}L_{1}r_{2}\omega_{i}e^{1.5s\cdot T_{s}} + 2C_{f}L_{2}r_{1}\omega_{i}e^{1.5s\cdot T_{s}} \\ b_{2} &= k_{p} + r_{d1} + r_{2}r_{d2} + r_{1}e^{1.5s\cdot T_{s}} + r_{2}e^{1.5s\cdot T_{s}} + 2L_{2}r_{d2}\omega_{i} \\ &+ 2L_{1}\omega_{i}e^{1.5s\cdot T_{s}} + 2L_{2}\omega_{i}e^{1.5s\cdot T_{s}} + C_{f}L_{2}r_{d1}\omega_{o}^{2} + 2C_{f}r_{2}r_{d1}\omega_{i} \\ &+ 2C_{f}r_{1}r_{2}\omega_{i}e^{1.5s\cdot T_{s}} + C_{f}L_{1}r_{2}\omega_{o}^{2}e^{1.5s\cdot T_{s}} + C_{f}L_{2}r_{1}\omega_{o}^{2}e^{1.5s\cdot T_{s}} \\ b_{1} &= 2k_{p}\omega_{i} + 2k_{r}\omega_{i} + 2r_{d1}\omega_{i} + L_{1}\omega_{o}^{2}e^{1.5s\cdot T_{s}} \\ &+ L_{2}\omega_{o}^{2}e^{1.5s\cdot T_{s}} + 2r_{2}r_{d2}\omega_{i} + 2r_{1}\omega_{i}e^{1.5s\cdot T_{s}} \\ &+ L_{2}\omega_{o}^{2}e^{1.5s\cdot T_{s}} + 2r_{2}r_{d2}\omega_{i}^{2} + C_{f}r_{1}r_{2}\omega_{o}^{2}e^{1.5s\cdot T_{s}} \\ &+ L_{2}r_{d2}\omega_{o}^{2} + C_{f}r_{2}r_{d1}\omega_{o}^{2} + C_{f}r_{1}r_{2}\omega_{o}^{2}e^{1.5s\cdot T_{s}} \\ &+ L_{2}r_{d2}\omega_{o}^{2} + r_{1}\omega_{o}^{2}e^{1.5s\cdot T_{s}} + r_{2}\omega_{o}^{2}e^{1.5s\cdot T_{s}} + r_{d1}\omega_{o}^{2} + r_{2}r_{d2}\omega_{o}^{2} \,. \end{split}$$

As shown in Fig. 5, although compared with the conventional SMC control law, the reconstructed SMC control law has been simplified, there are still three cascaded control loops and three feedforward terms in our control diagram. It is still a big challenge for engineers to design the controller.

Here, a three-loop step-by-step design procedure will be introduced. The design flowchart is shown in Fig. 6. Firstly, based on the nominal *LCL* filter parameters ( $L_1 = 1.2 \text{ mH}$ ,  $L_2 = 1.2 \text{ mH}$  and  $C = 6 \mu\text{F}$ ), the inner inverter-side current loop is designed and the controller parameter  $r_{d1}$  is selected. Secondly, based on the previous controller parameter  $r_{d1}$ , the middle loop of capacitor voltage feedback is designed and the controller parameter  $r_{d2}$  is chosen. Thirdly, the outer PR controller is designed, according to the previous controller parameters  $r_{d1}$  and  $r_{d2}$ . Finally, the verification of robustness against *LCL* filter parameters drift is addressed, after selecting the controller parameters.

It is worth mentioning that the following method can get the approximate stability regions of the inner and middle loop. The outer loop would ultimately determine the stability of the system. Due to the complexity of the control diagram, through this design procedure, only a set of better parameters can be obtained, but the optimal controller parameters is hardly chosen. During the stability analysis, the sample frequency is set as 12 kHz, which is equal to the switching frequency.

# B. Step-by-step Parameters Tuning

# Step 1-Design the Gain $r_{d1}$

The control block diagram of the inverter-side current loop is shown in Fig. 7. Here, the reference and the feedback of capacitor voltage are both regarded as perturbation terms. The closed-loop transfer function is





$$G_{1}(s) = \frac{i_{1}(s)}{i_{1}^{*}(s)} = \frac{L_{1}'s + r_{1}' + r_{d1}}{(L_{1}e^{1.5s * T_{s}})s + r_{d1} + r_{1}e^{1.5s * T_{s}}}$$
(29)

In order to facilitate the observation of the location of the poles and improve the accuracy of the digital realization, the transfer function in the above continuous domain is linearly discretized by MATLAB. Fig. 8 shows the discrete closed-loop pole locations of (29). It can be seen that the poles are located inside the unit circle, as long as the gain  $r_{d1}$  less than 12.

