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MASTER'S THESIS

OPTIMIZATION TECHNIQUES FOR CELL-FREE MASSIVE MIMO SYSTEM

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ABSTRACT

The problem of max-min signal-to-interference plus noise ratio (SINR) for uplink transmission of cell-free massive multiple-input multiple-output (MIMO) system is considered. We assume that the system is employed with local minimum mean square error (L-MMSE) detection. The objective is to preserve user fairness by solving max-min rate optimization problem, by optimizing transmit power of each user equipment (UE) and weighting coefficients at central processing unit (CPU), subject to a transmit power constraint at each UE. This problem is not jointly convex. Hence, we decompose original problem into two subproblems, particularly for optimizing power allocation and weight coefficients. Then, we propose an iterative algorithm to solve these two subproblems alternately. Weight coefficient subproblem is solved in the form of generalized eigen value problem while power allocation subproblem is solved by approximating as geometric programming (GP) problem.

Keywords: Cell-free massive MIMO, max-min SINR problem, geometric programming, generalized eigen value problem, L-MMSE.

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FOREWORD

The focus of this thesis is to solve max-min SINR problem in uplink for cell-free massive MIMO system with L-MMSE receiver by utilizing optimization techniques. This research work has been conducted as a part of High5 and MOSSAF project at Centre for Wireless Communication (CWC) of University of Oulu and it is a great stepping stone for my future career.

It is my great pleasure to acknowledge the roles of individuals who were instrumental for completion of my master's research. First and foremost, I would like to express the sincerest gratitude to my supervisors Prof. Nandana Rajatheva and Dr. Shashika Manosha for their immense support and guidance given throughout the thesis. I extend my appreciation to Prof. Matti Latva-aho for the opportunity given to join the research group in CWC. My warm thanks go to Dr. Pekka Pirinen, the project manager and other colleagues at CWC for their support in my research.

I remember with gratitude all my teachers who taught me during my master's studies and entire student life. This thesis is dedicated to the memory of my father, who believed in me and wished for my success in academics. Special thanks go to my mother and brother, for their unconditional love and support. I would acknowledge the citizens of Sri Lanka for investing in my free education from my childhood. Last but not least, I would like to thank all whose names I am unable to mention, but supported me in many aspects during the completion of the thesis.

Oulu, 23rd August, 2019

Chamalee Wickrama Arachchi

LIST OF ABBREVIATIONS AND SYMBOLS

3GPP	Third Generation Partnership Project
5G	Fifth Generation
6G	Sixth Generation
AO	Alternating Optimization
AP	Access Point
CoMP	Coordinated multi-point
CSI	Channel State Information
CWC	Center for wireless communications
DL	Downlink
DoF	Degrees of Freedom
FD-MIMO	Full-Dimension Multiple Input Multiple Output
GEP	Generalized Eigenvalue Problem
GP	Geometric Program
L-MMSE	Local Minimum Mean Square Error
LP	Linear Program
LS	Least Square
MF	Match Filtering
MIMO	Multiple Input Multiple Output
mMIMO	Massive MIMO
MMSE	Minimum Mean Square Error
MR	Maximum Ratio
MU-MIMO	Multi-User MIMO
NCOP	Nonlinearly Constrained Optimization Problem
SE	Spectral Efficiency
SINR	Signal to Interference and Noise Ratio
TDD	Time Division Duplexing
UE	User Equipment
UDN	Ultra Dense Networks
UL	Uplink

a_{kl}	weight coefficient of kth UE at lth AP
$\mathbf{A}_k, \mathbf{B}_k$	matrix pair of generalized eigenvalue problem
$\mathbf{C}_{kl}$	correlation matrix of the estimation error signal of kth UE at lth AP
D	square length of simulation area
d_{kl}	the distance between kth UE and lth AP
$\mathbf{g}_{ki}$	receive-combined channels between kth UE and each of the APs
$\mathbf{h}_{kl}$	channel coefficient between kth UE lth AP
$\hat{\mathbf{h}}_{kl}$	channel estimate between kth UE lth AP
$\tilde{\mathbf{h}}_{kl}$	channel estimation error between kth UE lth AP
$\mathbf{I}_n$	$n \times n$ identity matrix
L	number of access points
K	number of UEs
MSE_{kl}	mean squared error (MSE) of kth symbol at lth AP

N	number of antennas of an access point
$\mathbf{N}_l$	noise matrix at lth AP
$\mathbf{n}_l$	receiver noise at lth AP
p_i	the transmitted power at ith UE
$p_{max}^{(k)}$	maximum transmit power available at user k
$\mathbf{R}_{kl}$	spatial correlation matrix between kth UE lth AP
R_k	achievable rate of kth UE
$\check{s}_{kl}$	local estimate of$k$th UE at lth AP
s_i	information bearing signal at ith UE
$SINR_k$	effective SINR of kth UE
$SINR_{k,max}$	maximum effective SINR of kth UE under fixed power case
t	slack variable
$\mathbf{v}_{kl}$	local combining vector at lth AP to estimate s_k
$\mathbf{y}_l$	received signal at lth AP
$\mathbf{Z}_l$	received pilot matrix at lth AP
t_k	pilot index assigned to kth UE
$\mathbf{z}_{t_k l}$	correlated signal with $\mathbf{Z}_l$ and corresponding pilot signal at lth AP for kth UE
τ_p	length of pilot sequence
τ_c	length of coherence interval
β_{kl}	large scale fading coefficient between UE k and AP l
Φ_k	kth mutually orthogonal pilot signal
$\mathbf{\Psi}_{t_k l}$	correlation matrix of the received signal $\mathbf{z}_{t_k l}$
σ^2	variance
$\mathbb{C}^n$	the set of complex n dimensional vectors
$\mathbb{C}^{m\times n}$	the set of complex $m \times n$ matrices
$\nabla f(\mathbf{x})$	gradient of function f at $\mathbf{x}$
$\sim$	distributed according to
$\approx$	approximated according to
$\lvert x\rvert$	absolute value of the complex number x
$\lVert x\rVert$	Euclidean norm of the vector $\mathbf{x}$
$\mathbf{X}^H$	Conjugate transpose (Hermtian) of matrix $\mathbf{X}$
$\mathbf{X}^T$	Transpose of matrix $\mathbf{X}$
$\mathbf{X}^{-1}$	Inverse of matrix $\mathbf{X}$
$diag(x)$	diagonal matrix with the elements with the vector $\mathbf{x}$
$\mathbb{E}\{\mathbf{x}\}$	The expected value of $\mathbf{x}$
log_k	logarithm in base k
$\mathcal{N}_{\mathbb{C}}(\mathbf{0},\mathbf{R})$	multi-variate circularly symmetric complex Gaussian distribution with correlation matrix $\mathbf{R}$
$\mathcal{CN}(0,\sigma^2)$	circularly symmetric complex Gaussian distribution with zero mean, variance σ^2
$\mathcal{N}(0,\sigma^2)$	real-valued Gaussian distribution with zero mean, variance σ^2

1 INTRODUCTION

In this chapter, we briefly describe the background and motivation about next generation wireless networks and importance of mathematical optimization in sophisticated radio resource management. Further, we present thesis objectives and contribution related to cell-free massive MIMO, followed by the outline of the thesis.

1.1 Background and Motivation

Fifth generation (5G) networks were standardized to meet growing demands of high spectral efficiency (SE), low latency, ultra reliability, and massive machine type communications. While 5G is being deployed around the world, researchers from both academia and industry are paving the way towards sixth generation (6G) networks [1]. In particular, network densification, massive multiple-input multiple-output (MIMO), and millimeter-wave (mmWave) bands have recently emerged as the promising physical layer enablers for 5G and beyond [2]. Thus, there are open mathematical and computational challenging problems which are related to the three main pillars of physical layer: employing MIMO networks, utilizing ultra-dense networks (UDN), and exploiting new frequency bands.

Massive MIMO (mMIMO) has become one of the most promising candidates towards fifth generation (5G) systems and beyond, due to improved spectral efficiency (SE); but not sacrificing extra bandwidth and transmit power resources [3]. In contrast to multi-user MIMO, mMIMO system deploys hundreds or even thousands of antennas at a base station (BS) exploiting spatial degrees of freedom (DoF) which leads to a huge improvement of SE. Coordinated multi-point (CoMP), synonymously referred to as multicell operation, is one of the promising ways to improve SE through BS cooperation [4]. CoMP provides improved SE, specially for cell edge users by mitigating inter cell interference via the coordination between the interfering and serving BSs.

