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An Exact Optimization Approach for Personnel Scheduling Problems
in the Call Center Industry

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Escola de Engenharia
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## An Exact Optimization Approach for Personnel Scheduling Problems in the Call Center Industry

Dissertação de Mestrado<br>Mestrado Engenharia de Sistemas

Trabalho realizado sob orientação de
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"It all seems impossible until it's done."
Nelson Mandela
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I am forever grateful for your constant support and cheer for my success. Without you all this would certainly be impossible...

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RESUMO

Hoje em dia, a importância da indústria dos call centers tem vindo a aumentar, uma vez que estes são um grande meio de comunicação entre as empresas e os respetivos clientes. Nesse sentido, garantir um bom e otimizado escalonamento de pessoal é crucial e traz consigo bastantes vantagens: redução dos custos totais de trabalho, reduzindo excesso de trabalhadores, aumento da satisfação dos empregados, atendendo às suas preferências, e ainda aumento da satisfação dos clientes, apresentando tempos de espera aceitáveis.

O problema considerado envolve escalonamento de pessoal num call center que opera 24 horas por dia, 7 dias por semana. Atualmente, o processo de escalonamento é feito manualmente. Assim, o principal objetivo é explorar abordagens de resolução exata para obter soluções que apresentam qualidade preferivel às das soluções obtidas até ao momento e para reduzir o tempo gasto em todo o processo.

O modelo de otimização proposto é um modelo de Programação Inteira, cujo objectivo é associar turnos de trabalho aos trabalhadores, minimizando o total das penalizações associadas às preferências horárias dos mesmos. 0 modelo é implementado no ILOG CPLEX Optimization Studio 12.7.0.0, utilizando linguagem OPL, e testado com várias instâncias, incluindo instâncias geradas aleatoriamente e instâncias com dados reais. A análise da qualidade do modelo passou pelo estudo computacional da sua relaxação linear, podendo concluir-se que o modelo apresenta um intervalo de integralidade nulo em todas as instâncias testadas. Assim, o modelo proposto é um modelo forte, isto é, um modelo de boa qualidade. De forma a avaliar o desempenho do modelo a resolver instâncias grandes, várias instâncias geradas aletoriamente são testadas utilizando o software ILOG CPLEX Optimization Studio 12.10.0.0., apresentando bons resultados computacionais.

Palavras-chaves: escalonamento de pessoal, call centers, otimização, programação inteira


#### Abstract

Nowadays, the importance of the call center industry is increasing because they are a major mean of communication between organizations and their costumers. So, ensuring good and optimized personnel schedules in call centers is crucial and has several advantages: reduction of total labor costs, reducing overstaffing, employees' satisfaction, meeting their preferences, and costumers' satisfaction, presenting acceptable waiting times.

The considered problem concerns personnel scheduling in a $24 / 7$ call center where the scheduling process is done manually. So, the main goal is to explore exact solution approaches in order to obtain solutions whose quality is preferable to the manually achieved ones and to reduce the processing time.

The proposed optimization model is an Integer Programming model. The purpose of this model is to assign shifts to workers, while minimizing the total penalization that are associated to employees' time preferences. The model is implemented on ILOG CPLEX Optimization Studio 12.7.0.0, using OPL, and tested with various instances, including randomly generated and real-world data instances. In order to analyze the quality of the model, a computational study of its linear relaxation was carried out, concluding that the model presents null integrality gaps in all the tested instances. So, the proposed model has a strong formulation, that is, a good quality model. Additionally, to evaluate the performance of the model when running large instances, several randomly generated instances were tested using ILOG CPLEX Optimization Studio 12.10.0.0, achieving good computational results.


Keywords: personnel scheduling, call centers, optimization, integer programming
1 introduction ..... 1
2 state of the art ..... 3
2.1 Personnel characteristics, decision delineation and shifts definition ..... 4
2.2 Constraints, performance measures and flexibility ..... 5
2.3 Solution method and uncertainty incorporation ..... 6
2.4 Application area: scheduling in call centers ..... 11
3 review of computational tools ..... 13
3.1 Resource Guru ..... 13
3.2 Synchroteam ..... 14
3.3 When I Work ..... 14
3.4 Homebase ..... 15
3.5 Ximble ..... 15
3.6 Shiftboard ..... 16
3.7 Deputy ..... 16
3.8 SISCOG ..... 17
3.9 Critical Analysis ..... 18
4 problem's description ..... 19
4.1 Problem's Description ..... 19
4.2 Example of an instance ..... 21
5 mathematical model ..... 25
6 implementation ..... 29
7 computational experiments and analysis ..... 33
8 conclusion ..... 41
Bibliography ..... 41
Appendices ..... 47
a data of computational experiments ..... 49

## LIST OF FIGURES

Figure 1 Example: set of time periods with a planning horizon of a day ..... 20
Figure 2 Schedule of team 1 obtained manually ..... 20
Figure 3 Schedule of team 2 obtained manually ..... 21
Figure 4 Schedule of team 1 - IP solution ..... 40
Figure 5 Schedule of team 2 - IP solution ..... 40
Figure 6 Workforce's time preferences - pref0 ..... 49
Figure 7 Workforce's time preferences - pref1 ..... 50
Figure 8 Workforce's time preferences - pref2 ..... 50
Figure 9 Workforce's time preferences - pref3 ..... 51
Figure 10 Workforce's time preferences - pref4 ..... 52
Figure 11 Workforce's time preferences - pref5 ..... 53
Figure 12 Workforce's time preferences - pref6 ..... 54
Figure 13 Workforce's requirements - req0 ..... 54
Figure 14 Workforce's requirements - req1 ..... 55
Figure 15 Workforce's requirements - req2 ..... 55
Figure 16 Workforce's requirements - req3 ..... 56
Figure 17 Workforce's requirements - req4 ..... 56
Figure 18 Workforce's requirements - req5 ..... 57
Figure 19 Workforce's requirements - req6 ..... 57
Figure 20 Workforce's requirements - req7 ..... 58
Figure 21 Workforce's requirements - req8 ..... 58
Figure 22 Workforce's requirements - req9 ..... 59
Figure 23 Workforce's requirements - req10 ..... 59
Figure 24 Workforce's requirements - req $11_{\text {team } 1}$ ..... 60
Figure 25 Workforce's requirements - req $11_{\text {team } 2}$ ..... 60
Figure 26 Workforce's requirements - req $12_{\text {team } 1}$ ..... 61
Figure 27 Workforce's requirements - req $12_{\text {team } 2}$ ..... 61
Figure 28 Workforce's requirements - req $13_{\text {team } 1}$ ..... 62
Figure 29 Workforce's requirements - req $13_{\text {team }}$ ..... 62
Figure 30 Workforce's requirements - req $13_{\text {team }} 3$ ..... 63
Figure 31 Workforce's requirements - req $13_{\text {team } 4}$ ..... 63
Figure 32 Workforce's requirements - req $13_{\text {team } 5}$ ..... 64

## LIST OF TABLES

Table $1 \quad$ Instance example: teams, time periods and workers ..... 22
Table 2 Instance example: set of teams to which a worker $w$ can be assigned ..... 22
Table 3 Instance example: workforce requirements ..... 22
Table 4 Instance example: preferences' penalizations ..... 23
Table 5 Penalization System ..... 27
Table 6 Implementation code in OPL - IP model ..... 30
Table 7 Computational experiments 1 - instances' data ..... 34
Table 8 Computational experiments 1 - results ..... 36
Table 9 Computational experiments 2 - instances' data ..... 38
Table 10 Computational experiments 2 - results ..... 39
BLLS Bucket List Local Search
CPU Central Processing Unit
CMH Constructive Matheuristic Algorithm
DBLS Day-based Local Search
IP Integer Programming
IPSTrSP Integrated Personnel Shift and Task re-Scheduling Problem
LP Linear Programming
LBIH Local Branching Improvement Heuristic
OR Operational Research
OPL Optimization Programming Language
SMPTSP Shift Minimization Personnel Task Scheduling Problem
VNS Variable Neighborhood Search
WBLS Worked-based Local Search

## I

## INTRODUCTION

One of the main goals of an organization is to satisfy the needs of employees in staffing and scheduling decisions in a cost-effective manner. Nevertheless, it is known that labor costs, which include employee wages, benefits, payroll, and other related taxes, can account for as much as $70 \%$ of total business costs. So, when having a good scheduling system, it is possible for an organization to reduce that costs, even just a little.

Additionally, call centers have a crucial role in companies, since they are one of the main channels for companies to communicate with their customers. Because customers' satisfaction is also a major goal of an organization and call centers have a weight in that, it is important to have optimized staff schedules in call centers.

The problem to be considered involves personnel scheduling in a call center that operates 24 hours a day and 7 days a week, where a monthly personnel scheduling is carried out manually. In this sense, the aim of this project is to develop an efficient exact solution method to solve personnel scheduling problems at call centers by producing schedules of such quality that they are preferable to manually made ones and thus reduce the process time.

This dissertation is divided in seven chapters, besides the introductory one.
Chapter 2 presents a literature review of personnel scheduling problems, including an overview of their main features, such as workforce characteristics, shift flexibility, type of constraints and objective function used to model the problem, and the selection of the solution method to solve it. To finish this chapter, a brief literature review of personnel scheduling problems in call centers is also presented.

In Chapter 3 an analysis of some computational tools available for the resolution of personnel scheduling problems is carried out, in order to give a practical perspective of these problems.

Chapter 4 is divided in two subsections and concerns the problem's main characteristics and the description of an example of an instance.

In Chapter 5 the developed Integer Programming model is presented and described.
In Chapter 6 a concise description of the chosen software to implement the developed model is given, including its principal advantages. The written and implemented code is analyzed, describing the data structures used throughout it.

Chapter 7 concerns the computational experiments that were performed. First, the linear relaxation of the integer programming model is analyzed, taking into consideration the results of different instances, in order to obtain some information about the quality of the IP model. Additionally, to evaluate the performance of the model when running large instances, some instances are tested. To finish the chapter, examples of the final solution are given.

The last chapter, Chapter 8, consists of a summary of the principal findings and resulting conclusions, as well as suggestions of future work.

In the end of the dissertation, a appendix can be found, in order to complement the information and results given throughout the project.