In order to assist with selecting the value of  $r_{d1}$ , the step responses of  $G_1(s)$  with different  $r_{d1}$  are illustrated in Fig.9, where the preferred region of  $r_{d1}$  range is between 3 and 6. Here, the  $r_{d1}$  is chosen as 4.



Fig. 8. Closed-loop pole locations of  $G_1(s)$ , when  $r_{d1}$  varies from 1 to 12.







Fig. 10. Block diagram of the capacitor voltage loop.

The design of  $r_{d2}$  is similar to that of  $r_{d1}$ . The diagram of the middle loop of capacitor voltage feedback is depicted in Fig. 10, where the inverter-side current reference and the grid-injected current feedback are regarded as disturbance terms. The closed-loop transfer function is

$$G_{2}(s) = \frac{v_{c}(s)}{v_{c}^{*}(s)} = \frac{1 + r_{d2}}{(CL_{1}e^{1.5s \cdot T_{s}})s^{2} + (Cr_{d1} + Cr_{1}e^{1.5s \cdot T_{s}})s + r_{d2} + e^{1.5s \cdot T_{s}}}$$
(30)

Fig. 11 shows the closed-loop pole locations of  $G_2(s)$ , showing that the stable region of  $r_{d2}$  ranges from -0.9 to -0.2. This is an approximate range of stability region, mainly to avoid introducing unstable poles.

Similarly, the step responses of  $G_2(s)$  with different  $r_{d2}$  are illustrated in Fig.12. Here,  $r_{d2}$  is chosen as -0.4.



Fig. 11. Closed-loop pole locations of  $G_2(s)$ , when  $r_{d2}$  varies from -0.9 to -0.2.



Fig. 12. Step responses of  $G_2(s)$  with different  $r_{d2}$ .

# Step 3-Design the Proportional Gain k<sub>P</sub>

Although the PR controller is adopted, the stability of system mainly depends on the proportional term of  $k_{\rm P}$ , since the resonant frequency is 50 Hz and the desired control bandwidth is far higher than 50 Hz. Therefore, the PR controller is simplified as  $k_{\rm P}$ .

With  $r_{d1}$  and  $r_{d2}$  the closed-loop pole locations of G(s) are shown in Fig. 13, where it can be clearly seen that the stable region of  $k_P$  is between 1 to 12 and the critical value of  $k_P$  is about 13. Finally,  $k_P$  is selected as 10 to ensure sufficient control bandwidth and stable margin. The resonant term of  $k_r$  is just selected as an initial value. In our case,  $k_r$  is chose as 800. In particular, according to the actual situation, the harmonic compensators could be inserted into the PR controller to resist the low order current harmonics caused by background harmonic voltages.



Fig. 13. Closed-loop pole locations of G(s), when  $k_P$  varies from 1 to 12.

### C. Robustness Analysis against Parameters Drift

Let the parameters of the *LCL* filter vary in a wide range.  $L_1$  varies in the range of 0.8 mH-2.0 mH (-33%~+66% of  $L_1$ ), *C* varies in the range of 4 uF-9 uF (-33%~+50% of *C*), and the grid side inductance  $L_g$  varies in the range of 0 mH-6 mH (0%~+500% of  $L_2$ ), respectively. Then the locations of the closed-loop poles of the system are derived. If the closed-loop poles of G(s) are still located inside the unit

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circle, the system is stable, which means that the controller parameter is reasonable and feasible. If any closed-loop pole locations of the system are located outside the unit circle, the system is unstable. Then, we have to go back to redesign the gains of  $r_{d1}$ ,  $r_{d2}$ , and  $k_{p.}$ , and then test the robustness again. From Fig. 14, it can be clearly seen that all the closed-loop poles of G(s) are still located inside the unit circle when the parameters of the *LCL* filter vary in a wide range.

In order to further evaluate the robustness of the proposed control strategy, a comparison between the traditional capacitor-current-feedback active damping (AD) method and the proposed method is performed. Notably, if the resonance frequency is equal to one-sixth of the sampling frequency and the total delay time is equal to one and half sampling period, the system can be hardly stable, matter how much the capacitor-current-feedback no coefficient is [37]. Here, the LCL parameters are set as  $L_1=1.6$  mH,  $L_2=1.2$  mH,  $L_g=1.9$  mH and C=6 µF, where the resonance frequency is about 2000 Hz ( $f_s/6$ ). As shown in Fig. 15, some closed-loop poles of G(s) with the traditional AD method are always located on the unit circle or outside the unit circle with various capacitor-current-feedback coefficients, while the closed-loop poles of G(s) with the proposed control strategy are all located inside the unit circle. It can be clearly proven that the proposed control strategy has a better stable region than the traditional capacitor-current-feedback AD method.