Cell-free massive MIMO (Cell-free mMIMO) is a hybrid model which combines the features of distributed MIMO and mMIMO [5]. By design, it is a user-centric implementation to overcome inter cell interference and provide macro-diversity [6]. Moreover, the excessive handover issue in small-cell systems can be solved using cell-free topology [7]. Thus, cell-free massive MIMO has attracted a lot of research interest recently. In cell-free mMIMO system architecture, it deploys large number of distributed access points (APs) over a geographical area where number of users are much lower than number of APs and all users are simultaneously served in the same time-frequency resource block by spatially distributed APs. In the existing literature, each AP performs multiplexing with receiver processing techniques; for example see maximum ratio (MR) [8], zero forcing (ZF) [7], and MMSE processing [9]. In this thesis, we evaluate the performance of cell-free mMIMO system with multi-antenna APs and local-MMSE (L-MMSE) detector, in terms of max-min fairness policy.

On the other hand, sophisticated radio resource management strategies for next generation wireless communication networks have been identified as a key requirement with the accelerated demands for radio resources, such as channel capacity, spectrum, quality of service (QoS) requirements, delay requirements, and many others. Specifically, it is understood that a wide variety of resource management problems of recent interest, including power/rate control, beamformer design of MIMO networks, and many others are directly or indirectly reliant on the generic max-min optimization problems [10]. In this thesis, a greater emphasis is placed on developing algorithms for max-min rate optimization problem, which is known to be generally non convex [11].

1.2 Thesis Objectives and Contribution

- **Objective:** *Applying optimization techniques for resource management in wireless communication networks (5G and beyond).*

- **Contribution:** *Developing an iterative alternating algorithm to solve the max-min SINR problem for the uplink of a cell-free mMIMO system employed with L-MMSE detector.*

  The focus of the earlier paper [9, Corollary 2] was to maximize individual signal-to-interference plus noise ratio (SINR) by only changing weighting coefficients; but, taking powers as fixed. In contrast to previous work, our research contribution is to maximize the smallest SINR of the system by changing both transmit power and weight coefficients. Note that, considered max-min SINR problem is a non-convex joint optimization problem with respect to weight and power allocation coefficients. Thus, the proposed iterative algorithm is based on alternating computational framework where non-convex joint optimization problem is divided into two easier optimization problems and solve each one of them in a sequential manner. More specifically, first subproblem is derived as well-known generalized eigenvalue problem and second subproblem is derived as an approximated geometric program, such that modern optimization tools are capable of solving those well-structured problems; eventually, obtaining a suboptimal solution for the joint optimization problem.

1.3 Thesis Outline

The remaining of this thesis is organized as follows:

- **Chapter 2:** Includes the necessary theoretical background and provides an overview of the main constitutional parts of the thesis.

- **Chapter 3:** Provides the details of the work carried out, where the system model is described. Herein, details of the channel estimation and uplink payload transmission are visited, we provide detailed problem formulation and algorithm derivations related to cell-free system model.

- **Chapter 4:** Illustrates the performance and effectiveness of the proposed algorithm numerically.

- **Chapter 5:** Concludes this thesis and provides potential future research directions.

2 BACKGROUND AND LITERATURE

In this chapter, we present basic features associated with cell-free networks and optimization techniques used in our proposed algorithm.

2.1 Cell-free Massive MIMO

2.1.1 Overview of Network Deployments

In this section, we briefly discuss several different types of network deployments in the existing literature.

- **Conventional cellular network:** By design, conventional cellular architecture consists of disjoint sets with cell boundaries as illustrated in Figure 1 (top-left). Herein, each UE is exactly connected to one base station (except in handover scenario) and each base station handles different number of active UEs at a particular time instance. One of the main bottleneck of this network model is that UEs within one cell cause inter-cell interference to UEs in other cells. On the other hand, "network densification" has been recognized as a key player to cater high capacity requirement [12]. However, the more we densify the network, the more we pay for inter-cell interference. This is due to the fact that the implementation of conventional cellular network is more towards cell-centric than user-centric.

- **Network-centric network with CoMP-JT:** Another type of network deployment is to include "base station cooperation" for the network-centric implementation, as shown in Figure 1 (top-right). In contrast to conventional cellular network model, multiple base stations are located within a cell. Hence, UEs residing in each cell are served by multiple base stations leading to higher SE. See 1 (bottom-left).

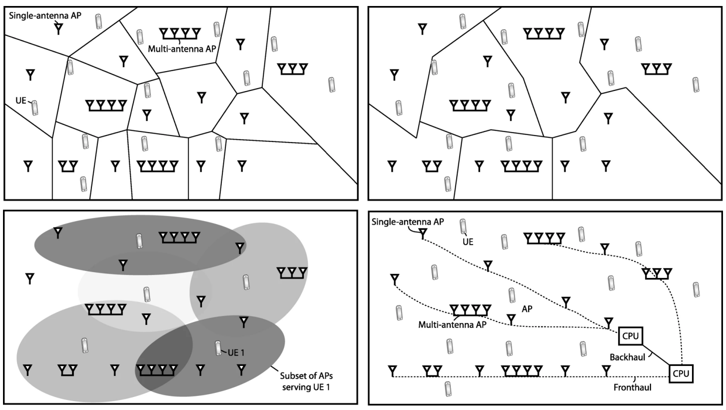

Figure 1. Example of network deployments. Top-left: A conventional cellular network. Top-right: A conventional network-centric implementation of CoMP-JT. Bottom-left: A user-centric implementation of CoMP-JT. Bottom-right: A cell-free mMIMO network. [6, Figure 1]

- **User-centric network with CoMP-JT:** In contrast to network-centric implementation of CoMP-JT, each UE communicates with its nearest set of APs facilitating a user-centric communication model.

- **Cell-free network:** The word *"Cell-free"* implies that there are no cell boundaries at least from user perspective [6]. In the cell-free literature, base station is referred as access point (AP). As illustrated in Figure 1 (bottom-right), all APs are connected to central processing unit (CPU) via fronthaul connections providing full cooperation with all APs. However, if there are multiple CPUs in the deployment area, then CPUs are connected by back-haul links. That is because those back-haul links enable user-centric communication in the areas covered by multiple CPUs [6].

  A major performance bottleneck of conventional cellular networks is the poor network coverage at cell edge users. But, cell-free networks are capable of alleviating inter-cell interference by cooperating between APs while conventional cellular networks suffer from poor coverage mostly at cell edge due to strong inter-cell interference near cell boundaries [6].

2.1.2 Pilot-based Channel Estimation in Cell-Free Massive MIMO

In contrast to conventional cellular networks, all users in cell-free networks are simultaneously served by all APs in the same time-frequency resource. Hence, pilot-based channel estimation which is inherited from mMIMO plays a critical role in cell-free networks. Generally, users are allocated with non-orthogonal pilots due to the fact that the length of the pilot sequence is limited by coherence time interval. Mostly in the cell-free literature, there is no channel state information (CSI) sharing among APs.

2.1.3 TDD Operation

In cell-free systems, uplink (from users to APs) and downlink (from APs to users) transmission are proceeded by time-division duplex (TDD) operation. As shown in Figure 2, each coherence interval is basically divided into three phases: uplink training, downlink payload data transmission, and uplink payload data transmission [5]. In addition to aforementioned TDD configuration, another possible configuration includes downlink training in which downlink pilots are used explicitly for downlink channel estimation.

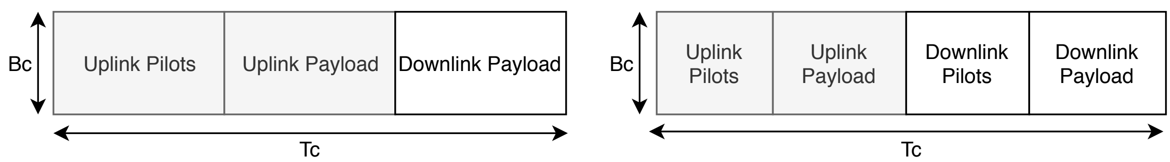

Figure 2. TDD frame structure. Left: The TDD frame without downlink pilots, Right: The TDD frame with downlink pilots. Guard intervals are not shown in the figure as it is deducted from the coherence time interval.