## STATE OF THE ART

Over the last years, the importance of studying personnel scheduling problems has been increasing. This is due to the impact of the labor cost on companies' costs [14]. Therefore, reducing this cost, even just a little, can be very advantageous and worthwhile. According to Bard et al. [4], weak personnel schedules can lead to an oversupply of workers with too much idle time, or an undersupply with an attendant loss of business. Thompson [26] reports three reasons for taking particular attention to staff scheduling. The first consists of the employee's preferences since a work schedule that comes reasonably close to meeting them can improve customer service. The second reason is related to the real time spent on developing a labor schedule, which can leave less time for the scheduler to manage the employees and to interact with customers. Finally, the third reason is profitability and effectiveness, since short-term overstaffing and long-term understaffing can be reduced.

One of the main goals of an organization is to satisfy the customer and also the employees in a costeffective manner. There are many concerns to consider, such as shift equity, staff preferences, flexible workplace agreements, and part-time work, and thus, it is expectable that, in order to achieve optimized staff schedules, decision support systems must be carefully implemented. Personnel scheduling consists of the assignment of shifts and days off to workers so that each one has a line-of-work. This process can be a complex task because schedulers must assign employees to shifts and tasks while accounting different rules and regulations, and also workers' availability. In the first phase of this process, it is not only necessary to have an idea of the number of staff needed to meet the service demand but also to know the particular skills of all individuals so that they can be allocated to shifts whose tasks meet their skills.

As time goes by, the relative importance of satisfying the needs of employees in staffing and scheduling decisions has increased. Nowadays, organizations want to offer some conditions to their employees that were not a concern in the past. Consequently, and as it was referred to above, organizations create work schedules taking into account employee preferences [5] and trying to offer part-time contracts or flexible work hours [4]. Mohan [22] considers a staff scheduling problem consisting entirely of part-time workers and also considers their shifts' preferences.

In the literature, several personnel scheduling problems with some different features can be found. This fact can be explained since, as it can be expected, different work environments imply different requirements.

Chapter 2. State of the art

Therefore, some features receive particular attention, such as workforce characteristics, shift flexibility, the type of constraints and objective function used to model the problem, and the selection of the solution method to solve the problem. These features will be presented below.

Several classification methods for personnel scheduling problems can be found, which provide a general framework for classifying the contributions that have been carried out in this area. The first one was proposed by Baker [3], and it is commonly studied in the literature for general labor scheduling [6].

Van den Bergh et al. [14] proposed a classification method which organizes the literature review using different perspectives. Therefore, the classification fields are the following: personnel characteristics, decision delineation, and shifts definition; constraints, performance measures and flexibility; solution method and uncertainty incorporation; and application area and applicability of research. Each field is described in detail as follows.

## 2.1 personnel characteristics, decision delineation and shifts definition

In order to classify personnel members, one can look at their labor contract. In this way, it is possible to distinguish two categories: full-time and part-time workers. Mohan [22] considered a scheduling problem where the workforce is composed entirely of part-time workers, claiming that more people are opting to work part-time due to the flexibility it offers. Bard et al. [4] considered both full-time and part-time workers. In some cases, casual workers, that are, for instance, workers from another company division, are also considered to deal with the lack of regular workforce. Another existing classification is based on the grouping of employees, where the workers are categorized in individual or crew entities. Usually, this last one can be found in scheduling problems in the transportation area. Besides that, one cannot forget that different employees have different availability and also different individual personal preferences, which are both important since one of the main goals of an organization is to satisfy the employees.

According to Felici and Gentile [17], a shift is a work duty characterized by a starting time and a duration. Shifts can be fixed or variable in terms of starting-time or length, for each employee and each day. In personnel shift scheduling, shifts and days-off are assigned to each worker in order that every employee has a line-of-work while taking into account some possible existing conditions or requirements.

As it can be expected, in personnel scheduling problems it is imperative and inevitable to take some decisions. The main ones are related to the assignment of tasks (e.g., a given employee is assigned to a certain task) and shift sequences (e.g., a given employee works on day 1 in the morning, but he/she does not work on day 2 and works on day 3 in the morning). The task assignment methods depend on whether tasks times are fixed or movable, breaks exist in shifts, overtime is allowed, or specific skills or qualifications are required to perform certain tasks. According to Smet et al. [25], tasks should be incorporated in the construction of rosters for employees in order to reduce operational expenses while still preserving a high
quality of service. Maenhout and Vanhoucke [20] claimed that when the scheduler integrates both personnel shift scheduling and personnel task scheduling, then the obtained personnel roster is improved.

## 2.2 constraints, performance measures and flexibility

Often there is a distinction between hard and soft constraints per category. The former must be satisfied while the latter may be violated, including penalty terms in the objective function.

Regarding the categories mention above, Van den Bergh et al. [14] classify the constraints in three different categories: coverage, time-related, and fairness and balance constraints. So, for example, the necessity of a specific skill, which is a coverage constraint, can be modeled either as a hard or a soft constraint, depending on the problem in hands, being that in the second case, people with other skills could take over as long as there is an associated penalization.

On the other hand, Blochliger [8] considers the following constraints categories:

- sequence constraints which are used when a specific pattern of working shifts and days-off is required or preferred by the employee;
- counting constraints whose main idea is to count different things over different time ranges in order to get a given range that is optimal or acceptable. For example, one can use these constraints to count the number of night shifts given to an employee or the number of working hours of an employee;
- job constraints that cover all the jobs' requirements;
- incompatibility constraints that are used to forbid two incompatible items to be assigned to the same task;
- local constraints which include sequence constraints and incompatibility constraints;
- global constraints that require a broad knowledge of a solution, like a schedule for a whole week or the schedule of multiple employees at a time. Examples of this type of constraints may include job constraints and counting constraints;
- intrinsic and extrinsic constraints. While the intrinsic ones are given by the nature of the items we want to schedule, like the fact that an employee can not be in two different places at the same time, the extrinsic constraints are those imposed by outside factors, like laws and regulations. Incompatibility constraints are extrinsic constraints examples. According to Blochliger [8], intrinsic constraints are usually modeled as hard constraints and extrinsic constraints are usually modeled as soft constraints.

It is also important to take into account the different skills that workers may possess. According to Bruecker et al. [10], skill is defined as the ability of a worker to perform certain tasks well. There are three
groups related to the extent to which skills are considered flexible: hierarchical, definable or categorical according to Bruecker et al. [10], and not allowed. In the first case, workers with a lower skill level can do less than workers with a higher skill level. Additionally, higher skilled people can perform tasks that are normally performed by a lesser skilled person but not the opposite. In the second case, skills are freely defined for every worker by the scheduler. A worker who possesses different definable skills is a crosstrained worker. In the last case, skills cannot be substituted. Therefore, a task that requires a specific skill can only be performed by a worker who possesses that specific skill.

In the literature, different types of objectives can be found in order to model the staff scheduling problem. Smet et al. [25] introduce a matheuristic approach to the problem of assigning tasks to multi-skilled employees while minimizing the number of employees. The problem discussed in this work is known as shift minimization personnel task scheduling problem (SMPTSP). Volland et al. [27] present a column generation based solution approach, where the objective function minimizes the total costs. According to Van den Bergh et al. [14], minimizing personnel cost is closely related to minimizing the number of employees, but has more possibilities because one can make a trade-off between hiring employees, casual workers, among others, by assigning a cost to all these factors. Mohan [22] propose an integer programming model where the objective function maximizes employee preferences while considering their seniority and availability. Felici and Gentile [17] formulate and solve staff scheduling problems where the objective function expresses an approximated measure of staff satisfaction by the proximity of certain shifts.

Maenhout and Vanhoucke [20] propose a heuristic optimization procedure for the integrated personnel shift and task re-scheduling problem (IPSTrSP). This problem can be divided into two different phases. In the first phase, the scheduler has to compose an integrated schedule, making a lot of deterministic assumptions, like the workers' availability, the number of tasks or even the timing of the tasks. These assumptions may not entirely represent the real context due to some existing operational variability. For each worker, a line-of-work is composed, where a particular task is assigned. It is important to keep in mind that a worker works according to shifts. So, a task is assigned to a worker, and different shifts are assigned to that task. The goal is to determine the minimum personnel cost. In the second phase, more accurate information is provided, and so some estimated values may turn out wrong. Therefore, the original personnel roster needs to be adapted, and a re-scheduling problem has to be solved. The goal of re-scheduling is to rebuild the schedule while minimizing the number of deviations to the original schedule and also to reestablish the feasibility of the personnel roster [20].

## 2.3 solution method and uncertainty incorporation

Different authors use different solution methods for solving specific problems. Nevertheless, a large number of approaches are classified into mathematical programming categories (e.g., mixed integer programming) or as constructive or improvement heuristics (e.g., tabu search).

Brucker, Qu, and Burke [9] affirm that the Dantzig set covering formulation can be used to formulate rostering problems. However, this is true only for simple problems because this formulation is difficult to be used to tackle some common characteristics in personnel scheduling problems [9].

According to Ernst et al. [15], algorithms based on mathematical programming approach generally achieve the lowest cost solutions. However, there are some difficulties with these approaches that prevent them from being universally applied. In many cases, the linear integer programming models have a huge number of variables and researchers end up using decomposition techniques and heuristic algorithms to overcome that, since they are iterative methods. Thereby, mathematical programming formulations are more commonly applied to simplified versions of the real-world scheduling problem or with few complications in the original problem. Blochliger [8] claims that, when formulating the problem as an integer linear program, the most significant disadvantage is that even if a solution is found, it can be optimal in the sense of the model, but probably not in reality because human factors are very hard to model correctly.
When dealing with complex optimization problems, one can use metaheuristics in order to be more efficient. These heuristics form an important class of methods that solve personnel scheduling problems. According to Van den Bergh et al. [14], the practical advantage of these methods is their effectiveness and general applicability. However, they cannot demonstrably produce optimal solutions, although they tend to be relatively robust, since reasonably good feasible solutions can be found, nor can they demonstrably reduce the search space (Burke et al., 2010, as cited in [14]).

Other solution methods include simulation methods, which can be used to help researchers to validate their deterministic optimization approach; constraint programming methods, that are ideal to deal with highly constrained personnel scheduling problems; and queueing methods, that are designed to achieve fairness when a limited resource is shared, and so these are mainly used in call center applications.

Jarray [19] presents an exact decomposition approach for an employee days-off scheduling problem, where all the employees are equally skilled, while considering variable demands and taking account an already defined number of workdays per employee over the planning horizon. The solution method proposed is a 3-step decomposition algorithm that the authors adapt to solve two different versions of the problem: the basic problem, where every employee is entitled to 3 or 4 consecutive days-off per week and the monthly rest problem, where every employee takes at least 4 consecutive days-off in each week per month. A polynomial algorithm is also proposed to solve the weekend rest problem, where each employee gets at least one weekend-off per month.

