

Astra Version 1.0: Evaluating Translations from Alloy to SMT-LIB

Ali Abbassi¹, Nancy A. Day¹, and Derek Rayside²

¹ David R. Cheriton School of Computer Science, University of Waterloo, Waterloo,
ON, Canada {aabbassi, nday}@uwaterloo.ca

² Department of Electrical and Computer Engineering, University of Waterloo,
Waterloo, ON, Canada drayside@uwaterloo.ca

Abstract. We present a variety of translation options for converting Alloy to SMT-LIB via Alloy’s Kodkod interface. Our translations, which are implemented in a library that we call Astra, are based on converting the set and relational operations of Alloy into their equivalent in typed first-order logic (TFOL). We investigate and compare the performance of an SMT solver for many translation options. We compare using only one universal type to recovering Alloy type information from the Kodkod representation and using multiple types in TFOL. We compare a direct translation of the relations to predicates in TFOL to one where we recover functions from their relational form in Kodkod and represent these as functions in TFOL. We compare representations in TFOL with unbounded scopes to ones with bounded scopes, either pre or post quantifier expansion. Our results across all these dimensions provide directions for portfolio solvers, modelling improvements, and optimizing SMT solvers.

1 Introduction

The Alloy Analyzer [12] is a well-used formal analysis tool because of its high-level modelling language based on first-order logic and sets, and because of its convenient tool support. Its tool support is easy-to-use in part because its analysis is fully automated, which is possible because it checks only for sets of finite scope. Finite model finding provides users with quick results for small size problems when debugging their models. However, as verification problems become more detailed or the scopes become larger, the analysis engine of Alloy called Kodkod [17] reaches performance limitations. The Kodkod engine treats all predicates and functions as relations and implements Alloy’s typing system via predicates. Kodkod expands quantified formulas over finite domains and then passes the problem to a SAT solver.

SMT (satisfiability modulo theories) solvers [5] continue to improve in performance. These are solvers for first-order logic (FOL) formulas enriched with decision procedures for specific datatypes such as equality and linear arithmetic. Even for formulas outside of a decidable subset of FOL, an SMT solver can often reach a conclusion of either a satisfying solution or that no solution exists.

In this paper, we investigate the performance of number of translation options for converting Alloy to typed FOL (TFOL). We call our library Astra (Alloy to SM-T-LIB translation). To enable future integration with the Alloy Analyzer, we start from the Kodkod interface. We compare using only one universal type to recovering Alloy type information from the Kodkod representation and using multiple types in TFOL. We compare a direct translation of the relations to predicates in TFOL to one where we recover functions from their relational form in Kodkod and represent these as functions in TFOL. We compare representations in TFOL with unbounded scopes to ones with bounded scopes, either pre or post quantifier expansion. TFOL is represented as formulas in the SMT-LIB format. We run our performance tests using the SMT solver Z3 [14] to compare the results of all of our translation options to each other and Kodkod. We categorized our tests by characteristics of the model such as depth of quantifiers, number of types, the use of join, number of functions, *etc.* and discuss whether there are any meaningful correlations between model characteristics and our translation options.

There are several previous efforts at translating Alloy to SMT solvers or other automated solvers (*e.g.*, [8,9,18]) for the purposes of analyzing Alloy models of unbounded scope particularly for built-in Alloy types. Our translation options cover some of this work, however, none of the previous efforts compare different translation options or try to correlate them with model characteristics. Also, we start from the Kodkod interface for ease of future integration with the Alloy Analyzer. There have also been efforts to create theories for finite model finding in SMT solvers (*e.g.*, [2,15]), but these have not yet been linked with Alloy.

Our results are useful to anyone who uses Alloy for verification. We have not yet implemented transitive closure, cardinality of sets, or support for built-in Alloy types. Our contribution comparing all of these translation options provides directions for portfolio solvers, modelling improvements, and optimizing SMT solvers.

2 Background

Alloy. Figure 1 shows an example of an Alloy model that we use for illustration throughout the paper. Signatures are declarations that create a set. Within the signature, relations from that set to other sets are declared, such as on line 3, which declares a relation from elements in the set `A` to elements in the set `ID`. These relations can have cardinality constraints within their declarations. For example, on line 3, the range has the keyword `one`, which means that `id` is a total function, *i.e.*, each domain element is associated with exactly one range element. Alternative cardinality constraints are `lone`, which means it is a partial function and `set`, which means it is a relation. Constraints in Alloy are formulas in FOL and set theory, such as line 12, which forces `id` to be an injective function.

