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Locating Emergency Shelters While Incorporating Spatial Factors

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Abstract

In the immediate response phase of a natural disaster, local governments and nonprofit agencies often establish shelters for affected populations. Decisions regarding at which locations to open shelters are made ad hoc based on available building inventory, and may result in high travel impedance to reach shelters and congestion. This thesis presents a shelter location optimization model based on the two-step floating catchment area (2SFCA) method. The 2SFCA method creates a shelter accessibility score for each areal unit (e.g., census block group) which represents the ability for persons in the unit to access shelter capacity with low travel impedance, relative to persons in other units competing for the same shelter capacity. A distance decay function within the 2SFCA method models the propensity of a person to visit a shelter based on the distance to the shelter. The optimization model recommends locations at which to open shelters so as to optimize some function of the 2SFCA accessibility scores. Three single-objective models and one bi-objective model are considered. Across all areal units, the alternative models: (i) maximize the sum of accessibility scores; (ii) minimize the disparity in accessibility scores; (iii) maximize the minimum accessibility score; and (iv) maximize the sum of all scores *and* minimize disparity. These models are demonstrated via a case study based on Hurricane Florence, which struck North Carolina in 2018. The optimization model outputs are compared with actual shelter openings during Hurricane Florence in four North Carolina cities, and also with outputs of classic p-Median and p-Center facility location models. Case study results demonstrate that, across the range of parameter values included in a sensitivity analysis, the bi-objective model achieves the best tradeoff between efficient and equitable shelter locations, while also achieving a higher minimum accessibility score than either of the two single objective models on their own.

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1. Introduction

Each year, approximately 70 volcanoes erupt (Smithsonian Institution National Museum of Natural History Global Volcanism Program, 2013), 50 hurricanes form (NCAR & UCAR News, 2010), 620 earthquakes with magnitudes over 5.5 occur (Endsley), and more than 1200 tornadoes touch down (NOAA). Natural disasters such as these impact an estimated 160 million people and are responsible for 90 thousand deaths annually (World Health Organization, 2012). Populations who are displaced before, during, or after a disaster may require public shelters. In 2017 alone, the American Red Cross opened over 1,100 emergency shelters and provided 658,000 overnight stays in response to 242 disaster or weather events (American Red Cross, 2017).

Post-disaster shelters are usually divided into two categories. Medical Special Needs Shelters (MSNS) are specialized shelters for people with chronic medical conditions and the level of service provided is similar to what would be provided in a hospital or nursing home (US Department of Health and Human Services, 2014). Mass shelters provide a safe place for victims who do not require extensive medical attention to receive food, water, cots, and first aid services, among other things (FEMA & American Red Cross, 2015; International Association of Venue Managers, Inc. & American Red Cross, 2010). Mass shelters also provide functional needs support services to people with communicative, mobility, cognitive, intellectual, and mental health disabilities (FEMA & American Red Cross, 2015). During Hurricane Katrina in Louisiana in 2006, 563 mass shelters and 10 MSNS were opened, serving almost 150,000 people combined. More recently, in response to Hurricane Dorian in 2019, 85 mass shelters and 25 MSNS were opened in Florida, and the American Red Cross reported a total of 171 shelters were in use across Florida, Georgia, and South Carolina (Kijewski-Correa et al., 2019). The focus of this thesis is location decision-making for mass shelters. From this point forward, we refer to mass shelters simply as shelters.

The choice of where to locate shelters in response to an event typically follows an ad hoc process at the local government level, such as a county or a city. The placement of these shelters may not consider the distances each person will travel and the potential demand at a shelter, and therefore could lead to high travel impedance, congestion, and/or the imbalance of demand across the sheltering sites. A shelter planning model which can simultaneously consider the supply-to-demand imbalances and the travel impedance could be useful for shelter planners.

Two classic facility location models which could be used to assist these planners are the p-Median and p-Center problems. The shortcoming of these models are they assign people to a shelter, which while easy to do, may be hard to implement. People may decide to go to a different shelter than they were assigned to for a variety of reasons, such as wanting to shelter with their family members or pets. Noncompliance with shelter assignment decisions can lead to unintended sub-optimal system behavior, such as higher travel distances and increased demand imbalance. Thus, a location model which does not assume a person will travel to the nearest shelter may be preferred by a planner.

Access to disaster response shelters may vary from person to person, depending on a number of factors. For example, the availability of a private mode of transportation, such as a personal vehicle, has been associated with greater ability to reach a shelter (Indrakanti, Mikler, O'Neill, & Tiwari, 2016). The spatial proximity between a displaced person and nearby shelters can also influence a person’s ability to reach a shelter, and limited shelter capacities can prevent a person from gaining access to them. The concept of *potential accessibility* can be used to describe the ability to receive sheltering services if needed. While shelters can be located as to maximize the potential accessibility of the population, both spatial and aspatial factors should be considered to fully understand the accessibility context. *Spatial factors* relate to distance-based variables which affect a person's access to a facility, such as the distance between a person and the nearest shelters, and the interactions between shelter capacities and demands across a geographic region. *Aspatial factors* are nongeographic variables related both to people's and facilities' characteristics, such as a person's access to transportation for reaching a shelter, whether a shelter allows pets, or how congested a shelter is. Together, these factors can help determine the ability and propensity of a person to visit one shelter over another.

Accessibility to a particular service has been studied in a variety of contexts. In healthcare, numerous studies have considered a population's access to medical services, such as primary care services and home healthcare agencies. Access to and attractiveness of emergency sheltering services increases the quality of urban life (Unal & Uslu, 2016). In the field of sustainable urban planning, accessibility of emergency sheltering is one of the top debated issues (Unal & Uslu, 2016). By focusing on achieving an equitable balance of supply and demand of emergency sheltering, one can reduce the spatial imbalance of sheltering services. Because both spatial and aspatial factors affect the potential accessibility of sheltering services, developing decision support models for shelter location planning that consider both spatial and aspatial dimensions of accessibility may improve quality of life for individuals in an area.

The first objective of this thesis is to develop a new measure for the potential accessibility of disaster response mass care shelters. The new accessibility metric will extend potential accessibility models from the healthcare services literature with the inclusion of spatial and aspatial factors specific to shelter-seeking behavior. The second objective of this thesis is to introduce a model that will determine an optimal shelter location configuration, as measured by the potential accessibility of those shelters. The potential accessibility measure and optimization models developed in this thesis are demonstrated via a case study based on Hurricane Florence, which struck the Carolinas in September 2018. The shelter location configuration recommended by our new models are contrasted with the actual location configuration of shelters opened in response to Hurricane Florence.

The unique contributions of this thesis are fourfold. First, a novel formulation of distance decay is proposed. Second, the two-step floating catchment area method (2SFCA) is adapted to the context of shelter locations by incorporating spatial factors. Third, while the focus of the modeling in this thesis is on spatial factors, we provide discussion on a set of aspatial factors which may be relevant to shelter-seeking behavior. We relate how these aspatial factors have been incorporated into 2SFCA variants in the literature. Fourth, an optimization model for choosing disaster response shelter locations for a specified accessibility measure is created and demonstrated in the context of a case study based on a recent hurricane event.

The remainder of this thesis is organized as follows. Section 2 provides a review of the potential accessibility measures in the literature as well as current shelter location models. In Section 3, the new shelter accessibility metric and optimization model are presented. Sections 4 and 5 provide the case study definition and results, respectively. Finally, key findings and suggested future research areas are discussed in Section 6.

2. Literature Review

In Section 2.1, measures of potential accessibility are discussed with a focus on how spatial and aspatial factors have been incorporated into past models. Section 2.2 discusses the current state of shelter location models in the operations research and industrial engineering fields.

2.1 Measures of Potential Accessibility

One way to measure shelter accessibility is to use the two-step floating catchment area method (2SFCA) (Luo & Wang, 2003). The 2SFCA is derived from a gravity-based accessibility model, which is a spatial interaction model. Unlike distance-based models, spatial interaction models consider person-to-service location distances as well as facility capacities and demand magnitudes when computing numerical accessibility scores for demand units. Section 2.1.1 provides additional details regarding the 2SFCA, and Sections 2.1.2 and 2.1.3 discuss how spatial and aspatial dimensions of accessibility have been incorporated into 2SFCA variants in the literature.

2.1.1 The 2SFCA

The 2SFCA was originally developed to improve the categorization of a person's potential spatial accessibility to health care facilities, however, it can be generalized as a measure of potential spatial accessibility of a given service over a geographic area. The 2SFCA uses two equations to find the accessibility of a demand center to a service where I is the set of demand centers and J is the set of candidate facility sites (Luo & Wang, 2003):

$$R_j = \frac{S_j}{\sum_{\{i\ in\ I:\ d_{ij} \leq d_0\}} P_i} \qquad \forall\, j \in J, \tag{1}$$

$$A_i = \sum_{\{j \text{ in } J:\, d_{ij} \leq d_0\}} R_j \qquad \forall\, i \in I. \tag{2}$$

Equation (1) computes the supply-to-demand ratio, R_j, for each facility j, where S_j represents the facility supply (i.e., capacity). The denominator of Equation (1) computes the total demand placed on facility *j* by summing the demand P_i of each demand center *i* for which the distance from *i* to *j* (d_{ij}) is at most the threshold distance d_0. Demand centers satisfying this distance condition are referred to as being inside the facility's catchment. Equation (2) computes a potential accessibility score A_i for each demand center *i* by summing the R_j values across all the facilities in its catchment.

2.1.2 Spatial Factors

The following subsections discuss the spatial elements of distance decay and variable catchment sizes, which are two ways to model how the distance between a person and a shelter affect the likelihood of a person visiting a particular shelter. Two notable 2SFCA variants in which these elements have been incorporated will also be discussed.

2.1.2.1 Distance Decay

Distance decay can be defined as the decreasing propensity of a person to travel to a service location as the distance to that location increases. The original formulation of 2SFCA includes a binary decay. That is, a demand center either is within the catchment of a service location or not. There is no decreasing propensity to visit alternative service locations within the catchment that are at differing distances from the demand center. The introduction of more sophisticated decay functions is common among 2SFCA variants in the literature (e.g. see (Luo & Qi, 2009; McGrail

& Humphreys, 2009; Wan, Zou, & Sternberg, 2012)). The decay functions are used in methods which either assume discrete or continuous decay. Examples of 2SFCA variants in the literature that employ continuous decay include McGrail & Humphreys (2009), McGrail & Humphreys (2015) and Wang (2018). In these, the only points that share an identical decay value are those equidistant to a service location. In contrast, discrete decay methods split a catchment into zones and points within the same zone share a decay value. Examples from the literature include Li, Serban and Swann (2015), Luo and Qi (2009), and Wan, Zou and Sternberg (2012).

The distance decay functions used to model the rate of decay can be power, exponential, Gaussian, log-logistic, or kernel functions (Wang, 2018). Exponential decay functions have the advantage of only requiring tuning for a single parameter (Tang, Chiu, Chiang, Su, & Chan, 2017; Wang, 2018). Its use has been justified for applications in which people are willing to travel long distances to reach a service, as the upper tail of the distribution covers a broad range of distance values (Li, Serban, & Swann, 2015). On the other hand, Gaussian functions can model normally distributed decay (Shi, Alford-Teaster, Onega, & Wang, 2012). The Gaussian function can also be approximated via a logistic function (Bauer & Groneberg, 2016). Alternatives to exponential and Gaussian rate of decay functions also exist. For example, McGrail & Humphreys (2015) formulate decay to include a window of indifference, which represents a range of distance a person is willing to travel before decay sets in. Letting x be the window of indifference, d_{max} the maximum allowable travel distance, and d the distance between a demand center and service location pair, the decay function $f(d)$ from McGrail & Humphreys (2015) is given in Equation (3):

$$f(d) = \left(\frac{d_{max} - d}{d_{max} - x}\right)^{1.5}. \tag{3}$$

McGrail & Humphreys (2015) used ten minutes of travel for the window of indifference, but this parameter can be adjusted based on the application. The parameter d_{max} is allowed to change based on a demand center's catchment, meaning the rate of decay is tailored for each demand center.

2.1.2.2 Variable Catchment

A *catchment* represents the radius a person is willing to travel to reach a service location and may differ from person to person. McGrail & Humphreys (2015) incorporates different catchment radii for rural and urban environments. McGrail & Humphreys (2009) bases catchment size on two terminating criteria: either a maximum distance is reached, or there are 100 service locations within the catchment. A Variable 2SFCA (V2SFCA) model introduced in the literature bases variable catchments on the supply-to-demand ratios within each catchment (Luo & Whippo, 2012). The catchments increase in size until a certain supply-to-demand threshold is reached.

In the above examples, catchments are based on geographic characteristics of a demand center – whether it is urban or rural or how many service locations are nearby. Alternatively, catchments can be based on demographic profiles and how travel behavior varies across those profiles. For example, Paez et al. (2010) examined the relationships between personal characteristics and the willingness or ability to travel. Income and vehicle ownership were found to be positively correlated with travel time, while family size was negatively correlated. Based on an analysis of the 2003 Montreal Household Travel Survey, the authors fit a logarithmic equation to predict a person's travel bandwidth (d_i) based on their personal profile. The logarithmic equation given in Equation (4), where i is a person, N_i is income in thousands, E_i indicates whether a person is a senior, V_i indicates whether a person owns a vehicle, θ and β are coefficients, ε_i is a residual term, and u_i and v_i are contextual factors:

$$\log(d_i) = \beta_0 + \beta_1 N_i + E_i(\theta_{11} + \theta_{12}u_i + \theta_{13}v_i) + V_i(\theta_{21} + \theta_{22}u_i + \theta_{23}v_i) + \varepsilon_i \qquad \forall\, i \in I\,. \quad (4)$$

Paez et al. (2010) uses the bandwidths from Equation (4) in a measure of accessibility which does not consider supply-to-demand ratios. To our best knowledge, the incorporation of personal bandwidths into the 2SFCA does not exist in the literature at the time of this writing.

2.1.2.3 Notable 2SFCA Variants

Two of the most notable variants of the 2SFCA are the Enhanced Two-Step Floating Catchment Area (E2SFCA) and the Three-Step Floating Catchment Area (3SFCA) methods. These methods are often referenced in the literature in discussions of discussing catchments and/or distance decay (e.g. see (Li et al., 2015; McGrail & Humphreys, 2015; Tang et al., 2017; Wang, 2018)). These methods extend the 2SFCA by incorporating distance decay in alternative ways.

The E2SFCA uses a catchment of 30 minutes as past healthcare research has used a 30 minute catchment (Luo & Qi, 2009). Within the 30 minute catchment, there are three equally sized travel zones of 0-10, 10-20, and 20-30 minutes (Luo & Qi, 2009). While the researchers link the idea of three travel zones to an unpublished model, the model they reference did not equally divide the area into equally sized zones. Each of the three zones has a different distance decay weight, with the weights decreasing the farther away a point is from a facility. For this method, r represents the travel zone, W_r represents the decay weight for zone r, and D_r is the maximum catchment distance for zone r. The two equations for the E2SFCA method are:

$$R_j = \frac{S_j}{\sum_{r=1}^{3} \sum_{\{i \; in \; I: \; d_{ij} \le D_r\}} P_i \, W_r} \qquad \forall \, j \in J, \tag{5}$$

$$A_i = \sum_{r=1}^{3} \sum_{\{j \; in \; J: \; d_{ij} \le d_0\}} R_j W_r \qquad \forall \, i \in I. \tag{6}$$

Both of the equations for the E2SFCA include the distance decay weight W_r, modeling the assumption that when a person is farther from a facility, the person is less likely to travel to it, and when a facility is farther from a person, the facility is less likely to be visited by that person.

Like the E2SFCA, the 3SFCA has three ten-minute catchments, however, it also has a fourth 30-minute catchment, added to include rural areas (Wan et al., 2012). In general, this fourth catchment should be much larger than other catchments if the purpose of the catchment is to include isolated rural areas (Wan et al., 2012). The equations for the 3SFCA method are:

$$G_{ij} = \frac{T_{ij}}{\sum_{\{k \; in \; J: \; d_{ik} < d_0\}} T_{ik}} \qquad \forall \, i \in I, j \in J, \tag{7}$$

$$R_j = \frac{S_j}{\sum_{r=1}^{4} \sum_{\{i \; in \; I: \; d_{ij} \le D_r\}} G_{ij} P_i \, W_r} \qquad \forall \, j \in J, \tag{8}$$

$$A_i = \sum_{r=1}^{4} \sum_{\{j \; in \; J: \; d_{ij} \le d_0\}} G_{ij} R_j W_r \qquad \forall \, i \in I. \tag{9}$$

Equation (7) determines the selection weight, G_{ij}, by dividing the assigned Gaussian weight of T_{ij} by the sum of Gaussian weights associated with a demand center. The selection weight represents a travel-time-based competition weight between a demand center and a facility. The Gaussian weights for T_{ij} are based on the Gaussian weights associated with a catchment zone, W_r. Equations (8) and (9) are analogous to equations (1) and (2), however they now include the selection weights.

2.1.3 Aspatial Factors

In Section 2.1.3.1, literature related to healthcare needs of sheltering and how it can be included in the 2SFCA will be discussed. Section 2.1.3.2 provides an overview of how shelter attractiveness can be formulated. Finally, Section 2.1.3.3 discusses how congestion can be incorporated into the 2SFCA.

2.1.3.1 Healthcare Needs

As shelters provide healthcare and functional needs support services, a population with high healthcare and functional support needs may cause the effective burden of work on staff at a shelter to increase. The current standard for shelters is to have 1 nurse for every 50 people (International Association of Venue Managers, Inc. & American Red Cross, 2010). This standard does not differentiate between the presenting case mix from one shelter to another.

A method to adjust the 2SFCA method to account for varying levels of health needs exists in the public health literature. It uses health needs as a multiplier to increase the effective demand a population places on a facility. Letting HC_i represent healthcare needs, the modified 2SFCA is as follows (McGrail & Humphreys, 2009; McGrail & Humphreys, 2015; Tang et al., 2017)**:**

$$R_j = \frac{S_j}{\sum_{\{i \text{ } in \text{ } I: \text{ } d_{ij} \le d_0\}} P_i HC_i} \qquad \forall \, j \in J. \tag{10}$$

Parameter values for HC_i of less than 1 decrease the effective demand a population center *i* places on facility j, and values greater than 1 increase it.

2.1.3.2 Shelter Attractiveness

Shelters are not uniform structures and each have their own size, layout, and policies, like whether pets are allowed (Douglas, Kocatepe, Barrett, Ozguven, & Gumber, 2019). As such, the size and policies of a shelter may affect how attractive a particular shelter is to a person. *Shelter attractiveness* can be defined as the desirability of a shelter for an individual based on the shelter's policies and services. One way to model shelter attractiveness is to view the shelters as competitors, allowing the use of a competition model. One such method is proposed by Huff (1964), and has been used to model the location of preventative health care facilities. The Huff-based competition model was developed to determine consumer preferences for retail stores and differs from past gravity-based retail models by focusing on the consumer, not the store, as the consumer is the one who is making the decision. Unlike past models, the Huff Model is not empirically derived but is rather "a theoretical abstraction of consumer spatial behavior," meaning the model may be suitable for use in other environments than retail competition (Huff, 1964). Letting C_{ij} represent the probability consumer i will go to facility j, Z_j represent the size of a facility, T_{ij}^{λ} represent the travel time to the facility, and λ a parameter which affects the importance of travel time for the specific type of trip, the Huff Model is as follows:

$$C_{ij} = \frac{\frac{Z_j}{T_{ij}^{\lambda}}}{\sum_{k \in J} \frac{Z_k}{T_{ik}^{\lambda}}} \qquad \forall\, i \in I, j \in J. \tag{11}$$

2.1.3.3 Congestion

Congestion refers to how busy a shelter is and represents a stress that is applied to a shelter. It is categorized in the literature as an aspatial dimension of access (Tang et al., 2017). Congestion has been incorporated into the 2SFCA in varying ways. Tang et al. (2017) define congestion as the ratio of the people visiting a facility to the total number of people visiting any facility. Using this definition, congestion can be incorporated as a variable into the first step of the 2SFCA. In contrast, Li et al. (2015) model congestion as the reciprocal of R_j in the first step of the 2SFCA. As a reminder, R_j is the supply-to-demand ratio, so the inversion is the demand-to-supply ratio. Li et al. (2015) then defines an alternative accessibility, where A_i is the percent of visits assigned to facility i divided by the congestion.

2.2 Shelter Location Models

In Section 2.2.1 and 2.2.2, the concepts of equity and user equilibrium in the disaster sheltering field is explored. Section 2.2.3 discusses single objective facility location models. In section 2.2.4, an overview of hierarchy models in disaster sheltering is discussed. Finally, section 2.2.5 provides an overview of multi-objective shelter location models.

