# Estimation of Churches Frequencies Based on Simplified Geometry Parameters

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**Abstract.** Nowadays, the modal properties of structures can be obtained by experimental methods that are still expensive for professionals who have to deal with the seismic assessment. Additionally, for heritage buildings, due to their complexity, the identification of the modal properties can be challenging and requires of experience on the field. This paper aims to develop a methodology to predict the frequency response of churches based on simplified geometrical parameters. In this context, an extensive literature review was carried out to collect data, aiming to define the principal typologies of churches. Afterwards, a parametric analysis with numerical simulations of the eigenvalue problem was performed to identify which are the main geometrical parameters that influence the dynamic behavior of the churches. The correlations between the natural frequencies and relevant geometrical parameters are illustrated and discussed in order to develop a proposal for the estimation of the natural frequencies of the churches. Churches with single nave and three naves were defined as the two principal types of churches, and six equations were proposed, aiming to estimate the first three natural frequencies for each type of church. Finally, the methodology was validated on real case studies found in literature, in which experimental identification of frequencies was performed. This comparison allowed to evaluate the accuracy of the method to estimate the natural frequencies.

**Keywords:** Heritage constructions · Historical church
Natural frequency of vibration · Eigenvalue analysis · Parametric analysis
Dynamic identification methods


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## 1 Introduction

The current research was developed on churches of Portugal, which is one of the oldest European countries; moreover, these structures constitute a part of the heritage constructions built in masonry with high patrimonial value.

In Portugal, the seismic assessment of heritage constructions constitutes an important task given that is located in a region of moderate seismic hazard [1, 2]. Additionally, churches usually present high seismic vulnerability due to their structural configuration and geometrical proportions, material conformation, and the accumulation of chemical, physical, and mechanical damage due to their age.

In one hand, it is important to know the seismicity of the region, as well as the site condition of the construction to evaluate the seismic hazard. On the other hand, the properties of the structure (geometry, materials and the connections between different structural elements) define the dynamic structural behavior of the construction, being a fundamental issue for the seismic vulnerability assessment of the construction [3].

It is challenging to predict the actual period of vibration of a building for assessing the seismic behavior due to many uncertain parameters (i.e., current material properties, the seismic mass, soil condition, and contribution of secondary elements to the lateral stiffness of a building). Therefore, it is a common practice to use, analytical, experimental, and approximate empirical methods to estimate the fundamental period/frequency for the design of new construction as well as the assessment of an existing building. Among these methods, the following might be found: Rayleigh approximation method, eigenvalue analysis of a numerical model, dynamic identification methods, and empirical relations.

Nowadays, the dynamic identification methods still continue to be expensive to the majority owners and designers who have to deal with the seismic assessment [4]. Additionally, churches commonly present complex geometry and significant dimensions, which usually difficult the use of the dynamic identification methods to estimate the main dynamic parameters (mode shapes, frequencies, and damping coefficient) [5, 6].

For conventional structures, many recommendations can be found in literature to estimate their natural period in a simple way using empirical correlations (i.e., Eurocode 8 [7], ASCE-07 [8] and NTC-2008 [9]) but the information about masonry structures and churches is scarce. The possibility of applying a simplified procedure to determinate the natural frequencies of the different churches typologies is, therefore, strategic to achieve useful information in the context of seismic risk assessment and retrofitting. Additionally, the simplified correlations allow an optimization on both, the costs and on the execution time. For the above reasons, it was decided to define a correlation to estimate the frequency response of churches, in order to address the seismic vulnerability assessment.

