THE PROCESS AND A PITFALL IN DEVELOPING BIOLOGY AND CHEMISTRY PROBLEMS FOR MATHEMATICS COURSES

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https://doi.org/10.25891/zjn2-wb86

ABSTRACT
In this paper, we describe our process for developing applied problems from biology and chemistry for use in a differential calculus course. We describe our conversations and curricular analyses that led us to change from our initial focus on college algebra to calculus. We provide results that allowed us to see the overlaps between biology and mathematics and chemistry and mathematics and led to a specific focus on problems related to rates of change. Finally, we investigate the problems that were developed by the partner disciplines for use on recitation activities in calculus and how those problems were modified by the calculus coordinator. We compare what partner disciplines emphasize in scientific applications with what mathematics instructors emphasize in calculus and consider what that means for students' understanding of science in mathematics. We also describe the role of the students, partner discipline colleagues, and calculus instructors in the development, refinement, and use of the problems.

KEYWORDS
biology, calculus, chemistry, partner disciplines

The National Consortium for Synergistic Undergraduate Mathematics via Multi-Institutional Interdisciplinary Teaching Partnerships (SUMMIT-P) project brings together institutions and disciplines in order to improve undergraduate experiences in mathematics courses. The project is based on work by the *Committee on Curriculum Renewal Across the First Two Years* of the Mathematical Association of America and uses recommendations from the *Curriculum Foundations Project* (CF) (Ganter & Barker, 2004) to inform collaborations among disciplinary partners. Based on this prior work, the SUMMIT-P project is designed to strengthen the connections between mathematics and partner disciplines in institutionally—and programmatically—relevant ways. For example, at Ferris State University, mathematics faculty have partnered with nursing and social work faculty to meet specific programmatic goals through a quantitative reasoning course. As another example, mathematics faculty at LaGuardia Community College have partnered with business and economics faculty to revise a college algebra course to meet broader institutional goals. (Information about these two projects and many others that have taken place through SUMMIT-P are outlined in other papers in this issue.)

Oregon State University (OSU) is one of the eleven participating SUMMIT-P institutions. OSU is a land grant, research intensive university with approximately 27,000 undergraduates, and more than 3,200 of those students intend to earn a degree in one of the sciences. The Department of Mathematics at OSU offers lower-division courses including College Algebra, Trigonometry, and a calculus sequence that consists of Differential Calculus, Integral Calculus, Infinite Sequences and Series, Vector Calculus, and Differential Equations. In a given academic year, about 800 students enroll in College Algebra and approximately 2,800 enroll in Differential Calculus. Many students enrolled in College Algebra and Differential Calculus also co-enroll in a biology course or chemistry course. Because of the focus on science, the OSU SUMMIT-P team chose biology and chemistry as disciplinary partners. The goal of OSU's SUMMIT-P team is to improve connections between mathematics and the disciplines of biology and chemistry (Ganter & Haver, 2020). In this paper, we describe the four-step process we followed in doing this work (i.e., creating a content map, developing in-depth problems for use in Differential Calculus, implementing the problems, and revising the problems).

Our SUMMIT-P team consists of a mathematics educator in the Department of Mathematics (Beisiegel), two instructors from the Department of Integrative Biology (Kayes and Quick), and an instructor from the Department of Chemistry (Nafshun). Over the course of the project, our team also included a graduate student in mathematics (Michael Lopez; during summer 2017) and two undergraduates (Steve Dobrioglo and Michael Dickens; beginning in summer 2018). Because of the importance of college algebra (Ganter & Haver, 2011), our initial focus was on the College Algebra course, which was in the midst of a significant curricular and pedagogical redesign at OSU and seemed to be a good fit for the SUMMIT-P project. However, our focus eventually turned to the Differential Calculus course and developing rich, in-depth biology and chemistry contexts that required differential calculus concepts to answer questions about those contexts.

In this paper, we first share some background on the importance of providing students with connections between mathematics and partner disciplines, along with some information about the Committee on the Undergraduate Program in Mathematics (CUPM). Then we report the steps we took in the process of conducting our work. We first describe the step of creating a content map that resulted from our conversations about which mathematics concepts and skills are used in biology and chemistry and how the mapping helped us to better understand the connections between mathematics, biology, and chemistry. We then provide details about how we developed the in-depth differential calculus problems with biology and chemistry contexts along with some examples. Once the problems were developed, our next step was to incorporate the problems into the differential calculus course. We describe how that process occurred. Based on the implementation of the problems in the third step, we explain our fourth step which was revising the problems. Finally, we reflect on our experiences, including what were positive features of the experience and what we learned by going through the process.