2.2.1 Equity Models

Equity in terms of facility location models can be defined as "when each group receives its fair share of the effect of the facility siting decision," however there is disagreement on how equity should be measured (Marsh & Schilling, 1994). Marsh and Schilling (1994) provides a review of how equity is incorporated in facility location models. Their review includes 20 metrics of inequity published in the timeframe of 1912 to 1992. These metrics range from center problems, where one aims to minimize the inequity of the people the worst off, to the Gini coefficient, used in economics and social welfare, to the range of inequity, the difference between the best and worst off. While all of these are valid ways to measure equity, the authors mention seven characteristics of a good equity measure. These measures are: analytic tractability; appropriateness; impartiality, whether the condition or status of a person affects the solution; the Pigou-Dalton Principle or the principle of transfers, which states a solution is less equitable if the best off becomes better off at the expense of the worst off becoming worse off; scale invariance; Pareto optimality, a balance between equity and efficiency; and normalization. These seven characteristics may not be suitable for every situation, with the authors stating the Pigou-Dalton Principle may not be appropriate for location models. We employ several of these inequity metrics in the models tested in this paper, including minimizing the inequity of the worst off people, minimizing the range of inequity, and balancing equity with efficiency.

2.2.2 User Equilibrium Models

One common user equilibrium definition in disaster sheltering models is the Wardrop Equilibrium (Gutjahr & Dzubur, 2016; Kongsomsaksakul, Yang, & Chen, 2005). The Wardrop Equilibrium has two principles: (1) people will pick the route with the travel time which is equal to or less than all other routes, and (2) the average travel time is minimized (Bayram, 2016). An alternative to the Wardrop Equilibrium are stochastic user equilibrium and the system optimal solution, which are respectively analogous to the first and second principles (Bayram, 2016). In reality, people may not make optimal routing decisions due to their lack of knowledge of factors like the traffic across a network (Bayram, 2016). The drawback of these models for this thesis is the models assign people to a shelter. As discussed previously, this stringent assumption regarding human behavior is likely inappropriate for shelter location models.

2.2.3 Single Objective Facility Location Models

Three examples of single objective facility location models in the literature include the p-Median, p-Center, and the set covering problem. The objective of a p-Median problem is to minimize the demand-weighted travel distance of demand points to their nearest supply points, and is used widely in the public and private sectors for site selection (Jia, Ordóñez, & Dessouky, 2007; Ma, Xu, Qin, & Zhao, 2019). In the context of sheltering, the p-Median problem could be used to minimize the total travel distance of persons to their nearest shelters. The objective of the p-Center problem is to minimize the maximum distance a demand unit must travel to a supply point (Jia et al., 2007; Ma et al., 2019). Ma et al. (2019) states the p-Center model may not be suitable for location decisions during disasters. Both the p-Center and p-Median problems require assignment decisions between demand centers and shelter sites; further, they only consider the distance from a person to their nearest shelter, which neglects the possibility of bypassing. As discussed previously, this stringent assumption regarding human behavior is likely inappropriate for sheltering applications.

The objective of the set covering problem is to minimize the cost to open facilities which cover all of the demand points (Ma et al., 2019). One variant of the set covering problem is the maximal covering problem, which aims to maximize the number of people in areas served by the opened facilities. In the context of sheltering, the maximal covering problem could be used if a disaster response agency had limited resources and wanted to place the resources so that the maximum number of areas could use the resources. We do not consider set covering or maximal covering models in this thesis as they neglect supply and demand balance considerations.

2.2.4 Hierarchy Models

In the disaster sheltering literature, bi-level models are one of the most common forms of hierarchy models. A bi-level model has an upper and lower level, with the solution of the upper level being dependent on the solution of the lower level. The two levels have different objective functions, with the lower level often related to user equilibrium, while the upper level minimizes total travel distance, cost of opening shelters, or uncovered demand (Gutjahr & Dzubur, 2016; Kongsomsaksakul et al., 2005; Ng, Park, & Waller, 2010). To solve these models, researchers in the disaster sheltering field typically use heuristics (Gutjahr & Dzubur, 2016; Kongsomsaksakul et al., 2005; Ng et al., 2010). Like user equilibrium models, hierarchy models assign people to a shelter, so a hierarchy model is not appropriate for this thesis.

2.2.5 Multi-objective Shelter Location Models

Multi-objective shelter location models build on established single-objective models by introducing two or more objectives simultaneously, with some models additionally incorporating evacuation routing (Esposito Amideo, Scaparra, & Kotiadis, 2019). Example objectives that have appeared in bi-objective shelter location models include: minimizing travel distance, minimizing the risk of traveling, minimizing the fire risk in shelters, minimizing cost, minimizing human suffering, and maximizing coverage (Esposito Amideo et al., 2019). At the time of this writing, there are no papers in the literature we are aware of which use spatial accessibility metrics in the context of disaster response shelter location decisions.

3. Methodology

Section 3.1 discusses which spatial factors are included in the quantitative models in this thesis. As mentioned previously, the inclusion of aspatial factors is reserved as an area of future work. Instead we focus on comparing outputs from spatial accessibility-based optimization models with classic location models from the literature. Section 3.2 introduces the optimization model for choosing shelter locations for an accessibility-based objective function.

3.1 New Potential Accessibility Measure

In examining the factors previously discussed, distance decay will be included in the developed model for this thesis. As distance decay is important to measuring access to healthcare services, it would follow that distance decay is also important to sheltering accessibility. While there is not one definitive distance decay formulation that is used, arguments could be made for using either continuous or discrete decay, as well as a Gaussian or Exponential function. Variable catchments will not be incorporated into the model as there is a lack of empirical evidence in the literature regarding appropriate catchment sizes for shelter-seeking behavior. Conducting a primary data collection to determine this ourselves is outside the scope of this thesis.

As a starting point for the model, we can use the two equations from the 2SFCA (Equations 1-2) to help measure the potential accessibility of an area, however, the equations will need to be changed in order to incorporate distance decay. Additionally, as only a certain number of shelters can be opened, there needs to be a way to limit shelter openings in the developed model.

3.1.1 Distance Decay

Including distance decay in the 2SFCA requires choosing between discrete or continuous decay, and choosing a function and parameter values which most closely model human behavior when seeking shelter during a disaster. Unfortunately, empirical data to support this choice is not available in the literature. We justify our choices below as best we can using the available literature and our own intuition. As such, the models we produce should be viewed only as an important first step to measuring shelter spatial accessibility. The quality of model outputs will improve as higher quality inputs become available.

The Gaussian function is used to model distance decay as the exponential function may be appropriate for when people are willing to travel far for services (Li et al., 2015), which may not be the case for sheltering. When using discrete distance decay, one has to create the bounds of the zones within a catchment. While in the healthcare literature this commonly consists of at least three 10-minute zones (Luo & Qi, 2009; Wan et al., 2012), having three 10-minute zones may not model shelter-seeking behavior. Additionally, when using discrete distance decay there are points which are next to each other which can have significantly different distance decay values, which may not model shelter-seeking behavior. Thus, a continuous Gaussian distance decay function is used in this thesis.

To stay consistent with the literature, distance decay serves as a multiplier to decrease the magnitude of demand a center places on a facility as the distance between the demand center and facility increase; a value of 1 represents no decay and a value of 0 represents full decay. The formula for the Gaussian function where a is the height of the peak, b is the center of the peak, and c is the width of the bell is:

$$f(d_{ij}) = ae^{-(d_{ij}-b)^2 / 2c^2}. \tag{12}$$

In papers which use and provide the Gaussian function formulation, a is set to 1 (Shi et al., 2012; Wan et al., 2012). By setting a to 1 and assuming b is 0, the maximum value of the function when $d_{ij} \geq 0$ is 1 when d_{ij} equals 0. This means when no distance is traveled, no decay is experienced. Because b is the center of the peak, by changing this value the distance where the function is equal to 1 is modified. For example, if b is set to 10, then at d_{ij} = 10, the function evaluates to one. A piecewise function is used so when d_{ij} is less than b, the value will be one. Because b impacts where the decay starts, we can view b as our window of indifference. This means we can use this concept from McGrail & Humphreys (2015) while still using the Gaussian function. Finally, c is the width of the bell and is used to control the rate of decay (McGrail & Humphreys, 2015). A small value will produce quick decay, while a large value will produce slow decay. The formulation of the Gaussian function used for this thesis is provided in equation below, where $decay_{ij}$ is the distance decay between centers i and j:

$$decay_{ij} = \begin{cases} 1 & if\ d_{ij} < b \\ e^{-(d_{ij}-b)^2 / 2c^2} & o.w. \end{cases} \qquad \forall\ i \in I, j \in J. \tag{13}$$

To the best of our knowledge, no formulation of a piecewise Gaussian distance decay function with a window of indifference exists in the literature. Distance decay is included in the 2SFCA model by substituting $decay_{ij}$ in for d_{ij} in the two equations for the 2SFCA (Equations 1-2). Section 4.3 discusses the choice of b and c parameters to include in a sensitivity analysis for this thesis.

3.2 Optimization Model

The formulation of the linear program is as follows where y_j is a decision variable representing whether facility j is opened and n is an input parameter representing the number of shelters to open. One can think of n as the p in the p-Median and p-Center problems.

$$
\begin{array}{llll}
\text{opt} & f(A_i) & & (14) \\
\text{s.t.} & y_j \dfrac{S_j}{\sum_{i \in I} P_i decay_{ij}} = R_j & \forall\, j \in J & (15) \\
 & \displaystyle\sum_{j \in J} R_j decay_{ij} = A_i & \forall\, i \in I & (16) \\
 & \displaystyle\sum_{j \in J} y_j = n & & (17) \\
 & A_i, R_j \geq 0 & \forall\, i \in I, j \in J & (18) \\
 & y_j \in \{0,1\} & \forall\, j \in J & (19)
\end{array}
$$

The objective function (14) optimizes some function $\boldsymbol{f}$ of accessibility scores, and will change depending on which accessibility metric is used. Constraints (15) and (16) mirror the two equations from the 2SFCA, modified to include the decay parameter, and decision variables. Constraint (17) ensures exactly n facilities will be opened. Constraint (18) enforces A_i and R_j are greater than or equal to zero, and constraint (19) enforces y_j to be a binary decision.

Three accessibility-based objective functions are created for this thesis:

$$f_1(A_i)\colon \max \sum_{i \in I} A_i\,, \qquad (20)$$

$$f_2(A_i)\colon \min(\max_{i\ in\ I} A_i - \min_{i\ in\ I} A_i)\,, \qquad (21)$$

$$f_3(A_i)\colon\ \max(\min{}_{i\ in\ I} A_i)\,. \qquad (22)$$

The objective in Equation (20) maximizes the sum of all accessibility scores. The objective in Equation (21) minimizes the difference between the highest and lowest accessibility scores, and as such, it attempts to minimize equity disparity. The objective function in Equation (22) maximizes the minimum accessibility score, and as such, tries to improve the score of the worst-off spatial unit. A bi-objective model is also created which combines $f_1(A_i)$ and $f_2(A_i)$. Section 5.3 provides discussion on why these two objectives are selected. In addition to the accessibility-based models, the p-Median and p-Center models are also included for this thesis.

A total of five single-objective models are solved for each city in the case study (three accessibility models, two classic facility location models), as well as one bi-objective model. Each of the five single-objective models are evaluated using the objective functions of the other four models for comparison purposes. They are also compared with the actual solution that was implemented during Hurricane Florence, as well as a best compromise solution from the bi-objective model.

4. Case Study

Hurricane Florence made landfall on September 14, 2018, near Wrightsville Beach, North Carolina and resulted in a state of emergency being declared in North Carolina, South Carolina, Virginia, Georgia, Maryland, and Washington D.C (Huber, 2018). Within a few days of landfall, over 5 million people were impacted by 10 inches or more of rain, and over a million homes lost power (Resnick, 2018). Florence broke the previous record in North Carolina for the most rain from a single storm, with Elizabethtown, North Carolina, experiencing the most rain of 35.93" (Martinez, 2018; US Department of Commerce, NOAA, 2019). Florence was responsible for 51 deaths, and in North Carolina alone, at least $1.3 billion of federal funds were allocated to deal with the impact of the Hurricane (Borter, 2018; FEMA, 2018). The case study developed for this thesis includes four cities in North Carolina: Fayetteville, Greenville, Jacksonville, and Wilmington.

In Section 4.1, the potential shelter location data is introduced. Section 4.2 discusses the population and size of the block groups used, and Section 4.3 provides an overview of the distance between block groups and potential shelter locations. Section 4.4 discusses how the b and c parameters for distance decay were estimated for this thesis. The data gathered for this case study are provided in the online open-source repository Mendeley (Taylor, 2019).

4.1 Potential Shelter Location Data

The American Red Cross National Shelter System (NSS) contains information for over 56,000 potential shelter facilities (American Red Cross, 2018). The NSS is used to report statistics such as shelter capacity and the estimated number of residents in opened shelters during a disaster (American Red Cross, 2018). *Table 1* provides summary statistics for the potential shelter locations from the NSS in the four cities in this case study. Row 1 provides the number of available shelters in the NSS inventory for each city, and row 2 provides the number of shelters which were opened. Row 3 provides the average post impact capacity (number of people a shelter can serve) of shelters in a city, and row 4 provides the average area of shelters in a city for shelters for which an area was reported.

Table 1. Summary of Red Cross Shelter Data

	Fayetteville	Greenville	Jacksonville	Wilmington
Available Shelters	17	21	7	20
Shelters to Open	9	6	3	9
Post Impact Capacity (Number of People)	182	133	107	185
Area (ft^2)	7349	6627	8773	6893

In addition to buildings from the NSS inventory, public schools in an area can serve as additional potential shelter sites (Hale et al., 2009). In this thesis, all of the public elementary, middle, and high schools are included as potential shelter locations in each city. Square footage and capacities could not be obtained for these new schools, so missing capacities were imputed as the average capacity of public schools buildings available as shelters in the NSS database for the four cities. There are a total of 32 public schools in the NSS database across the four cities, representing almost half of the shelters. These schools have a post impact capacity ranging from 50 to 605 people, with a mean and standard deviation of 207 and 139 people, respectively. In *Table 2*, the first row provides the number of public schools outside of the NSS database in each city and the second row indicates the imputed shelter capacity in number of people served.

Table 2. Summary of Public School Shelter Data

	Fayetteville	Greenville	Jacksonville	Wilmington
Additional Schools	58	11	20	29
Post Impact Capacity (Number of People)	207	207	207	207

4.2 US Census Data

Block groups are used for this research as it was the smallest level of population aggregation available for the selected areas. The land area and latitude/longitude coordinates of the centroid for each block group in the cities are from the TIGER/Line dataset (U.S. Census Bureau, 2016a).

The American FactFinder is used to obtain the 2016 estimate of the population in each block group (U.S. Census Bureau, 2016b). Demand for each block group is aggregated at its centroid. *Table 3* summarizes the data from the U.S. Census. The radii values reported in *Table 4* are computed from the land areas and assume block groups are circular in shape.

Table 3. Summary of U.S. Census Data for Selected Block Groups

	Land Area (mi^2)	Population	Density (people/mi^2)	Radius (mi)
Average	2.03	119	191.38	0.80
Standard Deviation	5.56	76	195.08	1.33
Max	54.26	807	2511.83	4.16
Min	0.10	18	1.44	0.18
Range	54.16	789	2510.39	3.98

4.3 Google API Distance

A distance matrix can be computed for the block groups and shelter location pairs by using the Google Cloud Platform's Distance Matrix API. The API can take a set of either latitude/longitude coordinates or addresses as input and return the road-network distance between pairs of points. Default settings were used for the API, meaning for each block group and shelter-location pair, the API returns the distance of the path from the demand location to the shelter through the road network which results in the shortest travel time. *Table 4* provides summary statistics for the block group to shelter location travel distances for each city. For example, the average distance between all block group centroids in Fayetteville to all potential shelter locations in Fayetteville is 8.14 miles.

Table 4. Summary of Distance Matrix In Miles

	Fayetteville	Greenville	Jacksonville	Wilmington
Average	8.14	8.32	6.42	6.05
Standard Deviation	4.36	5.51	3.37	3.02
Max	26.88	31.27	17.13	20.43
Min	0.00	0.10	0.13	0.10
Range	26.88	31.17	17.00	20.32

4.3 Estimating the *b* and *c* Parameters

As a reminder, *b* is the window of indifference and *c* controls the rate of decay for the distance decay equation. For this thesis, sensitivity analysis is conducted for a range of *b* and *c* parameter values. Empirical data regarding people's actual travel distances to reach shelters in past disasters is not available in the literature for estimating realistic values for the *b* and *c* parameters. Instead, *b* and *c* parameters are estimated by combining information from 2SFCA case studies in the literature, what is known about travel behavior to grocery stores, and the author's own intuition. Sections 4.3.1 and 4.3.2 explain how the *b* and *c* parameters are estimated for this thesis.

4.3.1 Selection of *b* Values for Sensitivity Analysis

Three parameter values for *b* are included in our sensitivity analysis to represent an appropriate range of windows of indifference. These values are 0, 2, and 4 miles. Zero was selected to represent the case where there is no window of indifference. According to the USDA, the average distance between all Americans and their closest SNAP-authorized grocery store is 2.14 miles and the average distance between a person and their preferred grocery store is 3.79 miles (Mentzer Morrison & Mancino, 2015). These distances are rounded to their nearest mile to obtain a window of indifference of 2 and 4 miles. Empirical data is available for travel times to hospitals, however the number of shelters distributed across a region is more similar to the number of grocery stores than to hospitals, which often serve a very large catchment.

4.3.2 Selection of c Values for Sensitivity Analysis

Nine parameter values of c are included in our sensitivity analysis to provide a variety of decay rates. Let Z denote the maximum distance a person is willing to travel to reach a service. The rate of decay parameter, c, is related to Z, as the decay function should asymptotically reach zero near distance Z on the x-axis (Wan et al., 2012). For the 3SFCA, the distance decay for the fourth catchment is always greater than 0.01 (Wan et al., 2012). Because this distance Z is easier to conceptualize than rate of decay c, we focus our efforts on determining meaningful values for Z, from which c is then computed. To find c from b and Z, the following formula can be used:

$$c = \sqrt{\frac{-(Z-b)^2}{2\ln(0.01)}}. \tag{23}$$

To the best of our knowledge, the literature does not contain any instances where the maximum distance people are willing to travel for sheltering services is presented. In the general body of 2SFCA literature, not pertaining to shelter services, there are two instances of travel times found. Specifically, a 30-minute maximum travel time is used in the E2SFCA, and a 60-minute maximum travel time is used in the 3SFCA (Luo & Qi, 2009; Wan et al., 2012). We disregard the 60-minute maximum travel time as this results in a travel radius which is larger than each of the four cities included in our case study. Instead, we combine a 30-minute maximum travel time with two alternative driving speeds to arrive at estimates for the maximum travel distance, Z. A 20 mile per hour speed, which is recommended by the National Association of City

Transportation Officials for neighborhoods, results in a 10 mile distance. A 35 mile per hour speed, which is recommended by the same organization as a maximum speed on urban and arterial streets, results in a 17.5 mile distance. Because the cities in the case study include highways in addition to urban and arterial streets, we choose distances of 15 and 20 miles to include in the case study instead of 17.5 miles. Thus, the three values of Z we consider are 10, 15, and 20 miles. These values of Z result in the c values provided in *Table 5*. Note as the distance matrix is in meters, meters are used for the calculation of c values but the values in *Table 5* are provided in miles.

Table 5. c Values In Miles by b and Z

	Z = 10 miles	Z = 15 miles	Z = 20 miles
b = 0 miles	3.30	4.94	6.59
b = 2 miles	2.64	4.28	5.93
b = 4 miles	1.98	3.62	5.27

5. Results

The results are organized as follows. Section 5.1 discusses the sensitivity analysis of the b and c parameters. The implementation of a bi-objective model for this thesis is presented in Section 5.2. In Section 5.3, the seven models are compared. Finally, Section 5.4 presents summaries of the impact of non-NSS inventory shelters as well as the shelters selected to be open by the optimization models which were not opened during the historical disaster event. For the ease of reading, the models will be referred to by their abbreviations in *Table 6*.

Table 6. Model Abbreviations

Model #	Abbreviation	Description
1	*Imp*	Implemented (during actual disaster event)
2	*pMed*	p-Median (from traditional facility location literature)
3	*pCen*	p-Center (from traditional facility location literature)
4	*sumAi*	Maximize sum of accessibility (objective function f_1)
5	*EqDisp*	Minimize equity disparity (objective function f_2)
6	*maximin*	Maximize minimum accessibility (objective function f_3)
7	*bi-obj*	Bi-objective model (combines objectives f_1 and f_2)

Results are collected for each city, model, and b and c pair. For naming sake, the Z value takes the place of the c parameter. A test instance is defined as a b and Z combination for a city. The instances will be referred to by the first letter of a city's name and a number. *Table 7* lists out the test instances for Fayetteville.

Table 7. Test Instance Naming Convention

Instance #	b (in miles)	Z (in miles)
F1	0	10
F2	2	10
F3	4	10
F4	0	15
F5	2	15
F6	4	15
F7	0	20
F8	2	20
F9	4	20

When discussing a model and instance, the model abbreviation will precede the instance number. For example, if discussing the *Imp* model and test instance F1 where b=10 and Z=10, the name for this would be *Imp*_F1. Test instances for other cities are referred to with the letter "G" for Greenville, "J" for Jacksonville, and "W" for Wilmington in place of "F" for Fayetteville.