Fig. 14. Closed-loop pole locations of G(s) when the filter parameter varies. (a)  $L_1$  varies in the range of 0.8 mH-2.0 mH. (b) C varies in the range of 4 uF-9 uF. (c)  $L_g$  varies in the range of 0 mH-6 mH.



Fig. 15. Closed-loop pole locations of G(s) with traditional AD method and proposed control strategy when the resonance frequency is about 2000 Hz (fs/6).

(32)

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#### V. BRIEF DESIGN OF DISCRETE STATE OBSERVER

In order to save the total sensors, a simple and easily implemented discrete state observer is also adopted.

Based on (1)-(6), the discrete model of the system is obtained as

$$x(k+1) = A_{d}x(k) + B_{d}u_{i}(k) + D_{d}v_{g}(k)$$
(31)

$$A_{d} = \begin{bmatrix} 1 - \frac{r_{1}T_{s}}{L_{1}} & -\frac{T_{s}}{L_{1}} & 0\\ \frac{T_{s}}{C_{f}} & 1 & -\frac{T_{s}}{C_{f}}\\ 0 & \frac{T_{s}}{L_{2}} & 1 - \frac{r_{2}T_{s}}{L_{2}} \end{bmatrix} B_{d} = \begin{bmatrix} \frac{T_{s}V_{dc}}{L_{1}}\\ 0\\ 0\\ \end{bmatrix} C_{d} = \begin{bmatrix} 0\\ 0\\ 1\\ \end{bmatrix} D_{d} = \begin{bmatrix} 0\\ 0\\ \frac{T_{s}V_{pcc}}{L_{2}} \end{bmatrix}$$

As shown in Fig. 16, the discrete state observer is given by

$$\begin{cases} \hat{x}(k+1) = A_{d}\hat{x}(k) + B_{d}u_{i}(k) + D_{d}(k) + L_{d}\left[\hat{y}(k) - y(k)\right] \\ \hat{y}(k) = C_{d}\hat{x}(k) \end{cases}$$

where the subscript " $^{"}$  denotes the estimated value,  $L_{d}$  is the observer gain vector.



Fig. 16. Structure of the discrete state observer.

The design of  $L_d$  is carried out via zero pole assignment. The poles of the closed-loop system consist of the union of the controller poles and the observer poles. A rule of thumb is to select the observer poles to be two to six times faster than the poles of the controller. Then, the observer dynamics do not limit the bandwidth determined by the controller.

Generally, if the poles of the observer are chosen as three to five times faster than those of the controller, the dynamic characteristics of the observer will not impose any restrictions on the bandwidth determined by the controller. However, the observer's poles should not exceed the Nyquist frequency.

The characteristic polynomial of the observer dynamics is selected as

$$\det(zI_3 - G + L_dC_d) = (z - p_1)(z - p_2)(z - p_3) \quad (33)$$

where  $p_1$ ,  $p_2$  and  $p_3$  are the desired poles of observer, then observer gain vector can be calculated.

According to the principle of separation, the design of state observer and current controller do not affect each other, which means the sliding mode controller and the state observer can be designed independently.

# VI. EXPERIMENTAL VERIFICATIONS

In order to validate the effectiveness of the proposed strategy, a prototype of 3 kW three-phase three-wire LCLfilter-based system with proposed control scheme has been implemented by using the DSpace DS1202. When constructing the control loop, the control law is discretized due to the digital implementation. The power grid is emulated with Chroma 61830. Yokogawa DL 1640 digital oscilloscope is used to measure the grid voltage waveform and grid-injected current waveform. The experimental setup is shown in Fig. 17, while the system parameters and nominal controller parameters are listed in TABLE I.