2.1.4 Power Control

To ensure user fairness, max-min power control is utilized in the cell-free networks [5]. This mechanism provides uniformly good service to all users, irrespective of their geographical location. Moreover, this power control is done at CPU on the *large-scale fading time scale* [5], since CPU does not have the channel estimates; but only channel statistics.

2.1.5 Local MMSE Processing

We choose the *combining vector* at each AP such that the conditional mean squared error between the data signal and received signal is minimized. The word "Local" is highlighted in the sense that each AP locally estimates its signal before passing to CPU for final decoding [9].

2.2 Eigenvalue Problems

Eigen value problems play a critical role as one of the fundamental mathematical problems in the field of communication. In this section, we briefly discuss different types of eigen value problems.

2.2.1 Basic Eigenvalue Problem

The basic eigenvalue problem can be stated as follows [13]:

Definition 1: *Given a square matrix* $\mathbf{A} \in \mathbb{C}^{n \times n}$*, we consider the problem of finding non zero vector* $\mathbf{x} \in \mathbb{C}^n$ *and a scalar* $\lambda \in \mathbb{C}$*.*

$$\mathbf{A}\mathbf{x} = \lambda \mathbf{x}, \tag{1}$$

where λ *can be considered as eigenvalue and* $\mathbf{x}$ *is referred to as eigenvector.*

Geometrically, the eigen vectors represent the directions of the spread whereas eigenvalues represent the magnitude of the spread [14]. More precisely, (1) is a right eigenvalue problem and $\mathbf{x}$ is a right eigenvector for λ. Similarly, left eigenvalue problem is defined as solving $\mathbf{x}^H\mathbf{A} = \lambda\mathbf{x}^H$. Unless otherwise stated, "eigenvector" is referred to "right eigenvector".

2.2.2 Generalized Eigenvalue Problem

In most applications, eigenvalue problem is not exactly in the form of standard eigenvalue problem as defined in (1) but of the *generalized* form. Generalized eigenvalue problem (GEP) consists of a pair of matrices which is referred to as *matrix pencil* in the literature [15].

The generalized eigenvalue problem can be stated as follows [16]:

Definition 2: *Given two matrices* $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{n\times n}$*, we consider the problem of finding non zero vector* $\mathbf{x} \in \mathbb{C}^n$ *and a scalar* $\lambda \in \mathbb{C}$*.*

$$\mathbf{A}\mathbf{x} = \lambda \mathbf{B}\mathbf{x}, \tag{2}$$

where λ *can be thought as generalized eigenvalue and* $\mathbf{x}$ *is the corresponding eigenvector.*

Easily note that when $B = I$, where I is the identity matrix, the generalized eigenvalue problem equals to the basic eigenvalue problem.

In many engineering applications, GEP has special structured forms: for example discretizing partial differential equations one can solve (2) with $\mathbf{A}$ is a Hermitian matrix and $\mathbf{B}$ is Hermitian and positive-definite matrix. Generally, this problem is called as *Generalized Hermitian Eigenvalue Problem.*

2.2.3 Rayleigh Quotient

The Rayleigh quotient of a vector $\mathbf{x} \in \mathbb{C}^n$ and Hermitian matrix $\mathbf{A} \in \mathbb{C}^{n\times n}$ is the scalar defined by [13]

$$r(\mathbf{x}) = \frac{\mathbf{x}^H \mathbf{A}\mathbf{x}}{\mathbf{x}^H \mathbf{x}}. \tag{3}$$

To further explore the behaviour of Rayleigh quotient in mathematical sense, the gradient of $R(\mathbf{x})$, which is denoted by $\nabla R(\mathbf{x})$ is calculated as follows:

$$\nabla r(\mathbf{x}) = \frac{(\mathbf{x}^H \mathbf{x})2\mathbf{A}\mathbf{x} - (\mathbf{x}^H \mathbf{A}\mathbf{x})2\mathrm{x}}{(\mathbf{x}^H \mathbf{x})^2} = \frac{2(\mathbf{A}\mathbf{x} - R(\mathbf{x})\mathbf{x})}{\mathbf{x}^H \mathbf{x}}. \tag{4}$$

From (4), we can see that the gradient of $R(\mathbf{x})$ goes to zero when $R(\mathbf{x})$ is equal to an eigenvalue of matrix $\mathbf{A}$ and $\mathbf{x}$ is equal to the corresponding eigenvector.

In geometrical sense, the eigenvectors of $\mathbf{A}$ are the *stationary points* of $R(\mathbf{x})$ and corresponding eigenvalues $\mathbf{A}$ are the values of $R(\mathbf{x})$ at these stationary points. For example, refer Figure 3.

2.2.4 Generalized Rayleigh Quotient

The generalized Rayleigh quotient for a vector $\mathbf{x} \in \mathbb{C}^n$ is the scalar given by [13]

$$R(\mathbf{x}) = \frac{\mathbf{x}^H \mathbf{A}\mathbf{x}}{\mathbf{x}^H \mathbf{B}\mathbf{x}}. \tag{5}$$

The generalized Rayleigh quotient is closely associated with GEP stated in subsection 2.2.2. The relationship between GEP and generalized Rayleigh quotient can be shown using differential calculus. Assuming $\mathbf{A}$ and $\mathbf{B}$ are symmetric matrices, gradient of $R(\mathbf{x})$ with respect to $\mathbf{x}$ is calculated as follows:

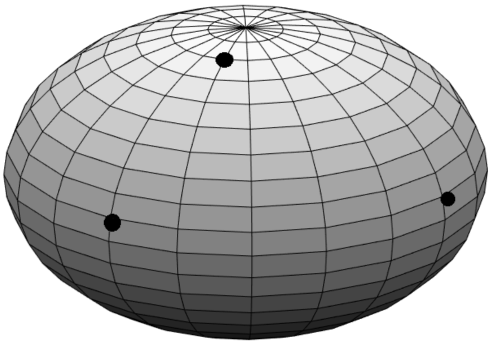

Figure 3. Suppose $X = \{\mathbf{x} \in \mathbb{R}^3 \mid \mathbf{x}^T\mathbf{x} = 1\}$ and $\mathbf{A}$ is a symmetric matrix. Then, $R(\mathbf{x})$ is a continuous function on X and three stationary points of $R(\mathbf{x})$ are orthonormal eigenvectors of $\mathbf{A}$.

$$\nabla R(\mathbf{x}) = \frac{2\mathbf{A}\mathbf{x}(\mathbf{x}^H\mathbf{B}\mathbf{x}) - 2\mathbf{B}\mathbf{x}(\mathbf{x}^H\mathbf{A}\mathbf{x})}{(\mathbf{x}^H\mathbf{B}\mathbf{x})^2} = \frac{2\mathbf{A}\mathbf{x} - 2R(\mathbf{x})\mathbf{B}\mathbf{x}}{\mathbf{x}^H\mathbf{B}\mathbf{x}}. \tag{6}$$

By setting $\nabla R(\mathbf{x}) = 0$ gives

$$\mathbf{A}\mathbf{x} = R(\mathbf{x})\mathbf{B}\mathbf{x}, \tag{7}$$

which is in the form of (2). Hence, maximum and minimum points of $R(\mathbf{x})$ is obtained as eigevalues of corresponding GEP. Likewise optimal $\mathbf{x}$ which maximize or minimize $R(\mathbf{x})$ becomes the eigenvector corresponding to optimal eigenvalue.

Solving *maximization* problem of generalized Rayleigh quotient with respect to $\mathbf{x}$, i.e., $\mathbf{x}^* = \mathop{\mathbf{arg\,max}}_{\mathbf{x}} R(\mathbf{x})$, is the eigenvector corresponding to the largest eigenvalue. Similarly, $x_* = \mathop{\mathbf{arg\,min}}_{x} R(\mathbf{x})$ is the eigenvector corresponding to the smallest eigenvalue.

2.3 Geometric Programming

Geometric programming (GP) is a powerful tool to solve wide variety of problems in field of science, engineering, finance, and statistics [17]. A geometric program is a special type of mathematical model with specific objective and constraint functions under the umbrella of optimization problems [17].

Following definitions on polynomials are important in our context.

Definition 3: *A monomial f is a real-valued function of x:*

$$f = c x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}, \tag{8}$$

where $x = (x_1, x_2, \ldots, x_n)$ is a vector of positive real variables, the coefficient c is positive, and the exponents $a_i \in \mathbb{R}$.