In the following sections, the objective function values of the five single-objective models are presented. As Fayetteville is the city with the most number of block groups and tied with the most number of shelters to be opened, and Jacksonville is the city with the least number of block groups and the least number of shelters to be opened, primarily these two cities are discussed in this section. Full lists of objective values and the shelters selected to be opened for each instance are provided in Appendix A.

5.1 Discussion of *b* and *c* Parameters

This section discusses the sensitivity analysis performed on the b and c parameters for the distance decay function. As R_j and A_i are a function of distance decay, changing a parameter in the distance decay function may cause R_j and A_i to change. This change may lead to the *sumAi*, *EqDisp*, and/or the *maximin* models to recommend a different set of shelters to be opened. The shelters selected to be opened are provided in Appendix A, and a summary of the percent of shelter locations changed is provided in *Tables 8* and *9*. These percentages were found by dividing the average number of shelters changed by the number of shelters opened. For example, row 1 of the table provides a comparison between instances where b is 0 with those where b is 2. Specifically, the 7% is computed by taking the average percentage of opened shelter locations that are different between *sumAi*_F1 and *sumAi* _F2, *sumAi* _F4 and *sumAi* _F5, and *sumAi* _F7 and *sumAi*_F8. Plugging values into this results in $\frac{1/9+1/9+0/9}{3}$, as the first two combinations have one out of the nine shelters different (8 of the 9 recommended shelter locations match) while the last combination has no shelters different. This sum is divided by three as there are three combinations for the *sumAi* model for Fayetteville where Z stays constant and b changes from 0 to 2.

Table 8. Percent of Shelters Changing When *b* Changes

	b Change	*sumAi*	*EqDist*	*maximin*
Fayetteville	0 to 2	7%	33%	4%
	2 to 4	7%	52%	22%
	0 to 4	15%	59%	22%
Greenville	0 to 2	0%	6%	17%
	2 to 4	11%	0%	22%
	0 to 4	11%	6%	17%
Jacksonville	0 to 2	33%	0%	0%
	2 to 4	0%	0%	22%
	0 to 4	33%	0%	22%
Wilmington	0 to 2	11%	15%	15%
	2 to 4	7%	15%	15%
	0 to 4	19%	15%	26%

It can be observed from *Table 8* how sensitive a model is to a change in the *b* parameter across cities. For Greenville and Wilmington the *maximin* model appears to be the most sensitive to *b*, for Fayetteville the *EqDisp* model appears to be the most sensitive, and for Jacksonville the *sumAi* model appears to be the most sensitive. In general across all four cities, the *maximin* model appears to be the most sensitive to the *b* parameter. Further, the Fayetteville test instance appears to be more sensitive to the choice of the *b* parameter than the other three cities.

As before, *Z* will be discussed as a proxy for the parameter *c* in the distance decay function. *Table 9* presents these values, computed following the same method as the values reported in Table 8.

Table 9. Percent of Shelters Changing When Z Changes

	Z Change	*sumAi*	*EqDist*	*maximin*
Fayetteville	10 to 15	26%	56%	41%
	15 to 20	22%	37%	26%
	10 to 20	41%	52%	67%
Greenville	10 to 15	17%	6%	39%
	15 to 20	17%	0%	11%
	10 to 20	28%	6%	39%
Jacksonville	10 to 15	44%	0%	0%
	15 to 20	11%	0%	22%
	10 to 20	56%	0%	22%
Wilmington	10 to 15	11%	11%	33%
	15 to 20	7%	11%	19%
	10 to 20	15%	15%	33%

As for the b parameter, the sensitivity to a change in c changes between each city and model pair. The model most sensitive to Z for each city is the same model most sensitive to b. Further, the most sensitive city and model overall for parameter Z are the same as for parameter b. In comparing the sensitivity from a change in b versus Z, the latter appears to have a larger effect on shelter location decisions for the *sumAi* and *maximin* models, while b has a larger effect for the *EqDisp* model.

Further analysis is carried out using the *maximin* model for Fayetteville as this city and model combination is determined to be the most sensitive to changes in the b and Z parameters. *Figure 1* presents the shelter and accessibility maps for the implementation of the *maximin* model for Fayetteville. As changing the b and Z parameters change how distance decay, and in turn accessibility, is calculated, we should not compare the raw accessibility scores for these instances. Instead, we should compare the relative ranking of a block group in relation to the other block groups for a particular instance. In Figure 1, a black ‘x’ represents an opened shelter and a grey ‘x’ represents an unopened shelter. Each circle represents a block group, where the size of a circle indicates the relative population in the block group and the color represents the access score quintile for the block group. The color green represents the top quintile (best), blue fourth, grey third, orange second, and red the bottom quintile (worst).

There are a few conclusions we can reach from these maps. First, as the b and Z parameters change for a model and city, the choice of shelters to open often changes. It can be observed how for *maximin*_F1 and *maximin*_F2 five shelters were opened near the coordinates (35.10, -79.00) while no shelters are opened in the region for *maximin*_F9. The set of shelter locations opened in *maximin*_F1 and *maximin*_F2 are identical, as are the set opened in *maximin*_F4 and *maximin*_F5. For *maximin*_F1 and *maximin*_F2, the change in distance decay does not result in the block groups being in different quintiles, unlike for *maximin*_F4 and *maximin*_F5 where it can be observed across Fayetteville block groups are placed in different quintiles. These two instances exemplify how changing the parameters used in the distance decay formula changes the accessibility of an area.

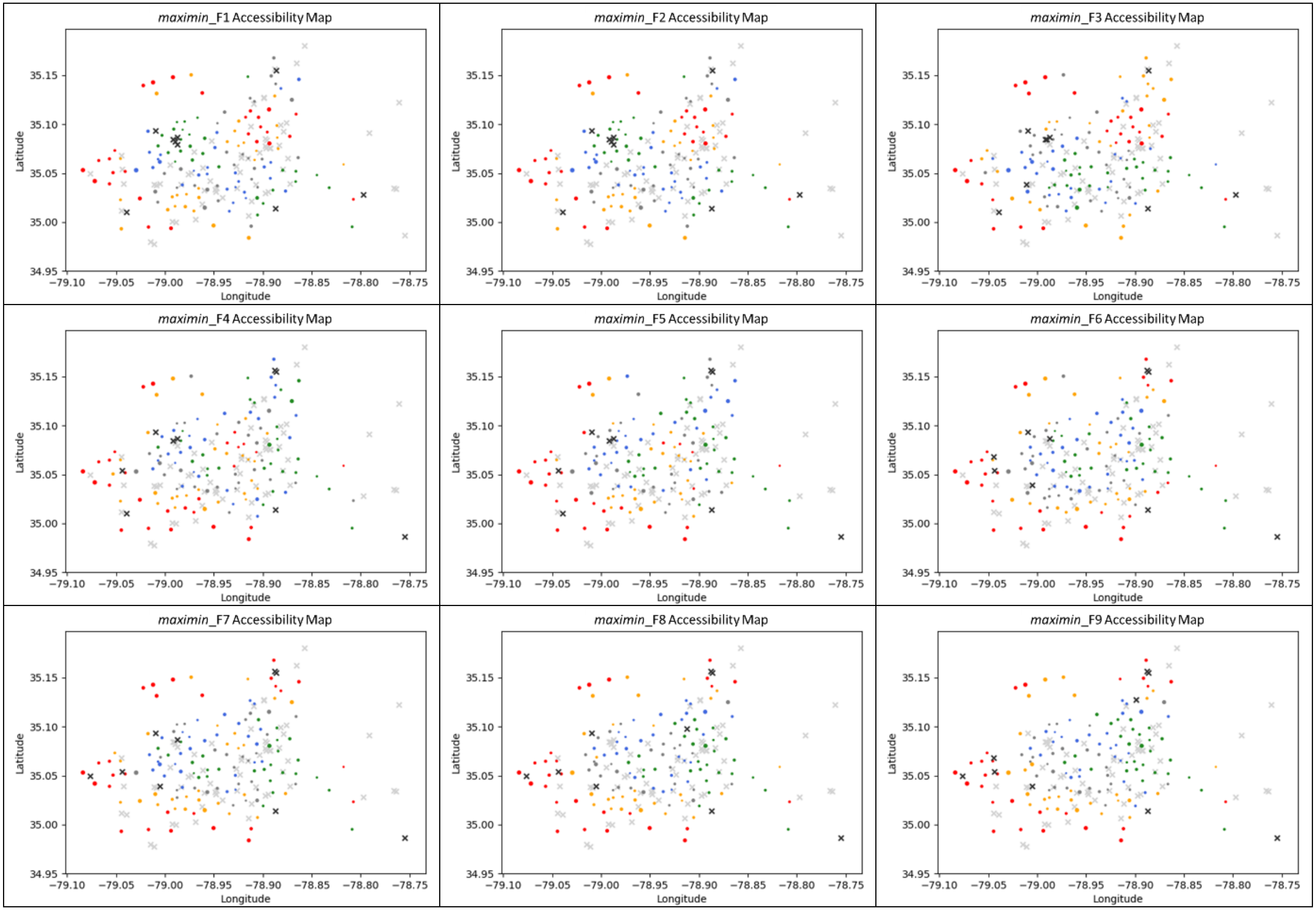

Figure 1. Fayetteville *minimax* Accessibility Maps

Figures 2-4 present the histogram of the accessibility scores for the implementation of the *maximin* model for Fayetteville. Even though the shelters mostly changed between the different instances, the distribution of accessibility of block groups change between every b and Z combination. First, the two sets of instances which opened the same shelters exemplify how a change in how the distance decay function is calculated changes the distribution of accessibility scores. For *maximin*_F1 and *maximin*_F2, we can see the solution for *maximin*_F1 results in slightly more block groups in the lower two buckets while the solution for *maximin*_F2 results in slightly more block groups in the upper two buckets. For *maximin*_F4 and *maximin*_F5, the main differences are with the [1.1, 1.25) and [1.25, 1.4) buckets, with the solution for *maximin*_F4 having more block groups in the [1.25, 1.4) bucket and the solution for *maximin*_F5 having more block groups in the [1.1, 1.25) bucket. These two examples seem to contradict each other on how b impacts accessibility; having a smaller b when Z is 10 miles causes the distribution of block groups to shift leftward while having a smaller b when Z is 15 miles causes the distribution of block groups to shift rightward. These examples show how there appears to be a tradeoff: while increasing the window of indifference increases the radius where people do not experience distance decay to visit a shelter, in turn there is now a higher effective demand on sheltering services as a whole. Unfortunately, there are no optimal shelter opening decisions where we can hold b constant and see how Z changes the distribution of accessibility scores. As a reminder Z represents how far people are willing to travel, and it would follow as people are willing to travel farther distances, the decision of which shelters to open will change. We can observe from *Figures 2-4* that as Z increases, the minimum accessibility does increase, and more block groups are now in the [1.1, 1.25) and [1.25, 1.4) buckets.

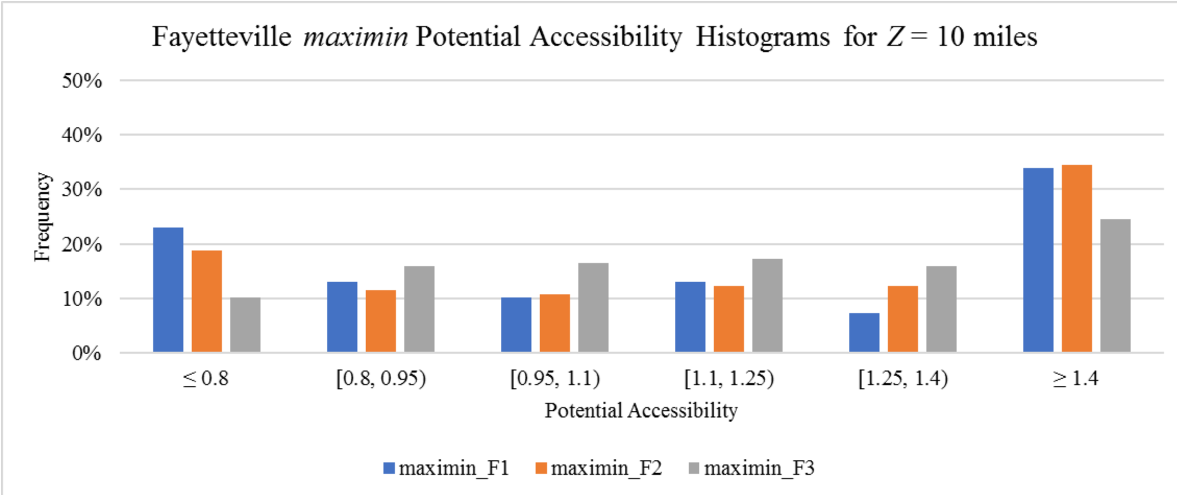

Figure 2. Fayetteville *maximin* Potential Accessibility Histograms for Z=10 miles

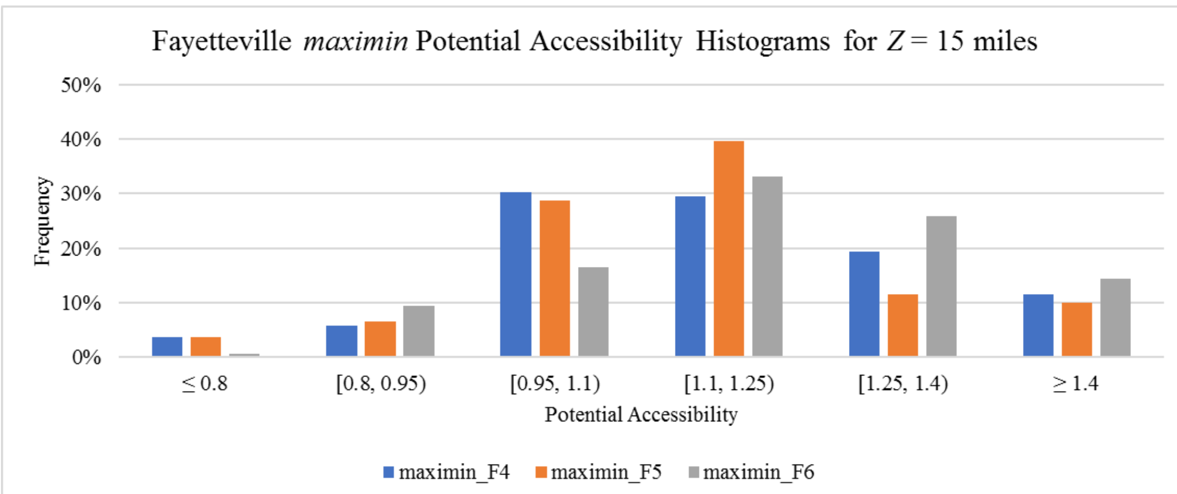

Figure 3. Fayetteville *maximin* Potential Accessibility Histograms for Z=15 miles

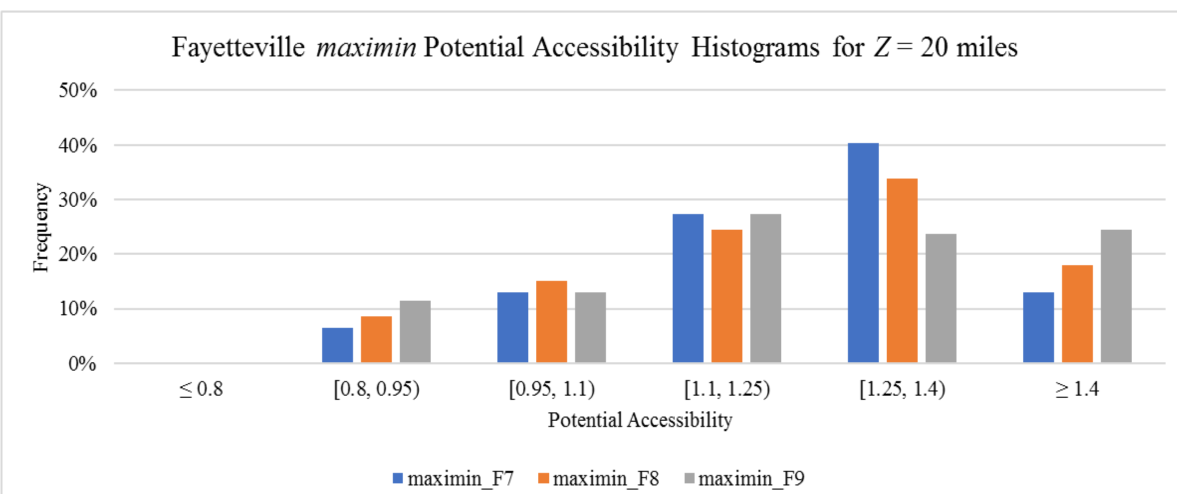

Figure 4. Fayetteville *maximin* Potential Accessibility Histograms for Z=20 miles

In summary, the b and Z parameters influence the which shelters are selected to be opened, and future work should attempt to develop meaningful estimates for these parameter values. To mitigate the effects of b and Z for future comparisons, situation 5 (b is 2 miles and Z is 15 miles) will be used as it represents the midpoint of the b and Z values tested.

5.2 ε-Constraint Method

Multi-objective optimization is appropriate for situations when there are multiple objectives a decision maker is trying to balance. For shelter location problems a decision maker uses efficiency and equity as guiding principles when deciding what shelters to open. The single-objective models previously discussed focus either on efficiency or effectiveness, but not both simultaneously. Therefore we chose to explore a bi-objective model which combines an effectiveness-based model and an equity-based model. Some methods available for solving bi-objective optimization problems include the weighted sum method and the ε-constraint method. The weighted sum method was not used for this thesis as the method is not suitable for non-convex Pareto frontiers (Bérubé, Gendreau, & Potvin, 2009). Pseudocode for the ε-constraint method is adapted from Veerapen et al. (2015) and is shown in *Figure 5*, with N representing the set of non-dominated solutions and δ representing the step size. For the purpose of this thesis, g_1 represents an efficiency objective function and g_2 represents an equity objective function.

```
Determine x^1, an optimal solution for g_1
N ← {x^1}
ε_2 ← g_2(x^1) − δ
while max_{x∈X}{g_1(x)|g_2(x) ≤ ε_2} is feasible do
   x̂ ← max_{x∈X}{g_1(x)|g_2(x) ≤ ε_2}
   H ← H ∪ x̂
   ε_2 ← g_2(x^1) − δ
end while
Filter dominated solutions in H
```

Figure 5. ε-Constraint Method Pseudocode from Veerapen et al. (2015)

Of the three accessibility-based models, *EqDisp* model is the only equity-based model, so it will be used as g_2. A decision had to be made about whether *sumAi* or *maximin* would be used as the efficiency objective. Let f_1^* be the optimal solution to function f_1, $\boldsymbol{A}^{f_1^*}$ be the set of A_i for the optimal solution to f_1, with similar definitions for f_2^* and $\boldsymbol{A}^{f_2^*}$ for function f_2, and for f_3^* and $\boldsymbol{A}^{f_3^*}$ for function f_3. The appropriate δ for when *sumAi* and *maximin* are g_1 can respectively be found by the equations when ten steps are used:

$$\delta_1 = \frac{f_2\left(\boldsymbol{A}^{f_1^*}\right) - f_2\left(\boldsymbol{A}^{f_2^*}\right)}{11}, \tag{24}$$

$$\delta_3 = \frac{f_2\left(\boldsymbol{A}^{f_3^*}\right) - f_2\left(\boldsymbol{A}^{f_2^*}\right)}{11}. \tag{25}$$

To have a fair comparison between the two bi-objective models, δ_3 was used for both the *sumAi* and *maximin* models. The histograms of selected steps were compared, with *Figure 6* provided for F5 when the equity disparity had to be less than or equal to 2.082.

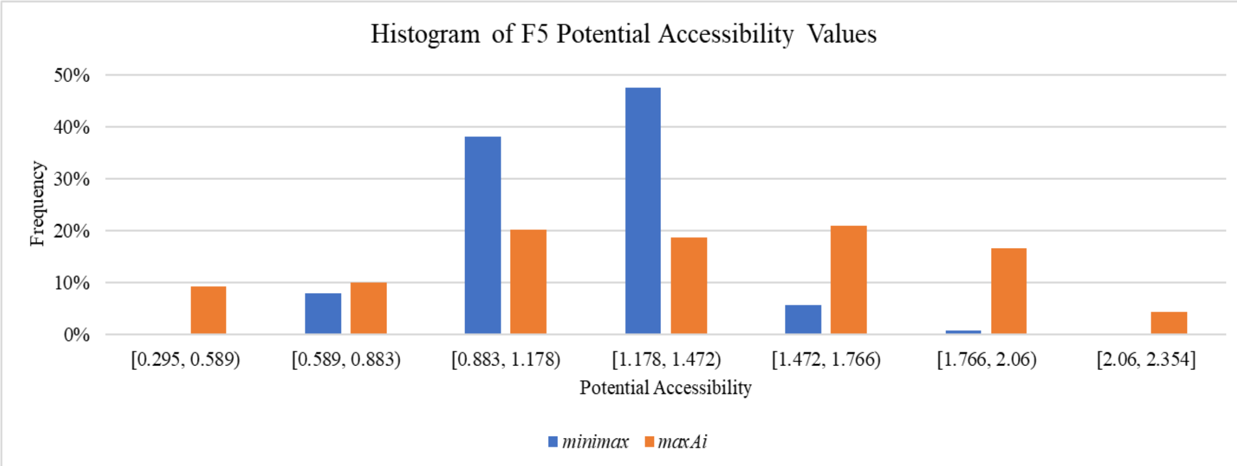

Figure 6. F5 Potential Accessibility Histogram Comparison

Using *maximin* as g_1 results in a higher minimum potential accessibility and a smaller equity disparity, with almost half of the block groups have a potential accessibility between 1.178 and 1.472. The median potential accessibility of all block groups using the *maximin* model is 1.192. While using *sumAi* as g_1 increases the inequity and decreases the minimum potential accessibility, both the average and median potential accessibilities increase. For the *sumAi* model, the median potential accessibility is 1.341. If we were to look only at Fayetteville, for every equal comparison we can make, the *sumAi* model results in a higher median potential accessibility than the *maximin* model. Thus, the *sumAi* model was used as g_1 for this thesis.