Alloy provides type inheritance where one signature can be a subset of another. In our example, `B` and `C` are subtypes of `A` (using the keyword `extends`). Since set `A` is declared as abstract, there are no elements in `A` that are not in

either B or C. The sets B and C are disjoint. For a signature that is not declared as abstract, there can be elements in the parent set that are not in the child sets.

Alloy models make frequent use of the join operator between relations since everything in Alloy is a set. For example on line 12, `a.id` is the join of the singleton set consisting of `a` with the `id` relation, resulting in any range elements of `id` that have `a` as their first element.

Using the Alloy Analyzer, a user can set finite scopes for the sets of the model and check their model using Alloy’s finite model finding analysis implemented in Kodkod. A `run` command (as on lines 15-16) asks the Alloy Analyzer to produce an instance of these sets and relations that satisfy the constraints. Using a `check` command, the Analyzer will determine if an assertion holds of the model.

```

1  sig ID {}
2  abstract sig A {
3    id: one ID
4  }
5  sig B extends A {
6    toC: one C
7  }
8  sig C extends A {
9    toB: set B
10 }
11 fact id_is_injective {
12   all a, a': A | a.id = a'.id => a = a'
13 }
14
15 run {} for exactly 6 A, exactly 3 B,
16       exactly 3 C, exactly 6 ID

```

Fig. 1. Example of an Alloy model.

Kodkod. Kodkod [17] is the component of the Alloy Analyzer that implements its finite model finding. An Alloy model is converted to a representation as a datatype in Kodkod. Kodkod supports only relations – there are no types and no functions. Types are represented as unary relations. Functions and relations with multiplicity constraints are all treated as relations with extra constraints to represent their multiplicity restrictions. Kodkod supports propositional logic operations, universal and existential quantification, and a set of common operators on sets including join. Kodkod is a Java library and the Alloy Analyzer can produce a Java program that creates the Kodkod representation of an Alloy model. This code has the following parts that are of interest to us: 1) an atomlist, a list of constants that represent values in the finite sets of the model; 2) a data structure called bounds, which populates each relation of the model with its possible tuples from the constants; and 3) a data structure representing the constraints of the Alloy model together with the problem to be solved (from

the `run` or `check` commands) as one conjunction. Kodkod performs quantifier expansion, simplifications and symmetry reductions to produce a problem for a SAT solver.

Fortress. One of our translation options is to use the existing library Fortress to represent models of finite scope. Fortress is a Java library created by Vakili and Day [19] that reduces the finite model finding problem for FOL to the logic of equality with uninterpreted functions (EUF). The idea is that an EUF solver can exploit the structure of functions and types used in the model to achieve better performance than Kodkod and other related finite model finding methods. Fortress introduces constants for the elements of the finite scope. Then, it creates range formulas, which are constraints on the outputs of each function to be one of the elements of the scope, which force the EUF problem to have solutions only within a finite scope. Fortress performs quantifier expansion, simplifications, and symmetry reductions before producing a problem for the SMT solver Z3 [14]. Vakili and Day [19] show Fortress has quite good results compared to Kodkod on a number of problems. We use Fortress’ abstract datatype for TFOL in our implementation.

SMT-LIB2 and SMT Solvers. SMT solvers [5] search for satisfying instances of FOL problems by using cooperating decision procedures for particular subsets of FOL that have standard interpretations (such as equality or linear arithmetic). The standard interface language for SMT solvers is SMT-LIB2 [4], which we use in this work. We use the SMT solver Z3 [14] although other solvers such as CVC4 [3] can be used.

3 Translating Kodkod to Typed First-Order Logic (TFOL)

Figure 2 presents an outline of the steps we use to translate Kodkod to TFOL. TFOL supports only total functions. Our goal is to create a very direct translation to TFOL from Alloy. Our translation is similar in parts to Ulbrich *et al.* [18]’s translation of Alloy to KeY [6] and El Ghazi *et al.* [8]’s translation of Alloy to SMT although both of those efforts implement only one combination of our options. Next, we describe the steps in our translation and the options we investigate for each step. To do the translation, we work from Kodkod’s atomlist, bounds, and formula data structures.

Step 1: Declaring Types. Alloy uses sets as types. In the Kodkod representation, there is one universe of elements and unary relations constrain the elements to be of a certain type when needed. In TFOL, we have the ability to use its type space to separate the elements into types and thereby reduce the size of the set of elements for each quantifier. *We hypothesize that using types will improve the performance of the SMT solver.* Our first translation step is to declare types in TFOL. We explore two options: untyped and typed.

