Dynamic Analysis and Obstacle Avoidance of Autonomous Tractor Semi-Trailers

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

Tractor semi-trailer, is a kind of multi-axle heavy-duty truck, which plays an important role in commercial transportation. With 5.6 million semi-trucks and 3.5 million truck drivers, commercial transportation is a essential and profitable business. These drivers drive more than a billion miles a year in total, hence there are many accidents related to the tractor semi-trailer. Thus, equipping tractor semi-trailer with autonomous driving technology could significantly improve road safety while drastically reduce driver's work load.

Different from the normal passenger vehicle, the tractor semi-trailer has multiple bodies which makes the dynamic responses of a tractor semi-trailer much more complicated. Dynamic responses of a tractor semi-trailer are not only affected by its parameter but also its payload. For example, liquid payload will cause the CG transferring. Moreover, instability happened more easily on a tractor semi-trailer than a passenger car. There are two instability categories: roll instability and yaw instability. Roll instability includes rollover, generally caused by a large lateral acceleration during turning or lane changing. Yaw instability includes jackknifing caused by unusual deceleration and trailer swinging caused by external lateral force. These three types of instabilities requires further research and attention during the autonomous vehicle designing process. This research is trying to solve an important problem: **stable obstacle avoidance for autonomous tractor semi-trailer**. In particular, this thesis fills the research gap of the tractor semi-trailer left turn problem at city intersection. Although obstacle avoidance is a classic topic in the field of autonomous driving, however, most research is focused on passenger cars or single body vehicles. For an autonomous driving tractor semi-trailer, obstacle avoidance is an essential function. When controlling a tractor semi-trailer, the dynamic response analysis must be performed to assure safety.

This thesis starts from a dynamic response analysis with a dynamic model and concludes some important results for controller designing. The Model Predictive Control(MPC) control algorithm is applied to the control part with a built-in obstacle avoidance algorithm. MPC uses a kinematic model to control a tractor semi-trailer to avoid obstacles in different scenarios. The conclusion conducted from the dynamic analysis will provide further constraints in MPC designing since the kinematic model does not include any dynamic information. Finally, the simulation of different scenarios will be presented based on the control algorithm and an obstacle avoidance algorithm. It verifies that the algorithm proposed by this thesis can contribute to the tractor semi-trailer obstacle avoidance.

Acknowledgements

I would like to thank all the little people who made this thesis possible.

It is been a long road to this two-year research, and now I am at the end of this road. Now, when I look back to the road I travelled, I still remember people who helped me, encouraged me and companied me during this journey.

First of all, I wish to express my deepest gratitude to my supervisor Dr. Dongpu Cao. It is my honour to have him as my supervisor during the past two years. He taught me a lot with his insights from autonomous driving industrial. Not only taught me knowledge in academia but also taught me how to overcome difficulties. This project would not have been realized without his patiently help.

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Friendship is something we cannot live without. I would like to thank my dear friend Jingyu Wen, Anqi Bian, Yaodong Cui, Bruce Huang, Rocky Liang, Zhenan Liu, Jianyu Pan, Sha Tong, Peizhi Wu and Shnegyuan Xu.

Finally, it is whole-heartedly appreciate my parents, Mrs. Mingli Gao and Mr. Nan Ye.

Dedication

To my parents Mr.Ye and Mrs.Gao

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List of Abbreviations

ML Machine Learning 10

MPC Model Predictive Control 3, 10, 34, 37, 40–45, 47, 49–51, 55, 56, 58, 60, 62, 65–67, 70–72, 74, 76

NLP Nonlinear Programming 34, 47, 48, 60

NMPC Nonlinear Model Predictive Control 47

OCP Optimal Control Problem 34, 60

PID Proportional–Integral–Derivative 9, 10, 49, 50

RL Reinforcement Learning 10

ROS Robot Operating System 76

Chapter 1

Introduction

1.1 Motivation and Objectives

Tractor semi-trailer plays an important role in commercial transportation nowadays. In the US, there are about 2 million semi-trucks registered in operation and 5.6 million semi-trailers. [1]. Based on statics performed by the United States Census Bureau, there are more than 3.5 million people working as truck drivers [13]. Compared with commercial transportation via railway, commercial transportation via semi-trailer is much more flexible. Until now, about 70 percent of goods are transported by tractor semi-trailer.

Meanwhile, the autonomous truck is becoming an irresistible trend of merchandise transportation with the ever-accelerated updating of science and technology. The development of autonomous driving will have an impact on the economy and the environment. For example, a truck team platoon driving would save 20%-30% fuel consumption [56]. With the help of autonomous driving, it will become a possible mission that one person leads a whole truck team. Also, with the development of electric trucks, the consumption of commercial transportation will be reduced further. The autonomous truck can also save drivers from killed by fatigue driving. Based on data provided by the Australian Transport Accident Commission, drowsy driving contributes 16% - 20% of all road crashes [18].

According to the autonomous driving classification standard defined by the National Highway Traffic Safety Administration of the United States, autonomous driving can be divided into 5 levels based on the intelligence level of an autonomous vehicle: from no autonomous driving function vehicle which required people driving without any assistant function (Level 1) to a fully autonomous driving vehicle which can completely drive itself without supervised (Level 5). Generally, there are four fundamental stages during autonomous driving: perception, decision making, path planning, and controlling. This project will focus on path planning and controlling. These four stages form a pipeline, contribute to autonomous driving together.

Designing an autonomous driving truck is much more complicated than designing an autonomous driving passenger car. According to the regulation of the Canada Ministry of Transportation, the width of commercial motor vehicles should not exceed 2.6 meters [43] while the limitation for the length of semi-trailer is 14.65 meters. No matter from the dimension aspect or from the special dynamic phenomena aspect, designing an autonomous driving function for a tractor semi-trailer is a challenging task.

Obstacle avoidance is a popular research area of autonomous driving. So far, there is a lot of research on how to improve the obstacle avoidance algorithm [64] [55], how to use a controller to achieve an ideal obstacle avoidance result and how to avoid small obstacles on the road. This thesis is focused on the real daily-problems and fills in gaps in the application of the obstacle avoidance algorithm in tractor semi-trailer in the daily driving scenarios. the thesis not only solves the crossroad left-turning problem but also solves the highway obstacle avoidance problem. In this thesis, people can find how a tractor semi-trailer handles the low-speed scenario as well as the high-speed scenario. Moreover, the dynamic response analysis will also give people a better understanding dynamic of the semi-trailer.

1.2 Contribution

The contribution of this thesis on dynamic analysis of tractor semi-trailer and controlling can be summarized as follows:

- By using a dynamic model to analyze the dynamic response of the tractor semi-trailer, a amplify and reduction phenomena of the semi-trailer under different speeds can be found. This thesis also conducts conclusions on how parameters of a semi-trailer affect phase delay in different states. Furthermore, it also researches how to assign parameters to a tractor semi-trailer to reduce instability risk.
- A new approach to defining a safety envelope has been proposed based on the off-tracking and dynamic analysis. It is related to velocity, acceleration and dimension of the tractor semi-trailer. By using the safety envelop, it can help people to design the planning algorithm more easily.

- Use a solver called “CasADi” to solve an optimal control problem, this solver outperforms other traditional solvers in speed when it comes to solving an optimal control problem.
- With the help of the Model Predictive Control (MPC) and obstacle avoidance algorithm, the thesis designs a new obstacle avoidance approach which is first used on a tractor semi-trailer. The control algorithm embedded an obstacle avoidance algorithm in MPC, and it achieves a balance between accuracy and efficiency. With the help of a control algorithm, the thesis solves a intersection left-turning problem which is common and has never been solved by others.
- Solve the classic but indispensable highway obstacle avoidance problem with the same approach.

1.3 Thesis Outline

Besides the first chapter, other chapters in this thesis outline as follows:

Chapter2 Literature view and background

Chapter 2 will start by introducing the current situation and prospect of tractor semi-trailer, and then discuss the difference between tractor semi-trailer and other cars. The instability of the tractor semi-trailer will follow up with the introduction. Moreover, the control techniques will be reviewed and then one of them will be chosen to aid controller design. Finally, this chapter will end with a review on the obstacle avoidance technique.

Chapter3 Dynamic Responses and Stability Analyses of Tractor-Semitrailer Systems

In chapter 3, a three degree of freedom dynamic model has been selected from an existing paper and upgraded for dynamic responses study. Three velocities including low velocity on a narrow road, moderate velocity on city road and high velocity on the highway have been used in the study. Instead of only analyzing dynamic responses for a specific tractor semi-trailer, this section uses different parameters to explore how parameters of the tractor semi-trailer are related to its dynamic responses. The simulation gives people a brief understanding of the behaviour of tractor semi-trailer under different velocity and how to take actions for risk reduction on yaw and roll instability.

Chapter4 Controller Development and Problem Formulation

Starting from implementing a kinematic model for the controller, this chapter mainly introduces how to design a controller to control a tractor semi-trailer. A popular optimal control (OC) method has been selected to take the responsibility of controlling the system. This chapter talks about the general ideas of the control algorithm and provides new approaches to solving optimal control problems (OCPs). After that, a software which is used to solve OCPs called “CasADi” has been presented. Finally, this chapter shows how to use the control algorithm to solve real-life problems. Since the kinematic model is used in controller design in order to reduce the computational burden, conclusions from chapter 3 will be used to aid controller design.

Chapter5 Applications and Simulation

This chapter will show the simulation results. Simulation results will be based on two problem categories: intersection left-turning problem and highway obstacle avoidance problem. Based on the different scenarios, there are three simulations in the highway obstacle avoidance problem: a static obstacle, a moving obstacle with constant velocity and a moving obstacle with an emergency deceleration. Furthermore, in this chapter, the reason for use model predictive control instead of optimal control will be discussed.

Chapter6 Conclusions and Future Work

All work introduced in previous chapters has been concluded in this chapter and potential work on this topic has been talked through in this chapter.

Chapter 2

Literature Review and background

2.1 Introduction

The global semi-trailer market size was valued at USD 21 billion USD in 2018 [54]. Total commercial trailer production in North America has increased four times from 2009 to 2016 [52]. Five mainstream types of semi-trailer are curtain, refrigerated, tipper, dry van and tankers that take up more than 80 percent share of semi-trailer market. Starting from section 2.2, a brief overview of the tractor semi-trailer will be given. Section 2.2 not only compares tractor semi-trailer with other road vehicles but also explains why the tractor semi-trailer is so popular in cargo transportation. Section 2.3 will talk about the instability, a dangerous situation that will lead to fatal injury accidents, as well as potential risks bringing to instability situation. Finally, the chapter will end with section 2.5, a review on a few popular obstacle avoidance algorithms.

2.2 Brief Overview on Tractor Semi-Trailer

Based on the standard developed by US Federal Highway Administration (FHWA) in the mid-1980s, vehicles on the road can be classified into thirteen categories as shown in Figure 2.2. This rule based on the vehicle characteristic that can be identified visually [4]. Even though the standard might be varied based on region, it still provides a rough reference on the vehicle classification for people. Category 8 to 10 which based on FHWA thirteen-category standard are the most three general vehicle categories used as trucks on the highway. Different from the single body truck, the tractor semi-trailer has multi-body: one tractor attached to at least one trailer. The tractor can hitch more than one semi-trailer and brings more flexibility to the tractor semi-trailer.

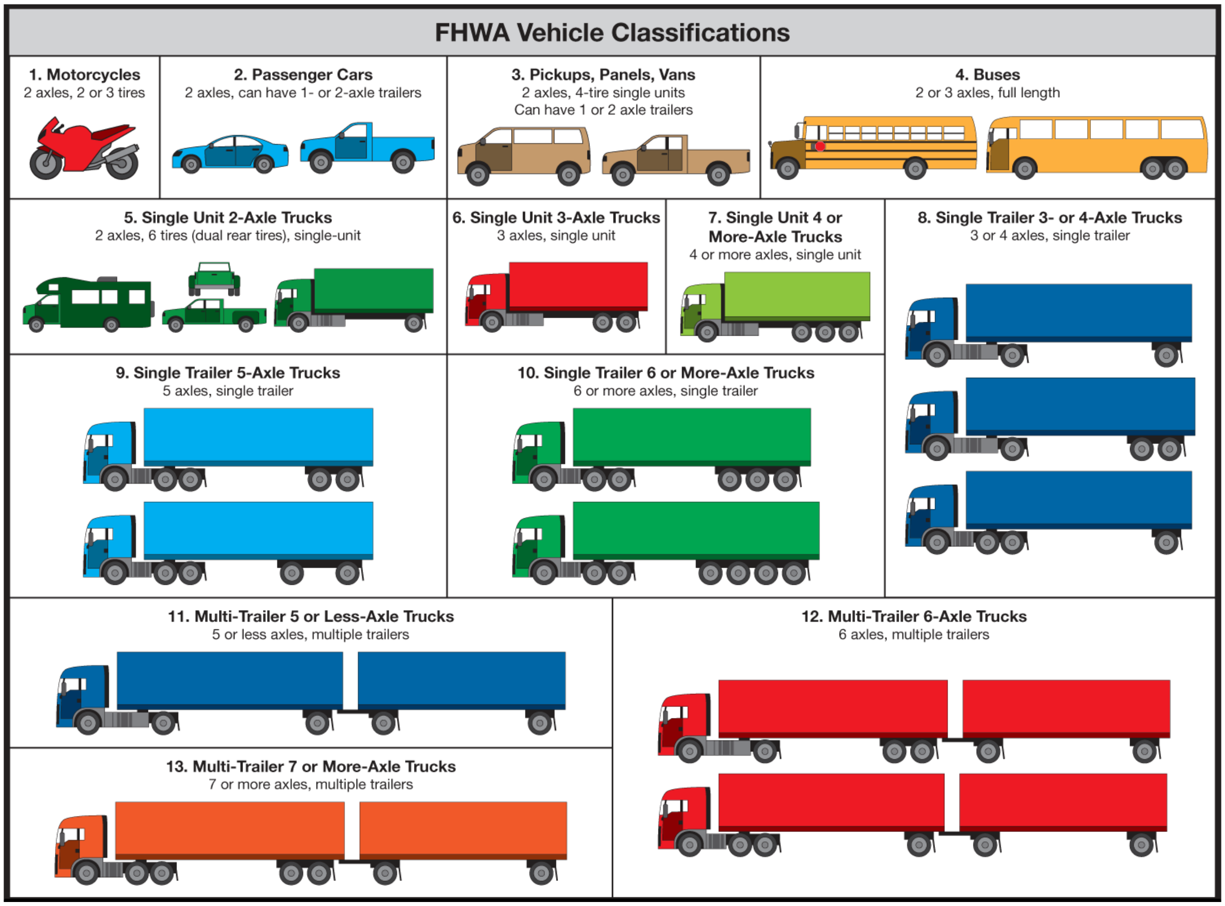

Figure 2.1: FHWA Thirteen-Categories Scheme [3]

Compare to the single-body truck, the tractor semi-trailer has significant advantages than the full trailer and single body truck. First of all, bodies of tractor semi-trailer can be hitched and unhitched without any effort, which will help people save a lot of loading time. A factory can load a semi-trailer before the tractor arrives, which will significantly improve the efficiency of transportation. With this advantage, the risk of delay in delivery will be reduced: if the tractor breaks down on the road, another tractor can replace it and transport the semi-trailer immediately without unloading payload. Secondly, since semi-trailers are hitched on the tractor, the transport capacity will increase easily if they are attached to another semi-trailer. Last but not least, the semi-trailer has a smaller turning radius comparing to the same size full trailer, which allows tractor semi-trailer driving in a narrower space.

Most tractor semi-trailers benefit from its multiple bodies in nature. However, like every coin has two sides, the great features also bring some problems. The semi-trailer is hitched on the tractor and it can move freely around the hitch point, which leads to instability such as jackknifing, an instability that will bring a series accident.

2.3 Instability

2.3.1 Rollover

Rollover is a kind of roll-plane instability, and there are over 15,000 rollovers of commercial trucks a year, including 9,400 cases involving tractor semi-trailers [57]. According to U.S. DOT data, even though rollover only occupied 4% of truck accidents, it caused 58 percent fatal injury to truck driver [44].

There are two types of rollovers: tripped rollover and untripped rollover [6]. Resulted from the large lateral acceleration, an untripped rollover happened during the sharp turning on the road. The tripped rollover results from the external force such as the wheel lift off the road by the obstacle, respectively. The tripped rollover can be avoided by obstacle avoidance. At the same time, the untripped rollover can be avoided by monitoring the lateral acceleration. According to research on the rollover threshold of a heavy-duty vehicle [58], the lateral acceleration should not exceed 0.35G to prevent the risk of untripped rollover.

2.3.2 Jackknifing and Swing

Another instability is jackknifing, a kind of yaw-plane instability that causes a large number of fatal injuries in tractor semi-trailer accidents. According to a large truck crash causation study performed by Federal Motor Carrier Safety Administration, 5% large truck crashes are caused by jackknifing [5].

Compare to the loaded semi-trailer, a system with an empty semi-trailer are easier to get into the yaw instability [40]. Besides, jackknifing mostly takes place during a deceleration such as an attempt to slow down a tractor semi-trailer [20]. Other factors such as the road

Figure 2.2: Tractor semi-trailer rollover [41]

surface, weather and momentum of tractor semi-trailer will also be potential risks to cause a jackknifing.

There are two types of jackknifing:jackknifing of tractor and semi-trailer jackknifing. When the tractor semi-trailer gets into jackknifing, one unit of tractor semi-trailer will slide and make the tractor semi-trailer lose control [38]. Finally, two units will fold together. Figure 2.3 shows how jackknifing, swing and oscillation look like.

Trailer swing is generally caused by an external force such as the side wind. Oversteer characteristics of the trailer will also lead the system to swing [21]. This phenomenon is expected to be avoided but it will not bring that much risk as jackknifing.

2.4 Control Techniques

In the past few decades, with the development of the control techniques, solid fundamental has been laid for obstacle avoidance. The important role of control in autonomous driving was beyond reproach. Most of them are model-based control, few of them are model-free

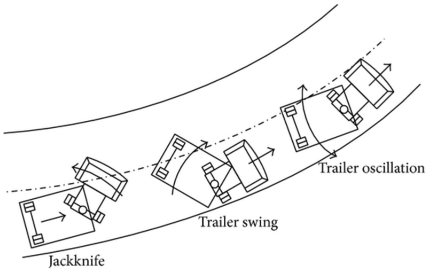

Figure 2.3: Jackknifing, swing and oscillation of a tractor-semitrailer [61]

control. There is no doubt that model-free control is fancy control techniques to some extent, however, it requires a large amount of time on the learning model and tuning the controller. In this section, some control techniques will be reviewed.

2.4.1 PID

Proportional–Integral–Derivative (PID) control stands for proportional, integral and derivative control [30]. If running a system without any control, there will exist a significant difference between the output of a system and the expected result which is called an error.

First of all, PID is a kind of feedback control, whose ultimate goal is calculating and minimizing error. By multiplying errors with the proportional term, which is also known as proportional gain, the system error can be reduced. When the error becomes small enough, after multiplying with a proportional gain, the small result will not sufficiently increase the controller input to eliminate the small error. Even though the error can be reduced, the residual error will still exist as the steady-state error if a controller only has a proportional gain.

Fortunately, the steady-state error can be reduced further after introducing the integral gain to the controller. The integral term can be regarded as a memory of the steady-state error and integral of steady-state error can be a large number after some time. The last problem surfaced as the controller will cause an overshoot problem if it only has a proportional (P) and an integral (I) term.

Overshoot is not expected and caused by the controller with an integral term that might output a larger value than expected. The derivative term will reduce the overshoot effect by monitor the changing rate error.

Even though PID is a simple control algorithm, it is one of the most robust control algorithms. It can handle countless different scenarios. The PID control is already a maturity control and has been used to control different systems such as robot manipulators [32], hydraulic system [36], truck ABS system [29] and lane-keeping system of vehicle [63]. Furthermore, comparing with other advanced control algorithms, the tuning parameter is not a stage that requires that much effort. There are a lot of tuning methods available for tuning PID for different control system[46, 47, 42, 48]. Even though PID has a lot of advantages, it still has its limitation in the unstable and integrating process [10].

2.4.2 Stanley

Stanley control, a specific controller designed for autonomous automobile trajectory tracking [26], firstly was used on a real vehicle by Stanford Racing Team in the 2005 DARPA Grand Challenge. It used a geometric method to track a given reference path. It tries to minimize the cross-track error which is defined by the speed of vehicle, yaw angle and the front-wheel steering angle.

Not like PID, this control algorithm can only use to handle the specific task: vehicle trajectory tracking. It is already proven by Stanford team that it can be used on a real vehicle with the limitation that the vehicle must have a given trajectory.

2.4.3 MPC

MPC refers to Model Predictive Control, which is another popular exclusive control algorithm. It is a model-based control algorithm which means that it needs a model to tell the controller how the controlled system works. An accurate model will bring a precise control result. Minimizing an objective function, a combination of a few quadratic functions has been considered as the first priority of the Model Predictive Control.

Similar to the PID controller, MPC is another popular control algorithm that has already been used in research areas such as power converters [34], quadcopter drones [62] and vehicle stabilization [11]. It can also combine with the Machine Learning (ML) algorithm such as Reinforcement Learning (RL) to improve the safety [33] and efficient [9] of RL. Furthermore, it provides a platform to fusion control algorithm and obstacle avoidance algorithm [28].