The appropriate δ was used for when g_1 is the *sumAi* model, however the steps found for the *maximin* model are also included to provide a better insight into the Pareto frontier as the whole frontier was not created. *Figures 7* and *9* show the partial Pareto frontier for F5 and J5, respectively, while *Figures 8* and *10* show the corresponding minimum potential accessibility values. Graphs for instances G5 and W5, and the results tables for each instance are provided in Appendix B. To pick a solution from the *bi-obj* model to use in comparisons, the level diagram technique from Blasco et al. (2008) via the Euclidean norm is used. The solutions picked from this technique are indicated by a star in Appendix B, and in the figures below are represented by a black dot.

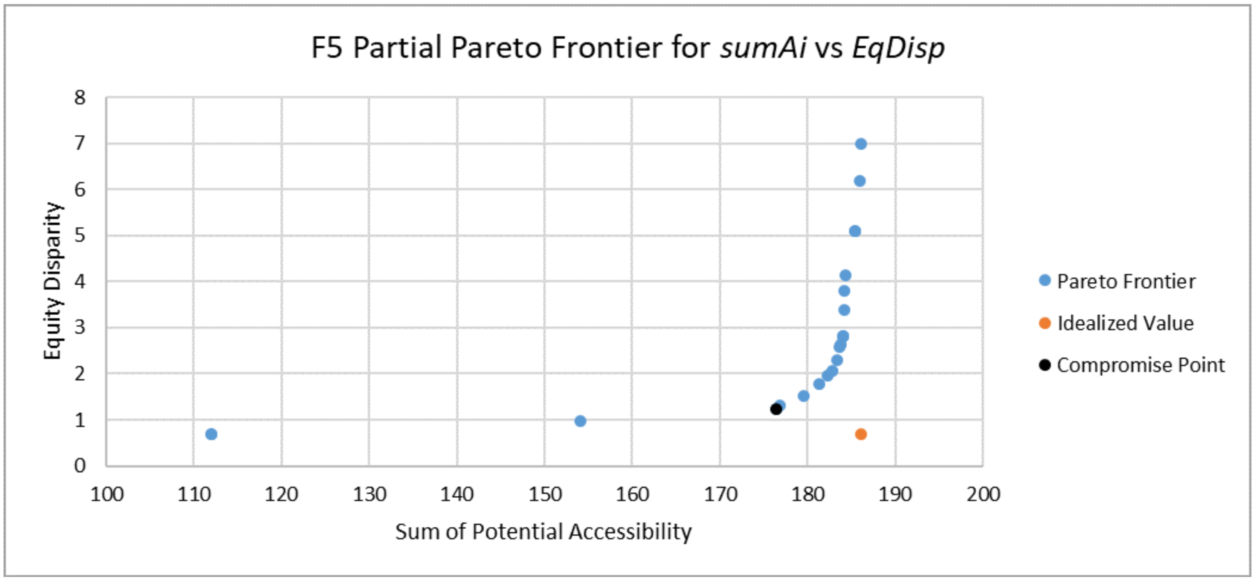

Figure 7. F5 Partial Pareto Frontier for *sumAi* versus *EqDisp*

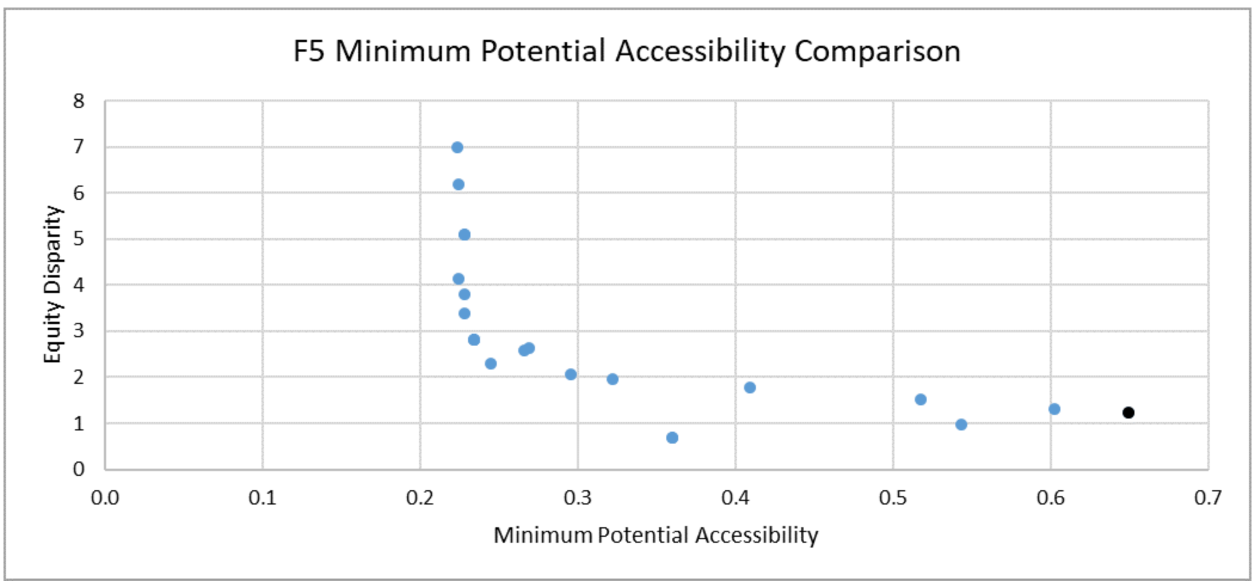

Figure 8. F5 Minimum Potential Accessibility Comparison

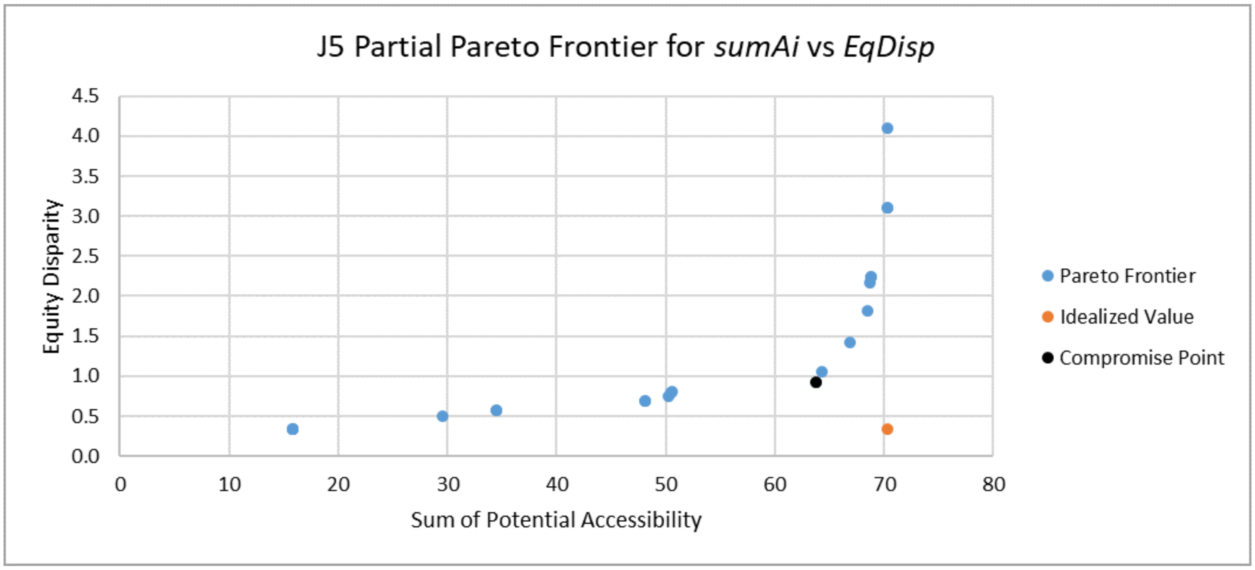

Figure 9. J5 Partial Pareto Frontier for *sumAi* versus *EqDisp*

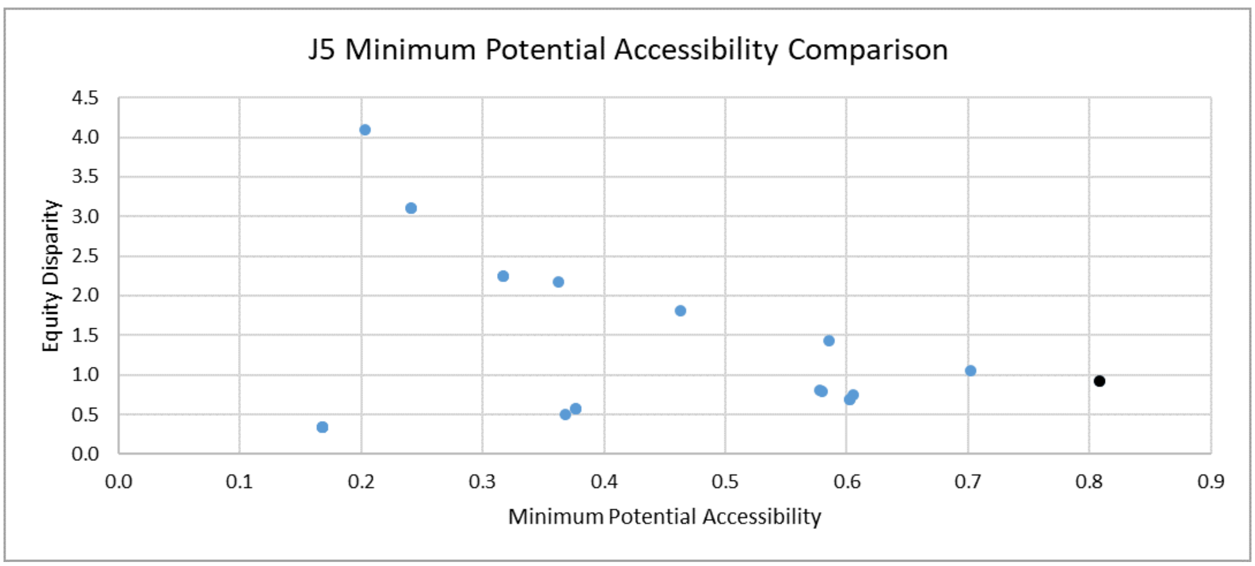

Figure 10. J5 Minimum Potential Accessibility Comparison

Even though we are not explicitly trying to improve the minimum potential accessibility, for each city we can find at least one epsilon which caused the minimum potential accessibility to be higher for that solution than for the *sumAi* or *EqDisp* model alone.

5.3 Comparison of Solutions

In this section, we compare how the different models perform across three accessibility-based functions, and from the performance recommend two models to explore in future research. To do this we compare the values of these models on the accessibility-based functions, the distribution of accessibility scores, and the accessibility maps for two of the cities. As a reminder, the b and Z values must be held constant between the models so an accurate comparison can be made. *Table 10* presents the optimal solutions for instance 5 (b is 2 miles and Z is 15 miles) across the different cities and models with the values for the accessibility-based objective functions. *Figures 11-14* presents resulting distribution of block groups' accessibilities.

Table 10. Optimal Solutions for F5 Evaluated Across Accessibility-Based Objective Functions

Model	Sum of Potential Accessibility	Equity Disparity	Minimum Accessibility
*Imp*_F5	159.527	1.824	0.252
*pMed*_F5	129.497	1.228	0.104
*pCen*_F5	128.640	1.126	0.381
*sumAi*_F5	186.076	6.989	0.224
*EqDisp*_F5	111.972	0.689	0.360
*maximin*_F5	166.322	3.474	0.731
*bi-obj*_F5	176.385	1.236	0.650
*Imp*_G5	78.570	1.856	0.027
*pMed*_G5	73.566	1.550	0.027
*pCen*_G5	80.316	3.630	0.063
*sumAi*_G5	135.594	3.036	0.002
*EqDisp*_G5	23.465	0.498	0.000
*maximin*_G5	104.096	4.467	0.240
*bi-obj*_G5	117.110	2.262	0.123
*Imp*_J5	27.708	0.600	0.232
*pMed*_J5	50.474	0.871	0.532
*pCen*_J5	50.474	0.871	0.532
*sumAi*_J5	70.326	4.102	0.203
*EqDisp*_J5	15.854	0.337	0.168
*maximin*_J5	63.794	0.924	0.808
*bi-obj*_J5	63.794	0.924	0.808
*Imp*_W5	166.183	2.047	1.007
*pMed*_W5	137.457	1.565	0.634
*pCen*_W5	123.348	1.381	0.590
*sumAi*_W5	260.596	3.208	1.304
*EqDisp*_W5	64.421	0.593	0.482
*maximin*_W5	251.997	2.569	1.943
*bi-obj*_W5	204.255	1.779	1.577

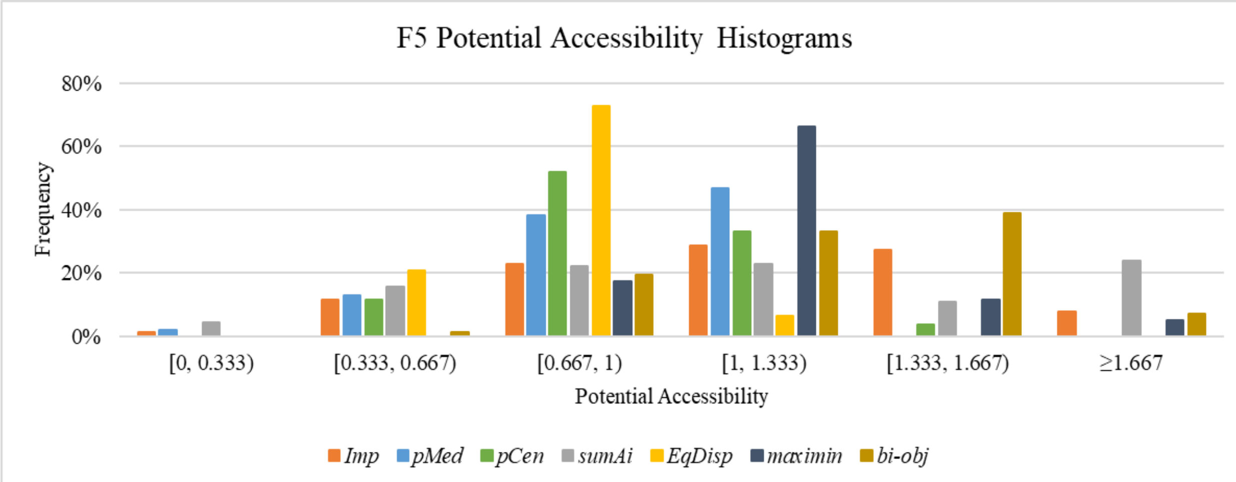

Figure 11. F5 All Models Potential Accessibility Histograms

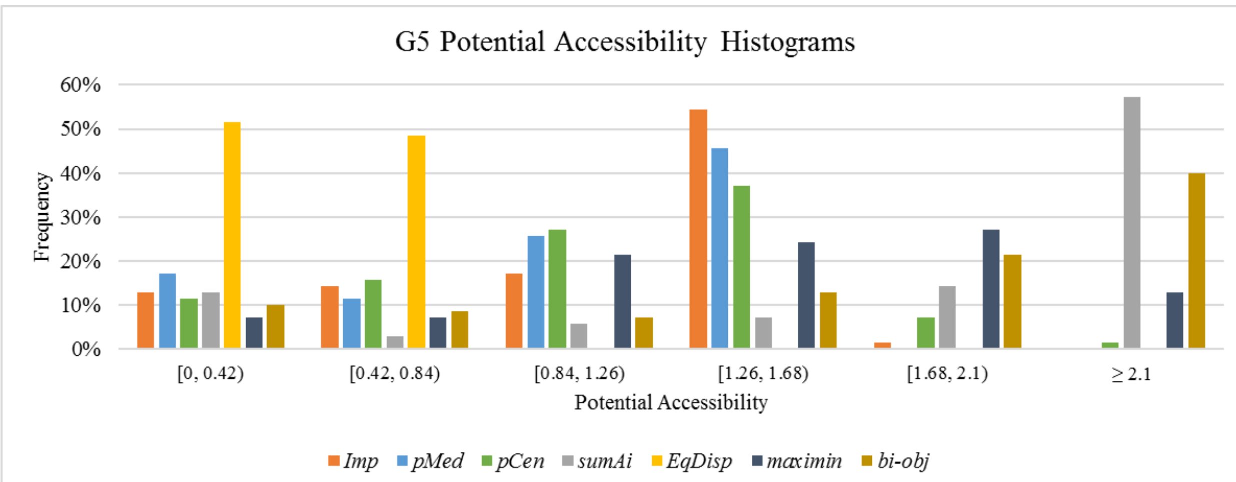

Figure 12. G5 All Models Potential Accessibility Histograms

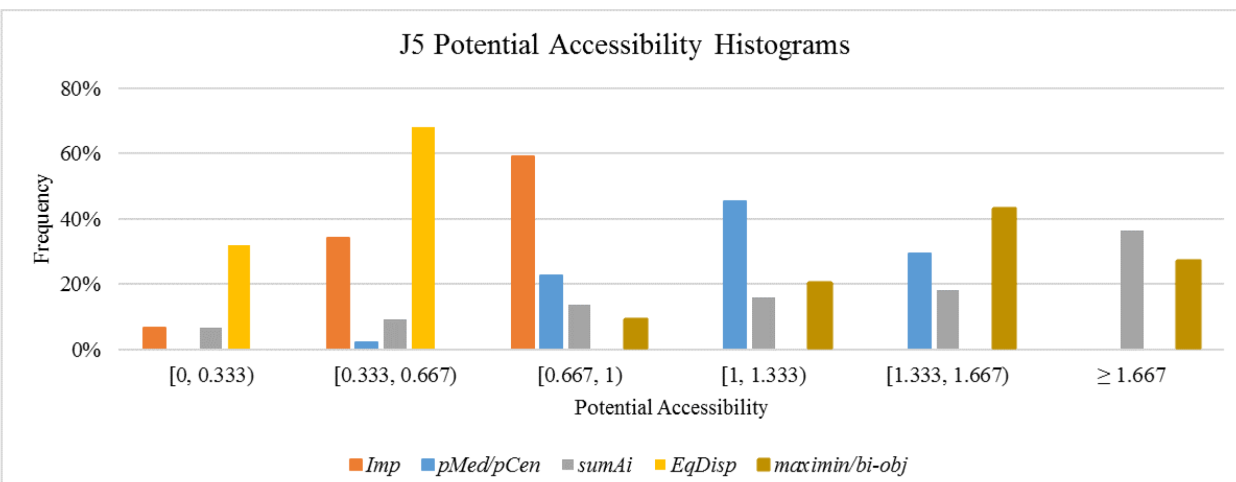

Figure 13. J5 All Models Potential Accessibility Histograms

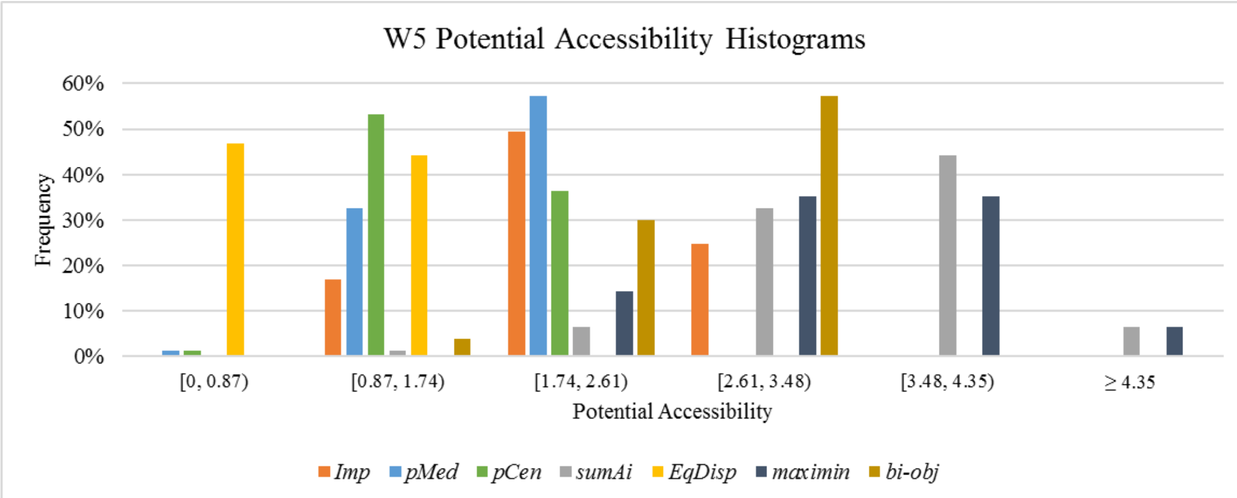

Figure 14. W5 All Models Potential Accessibility Histograms

First, even though the *pMed* and *pCen* models are included in the results, these models are not considered to be good models as they assign people to a shelter and only consider one aspect of accessibility, travel impedance. Capacity constraints could be added which may cause people to no longer be assigned to their closest shelter, however people are still assigned to shelters. These models in general seem to produce inferior results compared to the *sumAi*, *maximin*, and *bi-obj* models.