Felici and Gentile [17] discuss an integer programming approach for a class of staff scheduling problems. To deal with the symmetries in the solution space, a branching strategy is built, that is proven to contribute to a reduction of the optimality gap. The proposed algorithm was tested on eight instances coming from realworld cases, where the number of staff members ranges from 30 to 100 , the number of shifts considered was 11 , including rest shifts, and the number of incompatibility pairs of shifts was 28 . Additionally, the algorithm was also tested on extra instances generated from the first ones with random perturbations
concerning three parameters: the demand for each work shift, the number of shifts, and the weights of consecutive pairs in the objective function. These additional instances served to test the robustness and the stability of the algorithm. The results were obtained in reasonable computation times both in the realworld and extra instances. However, in a specific instance, the CPLEX 8.0, which was the optimization engine used, was not able to solve the problem within an hour of computation time against the 5 seconds required by the proposed approach. According the authors, the algorithm determines good solutions where commercial solvers fail.

Note that the solution methods mentioned above can be combined to increase the efficiency of the approach, leading to hybrid approaches. A hybrid approach combines various methods to improve or enhance overall search efficiency. It is important to keep in mind that the performance of a hybrid algorithm can be mixed: some can improve, but some may become worse if two algorithms are randomly selected to form a new one.

In order to deal with highly constrained nurse rostering problems, Burke et al. [11] introduce a hybrid multi-objective model that combines integer programming (IP) and variable neighborhood search (VNS). The main purpose of this research is to evaluate the performance of the combination of exact algorithms and metaheuristics considering some constraints and requirements from nurse rostering. According to Mladenovic and Hanson [21], a basic VNS algorithm can be divided into two steps: in the first step, one has to select the set of neighborhood structures that will be used in the search and have to find an initial solution; in the second step, a random point is generated from the neighborhood of the initial solution. Then, one has to apply some local search method to that random point to obtain a local optimum. If the local optimum is better than the initial solution, then the solution to be used from now on is the local optimum. If not, the second step must be repeated. Briefly, this metaheuristic explores several neighborhoods of an initial solution and jumps from there to a new one only on the condition that the latest is better than the former. The proposed approach is tested in two groups of experiments, each one having 12 real-world data instances corresponding to the 12 months of a year, using ILOG CPLEX 10.0 to solve the IP part and Java 2 to code the VNS part. While one of the experiment groups considers IP and VNS without decomposition, the other considers the proposed decomposition approach. As conclusions, the authors highlight the fact that, when considering realistic runtime restrictions, the full IP model for the entire problem cannot produce good solutions and, due to the high number of constraints of the problem, the basic VNS alone cannot produce feasible solutions for all data instances. However, the proposed hybrid approach can achieve good results on all instances: the IP is able to find several integer solutions within an acceptable time, and VNS is able to achieve local refinement quickly within the defined neighborhood. As an example, the authors refer to a particular instance, in which the IP found 12 integer solutions within 45 minutes and the VNS presented 53 improvements within half a minute from the best solutions of the IP.

Smet et al. [25] present a two-phase hybrid heuristic algorithm based on the principles of matheuristics to deal with a shift minimization personnel task scheduling problem. In the first phase, a constructive
heuristic generates an initial solution that is improved in the second phase. The authors introduce three different constructive approaches, first fit, best fit and a constructive matheuristic algorithm (CMH). In order to improve the solution given by one of the constructive approaches, a matheuristic based on the idea of local branching is presented. The advantages of this improvement heuristic are its versatility that allows its application to a large class of problems and the fact that it does not require a feasible starting solution. The proposed approach is tested in instances which dimensions range from small ( 23 employees and 40 tasks) to very large ( 245 employees and 2105 tasks). The computational tools chosen by the authors were Java, to code the algorithms, and Gurobi 5.1.0, as the optimization solver. In order to decrease the difficulty of the evalution of the algorithmic performance, a lower bound was defined, choosing the minimum number of employees needed to cover the largest number of overlapping tasks as that bound. The performance of the three constructive approaches proposed is then compared with two different block sizes that differ in the number of employees considered in one subproblem of the CMH: while one of the blocks considers 10 employees, the other considers 15 employees. The best fit constructive approach was able to achieve more optimal solutions than the first fit constructive approach when tested in 137 available benchmark instances. However, when the best fit could not solve the problem up to optimally, the first fit achieved better solutions. The CMH was the one that produced significantly more optimal solutions. On the other side, in order to achieve those solutions, an increase in the computation time was required. While the average computational time of the best fit and the first fit is, respectively, 0.02 and 0.11 seconds, the average calculation time of the CMH is 12.26 and 44.93 seconds when the block size is 10 employees and 15 employees, respectively. To continue the experiments, the block with 10 employees was chosen to generate the initial solution for the improvement heuristic. Additionally, to determine the impact of employee selection for the subproblems of the CMH , an experiment was set up in which three approaches were compared. 50 instances were considered, in each the number of employees differs from 88 to 415, the number of tasks ranging from 777 to 2105 and the employees were randomly skilled. As a conclusion, the three approaches compared were proved not to influence the performance of the matheuristic under the circumstances referred to above. When testing the performance of the local branching based improvement heuristic (LBIH), the authors concluded that, for instances with many tasks, the number of tasks allowed to be reassigned should not be too large because of the increasing of the calculation time. However, when more tasks are allowed to be moved, better solutions can be achieved. So, the combination $\mathrm{CMH}+\mathrm{LBIH}$, where the block size is 10 employees, is the most appropriate. When compared with a commercial general purpose mathematical solver (Gurobi 5.1.0) and with two methods recently reported in the literature, the combination considered above was able to perform significantly better, both in terms of solution quality and required calculation time, achieving 137 optimal solutions and an average calculation time of 55.52 seconds to find an optimal solution. The authors highlight the fact that the multi-skilling level, that is the percentage of the total number of tasks each employee is qualified for, on average, and the average task duration are the most influential factors to the hardness of the problem.

Maenhout and Vanhoucke [20] discuss a heuristic optimization procedure for the integrated personnel shift and task re-scheduling problem aiming to restore an already announced personnel roster that is subject to operational variability. The proposed meta-heuristic re-scheduling algorithm relies on a perturbation mechanism based on the local branching principles and a variable neighborhood that successively optimizes different subparts of the considered roster. The purpose of the perturbation mechanism is to create a new solution point in the neighborhood of a reference solution, escaping from local optima. This neighborhood is defined based upon the principles of local branching, that creates high-level branchings to define it and use certain branching strategies to explore it. The advantage of this method is the small computational stages required to find high-quality solutions. To test the proposed approach, an artificial set of problem instances is generated to cover a wide range of settings, considering both 7 and 28 days planning horizon and three non-overlapping working shifts per day with a duration of eight hours. The workforce considered is homogeneous, which means that all workers are equally skilled. In this particular work, no additional worker can be added and so the disruptions should be resolved by the existing workforce. Different experiments were conducted to demonstrate the robustness of the proposed algorithm. Firstly, the authors start to vary different input parameters of the perturbation method, in order to analyze the impact of them on the solution quality. The parameters considered were the minimum equality and diversity thresholds, which were both varied between $0 \%$ and $100 \%$. Due to the existing correlation between them, the sum of the thresholds cannot exceed $100 \%$. The optimal values of these thresholds are very dependent upon the size of the instances considered. So, for small instances, it is important to lay down on a greater diversification of the reference solution. On the other hand, for larger instances, a higher similarity with the reference solution is recommended. Selection strategies to select a reference solution were also tested, concluding that the best one is to select it from a limited set of unique high-quality solutions and the worst one is to select it randomly. Secondly, the impact of the perturbation objective on the solution quality was also analyzed, coming to the conclusion that, between the three strategies tested, the "mixed" one outperforms, which highlights the fact that it is crucial to introduce some randomness in the perturbation costs because this strategy considers both the original task assignment cost and random costs. The authors also evaluate the contribution of the different neighborhoods structures and the neighborhood size. These experiments proved that the bucket list local search (BLLS) method outperforms both worker-based local search (WBLS) method and day-based local search (DBLS). However, since a local improvement method is required, the combination of all three local searches methods referred to, i.e., the variable neighborhood search (VNS) method, can obtain better results, achieving a solution quality over the solved problem instances of 645.41 and a percentage deviation from the IP optimal solution of $0.59 \%$. Although the BLLS method alone was able to achieve a minor percentage deviation than the VNS method, the quality of the solution obtained is slightly worst. In order to prove that the algorithm can cope with different settings satisfactorily, additional scenarios with different objective function and disruption profiles were generated. The purpose of these extra scenarios was to compare four optimization techniques: the proposed procedure, a multi-start heuristic, an alternative

VNS, and an integer programming. Computational experiments revealed that the proposed algorithm can achieve a good performance, once it was able to obtain a low percentage deviation and an acceptable solution quality. When comparing with the other techniques, one can state that, when the solution quality is better, the percentage deviation suffers a significantly increase. The authors highlight the fact that, even though these conclusions were obtained with instances with 7 days, they are also valid to larger sized instances within a horizon of 28 days.

It is possible to distinguish three sources of operational variability: uncertainty of capacity, uncertainty of demand and uncertainty of arrival. The first one is caused by the unavailability of workers to work a scheduled task, the second one exists when predicted demand is different from the actual demand, and the last one has an impact in the duration or/and start times of the scheduled tasks. Maenhout and Vanhoucke [20] assume the three types of variability referred to above, describing them through a probability distribution with adequate parameters.

## 2.4 application area: scheduling in call centers

According to Ernst et al. [15], staff scheduling methods have been applied to transportation systems such as airlines and railways, health care systems, emergency services such as police, ambulance and fire brigade, call centers, and many other service organizations such as hotels, restaurants and retail stores. Van den Bergh et al. [14] add other application areas like military, supermarkets, festivals, and parking lots personnel scheduling problems. Even so, there are two applications of staff scheduling and rostering that stand out from the other: transportation services, particularly airline crew scheduling and rostering, because of its economic scale and impact, and nurse scheduling since is unacceptable not to fully support patient care needs.