TABLE I SYSTEM PARAMETERS								
Symbol	Value	Symbol	Value					
Grid voltage $v_g = 110$	V(RMS)	Switching frequency $f_{sw}$	12 kHz					
DC link voltage $U_{dc}$	350 V	Sampling frequency $f_s$	12 kHz					
Inverter-side inductor $L_1$	1.2 mH	Coefficient of control $k_p$	10					
Grid-side inductor $L_2$	1.2 mH	Coefficient of control $k_r$	800					
Filter capacitor C	$6\mu\mathrm{F}$	Coefficient of control $k_{d1}$	4					
Equivalent series resistance of $L_1$	0.2Ω	Coefficient of control $k_{d2}$	-0.4					

Equivalent series resistance of  $L_2 = 0.2\Omega$ 



Fig. 17. Experimental setup.

A. Performance under Nominal LCL Parameters Condition

Fig. 18 and Fig. 19 show the results under nominal LCL parameters condition.

Fig. 18 shows that the grid-injected current can keep tracking the reference with zero steady-state error when  $k_{\rm p}$ =10. In addition, the grid-injected current steps down from 12.86 A to 6.43 A, where the proposed strategy exhibits a fast dynamic response, meaning that the system has achieved a wide control bandwidth.

While, Fig. 19 shows that the measured grid voltage and grid-injected current oscillated, when the gain  $k_p$  increased to 13 (critical value obtained by theory analysis), which matches the theoretical analysis in Fig. 13 quite well.

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Fig. 18. Measured grid voltage waveform and grid-injected current waveform when  $k_p=10$ .



Fig. 19. Measured grid voltage waveform and grid-injected current waveform when  $k_p=13$ .

# B. Performance under LCL Filter Parameters Drift Condition

To evaluate the robustness of the proposed control strategy, Fig. 20, Fig. 21 show the grid-injected current under the condition of *LCL* filter parameters drift ( $L_1$  varies -33%, *C* varies -33% from the nominal values, respectively.). Despite of the variations of *LCL* filter parameters, the system can still be stable and achieve zero steady-state error, which is good agreement with the closed-loop pole locations of *G*(s) in Fig. 14.

In actual applications, the grid inductance may change in a large range, especially when multiple GCIs are in parallel connected with the power grid at the same common coupling point [9]. To emulate this situation, the external inductors ( $L_g$ = 6 mH) are adopted. As shown in Fig. 22, the system remains stable with zero steady-state error. It is worth mentioning that the large equivalent grid impedance usually leads to the decrease of the system control bandwidth. However, the transient response with the proposed control strategy is good (the settling time is within one cycle as shown in Fig. 22), proving that the proposed strategy can also achieve a good control bandwidth.

Fig. 23 shows grid-injected current with the capacitorcurrent-feedback AD method when the resonance frequency is about 2000 Hz (fs/6). The system is unstable which is consistent with conclusions in [37]. Fig. 24 shows gridinjected current with the proposed control strategy under the same condition, where the system is stable. It can be proven that the proposed control strategy has a wider stable region than that with the conventional capacitor-current-feedback AD method.







Fig. 21. Measured grid-injected current waveform when  $L_1 = 1.2$  mH,  $L_2+L_g = 1.2$  mH, and C = 4 µF.



Fig. 22. Dynamic performance when  $L_1 = 1.2$  mH,  $L_2+L_g = 7.2$  mH, and  $C = 6 \mu$ F.



Fig. 23. Measured grid-injected current waveform with the capacitorcurrent-feedback AD when the resonance frequency is about 2000 Hz ( $f_{s}/6$ ).



control strategy when the resonance frequency is about 2000 Hz ( $f_s/6$ ).



Fig. 25. Measured grid voltage waveform and grid-injected current waveform under a distorted grid.

# C. Performance under a Distorted Grid

Fig. 25 shows the measured grid-injected current and grid voltage under a distorted grid. A programmable AC

source (Chroma 61830) is utilized to emulate the grid voltage, which is distorted by the 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup>, 9<sup>th</sup> harmonics. The magnitudes of harmonics with respect to the grid fundamental voltage are 3%, 3%, 3% and 3%, respectively. Due to the PR term and the wide bandwidth of the system, the grid-injected current remains sinusoidal, which proves the proposed system has the strong ability to resist the adverse effect caused by the harmonic grid voltage.

# VII. DISCUSSION

## A. Comparison with the Existing SMC Methods

Through the experimental results, it can be observed that the proposed design guideline of robust PWM-SMC controller is practical. A summary of comparison between the existing SMC methods and the proposed SMC strategy is shown in Table II. The performance of the system is analyzed from the following perspectives: sampling frequency, switching frequency, number of measured state variables, power rating, *LCL* filter parameters and harmonic rejection.