In comparing the *Imp* solution to the optimization models shows the *Imp* never did the best or worst for the three accessibility functions examined. While for Fayetteville, and to a lesser extent Greenville, the *Imp* solution does not seem bad, for Jacksonville the sum of potential accessibility is almost or over half of the sum for every model except for the *EqDisp* model. For Wilmington, the *Imp* solution results in a summed accessibility much less than the *sumAi*, *maximin*, and *bi-obj* models.

Out of the four accessibility models, the *EqDisp* model appears to be the worst. In almost every instance the model results in the lowest sum of potential accessibilities, as well as often having the smallest minimum accessibility. As this model attempts to have each block groups have the same accessibility, the model causes the block groups to have relatively low accessibilities when compared to other models as the low accessibilities result in a smaller disparity. For this reason, this model by itself is not recommended for future research.

The final model worth discussing at this point is the *sumAi* model, which leads to what appears to be a high accessibility of a city with often a high disparity. This is because the model often tries to open shelters in remote areas. As the supply-to-demand ratio, R_j, is calculated by the supply of a shelter divided by the potential demand, if this potential demand is small, R_j will be large value. As a reminder, the second step of the 2SFCA multiplies R_j by distance decay, so if a shelter has a large R_j value but distance decay is effectively zero for all block groups to it, the model will not be rewarded for opening very remote shelters. There does appear to be a tradeoff between giving a few block groups high accessibility versus a large number of block groups a low accessibility, but that tradeoff analysis is outside of the scope of this thesis.

The remainder of this section further explores the resulting solutions from the models for F5 and J5. *Figure 15* presents the shelter and accessibility maps for each model for F5. As a reminder, a black 'x' represents an opened shelter, and a grey 'x' represents an unopened shelter. The size of a circle indicates the relative population in a block group and the color represents the quintile of accessibility scores of a block group throughout *all models* for F5, with green representing the top quintile, blue fourth, grey third, orange second, and red the bottom quintile. Maps for all cities are provided in Section 8.3.

For Fayetteville, the model with the most block groups in the top quintile is the *bi-obj* model, with 48% of its block groups in the quintile, followed by *Imp* then *sumAi*. The model with the most block groups in the bottom quintile is the *EqDisp*, with 37% of its block groups in the quintile, followed by *sumAi* then *pCen*.

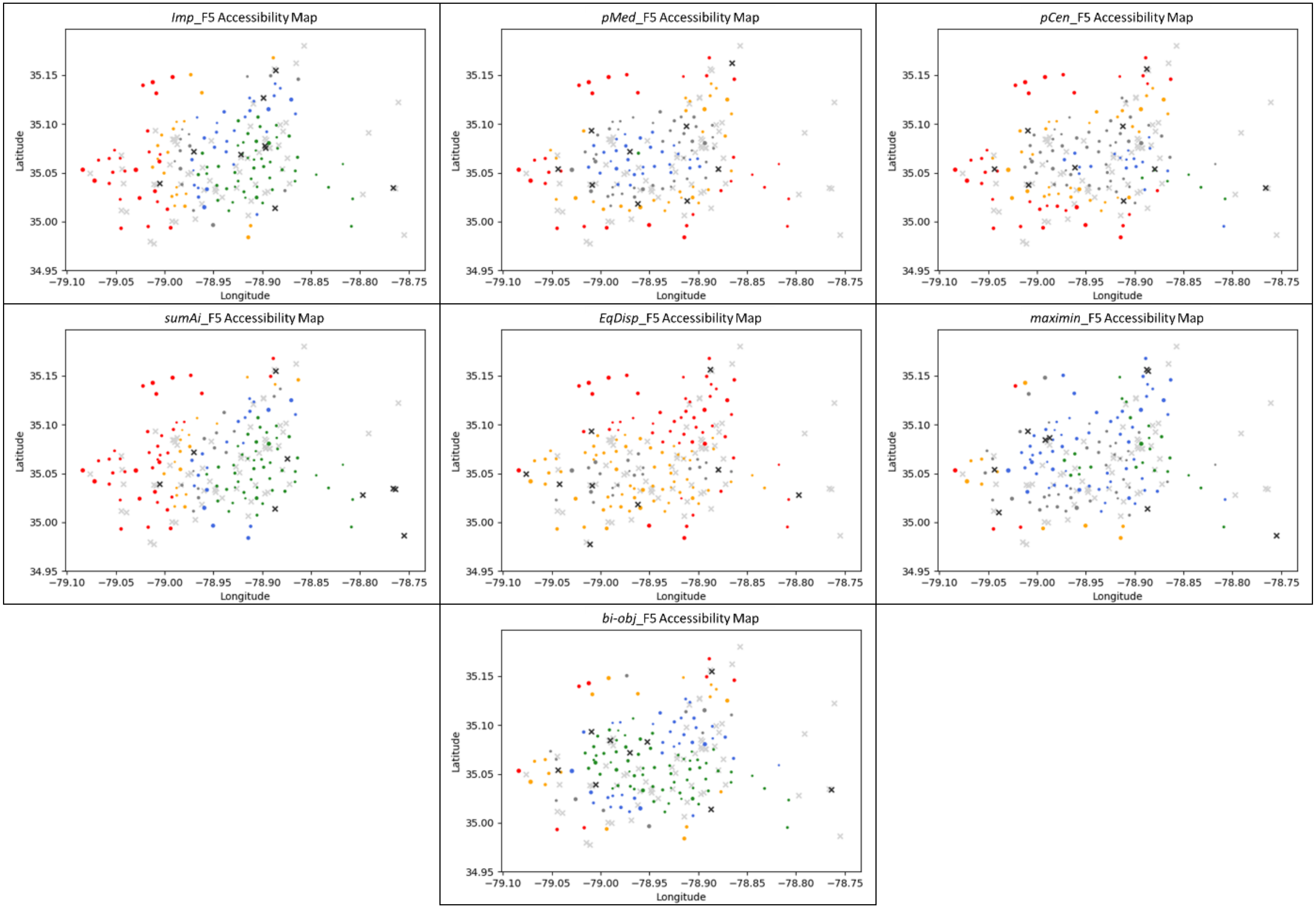

Figure 15. F5 All Models Accessibility Maps

In examining the *Imp*_F5 solution, we can see the shelter planners did not seem to do a poor job. This solution never did the best or worst for the three accessibility-based objective functions examined. The corresponding map of accessibility values also does not seem bad, however we can see the accessibility of block groups tends to be higher in the eastern part of the city than the western part. While the other models also appear to have a similar disparity between the eastern and western parts, the differences are more prominent for the *Imp* solution.

As stated before, the *sumAi* tends to open shelters in remote areas of the city. This is exemplified by the eastern part of the city, as the model choose to open four shelters in a lightly populated area, causing these block groups to have a high accessibility.

Out of all of the models, the *EqDisp* model appears to perform the worst. Examining the corresponding accessibility map shows us none of the block groups are in the top 40% of block groups, with most of the block groups being in the lower 40%. While the accessibility is slightly better in the middle part of the city, the accessibility is still poor.

The *maximin* and *bi-obj* model appear to be the best two models used for this thesis. We can see from *Figure 11* most of the block groups for these two models are in the last three buckets. The main difference appears to be the *maximin* has about 70% of its block groups in the fourth bucket and 10% in the fifth bucket, while the *bi-obj* model has about 35% of its block groups in the fourth bucket and 40% in the fifth. Examining the corresponding maps lets us visualize what these differences mean. The *bi-obj* model has a higher accessibility in the middle of the city while the *maximin* model has higher accessibility on the edge of the cities. Due to how many of the block groups for both of the models are in the top 40% of block groups, these models are recommended for future research.

As Jacksonville is the most constrained out of the four cities included in the case study as only three shelters can be opened, it is not surprising the *pMed* and *pCen* as well as the *maximin* and *bi-obj* models have the same solutions. For the *bi-obj* model, the best compromise solution was the same as the *maximin* solution. *Figure* 16 presents the shelter and accessibility maps for each model for J5. As a reminder, a black 'x' represents an opened shelter, and a grey 'x' represents an unopened shelter. The size of a circle indicates the relative population in a block group and the color represents the quintile of accessibility scores of a block group throughout *all models* for J5, with green representing the top quintile, blue fourth, grey third, orange second, and red the bottom quintile.

In examining *Figure 13*, the model with the most block groups in the top quintile are the *maximin* and *bi-obj* model, with 59% of its block groups in the quintile, with the only other model having block groups in the top quintile being *sumAi*. The model with the most block groups in the bottom quintile is the *EqDisp*, with 73% of its block groups in the quintile, with the only other models having block groups in this quintile being *Imp* and *maxAi*.

Unlike Fayetteville, the *Imp* solution does not appear to be good. This solution has the second lowest sum of accessibilities, as well as one of the smallest minimum accessibilities. None of the block groups have an accessibility in the top 40% across the models.

Like Fayetteville, the *EqDisp* model does not appear to create a good solution, and the *sumAi* model locates shelters in remote areas. This instance reinforces that these models by themselves are not recommended for future research. While the *maximin* and *bi-obj* models do not result in the highest summed accessibility, the models appear to create a solution where many of the block groups have high accessibility relative to the other models.

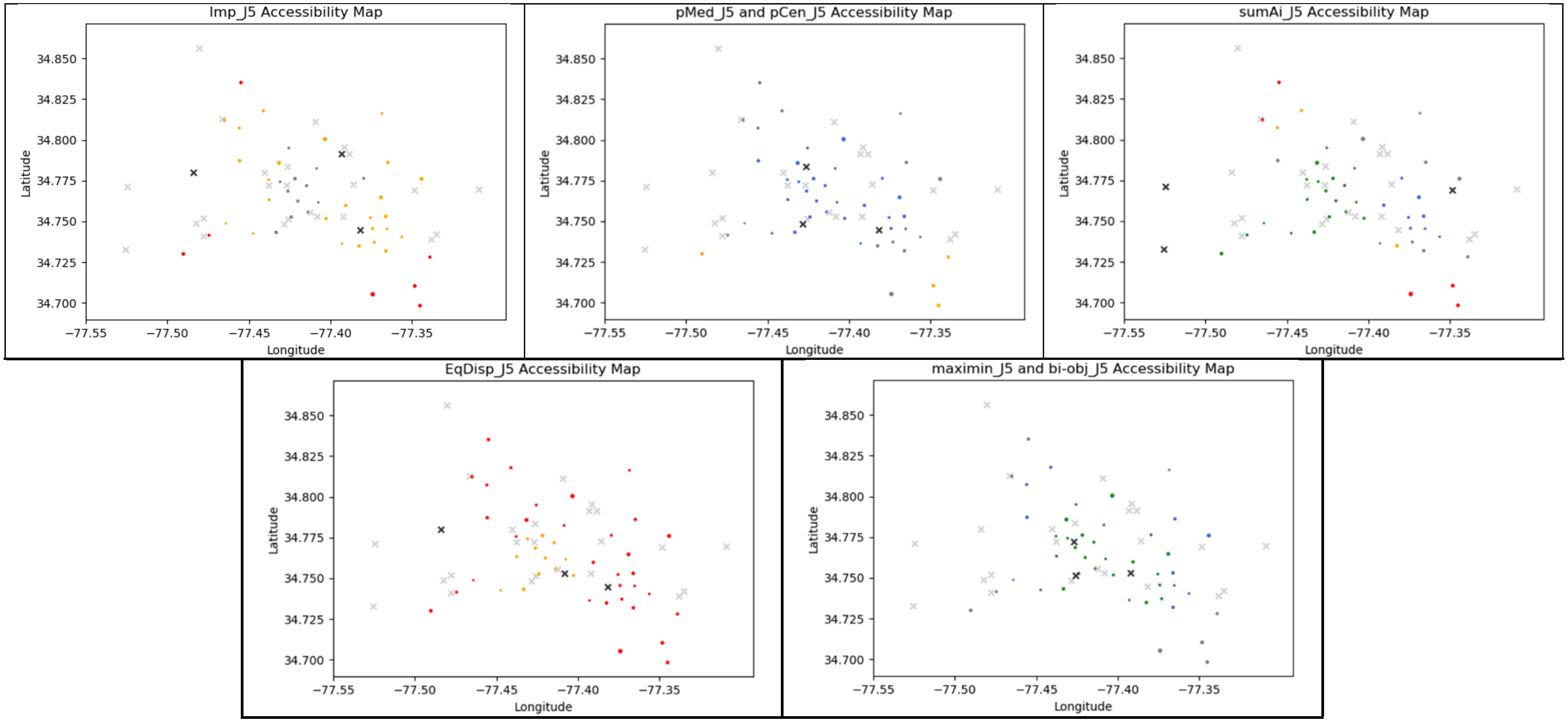

Figure 16. J5 All Models Accessibility Maps

5.4 Inclusion of Additional Shelters

Table 11 presents the percent of shelters opened for each model outside of the NSS inventory. From the table we can see Greenville was affected least by the addition of extra shelters, as well as the *EqDisp* model did not often open these shelters. Over half of the time the *maximin* model chose to open these extra shelters, with the *maximin* model being the most affected by the addition of these extra shelters. For *Table 11*, only the best compromise solution for the *bi-obj* model is considered.

Table 11. Percent of Shelters Opened Outside of the NSS Inventory

Model	Fayetteville	Greenville	Jacksonville	Wilmington
pMed	67%	0%	67%	67%
pCen	67%	33%	67%	67%
sumAi	42%	28%	70%	33%
EqDisp	47%	0%	0%	7%
maximin	70%	61%	100%	51%
bi-obj	44%	33%	100%	44%

We can also see how many shelters were selected to be opened which the *Imp* model also selected to be open. The percent of shelters to open for each model which were the same as the *Imp* solution is provided in *Table 12*. For *Table 12*, only the best compromise solution for the *bi-obj* model is considered.

Table 12. Percent of Shelters Opened The Same As *Imp* Solution

Model	Fayetteville	Greenville	Jacksonville	Wilmington
pMed	11%	50%	0%	0%
pCen	11%	17%	0%	11%
sumAi	47%	17%	0%	22%
EqDisp	19%	17%	67%	38%
maximin	27%	24%	0%	26%
bi-obj	44%	33%	0%	22%

6. Conclusion

This thesis presents an optimization model based on the 2SFCA method to place shelters in response to a disaster event. As people may not go to their closest shelter, the model considers supply-to-demand ratios and uses distance decay to indicate the likelihood of a person going to a particular shelter. A case study based on Hurricane Florence was created to test the model. Three shelter accessibility-based objective functions were created for the model, and were compared to the actual shelters opened, the p-Median solutions, and the p-Center solutions. A bi-objective model was also developed to balance the efficiency and equity of shelter accessibility. Out of the examined models, the *maximin* and *bi-obj* models perform the best on the accessibility metrics, as well as having block groups with relatively higher accessibilities than the other models. These models could show shelter planners areas where they should think about opening shelters, and if there are no shelters in the NSS database near recommended locations, areas where building could be examined for potential shelter locations. For this case study, these models were solved almost instantaneously, and the data used for these models should be easily accessible for emergency managers. It should be noted the actual shelters selected to be opened did not create poor solutions. Our research leads us not to recommend the *sumAi* and *EqDisp* models, as the *sumAi* model tends to overserve rural areas and underserve urban areas, while the *EqDisp* model tends to create a low overall accessibility for the population. While there are no aspatial factors included in the model, a discussion about a set of factors which may be relevant to shelter-seeking behavior is provided. Public schools from outside of the NSS database were included in the case study, and the model often picked some of these schools as good candidate shelter sites. This demonstrates the value of expanding the available building inventory in the NSS database and points to which school buildings should be prioritized for inspection and addition to the database.

While we analyzed different models across different scenarios, this work has some limitations. First, the distance decay formulation for this thesis is based on past research which has implemented distance decay as well as the author's own intuition. The choice of a distance decay function and the *b* and *c* parameters used were estimated to the best of the author's abilities, however sensitivity analysis showed how a change in the *b* and *c* parameters can change the choice of shelters to open. It would follow if a different distance decay function was used, the choice of which shelters the optimized models picked to open could also be different. Empirical research to determine the appropriate distance decay function and parameters is needed. Second, the models for this thesis only considered the number of shelters to open as the "budget" constraint for shelters. Another way to limit the ability to open a shelter is if each city had a certain capacity they could provide, and one was to determine which shelters should be opened and how much capacity those shelters should have. This constraint could change the choice of shelters to open. The final limitation to discuss are boundary effects. While *maximin*_F5 and *bi-obj*_F5 may seem to have relatively poor accessibility for block groups at the edge of a city, in reality these people may choose to visit shelters just outside of the city. To account for these boundary effects, potential shelter locations from the surrounding areas could be incorporated, however the block groups which could visit these shelters should then be incorporated into the dataset. This expanded dataset will still have boundary effects, however the area which does not experience these effects should increase. A city such as Wilmington may experience less boundary effects as the city is close to the Atlantic coast.

There are a variety of options for what could be considered to be the next step of this research. This thesis discusses different spatial and aspatial factors which may influence shelter-seeking behavior of people, however only one of these factors is included in the developed model. Future research could examine these factors and incorporate them into the model. For example, because the Z values in this thesis are used as the maximum potential travel distance, if these values changed across the block groups, the model would then include a variable catchment. The incorporation of these factors should change the decision of which shelters to open. For example, if healthcare needs are considered we would expect for shelters to open in areas of high healthcare needs. Second, knowledge about what constitutes a good sheltering strategy should be elicited from shelter planners. Specifically, the trade-off between efficient and equitable sheltering strategies is of importance. Models developed in this thesis can be shown to shelter planners as examples of different sheltering strategies and how the strategies affect accessibility. Finally, the parameters to use in the proposed distance decay formulation could be found from an actual situation. There are two problems which may be encountered for this. First, the collected travel distances may only be applicable for that particular situation. Second, depending on what data is collected, only one of the parameters may be estimated. For example, if only ZIP codes are collected from people who seek sheltering services, the data may provide insight into applicable Z values, but provide no insight into potential b values.

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8. Appendix

8.1 Appendix A

Table 13. Fayetteville Shelters Opened Per Instance

#	Shelter								
*Imp*_F_All	3	5	8	9	10	13	14	15	17
*pMed*_F_All	6	12	15	36	48	49	52	63	71
*pCen*_F_All	6	12	17	36	40	49	52	53	71
*sumAi*_F1	1	2	9	13	15	17	18	62	67
*sumAi*_F2	1	2	9	13	15	17	62	67	69
*sumAi*_F3	1	2	9	13	15	17	62	67	69
*sumAi*_F4	2	9	10	13	15	22	62	67	69
*sumAi*_F5	2	9	10	13	15	17	22	62	67
*sumAi*_F6	2	9	10	13	15	17	22	62	67
*sumAi*_F7	9	10	13	15	19	22	62	67	69
*sumAi*_F8	9	10	13	15	19	22	62	67	69
*sumAi*_F9	9	10	13	15	19	60	62	65	67
*EqDisp*_F1	3	5	6	7	12	14	16	34	46
*EqDisp*_F2	3	5	6	7	12	36	42	52	71
*EqDisp*_F3	3	5	6	7	12	27	49	55	70
*EqDisp*_F4	7	12	16	21	27	40	49	55	67
*EqDisp*_F5	6	7	12	24	40	46	49	63	67
*EqDisp*_F6	3	5	6	7	12	14	22	24	49
*EqDisp*_F7	3	6	7	12	40	41	46	66	67
*EqDisp*_F8	5	6	7	12	40	41	46	66	67
*EqDisp*_F9	3	5	6	7	14	16	22	24	71
*maximin*_F1	9	10	16	49	54	56	58	67	73
*maximin*_F2	9	10	16	49	54	56	58	67	73
*maximin*_F3	9	10	21	49	56	58	67	72	73
*maximin*_F4	9	10	40	49	52	56	58	62	73
*maximin*_F5	9	10	40	49	52	56	58	62	73
*maximin*_F6	9	10	13	40	41	49	52	58	62
*maximin*_F7	9	10	13	40	46	49	52	58	62
*maximin*_F8	9	10	13	40	46	49	52	62	71
*maximin*_F9	9	10	13	30	40	41	46	52	62
bi-obj_F5[1]	2	9	10	13	15	49	52	72	74

[1] Represents the best compromise solution between maximizing the sum of accessibility and minimizing the equity disparity for test instance F5.