Definition 4: *A posynomial g is any sum of monomials, i.e., a function of the form*

$$g = \sum_{k=1}^{t} c_k x_1^{\alpha_1 k} x_2^{\alpha_{2k}} \cdots x_n^{\alpha_{n,k}}, \tag{9}$$

where each coefficient c_k *is positive.*

Definition 5: *The standard form of GP is defined as follows [18]:*

$$\begin{aligned} \text{minimize} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \leq 1, \quad i = 1, \cdots, m, \\ & g_i(\mathbf{x}) = 1, \quad i = 1, \cdots, p \end{aligned} \tag{10}$$

where f_i *is a posynomial function and* g_i *is a monomial function. The optimization variables are denoted by* $\boldsymbol{x}$ *which is an n-dimensional vector.*

The GP problem (10) is not convex, however it can be shown that a GP can be converted into a convex optimization problem via a logarithmic and variable transformation. We can first transform the variables as $u_i = \log x_i$ and take the logarithm of the objective and constraint functions to obtain the following equivalent problem:

$$\begin{aligned} \text{minimize} \quad & \log f_0(e^{u_1}, ..., e^{u_n}) \\ \text{subject to} \quad & \log f_i(e^{u_1}, ..., e^{u_n}) \leq 1, \quad i = 1, \cdots, m, \\ & \log g_i(e^{u_1}, ..., e^{u_n}) = 1, \quad i = 1, \cdots, p. \end{aligned} \tag{11}$$

- The transformed monomial function $g_i(\mathbf{x})$ can be written as follows:

  $$\log g_i(e^{u_1}, ..., e^{u_n}) = \mathbf{a_i}^T \mathbf{u} + b_i,$$

  where $\mathbf{a_i} \in \mathbb{R}^{n \times 1}$, $\mathbf{u} \in \mathbb{R}^{n \times 1}$, and $b_i \in \mathbb{R}$.

- The transformed posynomial function $f_i(\mathbf{x})$ can be written as sum of $K_i \in \mathbb{Z}^+$ monomials:

  $$f_i(\mathbf{u}) = \sum_{k=1}^{K_i} e^{\mathbf{a_{ik}}^T \mathbf{u} + b_{ik}},$$

  where $\mathbf{a_{ik}} \in \mathbb{R}^{n \times 1}$, $\mathbf{u} \in \mathbb{R}^{n \times 1}$, and $b_{ik} \in \mathbb{R}$.

Note that above transformations would turn a monomial function into an *affine function* and a posynomial into a *sum of exponentials of affine functions.* Thus, GP (10) transforms into the following equivalent convex optimization problem:

$$\begin{aligned} \text{minimize} \quad & \log \sum_{k=1}^{K_i} e^{\mathbf{a_{0k}}^T \mathbf{u} + b_{0k}} \\ \text{subject to} \quad & \log \sum_{k=1}^{K_i} e^{\mathbf{a_{ik}}^T \mathbf{u} + b_{ik}} \leq 0, \quad i = 1, \cdots, m, \\ & Gu + d = 0. \end{aligned} \tag{12}$$

2.4 Alternating Optimization

Let $f(\mathbf{x})$ is the target optimization function and $\mathbf{x}$ is an n-dimensional vector of optimization variables. The underlying theory behind alternating optimization (AO) is to replace difficult joint optimization of $f(\mathbf{x})$ over all $\mathbf{x}$ variables, with a sequence of easier optimizations; easier in the sense that it involves only grouped subsets of the variables [19]. Thus, AO is an iterative procedure to optimize (minimize or maximize) the target function $f(\mathbf{x})$ in which minimization/maximization is performed jointly over all variables by iteratively optimizing $f(\mathbf{x})$ over subset of optimization variables. In state of the art, AO is being considered as an efficient computational framework to solve non-convex optimization problems [19, 20]; for max-min rate optimization problem, see [11].

3 SYSTEM MODEL, PROBLEM FORMULATION AND ALGORITHM DERIVATION

3.1 System Model and Analysis

3.1.1 Introduction

In this chapter, we derive a closed form expression for spectral efficiency (SE) in cell-free massive MIMO network in terms of weight and power coefficients. Here, we assume that local MMSE processing is used at each AP. To maximize achievable SE, prior published work [9, Corollary 2] treat power coefficients as fixed. However, in our study, we consider power coefficients as variables and propose an iterative algorithm to maximize achievable SE by changing both power and weight coefficients.

3.1.2 Cell-free System Model

We consider a cell-free massive MIMO system with L APs each equipped with N antennas and K single antenna users randomly distributed in a large area. We assume that $K \ll L$. Each AP is connected to the central processing unit (CPU) through fronthaul connection. The channel co-efficient vector between lth AP and kth UE is denoted by $\mathbf{h}_{kl} \in \mathbb{C}^N$. We model the channel using block fading model where $\mathbf{h}_{kl}$ is fixed during time-frequency blocks of τ_c samples. In other words, coherence block consists of τ_c number of samples. Channel coefficients are independent and identically distributed (i.i.d) random variables. In each block, $\mathbf{h}_{kl}$ is an independent realization from a correlated Rayleigh fading distribution defined as follows:

$$\mathbf{h}_{kl} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{R}_{kl}), \tag{13}$$

where $\mathbf{R}_{kl} \in \mathbb{C}^{N \times N}$ is the spatial correlation matrix.

3.1.3 Pilot Transmission and Channel Estimation

We consider uplink transmission where all users send uplink pilots and payload data in τ_p and $\tau_c - \tau_p$ samples respectively. In order to estimate the channel coefficients in the uplink, all users send simultaneously pilot sequences of length τ_p to the APs.

Let τ_p mutually orthogonal pilot signals $\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_{\tau_p}$ with $\|\boldsymbol{\phi}_t\|^2 = \tau_p$ are used for channel estimation. We consider a large network where $K > \tau_p$ so that each pilot signal is assigned to more than one UE . t_k represents the pilot index assigned to kth UE as $t_k \in \{1, \ldots, \tau_p\}$. Then, the received pilot matrix $\mathbf{Z}_l \in \mathbb{C}^{N \times \tau_p}$ at lth AP is given by

$$\mathbf{Z}_l = \sum_{i=1}^{K} \sqrt{p_i} \mathbf{h}_{il} \boldsymbol{\phi}_{t_i}^T + \mathbf{N}_l, \tag{14}$$

where $p_i \geq 0$ is the transmit power of ith UE , $\mathbf{N}_l \in \mathbb{C}^{N \times \tau_p}$ is the noise at AP. The elements of $\mathbf{N}_l$ are assumed to be independent and identically distributed as $\mathcal{CN}(0, \sigma^2)$ and σ^2 is the noise power.

After AP receives $\mathbf{Z}_l$, it first correlates $\mathbf{Z}_l$ with corresponding normalized pilot signal $\boldsymbol{\phi}_{t_k}/\sqrt{\tau_p}$, which is denoted as $\mathbf{z}_{t_k l} \triangleq \frac{1}{\sqrt{\tau_p}}\mathbf{Z}_l\boldsymbol{\phi}_{t_k}^* \in \mathbb{C}^N$. It can be simplified as follows:

$$\begin{aligned}\mathbf{z}_{t_k l} &= \sum_{i=1}^{K} \frac{\sqrt{p_i}}{\sqrt{\tau_p}}\mathbf{h}_{il}\boldsymbol{\phi}_{t_i}^T\boldsymbol{\phi}_{t_k}^* + \frac{1}{\sqrt{\tau_p}}\mathbf{N}_l\boldsymbol{\phi}_{t_k}^* \\ &= \sum_{i\in\mathcal{P}_k} \sqrt{p_i\tau_p}\mathbf{h}_{il} + \mathbf{n}_{t_k l},\end{aligned} \tag{15}$$

where $\mathbf{n}_{t_k l}$ is an N-dimensional vector distributed as $\mathcal{N}_{\mathbb{C}}(\mathbf{0}, \sigma^2\mathbf{I}_n)$. After performing correlation operation, the MMSE estimate of channel coefficient vector between lth AP and kth UE, $\hat{\mathbf{h}}_{kl}$ is given by [9],

$$\hat{\mathbf{h}}_{kl} = \sqrt{p_k\tau_p}\mathbf{R}_{kl}\boldsymbol{\Psi}_{t_k l}^{-1}\mathbf{z}_{t_k l}, \tag{16}$$