Since the case study to be considered concerns personnel scheduling in a call center environment, a brief literature review of this application area will be presented.
The importance of call centers for companies is widely increasing because, although they are not the only one, they are a vital channel for companies to communicate with their customers [16], having a crucial role in customer acquisition and retention [1]. As a consequence, having a good scheduling system is fundamental because inadequate number of agents for each shift has an effect on customer satisfaction, for instance because of long waiting times, and also on labor related cost. According to Cordone et al. [13], a major part of the call center's operational costs are related to labor cost. That is why the staff scheduling problem became the main problem in call center operation. However, it is not a easy one to solve due to uncertainty of the number of tasks that are needed to be performed.
According to Robbins and Harrison [23], some OR methodologies, such as topics of queuing theory, optimization, and simulation, can be found in the literature of call centers' problems. Usually, solving shift scheduling problems in call center can be divided in two phases. In the first one the goal is to determine the
number of required agents for each period, considering some customer service measures like customer waiting times or abandonment rates. For this task, some authors use queuing and simulation models to obtain the correct numbers, as it can be seen in [16] and [12]. The second phase is defined by the scheduling algorithm, where the goal is to allocate employees to the shifts, taking into consideration a particular objective function, as it was seen before.

Bhulai, Koole, and Pot [7] propose an integer programming model for the shift scheduling problem in a call center, where they considered a multiskill workforce and their objective is to obtain a good service-level against minimal costs. In this sense, the presented method is divided in two phases: determination of the required number of agents for each period, considering service-level constraints, and determination of an optimal set of shifts, minimizing costs while satisfying the required workforce calculated in the first step. The service-level constraints studied include a minimum bound of calls that have a waiting time of less than 20 seconds. Considering a workforce with two distinct skills, shifts with a length of five and six hours, and satisfying the service-level constraint for $80 \%$ of the calls, the optimal number of shifts is 167 . The authors checked the optimality of the methods from both steps and concluded that the solution obtained is less than $3 \%$ from the optimal objective value. In conclusion, the authors highlighted the short computation time required and easy implementation, as also referred to that, although the method was developed for call centers, it can be used for other service systems that involve distinguished multiple skills, agents working on different tasks, and employees with identical productivity within the same skill group.

Alfares [2] present a two-step solution method to minimize the labor cost in a call center, by determining the optimum number of agents and their schedules to meet demand. In the first step, queuing theory is used to convert extensive data into values of demand for a typical work week. Then, an integer programming model is built, through which detailed weekly schedules are constructed. These schedules include work hours, meal breaks times and off days for each agent. The problem approached considers a call center that operates 24 hours a day, 7 days of week. The 47 agents that work there are divided in three groups, having different pay scales and different work schedules. There are two problems in this organization to be tackled: customers are not completely satisfied because they are being placed on hold for several minutes during peek periods and the 1 -hour breaks that the workers are entitled for are unscheduled. Before the study of the solution method proposed, using no scientific methods but based on the management's observations, a schedule for each worker was built. Then, using LINDO optimization software, two instances were solved, differing in the minimum number of employees required in the considered hours. The solutions obtained and the a priori solution were compared. Although both of the solutions given by the integer programming model were better than the a priori schedule, since the workforce size is reduced, the workforce utilization is increased and the labor cost decreased, the management chose one of them because it was the one that minimizes most the waiting time.

REVIEW OF COMPUTATIONALTOOLS

Organizations are becoming more aware to staff scheduling problems. As one of its main goals is the employee's satisfaction, it is necessary to have a commercial platform that is able to address those problems. Therefore, several available commercial platforms are going to be analyzed, in order to understand their advantages and what they can offer to organizations.

Gathering some information provided by Capterra, which is a software database, such as most popular, most affordable software, and also software with more total costumers and users, seven commercial platforms were chosen to be analyzed. These software tools can also be found in Software Advice, that is another available software database. The last presented software is an optimization decision-support software that I got to know during a conference.

## 3.1 resource guru

Resource Guru was founded by Andrew Rogoff and Percy Stilwell in 2011, and it is a scheduling software suitable for small, medium and large businesses, used by different companies such as NASA, Apple, Uber, and Deloitte. This software works as a web application, so an installation is not required. However, using it has an associated cost: its price varies between $\$ 3$ and $\$ 8$ per user per month if paid monthly, and between $\$ 2.50$ and $\$ 6.65$ per user per month, if paid yearly. The price varies according to the features included in the chosen version. The complete version of Resource Guru includes:

- Management of vacations and other time-off like sick leave, maternity leave, and even public holiday;
- Filters and custom fields to focus on a specific group - for example, it is possible to filter the employees by skills, department or even by contract;
- Tools to maximize utilization of resources, allowing the scheduler to see when and who is available at a particular time and preventing over-booking by using a waiting list;
- Various reports that allow the scheduler to monitor utilization of the employees - through these reports, it is possible to know how much capacity is available for the next month, which resources are under

Chapter 3. Review of Computational Tools
or over-utilized, and which projects are consuming the most time and if that time is being wasted in billable or non-billable work.

## 3.2 synchroteam

Synchroteam is a cloud-based service management solution whose main goal is to schedule employees in real-time. This software is intended for small and midsize business and has an associated cost of $\$ 22$ per user per month if paid annualy, and $\$ 28$ per user per month, if paid monthly. This service includes:

- Weekly and monthly views, allowing the employees to have a better perception of their schedule;
- Tools that allow job scheduling optimization, analyzing the existing schedule and suggesting the best time for a new job, while taking into account various factors such as skills required and travel distance;
- Interactive daily schedule, which allows the scheduler to see who is available, when and for how long;
- Filters to focus in on a specific group - for example, it is possible to filter the employees by skills.


## 3.3 when i work

When I Work is also a cloud-based service the whose main goal is employee scheduling, delivering various tools to their costumers in order to do so. This software can be used by all business sizes and has no cost for up to 75 workers. However, if the service is used to schedule more than 75 employees, the price varies between $\$ 1.50$ and $\$ 2.25$ per user per month, depending on the chosen version. It is also possible to contact the company in order to obtain a budget to a fully integrated version. When I Work includes:

- Tools that allow the scheduler to find a particular work with the required skills in order to perform a certain shift;
- Specific views to easily see workers' schedule;
- Management of vacations and other time-off like sick leave, maternity leave, and even public holiday;
- Tools that allow the scheduler to see only all the qualified and available employees when it necessary to replace a shift;
- Filters to focus in on a specific group;
- Tools to maximize utilization of resources, allowing the scheduler to see when and who is available at a particular time.


## 3.4 homebase

Homebase is an online employee scheduling software and was launched to the public in 2015. This software is suitable for hourly employees based business. For only one location and an unlimited number of employees, the utilization of this software has no associated cost. However, if it is used for more than one location, the price can vary between $\$ 19.95$ and $\$ 99.95$ per location per month, if paid monthly, and between $\$ 16$ and $\$ 80$ per location per month, if paid yearly, depending on the features included. An example of an organization that uses this software is Pizza Hut. The full version of Homebase includes:

- Different views that allow the scheduler and employees to see their schedule by role, time period or employee, calculating their working hours;
- Tools that allow automatic scheduling, taking into account workers' availability and skills;
- Updated information about workers' availability, provided by workers themselves;
- Tools that allow workers to request shift trades, which are approved or not by the scheduler;
- Management of time-off;
- Management of labor costs, according to the organization's sales, over and break times and total working hours.


## 3.5 ximble

Ximble is an employee scheduling software with precise time tracking, suitable for small to midsized organizations that need an employee scheduling solution with pricing per employee such as restaurants, healthcare systems, education systems and also governmental services. The utilization of this software has an associated cost, which varies between $\$ 1$ and $\$ 2$ per user per month if paid monthly, and between $\$ 1.50$ and $\$ 2.50$ per user per month, if paid annually, depending on the features included in the chosen version. It is also possible to get a budget for an advanced version of the software, when contacting the company. The full version of Ximble includes:

- Daily, weekly and monthly views, allowing the employees to have a better perception of their schedule;
- Management of effective budget, providing tools that calculate the total working hours and respective labor costs;
- Tools that automatically updates the employees' schedule as needed, taking into account employee's requests;
- Instant communication of shifts' changes;
- Artificial Intelligence tools that use historical data to create smarter and cost-effective schedules, and also to allocate shifts in seconds;
- Data-driven forecasting tools that also use historical data such as sales records to predict and regulate overtime costs, while avoiding over and under-staffing;
- Management of vacations and other time-off like sick leave, maternity leave, and even public holiday, including tools that allow the employees to view their earned hours and to request time-off;
- Tools to maximize utilization of resources, allowing the scheduler to see when and who is available at a particular time.


## 3.6 shiftboard

Shiftboard is a workforce scheduling software which is appropriate to industries with a complex and large workforce scheduling, regardless of their size. It is used by many education institutions such as The University of Texas and Yale, health care systems such as Stanford Health Care, emergency services such as Metropolitan Police, among others. The utilization of this software has an associated cost of plans starting from $\$ 3$ per user per month, depending on the features included in the chosen version. Through Shiftboard it is possible to:

- Improve workforce efficiency by using historical data in order to make better predictions of demand, to optimize shift assignment for employee schedules, and to track labor costs;
- Achieve lower labor costs, using tools that allow assign and change shifts automatically and using advanced rules engine that align shifts in order to fit complex scheduling and overtime rules;
- Boost employee engagement, using instant communication between schedulers and employees in order to pick up or sign up for shifts;
- Track compliance, using features that track labor contracts, employee credentials, seniority rules, and mandatory rest periods in order to reduce and even eliminate scheduling errors;
- Analyze shifts, employees, budget, forecasts, and labor costs by detailed reports.


## 3.7 deputy

Deputy is a workforce management software that was officially launched in 2008. This software is suitable for every business size and like the ones referred to above it has an associated cost that can vary between
$\$ 2$ and $\$ 4$ per user per month, depending on the features included in the chosen version and also if it is paid monthly or yearly. There are several industries served by Deputy such as retail, restaurant, healthcare, entertainment, education, technology and software, among others. The full version of this software includes:

- Artificial Intelligence tools, which use historical data to forecast labor demand, and to automate the schedules in order to optimize the scheduling process;
- Management of vacations and other time-off like sick leave, maternity leave, and even public holiday, including tools that allow the employees to request time-off;
- Management of wage costs, weighing both cost staff and sales data so that it is possible to never go over budget;
- Tools and filters that allow the scheduler to schedule employees according to their skills and qualifications;
- Management of fatigue and overtime, allowing the scheduler to set boundaries on how many hours an individual can work per day, or week;
- Tools that calculate employee overtime, penalty rates, and salary costing with every shift;
- Tools that allow the scheduler to schedule part-time workers.