2.5 Obstacle Avoidance Techniques

Approximately ninety-three percent of car accidents are caused by human errors. [7] Therefore, fully autonomous vehicles without human interventions will significantly reduce accidents without the human factor impact. The autonomous driving is indispensable to obstacle avoidance. The obstacle avoidance is a sub-topic of planning and control. This section will briefly talk about the mainstream obstacle avoidance techniques and compare the pros and cons of these methods.

2.5.1 Circle

Circle-based obstacle avoidance has been widely used on autonomous robots such as unmanned air vehicles (UAV) [17] and mobile robot [16]. Circles will be used to represent an ego vehicles' outline and obstacles' outline in the basic circle-based obstacle avoidance algorithm. By continuously calculating the distance between two circles, the obstacle can be avoided. This algorithm is simple and easy enough to develop. However, the outline of a normal passenger car is rectangular. Under this situation, a circle will waste space, and the algorithm cannot adapt to the narrow space.

2.5.2 Rectangular

Similar to the circle-based obstacle avoidance algorithm, a rectangular-based obstacle avoidance algorithm using one or more rectangles to represent the vehicle and the obstacle based on the details of the algorithm. Even though the problem becomes more complicated in this algorithm compared to the circle approach since this algorithm needs more computational progress, the rectangular approach will not waste space. These advantages make the rectangular approach is able to handle the problem in a narrow space.

2.5.3 Potential Field

Potential Field is a path planning method that has been used for mobile robot path planning [27] [35]. The autonomous vehicle is a kind of mobile robot with a high moving speed. Since the potential field has been used and verified on the mobile robot in the past, there is no doubt that this fancy method can also be used on the vehicle's path planning [53] [50]. In obstacle avoidance, the potential field is used as a part of the cost function of the Model Predictive Controller. The controller is trying to find and follow a path that has the minimum cost. The controller can avoid obstacles with the help of cost function since it embeds a potential field function. If a path has an obstacle, following that path will have a higher cost than following another path without any obstacle. However, the cost function also depends on many other factors, and the potential field only increases the specific amount cost based on the obstacle category.

2.6 Summary

In this chapter, the current situation of tractor semi-trailer, control algorithm and obstacle avoidance algorithms has been reviewed. Since the Model Predictive can fusion with an obstacle avoidance algorithm, this thesis will use it to design the algorithm. The following thesis will study on dynamic of tractor semi-trailer and find a method to reduce the instability risk. Moreover, it will use the algorithm mentioned in this chapter to solve real-world problems.

Chapter 3

Dynamic Responses and Stability Analyses of tractor semi-trailer Systems

3.1 Introduction

Different from the passenger car, the tractor semi-trailer has two free bodies. These two bodies are related to each other, in other words, the behaviour of one body will affect the other one. However, a kinematic model can describe this relation basically but not accurately. Based on this fact, the study on dynamic responses and stability becomes important before controlling the tractor semi-trailer System with a kinematic model.

The Chapter is organized as follows. Section 3.2 begins with the dynamic model implementation, and then the model has been improved by using a more accurate tire model. Section 3.3 analyses the behaviour of tractor semi-trailer with different type inputs, these inputs aims to simulate different behaviours of tractor semi-trailer in a different scenario. Besides, each part of this section studies how will the parameter of the tractor semi-trailer affects the behaviour of the tractor semi-trailer. Section 3.4 studies the instability behaviours of the tractor semi-trailer.

3.2 Dynamic Model

3.2.1 Introduction

In this section, a three-degree of freedom dynamic model will be implemented for dynamic analysis. Because the stability during lane changing and parameter of tractor semi-trailer affect on lane change performance will be studied during this chapter and the load transfer should also be added to the dynamic model. And the tire model should be chosen carefully, it should describe the basic character of tire yet not too complex. Finally, a constant acceleration and steering angle will be used as input to verify the response of each state.

3.2.2 Model Implementation

Before studying and analyzing dynamic responses of the tractor semi-trailer, a dynamic model is required. Since only the yaw plane motion will be studied, the dynamic model has been implemented based on the single-track vehicle dynamic model, which is also known as the “bicycle model” [24].

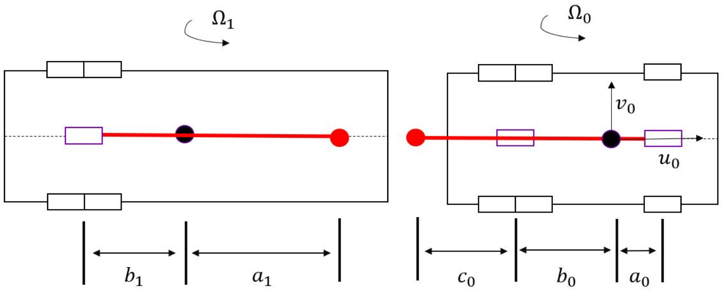

Figure 3.1: Sketch of tractor semi-trailer used for model implementation.

The “bicycle model” adds a semi-trailer part in order to describe the basic dynamic behaviour of tractor semi-trailer [59]. To simplify the problem, the dynamic model has the following assumptions:

- The tractor and semi-trailer only have a single axle at the rear part.
- The hitch point of the semi-trailer is located behind the rear axle of the tractor.

Equations have been derived as follow:

$$\dot{\Omega}_1 = \dot{\Omega}_0 + \ddot{\Gamma}_1 \tag{3.1}$$

$$\begin{aligned}\dot{u}_1 = \dot{u}_0 \cos\Gamma_1 - \Big((b_0 + c_0)\,\dot{\Omega}_0 - \dot{v}_0 \Big) \sin\Gamma_1 \\ - \dot{\Gamma}_1 \left(u_0 \sin\Gamma_1 + \left((b_0 + c_0)\,\Omega_0 - v_0 \right) \cos\Gamma_1 \right)\end{aligned} \tag{3.2}$$

$$\begin{aligned}\dot{v}_1 = \dot{u}_0 \sin\Gamma_1 - \Big((b_0 + c_0)\,\dot{\Omega}_0 - \dot{v}_0 \Big) \cos\Gamma_1 - a_1 \dot{\Omega}_1 \\ - \dot{\Gamma}_1 \left(u_0 \cos\Gamma_1 - \left((b_0 + c_0)\,\Omega_0 - v_0 \right) \sin\Gamma_1 \right)\end{aligned} \tag{3.3}$$

$$m_0 \left(\dot{v}_0 + u_0 \Omega_0 \right) = F_{f_0} \cos\delta + F_{r_0} + Y_{H_0} \tag{3.4}$$

$$I_{z_0} \dot{\Omega}_0 = F_{f_0} \cos\delta a_0 - F_{r_0} b_0 - Y_{H_0} \left(b_0 + c_0 \right) \tag{3.5}$$

$$I_{z_1} \dot{\Omega}_1 = m_1 \left(\dot{v}_1 + u_1 \Omega_1 \right) a_1 - F_{r_1} \left(a_1 + b_1 \right) \tag{3.6}$$

$$Y_{H_0} = \left(F_{r_1} - m_1 \left(\dot{v}_1 + u_1 \Omega_1 \right) \right) \cos\Gamma_1 - m_1 \left(\dot{u}_1 - v_1 \Omega_1 \right) \sin\Gamma_1 \tag{3.7}$$

$$X_{H_0} = - \left(F_{r_1} - m_1 \left(\dot{v}_1 + u_1 \Omega_1 \right) \right) \sin\Gamma_1 + m_1 \left(\dot{u}_1 - v_1 \Omega_1 \right) \cos\Gamma_1 \tag{3.8}$$

In equations 3.1 to 3.8, subscripts 0 and 1 are for tractor and semi-trailer respectively. u and v are longitudinal velocity and lateral velocity in the body coordinate system. δ is the steering angle, Ω is yaw angle and Γ_1 is the articulation angle. a_0 is the distance between front axle and center of mass (CG) of tractor, b_0 is the distance between rear axle of tractor and its CG, c_0 is the distance between rear axle and hitch point of tractor, a_1 is the distance between hitch point and CG of semitrailer, and b_1 is the distance between rear axle and CG of semitrailer. In addition, Y_{H_0} and X_{H_0} are coupling force on the hitch point, m and I_z represent mass and yaw inertia. Finally, F represents the lateral tire force which will be discussed further in the following section.

3.2.3 Tire Model

Tire plays one of the most important roles in the vehicle dynamic since the entire vehicle motion is controlled by the force applied to the tire. Tires can not only provide longitudinal forces for vehicle accelerating and braking but also provide the lateral force for vehicle cornering. Tires provide these forces through the contact patch, which is the small contact area between tire and road.

When the vehicle is driving on the road, there always exist longitudinal slips or lateral slips. These slips are caused by deformations and distortions on tires [19], and then shear stresses are generated at the contact patch. The force generation process of tire includes three zones: linear zone, transient zone (e.g. peak zone), and saturation zone. Since Visco-elastic behaviour of the tire, this nonlinear process is difficult for modelling while using a mathematical formula.

However, the magic formula [45] can describe this process with a complex mathematical formula. It is a semi-empirical tire model used to identify important forces generated at the contact patch by using a batch of coefficients.

The general form of the magic formula is described as following equation:

$$F_{lat} = D \sin[C \arctan\{B\alpha - E(B\alpha - \arctan(B\alpha))\}] \tag{3.9}$$

In equation 3.9, the stiffness factor B is used to control the cornering stiffness, shape factor C is used to determine the shape of the curve, curvature factor E can determine the position of the peak, and factor D is used to decide the maximum value of the tire force. However, the equation 3.9 needs to be adjusted a little bit in order to simulate the longitudinal load transfer. Finally, the equation 3.10 has been derived:

$$F_{lat} = F_z D \sin[C \arctan\{B\alpha - E(B\alpha - \arctan(B\alpha))\}] \tag{3.10}$$

In equation 3.10, α is the slip angle, and it can be calculated by using following equations:

$$\alpha_{f_{0_{rev}}} = -\arctan\left(\frac{v_0 + a_0\Omega_0}{u_0}\right) + \delta \tag{3.11}$$

$$\alpha_{r_{0_{rer}}} = -\arctan\left(\frac{v_0 - b_0\Omega_0}{u_0}\right) \tag{3.12}$$

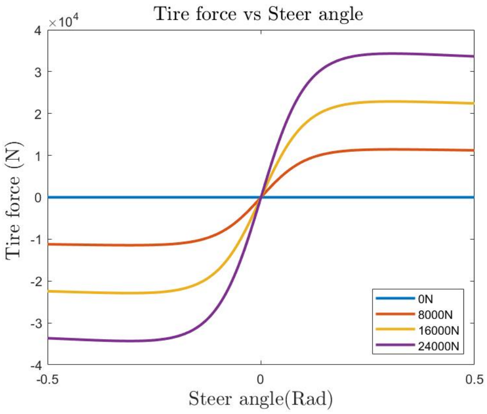

Figure 3.2: Magic formula with different loading

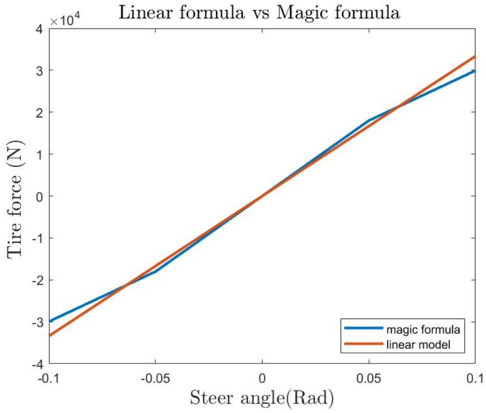

Figure 3.3: Magic formula vs linear tire model

$$\alpha_{r_{1rev}} = -\arctan\left(\frac{v_1 - b_1\Omega_1}{u_1}\right) \tag{3.13}$$

Figure 3.2 shows a typical tire model with a magic formula, which has the longitudinal load transfer and the tire force can be different due to the longitudinal load transfer. Figure 3.3 shows the linear tire model, the linear model has a minor difference compared with linear zone of the magic model. So, the linear tire model has been used in equation 3.14. The equation is also modified to adapt the longitudinal load transfer with coefficient c, the vertical force F_z, and the angle α.

$$F_{lat} = cF_z\alpha \tag{3.14}$$

Friction Ellipse

Both traction/braking force and the cornering force are supplied by the friction force. And this phenomenon can be described by the friction ellipse. Figure 3.4 shows the typical friction ellipse. The magnitude of traction/braking force and the cornering force should not exceed the friction force.

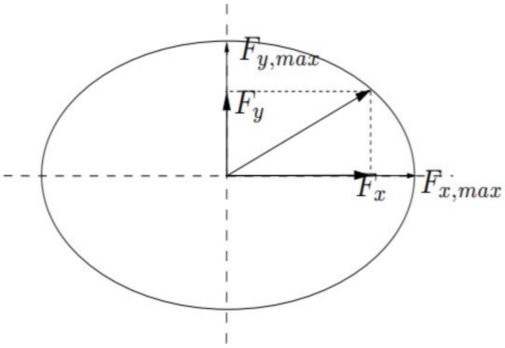

Figure 3.4: Friction ellipse [51]

Table 3.1: Tractor parameters for model verification

Parameter	**Symbol**	**Value**	**Units**
Front axle to CG	a_0	1.125	m
Rear axle to CG	b_0	2.575	m
Rear axle to hitch	c_0	0	m
Mass	m_0	6988	kg
Yaw Inertia	I_{z_0}	42147	kgm^2

3.2.4 Model Verification

After deriving the dynamic functions, the model needs verifying if it is correct. It has been verified with a constant steer input (shown in Figure 3.5) and a constant speed at $15m/s$. By using these two inputs, the tractor semi-trailer should have a circular motion. At the verification stage, the tractor semi-trailer uses similar parameters which are used in [59]. Even though most semi-trailers are hitched on the point in front of the center of rear axle of the tractor, this distance has been neglected since it was too small compared with wheelbase of the tractor and the semi-trailer. Table 3.1 and 3.2 show the detail of parameters used in model verification:

In figure 3.6, six essential states have been captured: yaw rate, lateral acceleration based on body frame and world frame, lateral position, articulation angle and articulation rate. Lateral acceleration is important to vehicle stability. In order to keep roll stability, the lateral acceleration based on the world frame should not exceed 0.35 G [2]. Articulation angle and articulation rate can basically describe the relation between tractor and trailer.

Table 3.2: Semi-trailer parameters for model verification

Parameter	**Symbol**	**Value**	**Units**
Front axle to CG	a_1	6	m
Rear axle to CG	b_1	1.85	m
Mass	m_1	8800	kg
Yaw Inertia	I_{z_1}	156860	kgm^2

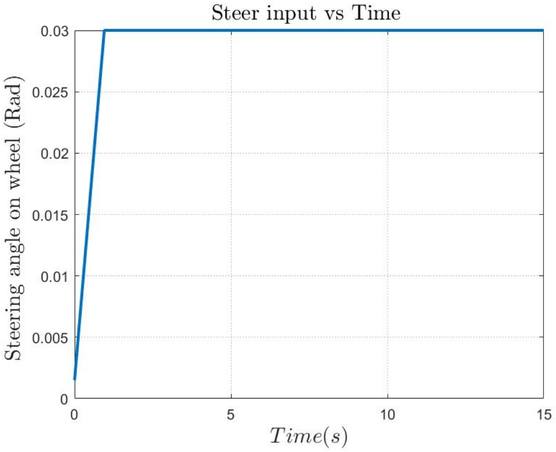

Figure 3.5: Steer input for model verification

A large articulation angle or articulation rate can easily cause instability like Jackknifing. Based on this fact, a small articulation angle and articulation rate are expected. Figure 3.6 shows that states of tractor semi-trailer raise from zero to steady-state.

3.2.5 Summary

Six states have been used in the model verification stage: yaw rate, lateral acceleration in body and world frame, lateral position, articulation angle and articulation rate. And three of them will be selected as essential states: yaw rate, lateral acceleration in world frame and articulation angle. Yaw angle can be implied from the yaw rate, and it can describe the angle between tractor and semi-trailer. The articulation angle can also imply from this information or from the articulation rate. No matter in which driving scenario, the articulation rate should be a minor value. The large articulation rate will lead tractor semi-trailer to the yaw instability zone. The world frame acceleration will be a measurement

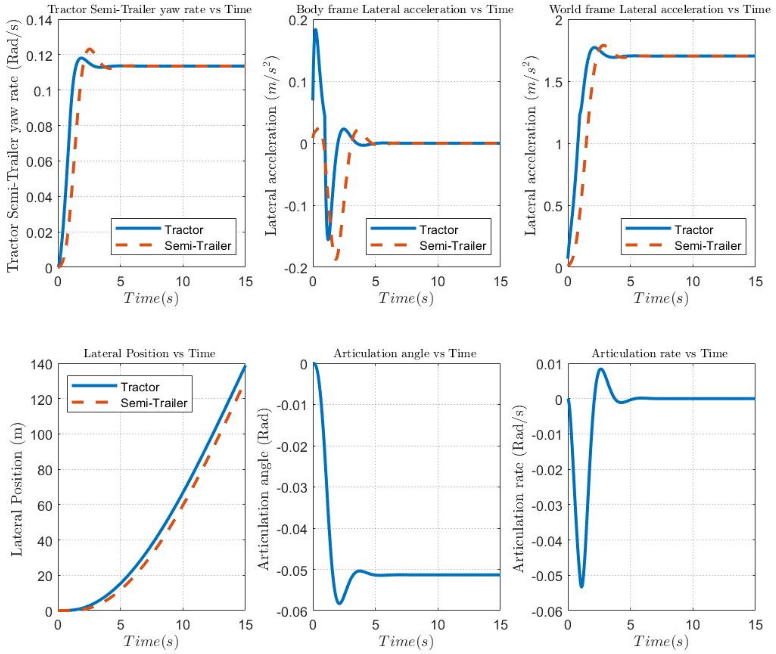

Figure 3.6: Six essential states for model verification

for roll instability. Besides, the lateral position will also be used in lane change analysis to prove that the truck has changed lane successfully.

3.3 Dynamic Analyses

3.3.1 Introduction

In this section, the tractor semi-trailer will be given different inputs in order to study the behaviour of different scenarios. The steering input must start from zero in order to prevent the sudden change in every state. After input becomes constant, each state will take a few time steps to reach a steady state. From the steady-state, the basic relationship between the behaviour of tractor and semi-trailer will be found. This section will give us a preliminary understanding of the responses of the tractor semi-trailer in the different driving scenarios. More importantly, from the lane change analysis, the dynamic responses during lane change will be studied, and from this subsection, we can learn the minimum safety distance for obstacle avoidance during the specific scenarios, and design the obstacle avoidance strategy for some critical conditions. Ratio analysis and phase delay are also two exciting areas, it will give us information on how semi-trailer responds with the tractor during different driving scenarios so that we can make wiser decisions during a lane change or obstacle avoidance.

3.3.2 Step Input Respond

This simulation is expected to analyze how the speed and steering angle affect dynamic behaviour of the tractor semi-trailer. The tractor semi-trailer use three different speed in this simulation: 5 m/s (18 km/h), 15 m/s (54 km/h) and 30 m/s (108 km/h). These three speeds simulate different driving scenarios: narrow road, city road and highway. Based on different scenarios, the tractor semi-trailer will be given different steering inputs. However, since the exact step inputs will bring a large lateral acceleration and leads to roll instability, all of these inputs have been designed to start from zero and then take one or two seconds to reach the desired magnitude in order to make it similar to a step input. Figure 3.7, 3.8 and 3.9 show the inputs used in this simulation.