Background

Introductory mathematics courses, such as College Algebra or Pre-Calculus, and subsequent courses, like Calculus, have historically been problematic for students (Fairweather, 2008; Seymour & Hewitt, 1997). In particular, college mathematics courses are "frequently uninspiring, relying on memorization and rote learning" (President's Council of Advisors on Science and Technology, 2012, p. 28). To add to this issue, faculty in departments of mathematics do not often collaborate with faculty outside of mathematics on curriculum development. As a result, students are unable to "see the connections between mathematics and their chosen disciplines; instead they leave mathematics courses with a set of skills that they are unable to apply in non-routine settings" (Ferguson, 2012, p. 187).

For several decades, the Mathematical Association of America (MAA) has aimed to address these issues through CUPM. As part of its charge, CUPM provides recommendations to help mathematics departments design meaningful materials for undergraduates taking mathematics courses. The *2004 CUPM Curriculum Guide* (Barker, Bressoud, Epp, Ganter, Haver, & Pollatsek, 2004) offered six recommendations, three of which were the focus of the OSU SUMMIT-P project. Our goal was to "continually strengthen courses and programs to better align with student needs, and assess the effectiveness of such efforts" (p. 1), to "promote awareness of connections to other subjects (both in and out of the mathematical sciences) and strengthen each student's ability to apply the course material to these subjects" (p. 2), and to "encourage and support faculty collaboration with colleagues from other departments to modify and develop mathematics courses" (p. 2).

The Process of Our Project

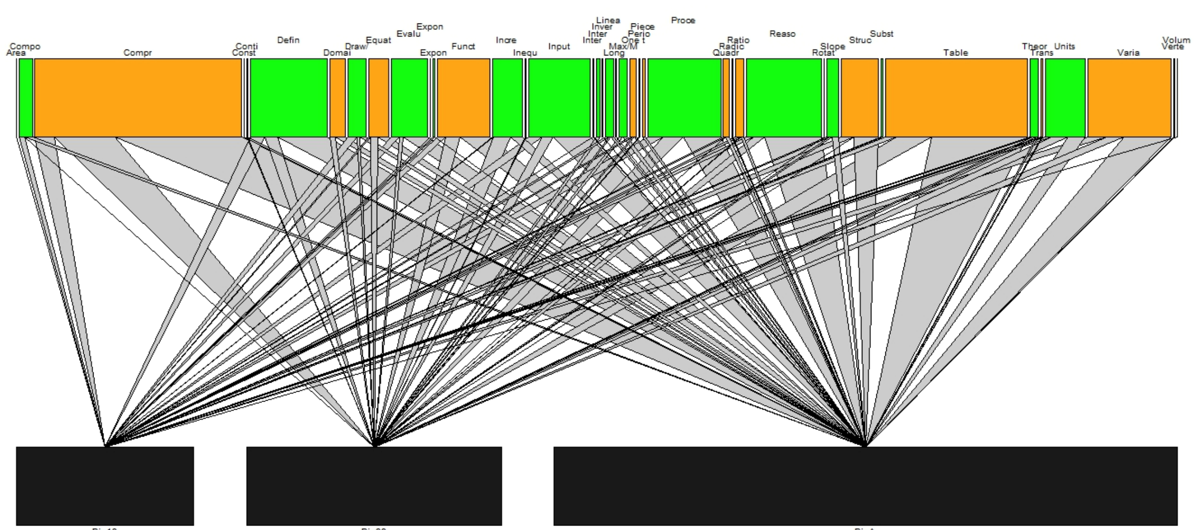
Step 1: Content Mapping

From the outset of the project, the OSU team focused on the partner disciplines of biology and chemistry. In the first year of the project (2016 – 2017) Beisiegel, Kayes, Nafshun, and Quick met bi-monthly and looked specifically at whether or not different mathematics topics appeared in biology and chemistry courses. If concepts were included in a course, we analyzed how those concepts were described and used in the partner discipline curriculum. We learned about what was important to the partner disciplines on a national level by reading publications of the CF project (Ganter & Barker, 2004) and locally via fishbowl discussions with OSU partner discipline faculty.

In spring 2017, Kayes from biology and Nafshun from chemistry met with faculty members from their departments using the fishbowl protocol (Hofrenning et al., 2020) and developed lists describing the mathematical "needs" for biology and chemistry courses. We noticed that the list outlined in CF reports (Ganter & Barker, 2004) and the one developed through the fishbowl activity aligned with the content covered in mathematics courses. Many mathematical concepts that the partner disciplines noted as important were concepts that were emphasized in mathematics courses. For example, the chemistry fishbowl included a discussion about proportional reasoning being a key skill that chemists hoped students would learn in mathematics courses. However, after several discussions about specific places in the mathematics curriculum to include problems with science contexts, we realized that language was a barrier to doing this. We noticed the different ways that mathematics describes and uses concepts compared to other disciplines like biology and chemistry.