For the untyped option, we introduce one “universal” type (called “Univ” in our examples in the paper) to use as the types of all functions in TFOL.

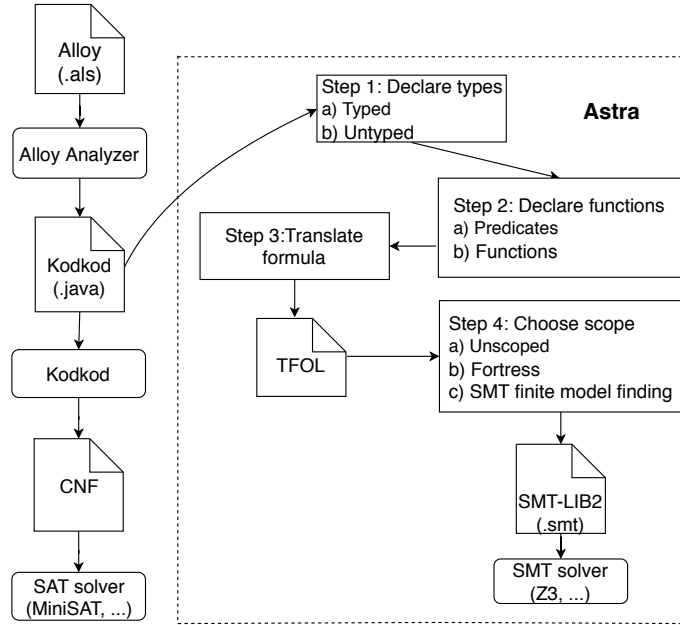


Fig. 2. Steps in translation.

In the typed option, there are various levels in the Alloy type hierarchy we could choose as FOL types. To investigate an opposite translation point from untyped, we choose the types at the leaves of the hierarchy as the TFOL types. For the example of Figure 1, the types declared are **B** and **C**.

Kodkod does not store the type hierarchy of the Alloy model directly. We have to reverse engineer it from the atomlist and bounds data structures. Kodkod names its constants prefixing them with type names of the leafs of the type hierarchy. This information tells us the names of the leaf types so we can declare them in TFOL. Some of these type names are for a set of elements that are not part of a declared leaf type in Alloy because the subtypes do not necessarily include all elements of the parent type.

Step 2: Declaring Functions. The next step of translation is to declare the total functions in TFOL. A predicate in TFOL is a function that returns a Boolean value.

In this step, we traverse Kodkod’s bounds datatype, ignoring unary relations that were used as types if the typed option was chosen for Step 1. For the remaining relations, we create function declarations in TFOL. *We hypothesize that solving a problem with theories is harder than solving a SAT problem in an SMT solver. Therefore, using predicates instead of functions can improve the performance, at least for UNSAT problems, since if the SMT solver cannot find a satisfying Boolean assignment, then it does not have to use its theories.* Thus, we implement two options for declaring TFOL functions. The first, which we call

“predicates”, is to make all Kodkod relations predicates in TFOL. The second, which we call “functions”, is to use as many non-predicate functions as possible in TFOL.

For the predicates option, we declare the predicates in TFOL from the information in the bounds datatype directly. For the untyped option of Step 1, we declare all relations to take arguments of the universal type and return a Boolean. The TFOL function declarations for the type hierarchy of Figure 1 for the untyped option are:

$$\begin{aligned} id &: Univ \times Univ \rightarrow Bool \\ toC &: Univ \times Univ \rightarrow Bool \\ toB &: Univ \times Univ \rightarrow Bool \end{aligned}$$

We then add constraints to describe the multiplicity restrictions.

For the functions option, we have to reverse engineer from the Kodkod datatypes which of these relations are total functions in Alloy. We traverse the Kodkod formula looking for multiplicity constraints. The information that a relation is total function is found as a constraint of a certain pattern in the formula data structure of Kodkod indicating the range of the function is `one x`, which means we can declare it as having a return type X in TFOL. For example, the TFOL declaration of the total function `toC` in Figure 1 for the functions option would be:

$$toC : Univ \rightarrow Univ$$

and fewer multiplicity restrictions are needed. For relations that have non-total function multiplicities (including `lone` multiplicities, which means it is a partial function), we declare them as predicates of the appropriate type and add their multiplicity constraints as formulas in our TFOL model. When we translate the formula in Step 3, we ignore these multiplicity constraints in Kodkod because they have already been translated.