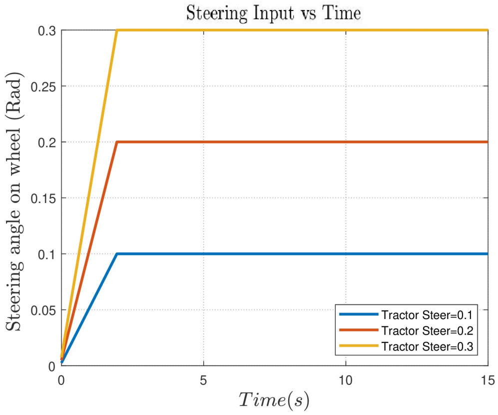

Figure 3.7: Steer input for velocity at 5 m/s

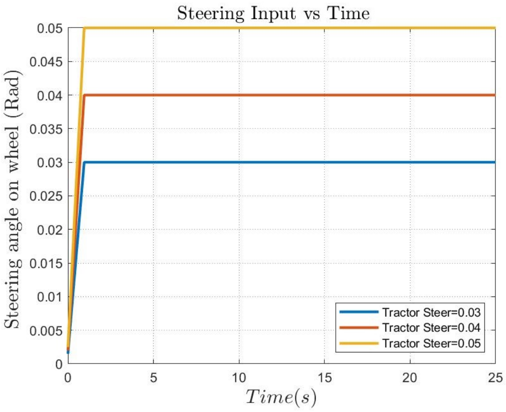

Figure 3.8: Steer input for velocity at 15 m/s

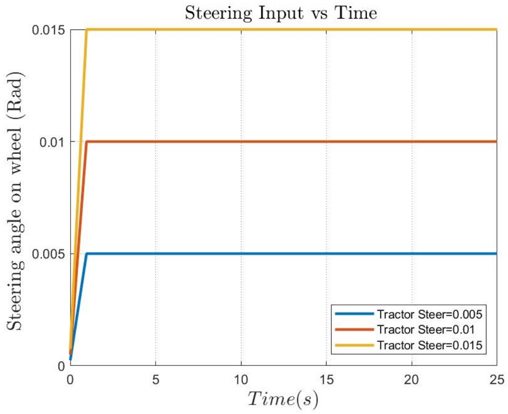

Figure 3.9: Steer input for velocity at 30 m/s

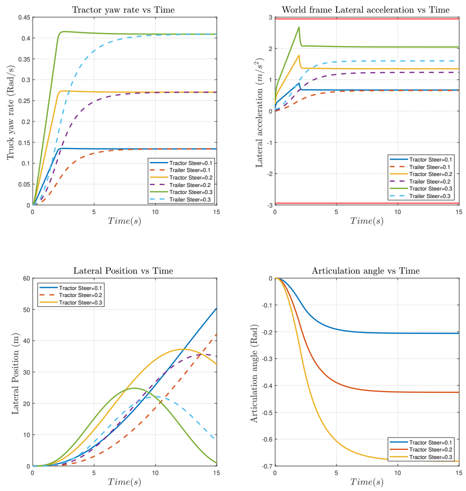

Figure 3.10: Four essential states for tractor semi-trailer driving at 5 m/s

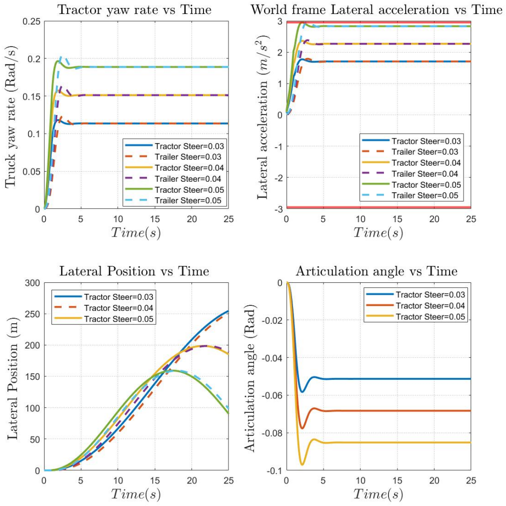

Figure 3.11: Four essential states for tractor semi-trailer driving at 15 m/s

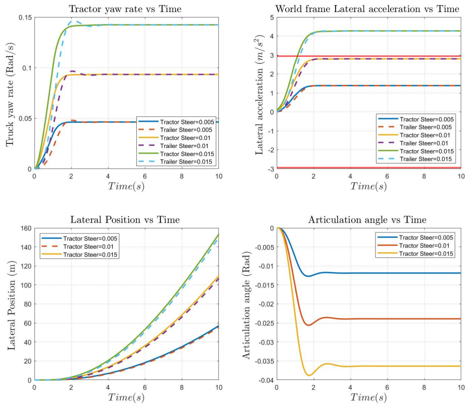

Figure 3.12: Four essential states for tractor semi-trailer driving at 30 m/s

Lateral Acceleration

Lateral acceleration is important to roll stability of tractor semi-trailer. Rollover can be caused by large lateral acceleration. The tractor semi-trailer should avoid large steer input during the high speed. The threshold for rollover lateral acceleration is 0.3 G to 0.35 G.

3.3.3 Lane Change

Lane change is one of the most important topics in obstacle avoidance. It requires a series of safe control inputs during obstacle avoidance in some critical scenarios. The steering input during lane change should be in a periodical shape like a sine function. Based on on [37], the trapezoidal-liked function has been used as the steering input for a lane change. The expected lateral position change during lane change is based on highway lane width in North America, which is 3.7 meters [25].

Lane Change Analyses

In the simulation, a tractor semi-trailer driving at 30 m/s and finish lane change in 5 seconds. The steering input used in the simulation is shown in Figure 3.19. The max steering input is from the previous analysis in order to keep roll stability. The simulation result has been shown in Figure 3.20. From the result, lateral acceleration of the tractor semi-trailer is just 0.15 G, which is half of the critical lateral acceleration for roll instability. Based on this fact, a more aggressive steering input strategy can be applied to the more challenging obstacle avoidance scenario. However, the questions are how to define “aggressive”, and how aggressive could it be?

For obstacle avoidance, the aggressive avoidance strategy means the control inputs used in obstacle avoidance are critical, and they almost lead tractor semi-trailer to instability zone. On the highway, if any instability happens to the tractor semi-trailer, the result will be extremely serious. Not only the tractor semi-trailer itself but also most vehicles close to tractor semi-trailer will be involved in a serious accident during emergency obstacle avoidance.

Most instability caused by large yaw rate (yaw instability) and large lateral acceleration (roll instability). The most difference between the tractor semi-trailer and passenger car is that the tractor semi-trailer has two bodies. From the lane change simulation result, it is not difficult to conclude that the amplified effect and phase delay effect are two risks leading to tractor semi-trailer to instability.

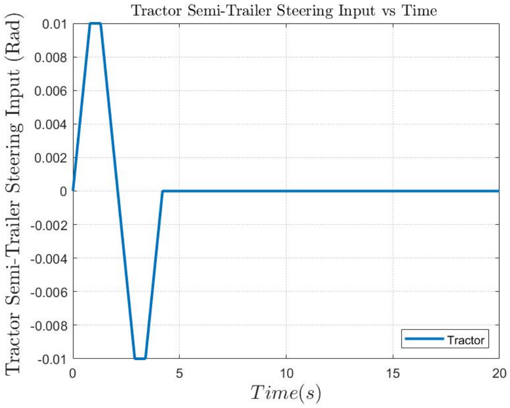

Figure 3.13: Steering input for tractor semi-trailer driving at 30 m/s

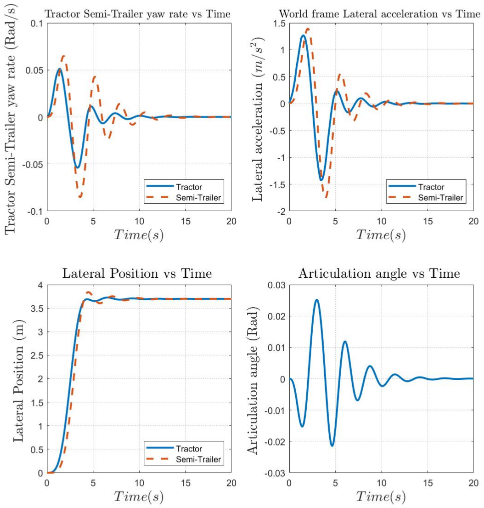

Figure 3.14: Lane change for tractor semi-trailer driving at 30 m/s

3.3.4 Ratio Analyses

This section mainly analyzes the ratio of the world frame-based lateral acceleration of the semi-trailer to lateral acceleration of the tractor. The ratio here is critical during the obstacle avoidance since if the ratio is too large, even a small steering input will result in yaw instability. Figure 3.15 shows the primary result with different speeds and different steer inputs. Since this is trend analysis, instability factors of the tractor semi-trailer will not be considered. After the simulation, the following conclusion has been summarized:

- Ratio is greater than 1 means that the semi-trailer amplifies the acceleration and the critical point is at 14 m/s.
- Ratio increase faster in low speed zone (below 8 m/s) than that in moderate (15 m/s to 20 m/s) and high speed zone (25 m/s to 32 m/s).
- Large steering angle input will lead a low ratio before critical point (velocity at 18 m/s) and that will lead to a higher ratio after a critical point.
- In the low-speed zone, making sure that lateral acceleration of the tractor does not exceed threshold of roll instability will bring a roll stability result for the whole tractor semi-trailer system (shown in Figure 3.16).

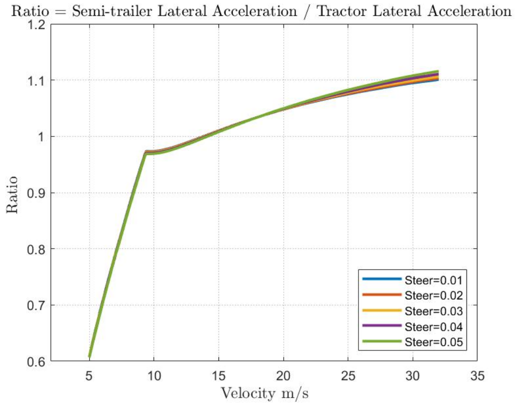

Figure 3.15: Primary ratio analysis result

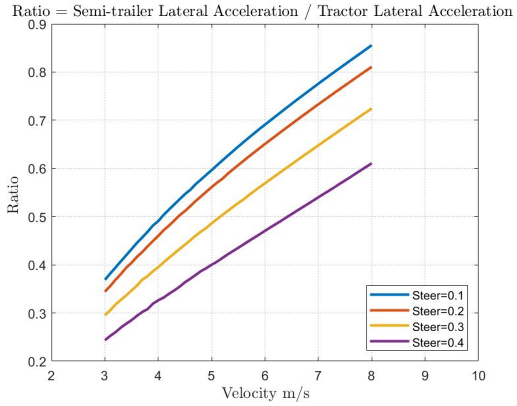

Figure 3.16: Ratio analysis result of low-speed

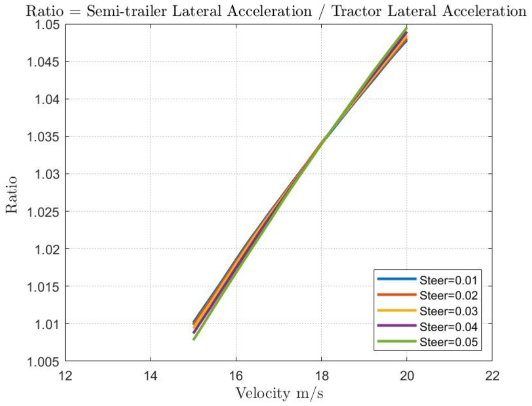

Figure 3.17: Ratio analysis result of moderate-speed

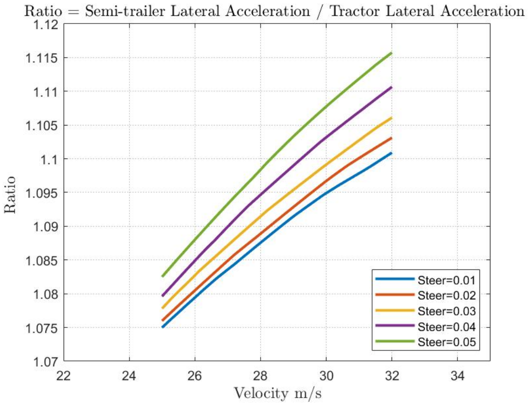

Figure 3.18: Ratio analysis result of high-speed

- In moderate speed zone and high-speed zone, the amplified effect should come into notice because there is a risk leading to rolling instability (shown in Figure 3.17 and Figure 3.18).

The ratio greater than one means the amplified effect, and the ratio smaller than one means the reduced effect. Since the lateral acceleration is directly related to the roll instability, the ideal ratio should be as small as possible. After analyzing ratio of lateral acceleration, it can be concluded that the amplified effect should be noticed in a moderate and high-speed zone, especially in the high-speed zone. Furthermore, the large steering input at high speed will easily lead tractor semi-trailer to roll instability.

3.3.5 Phase Delay

After ratio analysis, the amplified effect or reduced effect can be concluded. In ratio analysis, the maximum value has been compared, however, the problem is that the maximum value of lateral acceleration for tractor and semi-trailer does not reach at the same time. From the dynamic analysis, it seems that response of the semi-trailer is always after the tractor response. This process is called “phase delay”. Before analyzing the phase delay, a series of periodical responses should be created in order to provide data to analyze. In this simulation, the steering input is the same as lane change steering input. But after first lane change, the tractor semi-trailer will follow another lane change action.

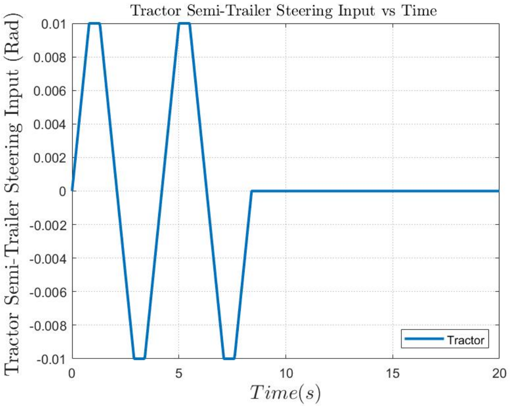

Figure 3.19: Steering input for tractor semi-trailer driving at 30 m/s

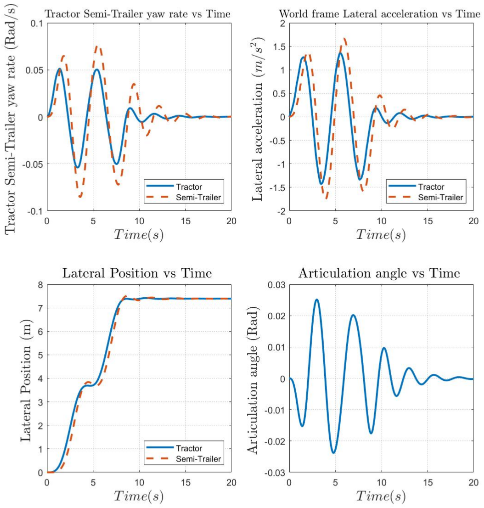

Figure 3.20: Lane change for tractor semi-trailer driving at 30 m/s

Phase Delay Analyses

In phase delay analyze, three speeds has been analyzed as previous sections: 5 m/s, 15 m/s and 30 m/s. The steering input of each simulation has the same periodic as Figure 3.19, the only difference is the magnitude of maximum steering input (0.01 rad in Figure 3.20). In lane change, if the tractor semi-trailer driving at a high speed, the steering input will be small in order to maintain stability. In this analysis, different speed take different maximum steering input. Figure 3.21, 3.22 and 3.23 show the result.

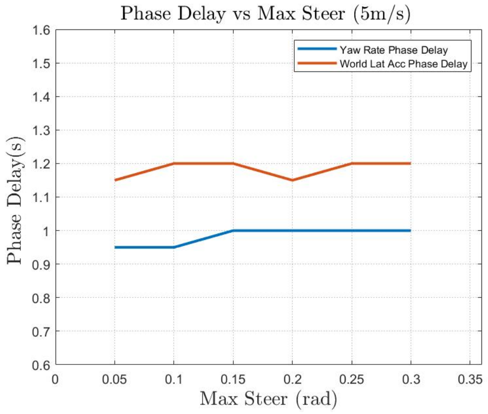

Figure 3.21: Phase delay vs max steer input for tractor semi-trailer driving at 5 m/s

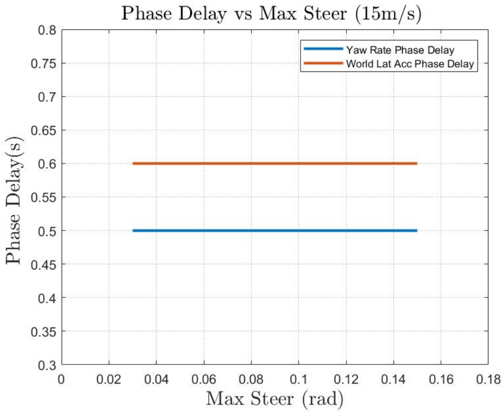

Figure 3.22: Phase delay vs max steer input for tractor semi-trailer driving at 15 m/s

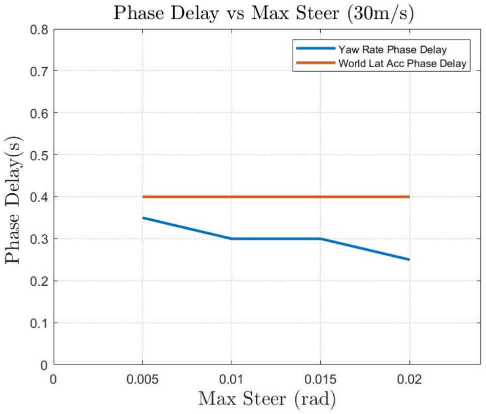

Figure 3.23: Phase delay vs max steer input for tractor semi-trailer driving at 30 m/s

Following results can be summarized through simulating result:

- The magnitude of steering input does not affect lateral acceleration phase delay.
- In high-speed zone (30 m/s), large steering input will reduce the yaw rate phase delay.

Parameter Studies

In the previous phase delay analysis, the parameter (center of gravity and weight) of the semi-trailer is constant. However, in the real world, these two parameters can always be changed. For example, a tractor semi-trailer full loaded at the departure station and it becomes half loaded after unloading in the first destination. It becomes meaningful to study how parameters affect the dynamic. This section study parameter from two parts: Center of gravity position of semi-trailer and weight of semi-trailer.

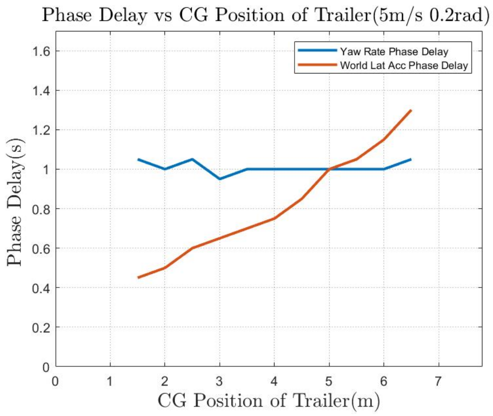

Figure 3.24: Phase delay vs CG position of semi-trailer for tractor semi-trailer driving at 5 m/s

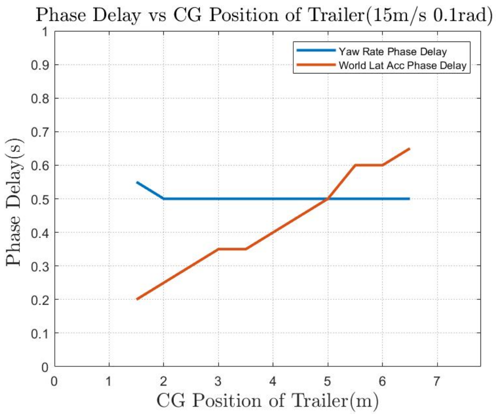

Figure 3.25: Phase delay vs CG position of semi-trailer for tractor semi-trailer driving at 15 m/s

Figure 3.24, 3.25 and 3.26 shows simulation result for effect of center of gravity position of semi-trailer on phase delay. Besides, Figure 3.27, 3.28 and 3.29 shows simulation result for effect of weight of semi-trailer on phase delay. Following conclusions can be found through simulation result:

- The CG position for the semi-trailer close to the rear axle will lead to a large phase delay on lateral acceleration.

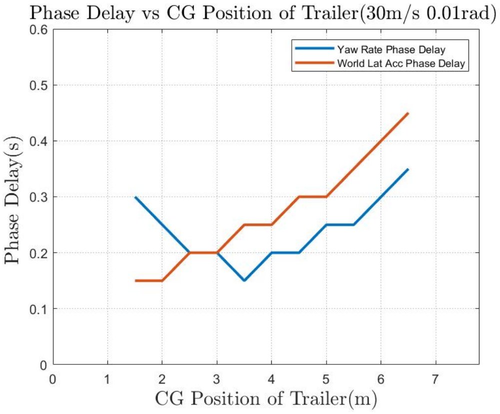

Figure 3.26: Phase delay vs CG position of semi-trailer for tractor semi-trailer driving at 30 m/s

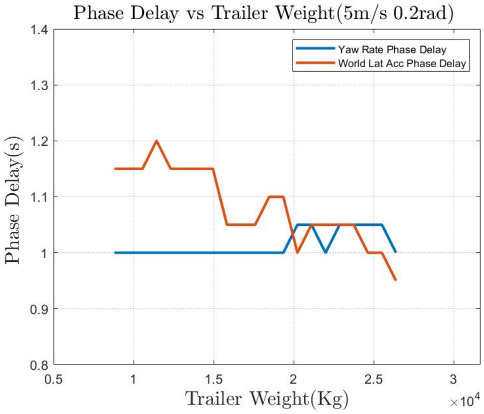

Figure 3.27: Phase delay vs weight of semi-trailer for tractor semi-trailer driving at 5 m/s

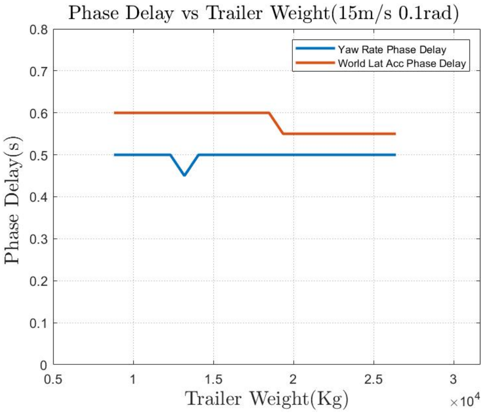

Figure 3.28: Phase delay vs weight of semi-trailer for tractor semi-trailer driving at 15 m/s

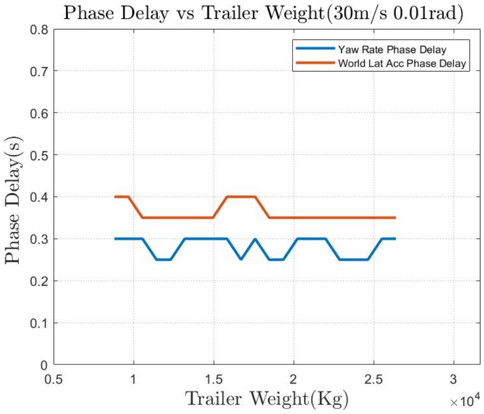

Figure 3.29: Phase delay vs weight of semi-trailer for tractor semi-trailer driving at 30 m/s

- In the high-speed zone, the CG position for a semi-trailer close to the rear axle will result in a large phase delay on the yaw rate. However, the CG position will not affect the yaw rate in other speed zones.
- The phase delay on lateral acceleration proportional to weight of semi-trailer in the low-speed zone, weight of semi-trailer has a small effect in the moderate and the high-speed zone.