where $\boldsymbol{\Psi}_{t_k l} = \mathbb{E}\{\mathbf{z}_{t_k l}\mathbf{z}_{t_k l}^H\} = \sum_{i\in\mathcal{P}_k}\tau_p p_i\mathbf{R}_{il} + \mathbf{I}_N$ is the correlation matrix of the received signal $\mathbf{z}_{t_k l}$. The estimate $\hat{\mathbf{h}}_{kl}$ and estimation error $\tilde{\mathbf{h}}_{kl} = \mathbf{h}_{kl} - \hat{\mathbf{h}}_{kl}$ are independent vectors distributed as $\hat{\mathbf{h}}_{kl} \sim \mathcal{N}_{\mathbb{C}}\left(\mathbf{0}, p_k\tau_p\mathbf{R}_{kl}\boldsymbol{\Psi}_{t_k l}^{-1}\mathbf{R}_{kl}\right)$ and $\tilde{\mathbf{h}}_{kl} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{C}_{kl})$ with

$$\mathbf{C}_{kl} = \mathbb{E}\{\tilde{\mathbf{h}}_{kl}\tilde{\mathbf{h}}_{kl}^H\} = \mathbf{R}_{kl} - p_k\tau_p\mathbf{R}_{kl}\boldsymbol{\Psi}_{t_k l}^{-1}\mathbf{R}_{kl}. \tag{17}$$

3.1.4 Uplink Data Transmission

During the uplink data transmission, all users send their signals simultaneously to all APs. The transmitted signal from ith UE is distributed as $s_i \sim \mathcal{CN}(0, p_i)$ and p_i is the transmitted power at ith UE. The N-dimensional receiver noise at lth AP is distributed as $\mathbf{n}_l \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, p_i)$. The received signal at lth AP is given by

$$\mathbf{y}_l = \sum_{i=1}^{K}\mathbf{h}_{il}s_i + \mathbf{n}_l. \tag{18}$$

We assume that an L-MMSE detector is employed at each AP and the received signal at lth AP is first pre multiplied by $\mathbf{v}_{kl}$ where $\mathbf{v}_{kl} \in \mathbb{C}^N$ is the local combining vector at lth AP to estimate s_k. Let $\check{s}_{kl}$ is the local estimate of kth user at lth AP. The local estimate of s_k is given by

$$\check{s}_{kl} \triangleq \mathbf{v}_{kl}^H\mathbf{y}_l = \mathbf{v}_{kl}^H\mathbf{h}_{kl}s_k + \sum_{i=1, i\neq k}^{K}\mathbf{v}_{kl}^H\mathbf{h}_{il}s_i + \mathbf{v}_{kl}^H\mathbf{n}_l. \tag{19}$$

The mean squared error (MSE) of kth symbol at lth AP is denoted by $\text{MSE}_{kl} = \mathbb{E}\{|s_k - \mathbf{v}_{kl}^H\mathbf{y}_l|^2 \big| \{\hat{\mathbf{h}}_{il}\}\}$. The combining vector that minimizes the MSE can be derived by computing the first derivative of conditional expectation and setting it to zero. The optimal combining vector which minimizes the MSE is given by

$$\mathbf{v}_{kl} = p_k\left(\sum_{i=1}^{K}p_i\left(\hat{\mathbf{h}}_{il}\hat{\mathbf{h}}_{il}^H + \mathbf{C}_{il}\right) + \sigma^2\mathbf{I}_N\right)^{-1}\hat{\mathbf{h}}_{kl}. \tag{20}$$

The pre processed signals using combining vectors at each AP are then forwarded to CPU for signal detection. The forwarded signals are further multiplied by weight coefficients at CPU to improve achievable rate. CPU does not have the knowledge of the channel estimates and therefore, only channel statistics are utilized to maximize SE.

Let a_{kl} is the weight coefficient of kth user at lth AP. The aggregated signal at CPU to detect s_k is given by

$$\hat{s}_k = \sum_{l=1}^{L} a_{kl}^{*} \check{s}_{kl}. \tag{21}$$

By substituting (19) in (21), we can derive that

$$\begin{aligned}\hat{s}_k &= \left(\sum_{l=1}^{L} a_{kl}^{*} \mathbf{v}_{kl}^{H} \mathbf{h}_{kl}\right) s_k + \sum_{l=1}^{L} a_{kl}^{*} \left(\sum_{i=1, i\neq k}^{K} \mathbf{v}_{kl}^{H} \mathbf{h}_{il} s_i\right) + \mathbf{n}_k' &\text{(22a)}\\ &= \mathbf{a}_k^{H} \mathbf{g}_{kk} s_k + \sum_{i=1, i\neq k}^{K} \mathbf{a}_k^{H} \mathbf{g}_{ki} s_i + \mathbf{n}_k', &\text{(22b)}\end{aligned}$$

where $\mathbf{g}_{ki} = [\mathbf{v}_{k1}^{H}\mathbf{h}_{i1} \, \ldots \, \mathbf{v}_{kL}^{H}\mathbf{h}_{iL}]^T \in \mathbb{C}^L$ is the receive-combined channels between kth UE and each of the APs, $\mathbf{a}_k = [a_{k1} \, \ldots \, a_{kL}]^T \in \mathbb{C}^L$ is the weighting coefficient vector, $\{\mathbf{a}_k^{H}\mathbf{g}_{ki} : i = 1, \ldots, K\}$ is the set of effective channels, and $\mathbf{n}_k' = \sum_{l=1}^{L} a_{kl}^{*} \mathbf{v}_{kl}^{H} \mathbf{n}_l$.

Using the channel statistics at the CPU, the effective SINR of kth UE can be expressed as [9],

$$\begin{aligned}SINR_k &= \frac{p_k \left|\mathbf{a}_k^{H}\mathbb{E}\{\mathbf{g}_{kk}\}\right|^2}{\sum\limits_{i=1}^{K} p_i \mathbb{E}\{|\mathbf{a}_k^{H}\mathbf{g}_{ki}|^2\} - p_k \left|\mathbf{a}_k^{H}\mathbb{E}\{\mathbf{g}_{kk}\}\right|^2 + \sigma^2 \mathbf{a}_k^{H}\mathbf{D}_k\mathbf{a}_k} &\text{(23a)}\\ &= \frac{p_k \left|\mathbf{a}_k^{H}\mathbb{E}\{\mathbf{g}_{kk}\}\right|^2}{\mathbf{a}_k^{H}(\sum\limits_{i=1}^{K} p_i \mathbb{E}\{|\mathbf{g}_{ki}|^2\} - p_k \left|\mathbb{E}\{\mathbf{g}_{kk}\}\right|^2 + \sigma^2 \mathbf{D}_k)\mathbf{a}_k} &\text{(23b)}\\ &= \frac{\mathbf{a}_k^{H}\left(p_k \mathbb{E}\{\mathbf{g}_{kk}\}(\mathbb{E}\{\mathbf{g}_{kk}\})^H\right)\mathbf{a}_k}{\mathbf{a}_k^{H}(\sum\limits_{i=1}^{K} p_i \mathbb{E}\{|\mathbf{g}_{ki}|^2\} - p_k \left|\mathbb{E}\{\mathbf{g}_{kk}\}\right|^2 + \sigma^2 \mathbf{D}_k)\mathbf{a}_k} &\text{(23c)}\end{aligned}$$

where $\mathbf{D}_k = (\mathbb{E}\{\|\mathbf{v}_{k1}\|^2\}, \ldots, \mathbb{E}\{\|\mathbf{v}_{kL}\|^2\}) \in \mathbb{C}^{L\times L}$ and the expectations are with respect to all sources of randomness. Note that the uplink effective SINR of kth UE can be formulated as a generalized Rayleigh quotient [21] with respect to $\mathbf{a}_k$.

