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Estimation of Stress Strength Reliability for Transmuted Exponentiated Inverse Rayleigh Distribution

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Abstract - The problem of estimation of reliability of systems in stress-strength set up is an important area of research in Statistics, particularly, in Statistical Inference on reliability. In this paper, the estimation of stress-strength reliability when the strength and stress variables are assumed to be independently distributed as transmuted exponentiated inverse Rayleigh distribution (TEIRD) is considered. The TEIRD is a general distribution which includes transmuted inverse Rayleigh distribution, exponentiated inverse Rayleigh distribution and inverse Rayleigh distribution as a particular cases. The maximum likelihood estimator of stress -strength model is derived. Also, asymptotic confidence interval for reliability is constructed. The real data analysis is considered and the simulation study is conducted.

Keywords: Stress-strength model, Reliability, Maximum likelihood estimator.

I. INTRODUCTION

The technique of adding a new parameter to generalize the well-known distributions using quadratic rank transmutation map is introduced by Shaw et al [9]. The resulting family of distributions are called transmuted family of distributions with reference to a base line distribution. The general form of cumulative distribution function (cdf) and probability density function (pdf) of transmuted family of distributions are given by

$$F(x) = (1+\lambda)F_1(x) - F_1^2(x) \tag{1}$$

and

$$f(x) = f_1(x)\left[(1+\lambda) - 2\lambda F_1(x)\right], \tag{2}$$

where $F_1(x)$ and $f_1(x)$ are the baseline cdf and pdf with λ being the transmuted parameter.

In this paper, stress-strength reliability is estimated by assuming stress and strength variables to follow transmuted family with exponentiated inverse Rayleigh (EIR) distribution (Rahman and Dar[10]) as baseline distribution and the distribution considered here is known as transmuted exponentiated inverse Rayleigh distribution (TEIRD) which is due to Muhammad [7]. The pdf and cdf of EIR distribution are in the following form:

$$f_1(x) = \frac{2\alpha\theta}{x^3} e^{-\frac{\theta\alpha}{x^2}}, x > 0, \theta, \alpha > 0 \tag{3}$$

$$F_1(x) = e^{-\frac{\theta\alpha}{x^2}}, x > 0, \theta, \alpha > 0 \tag{4}$$

The pdf and cdf of TEIR distribution is obtained by substituting (3) and (4) in (1) and (2).

$$f(x;\alpha,\theta,\lambda) = \frac{2\alpha\theta}{x^3} e^{-\frac{\theta\alpha}{x^2}} \left[1 + \lambda - 2\lambda e^{-\frac{\theta\alpha}{x^2}}\right], \; x > 0, \; \theta, \alpha > 0, -1 \le \lambda \le 1. \tag{5}$$

$$F(x;\alpha,\theta,\lambda)=e^{-\frac{\theta\alpha}{x^2}}\left[1+\lambda-\lambda e^{-\frac{\theta\alpha}{x^2}}\right],\ x>0,\ \theta,\alpha>0,-1\le\lambda\le 1. \qquad (6)$$

The TEIR distribution with parameters α, θ and λ is denoted as TEIR (α, θ, λ).
Some particular cases of TEIR distribution are given below:
i) If $\lambda = 0$, then (4) reduces to EIR distribution.
ii) The equation (4) reduces to transmuted inverse Rayleigh (TIR) distribution, when $\theta = 1$.
iii) If $\lambda = 0$ and $\theta = 1$, then equation (4) reduces to inverse Rayleigh (IR) distribution.

In the last decade, the transmuted family of distributions are studied extensively by many researchers. Transmuted Gumbel distribution is studied by Aryal and Tsokos [3]. Ashour et al [2] studied the transmuted Lomax distribution. Transmuted Lindley distribution is studied by Merovci [5]. The transmuted inverse Rayleigh and transmuted pareto distributions respectively, are studied by Ahmad [1] and Merovci et al [6].

The organized of the paper is as below. In Section II, stress strength reliability assuming the stress and strength variables to be TEIR distribution is derived and studied the estimation of stress-strength reliability. Asymptotic confidence interval for reliability is obtained in section III. The analysis of real data is given in section IV. The simulation study results are tabulated in section V and section VI gives conclusions and remarks.