After the biology and chemistry fishbowl conversations, one of our main questions became: What specific mathematics concepts are important to the study of biology and chemistry? After some discussion, an equally important question arose: Do these two disciplines talk about and use mathematical concepts in similar ways? To answer these questions, a mathematics graduate student (Lopez) conducted a rich, in-depth analysis of the mathematical concepts, skills, and vocabulary that are included in the biology and chemistry curriculum materials. As a first step, the OSU team listed the most common mathematical terms and concepts that are used in courses such as College Algebra, Trigonometry, and Differential Calculus. Examples of these terms include function, variable, input, composition, graph, and intercept, among many others. Mr. Lopez then examined OSU biology and chemistry materials as well as Advanced Placement materials, which are used as a guide for biology courses, to look for instances of these terms.

Figure 1
Map Between Biology Content and Mathematical Terms



Figures 1, 2, and 3 illustrate a mapping between mathematical terms and concepts found in biology and chemistry curriculum materials. The width of a rectangle at the top of the figure is a representation of how often a term appeared in the science content. For example, in Figure 3 (which is a zoomed in corner of Figure 1) the terms 'constant' and 'continuous' are represented by very narrow rectangles indicating that these terms appear very infrequently in the biology content. The width of the rectangles at the bottom of each figure illustrates the extent to which the set of curricular materials contained mathematical concepts and language. In Figure 1, Bio A represents the activities, and Bio 13 represents the lecture notes that are used in Principles of Biology at OSU; Bio 33 represents the materials used in Advanced Anatomy and Physiology at OSU. Note that the width of the Bio A rectangle compared to the width of the Bio 13 rectangle in Figure 1 reflects that mathematics terms appear much more frequently in activities than in the lectures in Principles of Biology.

Figure 2

Map Between Chemistry Content and Mathematical Terms

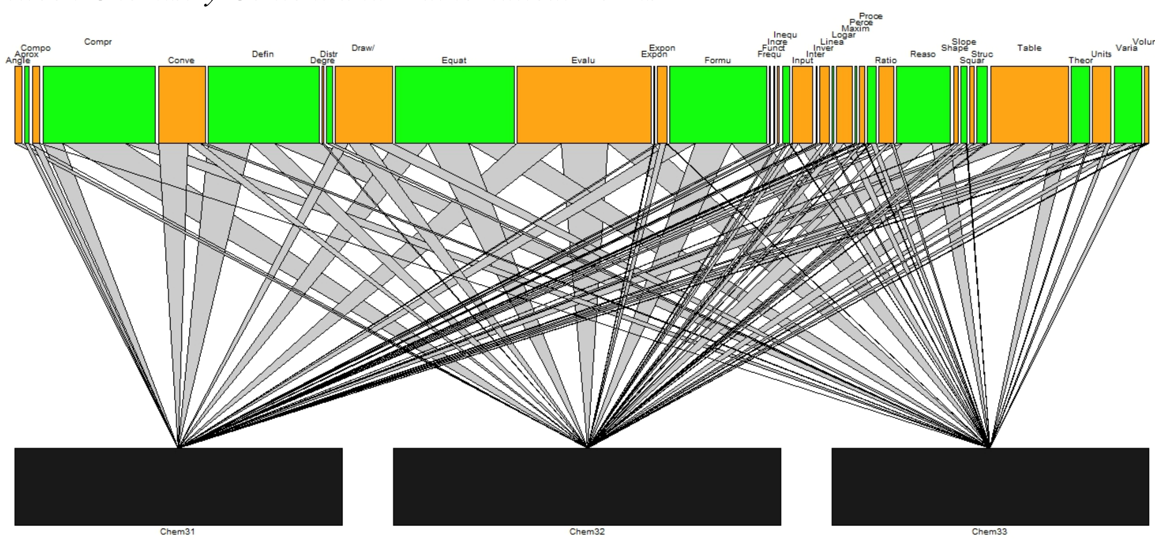
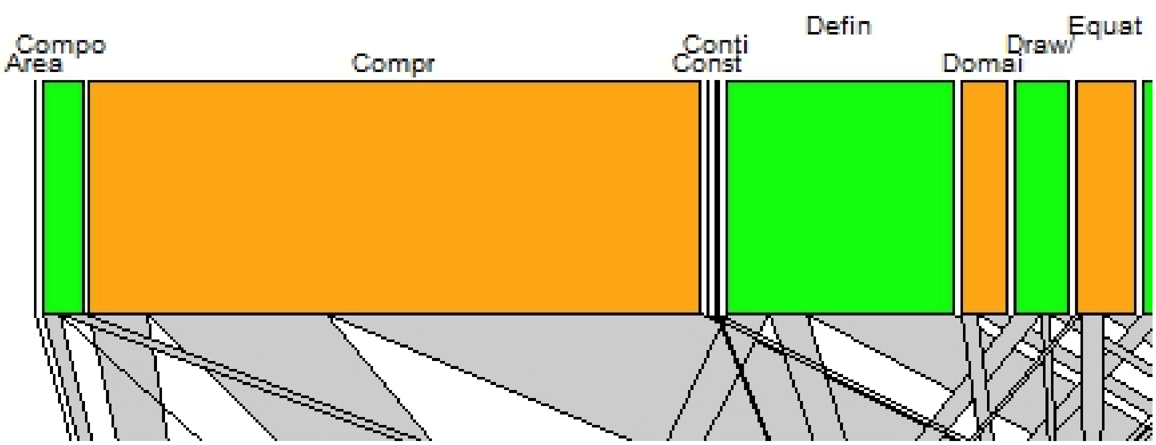