1) HM-SMC

In [26], [27] and [28], the hysteresis modulator is utilized to ensure the switching of the HM-SMC system. The width of the hysteresis band controls the switching frequency. Among these three cases, there are two obvious advantages: fast dynamic response (which means high control bandwidth) and highly strong robustness. This is mainly because the hysteresis modulator directly realizes the switching of the system, which reflects the invariance of SMC theory. However, the issue of variable switching frequency introduced by the hysteretic modulator will impose a burden on the design of LCL filter. For example, in [27] and [28], the total inductance is 7 mH and 12 mH, respectively, which may cost much. In addition, in order to achieve a good control effect, a high sampling frequency is needed. For example, the sampling frequency is 125 KHz in [26], which need high requirements for digital control.

Compared with the above HM-SMC methods, the proposed control strategy adopts the pulse width modulation, where the switching frequency is fixed and the sampling frequency is low. Thus, the total inductance is 2.4 mH and the sampling frequency is 12 kHz. Although the dynamic response is not so fast as the one with HM-SMC, the experiment results show that the transient performance is acceptable.

2) PWM-SMC

In [29] and [30], the PWM-SMC is used for controller design, where the Lyapunov stability theorem is utilized to verify the stability of the system. However, this theorem cannot obtain the accurate stability region. Thus, in [29] and [30], the design of SMC controller parameters had not been presented in detail.

Compared with the above PWM-SMC methods, the proposed control strategy analyzes the essence of the SMC and then provides an accurate and detailed design guideline, which can help engineers to precisely design a highly robust PWM-SMC controller.

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COMPARISONS BETWEEN THE EXISTING SMC METHODS AND THE PROPOSED ONE									
		HM-SMC			PWM-SMC				
Comparison category		[26]	[27]	[28]	[29]	[30]	Proposed		
Sampling frequency 125 kHz		60 kHz	40 kHz	15 kHz	12 kHz	12 kHz			
Switch	ing frequency	18.8 kHz	6 kHz	6 kHz	15 kHz	12 kHz	12 kHz		
Measured variables		3	2	2	4	4	2		
Power Rating		3.3 kW	1.5 kW	1.5 kW	5 kW	3.2 kW	3 kW		
LCL filter	$L_1$	1.74 mH	5 mH	7 mH	1.2 mH	1 mH	1.2 mH		
	С	50 µF	6.8 <i>µ</i> F	6.8 <i>µ</i> F	50 µF	62 µF	6 <i>µ</i> F		
	$L_2$	0.6867 mH	2 mH	5 mH	0.4 mH	0.3 mH	1.2 mH		
Harmo	nic rejection	Not reported	Good	Good	Good	Not reported	Good		

TABLE II

In addition, the performance of the proposed PWM-SMC strategy is satisfactory, including the robustness, harmonic rejection and steady-state error. Owing to the implementation of discrete state observer, only two state variables are measured.

#### B. Comparison with the Full-state Feedback Controller

It is worth mentioning that the proposed PWM-SMC strategy utilized three state variables, which is in the same way as the full-state feedback controller. Indeed, from the perspective of digital implementation, there is a little similarity between the two control methods.

However, from the perspective of the physical significance, the two control methods are totally different. The SMC strategy provides a nonlinear view on the control of the power electronics, where the physical significance, for example as shown in Figs 2 and 4, is much more clear.

Further, in the proposed PWM-SMC control law, there are some feedforward terms, which can have some effects on improving the dynamic response and eliminating the steady-state error. However, since the lack of the global system modeling, the full-state feedback controller does not have these feedforward terms.

## VIII. CONCLUSION

This paper presents a novel step-by-step design guideline of robust PWM-SMC controller using linearized model for three-phase GCI with LCL filter. The conclusions can be drawn as following:

- There are many ways to realize switching in SMC 1) system. Hysteresis modulator and pulse width modulator are two special switching modes in power electronics, which can ensure the switching of the system without extra nonlinear term in the control law.
- 2) Compared with the HM-SMC controller, the size of output filter and the sampling frequency can be much smaller for the proposed strategy.
- Different from the traditional PWM-SMC based on 3) Lyapunov stability or phase plane, the proposed strategy has adopted the linear tool to design SMC parameters, where the accurate stability region can be exactly obtained, especially when the delay issue is addressed.

A 3 kW/3-phase/110 V experimental lab setup has been constructed to verify the feasibility of proposed PWM-SMC controller using linearized model design strategy. The experimental results show that the proposed PWM-SMC controller has the excellent robustness against the filter parameters shift and satisfactory dynamic performance, even under the condition of seriously distorted grid.

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