Assuming that UEs transmit with fixed powers, we maximize generalized Rayleigh quotient in (23b). Hence, the optimal weight coefficient vector of kth UE, under fixed power constraints is given by,

$$\mathbf{a}_k = \left(\sum_{i=1}^{K} p_i \mathbb{E}\{\mathbf{g}_{ki}\mathbf{g}_{ki}^{H}\} + \sigma^2 \mathbf{D}_k\right)^{-1} \mathbb{E}\{\mathbf{g}_{kk}\} \tag{24}$$

which leads to the maximum value under fixed power constraints,

$$\begin{aligned}SINR_{k,max} &= p_k \mathbb{E}\{\mathbf{g}_{kk}^{H}\}\\ &\times \left(\sum_{i=1}^{K} p_i \mathbb{E}\{\mathbf{g}_{ki}\mathbf{g}_{ki}^{H}\} + \sigma^2 \mathbf{D}_k - p_k \mathbb{E}\{\mathbf{g}_{kk}\}\mathbb{E}\{\mathbf{g}_{kk}^{H}\}\right)^{-1} \mathbb{E}\{\mathbf{g}_{kk}\}.\end{aligned} \tag{25}$$

An achievable rate of kth UE is given by

$$R_k = \left(1 - \frac{\tau_p}{\tau_c}\right) \log_2 \left(1 + SINR_k\right). \tag{26}$$

Max-min rate problem can be formulated such that minimum uplink user rate is maximized subject to individual transmit power constraint at each UE. This max-min rate problem can be formulated as follows:

$$\begin{aligned} \underset{p_k,\, \mathbf{a}_k}{\text{maximize}} \quad & R_k \\ \text{subject to} \quad & ||\mathbf{a}_k|| = 1, \quad \forall\ k, \\ & 0 \le p_k \le p_{\max}^{(k)}, \quad \forall\ k, \end{aligned} \tag{27}$$

where $p_{max}^{(k)}$ is the maximum transmit power available at user k.

3.2 Problem Formulation and Algorithm Derivation

In this section, we design an iterative algorithm as a suboptimal solution to maximize the minimum SINR in cell-free massive MIMO system with L-MMSE detector.

Problem (27) is not jointly convex with respect to optimization variables, $\mathbf{a}_k$ and p_k. Thus, standard convex optimization tools cannot be directly applied to solve problem (27). Therefore, in the sequel, we propose an approach to find suboptimal solution for (27), by alternately solving two subproblems, as illustrated in Figure 4.

3.2.1 Weighting Coefficients Design

First, we fixed transmit powers of all UEs and solve the weighting coefficient subproblem to maximize uplink SINR of kth user (23c), for all k. These optimal weight coefficients are obtained by solving optimization problem (28) as follows:

$$\begin{aligned} \underset{p_k,\, \mathbf{a}_k}{\text{maximize}} \quad & \frac{\mathbf{a}_k^H \left(p_k \mathbb{E}\{\mathbf{g}_{kk}\}(\mathbb{E}\{\mathbf{g}_{kk}\})^H\right) \mathbf{a}_k}{\mathbf{a}_k^H (\sum\limits_{i=1}^{K} p_i \mathbb{E}\{|\mathbf{g}_{ki}|^2\} - p_k \left|\mathbb{E}\{\mathbf{g}_{kk}\}\right|^2 + \sigma^2 \mathbf{D}_k) \mathbf{a}_k} \\ \text{subject to} \quad & ||\mathbf{a}_k|| = 1, \quad \forall\ k. \end{aligned} \tag{28}$$

Problem (28) is a generalized eigenvalue problem [22]. The optimal coefficient values can be obtained by determining the generalized eigenvalue of the matrix pair $\mathbf{A}_k = p_k \mathbb{E}\{\mathbf{g}_{kk}\}\mathbb{E}\{\mathbf{g}_{kk}\}^H$ and $\mathbf{B}_k = \sum\limits_{i=1}^{K} p_i \mathbb{E}\{|\mathbf{g}_{ki}|^2\} - p_k \left|\mathbb{E}\{\mathbf{g}_{kk}\}\right|^2 + \sigma^2 \mathbf{D}_k$ corresponding to the maximum generalized eigenvalue.

3.2.2 Power Allocation

We solve the power allocation subproblem by fixing weight coefficients in master problem (27). The power allocation subproblem can be formulated as following max-min problem:

$$\begin{aligned} \max_{p_k} \quad & \min_{k=1,\cdots,K} \quad \text{SINR}_k \\ \text{subject to} \quad & 0 \leq p_k \leq p_{max}^{(k)}, \quad \forall\ k. \end{aligned} \tag{29}$$

Then, problem (29) can be rewritten by introducing a new slack variable as

$$\begin{aligned} \underset{t,\, p_k}{\text{maximize}} \quad & t \\ \text{subject to} \quad & 0 \leq p_k \leq p_{\max}^{(k)}, \quad \forall\ k, \\ & \text{SINR}_k \geq t, \quad \forall\ k. \end{aligned} \tag{30}$$

From (23c) the uplink effective SINR of kth UE can be approximated as follows.

$$SINR_k \approx \frac{\mathbf{a}_k^H \left(p_k \mathbb{E}\{\mathbf{g}_{kk}\} (\mathbb{E}\{\mathbf{g}_{kk}\})^H \right) \mathbf{a}_k}{\mathbf{a}_k^H (\sum\limits_{i=1 \neq k}^{K} p_i \mathbb{E}\{|\mathbf{g}_{ki}|^2\} + \sigma^2 \mathbf{D}_k) \mathbf{a}_k} \tag{31}$$

Proposition 1: With the SINR approximation in (31), problem (30) can be approximated into a GP.

Proof: The form of the SINR constraint in (30) is not a posynomial function. Therefore, it can be first rewritten and then approximated into a posynomial function as follows:

$$\frac{\mathbf{a}_k^H \left(\sum\limits_{i=1}^{K} p_i \mathbb{E}\{|\mathbf{g}_{ki}|^2\} - p_k \left| \mathbb{E}\{\mathbf{g}_{kk}\} \right|^2 + \sigma^2 \mathbf{D}_k \right) \mathbf{a}_k}{\mathbf{a}_k^H \left(p_k \mathbb{E}\{\mathbf{g}_{kk}\} (\mathbb{E}\{\mathbf{g}_{kk}\})^H \right) \mathbf{a}_k} \leq \frac{1}{t}, \forall k.$$

From (31), SINR constraint can be approximated as follows:

$$\frac{\mathbf{a}_k^H \left(\sum\limits_{i=1 \neq k}^{K} p_i \mathbb{E}\{|\mathbf{g}_{ki}|^2\} + \sigma^2 \mathbf{D}_k \right) \mathbf{a}_k}{\mathbf{a}_k^H \left(p_k \mathbb{E}\{\mathbf{g}_{kk}\} (\mathbb{E}\{\mathbf{g}_{kk}\})^H \right) \mathbf{a}_k} \leq \frac{1}{t}, \forall k. \tag{32}$$

With a simple rearrangement, (32) can be converted to following equivalent inequality.

$$p_k^{-1} \left(\sum_{i \neq k}^{K} a_{ki} p_i + c_k \right) < \frac{1}{t}, \forall k, \tag{33}$$

where

$$a_{ki} = \frac{\mathbf{a}_k^H (\mathbb{E}\{|\mathbf{g}_{ki}|^2\}) \mathbf{a}_k}{\mathbf{a}_k^H (\mathbb{E}\{\mathbf{g}_{kk}\} (\mathbb{E}\{\mathbf{g}_{kk}\})^H) \mathbf{a}_k}$$

and

$$c_k = \frac{\mathbf{a}_k^H \mathbf{D}_k \mathbf{a}_k}{\mathbf{a}_k^H (\mathbb{E}\{\mathbf{g}_{kk}\} (\mathbb{E}\{\mathbf{g}_{kk}\})^H) \mathbf{a}_k}.$$

The left-hand side of (33) is a posynomial function. Both inequality constraint and objective function are in the form of posynomial function. Therefore, approximated version of the power allocation problem (30) is a standard GP problem as defined in (10). Q.E.D.

Therefore, problem (30) can be formulated as a geometric programming problem which can be solved using convex optimization software.

3.2.3 Proposed Algorithm

Thus, we proposed an iterative algorithm to find suboptimal solution for master problem (27) by alternately solving these two subproblems. The proposed algorithm is summarized in Algorithm 1.