For the typed option of Step 1, the TFOL function declarations are more complicated. We read the leaf type names from the names of constants in the tuples Kodkod assigned to the relation. For relations that have domain and range types that are non-leaf types, we create multiple copies of the relation. For example, in Figure 1, `id` is a function which has a domain that is a non-leaf type, `A`, thus the TFOL declarations for the predicates option are:

$$\begin{aligned} id1 &: B \times ID \rightarrow Bool \\ id2 &: C \times ID \rightarrow Bool \\ toC &: B \times C \rightarrow Bool \\ toB &: C \times B \rightarrow Bool \end{aligned}$$

For the functions option, the TFOL declarations are:

$$\begin{aligned} id1 &: B \rightarrow ID \\ id2 &: C \rightarrow ID \\ toC &: B \rightarrow C \\ toB &: C \times B \rightarrow Bool \end{aligned}$$

Step 3: Translating Formulas. Step 3 is to translate the formulas in the Kodkod datatype to formulas in TFOL. In this step, we have to translate set operations (union, intersection, *etc.*) into their equivalent in FOL. Our translation is defined by the $[\cdot]$ operator, which takes a formula in Kodkod, and translates it into a TFOL formula. We define $[\cdot]$ in this section.

To create as direct a translation to TFOL as possible, we represent each set operation using the characteristic predicate for the set and the propositional operation that is the equivalent of the set operation. For example, the union of two sets is the disjunction of the characteristic predicate for each of the operands to the union.

This translation can be done in either a top-down or bottom-up traversal of the Kodkod formula data structure. To handle the generality of set expressions in Alloy, we choose a bottom-up traversal to facilitate its compositionality and so that the types of terms can be determined from their leaves on the way up. To make a bottom-up traversal possible, we have to provide a translation for each term, not just each formula. We describe our translation below for the typed option in Step 1.

The leaves of the Kodkod formula datatype are variables or relations of certain types. We need this type information in our translation to TFOL. Kodkod stores the types of its variables with the variable, so the translation of a Kodkod variable or relation is simply a term of the type in the Kodkod, as in $[v : t] := v : t$.

Next, we describe the translation for the set operations. A term in Kodkod must return a term in TFOL so we translate each non-leaf term into a helper relation and add a constraint for the meaning of the helper relation. For example, $[A \cup B]$ where A and B are both of type t is a new relation R of type $t \rightarrow Bool$ with an additional constraint of $\forall x : t \bullet R(x) \Leftrightarrow A(x) \vee B(x)$. Thus, the translation of the set operations results in a TFOL term plus declarations and additional constraints. The type of the Kodkod term must be determined as we walk up the data structure; we use the notation $\text{TYPE}(\cdot)$ to denote this calculation. In the following, each R_{new} is a new relation name:

$$\begin{aligned}
[A \cup B] &:= R_{new} : \text{TYPE}(A) \rightarrow Bool \\
&\quad \text{add } \forall x : \text{TYPE}(A) \bullet R_{new}(x) \Leftrightarrow [A](x) \vee [B](x) \\
[A \cap B] &:= R_{new} : \text{TYPE}(A) \rightarrow Bool \\
&\quad \text{add } \forall x : \text{TYPE}(A) \bullet R_{new}(x) \Leftrightarrow [A](x) \wedge [B](x) \\
[A - B] &:= R_{new} : \text{TYPE}(A) \rightarrow Bool \\
&\quad \text{add } \forall x : \text{TYPE}(A) \bullet R_{new}(x) \Leftrightarrow [A](x) \wedge \neg[B](x) \\
[\sim A] &:= R_{new} : \text{TYPE}(\text{ran}(A)) \times \text{TYPE}(\text{dom}(A)) \rightarrow Bool \\
&\quad \text{add } \forall x : \text{TYPE}(\text{ran}(A)), y : \text{TYPE}(\text{dom}(A)) \bullet \\
&\quad \quad R_{new}(x, y) \Leftrightarrow [A](y, x) \\
[((v_1 : t_1), (v_2 : t_2))] &:= v_3 : t_1 \times t_2
\end{aligned}$$

By using this method of helper relations (also used in [18], although not for a link with SMT solvers), our translation results in smaller clauses but more of them. This method reduces the size of formulas after quantifier expansion for finite model finding.