II. ESTIMATION OF STRESS-STRENGTH RELIABILITY

Let X and Y be two independent random variables having TEIR$(\alpha_1,\theta_1,\lambda_1)$ and TEIR$(\alpha_2,\theta_2,\lambda_2)$ distributions respectively. The stress-strength reliability R is given by

$$\begin{aligned} R &= P(X>Y) \\ &= \int_0^\infty \left\{\int_0^x f(y;\alpha_2,\lambda_2,\theta)dy\right\} f(x;\alpha_1,\lambda_1,\theta_1)dx \\ &= \frac{v^2[(1-\lambda_1)(2+\lambda_2)]+v[\lambda_1\lambda_2-\lambda_1+2\lambda_2+5]+2}{(v+1)(v+2)(2v+1)}, v=\frac{\theta_2\alpha_2}{\theta_1\alpha_1} \end{aligned} \qquad (7)$$

The stress- strength reliability R is estimated using the method of maximum likelihood. For that, first we obtain the maximum likelihood estimators (MLE) of $\theta_1, \lambda_1, \alpha_1, \theta_2, \lambda_2$ and α_2.

A. Maximum Likelihood Estimation:
Let $(X_1, X_2, ..., X_n)$ and $(Y_1, Y_2, ..., Y_m)$ be two independent random samples drawn from TEIR distribution. Then the likelihood function of $(\alpha_1,\lambda_1,\theta_1,\alpha_2,\lambda_2,\theta_2)$ for a given $(\underline{x},\underline{y})$ is

$$L(\alpha_1,\lambda_1,\theta_1,\alpha_2,\lambda_2,\theta_2 \mid \underline{x};\underline{y})=\prod_{i=1}^n f(x_i,\alpha_1,\lambda_1,\theta_1)\times\prod_{j=1}^m f(y_j,\alpha_2,\lambda_2,\theta_2)$$

$$L(\alpha_1,\lambda_1,\theta_1,\alpha_2,\lambda_2,\theta_2 \mid \underline{x};\underline{y})=\prod_{i=1}^n \frac{2\alpha_1\theta_1}{x_i^3}\exp\left(-\frac{\alpha_1\theta_1}{x_i^2}\right)\times\prod_{j=1}^m \frac{2\alpha_2\theta_2}{y_j^3}\exp\left(-\frac{\alpha_2\theta_2}{y_j^2}\right)$$

Let $\ell=\log L.$ Then, the likelihood equations to estimate the parameters $\alpha_1,\theta_1,\lambda_1,\alpha_2,\theta_2$ and λ_2, are

$$\frac{\partial \ell}{\partial \theta_1} = \frac{n}{\theta_1} - \alpha_1 \sum_{i=1}^{n} \frac{1}{x_i^2} + \sum_{i=1}^{n} \left[\frac{2\lambda_1 \alpha_1 \exp\left(-\frac{\alpha_1 \theta_1}{x_i^2}\right)}{x_i^2 \left(1 + \lambda_1 - 2\lambda_1 \exp\left(-\frac{\alpha_1 \theta_1}{x_i^2}\right)\right)} \right] = 0 \quad (8)$$

$$\frac{\partial \ell}{\partial \alpha_1} = \frac{n}{\alpha_1} - \theta_1 \sum_{i=1}^{n} \frac{1}{x_i^2} + \sum_{i=1}^{n} \left[\frac{2\lambda_1 \theta_1 \exp\left(-\frac{\alpha_1 \theta_1}{x_i^2}\right)}{x_i^2 \left(1 + \lambda_1 - 2\lambda_1 \exp\left(-\frac{\alpha_1 \theta_1}{x_i^2}\right)\right)} \right] = 0 \quad (9)$$

$$\frac{\partial \ell}{\partial \lambda_1} = \sum_{i=1}^{n} \left[\frac{1 - 2\exp\left(-\frac{\alpha_1 \theta_1}{x_i^2}\right)}{\left(1 + \lambda_1 - 2\lambda_1 \exp\left(-\frac{\alpha_1 \theta_1}{x_i^2}\right)\right)} \right] = 0 \quad (10)$$