# 2 Geometrical Survey and Analysis of the Data

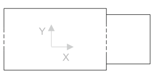
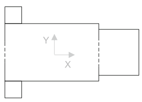
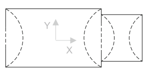
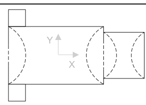
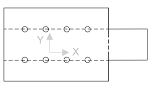
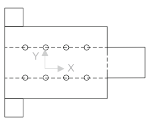
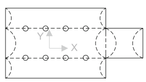
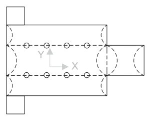
The selection of churches to perform the geometrical survey was performed taking into account the following aspects: geometric variety corresponding to the architectural styles, which represents the evolution of construction of churches in Portugal during 14 centuries, and their location in the country, aiming to obtain a representative sample of the churches along the country. The data of the churches was taken from [10, 11] and was complemented by the plans and drawings published by [12]. 50 churches located in 13 Districts of the 18 Districts were studied, most of them are located in Porto District (49%), which is one of the most occupied with religious buildings in Portugal. On the other hand, the Romanesque, Manueline and Baroque styles are the most common in the selected group of churches.

## 2.1 Typological Classification of the Churches

The characterization of churches by typologies was a major step for carrying out the numerical simulation and was performed by a geometrical survey. The definition of typologies was based on plan and spatial composition, in which the main structural elements and the range of geometric measures were considered.

Sampled churches had similar characteristics, in terms of both architectural features and construction details. Religious heritage is mainly represented by longitudinal churches, with a long nave connected by a main chapel. Most of the studied churches usually have one or three naves with Latin cross-section, with the central nave taller than the lateral ones. The bell tower is commonly present in most churches and it is located adjacent to the building, in some cases, the structures presented two towers. The roof system usually consists of masonry vaults or wooden roofs connected by timber beams embedded in the longitudinal walls, in both cases, the ceiling tiles are supported on those elements. The typologies of the churches in this research were defined based on the plan composition, number of naves, bell towers, and type of roof (with or without masonry vault). Table 1 shows that the typology of churches was divided into two types: single nave and three naves, and each type had four classes. The first one was class S that corresponded to churches with a structural system composed

**Table 1.** Typological classification of churches

| Type \ Class | Simple (S) | Bell Tower (T) | Vault (V) | Vault and Bell Tower (VT) |
|---|---|---|---|---|
| Single Nave (1N) |  |  |  |  |
| Three Naves (3N) |  |  |  |  |

by nave(s) and main chapel, and the second one was class T that corresponded to churches with nave(s), main chapel, and bell tower. The third one was class V that corresponded to churches with nave(s), main chapel, and vault. The last one class is class VT that was a mixture of the previous ones.

## 2.2 Relationship Between Geometrical Parameters

After defining the typologies, the task was to find the relation between geometric parameters of the sampled churches to set the main geometrical properties (width, length, height and others as shown in Fig. 1) of each type. The main geometrical parameters in plan and elevation of the structure were identified and correlations between them were found. Table 2 summarizes the established linear correlation and the coefficient of correlation ($R^2$) between the geometrical parameters. It can be observed that the correlations have good accuracy in most of the cases, thus, the best correlation between the LNV (Length of the nave) with the WNV (Width of the nave), where values of $R^2$ equal to 0.70 and 0.61 for single and three naves respectively were found. Since the number of geometrical parameters and how they were related to each other was known, several models of churches from a base model were generated.