Using this method, the general case of translating the very common join operation in Alloy is:

$$\begin{aligned} [A.B] &:= R_{new} : \text{TYPE}(\text{dom}(A)) \times \text{TYPE}(\text{ran}(B)) \rightarrow \text{Bool} \\ &\text{add } \forall x : \text{TYPE}(\text{dom}(A)), y : \text{TYPE}(\text{ran}(B)) \bullet \\ &R_{new}(x, y) \Leftrightarrow \exists z : \text{TYPE}(\text{ran}(A)) \bullet [A](x, z) \wedge [B](z, y) \end{aligned}$$

For the functions option of Step 2, there are special cases for join where the first argument, v , to join is a variable or the first argument is the result of a total function application (which results in a scalar), we can translate this expression to function application:

$$\begin{aligned} [(v : t).f] &:= (f(v : t)) : \text{TYPE}(\text{ran}(f)) \\ [(v : t).f_1.f_2] &:= (f_2(f_1(v : t))) : \text{TYPE}(\text{ran}(f_2)) \end{aligned}$$

The remaining formulas of Kodkod (Boolean operations and equality, quantification, \subseteq , and \in) have straightforward translations:

$$\begin{aligned} [true] &:= true \\ [false] &:= false \\ [\neg A] &:= \neg[A] \\ [A \wedge B] &:= [A] \wedge [B] \\ [A \vee B] &:= [A] \vee [B] \\ [A \Rightarrow B] &:= [A] \Rightarrow [B] \\ [A \Leftrightarrow B] &:= [A] \Leftrightarrow [B] \\ [A = B] &:= [A] = [B] \\ [\forall(x : t) \bullet A] &:= \forall x : t \bullet [A] \\ [\exists(x : t) \bullet A] &:= \exists x : t \bullet [A] \\ [A \subseteq B] &:= \forall x : \text{TYPE}(A) \bullet [A](x) \Rightarrow [B](x) \\ [(v : t) \in B] &:= [B](v : t) \end{aligned}$$

Everywhere that a relation of non-leaf domain type is used in a formula, it has to be replaced by an appropriate operation over the leaf copies of the relation.

If the untyped option is chosen in Step 1, this translation changes slightly. The types all become the universal type and unary predicates are added as appropriate to limit the formula to the correct type. For example, the translation for the union operator is:

$$\begin{aligned} [A \cup B] &:= R_{new} : \text{Univ} \rightarrow \text{Bool} \\ &\text{add } \forall x \bullet P_{\text{TYPE}(A)}(x) \Rightarrow (R_{new}(x) \Leftrightarrow [A](x) \vee [B](x)) \end{aligned}$$

where $P_{\text{TYPE}(A)}(x)$ is the predicate for the type of set A . The change also affects the translation of the multiplicity constraints in Step 2 in a similar manner.

The alternative of a top-down traversal would have been more difficult to correctly implement. It would result in longer formulas, but no extra quantified constraints. In a top-down traversal of a nested set expression, variables of unknown type would have had to be created on the way down the traversal so that formulas would always be passed back up the traversal.

Step 4: Choosing Scopes. In step 4, the scopes for the problem are set. We can determine the scopes the user set for each of the Alloy sets by counting the number of constants with a prefix of a set name in Kodkod’s atomlist. For the untyped option, the scope is the length of Kodkod’s atomlist.

We hypothesize that if the problem is within a decidable fragment of TFOL, the SMT solver can solve it quickly without any bounds since the formula is smaller without introduced constants and their expansion. We also want to investigate whether quantifier expansion with finite scopes is more efficiently done by the SMT solver or prior to solving. We investigate three methods for handling the scope.

Our first option is called unscoped because we leave the types unbounded. This SMT problem may not be decidable, but it is possible that we will get a result from the SMT solver.

Our second option is called Fortress because we use the existing Fortress library. Along with the formula, we need to pass to Fortress scopes for each type. Fortress first creates distinct constants for each element of the scope. Next, it creates range formulas for each function to constrain it to return one of the constants of the type. Finally, Fortress expands each quantifier, does symmetry reductions and simplifications before creating an SMT-LIB2 problem.

We call our third option SMT finite model finding (SMT FMF), where we let the SMT solver do any needed quantifier expansion over the constants and simplifications. We declare the constants and create range formulas and pass the problem and the range formulas directly to the SMT solver.

Step 5: SMT Solver. In the final step, Step 5, the SMT solver is called on the problem.

We have not yet proven the correctness of our translation, however, since the translation is from Kodkod (rather than Alloy) the number of cases to consider for correctness is reduced and can be seen more directly.

Implementation. Astra is implemented in Java as a solver that takes Kodkod’s data structures as arguments to facilitate easy future integration with the Alloy Analyzer. We use the Fortress library implemented by Vakili and Day [19] as our abstract datatype for TFOL declarations and formulas.