## 3.8 siscog

SISCOG is an optimization decision-support software for resource scheduling and management founded in 1986. This software is fully focused in transportation cases, having several railways companies as clients. There are three different SISCOG's products available: ONTIME, FLEET, and CREWS.

ONTIME is a standard decision-support product that creates and manages optimized schedules, with the creation of services and network allocation over time, considering passenger demand, and operational constraints. In this product:

- different kinds of scheduling problems are addressed, using state-of-the-art technology, such as genetic algorithms, integer linear programming or even constraint programming, so that optimized solutions can be obtained;
- several what-if scenarios are runned, just by changing parameter values, in order to gain more knowledge about different possible situations;
- the expected consequences are lower network usage costs and reduced process time.

On the other hand, FLEET creates and manages optimized vehicle schedules, considering expected passenger demand figures, timetables, network, vehicle specifications and needs, and operational constraints. In this product:

- different kinds of scheduling problems are addressed, using state-of-the-art technology, such as column generation, integer linear programming or even metaheuristics, so that optimized solutions can be obtained;
- significant drop of vehicle usage costs and reduction of process time are promised;
- different goals can be optimized at the same time;
- large scheduling and rescheduling problems can be addressed.

Last but not least, CREWS product creates and manages optimized staff schedules, considering some parameters among which staff skills and preferences can be highlight. In this product:

- different kinds of scheduling problems are addressed, using state-of-the-art technology, such as Lagrangian relaxation, integer linear programming or even constraint programming, so that optimized solutions can be obtained;
- significant drop of staff usage costs and reduction of process time are promised;
- different goals can be optimized at the same time;
- large and very large scheduling and rescheduling problems can be addressed.

The information relative to the different products described above can be consulted in [24].

## 3.9 critical analysis

When analyzing these software tools, there are two features that are common to almost all: real-time automated scheduling, and workforce and shift scheduling. This allows the schedulers to manage employee's work and to perform skills-based job allocation, and also allows the employees themselves to sign up or to volunteer for open shifts. Using these scheduling softwares can bring several benefits to a business such as improved staff efficiency by facilitating the whole scheduling process, centralized information, and increased accountability and visibility by allowing employees to select open shifts and automatically generating optimized schedules.

SISCOG software gives a different perspective of the computational tool, when comparing with the other analyzed tools, because it is explicit the optimization techniques that can be used during the process. This particular software has the advantage of allowing the optimization of different goals at the same time.

In general, using one of these scheduling softwares can reduce labor cost and process time.

## 4

## PROBLEM'S DESCRIPTION

As stated in Chapter 2, one of the main goals of an organization is to satisfy both customers and employees in a cost-effective way. Additionally, call centers have becoming a vital channel to communicate between companies and their customers [16]. As a consequence, having a good scheduling system is crucial because inadequate number of agents for each shift has an effect on customer satisfaction, for instance because of long waiting times, and also on labor related cost. In some organizations, all the scheduling process is done manually. Thereby, the problem considered involves personnel scheduling in a call center.

In this chapter, the problem's characteristics will be approached and an example of an instance will be described.

## 4.1 problem's description

The call center considered operates 24 hours a day and 7 days a week, having a new shift starting every 30 minutes. This means that, in a day, there are 48 different shifts: the first shift starts at 00 h 00 and the last shift starts at 23h30. Each shift has a fixed duration of 9 hours and some mandatory breaks. The planning horizon of the problem is one month, divided in time periods of 30 minutes that constitute set $H$. So, for example, considering a planning horizon of a single day, set $H$ is constitute by 48 elements, as it can be seen in figure 1.
$\left.\begin{array}{c|ccccccccccc}\text { hours } & 00: 00: 00 & 00: 30: 00 & 01: 00: 00 & 01: 30: 00 & 02: 00: 00 & 02: 30: 00 & 03: 00: 00 & 03: 30: 00 & 04: 00: 00 & 04: 30: 00 & 05: 00: 00 \\ 05: 30: 00 \\ \hline H & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10\end{array}\right] 11$

| hours | $06: 00: 00$ | $06: 30: 00$ | $07: 00: 00$ | $07: 30: 00$ | $08: 00: 00$ | $08: 30: 00$ | $09: 00: 00$ | $09: 30: 00$ | $10: 00: 00$ | $10: 30: 00$ | $11: 00: 00$ | $11: 30: 00$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| H | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |


| hours | $12: 00: 00$ | $12: 30: 00$ | $13: 00: 00$ | $13: 30: 00$ | $14: 00: 00$ | $14: 30: 00$ | $15: 00: 00$ | $15: 30: 00$ | $16: 00: 00$ | $16: 30: 00$ | $17: 00: 00$ | $17: 30: 00$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $H$ | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 |


| hours | $18: 00: 00$ | $18: 30: 00$ | $19: 00: 00$ | $19: 30: 00$ | $20: 00: 00$ | $20: 30: 00$ | $21: 00: 00$ | $21: 30: 00$ | $22: 00: 00$ | $22: 30: 00$ | $23: 00: 00$ | $23: 30: 00$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| H | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 |

Figure 1: Example: set of time periods with a planning horizon of a day

Each worker cannot work more than 5 consecutive working days every 7 days and can only be in one shift per day. Additionally, between two consecutive shifts, each worker must stop for at least 11 hours. Ideally, one worker has the same shift for a maximum of 2 weeks, creating a stability route for each one in order to provide a comfortable schedule for the workers.

The workforce is divided geographically between two industrial centers, where the workers are also divided by teams according to their language and team skills. This means that each worker $w$ will have a corresponding skill set $K_{w}$, composed by the different teams that he can be assigned because he has the required skills.

In order to guarantee workers' satisfaction, their time preferences are taken into consideration using a penalization system: the exact preferred time has a minimum penalization associated, i.e. 1 , and this value increases exponentially as the considered time gets away from the preferred time.

The scheduling goal is to assign workers to teams and shifts, minimizing the total penalization when considering workers' preferences and ensuring that the workforce requirements $r^{k h}$ for certain team $k$ at certain time period $h$ are covered.

In this organization, a monthly personnel scheduling is carried out and this process is done manually by a scheduler. So, the current process time is larger than expected, having a duration of some days. Figures 2 and 3 represent the schedules obtained by this process referring to two different teams of the considered call center.


Figure 2: Schedule of team 1 obtained manually


Figure 3: Schedule of team 2 obtained manually

The following notation is used in the development of the model:

## Indices and Sets

$k$ : index for the set of teams $K$;
$h$ : index for the set of time periods $H$;
$w$ : index for the set of workers $W$;
$H$ : set of time periods;
K: set of teams;
$W$ : set of workers

## Parameters

$K_{w}$ : set of teams to which a worker $w(w \in W)$ can be assigned because he has the required skills;
$p_{w}^{h}$ : penalization of assigning worker $w$ on time period $h ;$
$r^{k h}$ : number of workers that are needed at team $k$ and time period $h$;

## 4.2 example of an instance

An instance of this problem is defined based on the notation presented in the previous section. In this section, an example of an instance will be described.

Consider a scenario where the planning horizon is one day. There is only 1 working team, composed by 3 different workers. Table 1 shows the data referring to four parameters: teams, time periods, workers, and the sets associated with each worker that gives the list of teams where he can work in. Note that all the workers are identified by an ID number, composed only with integer numbers.

Table 1: Instance example: teams, time periods and workers

$$
\begin{gathered}
K=\{1\} \\
\frac{H=\{0, \ldots, 47\}}{W=\{1,2,3\}}
\end{gathered}
$$

Table 2: Instance example: set of teams to which a worker $w$ can be assigned

| $w$ | $K_{w}$ |
| :---: | :---: |
| 1 | $\{1\}$ |
| 2 | $\{1\}$ |
| 3 | $\{1\}$ |

In this particular case, only one worker is required to work the shift that starts at 8h00. When it cames to the preference's penalizations, we have three different situations: worker 1 prefers to work on 17 h 30 shift, which means that, in that hour, the associated penalization will be minimum (i.e., equals to 1 ); worker 2 prefers to work on 07 h 00 shift, which means that that hour has a penalization association of 1 ; and worker 3 prefers to work on 14 h 30 shift, which means that, in that hour, the associated penalization will also be minimum. The considered values of both necessities and preference's penalizations can be consulted in table 3 and table 4.

Table 3: Instance example: workforce requirements

| period time | team 1 | period time | team 1 | period time | team 1 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 16 | 1 | 32 | 1 |
| 1 | 0 | 17 | 1 | 33 | 1 |
| 2 | 0 | 18 | 1 | 34 | 0 |
| 3 | 0 | 19 | 1 | 35 | 0 |
| 4 | 0 | 20 | 1 | 36 | 0 |
| 5 | 0 | 21 | 1 | 37 | 0 |
| 6 | 0 | 22 | 1 | 38 | 0 |
| 7 | 0 | 23 | 1 | 39 | 0 |


| 8 | 0 | 24 | 1 | 40 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 9 | 0 | 25 | 1 | 41 | 0 |
| 10 | 0 | 26 | 1 | 42 | 0 |
| 11 | 0 | 27 | 1 | 43 | 0 |
| 12 | 0 | 28 | 1 | 44 | 0 |
| 13 | 0 | 29 | 1 | 45 | 0 |
| 14 | 0 | 30 | 1 | 46 | 0 |
| 15 | 0 | 31 | 1 | 47 | 0 |