$$\frac{\partial \ell}{\partial \alpha_2} = \frac{m}{\alpha_2} - \theta_2 \sum_{j=1}^{m} \frac{1}{y_j^2} + \sum_{j=1}^{m} \left[\frac{2\lambda_2 \theta_2 \exp\left(-\frac{\alpha_2 \theta_2}{y_j^2}\right)}{y_j^2 \left(1 + \lambda_2 - 2\lambda_2 \exp\left(-\frac{\alpha_2 \theta_2}{y_j^2}\right)\right)} \right] = 0 \quad (11)$$

$$\frac{\partial \ell}{\partial \theta_2} = \frac{m}{\theta_2} - \alpha_2 \sum_{j=1}^{m} \frac{1}{y_j^2} + \sum_{j=1}^{m} \left[\frac{2\lambda_2 \alpha_2 \exp\left(-\frac{\alpha_2 \theta_2}{y_j^2}\right)}{y_j^2 \left(1 + \lambda_2 - 2\lambda_2 \exp\left(-\frac{\alpha_2 \theta_2}{y_j^2}\right)\right)} \right] = 0 \quad (12)$$

$$\frac{\partial \ell}{\partial \lambda_2} = \sum_{j=1}^{m} \left[\frac{1 - 2\exp\left(-\frac{\alpha_2 \theta_2}{y_j^2}\right)}{\left(1 + \lambda_2 - 2\lambda_2 \exp\left(-\frac{\alpha_2 \theta_2}{y_j^2}\right)\right)} \right] = 0 \quad (13)$$

The maximum likelihood estimates of $\alpha_1, \theta_1, \lambda_1, \alpha_2, \theta_2$ and λ_2 are obtained by solving the nonlinear system of equations (8) to (13) using some iterative technique like Newton-Raphson method.

III. ASYMPTOTIC DISTRIBUTION OF R

The asymptotic distribution of the maximum likelihood estimator (MLE) $\hat{\underline{\mu}} = \left(\hat{\alpha}_1, \hat{\theta}_1, \hat{\lambda}_1, \hat{\alpha}_2, \hat{\theta}_2, \hat{\lambda}_2\right)'$ of $\underline{\mu}$ is multivariate normal $N_P\left(\underline{\mu}, I^{-1}(\mu)\right)$, where $I^{-1}(\mu)$ is the observed information matrix evaluated at $\hat{\mu}$ .That is, $\sqrt{n}\left(\hat{\underline{\mu}} - \underline{\mu}\right) \rightarrow N_P\left(0, I^{-1}(\mu)\right)$ where $I^{-1}\left(\underline{\mu}\right)$ is the variance-covariance matrix of the unknown parameters $\mu = (\alpha_1, \theta_1, \lambda_1, \alpha_2, \theta_2, \lambda_2)'$.
Then $100(1-\alpha)\%$ confidence intervals for $\alpha_1, \theta_1, \lambda_1, \alpha_2, \theta_2$ and λ_2 are, respectively, given by

$$\left(\hat{\alpha}_1 - Z_{\frac{\alpha}{2}}\sqrt{I_{11}^{-1}},\ \hat{\alpha}_1 + Z_{\frac{\alpha}{2}}\sqrt{I_{11}^{-1}}\right), \quad \left(\hat{\lambda}_1 - Z_{\frac{\alpha}{2}}\sqrt{I_{22}^{-1}},\ \hat{\lambda}_1 + Z_{\frac{\alpha}{2}}\sqrt{I_{22}^{-1}}\right), \left(\hat{\theta}_1 - Z_{\frac{\alpha}{2}}\sqrt{I_{33}^{-1}},\ \hat{\theta}_1 + Z_{\frac{\alpha}{2}}\sqrt{I_{33}^{-1}}\right),$$

$$\left(\hat{\alpha}_2 - Z_{\frac{\alpha}{2}}\sqrt{I_{44}^{-1}},\ \hat{\alpha}_2 + Z_{\frac{\alpha}{2}}\sqrt{I_{44}^{-1}}\right), \left(\hat{\lambda}_2 - Z_{\frac{\alpha}{2}}\sqrt{I_{55}^{-1}},\ \hat{\lambda}_2 + Z_{\frac{\alpha}{2}}\sqrt{I_{55}^{-1}}\right) \quad \text{and}$$

$$\left(\hat{\theta}_2 - Z_{\frac{\alpha}{2}}\sqrt{I_{66}^{-1}},\ \hat{\theta}_2 + Z_{\frac{\alpha}{2}}\sqrt{I_{66}^{-1}}\right).$$