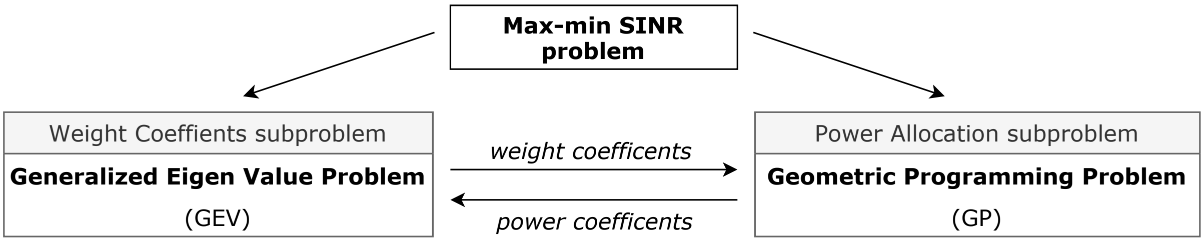

Figure 4. The basic idea of Algorithm 1.

Algorithm 1

1. Initialize $\mathbf{p}^{(0)} = [p_1^{(0)}, p_2^{(0)}, \cdots, p_K^{(0)}]$, $i = 0$
2. Repeat
3. $i = i + 1$
4. Set $\mathbf{p}^{(i)} = \mathbf{p}^{(i-1)}$ and find the optimal weight coefficients $\mathbf{a}^{(i)} = [\mathbf{a}_1^{(i)}, \mathbf{a}_2^{(i)}, \cdots, \mathbf{a}_K^{(i)}]$ through solving the generalized eigenvalue Problem (28)
5. Compute $\mathbf{p}^{(i+1)}$ through solving Problem (30)
6. Go back to Step 3 and repeat until required accuracy

4 SIMULATION SETUP AND NUMERICAL RESULTS

In this chapter, we present numerical results to evaluate the performance and convergence of our proposed iterative algorithm with different simulation settings.

4.1 Parameters and Setup

We define the simulation setup of cell-free mMIMO model with parameters summarized in Table 1. The system consists of L number of M-antenna APs which are uniformly distributed in a grid, covering a square area of 1000×1000 m ($D \times D$). Initially, we divide simulation area into 4 virtual cells in order to facilitate UE placement. It is assumed that total number of UEs as K in which $K/4$ number of users are randomly dropped in each cell. Eventually, simulation results are averaged over 100 UE distributions. Here, we consider communication over a 20 MHz bandwidth with a receiver noise power σ^2 of -96 dBm [9]. It is assumed that maximum transmit power of each UE lies between 90 mW - 120 mW and all UEs transmit at their maximum transmit power in channel estimation phase. Moreover, simulations are performed over three cases of pilot reuse factors ($f = 1, 4, 8$). For pilot reuse factor of 1 (mutually orthogonal pilot assignment), we assume that $\tau_p = K$, however τ_p is reduced with higher reuse factors.

Similar to [9], 3GPP urban microcell model is considered as propagation model with 2 GHz carrier frequency. Small scale fading coefficients are generated using correlated Rayleigh fading in which Gaussian local scattering model with 15° angular standard deviation [21, Sec. 2.6] contributes for the spatial correlation matrix. Moreover, large scale fading coefficients are generated independently as follows [9]:

$$\beta_{kl}\,[\text{dB}] = -30.5 - 36.7 \log_{10}\left(\frac{d_{kl}}{1\,\text{m}}\right) + F_{kl} \tag{34}$$

where d_{kl} is the distance between UE k and AP l and $F_{kl} \sim \mathcal{N}(0, 4^2)$ is the shadow fading.

Table 1. Cell-free Massive MIMO network.

Simulation area	$1\,\text{km} \times 1\,\text{km}$
Bandwidth	20 MHz
Number of APs	L
Number of UEs	K
Number of Antennas per AP	N
UL noise power	-96 dBm
Samples per coherence block	$\tau_c = 200$
Pilot reuse factors	f

4.2 Results and Discussions

4.2.1 Performance Analysis

In this subsection, we evaluate the performance of our proposed algorithm with respect to fixed power scenario for few simulation setups.

- Simulation setup 1

  We first compare the cumulative distribution of the achievable uplink rate of given simulation setup with hundred of four-antenna APs, forty users ($L = 100$, $M = 4$, $K = 40$) and pilot reuse factor of 4. As illustrated in Figure 5, the proposed algorithm significantly outperforms due to the fact that optimization is performed over both power and weight coefficients.

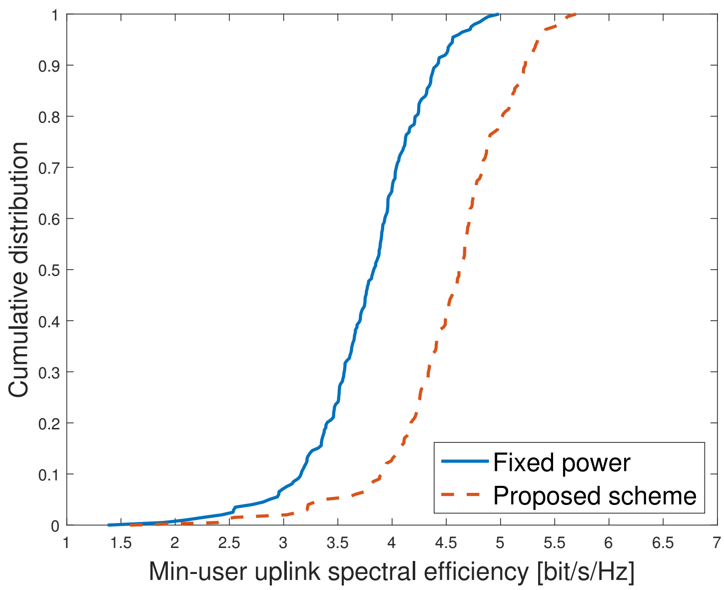

Figure 5. The cumulative distribution of the per-user uplink rate, with random pilots assignments of f = 4 for $L = 100$, $K = 40$, $N = 4$ and $D = 1$ km.

- Simulation setup 2

  Next, we evaluate the cumulative distribution for three cases of mutually orthogonal pilots, $f = 2$ and $f = 4$. For this simulation, cell-free massive MIMO system is considered with sixty-four number of two-antenna ($L = 64$, $M = 2$) APs and sixteen number of users ($K = 16$). Figure 6 presents the cumulative distribution of the achievable uplink rate for the proposed algorithm and fixed power scheme for three different pilot assignments. The results show that even for different pilot assignments, the performance of proposed scheme is higher compared to fixed power case. However, in the case of orthogonal pilot assignment, there is a prominent increase in the min-user uplink rate compared to non-orthogonal pilot schemes. Among non-orthogonal pilot assignments, $f = 2$ case outperforms over $f = 4$ case due to the fact that channel estimation error increases with pilot reuse factor.

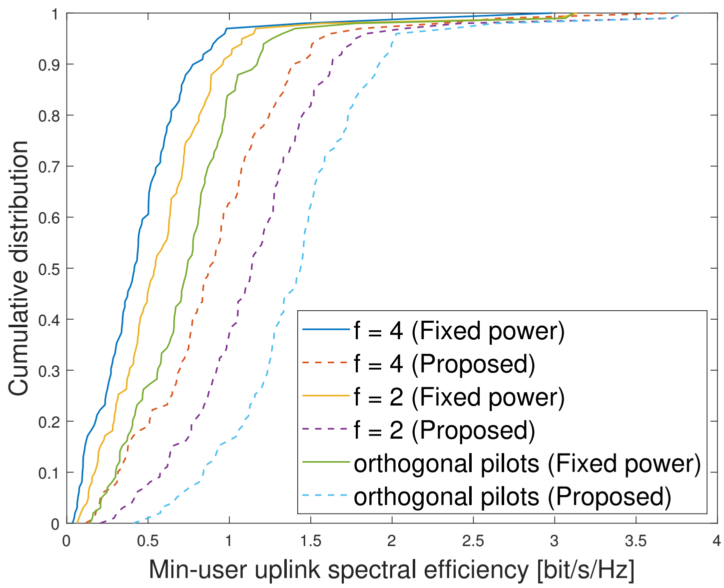

Figure 6. Min-user uplink rate with different pilot reuse factors $L = 64$, $K = 16$, $N = 2$ and $D = 1$ km. The dashed curves refer to the proposed Algorithm 1, while the solid curves present the fixed power case.

- Simulation setup 3

  Next, we evaluate the cumulative distribution of the achievable uplink rate of another simulation setup with hundred of four-antenna APs and forty users ($L = 100$, $M = 4$, $K = 40$) for both orthogonal pilots and pilot reuse factor of 8. Figure 7 presents the cumulative distribution of the achievable uplink rate for the proposed algorithm and fixed power scheme. The results show that performance of our proposed scheme is higher compared to fixed power scheme and $f = 1$ outperforms over $f = 8$.