Table 4: Instance example: preferences' penalizations

| period time | worker $\mathbf{1}$ | worker $\mathbf{2}$ | worker 3 |
| :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | 64 | 128 | 512 |
| $\mathbf{1}$ | 64 | 64 | 512 |
| $\mathbf{2}$ | 128 | 64 | 1024 |
| $\mathbf{3}$ | 128 | 32 | 1024 |
| $\mathbf{4}$ | 256 | 32 | 2048 |
| $\mathbf{5}$ | 256 | 16 | 2048 |
| $\mathbf{6}$ | 512 | 16 | 2048 |
| $\mathbf{7}$ | 512 | 8 | 1024 |
| $\mathbf{8}$ | 1024 | 8 | 1024 |
| $\mathbf{9}$ | 1024 | 4 | 512 |
| $\mathbf{1 0}$ | 2048 | 4 | 512 |
| $\mathbf{1 1}$ | 2048 | 2 | 256 |
| $\mathbf{1 2}$ | 2048 | 2 | 256 |
| $\mathbf{1 3}$ | 1024 | 1 | 128 |
| $\mathbf{1 4}$ | 1024 | 1 | 128 |
| $\mathbf{1 5}$ | 512 | 1 | 64 |
| $\mathbf{1 6}$ | 512 | 1 | 64 |
| $\mathbf{1 7}$ | 256 | 1 | 32 |
| $\mathbf{1 8}$ | 256 | 2 | 32 |
| $\mathbf{1 9}$ | 128 | 2 | 16 |
| $\mathbf{2 0}$ | 128 | 4 | 16 |
| $\mathbf{2 1}$ | 64 | 4 | 8 |


| $\mathbf{2 2}$ | 64 | 8 | 8 |
| :--- | :---: | :---: | :---: |
| $\mathbf{2 3}$ | 32 | 8 | 4 |
| $\mathbf{2 4}$ | 32 | 16 | 4 |
| $\mathbf{2 5}$ | 16 | 16 | 2 |
| $\mathbf{2 6}$ | 16 | 32 | 2 |
| $\mathbf{2 7}$ | 8 | 32 | 1 |
| $\mathbf{2 8}$ | 8 | 64 | 1 |
| $\mathbf{2 9}$ | 4 | 64 | 1 |
| $\mathbf{3 0}$ | 4 | 128 | 1 |
| $\mathbf{3 1}$ | 2 | 128 | 1 |
| $\mathbf{3 2}$ | 2 | 256 | 2 |
| $\mathbf{3 3}$ | 1 | 256 | 2 |
| $\mathbf{3 4}$ | 1 | 512 | 4 |
| $\mathbf{3 5}$ | 1 | 512 | 4 |
| $\mathbf{3 6}$ | 1 | 1024 | 8 |
| $\mathbf{3 7}$ | 1 | 1024 | 8 |
| $\mathbf{3 8}$ | 2 | 2048 | 16 |
| $\mathbf{3 9}$ | 2 | 2048 | 16 |
| $\mathbf{4 0}$ | 4 | 2048 | 32 |
| $\mathbf{4 1}$ | 4 | 1024 | 32 |
| $\mathbf{4 2}$ | 8 | 1024 | 64 |
| $\mathbf{4 3}$ | 8 | 512 | 64 |
| $\mathbf{4 4}$ | 16 | 512 | 128 |
| $\mathbf{4 5}$ | 16 | 256 | 128 |
| $\mathbf{4 6}$ | 32 | 256 | 256 |
| $\mathbf{4 7}$ | 32 | 128 | 256 |

## MATHEMATICALMODEL

The aim of this project is to develop an efficient exact solution method to solve personnel scheduling problems at call centers by producing schedules of such quality that they are preferable to manually made ones and thus reduce the process time.

So, considering the problem described in Chapter 4, the following integer programming (IP) model was developed:

$$
\begin{equation*}
\min . \quad \sum_{w \in W} \sum_{k \in K_{w}} \sum_{h \in H} p_{w}^{h} x_{w}^{k h} \tag{1}
\end{equation*}
$$

s. t.

$$
\begin{align*}
& \sum_{h=i}^{\min (|H|-1, i+335)} \sum_{k \in K_{w}} x_{w}^{k h} \leq 5, \forall w \in W, \forall i \in I=\{h \in H: h \bmod 336=0\}, \\
& \sum_{h=f}^{f+47} \sum_{k \in K_{w}} x_{w}^{k h} \leq 1, \forall w \in W, \forall f \in F=\{h \in H: h \bmod 48=0\},  \tag{2}\\
& \sum_{\left\{w \in W: k \in K_{w}\right\}} \sum_{h=\max ^{k}(0, b-17\}}^{b} x_{w}^{k h} \geq r^{k b}, b \in H, \forall k \in K,  \tag{3}\\
& \sum_{k \in K_{w}}^{i+39} \sum_{h=i}^{k h} x_{w}^{k h} \leq 1, \forall w \in W, i \in J=\{h \in H: h \leq|H|-40\} . \tag{4}
\end{align*}
$$

## Decision Variables

$$
x_{w}^{k h}= \begin{cases}1, & \text { if worker } w \text { is assigned to team } k, \text { and to the shift starting at time period } h,  \tag{6}\\ 0, & \text { otherwise. }\end{cases}
$$

The purpose of this IP model is to assign workers to teams and shifts. To formulate the problem, a set of decision variables (6) is used to indicate if a worker is assigned to a certain team and to the shift starting at a certain period time. The objective function (1) is to minimize the total penalization when taking into account workers' preferences.

Constraints (2) and (3) ensure that each worker cannot work more than 5 days per week and can only be in one shift per working day, respectively. In (2), the set $I=\{h \in H: h \bmod 336=0\}$ is composed by all the time periods $h \in H$ that are divisors of 336. In practice, all the time periods that belong to this set are the first period time of every 7-day week, because 336 time periods of 30 minutes corresponds to a whole week, starting at $00 h 00$ of day 1 to $23 h 30$ of day 7 . So, in order to ensure that each worker only works a maximum of 5 days per week, the set $I$ is used to guarantee that the summed shifts correspond to only a week. Additionally, the upper bond of the summation in $I$ is given by $\min (|H|-1, i+335)$, where $i \in I$, because of the possibility of the month finishing in the middle of a week. If the considered month does not include the whole week, then the upper bound of the summation will be $|H|-1$. Otherwise, the upper bound will be $i+335$, where $i \in I$. In (3), the logic is the same but the time horizon is only a day, that is, 48 time periods of 30 minutes. Therefore, the set $F=\{h \in H: h \bmod 48=0\}$ is composed by all the time periods $h \in H$ that are divisors of 48 , that is, all the time periods that belong to this set are the first period time of every day. In this case, the upper bond of the summation in $F$ is only given by $f+47$, where $f \in F$, because all the days that are considered are complete.

Constraints (4) force the number of allocated workers to a certain shift in a certain period time and team to be at least equal to the number of required workforce in that same period time and team. In this constraint, the lower and upper bounds of the summation in $H$ are, respectively, $\max \{0, b-17\}$ and $b$, where $b \in H$. This is because, in order to know if the required number of workers in a certain time period is covered, it is necessary to sum all the workers that started working a maximum of 08h30, i.e. 17 time periods, before the time period considered until the ones that start working in the exact time period considered. In the beginning of every month, until the considered time period is less that 17 , the sum is done from the time period 0 , not considering the data of the previous month.

Constraints (5) establish, for each worker, a mandatory break of at least 11 hours between two consecutive shifts. In order to ensure that, a time interval of 40 time periods is considered. This interval is composed by 18 time periods corresponding to a 9 hour shift plus 22 time periods corresponding to a 11
hour break. To deal with the end of the month, the set $J=\{h \in H: h \leq|H|-40\}$ is used, guaranteeing that the limit is not overpast.

In order to include workers' preferences, and based on the quantification of preference violations proposed by Bard and Purnomo [5], a penalization system was build.

Let $p$ be the time preference of a worker. Considering that $h \in\{0, \ldots, 47\}$ represents the $(h+1)$ th 30 minutes time period of a day, a penalty point $v$ is fixed in order to calculate a penalization value $2^{v-1}$, according to the difference between $h$ and $p$, as seen in table 5 .

Table 5: Penalization System
violation
penalty points $v \quad$ penalization $2^{v-1}$

| $0<\|h-p\| \leq 2 \vee\|h-p\|>45$ | 1 | 1 |
| :---: | :---: | :---: |
| $2<\|h-p\| \leq 4 \vee 43<\|h-p\| \leq 45$ | 2 | 2 |
| $4<\|h-p\| \leq 6 \vee 41<\|h-p\| \leq 43$ | 3 | 4 |
| $6<\|h-p\| \leq 8 \vee 39<\|h-p\| \leq 41$ | 4 | 8 |
| $8<\|h-p\| \leq 10 \vee 37<\|h-p\| \leq 39$ | 5 | 16 |
| $10<\|h-p\| \leq 12 \vee 35<\|h-p\| \leq 37$ | 6 | 32 |
| $12<\|h-p\| \leq 14 \vee 33<\|h-p\| \leq 35$ | 7 | 64 |
| $14<\|h-p\| \leq 16 \vee 31<\|h-p\| \leq 33$ | 8 | 128 |
| $16<\|h-p\| \leq 18 \vee 29<\|h-p\| \leq 31$ | 9 | 256 |
| $18<\|h-p\| \leq 20 \vee 27<\|h-p\| \leq 29$ | 10 | 512 |
| $20<\|h-p\| \leq 22 \vee 25<\|h-p\| \leq 27$ | 11 | 1024 |
| $22<\|h-p\| \leq 25$ | 12 | 2048 |

## 6

IMPLEMENTATION

The chosen software to implement the model presented in the previous chapter was IBM ILOG CPLEX Optimization Studio. This software is an analytical decision support toolkit that allow us to develop and design optimization models, using mathematical and constraint programming. One of the advantages of this toolkit is that the user can use OPL (Optimization Programming Language) or one of the application programming interfaces available, such as Python, Java, C, C + + or C $\#$. In this project, the model will be implemented using OPL. The version of the software used to implement the code is version 12.7.0.0.

Optimization Programming Language, or OPL, is an algebraic modeling language for mathematical optimization models. Because of the similarity of its syntax to the mathematical notation of optimization problems, using OPL can make coding easier. This programming language has also another advantages: allows the definition of decision variables and decision expressions over index sets to represent choices affected by them, allows the development, debug, test and tune of mathematical programming models, and is able to specify constraints, sums and other mathematical operations over index sets. According to Heisig and Minner [18], the advantages of OPL are:

- the wide range of addressed problems;
- the general modeling capabilities provided;
- the separation of a project into the modeling and data art, simplifying the execution of the same problem for several instances;
- the less CPU time required when comparing to traditional modeling languages;
- the several examples that are included, allowing the users to use and adapted them in order to solve their problems.

On the other hand, the disadvantages associated with the use of OPL include the difficultly in understanding error message if one is not experienced in computer programming languages and the fact that some desirable additional information for output and sensitivity analysis can only be obtained using OPL script language or specific output functions [18].

To make the management of the data easier, Excel files will also be use, since it is possible to make a connection between the code written and the sheets in that file.

The code of the model developed can be consulted in 6 .