4 Evaluation

Our goal in this work is to investigate whether alternative methods to Kodkod have better performance. We expect that no one method will always have better performance, but perhaps we can learn which solving method works better for models of certain characteristics.

4.1 Tests

We chose a number of Alloy models that cover a range of interesting characteristics: 1) number of types (# T); 2) maximum depth of applications (Application); 3) maximum number of connected joins (join length); 4) maximum nesting of

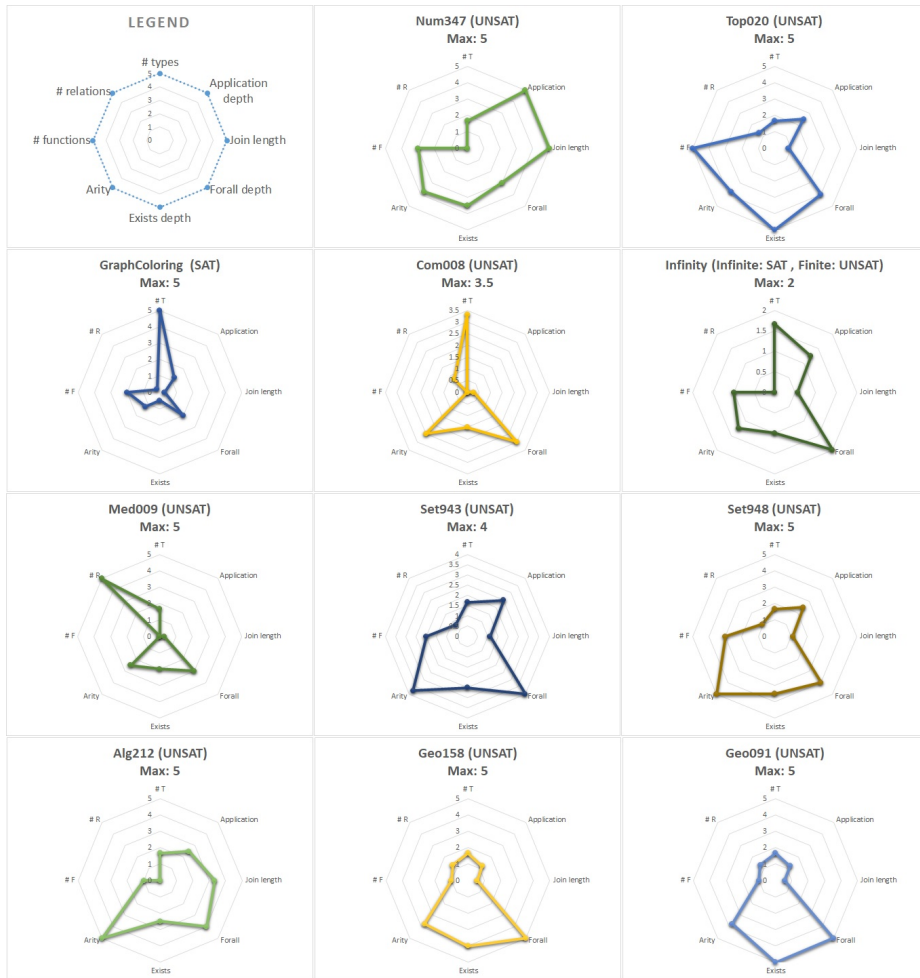


Fig. 3. Characteristics of tests.

universal quantifiers (forall); 5) maximum nesting of existential quantifiers (exists); 6) maximum arity of a relation (arity); 7) number of total functions (# F); and 8) number of relations (# R). We manually measured these characteristics for each of our models. Figure 3 uses a radar chart to illustrate how our different models covered these characteristics. We have eleven models that are tested for different scopes for a total of twenty-nine tests. Most are not satisfiable because these tend to be harder problems for solvers, but a few tests have satisfying solutions. These models originated from Kodkod benchmarks [17], which were also used to test Fortress, and we created some additional models to ensure that we had models that contained interesting instances of the above characteristics.

4.2 Performance

We ran our models for different scopes with Kodkod and with all the twelve combinations of options we described for translation. We immediately rejected a number of combinations due to poor performance. Table 1 shows the combinations with interesting results and those that we immediately rejected. Some of these combinations with poor performance can be explained by considering the steps in the process. For example, there is more quantifier expansion with relations than functions, thus the combination of Fortress and relations has poor performance.