Figure 3

Zoomed in View of the Map Between Biology Content and Mathematical Terms



These figures provided insight into where we could explore the correlations between College Algebra and partner discipline content instead of wading through the content without any specific direction. It also allowed us to explore ideas that appear to be very important in mathematics but on the surface appear to be not as important to the disciplinary partners. For example, “function” is a salient concept in mathematics courses, and yet it did not appear as important in biology or chemistry. Notice the somewhat narrow rectangle representing the function concept in Figure 1 and the extremely narrow rectangle representing the function concept in Figure 2. In our discussions about this discrepancy, we found that biologists and chemists might use the terms ‘formula’ or ‘equation’ instead of ‘function.’ More importantly, these discussions allowed us to understand some essential differences in how mathematical concepts are addressed in the partner disciplines. This was critical to our understanding of how biology and chemistry problems could be incorporated into a mathematics course in a meaningful way.

Through the analysis of terms and concepts and the ensuing conversations, we found the concept in the partner disciplines that would be most appropriate to highlight in mathematics courses was *change*; in particular, we saw that biology and chemistry problems that address change (e.g., rates of change) provided a strong connection between mathematics and the sciences. While linear slope is a rate of change that is studied in College Algebra, it is only highlighted briefly in the course. Given that we wanted to create in-depth, robust connections between mathematics and biology and chemistry, we decided to move to project’s focus to the Differential Calculus course, in which change is a predominant theme.

Step 2: Developing In-depth Biology and Chemistry Contexts for Use in Differential Calculus

In the second year of our project (2017–2018), our goal was to develop specific problems and activities for Differential Calculus that were strongly rooted in the biology and chemistry content. During the academic year, we met monthly, and as we began our work in earnest, we realized that involving undergraduate students who had taken courses in all three disciplines would be advantageous for the project. Students with experiences in these courses would have unique insights that the faculty might not have. At the beginning of summer 2018, we met with the students (Dobrioglo and Dickens) to share the purpose, goals, and intended outcomes of the SUMMIT-P project.

We described the vision of developing calculus problems that could be addressed from both the mathematical and partner discipline (biology or chemistry) perspectives. We posed the following questions to the students: How do mathematics and biology or chemistry faculty represent problems in class? What questions do they ask during class? How are the representations in mathematics different or similar from the representations in biology or chemistry? What different terminology do they use? For example, in one of the initial meetings with the Dobrioglo and Dickens, we talked about the purpose of the logistic function in calculus compared to how the function is presented and used in biology courses. More broadly, we talked about wanting to develop a specific set of applied problems that could be explored at different points in Differential Calculus. In this way, students would understand how different calculus concepts can be used to understand different parts of contextually rich problems. We felt that this would provide a much deeper experience for students than the more typical “one-off” problems.

We met with the students weekly and gave them “homework assignments.” Their first assignment was to explore the logistic function and to find contexts in biology, chemistry, general science, and other human-interest situations of interest to undergraduates that could be used to introduce the function in Differential Calculus. In addition to emphasizing contexts from partner disciplines, the problems that were developed also highlighted the importance of understanding the behavior of this function by determining its derivatives. We created a Google document that the entire team could access as we developed the set of problems. In subsequent meetings, we discussed the problems and contexts the students found, the terminology used in different applications, the significance of those terms in the partner disciplines, and the appropriateness of those applications for use in Differential Calculus.