Table 6: Implementation code in OPL - IP model

```
File.MOD
int h=\ldots; /*time period*/
range H=0..h;
int k=_..; /*eam*/
range K=1..k;
{int} W=\ldots; /*set of workers*/
int r [K][H]=...;
/*number of workers that are needed at team k and period time h*/
int p[W][H]=...;
/*
{int} Kw[W] = ...;
/*set of teams to which a worker w can be assigned because he has the required skills*/
{int} F}={\textrm{h}|\textrm{h}\mathrm{ in H : h mod 48== 0};
{int} I = {h| h in H:h mod 336== 0};
{int} J = {h| h in H:h <= card(H)-40};
/*Decision variables:*/
dvar boolean x[W][K][H]; /*defines if a worker w has shift on hour h on team k*/
/*Objective Function:*/
minimize sum(w in W, k in Kw[w], h in H) x[w][k][h]*p[w][h];
/*minimize the total assignment cost of assigning works to shifts*/
/*Constraints:*/
subject to {
forall(w in W, i in I) c1: sum(h in i..minl(i+335, card(H)-1), k in Kw[w]) x[w][k][h] <= 5;
/*5 working days per week*/
forall(w in W, f in F) c2: sum(h in f..f+47, k in Kw[w]) x[w][k][h] <= 1;
/*each worker can only be in one shift per day*/
forall(k in K, b in H) c3: sum(w in W: k in Kw[w],h in maxl(0, b-17)..b) x[w][k][h] >= r[k][b];
/*workforce requirement for team k at time period h*/
forall(w in W, j in J) c4: sum(k in Kw[w], h in j..j+39) x[w][k][h]<= 1;
/*each worker must stop for at least 11h between 2 consecutive shifts*/
}
tuple someTuple{
int w;
int k;
int h;
int value;
}
{someTuple} someSet ={<i, j, l, x[i,j,l]> | i in W, j in K, l in H};
Data File
SheetConnection sheet("name of the file to import solution.xlsx");
someSet to SheetWrite(sheet, "interval to considered");
h = #;
k = #;
Kw= #;
SheetConnection sheet1("name of the file to export data.xlsx");
W from SheetRead(sheet1, "interval to considered");
p from SheetRead(sheet1, "interval to considered");
r from SheetRead(sheet1, "interval to considered");
```

In the written code, several data structures are used in order to combine different data and to make management of data easier.

To declare both time periods and teams two different ranges are used, allowing the definition of lower and upper bounds for each one of them. In these two cases, the upper bound is given by an expression, corresponding to the number of 30 minutes period in the planning horizon and the number of teams of the problem to be solved, respectively.

The set of workers and sets $F, J$, and $I$ are declared using data structure sets, which have the advantage of not allowing duplicate values and allowing different operations. These four sets are all composed by integers only.

In the case of the workforce requirements and the penalizations associated with time preferences, these two parameters are declared using two-dimensional arrays. The former is indexed by teams and workers while the latter is indexed by workers and time period. The set of decisions variables is also defined as an array, indexed by workers, teams and time periods. Using multidimensional arrays is a better and convenient way to represent multiple data items of the same type and size. Also, iterating the arrays using their indexes is faster when compared with other methods. In the first two cases, workforce requirements and penalizations, the two-dimensional arrays are used to represent matrices.

To declare the set of teams to which a specific worker can be assigned because he or she has the required skills an one-dimensional array is used, indexed by workers. In this case, for each value of workers, there is a set of integers corresponding that indicates the teams where he can work in.

In order to obtain the solution of the model in an Excel file, it was necessary to declare a tuple to aggregate some values. Through the declared tuple, a set was externally initialized, giving the set someSet, in which every element is composed by the identification of the worker, the identification time, the time period, and the value of the decision variable.

## 7

## COMPUTATIONAL EXPERIMENTS AND ANALYSIS

To evaluate and analyze the performance of the model, several computational experiments were executed. The code is implemented in IBM ILOG CPLEX Optimization Studio 12.7.0.0 and the tests are carried out on an Intel Core i7-6500U processor and 8 GB of memory RAM, with Windows 10 operating system.

A way to obtain information about the IP problem is through the solution of its linear relaxation. The linear programming (LP) relaxation of an integer programming model is the problem that arises when the integrality constraints are relaxed, which means that all variables are allowed to take non-integer values. Hence, the optimal solution to the LP is not necessarily integer. However, since the feasible region of the LP is larger than the feasible region of the IP, the optimal value of the former is no worse than the optimal value of the latter. This implies that the optimal value to the LP is a lower bound on the optimal value for the problem we started out with. The gap between the optimal LP value and the optimal integral solution is called the integrality gap of the linear program and it is given by

$$
\begin{equation*}
I G=\frac{O P T_{i n t}}{O P T_{f r a c}} \tag{7}
\end{equation*}
$$

where $O P T_{\text {int }}$ is the optimal value of the IP and $O P T_{\text {frac }}$ is the optimal value of the LP.
In order to analyze the integrality gap of the linear program and to evaluate the quality of the IP model, a set of 55 randomly generated instances and a set of 3 real-data instance were tested, in which the planning horizon is a 28 -day month or a 31 -day month. While the number of workers in the first set varies between $30,50,75,100$, and 150 , in the second set it only varies between 55 and 71 workers. More information about which one of the instances, including number of teams, number of workers required, and values of penalizations are described in table 7. Note that in the columns of the workforce requirements and preferences' penalizations, the values are referred to different sheets of the Excel files used, that can be consulted in A .

Table 7: Computational experiments 1 - instances' data

| instance | planning <br> horizon | number of <br> workers | number of <br> teams | workforce <br> requirements | preferences' <br> penalizations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 31 days | 30 | 1 | req0 | pref0 |
| 2 | 31 days | 30 | 1 | req1 | pref0 |
| 3 | 31 days | 30 | 1 | req2 | pref0 |
| 4 | 31 days | 30 | 1 | req3 | pref0 |
| 5 | 31 days | 30 | 1 | req4 | pref0 |
| 6 | 31 days | 30 | 1 | req5 | pref0 |
| 7 | 31 days | 30 | 1 | req6 | pref0 |
| 8 | 31 days | 30 | 1 | req7 | pref0 |
| 9 | 31 days | 30 | 1 | req8 | pref0 |
| 10 | 31 days | 30 | 1 | req9 | pref0 |
| 11 | 31 days | 30 | 1 | req10 | pref0 |
| 12 | 31 days | 50 | 1 | req0 | pref1 |
| 13 | 31 days | 50 | 1 | req1 | pref1 |
| 14 | 31 days | 50 | 1 | req2 | pref1 |
| 15 | 31 days | 50 | 1 | req3 | pref1 |
| 16 | 31 days | 50 | 1 | req4 | pref1 |
| 17 | 31 days | 50 | 1 | req5 | pref1 |
| 18 | 31 days | 50 | 1 | req6 | pref1 |
| 19 | 31 days | 50 | 1 | req7 | pref1 |
| 20 | 31 days | 50 | 1 | req8 | pref1 |
| 21 | 31 days | 50 | 1 | req9 | pref1 |
| 22 | 31 days | 50 | 1 | req10 | pref1 |
| 23 | 31 days | 75 | 1 | req0 | pref2 |
| 24 | 31 days | 75 | 1 | req1 | pref2 |
| 25 | 31 days | 75 | 1 | req2 | pref2 |
| 26 | 31 days | 75 | 1 | req3 | pref2 |
| 27 | 31 days | 75 | 1 | req4 | pref2 |
| 28 | 31 days | 75 | 1 | req5 | pref2 |
| 29 | 31 days | 75 | 1 | req6 | pref2 |
| 30 | 31 days | 75 | 1 | req7 | pref2 |
| 31 | 31 days | 75 | 1 | req8 | pref2 |
| 32 | 31 days | 75 | 1 | req9 | pref2 |
| 2 |  |  |  |  |  |


| 33 | 31 days | 75 | 1 | req10 | pref2 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 34 | 31 days | 100 | 1 | req0 | pref3 |
| 35 | 31 days | 100 | 1 | req1 | pref3 |
| 36 | 31 days | 100 | 1 | req2 | pref3 |
| 37 | 31 days | 100 | 1 | req3 | pref3 |
| 38 | 31 days | 100 | 1 | req4 | pref3 |
| 39 | 31 days | 100 | 1 | req5 | pref3 |
| 40 | 31 days | 100 | 1 | req6 | pref3 |
| 41 | 31 days | 100 | 1 | req7 | pref3 |
| 42 | 31 days | 100 | 1 | req8 | pref3 |
| 43 | 31 days | 100 | 1 | req9 | pref3 |
| 44 | 31 days | 100 | 1 | req10 | pref3 |
| 45 | 31 days | 150 | 1 | req0 | pref4 |
| 46 | 31 days | 150 | 1 | req1 | pref4 |
| 47 | 31 days | 150 | 1 | req2 | pref4 |
| 48 | 31 days | 150 | 1 | req3 | pref4 |
| 49 | 31 days | 150 | 1 | req4 | pref4 |
| 50 | 31 days | 150 | 1 | req5 | pref4 |
| 51 | 31 days | 150 | 1 | req6 | pref4 |
| 52 | 31 days | 150 | 1 | req7 | pref4 |
| 53 | 31 days | 150 | 1 | req8 | pref4 |
| 54 | 31 days | 150 | 1 | req9 | pref4 |
| 55 | 31 days | 150 | 1 | req10 | pref4 |
| 56 | 28 days | 71 | 2 | req11 | team1 |

The results are given in table 8. Column $O P T_{\text {int }}$ and $O P T_{\text {frac }}$ give the optimal value of the IP model and the optimal value of the LP relaxation, respectively. Assuming the same thinking, column $C P U_{i n t}$ time
gives the process time (minutes) for each one of the instances executed with the IP model and column $C P U_{f r a c}$ time the process time (minutes) for each one of the instances executed with the LP relaxation. At last, the last column LP solution is integer? tells us if the obtained solution of LP relaxation allows noninteger variables or not. In all instances, the optimal value of both models are the same, which means that the integrality gap is 1 , that is, null integrality gap. We can not state that the model has null integrality gap but that the model has a null integrality gap for a large set of instances. Additionally, in some cases, the optimal solution allows non-integer variables. So, this means that, although the LP relaxation gave us good approximation solutions of IP model, the solution may turn out to be integer infeasible and, if we round it, the solution may not be the optimal integer solution.