The asymptotic distribution of MLE of stress strength reliability $\hat{R}$ as $n, m \rightarrow \infty$ is

$$\frac{\hat{R} - R}{\sqrt{AV(\hat{R})}} \rightarrow N(0,1), \text{ where } AV(\hat{R}) = \sum_{i=1}^{6}\sum_{j=1}^{6} \frac{\partial R}{\partial \mu_i}\frac{\partial R}{\partial \mu_j} I_{ij}^{-1}.$$

Here, I_{ij}^{-1} is the (i, j)th element in the inverse of the matrix $I\left(\underline{\mu}\right)$.
Using the asymptotic distribution of R, we construct the $100(1-\alpha)\%$ asymptotic confidence interval for reliability R and it is obtained as

$$\left(\hat{R} - Z_{\frac{\alpha}{2}}\sqrt{AV(\hat{R})},\ \hat{R} + Z_{\frac{\alpha}{2}}\sqrt{AV(\hat{R})}\right).$$

A. Likelihood Ratio Test:

To check whether the Transmuted G model is a better fit to the data as compared to G model, we test $H_0 : \lambda = 0$ against $H_1 : \lambda \neq 0$. Then, the likelihood ratio statistic for testing H_0 against H_1 is given by

$$w = 2\left\{\ell\left(\hat{\lambda}, \hat{\alpha}, \hat{\theta}\right) - \ell\left(0, \tilde{\alpha}, \tilde{\theta}\right)\right\},$$

where $\hat{\lambda}, \hat{\alpha}$ and $\hat{\theta}$ are the MLEs of λ, α and θ under H_1 and $\tilde{\alpha}, \tilde{\theta}$ are the MLES of α and θ under H_0. The test criterion is to reject H0, if $w > \chi^2_{(k)}(1-\alpha)$. The test procedure is applied for both data sets.

IV. REAL DATA ANALYSIS

In this section, we consider the data sets used by Xia et al [11] and Saracoglu et al [8]. Recently, these data sets were also used by Hassan et al [4] to study the estimation of reliability in a multicomponent stress strength model assuming generalized linear failure rate distribution for the distribution of stress and strength variables. The data sets are given below.

Data set I: Breaking strength of jute fiber length 10 mm (Variable X)
693.73, 704.66, 323.83, 778.17, 123.06, 637.66, 383.43, 151.48, 108.94, 50.16, 671.49, 183.16, 257.44, 727.23, 291.27, 101.15, 376.42, 163.40, 141.38, 700.74, 262.90, 353.24, 422.11, 43.93, 590.48, 212.13, 303.90, 506.60, 530.55, 177.25.

Data set II: Breaking strength of jute fiber length 20 mm (Variable Y)
71.46, 419.02, 284.64,585.57, 456.60, 113.85, 187.85, 688.16, 662.66, 45.58,578.62, 756.70, 594.29, 166.49, 99.72,707.36, 765.14, 187.13, 145.96, 350.70, 547.44, 116.99, 375.81, 581.60, 119.86, 48.01, 200.16, 36.75, 244.53, 83.55.

The goodness of fit of different models to the data sets are tested using the log-likelihood (logL), Akaike information criteria (AIC), Akaike information criteria corrected (AICc) and Bayesian information criteria. The table 1 and table 2 gives the MLEs of parameters along with its standard error, -logL, AIC, AICc, BIC for the data set I and II respectively.