- The weight will not have a sufficient effect on phase delay on the yaw rate.

In the real-life, drivers should expect the low phase delay in various driving scenarios. The semi-trailer will easily control if the semi-trailer response right after the tractor response. Based on this fact and parameter study in this section, the CG position of the semi-trailer should close to its front axle. However, this conclusion just considers the effect of phase delay. To answer which CG position is a good choice, other factors such as yaw instability should also be considered.

3.3.6 Summary

This section shows the dynamic response during lane change, the typical steering input has been used during a normal lane change. However, this input can be changed to a triangle input during the emergency or aggressive lane change. With the dynamic analysis, it can help people to decide what is the maximum steering input can be used during this scenario to make sure the whole process is safe enough. Also, the minimum distance can be calculated in order to decide whether should the tractor semi-trailer avoid obstacles or not.

In addition, there are two things that we care about most: amplify or reduction phenomena, and phase delay. There is a critical point for amplifying or reducing phenomena which is in a moderate speed zone. Before the critical point, the semi-trailer will reduce the lateral acceleration and yaw rate of the tractor. After that point, the semi-trailer will amplify the lateral acceleration and yaw rate. Based on this fact, what we really care about is the amplified effect (ratio over one). That will easily bring tractor semi-trailer to a roll instability and yaw instability zone. Besides, from ratio analysis, the relationship between speed, ratio and steering angle has also been found. This fact can help the controller control the tractor semi-trailer more wisely.

Finally, from the phase delay part, the phase delay under three different driving scenarios has been analyzed. Also, the parameter study can help people find how does CG position of the trailer, weight of the trailer and steer input of tractor will affect the phase delay. Since people expect the small phase delay in real life, this fact can help people to decide how to assign CG position to a trailer and weight of trailer under different driving scenarios to ensure enough safety margin for tractor semi-trailer.

3.4 Instability Analyses

3.4.1 Introduction

For the tractor semi-trailer, the roll instability and yaw instability are the two most important fields that directly related to driving safety. In dynamic analysis, three models has been implemented. The roll instability can be observed from lateral acceleration, and the yaw instability can be simulated from a dynamic model. There are two kinds of yaw instability: sneaking and swing. Sneaking is normally caused by the external force, so only jackknifing will be analyzed in this section.

3.4.2 Yaw Instability Analyses

Jackknifing/Swing

There two thresholds for jackknifing: The first threshold can be described by equation 3.15, and the second threshold can be described by equation 3.16. In these two equations, K_{us} is the understeer coefficient, notation t stands for tractor, notation s stands for semi-trailer, and L is the wheelbase. The understeer coefficient can be calculated by equation 3.17. W is weight distribution and C_α is cornering stiffness. From these two equations for threshold, the understeer coefficient [38] should always be positive for the tractor semi-trailer in order to avoid any risk to yaw instability.

$$K_{us,t} < 0 \qquad\qquad K_{us,s} > 0 \tag{3.15}$$

$$K_{us,t} < 0 \qquad K_{us,s} < 0 \qquad (K_{us,s}/K_{us,t}) < (L_s/L_t) \tag{3.16}$$

$$K_{US} = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \tag{3.17}$$

The tractor semi-trailer should be understeer in general driving condition. However, from equation 3.15, 3.16 and 3.17, the root cause of jackknifing is the tractor or semi-trailer becoming oversteer, and the following statement can still be implemented:

- The CG position for semi-trailer close to the front axle and the vertical force of the trailer applied on the tractor is to exceed critical point and make tractor become oversteer.
- In an emergency scenario, deceleration of the tractor becomes too large, the rear wheel of the tractor loses traction force.
- Wheels of the semi-trailer locked up during the emergency braking.
- In snow and rain, the tractor loses grip due to the low friction coefficient.

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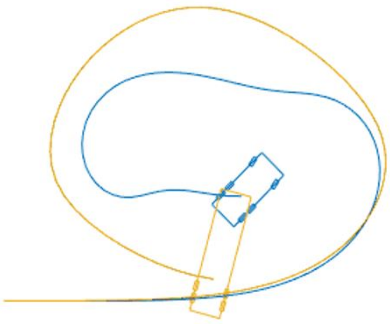

Figure 3.30: Simulation for jackknifing

In Figure 3.30, the jackknifing has been created by making the tractor semi-trailer losing the cornering force. The trajectory of jackknifing is unpredictable. So, this instability must be avoided by:

- Assign CG position and weight to semi-trailer wisely. In another word, the CG position should not be too close to the hitch point and the semi-trailer should never be overloaded.
- In an emergency scenario, the tractor semi-trailer should not have large steer input or large deceleration. These two inputs will cause longitudinal and lateral load transfer, especially on oil tank trucks. Furthermore, emergency brake will place a tractor semi-trailer in the risk to loose traction force.

- Do not take any large input on the road with a low friction coefficient.

3.4.3 Summary

From the simulation, the jackknifing can be easily caused by the bad CG location. The CG location should be close to the rear axle of semi-trailer as much as possible. However, this will cause a sightly larger phase delay. That becomes a trade-off in how to assign CG location for semi-trailer.

3.5 Summary

In this chapter, a dynamic model has been developed. With the help of a new tire model, the model can simulate weight distributed cross the longitudinal direction and different road friction coefficient. After that, the dynamic responses in different velocity have been analyzed with the help of the dynamic model. The amplify phenomena and phase delay phenomena have been found in this stage. Furthermore, Chapter 3 presented a parameter study and give people a deep understanding of how does weight of semi-trailer and weight distribution affect the amplify phenomena and phase delay phenomena. Moreover, how to assign parameters for a tractor semi-trailer has been analyzed at the end of the chapter. These valuable conclusions will be used in the next chapter and help us design a better controller.

Chapter 4

Controller Development and Problem Formulation

4.1 Introduction

Now, the dynamics responses have been analysed, and it provides various valuable information for controller design. The controller is based on the optimal control algorithm. First of all, a kinematic model has been implemented in order to simplify the control problem in section 4.2. Then section 4.3 will introduce an optimal control algorithm called Model Predictive Control and explain the general idea of it. Furthermore, this section will also introduce how to transfer a MPC problem to an optimal control problem and to a nonlinear programming problem. Section 4.4 explains how to transfer an Optimal Control Problem (OCP) to Nonlinear Programming (NLP), and present a new software called "CasADi" to solve the NLP problem.

4.2 Kinematic Model

4.2.1 Introduction

Since the controller will be based on a type of model-based control algorithm - MPC. Thus the implementation of a model is inevitable. The reference model is required to have essential states of the control system and the model can describe how does the system work in the real world. However, the model cannot be too complex since that would increase the complexity of the problem, in other words, this is a trade-off between computational complexity and model accuracy. Based on previous rules, a three-degrees of freedom kinematic model will be implemented for optimal control in this section.

4.2.2 Model Implementation

The kinematic model would not consider any dynamic information such as side slip angle of tire, yet it contains all essential states of the tractor semi-trailer. Similar to the dynamic model, three degrees of freedom is enough for this model to be used in the controller to control a tractor semi-trailer. The kinematic model is only based on the geometrical relationship. In this kinematic model, there are two inputs: steering angle and longitudinal velocity of the tractor. Equation 4.1 to Equation 4.5 are mainly equations of this kinematic model. They include steering angle (δ), yaw angle (θ_f) of tractor, yaw angle (θ_r) of semi-trailer, articulation angle (γ), longitudinal velocity of the tractor (v_f) and the global position of hitch point (x_f, y_f). Equation 4.6 to Equation 4.13 describe the global position of different points on the tractor semi-trailer. These points are shown in Figure 4.1.

$$\dot{\theta}_f = (v_f \tan\delta)/l_{fb} \tag{4.1}$$

$$\dot{\theta}_r = (v_f \sin\gamma)/l_{rb} \tag{4.2}$$

$$\dot{\gamma} = \theta_f - \theta_r \tag{4.3}$$

$$\dot{x}_f = v_f \cos\theta_f \tag{4.4}$$

$$\dot{y}_f = v_f \sin\theta_f \tag{4.5}$$

$$\dot{x}_{ff} = v_f \cos\theta_f - (l_{fa} + l_{fb})\,\dot{\theta}_f \sin\theta_f \tag{4.6}$$

$$\dot{y}_{ff} = v_f \sin\theta_f + (l_{fa} + l_{fb})\,\dot{\theta}_f \cos\theta_f \tag{4.7}$$

$$\dot{x}_{fr} = v_f \cos\theta_f + l_{fc}\dot{\theta}_f \sin\theta_f \tag{4.8}$$

$$\dot{y}_{fr} = v_f \sin\theta_f - l_{fc}\theta_f \cos\theta_f \tag{4.9}$$

$$\dot{x}_{rf} = v_f \cos\gamma \cos\theta_r - (l_{ra} + l_{rb})\,\dot{\theta}_r \sin\theta_r \tag{4.10}$$

$$\dot{y}_{rf} = v_f \cos\gamma \sin\theta_r + (l_{ra} + l_{rb})\,\dot{\theta}_r \cos\theta_r \tag{4.11}$$

$$\dot{x}_{rr} = v_f \cos\gamma \cos\theta_r + l_{rc}\dot{\theta}_r \sin\theta_r \tag{4.12}$$

$$\dot{y}_{rr} = v_f \cos\gamma \sin\theta_r - l_{rc}\dot{\theta}_r \cos\theta_r \tag{4.13}$$

In order to use the conclusion made from Chapter 3, the dimension of the tractor semi-trailer in the kinematic model should be the same as which in the dynamic model. The value assigned to each parameter of the kinematic model is shown in table 4.1.

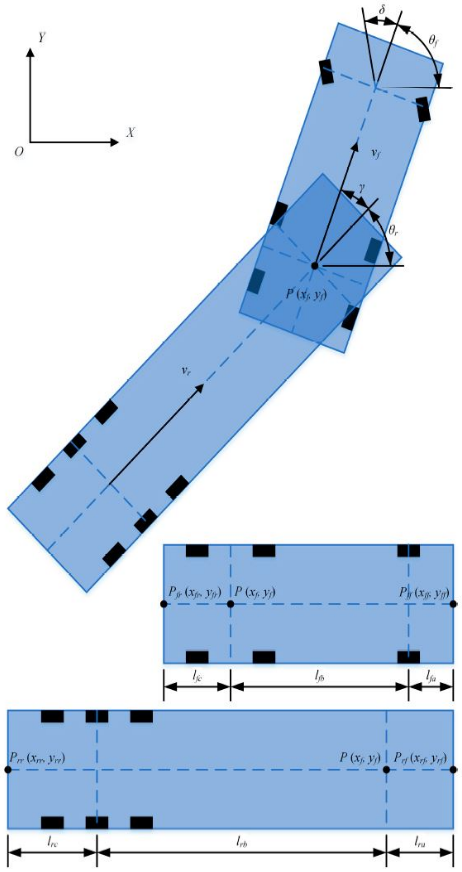

Figure 4.1: Sketch of tractor-semitrailer used for model implementation [23]

4.3 Model Predictive Control

4.3.1 Introduction

As a kind of high-speed mobile robot, the autonomous vehicle has to percept the driving environment, make the decision, plan its path to follow the specific path and arrive at the pre-setting destination. All unmanned ground vehicles have multiple sensors, including but not limiting, camera, GPS and radar. These sensors provide different information for the self-driving. And this project is only focusing on the path following in a high-speed travelling scenario.

Table 4.1: Parameters of tractor semi-trailer in kinematic model

Parameter	**Symbol**	**Value**	**Units**
Front axle of tractor to its front end	l_{fa}	1	m
Wheelbase of tractor	l_{fb}	3.7	m
Front axle of tractor to its rear end	l_{fc}	1	m
Front axle of trailer to its front end	l_{ra}	1	m
Wheelbase of trailer	l_{rb}	7.85	m
Front axle of trailer to its rear end	l_{rc}	1	m

In this project, the main controller is designed by the MPC. MPC is an advanced process control method that was proposed in the 1980s. Comparing with the LQR and other traditional control methods, MPC has a lot of advantages. For now, most vehicles are using MPC as their primary control method in the research area. First of all, it is friendly to multiple inputs and multiple outputs. After that, MPC has been widely used throughout the industry. MPC highly relies on the system model; it can predict the future based on the system model. Based on the prediction, the controller generates a control strategy with the minimum cost for the system.

4.3.2 General Idea

Figure 4.2 shows the general idea of Model Predictive Control. First of all, the MPC has a reference. This reference can be a line or a point. If the reference is a line, the MPC will become a trajectory tracking problem. If the reference is a point, the MPC will become a point stabilization problem. The ultimate objective for the MPC controller is to help the system tracking the reference. In other words, the controller is to minimize the error between predicted future output (blue dash line in Figure 4.2) and reference (green line in Figure 4.2).

Second of all, the MPC has a reference model to help it to predict the future output. The prediction horizon P decides how many time steps will the MPC predict which cannot be small because it will not have enough time to respond rapidly to the environment, also cannot be a large number because the prediction result will be inaccurate after a long time step. Control horizon M is similar to prediction horizon P, however, only a few control inputs at the beginning will change the states of the system significantly. A control horizon M has a marginal effect in the control process: an increasing control horizon will improve the performance (accuracy) of MPC at the beginning, and reduce the performance (computation efficiency) at the end.

After the controller calculating the input strategy for the system, the system will only use the first set of numbers from the input strategy as the system input. The real output will be given by the system, then this process “roll out” to the next time step until the terminal state.

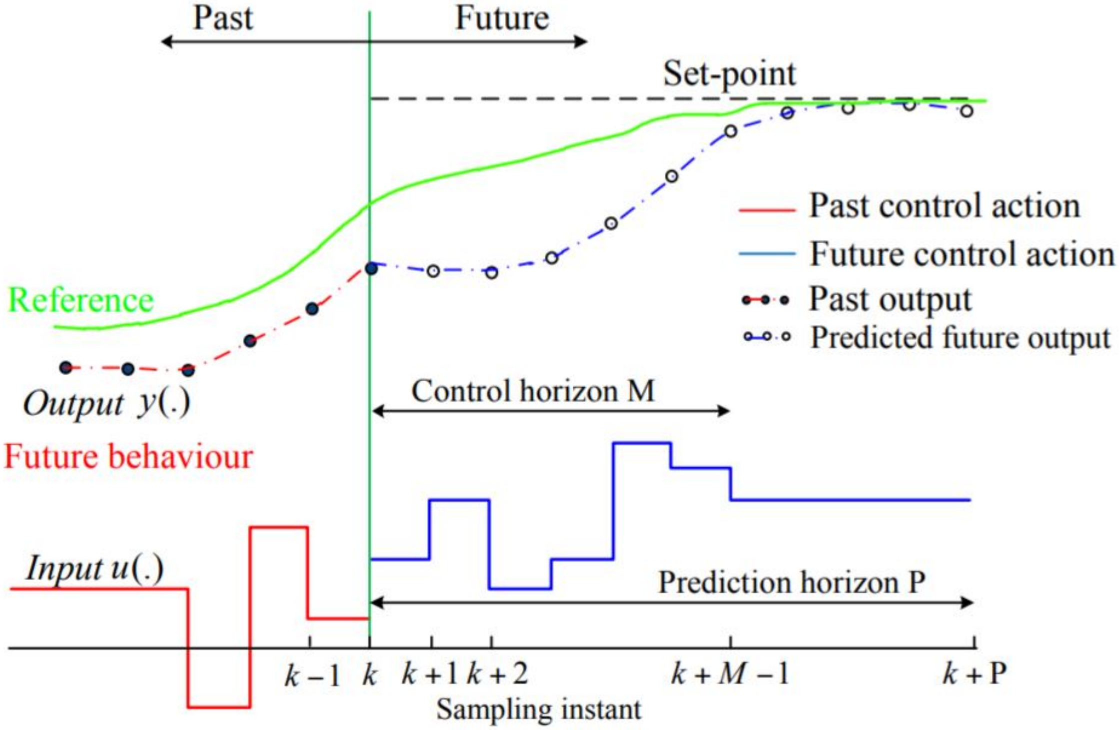

Figure 4.2: General concept of MPC [60]

4.3.3 Runge-Kutta Method

Runge-Kutta Method is used to solve the ordinary differential equation (ODE). Comparing with traditional approaches such as the Euler method, the Runge-Kutta method provides an approximate solution with higher accuracy. Many further developments rose after the Runge-Kutta method which was invented in the 1900s. [15] However, the classic Runge-Kutta method usually refers to the 4th order Runge-Kutta method; and it has already been able to improve the accuracy significantly comparing with the Euler method.

The kinematic model can be rewritten as following form as Equation 4.14:

$$\dot{x} = dyn(x, u(t)), \tag{4.14}$$

Equation 4.14 will be used as the dynamic function to design the controller, the “dynamic” of dynamic function means that states in the function changing over time. The dynamic function is different from the dynamic model and can be an arbitrary function that can describe a real-world property. However, the dynamic model of the automobile is an equation that describes the time-varying phenomena of a system, and it includes the interaction between motions and forces of a vehicle system.

Since the kinematic model implemented in the previous section is a continuous model, it has to be discretized before use it in the controller. The equation can be discretized by solving a differential equation. However, when solving the differential equation with the Euler method, the time steps ΔT has to be extremely small to avoid the error. Even though the error is extremely small in each time step, the accumulated error would have a great impact on the final result after a while. In the Runge-Kutta method, step size h is used to represent the sample time. Relation between h and t can be described by Equation 4.15.

$$t_{n+1} = t_n + h \tag{4.15}$$

A typically Runge-Kutta 4th order method can be formulated as Equation 4.16. Rather than only considering the slope at a single point, the Runge-Kutta method considers slope at four points and get an average for that. No matter in optimal control problem or nonlinear programming problem, the Runge-Kutta 4th order method will be used to satisfy the dynamic constraint which described by the dynamic function.

Equation 4.16, k_1 to k_4 are slopes which calculated based on the dynamic function, k is the number of time step, and h is the sample time. Use slopes at different points, the differential equation can be solved within a acceptable error.

$$\begin{gathered} k_1 = dyn\left(x_u(k-1), u\right) \\ k_2 = dyn\left(x_u(k-1) + \tfrac{h}{2}k_1, u\right) \\ k_3 = dyn\left(x_u(k-1) + \tfrac{h}{2}k_2, u\right) \\ k_4 = dyn\left(x_u(k-1) + hk_3, u\right) \end{gathered} \tag{4.16}$$

After have slops, states of dynamic function at current time step can be found by using Equation 4.17. Since the Equation 4.16 is a function of states of dynamic function of previous time step $x_u(k-1)$, system input $u(k-1)$ and sample time h, the Equation 4.17 can rewritten as Equation 4.18.

$$x_u(k) = x_u(k-1) + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) \tag{4.17}$$

At this subsection, a dynamic function of the system has been discretized successfully by using Runge-Kutta 4th order method described by Equation 4.18.

$$x_u(k) = f\left(x_u(k-1), u(k-1), h\right) \tag{4.18}$$

4.3.4 Optimal Control Problem

An effective system model is indispensable to design an MPC controller. The kinematic model will be undoubtedly used in the MPC as a reference model. Based on implemented equations, the kinematic model would have thirteen states, which are shown in Equation 4.19. Furthermore, five of them are essential states, which can be used to implement other sates, and are shown in Equation 4.21. Besides, the input of the system is shown in Equation 4.20. Even though a reference model can help the controller to predict future motion during its prediction horizon, a well-designed objective function is also crucially important.

$$x = \left[\begin{array}{ccccccccccccc} \theta_f & \theta_r & \gamma & x_f & y_f & x_{ff} & y_{ff} & x_{fr} & y_{fr} & x_{rf} & y_{rf} & x_{rr} & y_{rr} \end{array}\right]^T \tag{4.19}$$

$$u = \left[\begin{array}{cc} v_f & \delta \end{array}\right]^T \tag{4.20}$$

$$y = \left[\begin{array}{ccccc} \theta_f & \theta_r & \gamma & x_f & y_f \end{array}\right]^T \tag{4.21}$$

The running cost of an optimal control problem shows in Equation 4.22. In Equation 4.22, x_u is current states of the system, x^r is reference states of the system, u is control input of the system and Δu is the increment of the control input. Q, R and S are penalizing matrix for the running cost. The cost can calculate through errors between actual states and desire states with different weights in the weighting matrix. The error caused by the state with large weighting will eliminate fist in the control process. By tuning weights in the weighting matrix, the priority and penalize level of error will be determined. Equation

4.22 is a quadratic function that includes three parts: penalization on tracking error, the magnitude and increment of input. The first term of the objective function is to penalize the error between current states of the tractor semi-trailer and reference states of the tractor semi-trailer. The second term of the objective function is to penalize the amount of input since it can help the controller use a minimum input effort to reach the reference state. Moreover, the last term is to penalize the increment of input, since it can help the controller avoid suddenly request a large input that may cause instability.

$$\ell(x,u) = \|x_u - x^r\|_Q^2 + \|u\|_R^2 + \|\Delta u\|_S^2 \tag{4.22}$$

MPC has a character called "rollover". The problem can be treated as a batch of optimal control problems. The cost function of the MPC can be written as a combination of running costs in Equation 4.23.

$$J_N(x,u) = \sum_{k=0}^{N-1} \ell\left(x_u(k), u(k)\right) \tag{4.23}$$

There are two types of constraints in MPC: soft constraint and hard constraint. The soft constraint can be violated in some specific situations though it is not recommended. The hard constraint, which cannot be violated in any condition, in this controller should be based on the traffic rule and conclusion of dynamic analysis. Traffic rule includes the width and the speed limit of the road. Since the kinematic model does not include any dynamic information, dynamic constraints of the controller will be based on the dynamic analysis which is in Chapter 3, and it includes risk input that will lead tractor semi-trailer to yaw instability and roll instability.