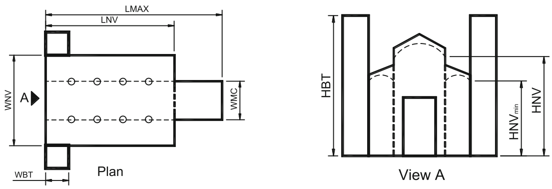


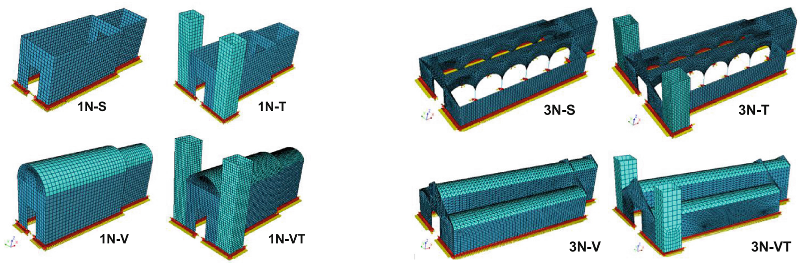

**Fig. 1.** Geometric parameters in plan and elevation

**Table 2.** Linear correlation and the coefficient of correlation ($R^2$) between geometrical parameters

| Single nave | $R^2$ | Three naves | $R^2$ |
|---|---|---|---|
| LNV = 2.53 × WNV − 1.61 | 0.7 | LNV = 2.02 × WNV − 1.64 | 0.61 |
| LMAX = 1.33 × LNV + 2.51 | 0.96 | LMAX = 1.45 × LNV − 4.95 | 0.89 |
| WMC = 0.62 × WNV + 0.83 | 0.74 | WMC = 0.85 × WNV − 6.09 | 0.74 |
| LNV = 1.49 × HNV + 2.84 | 0.71 | LNV = 1.37 × HNV + 16.37 | 0.32 |
| HNV = 9.90 × Thickness + 2.04 | 0.51 | HNV = 9.49 × Thickness + 3.00 | 0.57 |
| WBT = 0.28 × WNV + 0.80 | 0.61 | HNVmin = 0.72 × HNV − 0.83 | 0.69 |
| HBT = 1.7 × HNV | - | WBT = 0.60 × WNV − 4.58 | 0.75 |
| *Thickness: Thickness of the walls.* | | HBT = 1.4 × HNV | - |

## 3 Numerical Simulation and Parametric Analysis

The first step to perform numerical simulations of the eight typologies defined in Table 1 was to generate a base numerical model of each typology. These models were developed using the Finite Element Method (FEM) and constructed in the advanced structural analysis software iDiana 10.1 [13] (see Fig. 2).




(a) (b)

**Fig. 2.** Views of the numerical models of (a) single nave type and (b) three naves type

The following general hypothesis and assumptions were used in the models: Homogeneous masonry material in all elements of the structure, the density of the masonry material equal to 2000 kg/m3, the Young's Modulus was taken as 1 GPa and the Poisson's ratio equal to 0.2. All the walls of the structure had the same thickness in each model, the external wall openings (windows, the rose window, minor doors, etc.) were not considered in the model, and a rigid ground foundation (fixed on the base) was assumed.

After defining a base model for each typology, more models for each typology through a parametric analysis were developed, increasing the dimensions of the base model in function to the estimated correlations from the geometrical parameters. The parametric analysis was performed using a code implemented in MatLab R2013 [14] together with iDiana 10.1 to build different models and compute the numerical modes trough the eigenvalues analysis in a reduced time. Thus, a new model was generated for each typology of the churches when the parameters were updated. The analysis considered the variation of up to nine geometrical parameters at the same time in the case of the more complex-base models. Eleven and thirteen numerical models for each typology of the single nave and three naves were considered, respectively, obtaining in total 44 and 52 models for each typology. Furthermore, 55 additional models were made to study the influence of the Young's modulus on the response of churches type 1N-S.

Finally, by performing the eigenvalue analysis the main modes of vibration and frequencies of the numerical models were identified and compared. The first mode of vibration (mode 1_TY) was related to a global translation mode with single curvature in the transversal direction of the structure. The second one (mode 2_TY) corresponds to a global translation mode of the structure with double curvature in the transversal direction. Finally, the third one (mode_TX) is a global translation mode with simple curvature in the longitudinal direction of the structure suggests an overturning mechanism of the façade.

Figure 3 presents an example of frequencies obtained in this analysis and shows the mode 1_TY for the single and three naves models. Figure 3a shows that typologies with towers, vaults or both, have higher frequencies than the simple model (1N-S). Additionally, the models with towers and vaults (1N-VT) have the highest frequencies of all models, this is because the vaults increase the lateral stiffness of the churches. Nevertheless, when the HNV is higher than 24 m, the presence of towers and vaults does not influence the frequencies of the churches significantly. In the case of three naves models, the frequencies increase when the churches have vaults, up to 6 times compared to the models without vaults. In contrast, the models without vaults (S and T) do not change their frequencies considerable (Fig. 3b).