Table 1. Translation options. ✓ = interesting combination, ✗ = rejected combination

	Functions			Relations		
	Unscoped	Fortress	SMT FMF	Unscoped	Fortress	SMT FMF
Typed	✗	✓	✗	✓	✗	✗
Untyped	✗	✗	✗	✓	✗	✓

All of our tests were completed on a computer with 2.6-GHz Intel Core i5 CPU, with a 2500MB Memory limit and a 2000 second time limit for each process. Figure 4 shows our results using a logarithmic scale for time on the y-axis with the twenty-nine tests across the x-axis. The times include the time for translation and the time for SMT solving (or SAT solving in Kodkod). The five option combinations with interesting results are shown by different lines in the graph. The lowest line for a test on the graph means the best performance. Any lines that hits the uppermost point on the graph mean that we stopped the test after it had taken 2000 seconds or had run out of memory.

The differences in the performance of the methods is quite considerable in most tests, ranging from less than a second for the best performing method to over 2000 seconds for the worst performing method.

In nine of the eleven models, one of our translation combinations produces better results than Kodkod. The typed, predicates, unscoped option had the

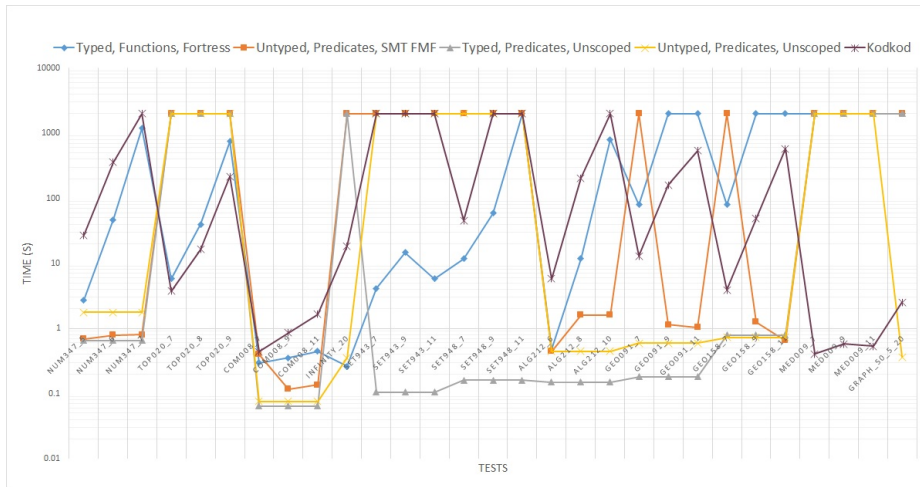


Fig. 4. Performance results for tests.

best performance in five models and tied for the best performance on three other models making it the clear winner in performance. Every combination of options that we included won at least one test. The combination of typed, functions, Fortress performed the best for the scoped combinations (including Kodkod).

4.3 Analysis of Performance Results

We hypothesized that using types would improve the performance of the SMT solver. When working with the Fortress option, the typed option works much better than untyped. However, although there are slightly better results with the typed option for other option combinations, the results are not conclusive.

Second, we hypothesized that using predicates instead of functions, can improve performance, at least for UNSAT problems. Most of the UNSAT problems are solved very quickly when predicates are used, however, they timed out, or ran out of memory, when the functions option is used.

Third, we hypothesized that the unscoped option would often be faster than finite scopes. Many of the tests with unscoped option are solved within seconds. However, it is unclear whether this good performance is due to the unscoped option or the predicates option.

Although we do not have a large benchmark, we tried the linear regression method to find a correlation between the model characteristics and the performance of a method. Since the number of tests is relatively small for such method, we used it only to rank the characteristics, and we present a model only for the ones with an R-squared larger than 0.6. This threshold eliminated the models for Kodkod and the untyped, predicates, SMT FMF method.

The first model we present is for the option combination of typed, functions, Fortress combination. This model's R-squared is 0.76, and it points out that the number of types, the depth of function applications, and the number of relations, play the most important roles in its performance.

The second model is for the typed, predicates, unscoped combination, which has an R-squared of 0.65. This model points out that the number of types and the number of functions and relations play the biggest role in performance time of the tool.

Lastly, the untyped, predicates, unscoped combination has the best suiting model with an R-squared of 0.86. This model, as expected, does not value the number of types as much. This model mostly emphasizes the arity of the functions and relations, the depth of function applications, and the number of functions and relations.

These results give us insight about which options may be more suitable for problems of certain characteristics. For example, we hypothesize that tests with many types may be better solved by the untyped option, while tests with functions of large arities may be better solved with other options. These insights can be used as guidelines in the Alloy modelling process to create models that can be solved more easily by a specific option.