Besides the objective function, the constraints of MPC are also important segments in controller design, where the constraint includes:

The dynamic constraint is shown in Equation 4.24:

$$\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k), \boldsymbol{u}(k), h) \tag{4.24}$$

The constraint on state variables and control input which is shown in Equation 4.25:

$$\begin{aligned} &u(k) \in U, \forall n \in [0, N-1] \\ &x_u(k) \in X, \forall n \in [0, N] \end{aligned} \tag{4.25}$$

The algebraic path constraint input which is shown in Equation 4.26:

$$g(x(k), u(k), h) \in G, \forall n \in [0, N] \tag{4.26}$$

Dynamic constraints are the reference model implemented previously, these constraints assure the MPC of solving a specific problem and controlling an expected system. The state variables and control input constraints assure that the real-world system will not reach its performance limitation, and other constraints such as obstacle formed constraints meet algebraic path constraints.

The controller will have but not limited to the following constraints:

- Constraint on articulation angle since a large articulation angle will bring yaw instability problems such as jackknifing.
- Constraint on longitudinal acceleration since the emergency braking will lead tractor semi-trailer to a yaw instability zone.
- Constraint on steering angle since the large steering input will bring roll instability problems such as rollover.
- Constraint on the increment of steering angle since it will generate a large lateral acceleration and lead tractor semi-trailer to a roll instability zone, besides, it might be caused an unnecessary large articulation rate.
- Constraint on lateral position since the tractor semi-trailer should drive inside the given road.
- Constraint on heading velocity since the tractor semi-trailer should drive based on speed limited in a different scenario.
- Constraint formed by the obstacle.

After the controller possessing the reference model, objective function and constraints, it can be used to control a tractor semi-trailer to achieve different tasks. Moreover, with the help of the MPC handling some complex scenarios such as obstacle avoidance, the safety factor of the tractor semi-trailer can be improved.

At this point, the rough framework of MPC has been derived and shown in Equation 4.27 :

$$
\begin{gathered}
\min J_N(x,u) = \min \textstyle\sum_{k=0}^{N-1} \ell\left(x_u(k), u(k)\right) \\
\\
\text{s. t.} \quad \boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k), \boldsymbol{u}(k), h) \\
x_u(0) = x_0 \\
u(k) \in U, \forall n \in [0, N-1] \\
x_u(k) \in X, \forall n \in [0, N] \\
g(x(k), u(k), h) \in G, \forall n \in [0, N]
\end{gathered}
\tag{4.27}
$$

Solving a problem described as Equation 4.27 is equal to solving a quadratic programming problem. The quadratic programming problem has a general form known as a nonlinear programming problem. The following two subsections will talk about what is a nonlinear programming problem and how to solve it.

4.3.5 Nonlinear programming Problem

MPC is a kind of optimization problem, and handling a system with multiple variables is an advantage of MPC comparing to other lower-level control approaches. The actuator constraints which include soft constraints and hard constraints can be added to the optimization problem easily.

However, in real life, a system will have disturbance (white noise) and uncertainty. Generally, the linear-time-varying (LTV) system model is in the state space form which is shown in Equation 4.28. In this equation, $x(k)$ is states of the kinematic model, $u(k)$ is input such as velocity and steering input, $d(k)$ is the disturbance, and $y(k)$ is the output of the system. However, if the equation writes in the state space form, the equation will be linear. For the tractor semi-trailer kinematic model, the linearized equation will have to assume the yaw angle extremely small.

$$
\begin{aligned}
&x(k+1) = A_d x(k) + B_d u(k) + d(k) \\
&y(k) = C_d x(k) + D
\end{aligned}
\tag{4.28}
$$

Equation 4.29 describes a general form of the nonlinear programming problem. $\Phi(w)$ called the objective function or cost function, can be combined by various functions which will be talked about in later sections. Besides the objective function, there are two types of constraints: equality constraints(g_1) and inequality constraints(g_2). The objective of solving an optimization problem is to minimize the cost function under some constraints.

There are two special cases: linear programming problem and quadratic programming problem. For linear programming problems, Φ and constraints can be expressed as linear combinations of these elements of w. For quadratic programming problems, constraints can be expressed as linear combinations of these elements of w, where Φ is a linear-quadratic function, respectively.

$$\begin{aligned} &\min_w \Phi(w) \\ \text{s. t. } &g_1(w) = 0 \\ &g_2(w) \leq 0 \end{aligned} \tag{4.29}$$

4.3.6 Solve the Nonlinear programming problem

The essential of MPC is an optimization problem, and it can be divided into a lot of Optimal Control Problems. Beginning with a simple optimization problem, this section will briefly introduce how to solve an optimization problem. First of all, consider the following function (Equation 4.30):

$$f(w) = x^2 - 10x + 30 \tag{4.30}$$

In order to solve this question, rewrite the Equation 4.30 to an standard Non-linear programming form:

$$\min_x f(x) = x^2 - 10x + 30 \tag{4.31}$$

In order to minimize the Equation 4.31, the gradient expression of the function has to be derived. In other words, the minimum point of the function is at the point which the first-order derivative (Equation 4.32) of the function is equal to zero.

$$\frac{df(x)}{dx} = 2x - 10 \tag{4.32}$$

Figure 4.3 shows the plot of Equation 4.30, the gradient is the slope of the function. The red point in the figure is located at the global minimum(5, 5) of the function. After adding the optimization constraints, the simple optimization example will become an integrated optimization problem.

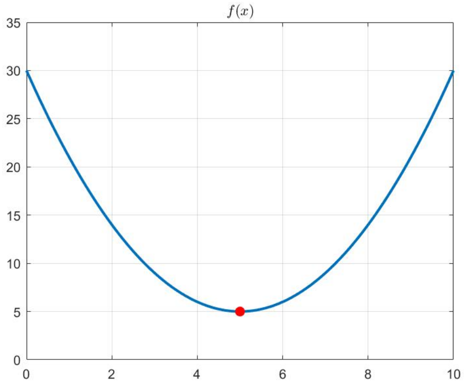

Figure 4.3: Plot of Equation 4.30

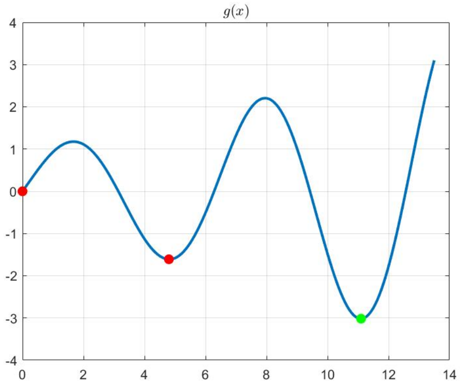

Figure 4.4: Local minimum vs. Global minimum

Furthermore, the initial value of the problem will also affect the solution. Now consider another function $g(x)$ shown in Figure 4.4. There are three minimum points in this function: two local minimum (red points) and one global (green point) minimum. All of these three points have the chance to be used as a solution in optimization problems, and it depends on the initial condition. If the initial condition at point $(\pi, 0)$, the second red point will be used as a solution, however, if the initial condition at point $(3\pi, 0)$, the global minimum will be the final solution.

In the MPC problem, the cost function is similar to the function shown in the previous example. Although the MPC problem is a kind of optimization problem, sometimes it can become an extremely complex problem with large scale equations. To solve the complex problem, an optimization solver with high efficiency and accuracy becomes an indispensable tool. In this project, an optimization solver called "CasADi" will assist the program to solve the optimization problem.

4.4 From Optimal Control to Nonlinear Programming

Consider following optimal control problem described in Equation 4.33:

$$
\begin{array}{c}
\min J_N(x,u) = \min \sum_{k=0}^{N-1} \ell\left(x_u(k), u(k)\right) \\
\text{s. t.} \quad \boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k), \boldsymbol{u}(k), h) \\
x_u(0) = x_0 \\
x_u(k) \in X, \forall n \in [0, N] \\
u(k) \in U, \forall n \in [0, N-1] \\
g(x(k), u(k), h) \in G, \forall n \in [0, N]
\end{array}
\tag{4.33}
$$

The decision variables will include both X (system state within the prediction horizon which calculate through dynamic constraint) and U (control input within the prediction horizon). The software will optimize each term in the decision variable (Equation 4.34). The objective function of the optimal control problem has been transformed into the objective function of the nonlinear programming problem.

$$
W = \left[\begin{array}{cccccccc} x_0 & x_1, & \cdots & x_N, u_0, & u_1 & \cdots & u_{N-1} \end{array}\right] \tag{4.34}
$$

Since CasADi breaks down the system and the whole dynamic equation will transform a continuous equation to discontinuous equation. Another path constraint would be added to the problem called equality constraint (Equation 4.35). In order to minimize the error, the Runge-Kutta method will be used to replace the Euler method in the discretization progress.

$$
x(k+1) - f(x(k), u(k), h) = 0 \tag{4.35}
$$

The equality constraint will be written as a matrix form (Equation 4.36). Now, the dynamic constraint mentioned in the optimal control problem has been transformed into the equality constraint in nonlinear programming problem.

$$
g_1(w) = \left[\begin{array}{c} \overline{x}_0 - x_0 \\ f\left(x_0, u_0, h\right) - x_1 \\ \vdots \\ f\left(x_{N-1}, u_{N-1}, h\right) - x_N \end{array}\right] = 0 \tag{4.36}
$$

Finally, the algebraic path constraint and control input constraint will be transformed into inequality constraint (Equation 4.37). In the algebraic path constraint, each term can be written as a function of control variables and state variables.

$$g_2(w) = \begin{bmatrix} g_2\left(x_0, u_0, h\right) \\ \vdots \\ g_2\left(x_{N-1}, u_{N-1}, h\right) \\ g_2\left(x_N, h\right) \end{bmatrix} \leq 0 \tag{4.37}$$

Now, the optimal control problem has been transformed into a standard form nonlinear control problem (Equation 4.29):

- The MPC cost function has been transformed into optimal control cost function and decision variables $\Phi(w)$.
- Dynamic constraint has been transformed into equality constraint g_1.
- Algebraic path constraints, control input constraints and state variables constraints has been transformed into inequality constraint g_2.

4.4.1 CasADi

CasADi is an open-source tool for nonlinear optimization and algorithmic differentiation [8]. It can provided a efficiency way to solve the optimization problem such as Nonlinear Model Predictive Control (NMPC).

Figure 4.5 shows the framework of the program: the first part of the program is user-defined variables and functions shown in the yellow block. The second part is the NLP problem formulation and the third part is called the solver to solve the NLP problem.

At beginning, user have to define the basic variables and function for the MPC problem; and it includes: initial condition x_0, terminal condition x_{des}, simulation time T, sample time ΔT, prediction horizon N, objective function J and dynamic constraint $f(x, u, t)$. After defining the basic framework of the MPC problem, user can add state variable constraints (x_{max} and x_{min}), control input constraints (u_{max} and u_{min}) and algebraic path constraints (g_{max} and g_{min}) to the MPC problem in order to form a completed NLP problem. Finally, user can assign an optimizer (ipopt in this program) for program and define setting for optimizer.

Second part of this program converts the NLP problem to a "CasADi format" NLP problem: A parameter matrix "P" collects x_0 (a 13×1 matrix), x_{des} (a 13×1 matrix) and

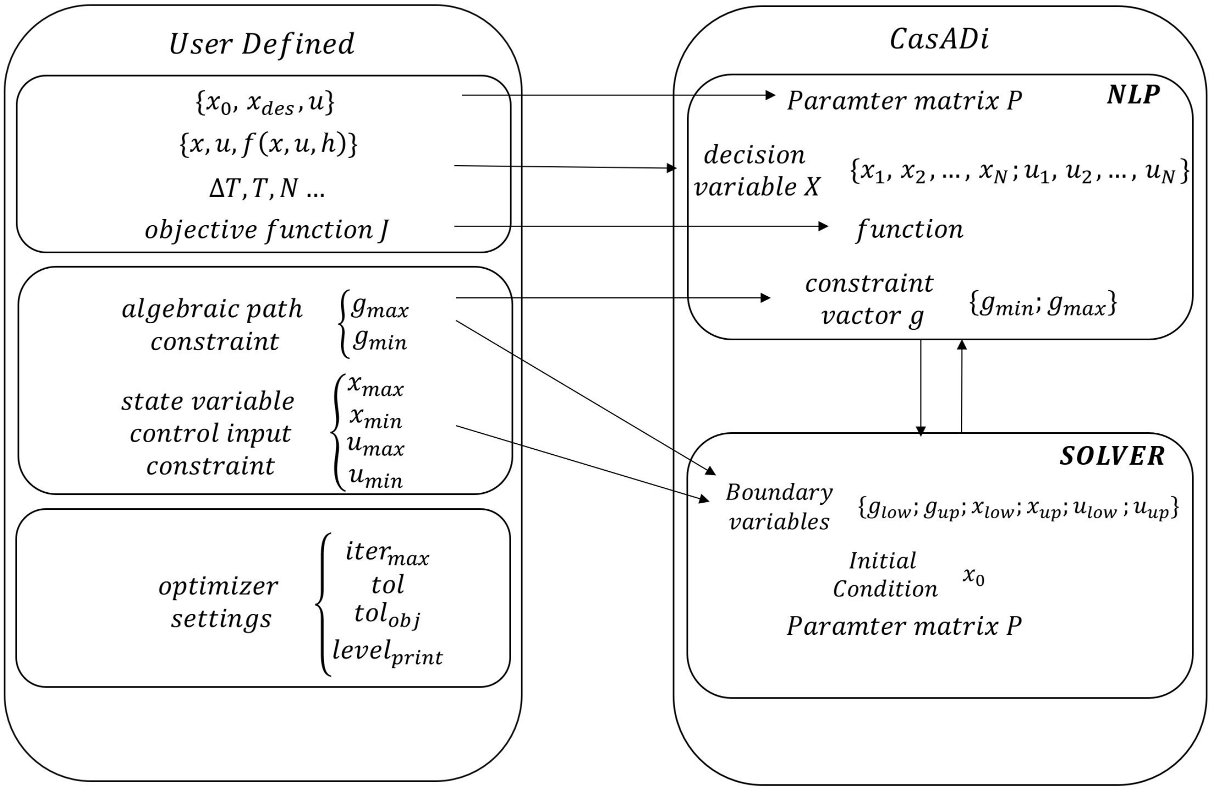

Figure 4.5: CasADi Framework

u (a 2×1 matrix). With this information, the matrix "P" has a dimension of 28×1, and x_0 in P referring to initial condition at each time step, whose value will be continuously updated within the simulation. The decision variable W includes all states and inputs calculated within the prediction horizon N at each time step. This matrix will also be updated within the simulation and has a dimension of $28 \times N$. The function is from the objection function defined by the user, and it allows the user to define different objective functions for a different step in the prediction horizon. The last term is constraint matrix g, the dimension of this matrix depends on how many algebraic path constraints are defined by the user.

The third part of the program is to use a solver to solve the NLP problem; and the result will be feedback and be used to update variables in the NLP problem. This process will not stop until reach final time step T or terminal state x_{des}.

4.5 Optimal Control Problem and Model Predictive Control

Trajectory tracking is one of the fundamental problems in autonomous driving. There are a lot of approaches for trajectory tracking: PID [30], Stanley control [26], LQR [31] and MPC. Simple control methods such as PID and Stanley can handle the trajectory tracking problem, and they even have better performance on robustness and computational efficiency comparing with complex control approaches. However, these primary control methods cannot handle multiple tasks at the same time such as tracking a trajectory and avoid obstacles. They have to rely on other path planning algorithms to planning a path and tracking the trajectory. It becomes reasonable that use MPC to tracking a trajectory: tracking a given trajectory and avoid obstacles on the trajectory without reliance on any other path planning algorithms.

The problem are formulated the same way as the MPC problem mentioned in the previous section. The only difference is the system is tracing a trajectory but not a point. The trajectory can be treated as a set of points, at each time step, and the system has to reach a specific point. By writing trajectory as a function of time, it can create a set of points manually. The first term of objective function has been changed: x_{des} will be continuously updated within the prediction horizon to track the trajectory with rest parts remaining the same. The state constraint and input constraint come from the dynamic analysis in Chapter 3.

$$
\begin{aligned}
&\min J_N(x,u) = \min \textstyle\sum_{k=0}^{N-1} \ell\left(x_u(k), u(k)\right) \\
&\text{s.t.} \quad x(t+1) = f(x(t), u(t), h) \\
&\qquad -\pi/3 \leq \theta_f \leq \pi/3 \\
&\qquad -\pi/3 \leq \theta_r \leq \pi/3 \\
&\qquad -\pi/12 \leq \gamma \leq \pi/12 \\
&\qquad -0.02 \leq u_{Steering} \leq 0.02 \\
&\qquad 0 \leq u_{Velocity} \leq 30 \\
&\qquad -0.005 \leq \Delta u_{Steering} \leq 0.005 \\
&\qquad -0.1 \leq \Delta u_{Velocity} \leq 0.1
\end{aligned}
\tag{4.38}
$$

Figure 4.6 shows the simulation results of trajectory tracking: the tractor semi-trailer tracks the given path under the given constraint. In the figure, the red line represents the reference path of the tractor semi-trailer, the black dash line represents the actual

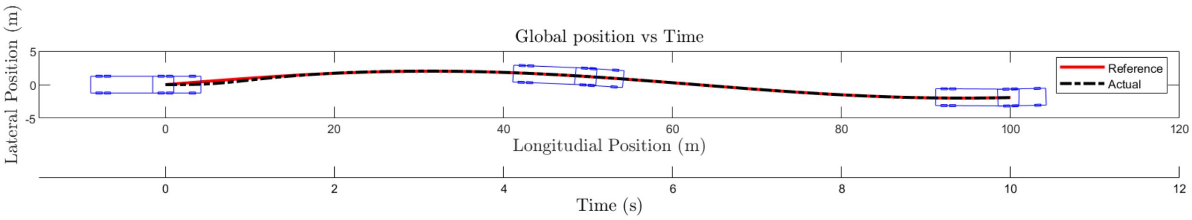

Figure 4.6: Trajectory tracking simulation result for tractor semi-trailer

path of the tractor semi-trailer. The tractor semi-trailer does not track the reference path accurately because there exists a hard constraint in the controller. The controller does not allow the tractor semi-trailer has that much steering input during the path tracking. However, this error has been eliminating after the initial tracking stage.

4.6 Highway Obstacle Avoidance Problem

4.6.1 Introduction

The obstacle avoidance problem is always an important topic in autonomous driving. There are two approaches to path planning. The first approach is based on the path planning algorithms. The tractor semi-trailer(ego vehicle) has the information on obstacle states and the road, and it will use one of the path planning algorithms to plan a path. After the ego vehicle having a reference path, it can use a simple control method such as PID and Stanley control to help the vehicle to follow the reference path. In another word, the ego vehicle has two tasks: path planning first and then control.

Besides, the second approach is based on MPC algorithms. Similar to the first approach, the ego vehicle has the same information. Besides, it already has a reference path, and it will follow that path. However, at this approach, the obstacle will become a kind of constraint and the controller has to figure out a new path for ego vehicle without violated constraints. In another word, the ego vehicle has one task: path planning and control at the same time. Since the second approach fusion a path planning algorithm and a control algorithm, this approach will be used to set up the problem.

4.6.2 Problem Setup

In this section, a classic problem will be solved: an ego vehicle is driving on a highway, then it decides to overtake another slow vehicle through changing a lane. The problem will be solved with the help of MPC, in other words, path planning and control will happen at the same time.