By the end of summer 2018, our team had developed over 25 problems with contexts in biology or chemistry for use in Differential Calculus. We provide two examples here:

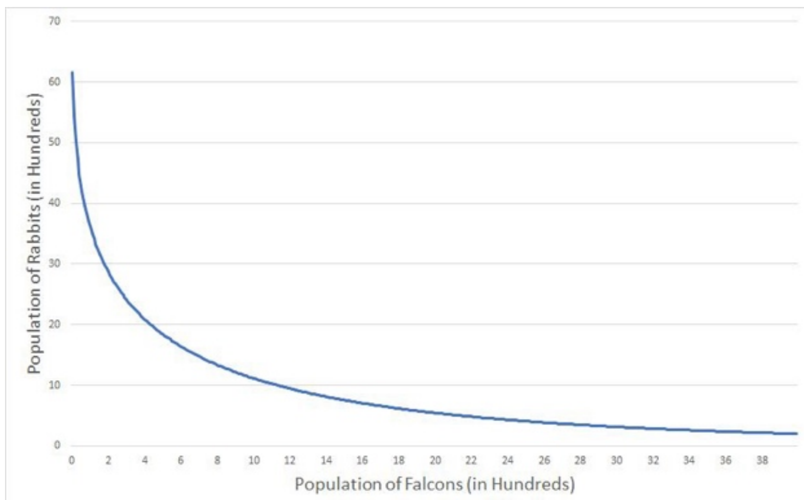
Example 1: Falcon-Rabbit, Predator-Prey in Biology

There is a large population of Mountain Cottontail rabbits in the woods of Oregon. A family of falcons (of the peregrine variety) moved into the area and preyed on the population of rabbits, devastating the rabbit population. The function below (see Figure 4) represents the number of rabbits, with *f* representing the number of falcons. The graph illustrates the change in the rabbit population. Use the function and data to determine how devastated the rabbit population was by the introduction of the falcons into their habitat.

Figure 4

Population of Falcons and Rabbits

$$r(f) = -\frac{200\, ln\left(\frac{f}{40}\right)}{f + 20}$$



1. Is the function continuous for all real numbers?
2. Find and interpret the value of $\frac{dr}{df}$ for $f = 10$.
3. Find and interpret the value of $\frac{d^2r}{df^2}$ for $f = 10$.
4. For what population of falcons will $\frac{dr}{df}$ be greatest?

This problem met our established goals for incorporating it into the course: (1) the problem could be used at different points across the ten-week term, and (2) it required concepts and skills covered in three chapters in the course textbook. The question about continuity could be addressed when instructors are developing the concepts of limits and continuity, the questions about the first and second derivatives could be addressed when the derivative is defined and the rules for differentiation are introduced, and the last question could be addressed when students are exploring optimization.

The biologists and the chemist on our team noted that there are many situations in their disciplines where one variable is changing in relation to another and neither variable represents time. In the example above, the population of prey depends on the population of predators and vice versa; the biology models that capture population growth and decline use one population as the dependent variable and the other population as the independent variable. In comparison, for most of the rate of change problems in a calculus course, the independent variable represents time. Thus, the team felt that this predator-prey problem represented a novel approach to exploring derivatives. The team also felt that gaining a broader understanding of these types of problems was important for students to make meaningful connections between calculus, biology, and chemistry.

Example 2: Effusion

Effusion in chemistry is the process in which two or more particles are escaping through a small hole with the lighter particles leaving the container faster than the heavier particles. Heavy particles move slower than light particles with the same kinetic energy. This gives the lighter particles a higher probability of escaping every second. This relationship can be determined by the equation where m represents the mass and $k > 0$ is a constant:

$$\frac{k}{\sqrt{m}} = r_e, \text{the rate of effusion}$$

A ratio can be made of effusion rates by Graham's Law:

$$\frac{rate\ of\ first\ particle}{rate\ of\ second\ particle} = \frac{\sqrt{MM\ of\ second\ particle}}{\sqrt{MM\ of\ first\ particle}}$$

where MM is the particle's molar mass. This formula can be used to find the relative rate of one particle to another. This can be very useful in chemistry when determining how much of one concentration will be present given the concentration of another gas after a certain amount of time.

A chemist has filled a balloon with equal proportions of Argon (MM = 39.948) and Hydrogen (MM = 1.008) he proceeds to poke a hole in the balloon, slowly letting the gas escape.

1. What is the proportion of the rate that Hydrogen escapes to the rate that Argon escapes?
2. If the concentration of Argon in the balloon can be modeled by the equation $A(t) = 20\ln(e^t+1) - 20t$ where time, t, is in seconds and concentration is in grams per liter, then what is the equation for the rate at which Hydrogen is escaping from the balloon?
3. What is the rate of change of the Hydrogen concentration at time $t = 4$ seconds?
4. What is the limit as t approaches infinity of the rate of change of Argon and Hydrogen? (use the equation and answer in part 2).