Table 14. Fayetteville Metrics Per Instance

#	Sum of Potential Accessibility	Equity Disparity	Minimum Accessibility	Weighted Travel Distance	Max Distance To Nearest Shelter
*Imp*_F1	164.629	3.968	0.077		
*Imp*_F2	164.495	4.436	0.070		
*Imp*_F3	163.644	4.739	0.071		
*Imp*_F4	159.782	1.873	0.237		
*Imp*_F5	159.527	1.824	0.252	6263	7.95
*Imp*_F6	158.880	1.731	0.291		
*Imp*_F7	156.549	1.320	0.408		
*Imp*_F8	156.164	1.253	0.430		
*Imp*_F9	155.509	1.138	0.471		
*pMed*_F1	129.518	1.553	0.022		
*pMed*_F2	129.780	1.574	0.021		
*pMed*_F3	129.925	1.596	0.026		
*pMed*_F4	129.445	1.266	0.099		
*pMed*_F5	129.497	1.228	0.104	4661	7.49
*pMed*_F6	129.405	1.141	0.118		
*pMed*_F7	128.718	1.017	0.205		
*pMed*_F8	128.612	0.970	0.216		
*pMed*_F9	128.347	0.891	0.237		
*pCen*_F1	131.135	3.432	0.215		
*pCen*_F2	131.230	3.800	0.223		
*pCen*_F3	131.556	3.880	0.260		
*pCen*_F4	128.565	1.113	0.375		
*pCen*_F5	128.640	1.126	0.381	4974	5.70
*pCen*_F6	128.615	1.049	0.389		
*pCen*_F7	126.742	0.688	0.512		
*pCen*_F8	126.644	0.682	0.521		
*pCen*_F9	126.421	0.701	0.501		
*sumAi*_F1	200.441	22.872	0.003	8228	12.75
*sumAi*_F2	201.230	25.290	0.001	8395	12.52
*sumAi*_F3	200.042	26.097	0.000	8395	12.52
*sumAi*_F4	185.984	5.837	0.216	6872	7.95
*sumAi*_F5	186.076	6.989	0.224	6900	7.95
*sumAi*_F6	185.754	6.395	0.258	6900	7.95
*sumAi*_F7	180.705	2.252	0.388	6887	7.95
*sumAi*_F8	180.368	2.183	0.401	6887	7.95
*sumAi*_F9	179.824	2.088	0.424	6886	7.95

Table 14 (Cont.)

*EqDisp*_F1	93.445	0.995	0.034	5477	7.49
*EqDisp*_F2	98.621	0.976	0.021	5777	9.04
*EqDisp*_F3	98.783	0.918	0.035	5599	7.49
*EqDisp*_F4	119.366	0.710	0.424	6922	7.43
*EqDisp*_F5	111.972	0.689	0.360	5950	6.71
*EqDisp*_F6	96.220	0.642	0.268	6089	7.49
*EqDisp*_F7	103.851	0.531	0.388	6737	8.99
*EqDisp*_F8	106.168	0.518	0.421	6755	8.99
*EqDisp*_F9	98.975	0.457	0.399	6606	7.49
*maximin*_F1	172.563	2.533	0.367	7775	7.07
*maximin*_F2	171.135	2.555	0.390	7775	7.07
*maximin*_F3	168.976	2.084	0.472	7365	7.07
*maximin*_F4	166.455	3.385	0.693	7489	9.12
*maximin*_F5	166.322	3.474	0.731	7489	9.12
*maximin*_F6	174.074	3.115	0.779	7065	9.12
*maximin*_F7	172.385	1.011	0.846	6994	9.12
*maximin*_F8	173.153	1.079	0.847	6077	9.12
*maximin*_F9	173.541	1.121	0.837	7493	9.12
bi-obj_F5	176.385	1.236	0.650	6262	7.12

Table 15. Greenville Shelters Opened Per Instance

#	Shelter					
*Imp*_G_All	5	7	8	12	20	21
*pMed*_G_All	3	5	15	18	20	21
*pCen*_G_All	3	15	18	21	27	28
*sumAi*_G1	1	3	8	10	24	30
*sumAi*_G2	1	3	8	10	24	30
*sumAi*_G3	1	3	8	10	24	28
*sumAi*_G4	1	3	8	10	28	30
*sumAi*_G5	1	3	8	10	28	30
*sumAi*_G6	1	3	8	10	26	28
*sumAi*_G7	1	3	6	8	10	28
*sumAi*_G8	1	3	6	8	10	28
*sumAi*_G9	1	3	6	8	10	28
*EqDisp*_G1	9	12	14	15	18	19
*EqDisp*_G2	4	9	12	15	18	19
*EqDisp*_G3	4	9	12	15	18	19
*EqDisp*_G4	4	9	12	15	18	19
*EqDisp*_G5	4	9	12	15	18	19
*EqDisp*_G6	4	9	12	15	18	19
*EqDisp*_G7	4	9	12	15	18	19
*EqDisp*_G8	4	9	12	15	18	19
*EqDisp*_G9	4	9	12	15	18	19
*maximin*_G1	12	13	21	23	27	28
*maximin*_G2	12	14	21	23	27	28
*maximin*_G3	8	12	13	21	27	28
*maximin*_G4	10	21	23	26	27	28
*maximin*_G5	21	22	23	26	27	28
*maximin*_G6	10	21	22	23	27	28
*maximin*_G7	10	21	23	26	27	28
*maximin*_G8	10	21	23	27	28	29
*maximin*_G9	10	21	22	23	27	28
bi-obj_G5[2]	3	7	10	21	28	32

[2] Represents the best compromise solution between maximizing the sum of accessibility and minimizing the equity disparity for test instance G5.

Table 16. Greenville Metrics Per Instance

#	Sum of Potential Accessibility	Equity Disparity	Minimum Accessibility	Weighted Travel Distance	Max Distance To Nearest Shelter
*Imp*_G1	77.338	4.835	0.001	3185	12.92
*Imp*_G2	77.392	5.615	0.000		
*Imp*_G3	77.345	6.231	0.000		
*Imp*_G4	78.621	1.850	0.031		
*Imp*_G5	78.570	1.856	0.027		
*Imp*_G6	78.306	1.784	0.025		
*Imp*_G7	78.245	1.435	0.120		
*Imp*_G8	78.111	1.398	0.121		
*Imp*_G9	77.828	1.339	0.130		
*pMed*_G1	74.246	2.079	0.001	2854	12.92
*pMed*_G2	74.586	1.972	0.000		
*pMed*_G3	74.499	1.783	0.000		
*pMed*_G4	73.651	1.590	0.031		
*pMed*_G5	73.566	1.550	0.027		
*pMed*_G6	73.177	1.474	0.026		
*pMed*_G7	72.803	1.321	0.107		
*pMed*_G8	72.608	1.275	0.110		
*pMed*_G9	72.235	1.211	0.119		
*pCen*_G1	79.302	8.713	0.004	3130	10.49
*pCen*_G2	79.380	10.347	0.002		
*pCen*_G3	79.908	11.900	0.002		
*pCen*_G4	80.376	3.375	0.075		
*pCen*_G5	80.316	3.630	0.063		
*pCen*_G6	80.251	3.849	0.058		
*pCen*_G7	80.442	1.887	0.200		
*pCen*_G8	80.408	1.878	0.210		
*pCen*_G9	80.338	1.824	0.231		
*sumAi*_G1	137.467	5.240	0.000	3556	17.53
*sumAi*_G2	136.729	5.005	0.000	3556	17.53
*sumAi*_G3	136.213	6.725	0.000	3472	17.53
*sumAi*_G4	136.171	3.043	0.003	3479	17.53
*sumAi*_G5	135.594	3.036	0.002	3479	17.53
*sumAi*_G6	134.910	2.899	0.001	3486	17.53
*sumAi*_G7	134.576	2.504	0.126	3437	15.57
*sumAi*_G8	134.256	2.471	0.121	3437	15.57
*sumAi*_G9	133.872	2.432	0.119	3437	15.57

Table 16 (Cont.)

*EqDisp*_G1	26.578	0.649	0.000	3151	16.14
*EqDisp*_G2	23.874	0.605	0.000	3210	16.14
*EqDisp*_G3	23.741	0.572	0.000	3210	16.14
*EqDisp*_G4	23.564	0.516	0.000	3210	16.14
*EqDisp*_G5	23.465	0.498	0.000	3210	16.14
*EqDisp*_G6	23.314	0.466	0.000	3210	16.14
*EqDisp*_G7	23.291	0.454	0.005	3210	16.14
*EqDisp*_G8	23.199	0.439	0.005	3210	16.14
*EqDisp*_G9	23.075	0.416	0.005	3210	16.14
*maximin*_G1	80.814	12.017	0.037	4967	11.75
*maximin*_G2	82.677	10.448	0.016	4109	10.49
*maximin*_G3	82.772	16.933	0.009	4752	10.49
*maximin*_G4	120.364	3.535	0.263	4023	10.75
*maximin*_G5	104.096	4.467	0.240	4801	11.75
*maximin*_G6	119.197	4.731	0.236	4100	11.28
*maximin*_G7	119.623	2.024	0.503	4023	10.75
*maximin*_G8	119.258	2.104	0.503	3701	11.28
*maximin*_G9	119.033	2.299	0.540	4100	11.28
bi-obj_G5	117.110	2.262	0.123	3621	10.71

Table 17. Jacksonville Shelters Opened Per Instance

#	Shelter		
*Imp*_J_All	2	4	5
*pMed*_J_All	5	14	27
*pCen*_J_All	5	14	27
*sumAi*_J1	10	24	27
*sumAi*_J2	7	23	24
*sumAi*_J3	7	23	24
*sumAi*_J4	7	23	24
*sumAi*_J5	7	15	23
*sumAi*_J6	7	15	23
*sumAi*_J7	7	15	23
*sumAi*_J8	7	15	23
*sumAi*_J9	7	15	23
*EqDisp*_J1	3	4	5
*EqDisp*_J2	3	4	5
*EqDisp*_J3	3	4	5
*EqDisp*_J4	3	4	5
*EqDisp*_J5	3	4	5
*EqDisp*_J6	3	4	5
*EqDisp*_J7	3	4	5
*EqDisp*_J8	3	4	5
*EqDisp*_J9	3	4	5
*maximin*_J1	8	17	19
*maximin*_J2	8	17	19
*maximin*_J3	8	17	19
*maximin*_J4	8	17	19
*maximin*_J5	8	17	19
*maximin*_J6	8	17	19
*maximin*_J7	8	17	19
*maximin*_J8	8	17	19
*maximin*_J9	10	17	27
bi-obj_J5[3]	8	17	19

[3] Represents the best compromise solution between maximizing the sum of accessibility and minimizing the equity disparity for test instance J5.

Table 18. Jacksonville Metrics Per Instance

#	Sum of Potential Accessibility	Equity Disparity	Minimum Accessibility	Weighted Travel Distance	Max Distance To Nearest Shelter
*Imp*_J1	27.517	1.019	0.080	1489	6.37
*Imp*_J2	27.762	0.909	0.091		
*Imp*_J3	27.965	0.866	0.131		
*Imp*_J4	27.617	0.632	0.210		
*Imp*_J5	27.708	0.600	0.232		
*Imp*_J6	27.747	0.513	0.279		
*Imp*_J7	27.465	0.428	0.328		
*Imp*_J8	27.479	0.382	0.357		
*Imp*_J9	27.444	0.298	0.406		
*pMed*_J1	52.159	1.930	0.165	1165	5.96
*pMed*_J2	51.805	1.592	0.183		
*pMed*_J3	50.941	1.167	0.256		
*pMed*_J4	50.832	1.110	0.467		
*pMed*_J5	50.474	0.871	0.532		
*pMed*_J6	49.920	0.580	0.660		
*pMed*_J7	50.195	0.696	0.684		
*pMed*_J8	49.926	0.518	0.755		
*pMed*_J9	49.579	0.316	0.864		
*pCen*_J1	52.159	1.930	0.165	1165	5.96
*pCen*_J2	51.805	1.592	0.183		
*pCen*_J3	50.941	1.167	0.256		
*pCen*_J4	50.832	1.110	0.467		
*pCen*_J5	50.474	0.871	0.532		
*pCen*_J6	49.920	0.580	0.660		
*pCen*_J7	50.195	0.696	0.684		
*pCen*_J8	49.926	0.518	0.755		
*pCen*_J9	49.579	0.316	0.864		
*sumAi*_J1	68.724	3.477	0.163	1624	7.22
*sumAi*_J2	69.371	7.634	0.012	2595	10.54
*sumAi*_J3	72.058	5.630	0.006	2595	10.54
*sumAi*_J4	69.736	3.589	0.245	2595	10.54
*sumAi*_J5	70.326	4.102	0.203	2838	11.34
*sumAi*_J6	71.174	3.297	0.210	2838	11.34
*sumAi*_J7	68.794	2.283	0.531	2838	11.34
*sumAi*_J8	69.003	2.014	0.546	2838	11.34
*sumAi*_J9	69.118	1.874	0.586	2838	11.34

Table 18 (Cont.)

*EqDisp*_J1	16.417	0.655	0.041	1487	7.48
*EqDisp*_J2	16.180	0.626	0.039	1487	7.48
*EqDisp*_J3	15.837	0.620	0.049	1487	7.48
*EqDisp*_J4	15.972	0.383	0.150	1487	7.48
*EqDisp*_J5	15.854	0.337	0.168	1487	7.48
*EqDisp*_J6	15.734	0.262	0.208	1487	7.48
*EqDisp*_J7	15.753	0.231	0.223	1487	7.48
*EqDisp*_J8	15.675	0.187	0.245	1487	7.48
*EqDisp*_J9	15.601	0.126	0.280	1487	7.48
*maximin*_J1	65.911	2.295	0.292	1283	6.33
*maximin*_J2	65.222	1.829	0.332	1283	6.33
*maximin*_J3	63.997	1.226	0.494	1283	6.33
*maximin*_J4	64.295	1.246	0.708	1283	6.33
*maximin*_J5	63.794	0.924	0.808	1283	6.33
*maximin*_J6	63.154	0.542	0.993	1283	6.33
*maximin*_J7	63.582	0.758	0.964	1283	6.33
*maximin*_J8	63.238	0.534	1.054	1283	6.33
*maximin*_J9	62.853	0.276	1.195	1690	7.22
bi-obj_J5	63.794	0.924	0.808	1283	6.33

Table 19. Wilmington Shelters Opened Per Instance

#	Shelter								
*Imp*_W_All	4	5	7	8	9	13	15	17	18
*pMed*_W_All	3	19	20	22	24	31	33	41	45
*pCen*_W_All	3	8	19	24	31	33	41	45	47
*sumAi*_W1	10	11	15	16	17	20	39	42	48
*sumAi*_W2	10	11	15	16	17	20	39	42	48
*sumAi*_W3	10	11	15	16	17	20	21	42	48
*sumAi*_W4	10	11	15	16	17	20	39	42	48
*sumAi*_W5	10	11	15	16	17	20	28	42	48
*sumAi*_W6	10	11	15	16	17	20	28	29	48
*sumAi*_W7	10	11	15	16	17	20	39	42	48
*sumAi*_W8	10	11	15	16	17	20	28	37	48
*sumAi*_W9	10	11	15	16	17	20	28	37	48
*EqDisp*_W1	1	2	3	4	7	8	12	13	19
*EqDisp*_W2	1	2	3	4	7	8	12	13	19
*EqDisp*_W3	1	2	3	4	7	8	12	13	19
*EqDisp*_W4	1	2	3	4	7	8	12	13	19
*EqDisp*_W5	1	2	3	4	5	8	13	19	49
*EqDisp*_W6	1	2	3	4	8	12	13	19	24
*EqDisp*_W7	1	2	3	7	8	12	13	19	40
*EqDisp*_W8	1	2	3	4	8	12	13	19	49
*EqDisp*_W9	1	2	3	4	8	12	19	24	47
*maximin*_W1	8	15	16	17	33	36	39	40	49
*maximin*_W2	8	11	15	16	17	36	40	48	49
*maximin*_W3	8	15	16	17	36	40	44	48	49
*maximin*_W4	15	16	17	20	36	37	40	44	49
*maximin*_W5	15	16	17	20	36	37	40	44	49
*maximin*_W6	11	15	16	17	20	21	36	40	49
*maximin*_W7	15	16	17	20	36	37	38	40	49
*maximin*_W8	11	15	16	17	20	36	40	47	49
*maximin*_W9	11	15	16	17	20	22	40	47	49
bi-obj_W5[4]	6	8	16	17	20	22	24	44	49

[4] Represents the best compromise solution between maximizing the sum of accessibility and minimizing the equity disparity for test instance W5.

Table 20. Wilmington Metrics Per Instance

#	Sum of Potential Accessibility	Equity Disparity	Minimum Accessibility	Weighted Travel Distance	Max Distance To Nearest Shelter
*Imp*_W1	166.134	4.025	0.605	2345	6.19
*Imp*_W2	166.070	3.832	0.633		
*Imp*_W3	167.650	2.952	0.817		
*Imp*_W4	166.023	2.273	0.960		
*Imp*_W5	166.183	2.047	1.007		
*Imp*_W6	166.529	1.657	1.095		
*Imp*_W7	164.930	1.468	1.259		
*Imp*_W8	164.846	1.289	1.317		
*Imp*_W9	164.691	1.023	1.412		
*pMed*_W1	139.258	2.687	0.207	1515	6.74
*pMed*_W2	138.488	2.529	0.220		
*pMed*_W3	137.466	2.056	0.294		
*pMed*_W4	137.952	1.772	0.573		
*pMed*_W5	137.457	1.565	0.634		
*pMed*_W6	136.913	1.224	0.766		
*pMed*_W7	137.259	1.209	0.897		
*pMed*_W8	136.907	1.020	0.983		
*pMed*_W9	136.540	0.754	1.126		
*pCen*_W1	124.545	2.226	0.245	1559	6.19
*pCen*_W2	124.220	2.127	0.260		
*pCen*_W3	123.576	1.810	0.336		
*pCen*_W4	123.656	1.536	0.541		
*pCen*_W5	123.348	1.381	0.590		
*pCen*_W6	122.919	1.115	0.694		
*pCen*_W7	122.937	1.069	0.810		
*pCen*_W8	122.661	0.918	0.881		
*pCen*_W9	122.312	0.698	0.999		
*sumAi*_W1	265.134	4.831	0.490	8228	6.57
*sumAi*_W2	263.264	4.876	0.507	8395	6.57
*sumAi*_W3	262.256	4.276	0.665	8395	6.57
*sumAi*_W4	261.384	3.236	1.173	6872	6.57
*sumAi*_W5	260.596	3.208	1.304	6900	6.57
*sumAi*_W6	260.911	2.847	1.523	6900	6.57
*sumAi*_W7	258.986	2.223	1.750	6887	6.57
*sumAi*_W8	258.561	2.056	2.020	6887	6.57
*sumAi*_W9	258.385	1.651	2.225	6886	6.57

Table 20 (Cont.)