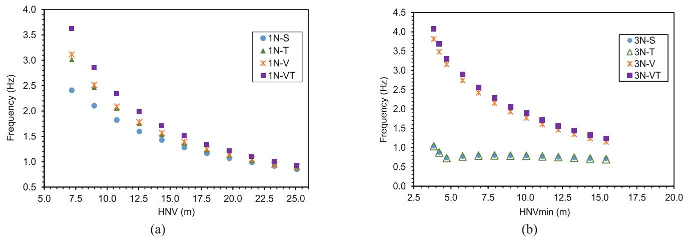



**Fig. 3.** Results of frequencies - mode 1_TY of (a) single nave and (b) three nave models.

Additionally, aiming to find the influence of Young's modulus in the frequencies of the churches, this parameter was evaluated in model 1N-S with values ranging from 0.5 GPa to 5 GPa. It was found that the frequencies calculated with Young's modulus equal to E were directly proportional to $\sqrt{E}$.

## 4 Proposed Formulations and Validation with Real Cases Studies

Based on the data of frequencies of the single and three naves models, and due to the scattering between the results of these models, formulations for each group of typologies were proposed including the influence of the class (S, T, V and VT) into the formulation as a parameter. The formulations proposed were developed by multivariable regression analysis using the program CurveExpert Professional [15]. Several regression models were evaluated, including cubic model, rational model, quadratic models, and power models to find the best approach. As shown in Eq. (1), the power model was chosen because it presents fewer constants on its form and matches better with the classical equations to calculate the fundamental period of structures.

$$\mathrm{Freq} = a(H)^{b}(\mathrm{Typ})^{c}(E)^{0.5} \tag{1}$$

where a, b and c are coefficients of the regression determined for each case; H is the height of the nave in meters (HNV or HNVmin depending on the case). Typ is a variable that depends on the class (S, T, V and VT) of the church, E is the average Young's modulus of the masonry in GPa, and Freq is the frequency in Hz.

The parameter Typ is a qualitative variable that considers the classes of the studied typologies of the churches and was defined considering the level of complexity of the structure from 1 to 4 where 4 is the most complex. Thus the value of Typ for class S is 1, class T is 2, class V is 3, and class VT is 4. Moreover, Table 3 presents the proposed equations to estimate the three main frequencies for churches with the best-fit parameters a, b and c from Eq. (1). A reasonable correlation was found between the studied parameters and the frequencies of the churches according to the results of $R^2$. Thus, the best correlation was obtained in the estimation of the frequencies of the Mode 1_TY for the churches with a single nave, where a value of $R^2$ equal to 0.99 was found.

**Table 3.** Proposed equation to estimate the main natural frequencies of churches.

| Mode shape | Type of church | Proposed equation | $R^2$ |
|---|---|---|---|
| Single nave | Mode 1_TY | $1\text{st Freq} = 18.509(\text{HNV})^{-0.985}(\text{Typ})^{0.178}(\text{E})^{0.5}$ | 0.99 |
| | Mode 2_TY | $2\text{nd Freq} = 34.234(\text{HNV})^{-0.929}(\text{Typ})^{-0.203}(\text{E})^{0.5}$ | 0.91 |
| | Mode_TX | $3\text{rd Freq} = 35.177(\text{HNV})^{-0.912}(\text{Typ})^{-0.096}(\text{E})^{0.5}$ | 0.85 |
| Three naves | Mode 1_TY | $1\text{st Freq} = 2.437(\text{HNVmin})^{-0.743}(\text{Typ})^{1.101}(\text{E})^{0.5}$ | 0.94 |
| | Mode 2_TY | $2\text{nd Freq} = 11.468(\text{HNVmin})^{-0.821}(\text{Typ})^{0.368}(\text{E})^{0.5}$ | 0.89 |
| | Mode_TX | $3\text{rd Freq} = 15.461(\text{HNVmin})^{-0.836}(\text{Typ})^{0.197}(\text{E})^{0.5}$ | 0.87 |

In general, a new correlation to approximate the principal frequencies based on church parameters as the height of the nave, typology and Young's modulus average of the material was obtained. These equations might be proposed for practical applications when a reasonable estimation of the frequencies of the churches are required.