While the first question for this problem is not directly related to any of the topics in Differential Calculus, proportional reasoning was an important concept included in the partner discipline wish list. The second and third questions can be used when students are learning about derivatives, and the final question can be used when exploring limits.

Step 3: Implementing the Problems in the Calculus Course

With the 25 problems developed over the summer and ready for implementation, during the 2018 – 2019 academic year we asked the coordinator of Differential Calculus to use some of them in the course. The course format is three, 50-minute periods that are taught by an instructor and an 80-minute recitation period led by a graduate teaching assistant. During the recitation, students typically work in groups on activities. The coordinator was asked to incorporate at least two to three of the problems in recitation activities over multiple weeks as new calculus concepts were presented in the course. Beisiegel was on sabbatical during the academic year, which meant that the SUMMIT-P team had fewer meetings, and most of our communication about the project took place via email.

It was at this stage in our process that we experienced a pitfall. Specifically, the coordinator modified the problems in such a way that they no longer attended to the science in a meaningful way. Without regular meetings, these changes went unchecked, unfortunately. We revisit the two examples we provided above to illustrate the modifications that were made to share how this had an impact on the science featured in the problems.

Revisiting Example 1

The modified predator-prey problem that was included in the recitation activity was the following:

Twelve rabbits, some male and some female, escape and begin a wild population. Suppose that population is modeled by:

$$P(t) = 12 + \frac{40000t^2}{t^2 + 1500}$$

where $t \geq 0$ is measured in years. When does the maximum growth rate occur? Does it correspond to a point of inflection in $P(t)$?

The changes degraded both the calculus and the science in the problem in the following ways: The questions about the function can now be answered without calculus skills. The context of the problem is better represented by a logarithmic curve because the rabbit population would level out at a carrying capacity at some time. Finally, rabbit population growth now only depends on time and not on a predator.

Revisiting Example 2

The modified effusion problem used in the recitation activity was the following:

Effusion in chemistry is the process in which two or more particles are escaping through a small hole with the lighter particles leaving the container faster than the heavier particles. Heavy particles move slower than light particles with the same kinetic energy giving lighter particles a higher probability of escaping every second. This relationship can be determined by the equation:

$k\frac{1}{\sqrt{m}} =$ the rate of effusion where m is the mass and $k > 0$ is a constant.

1. Show that the proportion of gas A escaping to gas B escaping (when equal numbers of particles are present) is $\frac{\sqrt{mm_1}}{\sqrt{mm_2}}$ where mm_1 is the mass of one mole of gas A and mm_2 is the mass of one mole of gas B (called "molar masses") measured in the same units.

2. If a balloon is filled with equal numbers of Argon atoms and Hydrogen atoms then immediately after poking a small hole, what is the proportion of the rate of escaping Hydrogen atoms to the rate of escaping Argon atoms?

The changes to the problem that were problematic with respect to the science included: (1) the important phrase "same kinetic energy" which implies that both gases are at the same temperature and that $\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2$ is lost; (2) both problems use $\frac{k}{\sqrt{m}}$ for background but the modified problem only uses r_e, which may leave students confused about what that variable represents; (3) the modified problem is about finding the ratio of rates, which is an algebra problem and no longer requires the use of calculus; and (4) the modified problem does not provide all the necessary information because molar masses are not given.

Step 4: Revising the Development and Implementation Process

As a result of the implementation issues we had with the calculus problems, during summer 2019 as well as during the 2019–2020 academic year, we returned to bi-monthly meetings with our entire project team and added two new steps to our process: (a) developing teaching guides for instructors and graduate teaching assistants who are assigned to lecture or run recitations for the course and (b) creating a professional learning community including the calculus coordinator, instructors, and graduate teaching assistants assigned to the course.

After discussions about what the teaching guides should contain, we designed a template (see Figure 5) to ensure that each guide is institutionally relevant and includes the same components.

Figure 5

Template for Teaching Guide for OSU SUMMIT-P

Learning Outcomes for the course

- Syllabus Level
    - Baccalaureate core outcomes*
    - Mathematics department outcomes**
- Problem/Recitation level

Teaching Notes

- What biological and mathematical background information are relevant for this problem. Include web links.

Representations in Biology/Chemistry versus Mathematics

- How are concepts represented in the different disciplines and what are some reasons for the different representations?

Trouble shooting/Common misconceptions

- What might students struggle with? How can you help them?

*Note: OSU has baccalaureate core outcomes defined for each discipline. The rationale for the baccalaureate core in mathematics is stated as: Everyone needs to manipulate numbers, evaluate variability and bias in data (as in advertising claims), and interpret data presented both in numerical and graphical form. Mathematics provides the basis for understanding and analyzing problems of this kind. Mathematics requires careful organization and precise reasoning. It helps develop and strengthen critical thinking skills.