Table 1

Name of the distribution	Estimates (standard error)	-logL	AIC	AICc	BIC
TEIRD	$\hat{\alpha}_1 = 11.5427(2.8367)$ $\hat{\theta}_1 = 1356.3496(0.1675)$ $\hat{\lambda}_1 = -0.7598(4.4469)$	215.4556	436.9112	437.8343	435.3426
TIRD	$\hat{\theta}_1 = 35741.1542(5.9316)$ $\hat{\lambda}_1 = -0.3312(0.2455)$	225.5907	455.1814	455.6258	454.1356
EIRD	$\hat{\alpha}_1 = 3.8110(0.6991)$ $\hat{\theta}_1 = 5274.7174(5.9316)$	220.3448	444.6897	445.134	443.6438
IRD	$\hat{\theta}_1 = 29374.155(5.932)$	222.8041	447.6083	447.7511	447.0853

Table 2

Name of the distribution	Estimates (standard error)	-logL	AIC	AICc	BIC
TEIRD	$\hat{\alpha}_2 = 68.4968(3.4421)$ $\hat{\theta}_2 = 139.1925(5.9720)$ $\hat{\lambda}_2 = -0.6789(0.1753)$	214.6822	435.3644	439.3644	432.3644
TIRD	$\hat{\theta}_2 = 11803.4889(0.1928)$ $\hat{\lambda}_2 = -0.5901(2.9662)$	218.2355	440.4711	440.9154	439.4252
EIRD	$\hat{\alpha}_2 = 3.8860(0.7193)$ $\hat{\theta}_2 = 3016.1266(5.9751)$	221.6141	447.2282	447.6726	446.1824
IRD	$\hat{\theta}_2 = 12436.336(2.966)$	221.6679	445.3357	445.4787	444.8129

From the above results, it is observed that the underlined TEIR distribution is betterfit than the other distributions such as transmuted inverse Rayleigh distribution (TIRD), exponentiated inverse Rayleigh distribution (EIRD) and inverse Rayleigh distribution (IRD). The values of LR statistics discussed in section 3.1 for the data set I and II are $w_1 = 9.7784$ and $w_2 = 7.8577$ respectively. The corresponding p-values are 0.00176 and 0.0051. Hence, reject the null hypotheses and it can be concluded that the underlined TEIR distribution fits well to the data as compared to baseline distribution.

Now, the estimators derived in this paper, can be used to estimate stress strength reliability R based on these data sets. The estimate of R is 0.6290 and the asymptotic 95% confidence interval of R is (0.4690, 0.7829).

V. SIMULATION STUDY

A simulation study is conducted by generating 10000 samples of size (n, m) = (10, 10), (10, 15), (20, 20), (20, 25), (30, 30), (30, 35) and (40, 40) to evaluate the performance of estimator of system reliability by computing mean squared error (MSE) of MLE of reliability. The samples are generated from TEIR distribution with parameters $\alpha_1, \theta_1, \lambda_1, \alpha_2, \theta_2$ and λ_2 .

Table 3: Maximum likelihood estimates and their MSEs

$\alpha_1 = 3$, $\lambda_1 = 0.5$, $\theta_1 = 4.5$, $\alpha_2 = 3.2$, $\lambda_2 = 0.8$ and $\theta_2 = 5.6$			
Sample size	R	$\hat{R}$	MSE($\hat{R}$)
$n=10, m=10$	0.4923	0.4736	0.0043
$n=10, m=15$		0.4778	0.0034
$n=20, m=20$		0.4705	0.0025
$n=20, m=25$		0.4725	0.0021
$n=30, m=30$		0.4728	0.0016
$n=30, m=35$		0.4735	0.0014
$n=40, m=40$		0.4761	0.0012
$\alpha_1 = 1.5$, $\lambda_1 = -1$, $\theta_1 = 2$, $\alpha_2 = 2$, $\lambda_2 = -0.5$ and $\theta_2 = 3$			
$n=10, m=10$	0.7493	0.7182	0.0039
$n=10, m=15$		0.7198	0.0031
$n=20, m=20$		0.7228	0.0022
$n=20, m=25$		0.7219	0.0020
$n=30, m=30$		0.7231	0.0015
$n=30, m=35$		0.7245	0.00136
$n=40, m=40$		0.7230	0.00138

VI. CONCLUSIONS

In this paper, we have considered the estimation of stress strength reliability for transmuted exponentiated inverse Rayleigh distribution. The system reliability of stress strength model is derived and it is estimated using the method of maximum likelihood. A real data analysis reveals that the underlined TEIR distribution gives better fit than other distributions, which are considered above. MLEs of stress strength reliability are obtained for simulated observations with different sample sizes and MSEs of $\hat{R}$ is decreasing as sample size increases.

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