Once the equations to estimate the frequencies were obtained, it was necessary to validate the proposed equations comparing the analytical results with measured frequencies by experimental tests in real cases. Limited real cases were found due to the lack of experimental studies performed in churches. The selected case studies are important religious monuments in Portugal, in which experimental tests and numerical models were performed. The case studies were: Our Lady of Conception [16] – Case 1, Saint Torcato [17] – Case 2, Monastery of São Miguel de Refojos [18] – Case 3, Monastery of Jerónimos [19] – Case 4, and monastery of Jesus [20] – Case 5. The experimental mode shapes and their respective natural frequencies were identified, additionally, the estimated Young's modulus of the constituent material of the structures was taken from the calibrated numerical models of each case, respectively. The experimental results were used to compare with the results obtained with the proposed equations.

Figure 4 shows the 1st and 2nd frequency for each case study. The estimation of the frequencies of the two first cases present the greatest differences of all the cases (around 40%), in the other cases, the difference is smaller. Additionally, is noted that for the 2nd frequency, the difference between the analytical and experimental method is (around 30%). The difference found between the two methods in Case 1 and 2, could be due to the structural configuration of these churches, which have significate differences to the established typologies. These churches present chapels or transept and dome, making the application of the methodology difficult.

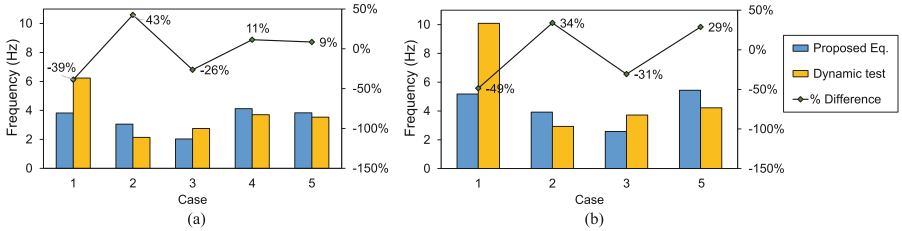



**Fig. 4.** Results of frequencies (a)1st and (b) 2nd frequency for each case, estimated with both methods

The lack of real data does not allow comparing the proposed equations considering the four values of the variable Typ. Even though several limitations were found to apply the proposed methodology, interesting results in the estimation of frequencies were obtained. The estimated frequencies of the first mode on the churches with three naves were close to the experimental values. In this case, it is probable that the presence of annex structures in the churches does not have a significant influence on the first frequency since the proposed methodology does not consider the existence of annex structures such as chapels. In the same way, it is possible that in the churches with a single nave, in which the estimation was not so close, the influence of the annex structures together with the variability of the Young's modulus may be significant.

## 5 Conclusions

Several methods were found in literature to determinate the natural periods of conventional structures using empirical correlations, but there are few works on masonry structures like churches. This paper presents a methodology to estimate the three main frequencies of churches based on simplified geometrical parameters. The geometrical survey of fifty existing churches was carried out, and eight typologies of churches grouped on two types (single nave and three naves) were defined. Additionally, the established linear correlations between the geometrical parameters allowed to generate models of churches and to identify which are the geometrical parameters that have an important influence on the dynamic behavior by means of parametric analysis. Six equations to estimate the three main frequencies of two types of churches that fit with a good accuracy the data of the numerical simulation were found and were validated with real cases studies of the literature. The estimated frequencies of the first mode of churches with three naves were more accurate when comparing with experimental results. Finally, the proposed equations require further validation comparing the analytical frequencies with experimental results from case studies that include all the typologies in order to improve their reliability and accuracy of the developed methodology. Also, performing a more extensive validation might be useful to propose correction factors for each formulation proposed.

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