**Every mathematics course at OSU has specific outcomes. These include: (1) identifying situations that can be modeled mathematically; (2) calculating and/or estimating the relevant variables and relations in a mathematical setting; and (3) critiquing the applicability of a mathematical approach or the validity of a mathematical conclusion.

Here we provide some examples from different teaching guides to illustrate what is included in the second half of the guide: "Representations in Biology/Chemistry Versus Mathematics and Trouble Shooting/Common Biology or Chemistry Misconceptions." We illustrate using a different predator-prey problem than the one shared earlier in the paper.

Example 3

Predator-prey models are usually based on the Lotka-Volterra equations, which are a pair of equations whose solutions cannot be modeled by a single equation. For this problem, an *approximation* was used as a model. Biologists use predator-prey models to predict how increasing or decreasing a predator or prey population will affect the other population. These predictions often lead to changes in fishing or hunting policies. In the Oregon ecosystem, there are a multitude of bears (Ursidae) that can be considered the apex predators of the ecosystem. Salmon (*Oncorhynchus*) are their prey. Below is a graph (see Figure 6) representing the population of bears compared to the population of salmon. Answer the following questions regarding the equations and the graph.

Figure 6

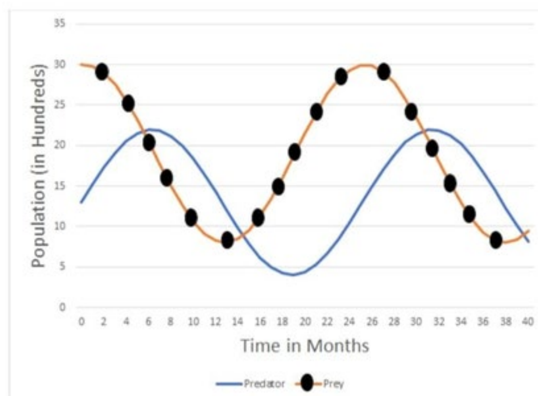
Predator Versus Prey Problem

Predator (bears):

$$B(t) = 9\sin\left(\frac{t}{4}\right) + 15$$

Prey (salmon):

$$S(t) = 11\cos\left(\frac{t - 0.249}{4}\right) + 19$$



1. Find the first derivative for the predator equation and the prey equation.
2. Find the point(s) on the graph that illustrate the greatest number of predators in the ecosystem. How does the prey graph compare at this point? Explain possible reasons for this.
3. Determine $\frac{d}{dt}$ for the prey equation at 14 months, where *t* = time in months.
4. Determine $\frac{d}{dt}$ for the predator equation at 14 months.
5. What is the equation that models the difference in the rate of change of prey and predator? Think about what it means for the difference to be negative and positive. What does that mean in this ecosystem?
6. When predator populations are at local maxima, what is the sign of the first derivative of the prey equation? Interpret what this means for the ecosystem.
7. When predator populations are at local minima, what is the sign of the first derivative of the prey equation? Interpret what this means for the ecosystem.

The teaching guide includes the following information that we hope will be useful to the instructors and teaching assistants who will use this problem in a recitation activity. We believe the description of the work of biologists is critical information for implementers of the problem. Indeed, the phrasing of the problem itself (i.e., "a pair of equations whose solutions cannot be modeled by a single equation") might be confusing for those unfamiliar with biology. It is useful to mathematics instructors and graduate teaching assistants to be familiar with biology problem solving methods.

Representations in Biology/Chemistry versus Mathematics

- Equations are often represented differently in biology than in mathematics in order to demonstrate the relationship of interest more clearly. See the Lotka-Volterra model (Yorke & Anderson, 1973).
- Using mathematical models in biology requires that biologists make a number of assumptions that may not actually hold true. For example, in predator-prey models assumptions that may not be true in nature include:
  1. The prey population finds ample food at all times.
  2. The food supply of the predator population depends entirely on the size of the prey population.
  3. The rate of change of population is proportional to its size.
  4. During the process, the environment does not change in favor of one species, and genetic adaptation is inconsequential.
  5. Predators have limitless appetites.

Troubleshooting/Common Biology Misconceptions

- Many students think that they do not need mathematics to do biology. In fact, the field of ecology is based on a great deal of mathematical modeling. These models are one example of that.
- Many students view ecology as a study that is not connected with the human species, but ecology can be applied to humans because it is the study of living organisms.
- Students may think that food webs only involve predators and prey but not plants (i.e., producers). Producers are the base of the food web and the source of all energy on earth. These models do not consider the food that is available to the prey.
- Students may incorrectly think that predator and prey populations are similar in size. This is not necessarily true; in fact, prey populations by definition have to be larger than predator populations. Why? We lose energy as we move up the food web due to the maintenance of the organisms at lower levels (i.e., 10% of energy is passed from herbivores to first-order carnivores).
- Students may think that the relative size of one population (predator or prey) has no bearing on the size of the other population. The point of the models is to make predictions about these relationships because they are connected. By looking at specific predators and prey, however, we have to simplify the food web and just look at the relationships between two species. In reality, there is a much more complicated set of interactions between all organisms in the ecosystem.