*EqDisp*_W1	54.437	0.856	0.162	5477	6.19
*EqDisp*_W2	53.966	0.817	0.167	5777	6.19
*EqDisp*_W3	53.474	0.795	0.202	5599	6.19
*EqDisp*_W4	53.788	0.604	0.261	6922	6.19
*EqDisp*_W5	64.421	0.593	0.482	5950	6.19
*EqDisp*_W6	62.894	0.507	0.481	6089	6.19
*EqDisp*_W7	64.693	0.440	0.578	6737	6.19
*EqDisp*_W8	62.920	0.395	0.574	6755	6.19
*EqDisp*_W9	73.816	0.322	0.715	6606	6.19
*maximin*_W1	238.306	5.644	1.127	7775	6.19
*maximin*_W2	240.793	4.618	1.187	7775	6.19
*maximin*_W3	241.469	3.239	1.482	7365	6.19
*maximin*_W4	251.704	2.918	1.829	7489	6.43
*maximin*_W5	251.997	2.569	1.943	7489	6.43
*maximin*_W6	254.115	1.893	2.119	7065	6.43
*maximin*_W7	250.104	1.850	2.228	6994	7.22
*maximin*_W8	252.617	1.506	2.332	6077	6.43
*maximin*_W9	252.217	1.104	2.514	7493	6.43
bi-obj_W5	204.255	1.779	1.577	1998	6.19

8.2 Appendix B

Table 21. F5 ε-Constraint Solutions

Point Number	Enforced Disparity	Sum of Potential Accessibility	Equity Disparity	Minimum Potential Accessibility	Shelters								
0	6.989	186.076	6.989	0.224	2	9	10	13	15	17	22	62	67
1	6.359	186.003	6.193	0.224	1	2	9	10	13	15	22	62	67
2	5.729	185.363	5.090	0.228	1	9	10	13	15	22	62	67	69
3	5.099	185.363	5.090	0.228	1	9	10	13	15	22	62	67	69
4	4.469	184.258	4.132	0.224	1	2	9	10	13	15	17	22	67
5	3.839	184.233	3.790	0.228	2	9	10	13	15	17	22	67	69
6	3.474	184.159	3.398	0.228	1	2	9	10	13	15	22	67	69
7	3.209	184.066	2.804	0.234	2	9	10	13	15	22	64	67	69
8	3.195	184.066	2.804	0.234	2	9	10	13	15	22	64	67	69
9	2.917	184.066	2.804	0.234	2	9	10	13	15	22	64	67	69
10	2.638	183.756	2.622	0.269	2	9	10	13	15	22	35	67	75
11	2.579	183.656	2.572	0.266	2	9	10	13	15	18	22	61	67
12	2.360	183.357	2.285	0.245	9	10	13	15	22	33	64	67	69
13	2.082	182.887	2.059	0.295	9	10	13	15	22	35	61	67	75
14	1.949	182.267	1.949	0.322	9	10	13	15	22	35	61	67	74
15	1.803	181.388	1.791	0.409	9	10	13	15	22	29	67	74	75
16	1.525	179.574	1.517	0.517	9	10	13	15	29	49	67	74	75
17	1.319	176.781	1.318	0.602	2	9	10	13	15	21	49	53	72
18*[5]	1.246	176.385	1.236	0.650	2	9	10	13	15	49	52	72	74
19	0.968	154.136	0.965	0.543	10	13	15	39	41	49	67	69	74
20	0.689	111.972	0.689	0.360	6	7	12	24	40	46	49	63	67

[5] The star represents the best compromise solution between maximizing the sum of accessibility and minimizing the equity disparity when the level diagram technique from Blasco et al. (2008) via the Euclidean norm method.

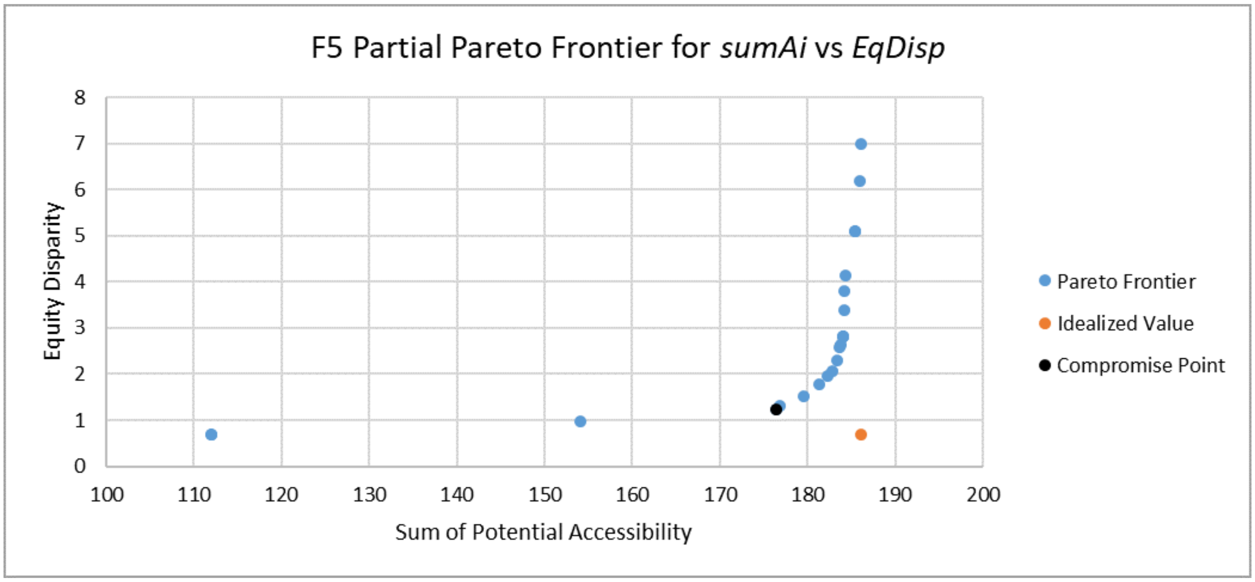

Figure 17. F5 Partial Pareto Frontier for *sumAi* versus *EqDisp*

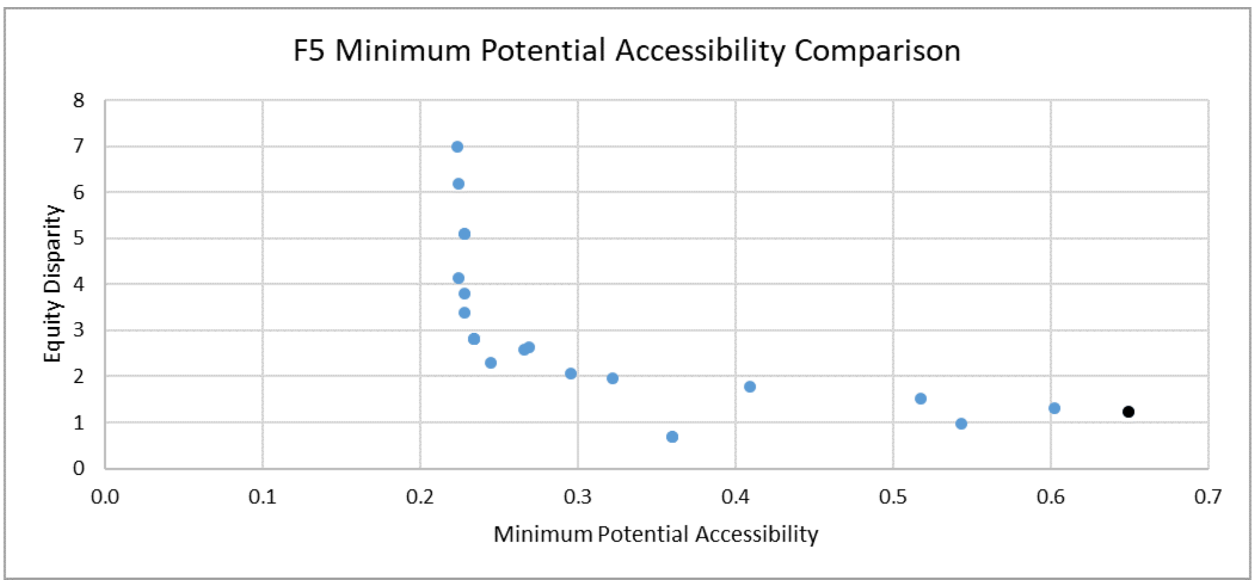

Figure 18. F5 Minimum Potential Accessibility Comparison

Table 22. G5 ε-Constraint Solutions

Point Number	Enforced Disparity	Sum of Potential Accessibility	Equity Disparity	Minimum Potential Accessibility	Shelters					
0	4.467	135.594	3.036	0.002	1	3	8	10	28	30
1	4.070	135.594	3.036	0.002	1	3	8	10	28	30
2	3.673	135.594	3.036	0.002	1	3	8	10	28	30
3	3.276	135.594	3.036	0.002	1	3	8	10	28	30
4	3.036	135.594	3.036	0.002	1	3	8	10	28	30
5	2.880	135.142	2.776	0.014	1	3	8	10	22	28
6	2.782	135.142	2.776	0.014	1	3	8	10	22	28
7	2.528	126.813	2.511	0.002	1	10	23	25	26	28
8	2.483	126.627	2.463	0.014	1	10	22	23	28	30
9*	2.275	117.110	2.262	0.123	3	7	10	21	28	32
10	2.086	112.030	2.050	0.069	5	10	13	23	26	28
11	2.021	107.102	2.008	0.040	7	10	11	23	28	29
12	1.767	89.104	1.767	0.010	3	5	7	12	25	29
13	1.689	84.440	1.688	0.007	4	5	13	29	31	32
14	1.513	75.626	1.512	0.010	3	5	7	12	15	30
15	1.292	64.542	1.289	0.001	1	5	9	15	18	25
16	1.260	62.687	1.259	0.001	4	5	11	12	15	25
17	1.006	49.621	1.004	0.001	4	11	12	15	19	29
18	0.895	44.503	0.894	0.001	4	9	11	12	18	29
19	0.752	36.457	0.748	0.000	4	5	9	12	15	18
20	0.499	23.465	0.498	0.000	4	9	12	15	18	19

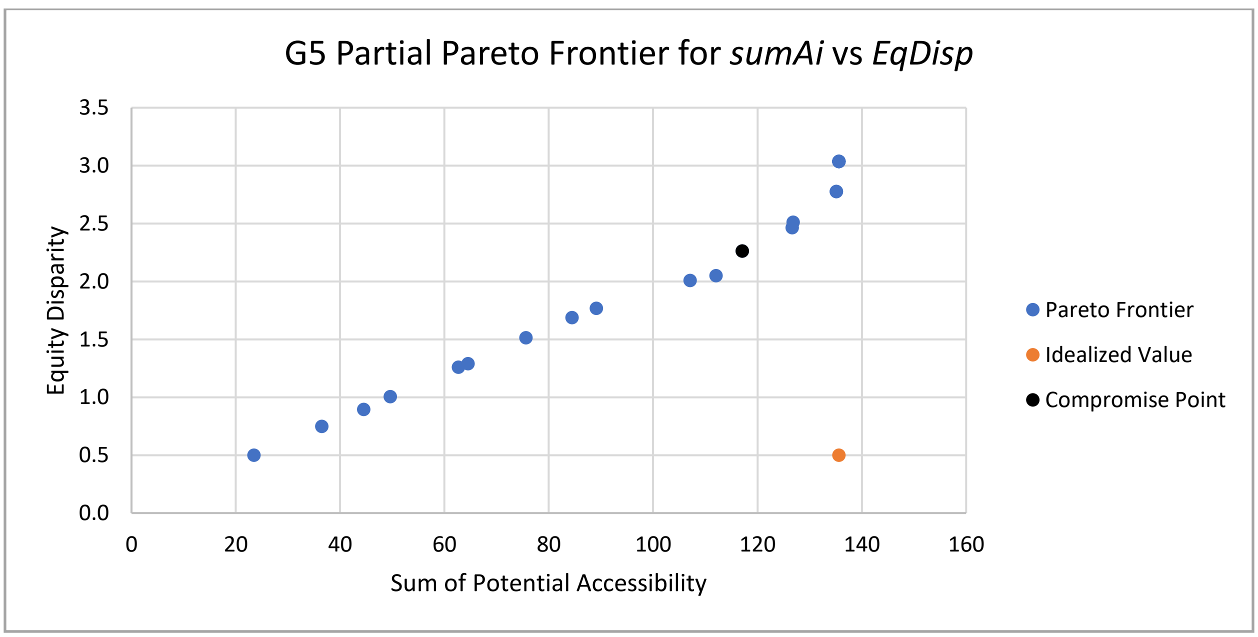

Figure 19. G5 Partial Pareto Frontier

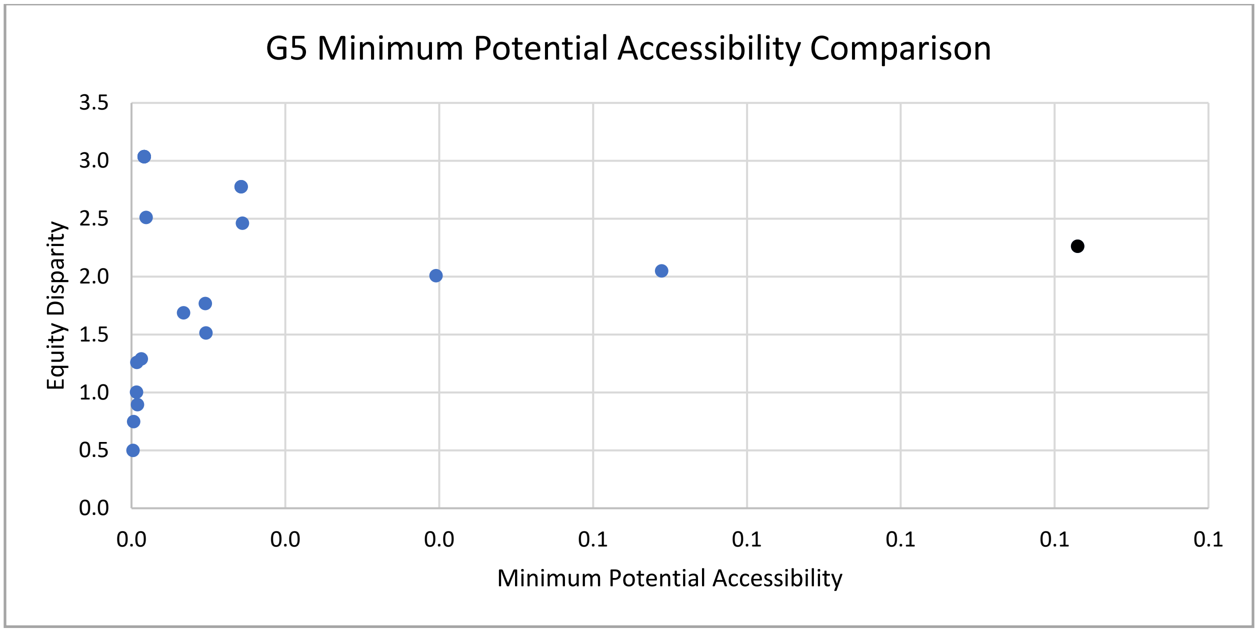

Figure 20. G5 Minimum Potential Accessibility Comparison

Table 23. J5 ε-Constraint Solutions

Point Number	Enforced Disparity	Sum of Potential Accessibility	Equity Disparity	Minimum Potential Accessibility	Shelters		
0	4.102	70.326	4.102	0.203	7	15	23
1	3.725	70.288	3.102	0.241	7	23	24
2	3.349	70.288	3.102	0.241	7	23	24
3	2.972	68.842	2.245	0.317	7	20	23
4	2.596	68.842	2.245	0.317	7	20	23
5	2.219	68.720	2.168	0.363	7	19	23
6	1.843	68.460	1.814	0.463	7	24	27
7	1.467	66.892	1.425	0.585	7	19	27
8	1.090	64.297	1.053	0.702	17	19	27
9*	0.924	63.794	0.924	0.808	8	17	19
10	0.865	50.544	0.811	0.577	5	19	27
11	0.807	50.448	0.792	0.580	5	17	27
12	0.748	50.293	0.746	0.605	5	10	17
13	0.714	48.132	0.686	0.602	3	10	17
14	0.689	48.132	0.686	0.602	3	10	17
15	0.631	34.534	0.567	0.377	3	5	27
16	0.572	34.534	0.567	0.377	3	5	27
17	0.513	29.578	0.504	0.368	3	4	10
18	0.455	15.854	0.337	0.168	3	4	5
19	0.396	15.854	0.337	0.168	3	4	5
20	0.337	15.854	0.337	0.168	3	4	5

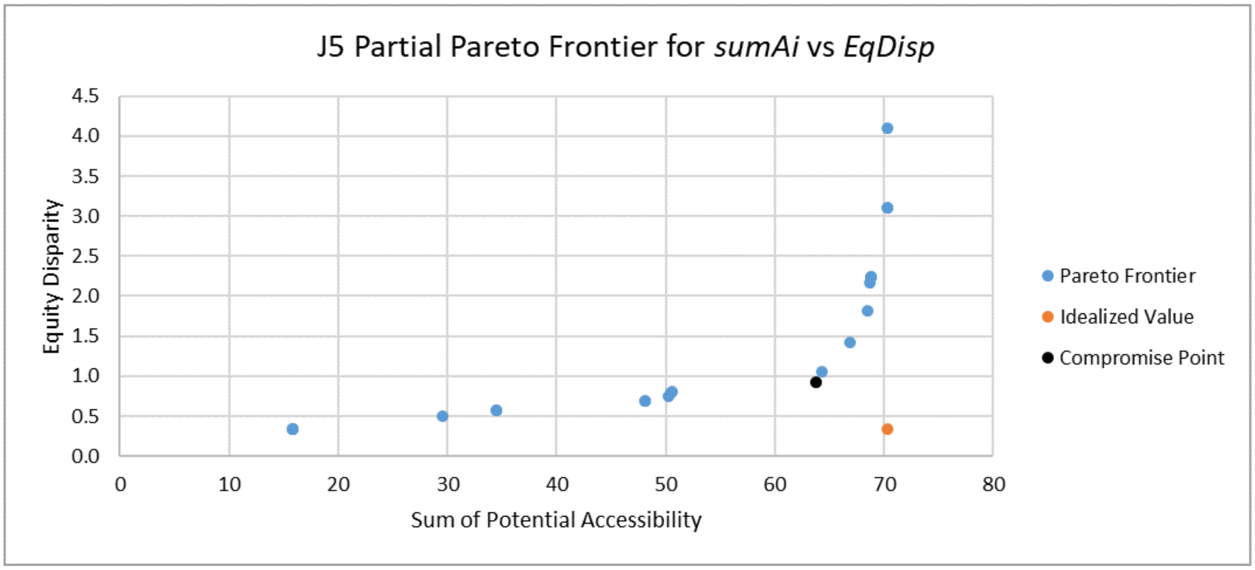

Figure 21. J5 Partial Pareto Frontier for *sumAi* versus *EqDisp*

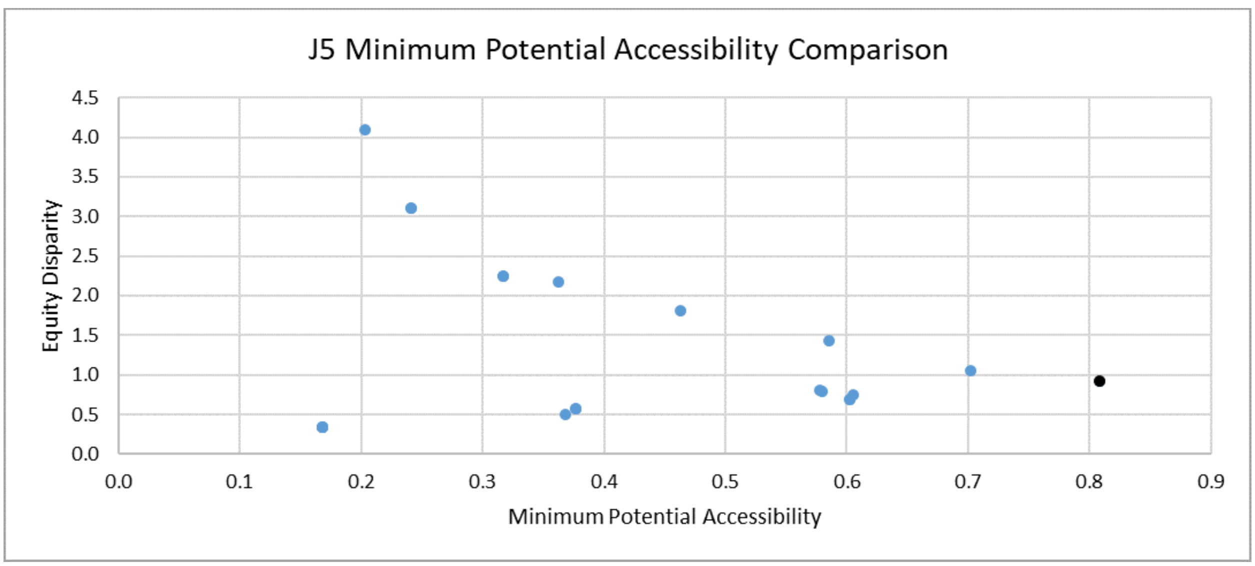

Figure 22. J5 Minimum Potential Accessibility Comparison

Table 24. W5 ε-Constraint Solutions

Point Number	Enforced Disparity	Sum of Potential Accessibility	Equity Disparity	Minimum Potential Accessibility	Shelters								
0	3.208	260.596	3.208	1.304	10	11	15	16	17	20	28	42	48
1	2.946	260.254	2.933	1.540	10	11	15	16	17	20	40	42	48
2	2.685	256.702	2.651	1.779	10	11	15	16	17	20	22	36	49
3	2.569	255.789	2.536	1.618	11	15	16	17	20	36	40	44	48
4	2.424	253.427	2.415	1.829	11	15	16	17	20	36	40	44	49
5	2.372	252.543	2.367	1.763	11	15	16	17	20	22	40	44	49
6	2.174	236.801	2.149	1.679	6	8	15	16	17	20	22	40	44
7	2.162	236.801	2.149	1.679	6	8	15	16	17	20	22	40	44
8	1.976	224.355	1.976	1.625	6	15	17	20	22	27	36	44	49
9	1.901	217.245	1.899	1.571	6	8	15	17	20	22	27	44	49
10*	1.779	204.255	1.779	1.577	6	8	16	17	20	22	24	44	49
11	1.639	196.589	1.634	1.497	17	20	22	24	27	30	39	40	44
12	1.581	190.991	1.567	1.477	6	17	20	22	24	25	27	40	44
13	1.384	170.191	1.382	1.394	6	8	17	19	20	22	27	44	49
14	1.378	169.894	1.374	1.352	3	6	17	22	24	25	27	40	44
15	1.186	146.972	1.185	1.203	1	2	6	8	17	22	24	27	44
16	1.116	140.702	1.115	1.142	6	7	22	24	27	36	40	44	49
17	0.989	123.841	0.988	0.948	1	7	8	22	24	27	40	44	47
18	0.855	104.041	0.854	0.768	1	3	12	19	22	27	36	40	44
19	0.791	96.285	0.788	0.756	1	3	6	7	19	22	27	36	40
20	0.594	64.421	0.593	0.482	1	2	3	4	5	8	13	19	49