Additionally, these results raise some questions regarding the internal process of SMT solvers. We had some unexpected results, such as models “Geo091” and “Geo158”, which could be solved with the untyped, predicates, SMT FMF combination easily for scopes of 9 and 11, but could not be solved within the 2000 seconds time limit for a scope of 7. Also, in model “Set943”, the typed, functions, Fortress combination solved the larger scope of 11 faster than the smaller scope of 9. This test was repeated multiple times, and the same result was observed each time.

5 Related Work

In this section, we discuss related efforts to translate Alloy to SMT-LIB. Compared to previous work, we investigate and evaluate multiple options for the translation and try to correlate them with model characteristics. While other work has evaluated the solving performance of their own translation, none of these works compare solving time for unbounded with bounded scopes in SMT solvers. Also, we start from the Kodkod interface for ease of future integration with the Alloy Analyzer. Translating from Kodkod rather than the Alloy language was easier with respect to having a more basic language to work with, but harder because we had to reverse engineer from Kodkod the types and functions of the Alloy model. We do not yet support the transitive closure operators, set cardinality or built-in types and some of these related efforts do support these operations/types.

El Ghazi *et al.* [9] describe a translation directly from Alloy to the SMT solver Yices [7]. Since Yices Version 1 supported subtypes, Alloy’s subtyping could be directly translated into its Yices equivalent. Partial functions in Alloy are translated to total functions in Yices by including an empty range value. The focus of their work is on using Yices’ theories for Alloy’s built-in types in order to leave these types unbounded. They evaluate their translation using one case study.

In El Ghazi *et al.* [8], a translation directly from Alloy to Z3 is described. It corresponds to our typed, unscoped option. They use relations at first and then do some simplifications for Alloy functions. Their work supports Alloy’s built-in types and the closure operations for relations. Their results shows Z3 performed well, solving a number of problems in Alloy. Rather than using helper functions to translate the set expressions, they take a top-down approach to translation, passing arguments down to the leaf relations.

Ulbrich *et al.* [18] describe a translation from Alloy to the KeY theorem prover [6] for first-order logic to check Alloy models over unbounded scopes. Their translation matches our untyped, relations, and unscoped option. They introduce helper relations to translate the set expressions using axioms similar to our constraints on the helper relations. KeY integrates automatic and interactive proof and includes support for some of Alloy’s built-in types. Their translation handles transitive closure and set cardinality, but these may require interactive

proof methods. Their results show that a number of Alloy assertions could be proven automatically in the KeY prover.

Reynolds et al. [16] propose a theory called Finite Cardinality Constraints (FCC) for doing finite model finding within an SMT solver. The theory is based on the EUF subset of FOL. Vakili and Day [19] report that this method did not have good performance compared to Fortress.

Bansal *et al.*[2] introduces a new theory for solving relational FOL (including set cardinality) of unbounded scope problems in SMT solvers. This is a calculus for relational logic in SMT which can be combined with their finite model finding feature. It has been implemented in CVC4 [3] and evaluated on some problems but not yet linked with Alloy for evaluation.

Alloy2B[13] is a tool that translates Alloy models to the B language [1], making a variety for B tools available for use on Alloy models including model checkers and interactive proof tools for examining a model of unbounded scope.

We have not yet translated the transitive closure operators. For a finite scope, it is possible to expand the meaning of transitive closure as is done by Kodkod. El Ghazi *et al.*[10] addresses this problem for unbounded scope by axiomatizing transitive closure in FOL, in an iterative manner.

6 Conclusion

We have presented an evaluation of various options for translating relational FOL as it is represented in Kodkod to typed FOL in SMT-LIB. We considered many options for the translation including: typed vs untyped, predicates vs functions, and unbounded vs bounded scopes where the formulas are either expanded pre-solving or during SMT solving. Our results show that with the Z3 SMT solver, the typed, predicates, unscoped combination is the best combination in general for unbounded scopes; and the typed, functions, Fortress translation combination is the best for bounded scopes. There are many interesting directions from our work to understand how model characteristics relate to solver performance, which could provide the basis for a portfolio of solvers for Alloy (perhaps based on Why3 [11]).

We have several directions for future work. We plan to investigate translations for the transitive closure operator, set cardinality, and the mapping of Alloy's built-in types to SMT theories. We hypothesize that the use of SMT solvers for these built-in types may provide better performance with unbounded scopes on these types. Also, for a satisfiable instance, we do not yet return the instance from the SMT solver to Alloy. This step becomes relevant when we integrate with the Alloy Analyzer, which is our next step. And we would like to broaden our analysis to include more SMT solvers or finite model finding techniques.

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