In our most recent iteration of problem implementation, our goal has been to organize a professional learning community that includes our SUMMIT-P team and the calculus coordinator, instructors, and graduate teaching assistants who are assigned to the course. In spring 2019, the SUMMIT-P team met with the calculus team to discuss the overall goals of the project, the problems that we have developed, and their use in Differential Calculus. In these meetings, we provided opportunities for the partner disciplines to explain the contexts of the problems and the reasons that certain problem features are critical. The group is also discussing the struggles of calculus students with the problems and how to refine the problems, as well as developing further clarifications for the information in the teaching guides. We hope to continue to expand our collaboration with the calculus team and provide ongoing support for them in the future.

Reflecting on Our Process

Our work has been fruitful. We have learned a significant amount about what aspects of our project worked well and how we can continue to improve the steps we take to achieve the overarching goals of the SUMMIT-P project. Here we summarize some of the positive aspects of this experience, the pitfall, and what we aim to do as we move forward.

Meetings with Disciplinary Partners and Users of the Materials

The meetings in which we explored the presence of mathematics content in biology and chemistry were incredibly useful. The learning curve was fairly steep for the SUMMIT-P team, but we have all increased our knowledge and understanding of each other’s disciplines significantly, which will help the work going forward. We plan to have meetings with the calculus course coordinator, instructors, and graduate teaching assistants. They can provide feedback on the course materials (i.e., the problems, instructional guides, etc.) and share their students’ experiences with the problems. Developing a better understanding of the student experience will help us to continue to improve the problems and determine how we can support the users of the problems. Our goal is to meet with the calculus coordinator, instructors, and graduate teaching assistants after every implementation of problems in Differential Calculus (approximately three meetings per 10-week term).

Employing Students

Employing the graduate student and undergraduate students was a critical part of the process. We would definitely do it again. Their seemingly endless energy and enthusiasm for the work, which is directly related to their fields of study, helped to propel the project forward. We could not have done this work without them. We were lucky to have found students who could make this work a priority. They genuinely wanted to contribute to the project. We were constantly impressed by the amount of time and effort they brought to the project.

Pitfall

The pitfall we experienced was unexpected. In hindsight, however, we should have known that the coordinator and instructors would need support to understand the problems—including why the problems were phrased as they were—and the importance of specifically including certain concepts from chemistry and biology in order to preserve the integrity of the disciplinary context. The breakdown in the process was not providing the calculus coordinator and instructors with the same conversational experiences as the SUMMIT-P team who developed the problems. Moving forward, we hope to minimize this issue with the teaching guides that we have described in this paper. While we expect the teaching guides will be useful to the users of the problems, we also expect that the instructors who teach Differential Calculus will continue to need support in order to use the problems as they are intended. As we mentioned above, we will aim to include the calculus team in our SUMMIT-P meetings more often and also plan more meetings in which they can provide input.

Moving Ahead

As we continue our work on the SUMMIT-P project, we plan to continue working to understand how mathematicians, biologists, and chemists see the same problems from different perspectives. As one example, we revisit the logistic regression problem discussed with the undergraduate students. During the conversation, we talked about scenarios in which the growth of an organism would level off at a certain point; for example, the spread of disease would reach a limit once most people had contracted the disease. In a mathematics class, the function that could be used to model this is:

$$P(t) = \frac{4000e^{-2t}}{1 + 2000e^{-2t}}$$

In contrast, for the same scenario the biologists would use an equation like this:

$$\frac{dN}{dt} = 2N\left(\frac{2000 - N}{2000}\right)$$

Students are likely to struggle to understand how these are related and, as a result, not understand the usefulness of mathematics in biology problems. Thus, our team would like to continue to explore these differences, including addressing questions like: Why are these differences in approach important to the disciplines? How we can support students in understanding the connections between different representations of the same problem? What problems we can design that will help students realize the connections between the disciplines?

Acknowledgment

This paper was developed in part through the project *Collaborative Research: A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships* (SUMMIT-P, www.summit-p.com) with support from the National Science Foundation, EHR/IUSE Lead Awards 1625771, 1822451, 1942808. The opinions expressed here are those solely of the authors and do not reflect the opinions of the funding agency.

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