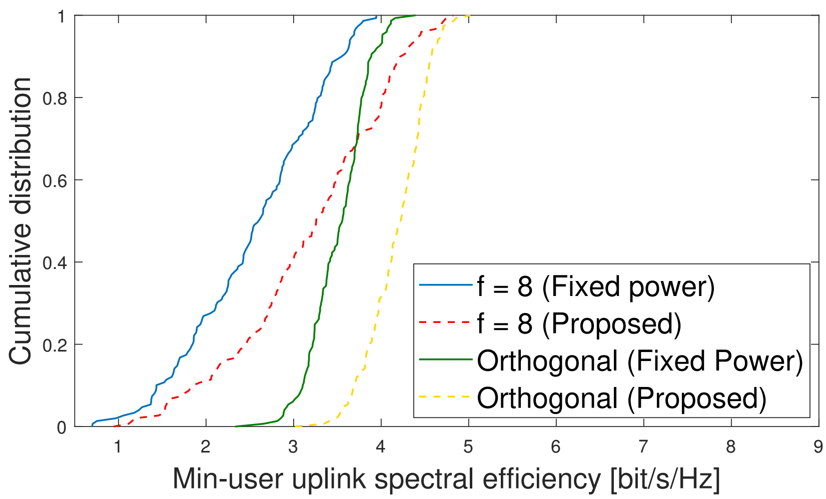

Figure 7. Min-user uplink rate with different pilot reuse factors $L = 100$, $K = 40$, $N = 4$ and $D = 1$ km. The dashed curves refer to the proposed Algorithm 1, while the solid curves present the fixed power case.

4.2.2 Convergence of proposed algorithm

Next, we investigate the convergence of our proposed algorithm over set of different channel realizations.

- Simulation setup 1

  We consider a simulation setup with hundred of four-antenna APs, forty users ($L = 100$, $M = 4$, $K = 40$), and pilot reuse factor of 4. As shown in Figure 8, the algorithm finds fast suboptimal solution within two number of iterations.

  The effectiveness of the proposed algorithm is illustrated in the simulation result which maximizes the smallest SINR of the system, in each iteration. The proposed algorithm requires no additional precautions in the initialization, and convergence to a suboptimal solution is possible within a very small number of iterations.

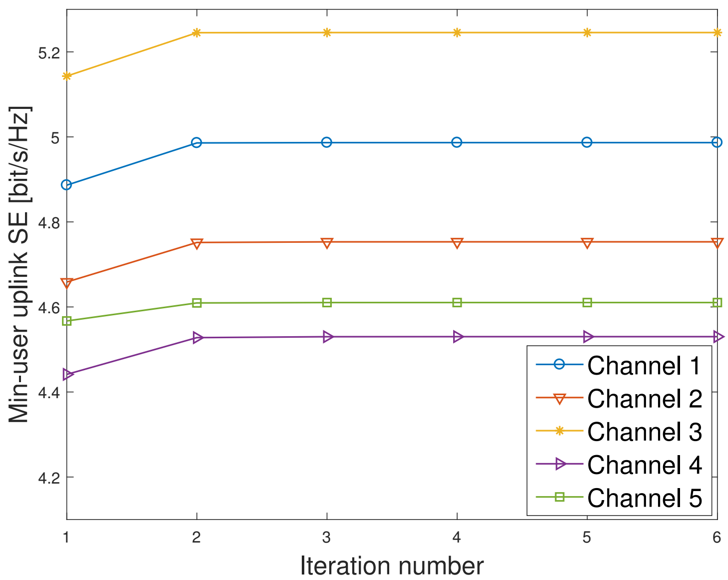

Figure 8. The convergence of the proposed max-min SINR approach for $L = 100$, $K = 40$, $N = 4$, $f = 4$ and $D = 1$ km.

- Simulation setup 2

  The convergence of proposed algorithm is further investigated for another simulation setup with hundred of two-antenna APs, sixteen users ($L = 64$, $M = 2$, $K = 16$), and pilot reuse factor of 2.

  As illustrated in Figure 9 , the convergence to a suboptimal solution is possible within a very small number of iterations.

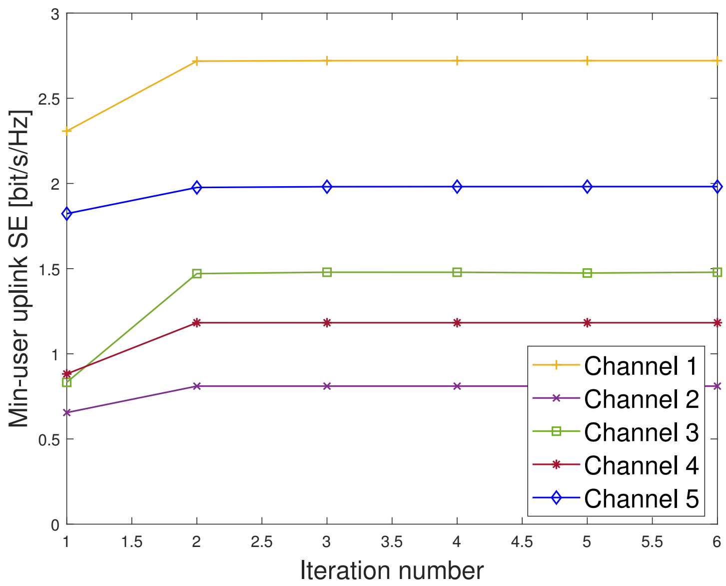

Figure 9. The convergence of the proposed max-min SINR approach for $L = 64$, $K = 16$, $N = 2$, $f = 2$ and $D = 1$ km.

5 CONCLUSION AND FUTURE WORK

5.1 Summary and conclusion

We studied cell-free mMIMO which has been identified as a potential candidate to cater the high capacity requirements of next generation networks; 5G and beyond. Compared to collocated mMIMO, coordinated multi-point (CoMP) which is a distributed version of MIMO provides improved spectral efficiency specially, for cell edge users via the coordination between the interfering and serving BSs. In the sequel, cell-free mMIMO, a hybrid model, that combines features of distributed MIMO and mMIMO, improves the spectral efficiency by mitigating inter-cell interference with its user centric implementation.

The max-min optimization problem in the uplink cell-free mMIMO system with L-MMSE reciever was considered. We proposed an alternating, iterative and suboptimal solution to maximize the smallest SINR by changing both weight and power coefficients. Although the original problem is nonconvex, the proposed algorithm finds a suboptimal solution. The original problem was decomposed in to two subproblems; weight coefficient subproblem was solved using generalized eigen value problem whereas approximated version of power coefficient problem was solved by geometric programming problem. Effectiveness of the proposed algorithm was discussed in the simulation result which maximizes the smallest SINR of the system, in each iteration. Further, we provided convergence results for a set of different channel realizations of two different simulation setups. As evidenced from these numerical results, the convergence of our proposed algorithm was validated. However, the proposed algorithm requires no additional precautions in the initialization, and convergence to a suboptimal solution is possible within a very small number of iterations. These simulation results validated that the smallest SINR of the system is higher with proposed algorithm, with respect to fixed power scheme.

In addition, we studied the behaviour of minimum uplink rates for three different pilot assignments and results were numerically presented. These results empirically showed that even for different pilot assignments, the performance of proposed scheme is higher compared to fixed power case. Further, it was understood that orthogonal pilot assignment outperforms with respect to non-orthogonal pilot assignments.

5.2 Future work

The analysis of the convergence and optimality of the proposed alternating algorithm is one of possible research work to be carried out in the future. Moreover, the performance analysis of “cell-free” networks have basically utilized correlated Rayleigh fading and 3GPP microcell urban channel model with 2GHz carrier frequency. However, new performance evaluations under realistic channel conditions is an interesting topic for future work.

Moreover, Full-dimension (FD) MIMO system is considered to be the official MIMO enhancement in third generation partnership project (3GPP) [23]. FD-MIMO system consists of active antenna elements which are capable of dynamically adjusting the gain of the antenna elements, providing an additional gain with respect to conventional MIMO. Therefore, another possible extension is to explore optimization techniques and signal processing aspects for novel hybrid system model which combines cell-free massive MIMO and FD-MIMO systems.

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