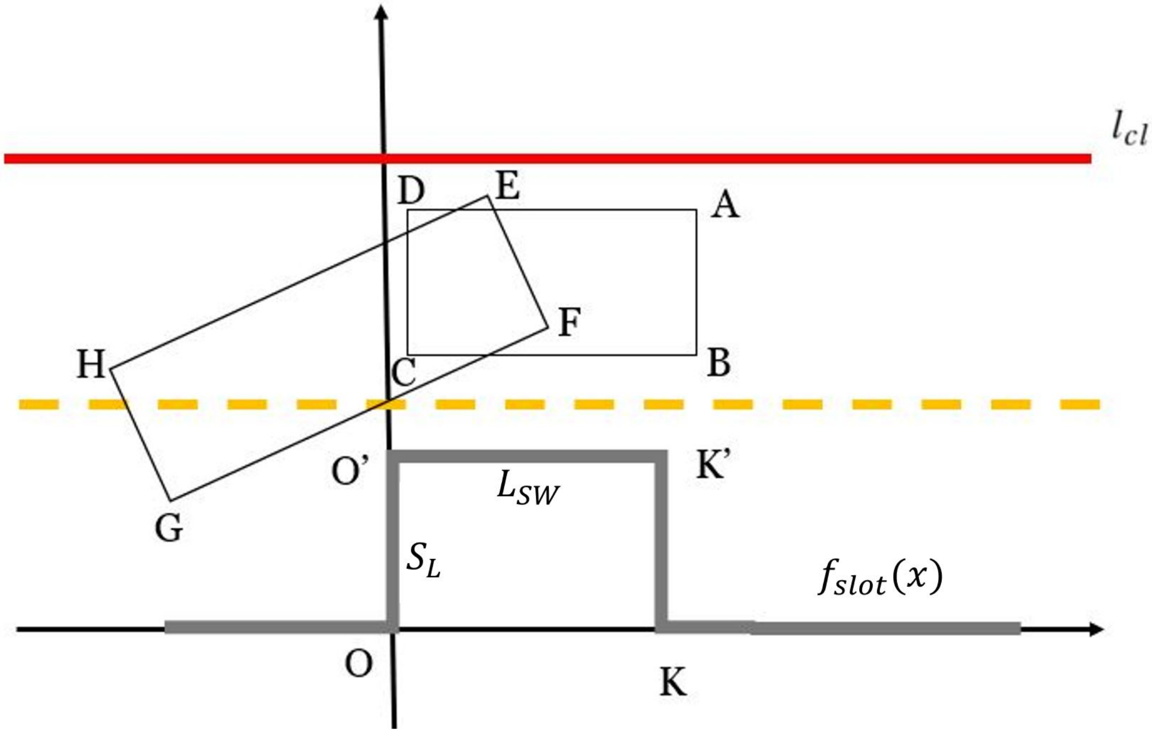

Figure 4.7: Obstacle avoidance problem formulation

To clarify how does the problem setup. A simple problem set up shown in Figure 4.7. The tractor semi-trailer has been split into two parts: tractor part and semi-trailer part. And then, two rectangles (rectangle ABCD and rectangle EFGH) have been used to estimate the shape of these two parts. Rectangle ABCD and Rectangle EFGH will be hitched together and they might have different yaw angles. The obstacle can be written as a continuous function $f_{slot}(x)$. $f_{slot}(x)$ which is shown in Equation 4.15 is a combination of a various unit step functions $S(x(t))$, l_{SW} and l_{SL}. In Equation 4.39, l_{SW} is the width of the obstacle and l_{SL} is the length of the obstacle, and these two variables defined the dimension of inflated obstacle together. The inflation distance depends on the safety distance in different scenarios, in obstacle avoidance, where the safety distance would be 5 to 10 meters. Figure 4.8 explains how inflation works: the blue block is the obstacle such as a passenger car, and the $f_{\text{slot}}(x)$ is larger than the outline of the obstacle. Not only ensuring enough safety distance, but the inflation can also block the lane which has obstacles. After blocking a lane, the controller can make sure the tractor semi-trailer will get enough distance to obstacles during the overtaking action.

$$f_{\text{slot}}(x) = \left(S(x(t)) - S\left(x(t) - l_{SL}\right)\right) l_{SW} \tag{4.39}$$

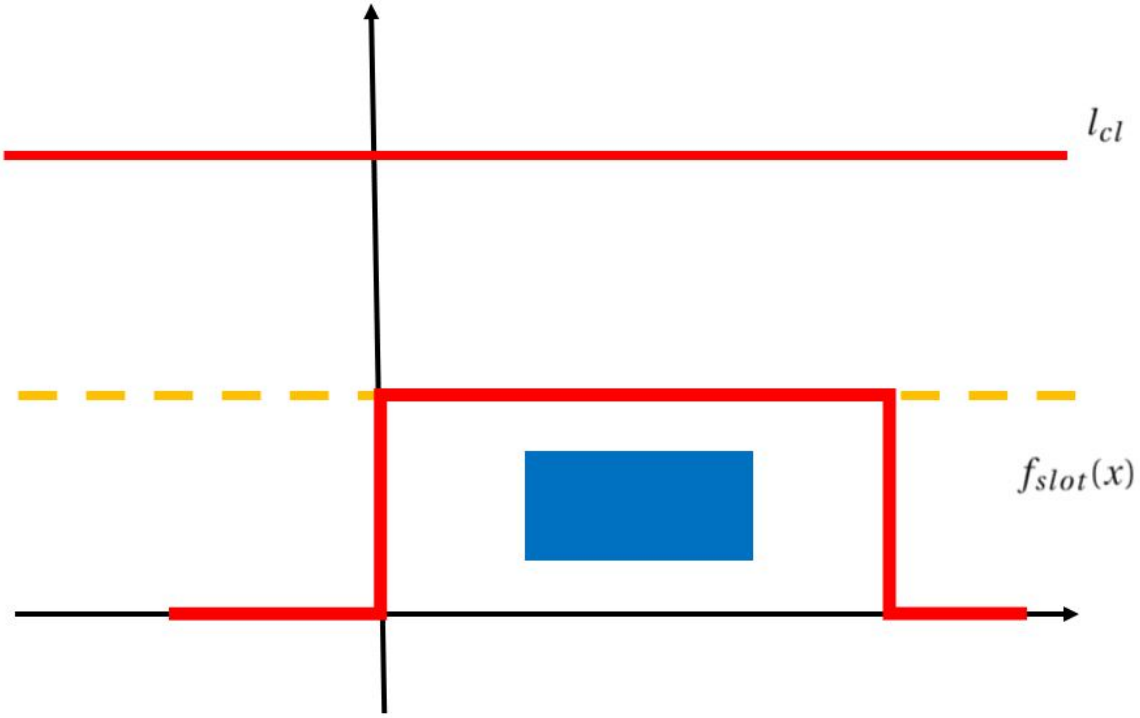

Figure 4.8: Obstacle avoidance problem formulation - inflation on obstacle

In the scenario shown in Figure 4.7, there are two lanes where the tractor semi-trailer is supposed to run, in other words, there are three position constraints for the tractor semi-trailer which are shown in Equation 4.40. The position of eight corners of the tractor semi-trailer should be above the obstacle function and the road boundary and below the other road boundary.

$$\begin{array}{lll} A_y(t) \leq l_{cl} & A_y(t) \geq X_{axis} & A_y \geq f_{slot}\left(A_x\right) \\ B_y(t) \leq l_{cl} & B_y(t) \geq X_{axis} & B_y \geq f_{slot}\left(B_x\right) \\ C_y(t) \leq l_{cl} & C_y(t) \geq X_{axis} & C_y \geq f_{slot}\left(C_x\right) \\ D_y(t) \leq l_{cl} & D_y(t) \geq X_{axis} & D_y \geq f_{slot}\left(D_x\right) \\ E_y(t) \leq l_{cl} & E_y(t) \geq X_{axis} & E_y \geq f_{slot}\left(E_x\right) \\ F_y(t) \leq l_{cl} & F_y(t) \geq X_{axis} & F_y \geq f_{slot}\left(F_x\right) \\ G_y(t) \leq l_{cl} & G_y(t) \geq X_{axis} & G_y \geq f_{slot}\left(G_x\right) \\ H_y(t) \leq l_{cl} & H_y(t) \geq X_{axis} & H_y \geq f_{slot}\left(H_x\right) \end{array} \tag{4.40}$$

Even though there are three constraints for the tractor semi-trailer, there is still one exception shown in figure 4.9. Four corners of the tractor satisfy the position constraints, however, the risk of tractor colliding with the obstacle still exists. Using four triangles formed by four corners of the tractor and the corner of obstacle can avoid this scenario from happening. These four triangles are: $\Delta AK'B$, $\Delta BK'C$, $\Delta CK'D$ and $\Delta DK'A$. This constraint described by Equation 4.41, and when the point K' is outside rectangular ABCD, the total area of $\Delta AK'B$, $\Delta BK'C$, $\Delta CK'D$ and $\Delta DK'A$ must be larger than area $ABCD$. A constant number C adds to the equation to give protection for the tractor semi-trailer. Similarly, the same constraint can be applied to a semi-trailer as well. Equation 4.41 to Equation 4.44 describe the area constraint for this problem.

$$Area(AK'B) + Area(BK'C) + Area(CK'D) + Area(DK'A) > Area(ABCD) + C \tag{4.41}$$

$$Area(AO'B) + Area(BO'C) + Area(CO'D) + Area(DO'A) > Area(ABCD) + C \tag{4.42}$$

$$Area(EK'F) + Area(FK'G) + Area(GK'H) + Area(HK'E) > Area(EFGH) + C \tag{4.43}$$

$$Area(EO'F) + Area(FO'G) + Area(GO'H) + Area(HO'E) > Area(EFGH) + C \tag{4.44}$$

Collaborating with the inflation of $f_{\text{slot}}(x)$, the area constraint can provide a dual protection for tractor semi-trailer. First of all, the constant number C in Equation 4.41

can decide the area between function $f_{\text{slot}}(x)$. In other words, a large C will give more safety distance for tractor semi-trailer when it is driving parallel to the obstacle. However, the constant number C cannot control how aggressive the action will be taken by the controller since this number cannot be too large. An unreasonable large number will provide a large repulsive force to the tractor semi-trailer to the other lane; the controller will not be able to find an optimal solution for this non-linear programming problem if the tractor semi-trailer is only allowed to use an adjacent lane during overtaking. The inflation of function $f_{\text{slot}}(x)$ is added to the controller and gives the controller a glimmer of hope to solve this issue. Not only providing dual protection for tractor semi-trailer but also bringing flexibility for the controller, the dimension of inflation can be varied based on the different scenarios: during normal obstacle avoidance scenario, large longitudinal inflation and lateral inflation will provide enough space between tractor semi-trailer and obstacle; during the critical or emergency obstacle avoidance scenario, small longitudinal inflation and lateral inflation will provide enough time and more choice for tractor semi-trailer to avoid obstacle yet not bringing any risk of instability.

All in all, the obstacle avoidance problem will become various constraints and build into the optimal control algorithm. Besides the normal constraints for states and input of tractor semi-trailer, the controller will also have the following constraints:

- Position constraint to prevent tractor semi-trailer driving outside road boundary.
- Position constraint to prevent tractor semi-trailer contact obstacle.
- Area constraint to prevent the special case that will lead position to constrain failed.

4.6.3 Problem Formulation

Before moving to next, recall the following facts found from the dynamic analysis in Chapter 3:

- The maximum steering angle input under specific speed is based on the dynamic analysis in Chapter 3.
- The tractor semi-trailer cannot give a large deceleration during the lane change.
- The magnifying effect on lateral acceleration during high speed.
- There exists a time delay between tractor and semi-trailer.

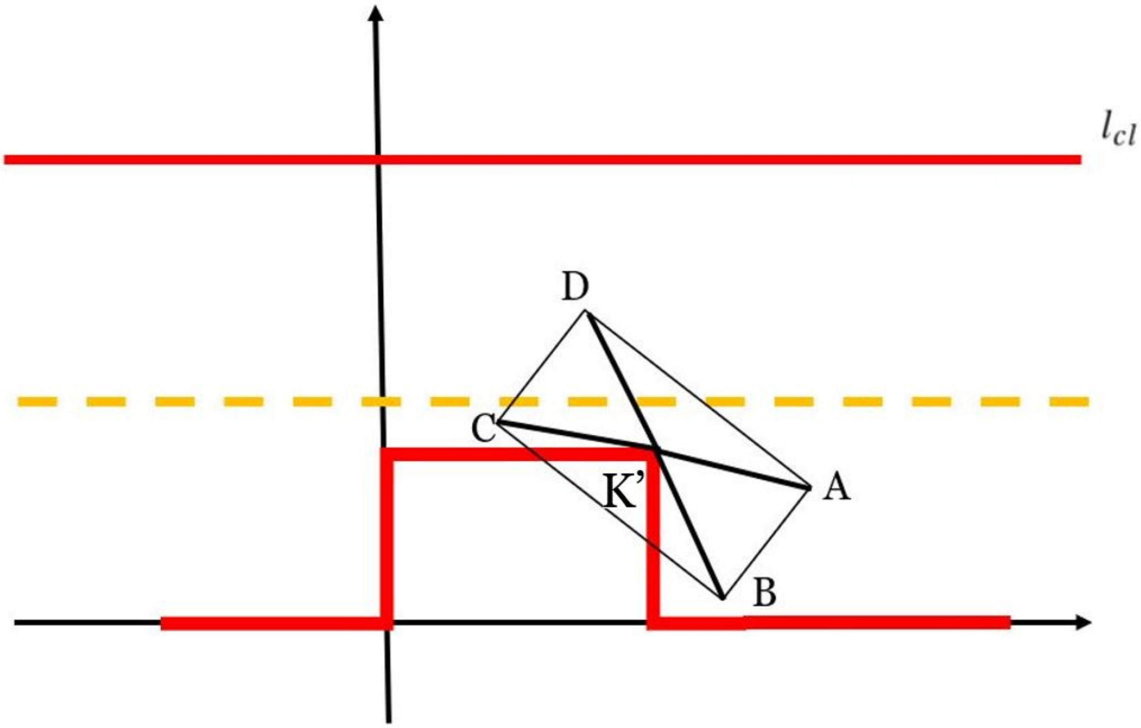

Figure 4.9: Obstacle avoidance - collision without violate position constraint

- How do parameters of tractor semi-trailer caused instability and parameters of tractor semi-trailer is the same as the dimension used in Chapter 3.
- Arrange CG position wisely can help reduce the risk of instability.

Moreover, result from Chapter 3 will provide extra valuable information for MPC in order to fill gaps in missing dynamic information. The problem described above has the following assumption and parameters:

- The highway is dry, and it can provide enough friction force.
- The controller can fully control the ego vehicle, and ego vehicle driver will not interference command of controller.
- The highway has two lanes, each lane is 3.75 meters wide.

- The highway has a speed limit at 100 km/h (approximate 28 m/s).
- The dimension of a passenger car (obstacle) is 5 meters in length and 2 meters in width.
- The absolute safety distance (inflation) is 15 meters (7.5 meters to the front of the obstacle and rear of the obstacle).
- Ego vehicle and passenger car are driving on the right lane.

Based on the methodology described in the previous section, the MPC is trying to solve a Nonlinear programming problem which is shown in Equation 4.45. In this problem, the sample time (δt) is 0.05 seconds, and the prediction horizon (N_p) and control horizon (N_c) is 30.

$$\min J = \min_{u_c} \left\| x_{t+k,t} - xdes_{t+k,t} \right\|_Q^2 + \left\| u_{c_{t+k-1,t}} \right\|_R^2 + \left\| u_{c_{t+k-1}t} - u_{c_{t+k-2,f}} \right\|_S^2$$

$$\begin{aligned} \text{s.t.} \quad & -\pi/3 \leq \theta_f \leq \pi/3 \\ & -\pi/3 \leq \theta_r \leq \pi/3 \\ & -\pi/12 \leq \gamma \leq \pi/12 \\ & -0.005 \leq \Delta u_{Steering} \leq 0.005 \\ & -0.1 \leq \Delta u_{Velocity} \leq 0.1 \\ & -0.02 \leq u_{Steering} \leq 0.02 \\ & 0 \leq u_{Velocity} \leq 30 \\ & \mathit{Position\ Constraints} \\ & \mathit{Area\ Constraints} \end{aligned} \tag{4.45}$$

4.7 Intersection Left Turn Problem

4.7.1 Introduction

Another challenging problem during daily driving is how to drive through a intersection/crossroad, especially left or right turning in a intersection. Figure 4.10 shows a typical intersection. The intersection have two directions, each direction has two lanes. With 3.5 meters road width, the lane width of the intersection is narrower than a 3.75-meter lane width of the highway since it is always built-in cities. There are two challenging points in this problem: the first one is based on the traffic rule: the vehicle should keep on the same lane before and after the left turning; the second one is the conflict between the narrow road width and the off-tracking problem caused by the different turning radius of different units of the tractor semi-trailer. These two challenging points make this problem become a meaningful segment in the autonomous driving of tractor semi-trailer.

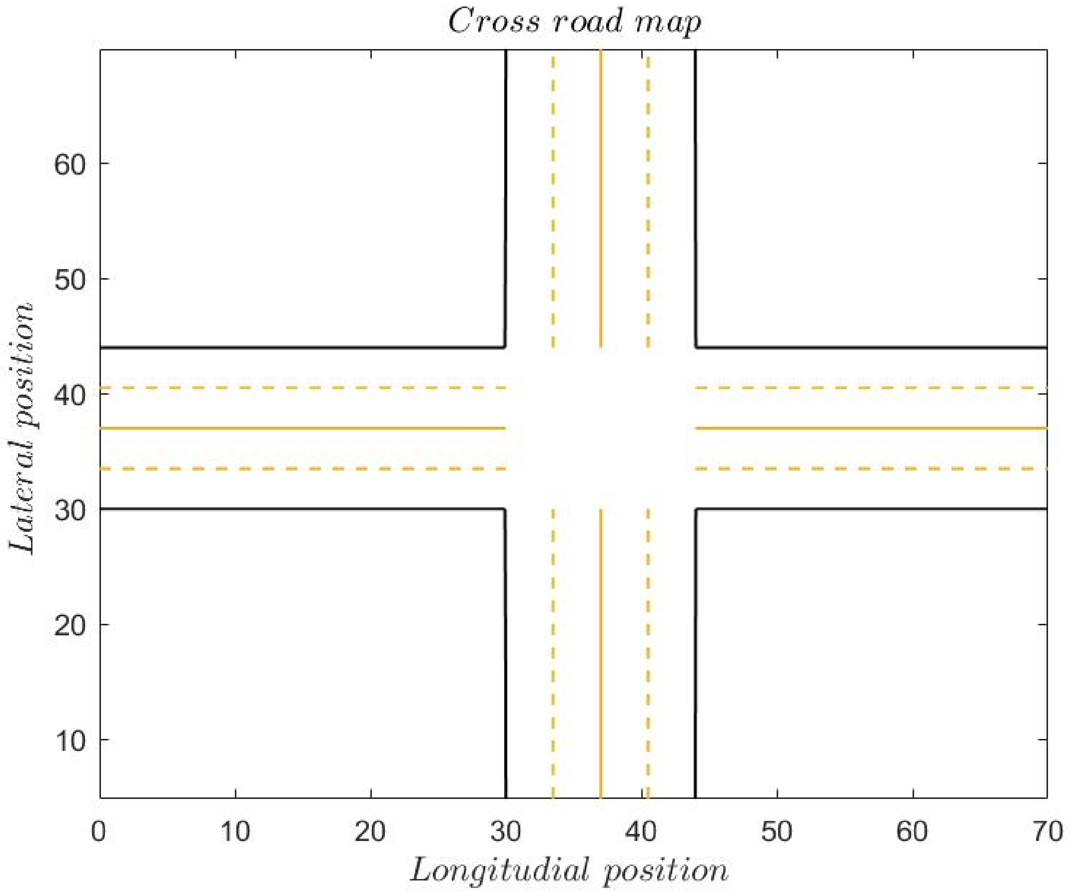

Figure 4.10: Problem formulation of left turn problem - crossroad map

4.7.2 Problem Setup

Figure 4.11 shows how this problem setups. The intersection are shown by the black line. From the starting point denoted by the blue star point, the tractor semi-trailer has an objective to arrive at which is denoted by red star point. During the left turning, there are few passenger cars stop in the opposite direction and the tractor semi-trailer must not collide with these cars. Driving through the top red line is not a wise choice since it is hard to reach the destination point without violating any traffic rule; colliding with the road boundary is not an option as well.

The problem becomes a point stabilization problem connecting to an obstacle avoidance problem. First of all, obstacles during the left turn can be divided into two categories: road and other vehicles. The red line in Figure 4.11 can wrap all obstacles during the left turning. The bottom red line represented by the $f_1(x)$ uses a similar function in the highway obstacle avoidance problem, and the only difference is on parameter within the function. Even though the function $f_1(x)$ is in the "concave" shape which closes at the bottom, it will not affect the problem setup since the tractor semi-trailer will not backward to the bottom of $f_1(x)$ during the left turning. Another contribution of the function $f_1(x)$ is that it blocks the road of the opposite direction, which will help the tractor semi-trailer not to collide with black cars shown in Figure 4.11. Second of all, the top red line is just a straight line to avoid the tractor semi-trailer colliding with the road boundary.