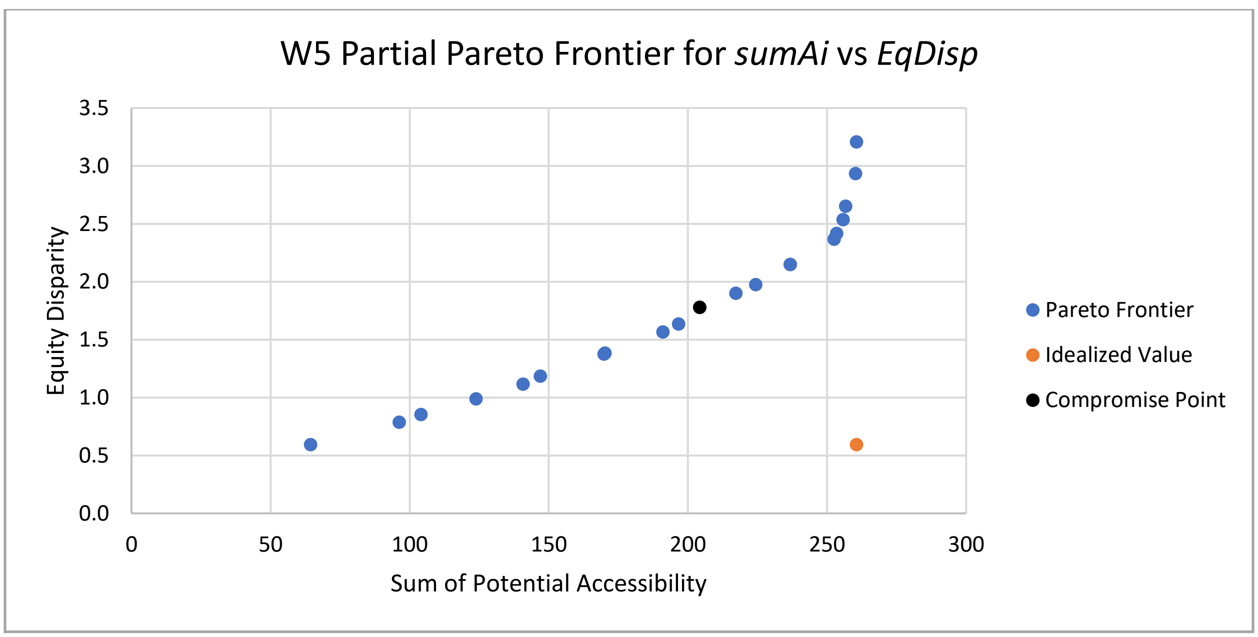

Figure 23. W5 Partial Pareto Frontier

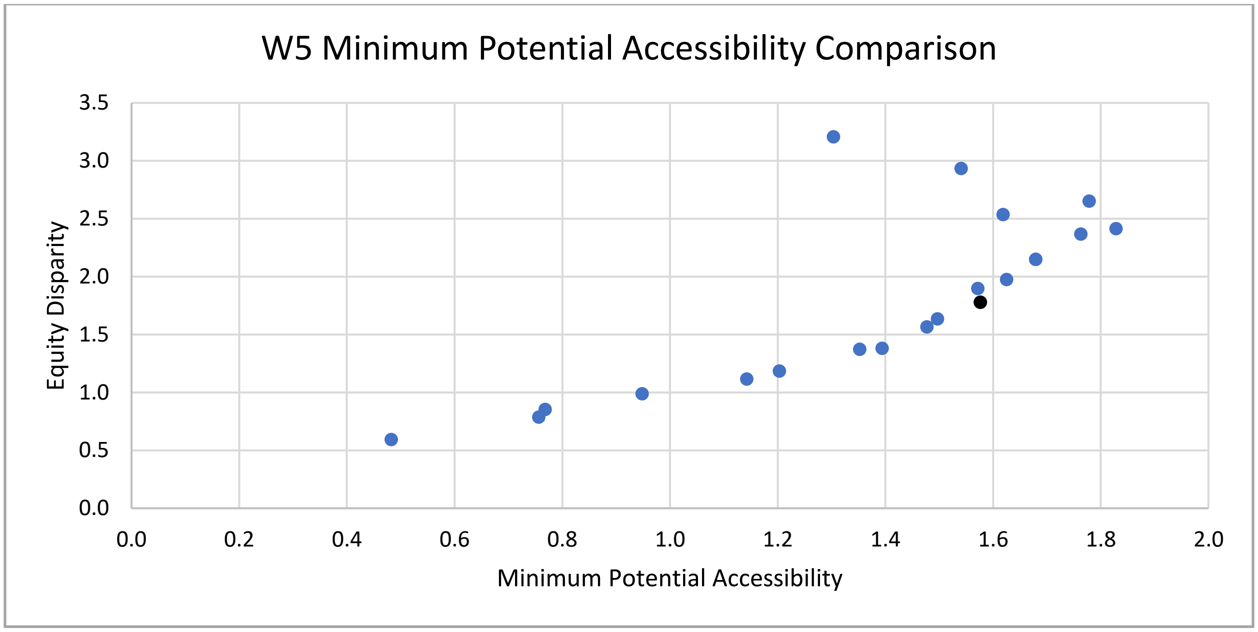

Figure 24. W5 Minimum Potential Accessibility Comparison

8.3 Appendix C

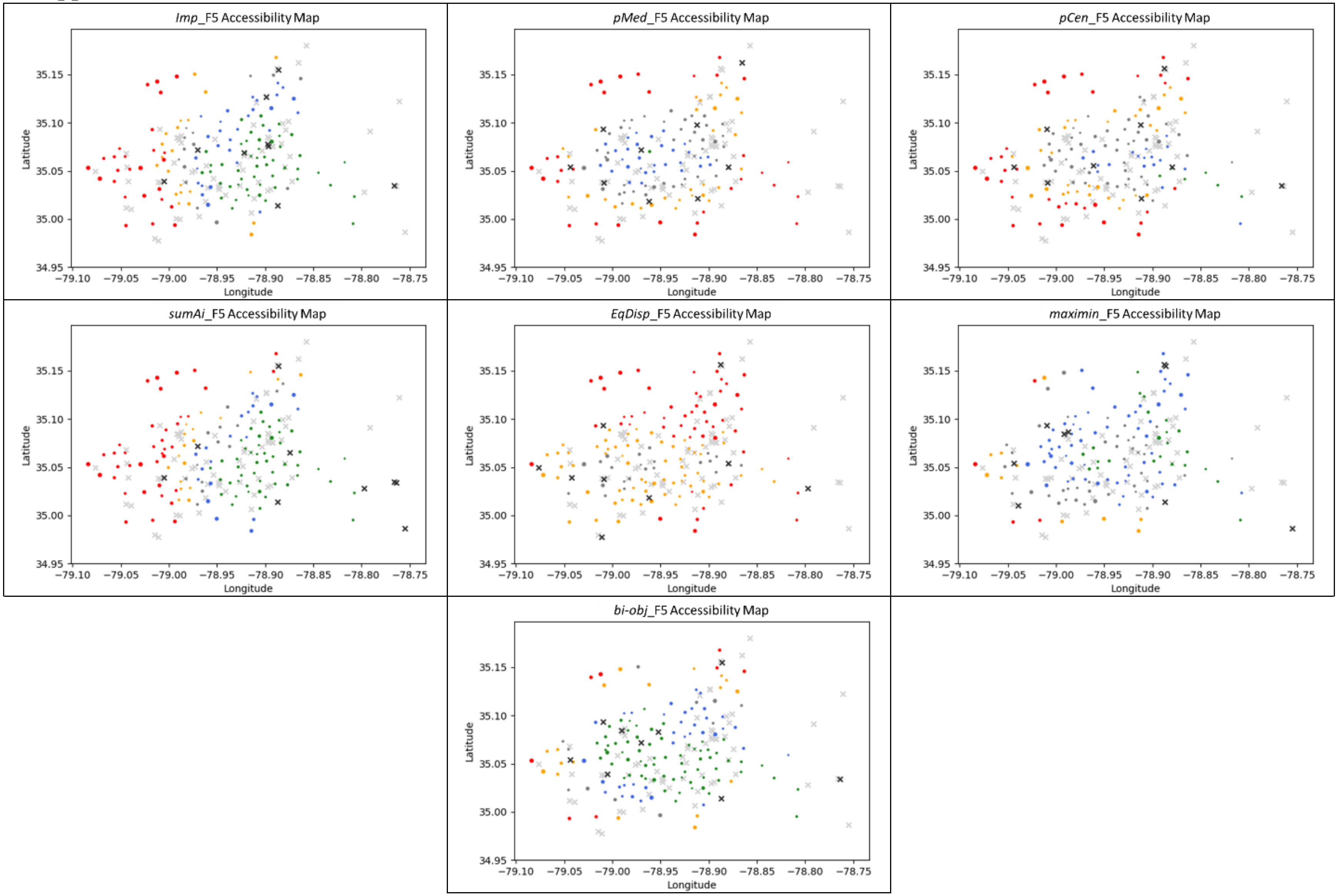

Figure 36. F5 All Models Accessibility Maps

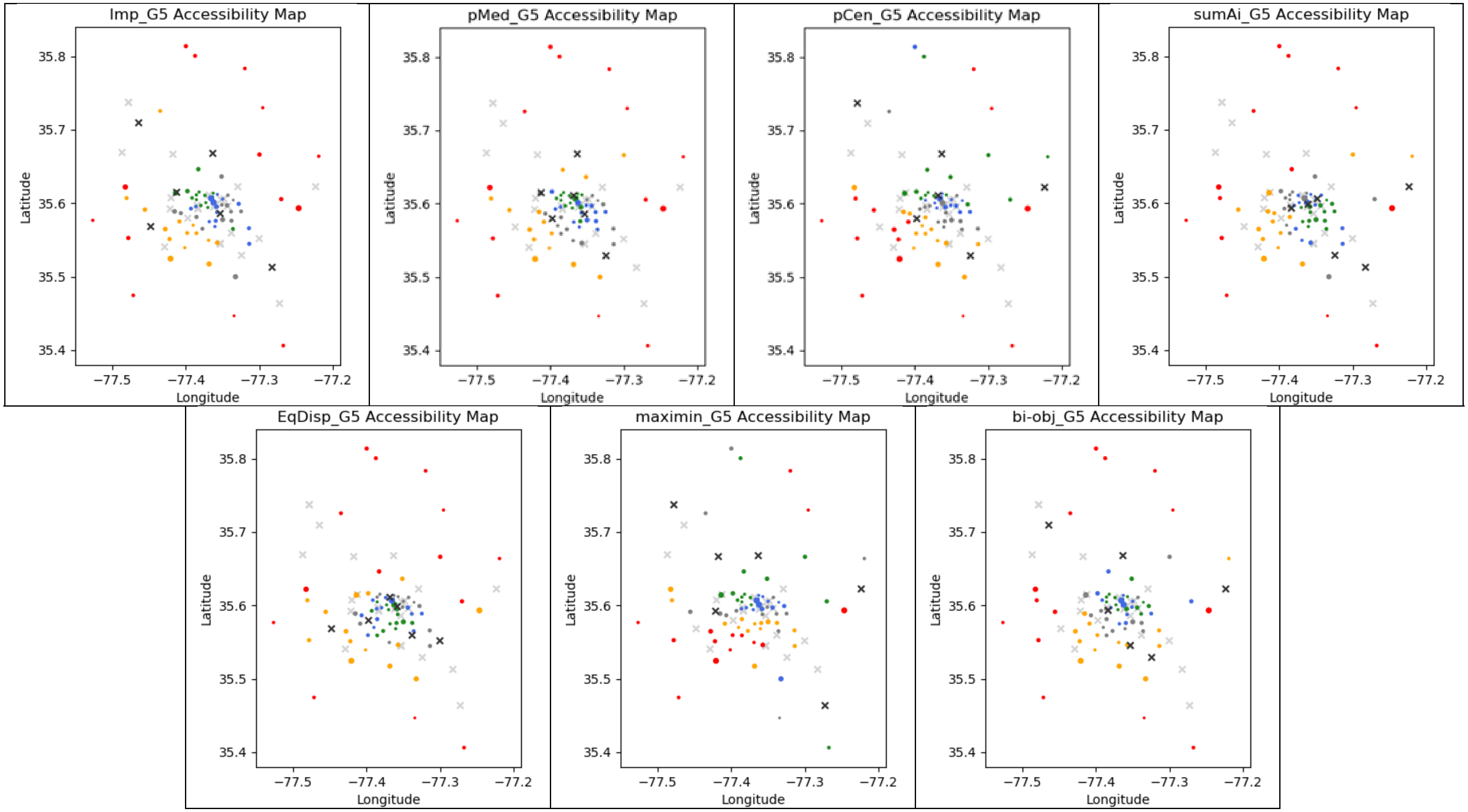

Figure 37. G5 All Models Accessibility Maps[6]

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[6] For Figures 36-39, a black 'x' represents an opened shelter and a grey 'x' represents an unopened shelter. Each circle represents a block group, where the size of a circle indicates the relative population in the block group and the color represents the access score quintile for the block group. The color green represents the top quintile (best), blue fourth, grey third, orange second, and red the bottom quintile (worst).

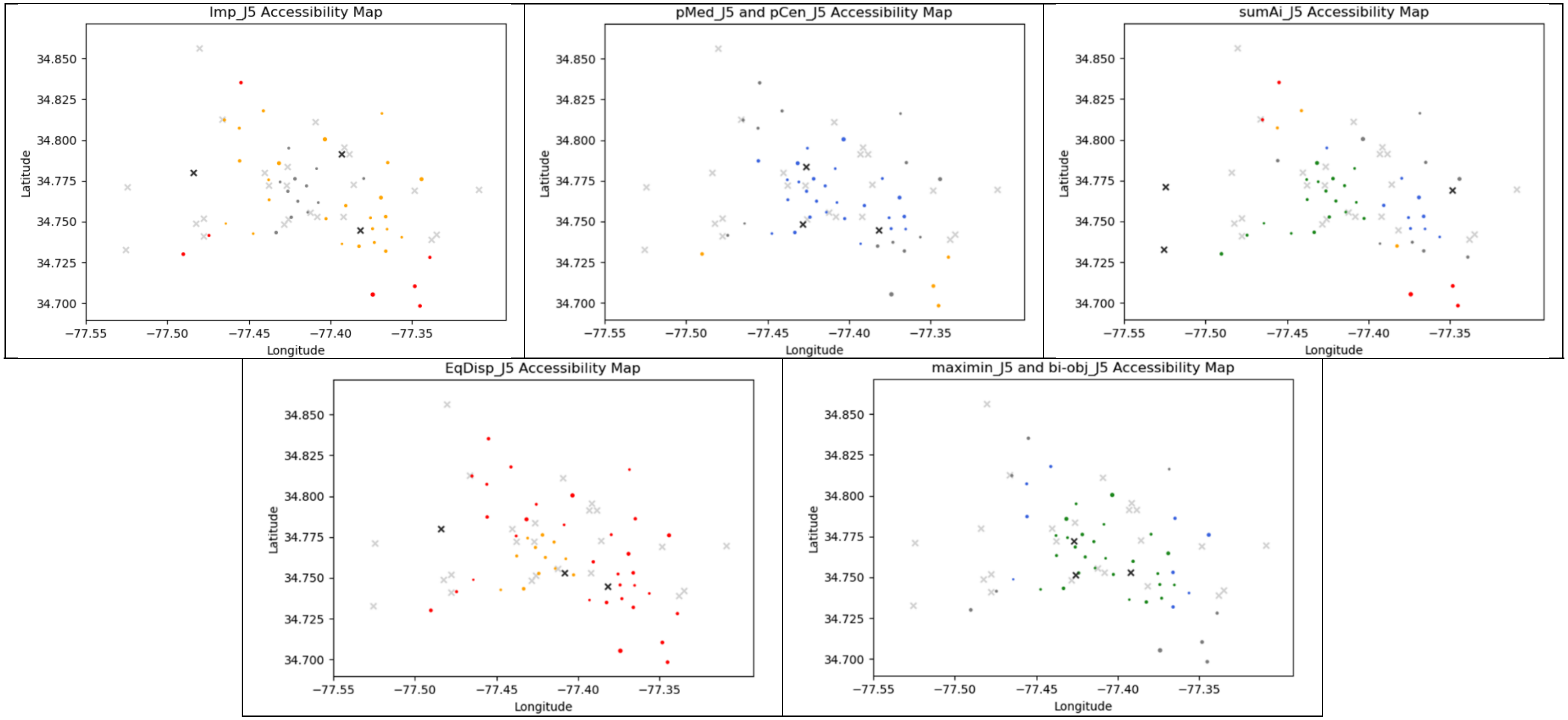

Figure 38. J5 All Models Accessibility Maps

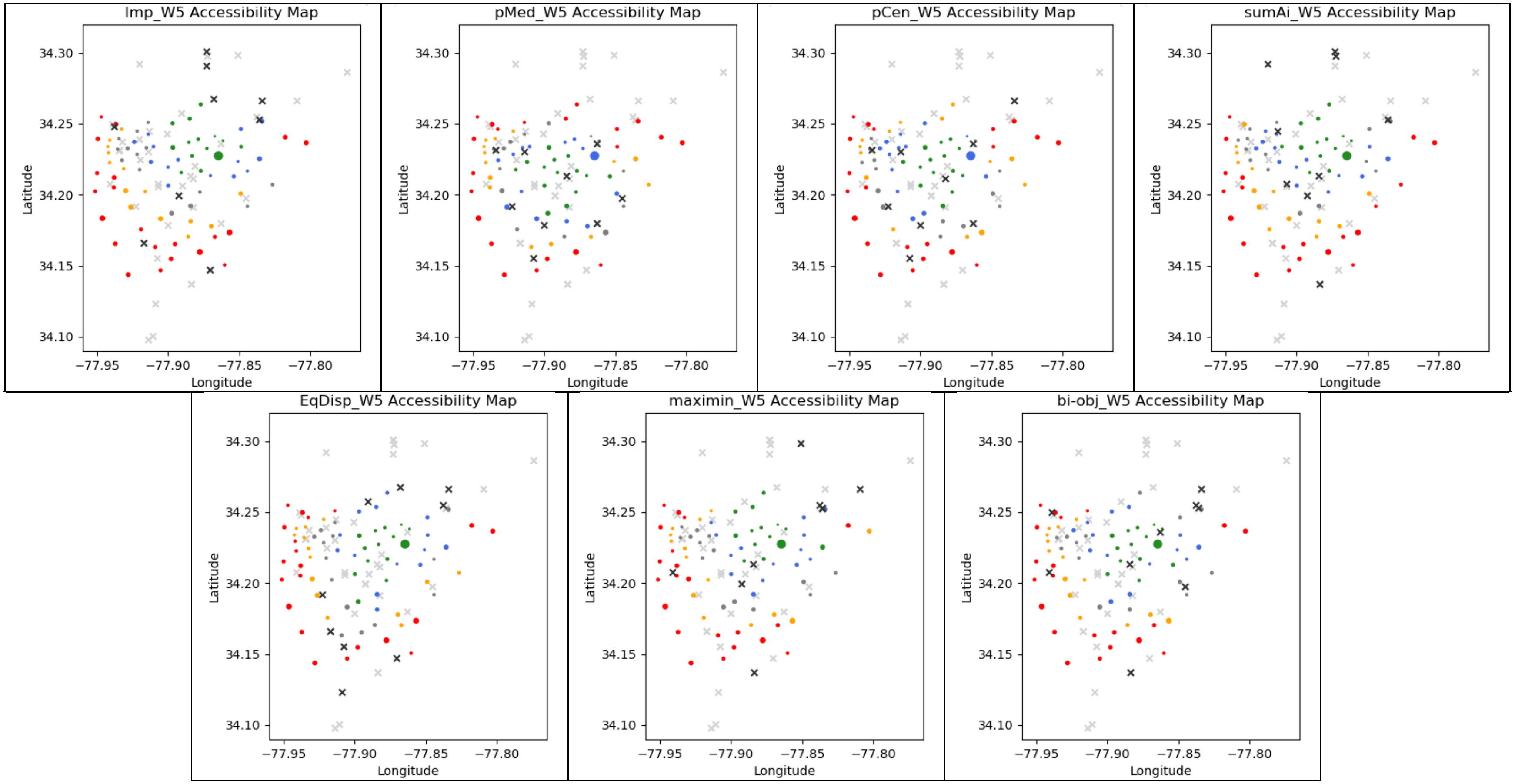

Figure 39. W5 All Models Accessibility Maps