Similar to the obstacle avoidance problem on the highway, MPCis chosen to solve this problem. Four corners of the tractor have been defined as point A, point B, point C and point D respectively. Similarly, four corners of semi-trailer have been defined as point E, point F, point G and point H. Point M and point N are used to define two corners of the road. With all of the ten points, the area constraint can be formulated as Equation 4.46.

$$\begin{aligned}
&Area(AMB) + Area(BMC) + Area(CMD) + Area(DMA) > Area(ABCD)\\
&Area(ANB) + Area(BNC) + Area(CND) + Area(DNA) > Area(ABCD)\\
&Area(EMF) + Area(FMG) + Area(GMH) + Area(HME) > Area(EFGH)\\
&Area(ENF) + Area(FNG) + Area(GNH) + Area(HNE) > Area(EFGH)
\end{aligned} \tag{4.46}$$

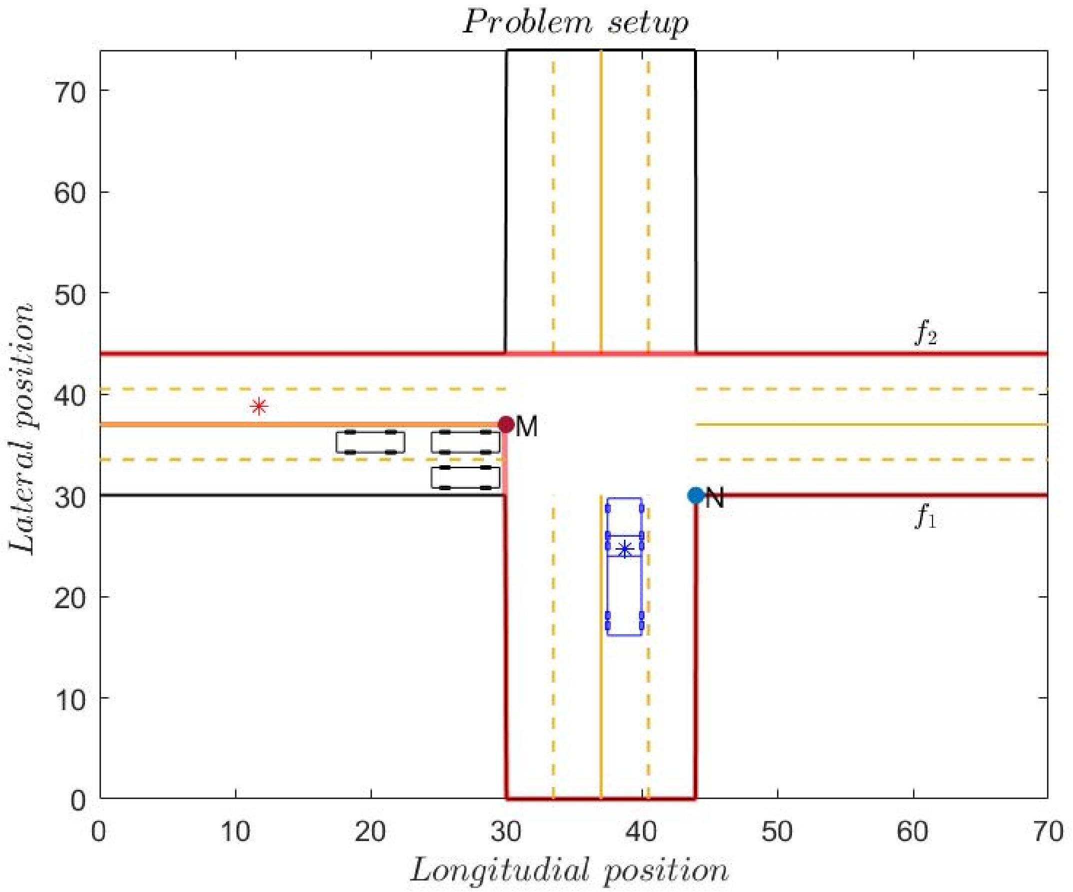

Figure 4.11: Problem formulation of left turn problem - problem setup

The position constraint is formulated as Equation 4.47.

$$\begin{array}{ll} A_y \geq f_1(A_x) & A_y \leq f_2(A_x) \\ B_y \geq f_1(B_x) & B_y \leq f_2(B_x) \\ C_y \geq f_1(C_x) & C_y \leq f_2(C_x) \\ D_y \geq f_1(D_x) & D_y \leq f_2(D_x) \\ E_y \geq f_1(E_x) & E_y \leq f_2(E_x) \\ F_y \geq f_1(F_x) & F_y \leq f_2(F_x) \\ G_y \geq f_1(G_x) & G_y \leq f_2(G_x) \\ H_y \geq f_1(H_x) & H_y \leq f_2(H_x) \end{array} \tag{4.47}$$

4.7.3 Problem Formulation

With the information mentioned above, the OCP problem can be formulated as Equation set 4.48. The dynamic constraint is based on the kinematic equation, and the 4th order Runge-Kutta method will benefit it to reduce errors during the calculation. Algebraic path constraint includes area constraint and position constraint represented by g in Equation set 4.48. Finally, the state variable constraint will be based on dynamic analysis.

$$\begin{gathered}
\min J_N(x,u) = \min \textstyle\sum_{k=0}^{N-1} \ell\left(x_u(k), u(k)\right) \\
\text{s.t.} \quad \boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k), \boldsymbol{u}(k), h) \\
x_u(0) = x_0 \\
u(k) \in U, \forall n \in [0, N-1] \\
x_u(k) \in X, \forall n \in [0, N] \\
g(x(k), u(k), h) \in G, \forall n \in [0, N]
\end{gathered} \tag{4.48}$$

In the MPC, the position of front-axle of the tractor will be tracked.

4.8 Summary

In this chapter, the kinematic model has been selected from an existing paper. After that, the advanced control method - MPC has been introduced. From the general idea of the MPC to the internal relationship with the Optimal Control Problem, it gives out a unique understanding of this advanced control approach.

A general idea on how to solve an MPC problem has also been introduced in the following section: how to transfer an Optimal Control Problem to a Nonlinear Programming Problem, how to use the Runge-Kutta method to improve the accuracy of the controller and how to solve a Nonlinear Programming Problem. Finally, an NLP problem solver called "CasADi" has been presented in the same section.

The MPC has been applied to the obstacle avoidance problem in the following section. Starting from two obstacle avoidance approaches, two problems including highway obstacle avoidance problem and intersection left-turning problem have been proposed, analyzed and formulated to an MPC problem. From these two approaches, they also show people a brief overview of the difference between point stabilization problem and trajectory tracking problem.

In the next chapter, simulations of these two problems will be presented.

Chapter 5

Applications and Simulation

5.1 Introduction

In previous chapter, two different problems has been mentioned and analyzed. Problems already set up based on the obstacle avoidance algorithm and control algorithm in Chapter 4. In this chapter, simulation results of two problems will be presented and analysed.

In section 5.3, simulation results of the intersection left turn problem has been presented. This simulation not only uses the MPC but also use Optimal Control. The results of the two control approaches will be compared in this section. Section 5.2 will describe the highway obstacle avoidance scenario. Different from the urban environment, the highway scenario is more challenging and have stricter constraint during obstacle avoidance.

5.2 Highway Obstacle Avoidance

5.2.1 Introduction

In this simulation, three common scenarios on the highway have been used to test the performance of the algorithm. All of these three simulations are based on the controller mentioned in Chapter 4. The controller uses a kinematic model to calculate the steering angle and longitudinal velocity as input for the tractor semi-trailer.

Based on the damage that obstacles would make if collided, the obstacle on the road can be classified as crossable obstacles and non-crossable obstacles. Crossable obstacle refers to small obstacles such as bottles on the road, meanwhile, non-crossable obstacle refers to vehicles, pedestrians, and other obstacles that would cause life or property loss. Since detecting and avoiding crossable obstacles are unnecessary tasks, the obstacle avoidance will not consider any crossable obstacles on the road. Moreover, a series of large steering inputs will bring instability risks to the tractor semi-trailer, hence the obstacle avoidance action should not be considered when it is not necessary. Based on these two reasons, in the simulation section, every obstacle has been assumed to be non-crossable. A passenger car is fairly representative of the non-crossable obstacle.

5.2.2 Static obstacle

The highway with a static obstacle might be one of the most common scenarios during daily driving. The static obstacle can be any non-crossable obstacle without velocity: the flat tires, the broke down vehicles and many other obstacles. Colliding with these non-crossable static obstacles would cause serious results, and a controller that can help the tractor semi-trailer handle this scenario becomes necessary.

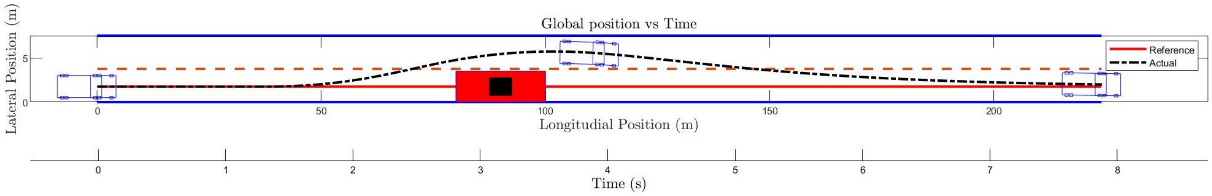

Figure 5.1: Trajectory during obstacle avoidance - static obstacle

In this simulation, the tractor semi-trailer is driving on a highway at 100 km/h, and a passenger car which is 5 meters long and 2 meters wide stops in front of it. The obstacle has inflation which dimension is 20 meters by 3.5 meters. The tractor semi-trailer has been allowed to exceed the speed limit of the highway a little bit during obstacle avoidance. The whole simulation is lasted for 15 seconds. Only fist 8 seconds shown in the Figure 5.1 and Figure 5.2

Figure 5.1 shows the trajectory of the tractor semi-trailer during obstacle avoidance. The red line is the reference line for trajectory tracking, which is located in the middle of the right lane. The black dash line is the actual trajectory of the tractor semi-trailer since the tractor semi-trailer must track the reference line under constraint. Figure 5.2 shows steering input and velocity input during obstacle avoidance. The trapezoidal-shaped steering input is caused by the maximum steering input constraint. The control input shows the fact

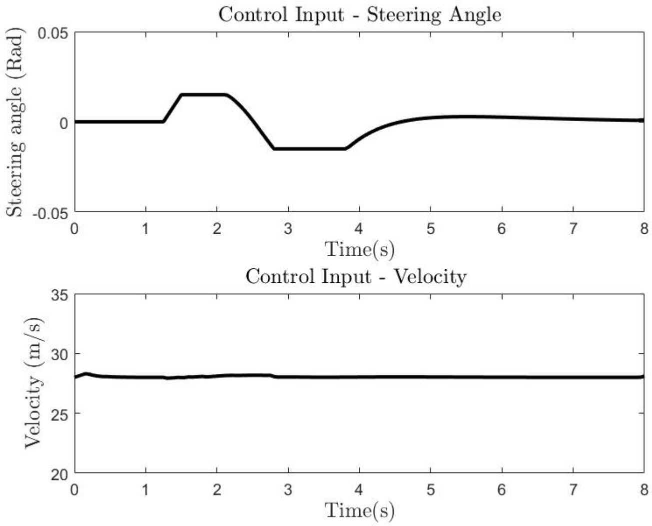

Figure 5.2: Control input during obstacle avoidance

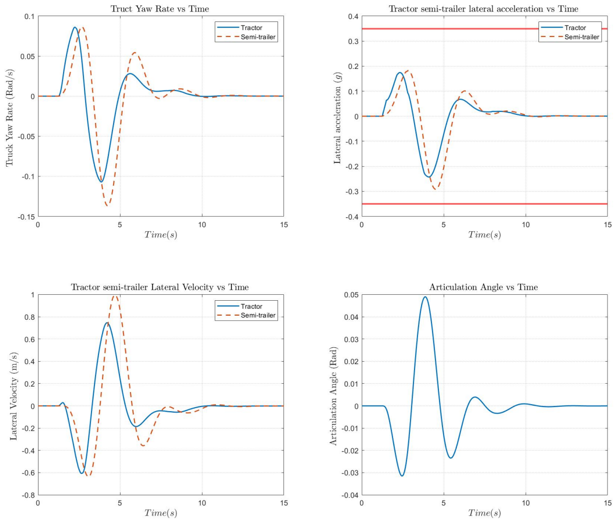

Figure 5.3: Dynamic response of the tractor semi-trailer

that the tractor semi-trailer uses a critical strategy to avoid the obstacle and the steering input provides evidence to prove this statement: “the tractor semi-trailer takes the largest possible steering increment rate and reaches the maximum steering input". The tractor semi-trailer avoids the obstacle successfully, and get back to the right lane after obstacle avoidance. In this scenario, the obstacle is static and if there are no other obstacles, this problem will also be able to be solved by OCP. However, the following scenarios cannot be solved by OCP and must be solved by MPC.

The Figure 5.3 shows the dynamic response during the obstacle avoidance, lateral acceleration of the tractor semi-trailer stay within the threshold of roll instability. And the amplify phenomena can also be detected from the subplot of lateral acceleration. That proves the algorithm of the obstacle avoidance can help the tractor semi-trailer dealing with static obstacles on highway.

5.2.3 Moving obstacle with constant velocity

The controller is trying to solve an obstacle avoidance problem on the highway. In order to use the advantage of the MPC algorithm and be close to reality, a moving vehicle has been used as an obstacle. The second case is an overtaking problem. In this scenario, a passenger car is driving in front of the tractor semi-trailer with an extremely low velocity and the tractor semi-trailer will apply an overtaking action. When a tractor semi-trailer is driving on the highway, it has plenty of reasons to overtake. First of all, driving at a constant speed is fuel-efficient. Second of all, with a heavy semi-trailer, it will have a risk to decelerate. After deceleration, the tractor semi-trailer will take a large amount of time to revive to the initial speed. Last but not least, if the tractor semi-trailer chooses following the passenger car instead of overtaking it, it will waste time and cause a delay in delivery. With reasons stated above, overtaking is always a wise choice and it becomes a must-have function of an autonomous vehicle.

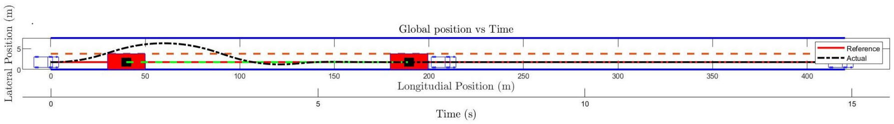

Figure 5.4: Trajectory during obstacle avoidance - moving obstacle with constant velocity

Figure 5.4 shows the trajectory of the tractor semi-trailer during obstacle avoidance. The black rectangular represents a passenger car, and the red rectangular represents the safety distance. The red line is the reference trajectory of the tractor semi-trailer, the black dash line is the actual trajectory of the tractor semi-trailer and the green dash line is the trajectory of the obstacle. Figure 5.5 shows control input during obstacle avoidance.

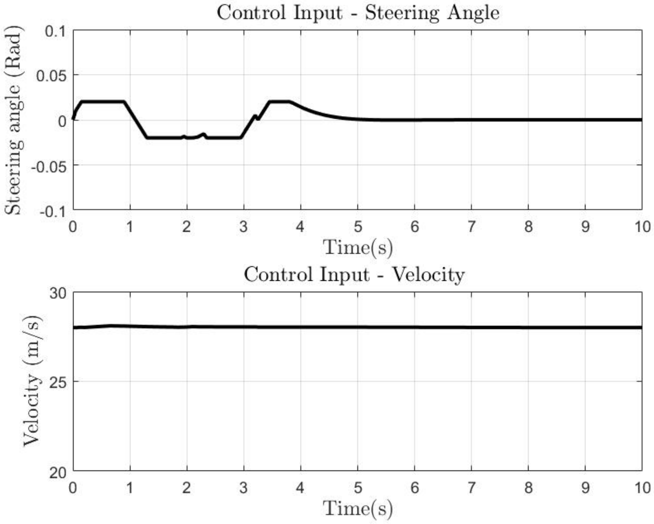

Figure 5.5: Control input during obstacle avoidance

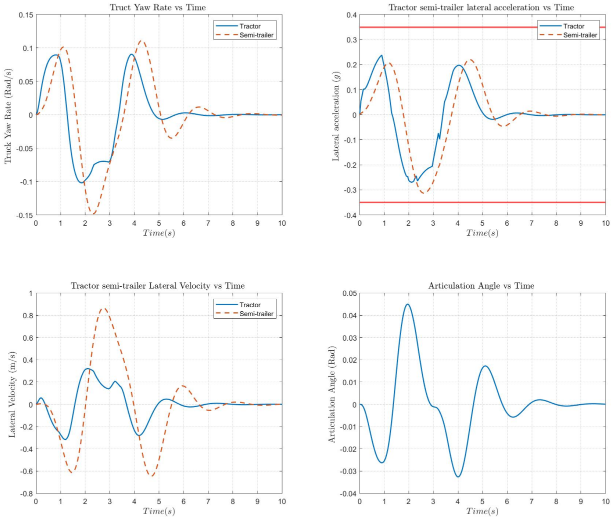

Figure 5.6: Dynamic response of the tractor semi-trailer

In this problem, optimal control will not be used to replace MPC since the position of the obstacle is continuously changing. Optimal control can only be planned for a time slot with a constant environment, in other words, states of obstacles cannot change. The problem must adapt to the new environment, which is fundamental to the MPC algorithm. From this scenario, the difference between optimal control problem and MPC becomes more clear: the MPC, from a macro scale, is equivalent to a series of optimal control problems, and it is continuously updated and forms a new optimal control problem at each new time step. Only by only taking the first control input calculated by optimal control problem, formulating and solving another optimal control problem, can MPC achieve "rollover" in its algorithm. Combined with conclusions conducted from the previous chapter, MPC must be used in the planning and control of the vehicle since the environment is continuously changing.

Figure 5.6 is the dynamic response of the tractor semi-trailer with given control input during obstacle avoidance when it avoids colliding with a moving obstacle with constant velocity. The simulation result shows that the tractor semi-trailer remains stable during obstacle avoidance.

5.2.4 Moving obstacle with emergency deceleration

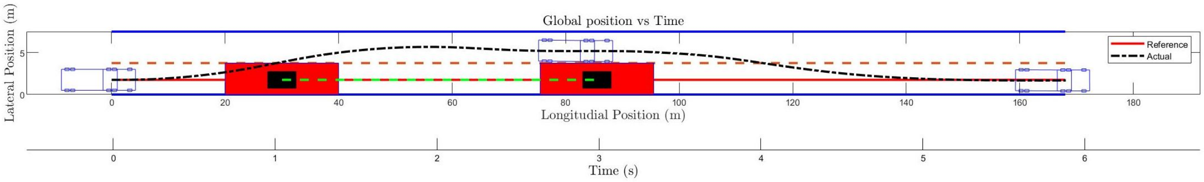

Figure 5.7: Trajectory during obstacle avoidance - moving obstacle with emergency deceleration

The third scenario is also common on the highway. Considering the following scenario: a tractor semi-trailer (ego vehicle) is driving on the highway, and it follows a passenger car that is driving in the middle of the lane and in front of the ego vehicle. Suddenly, the passenger car emergency decelerates with a large deceleration which exceeds the maximum deceleration of the tractor semi-trailer. When a driver of the tractor semi-trailer faces such a scenario, he/she barely can give the desired input to the tractor semi-trailer. The driver has to make a decision within the second, and the subconscious command is decelerated the tractor semi-trailer and change to another lane. These actions will cause two serious problems. The first problem is that the large deceleration will cause yaw instability - jackknifing. The semi-trailer has larger momentum of inertia, it will push the tractor and form a "V" shape during braking. Furthermore, large steering input will easily expose tractor semi-trailer to the risk of rollover. With these two potential risks that might be caused by the human driver, the driver of the tractor semi-trailer barely survives in such a critical scenario. The autonomous tractor semi-trailer which equipped with obstacle avoidance function can help people handle such critical situation and increase survive possibilities of the human driver. In this situation, the tractor semi-trailer has to figure out a path to avoid the passenger car in front of it, which is using the other lane to overtake, and remaining its stability.

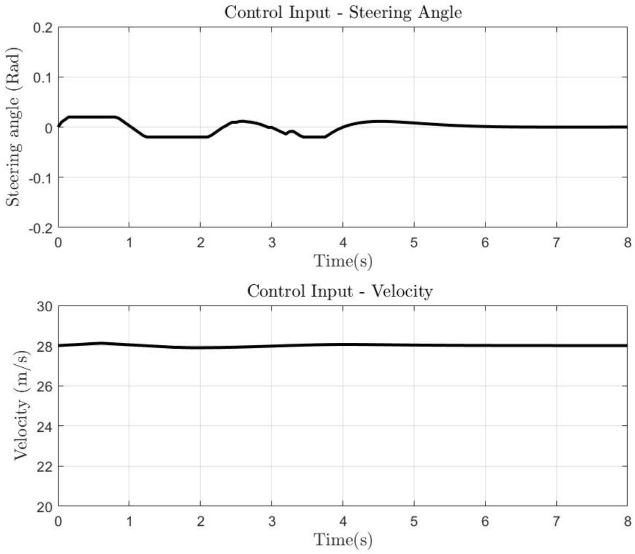

Figure 5.8: Control input during obstacle avoidance

Figure 5.7 shows the simulation, and Figure 5.8 shows control input during the obstacle avoidance. The simulation takes 15 seconds. During the simulation, the tractor semi-trailer driving along a straight line after 8 seconds of the simulation. Figure 5.7 only shows the most important time interval, in which the path tractor semi-trailer has a significant change.

The passenger car in front of the tractor semi-trailer is driving at 30 m/s initially, however, it decelerates at -8 m/s^2. The tractor semi-trailer cannot decelerate with the same deceleration for two reasons: the momentum of a fully-loaded semi-trailer will need a lot of effort to decelerate and decelerating with a large deceleration will bring yaw instability. The only option for the tractor semi-trailer is to change the lane and overtake the obstacle.

Figure 5.9 shows the dynamic response during obstacle avoidance in the current sce-

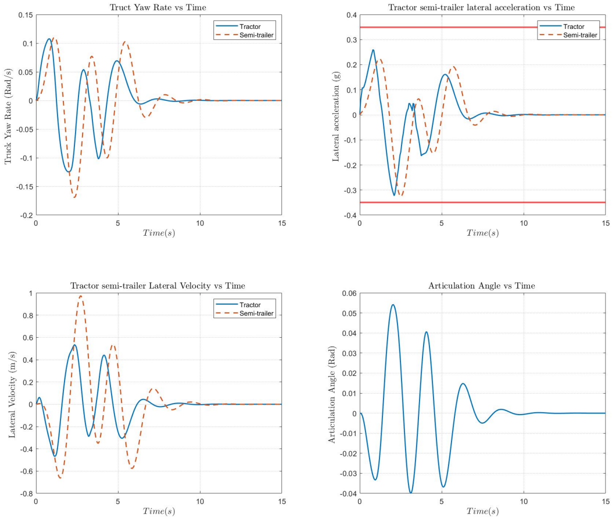

Figure 5.9: Dynamic response of the tractor semi-trailer

nario. Since the tractor semi-trailer did not change its speed too much, in other words, use a braking strategy, the risk of "Jackknifing" has been eliminated. Furthermore, the steering input restrained to prevent roll instability. By taking these two actions, the autonomous tractor semi-trailer avoid the obstacle and remain stability.

5.3 Intersection Left Turn

5.3.1 Introduction

Left turn at the intersection is a challenging task for drivers of tractor semi-trailers. Different from the passenger vehicle, the size of the tractor semi-trailer can bring various potential problems. Even an experienced driver of the tractor semi-trailer could make mistakes during the left turn at the intersection.

The simulation of intersection left turn problem is set up in a city environment. The tractor semi-trailer on the left lane of the road is heading to the north at the beginning and switching to the left lane heading to the west. However, the tractor semi-trailer cannot turn to the left since there are three vehicles (obstacles) on the counter-direction of the same road. The off-tracking phenomena will bring the collision risk to the tractor semi-trailer. In order to turn left without any collision, the tractor semi-trailer has to figure out a different path.

5.3.2 Left turn

The simulation result shown in Figure 5.10 and Figure 5.11. The problem has been designed as a point stabilization problem. The tractor semi-trailer will track the hitch point of it which starts at (38.5, 25) and terminate at (0, 38.5). Three vehicles with dimension 2 meters width and 5 meters length parking at the left side and waiting for the traffic light. In such a scenario, the tractor semi-trailer has the right of the road at the intersection. Figure 5.12 shows the control input during the left turn for MPC, and Figure 5.13 shows the control input during the left turn for optimal control, respectively.

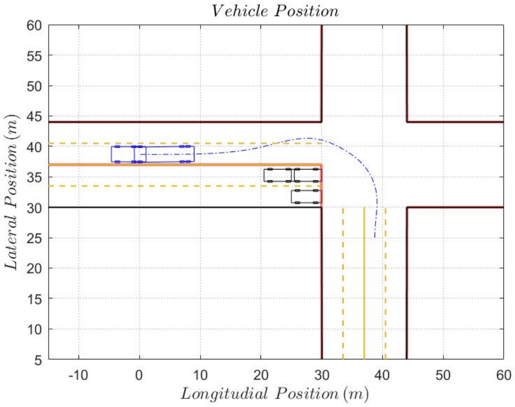

Figure 5.10: Trajectory during intersection left turn - Model Predictive Control

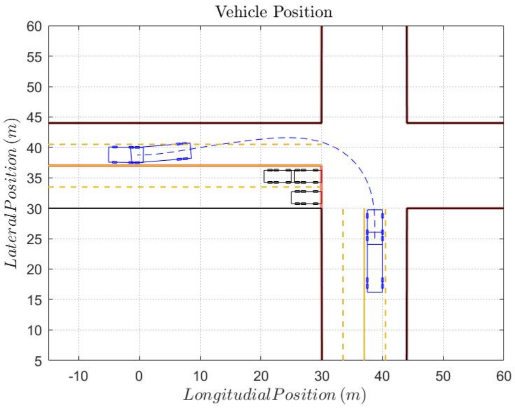

Figure 5.11: Trajectory during intersection left turn - Optimal Control

In the simulation, there are two types obstacle: vehicle and road boundary. There are two approaches to avoid collision and to pass through the intersection. First approach is driving forward as normal passenger car and backward a little bit to eliminate risks bring by off-tracking. Second approach is heading to the right lane and change the lane to left immediately in order to avoid block right lane of the road. Even though first approach will not block right lane of road, a busy intersection will not allow a tractor semi-trailer backward in the middle of road. However, compare to first approach, second approach is more feasible for the scenario in this simulation environment since it will take less time.

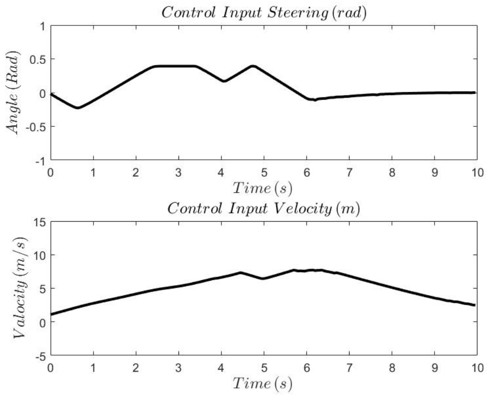

Figure 5.12: Input of tractor semi-trailer during intersection left turn - Model Predictive Control

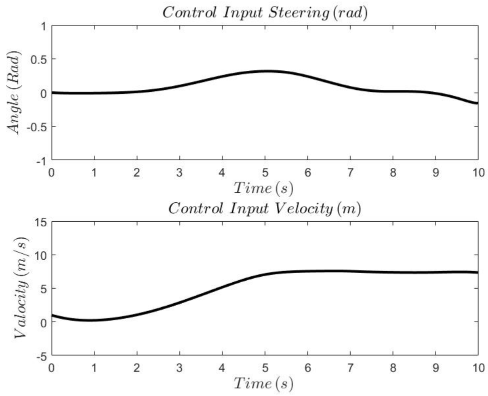

Figure 5.13: Input of tractor semi-trailer during intersection left turn - Optimal Control

In the MPC problem, the trajectory of the tractor semi-trailer during the left turn shown in Figure 5.10. The tractor semi-trailer avoids collision with three vehicles on the left by driving to its right side a little bit at the beginning and heading to the right lane of the target road. By this action, the tractor semi-trailer can offset the effect bringing by off-tracking. After passing the intersection, the tractor semi-trailer drives back to the left lane and give the right lane to other vehicles. In the optimal control problem, the trajectory of the tractor semi-trailer similar to which in MPC problem. Even though two bodies of the tractor semi-trailer do not completely parallel to the road, the control input much smoother than that in the MPC problem. That because MPC does not have as much prediction horizon as the optimal control. This revealed the pros and cons of these two approaches.

- MPC can handle a complex and changing environment easily, however, Optimal Control can only solve a problem in the environment with constant states.
- Optimal control has a more prediction horizon than the MPC, which can make the control input become much smoother.
- MPC has a fewer prediction horizon, it can adapt to the environment and get a more accurate control result than optimal control result.
- Once the control input has been decided, optimal control cannot change the control input, while MPC only uses the first step of the control input and recalculate in next

time step.

- MPC generally takes more time than optimal control when solving a problem.

In the simulation, both MPC and optimal control can solve the intersection' left turn problem. Furthermore, the difference between thees two approaches has been concluded. In the following section, another problem will be solved by MPC, it also will give people a deepen understanding of the difference between these two approaches.

5.4 Summary

In this chapter, simulation for two problems proposed by the previous chapters has been performed. In the intersection left turn problem, two control approaches including MPC and optimal control has been used to solve the problem. Differences between MPC and optimal control have also been discussed in this section.

Following the highway obstacle avoidance problem, three scenarios have been designed, and each one of them is a common scenario that tractor semi-trailer has to be faced in daily driving: static obstacle, moving obstacle with a constant low speed and moving obstacle with emergency deceleration. The simulation has proved that the MPC and obstacle avoidance algorithms used by this thesis are able to handle these scenarios. In simulations of these problems, it also shows when should MPC and optimal control be used by the controller.

Chapter 6

Conclusions and Future Work

This thesis mainly solved a crossroad left turn problem with a optimal control and a obstacle avoidance algorithm. Different from normal passenger vehicles, states transfer of tractor semi-trailer is much more complicated. However, the thesis start from analysis the dynamic of the tractor semi-trailer, and use conclusions conducted from dynamic analysis to design a controller to solve the problem.

6.1 Conclusions

At the beginning of dynamic respond analysis, the existing three degree of freedom dynamic model has been updated with a more accurate tire model to handle various simulation parameters. The friction circle is added to the dynamic model to make sure the analysis result is accurate enough. After that, three velocity has been chosen: low velocity for driving in a narrow space, moderate velocity for driving in city and high velocity for driving on highway. The simulation also consider yaw instability and roll instability. From the dynamic analysis, the amplify phenomena of the tractor semi-trailer: there existing a critical velocity; and the magnitude of yaw rate and lateral acceleration on semi-trailer smaller than which on tractor before the critical velocity, while the magnitude of yaw rate and lateral acceleration on semi-trailer larger than which on tractor before after critical velocity. Ratio analysis also concluded how "large" is the effect.

After getting the dynamic responds of the tractor semi-trailer with a specific parameter, a parameter study on the tractor semi-trailer is performed. By changing the semi-trailer's weight and CG position, more conclusion on phase delay has been conducted. More importantly, parameter study also provide information on how to assign parameter to tractor semi-trailer in order to reduce instability risk. In other words, a tractor semi-trailer's CG position should be close to rear axle of trailer to avoid potential risk on yaw instability.

In the controller development part, an advanced control approach called MPC has been used to aid controller design. In the same chapter, the fundamental of MPC, optimal control problem has been presented. MPC has been chosen for two reasons, first of all, it can accomplish planning and control while performing obstacle avoidance. Second of all, it can adapt for a changing environment. Some important segments of the MPC includes cost function, dynamic constraint, algebraic path constraint, control input constraint and state variables constraints has been introduced. Finally, two important obstacle avoidance problems has been formulated with MPC and obstacle avoidance algorithm. In the problem formulation part, the difference between point stabilization and trajectory tracking has been discussed. In point stabilization problem, tractor semi-trailer only track one stable point, while in trajectory tracking problem, tractor semi-trailer track different points at different time step.

Finally, this thesis shows simulation results on two problems. The cross road left turn problem is a novel problem and this is first time it has been solved by optimal control algorithm. Another problem is the classical highway obstacle avoidance problem, where this thesis design three most common highway scenarios to solve. From these four simulations, MPC and obstacle avoidance has been proved that they can handle these problems. Furthermore, they gives people deeper understanding on the difference between MPC and optimal control. MPC can be divided into a class of optimal control problem, where the states are changing. Controller design via MPC is better than design via optimal control.

6.2 Future Work

Improvement on Dynamic Model

In the dynamic analysis part, a three degree of freedom dynamic model has been selected. The roll instability only decided by the lateral acceleration. To achieve a better result, a complex dynamic model with roll plane information is developed. Furthermore, a dynamic model with pitch plane information can be use to analysis the riding comfort. In the yaw instability analysis, the dynamic model can even add a braking function on the semi-trailer. This function is already equipped on some modern tractor semi-trailer, and can help tractor semi-trailer get rid of potential risk on yaw-plane instability.

Oil/Chemical tank semi-trailer

There exist one category semi-trailer that use to transport oil and other liquid chemicals. Since the liquid can move in the tank without any effort, the weight distribution of the oil tank semi-trailer can continuously change with the time. That will add more uncertainty to the stability of the semi-trailer. Also, one tractor semi-trailer might have multiple destinations. The weight of the semi-trailer will change after passing by each destination. These problems easily exposed tractor semi-trailer to the risk of yaw instability.

In previous chapters, the parameter of a tractor semi-trailer assumes to be a constant number. So, the dynamic result conduct from Chapter 3 cannot use to handle such a problem. After the weight of the semi-trailer changed, the control strategy will also change. That becomes a challenging problem and can use as a potential future work.

There are three potential possible approaches to solve the problem. First of all, using a more complex dynamic model with roll plane information and pitch plane information to repeat the dynamic analysis process. Another approach might directly use a dynamic model to control the tractor semi-trailer. With the help of high degrees of freedom dynamic model, it easy to detect potential instability risk. However, that is a trade-off between computational efficiency and control accuracy. The third approach could be analysis the dynamic of the chemical tank semi-trailer further, find more dynamic relationships between tractor and semi-trailer. After the tractor semi-trailer can seem as one body. These potential possible approaches are based only on the control algorithm of this thesis. It still might exist other solutions based on other algorithms.

Environment

In the cross road left turn problem, the tractor semi-trailer has the right of the road. However, in some cross road, vehicles from counter direction has the right of the road. The tractor semi-trailer has to yield for other vehicles during this left turn, which makes the left turn problem more complex and harder to solve.

In the highway obstacle avoidance problem, it also can combined different obstacles in a single simulation. For example, a stable obstacle on right lane and a moving obstacle on the other lane. Furthermore, the critical obstacle avoidance scenario on the highway is also worth to research.

The cross road left turn problem can be updated to a narrow alley left turn problem. In this new problem, the road can be one lane. The tractor semi-trailer must reverse before pass the narrow alley.

Narrow alley problem

In the thesis, the controller shows its potential to help a tractor semi-trailer avoid obstacles and drive within an intersection. There exist another challenging problem: Narrow alley problem. The form of a narrow alley is similar to which of an intersection road, but it might only have one lane. Driving an oversize tractor semi-trailer in a narrow alley could be a challenging task. The tractor semi-trailer needs reverse in the alley. This problem can solve by formulating a similar problem to the intersection problem.

Experiment

The algorithm can also be used on a real tractor semi-trailer to verify the feasibility of on-line control in the future. The advantage of using MPC is that it can combine planning and controlling. The planning stage can omit with the help of MPC. First of all, the algorithm can integrate with other autonomous driving segments such as perception and decision making in Robot Operating System (ROS). By connecting ROS with truck-sim via the Simulink function of Matlab, the system can get feedback similar to which in the real-world. The algorithm can test in a close road from the simplest scenario: obstacle avoidance during low-speed driving. In the future, the algorithm can help truck drivers avoid obstacles on the highway under some critical scenarios. Furthermore, this algorithm will save lives by giving truck drivers more response time during critical scenarios.

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APPENDICES

Appendix A

Safety Envelope

A.1 Safety Envelope

A.1.1 Introduction

A safety envelope is an over-approximation of the system, and it describes states that are reachable under nominal conditions.[22] There are some concepts for safety envelope. Most of them consider the yaw rate and yaw angle [39][14]. The safety can be continuous change based on the yaw angle on the different time steps. Each unit can be used a simple shape to represent it. After considering the off-tracking problem for the tractor-trailer, one shape is not enough due to the risk of overestimation. A new concept of safety envelop is needed.

A.1.2 Safety Envelope for Tractor-Semitrailer

The safety envelope should contain all essential states of the tractor semi-trailer system. These essential states included the dimension of the system(CG position, wheelbase, width, hitch point position), yaw angles, yaw rates, articulation angle, speed and acceleration. Besides, it must solve the off-tracking problem related to the articulation angle. Furthermore, the safety envelope is dynamic, it can adaptive to the system's current states. Finally, it must make sure to waste as few spaces as possible.

- The safety envelope considers the off-tracking/sway.
- These phenomena cause a different turning radius.

- Safety Envelope is trying to calculate the difference turning radius between the head and tail of tractor and trailer to predict the safety distance.

‘

In order to calculate turning radius on different points on vehicle, the following equation has been developed. First of all, a passenger car(Shown in Figure A.1) used to implement the equation. x_p and y_p are the coordinate of a point "p" on the vehicle. At $t = 0$, the longitudinal and lateral velocity can be express by Equation A.1. At $t = \delta t$, longitudinal and lateral velocity of "p" can be express by Equation A.2. The increment of velocity after δt can be found through project velocity vectors at $t = \delta t$ on to coordinate. So, Equation A.3 can be implemented. Combine Equation A.1, A.2 and A.2, Equations A.4 and A.5 can be implemented. After take derivative, the turning radius of vehicle can be easily calculated.

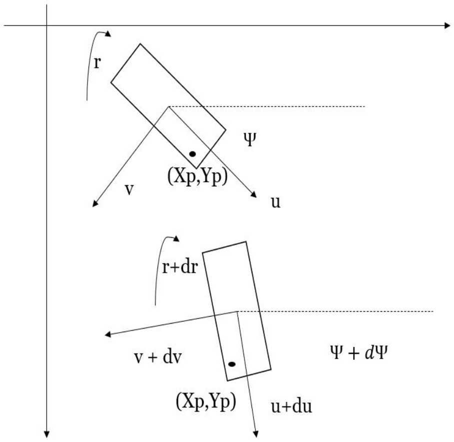

Figure A.1: Equation Implementation

$$u_p = u - y_p \cdot \Omega \qquad v_p = v + x_p \cdot \Omega \tag{A.1}$$

$$u'_{\mathrm{p}} = (u + \delta u) - y_{\mathrm{p}}(\Omega + \delta\Omega) \qquad v'_{\mathrm{p}} = (v + \delta v) + x_{\mathrm{p}}(\Omega + \delta\Omega) \tag{A.2}$$

$$\delta u_p = u'_p \cos\delta\psi - v'_p \sin\delta\psi - u_p \qquad \delta v_p = u'_p \sin\delta\psi + v'_p \cos\delta\psi - v_p \tag{A.3}$$

$$\delta u_p = \{(u+\delta u) - y_p(\Omega+\delta\Omega)\}\cos\delta\psi - \{(v+\delta v) + x_p(\Omega+\delta\Omega)\}\sin\delta\psi - (u - y_p\Omega) \quad \text{(A.4)}$$

$$\delta v_p = \{(u+\delta u) - y_p(\Omega+\delta\Omega)\}\sin\delta\psi - \{(v-\delta v) + x_p(\Omega+\delta\Omega)\}\cos\delta\psi - (v - x_p\Omega) \quad \text{(A.5)}$$

Figure A.2 shows the safety envelope for the tractor semi-trailer, the envelope is the trapezoidal shape, it has two parts: a rectangular to fit the shape of the tractor or semi-trailer, and a right triangle to describe the dangerous area. The height of the triangle is the length of the tractor or semi-trailer, and the width of the triangle is defined as ”inflation”. For a semi-trailer, inflation is the difference between the turning radius of point a and point b. For tractor, the inflation is the difference between these two points on tractor plus the inflation of semi-trailer respectively. Equation A.6 and A.7 describe this process.

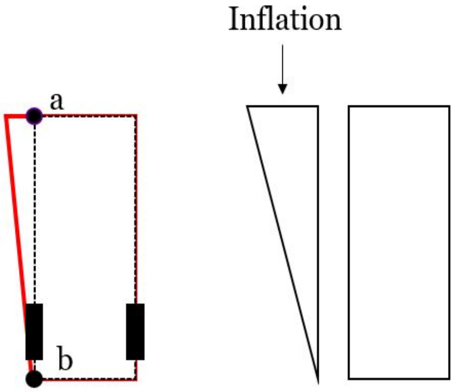

Figure A.2: Safety Envelope for tractor semi-trailer

Figure A.3 shows the simulation result for the safety envelope. This envelope used for low-speed scenarios (the off-tracking in high-speed scenario is really small, the envelope close to a rectangular.) The tractor semi-trailer in the simulation driving at $5m/s$ and has a constant steer input. The body of the tractor semi-trailer does not collide with black obstacles since the black obstacle outside the envelope. However, the red obstacle inside the envelope at the beginning and it collided with the semi-trailer.

$$Trailer_{inflation} = \Delta R_{Trailer} \quad \text{(A.6)}$$

$$Tractor_{inflation} = \Delta R_{Tractor} + \Delta R_{Trailer} \quad \text{(A.7)}$$

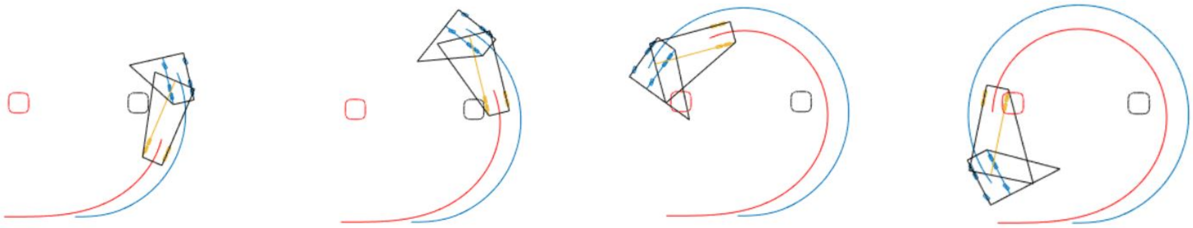

Figure A.3: Safety Envelope for tractor semi-trailer driving at $5m/s$

Appendix B

Weighting Matrix

B.1 Weighting Matrix for MPC Cost Function

B.1.1 Cross Road Problem

$$Q = \begin{bmatrix} 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.01 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{B.1}$$

$$R = \begin{bmatrix} 2e^{-5} & 0 \\ 0 & 0 \end{bmatrix} \tag{B.2}$$

B.1.2 Highway Obstacle Avoidance

$$Q = \begin{bmatrix} 0.01 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{B.3}$$

$$R = \begin{bmatrix} 100 & 0 \\ 0 & 0.01 \end{bmatrix} \tag{B.4}$$

$$S = \begin{bmatrix} 100 & 0 \\ 0 & 0.1 \end{bmatrix} \tag{B.5}$$