Table 8: Computational experiments 1 - results

| instance | $O P T_{\text {int }}$ | $\mathbf{C P U}{ }_{\text {int }}$ time | $O P T_{\text {frac }}$ | CPU frac $^{\text {time }}$ | LP solution is integer? |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 329 | 00:26:93 | 329 | 00:20:20 | yes |
| 2 | 329 | 00:24:73 | 329 | 00:19:85 | yes |
| 3 | 322 | 00:24:49 | 322 | 00:20:07 | yes |
| 4 | 322 | 00:26:89 | 322 | 00:19:94 | yes |
| 5 | 366 | 00:27:25 | 366 | 00:19:98 | yes |
| 6 | 366 | 00:26:85 | 366 | 00:19:96 | yes |
| 7 | 364 | 00:26:03 | 364 | 00:19:65 | yes |
| 8 | 366 | 00:25:02 | 366 | 00:20:23 | yes |
| 9 | 343 | 00:24:27 | 343 | 00:19:98 | yes |
| 10 | 343 | 00:24:05 | 343 | 00:20:00 | yes |
| 11 | 345 | 00:24:24 | 345 | 00:19:72 | yes |
| 12 | 293 | 00:37:94 | 293 | 00:29:80 | yes |
| 13 | 293 | 00:36:71 | 293 | 00:29:91 | yes |
| 14 | 296 | 00:37:35 | 296 | 00:30:32 | yes |
| 15 | 296 | 00:37:19 | 296 | 00:29:93 | yes |
| 16 | 295 | 00:36:98 | 295 | 00:30:86 | yes |
| 17 | 295 | 00:37:15 | 295 | 00:30:08 | yes |
| 18 | 294 | 00:39:91 | 294 | 00:30:50 | yes |
| 19 | 295 | 00:37:92 | 295 | 00:30:36 | yes |
| 20 | 292 | 00:37:56 | 292 | 00:30:52 | yes |


| 21 | 292 | 00:36:93 | 292 | 00:29:69 | yes |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 22 | 294 | 00:36:84 | 294 | 00:30:21 | yes |
| 23 | 293 | 00:55:72 | 293 | 00:43:96 | yes |
| 24 | 293 | 00:52:93 | 293 | 00:43:37 | yes |
| 25 | 296 | 00:54:84 | 296 | 00:43:01 | yes |
| 26 | 296 | 00:54:95 | 296 | 00:43:60 | yes |
| 27 | 295 | 00:52:90 | 295 | 00:43:57 | yes |
| 28 | 295 | 00:56:06 | 295 | 00:43:16 | yes |
| 29 | 294 | 00:51:29 | 294 | 00:43:53 | yes |
| 30 | 295 | 00:50:51 | 295 | 00:43:50 | yes |
| 31 | 292 | 00:52:37 | 292 | 00:46:89 | yes |
| 32 | 292 | 00:53:94 | 292 | 00:42:89 | yes |
| 33 | 294 | 00:52:32 | 294 | 00:43:26 | yes |
| 34 | 293 | 01:13:05 | 293 | 00:57:15 | yes |
| 35 | 293 | 01:14:72 | 293 | 00:57:05 | yes |
| 36 | 296 | 01:07:53 | 296 | 00:57:31 | yes |
| 37 | 296 | 01:07:97 | 296 | 00:56:66 | yes |
| 38 | 295 | 01:44:39 | 295 | 00:56:94 | yes |
| 39 | 295 | 01:57:52 | 295 | 00:56:92 | yes |
| 40 | 294 | 01:55:99 | 294 | 00:57:16 | yes |
| 41 | 295 | 01:58:96 | 295 | 00:57:46 | yes |
| 42 | 292 | 01:56:45 | 292 | 00:57:71 | yes |
| 43 | 292 | 01:57:10 | 292 | 00:57:74 | yes |
| 44 | 294 | 01:43:72 | 294 | 00:57:75 | yes |
| 45 | 293 | 02:19:79 | 293 | 01:27:43 | yes |
| 46 | 293 | 01:51:79 | 293 | 01:26:88 | yes |
| 47 | 296 | 02:00:22 | 296 | 01:28:97 | yes |
| 48 | 296 | 01:56:59 | 296 | 01:26:70 | yes |
| 49 | 295 | 02:04:18 | 295 | 01:26:19 | yes |
| 50 | 295 | 02:03:17 | 295 | 01:27:55 | yes |


| 51 | 294 | $02: 02: 82$ | 294 | $01: 26: 52$ | yes |
| :--- | :---: | :---: | :---: | :---: | :--- |
| 52 | 295 | $01: 39: 51$ | 295 | $01: 26: 30$ | yes |
| 53 | 292 | $02: 01: 27$ | 292 | $01: 27: 11$ | yes |
| 54 | 292 | $02: 00: 58$ | 292 | $01: 31: 75$ | yes |
| 55 | 294 | $02: 00: 58$ | 294 | $01: 31: 95$ | yes |
| 56 | 1107 | $00: 53: 23$ | 1107 | $00: 45: 66$ | yes |
| 57 | 1077 | $1: 21: 57$ | 1077 | $00: 59: 21$ | no |
| 58 | 1099 | $00: 46: 48$ | 1099 | $00: 38: 67$ | yes |

To evaluate the performance of the model with large instances, 15 randomly generated instances were tested, considering the planning horizon, the number of workers, and the preferences' penalizations equal in all cases, varying only the workforce's requirements. So, all instances have a planning horizon of a 31-day month and 500 workers, organized in a single team. The preferences' penalizations follow the same logic of the one used in figure 6, but applied to 500 workers. Regarding workforce's requirements, they were randomly generated between 0 and 50 or between 0 and 100, as it can be seen in table 9 .

Table 9: Computational experiments 2 - instances' data

| instance | planning <br> horizon | number of <br> workers | number of <br> teams | workforce <br> requirements |
| :---: | :---: | :---: | :---: | :---: |
| 59 | 31 days | 500 | 1 | randbetween $(0,50)$ |
| 60 | 31 days | 500 | 1 | randbetween $(0,50)$ |
| 61 | 31 days | 500 | 1 | randbetween $(0,50)$ |
| 62 | 31 days | 500 | 1 | randbetween $(0,50)$ |
| 63 | 31 days | 500 | 1 | randbetween $(0,50)$ |
| 64 | 31 days | 500 | 1 | randbetween $(0,50)$ |
| 65 | 31 days | 500 | 1 | randbetween $(0,50)$ |
| 66 | 31 days | 500 | 1 | randbetween $(0,50)$ |
| 67 | 31 days | 500 | 1 | randbetween $(0,50)$ |
| 68 | 31 days | 500 | 1 | randbetween $(0,50)$ |
| 69 | 31 days | 500 | 1 | randbetween $(0,100)$ |
| 70 | 31 days | 500 | 1 | randbetween $(0,100)$ |
| 71 | 31 days | 500 | 1 | randbetween $(0,100)$ |
| 72 | 31 days | 500 | 1 | randbetween $(0,100)$ |
| 73 | 31 days | 500 | 1 | randbetween $(0,100)$ |

The combination of the memory RAM of the computer used in the computational experiments and the version of the IBM ILOG CPLEX Optimization Studio initially used, i.e. 12.7.0.0, makes it impossible to achieve any results when testing instances of table 9, forcing an actualization of the version of the software. So, to test these 15 instances, the code is implemented in IBM ILOG CPLEX Optimization Studio 12.10.0.0.

The results are given in table 10. Column OPT gives the optimal value of the IP model, followed by column CPU time that gives the process time (minutes) for each one of the instances executed. In instance 65 the highest CPU time is achieved, having 744000 variables and 743988 constraints. So, taking into account the size of the tested instances, the model presents a good performance, achieving good computational results.

| Table 10: Computational experiments 2 - results <br> instance | OPT | CPU time |
| :---: | :---: | :---: |
| 59 | 3414 | $06: 47: 00$ |
| 60 | 3374 | $06: 49: 45$ |
| 61 | 3438 | $06: 32: 41$ |
| 62 | 3405 | $06: 55: 49$ |
| 63 | 3360 | $07: 01: 94$ |
| 64 | 3374 | $06: 53: 62$ |
| 65 | 3387 | $07: 12: 48$ |
| 66 | 3465 | $07: 12: 06$ |
| 67 | 3348 | $06: 56: 46$ |
| 68 | 3382 | $06: 36: 81$ |
| 69 | 6772 | $06: 51: 57$ |
| 70 | 6757 | $06: 34: 06$ |
| 71 | 6770 | $06: 26: 12$ |
| 72 | 6868 | $07: 03: 08$ |
| 73 | 6768 | $06: 31: 35$ |
|  |  |  |

Considering the solution given when executing real-data instance 56, and using Excel's tools, figures 4 and 5 illustrate the final schedule of team 1 and team 2, respectively. The CPU time in this scenario is lower than 1 minute, having 190848 variables and 97615 constraints. The goal of reducing the process time was obviously achieve.

When compared to schedules represented in figures 2 and 3 , schedules of figures 4 and 5 appear to have less stability than the former. However, the penalization systems used to obtain them are different: the one used in the proposed method is more relaxed. So, consider, for example, a worker that prefers to work in 07 h 00 shift. In the IP developed model, assigning this worker to the 07 h 00 shift or assigning the
same worker to the 08 h 00 shift is identical in the sense that the penalization associated to these two time periods is the same.


Figure 4: Schedule of team 1 - IP solution


Figure 5: Schedule of team 2 - IP solution

CONCLUSION

The personnel scheduling problem is a critical problem in the call center industry since call centers ensure communication between companies and costumers. So, it is important for call centers to be staffed so that the waiting times experienced by costumers are acceptable. In general, good and optimized schedules are advantageous for companies because they are a way to avoid overstaffing, minimizing the total labor costs.

In order to automate a manually scheduling process done in companies and, therefore, reduce the process time, the main goal of this project was to contribute with a new solution approach that rely on exact methods. In this sense, an integer programming model for assigning work shifts to each employee in a $24 / 7$ call center, while taking into account their time preferences, is proposed and implemented on IBM ILOG CPLEX Optimization Studio 12.7.0.0.

The model and its linear relaxation were tested, in order to analyze the quality of the original model, with both randomly generated and real-world instances. The integer programming model was proven to have null integrality gap in all instances that were tested, which means that its formulation is strong. So, the proposed model is a good quality model.

To evaluate the performance of the model when running large instances, more instances were tested, using a more actualized version (version 12.10.0.0) of the chosen software since the older one was not able to run such large instances with the proposed method. The model was proven to have a good performance, achieving low CPU times considering the size of the instances that were tested.

An advantage of the develop model is that it can be applied to scheduling problems in other areas.
As future work, it would be interesting to add more constraints in the proposed IP model, such as scheduling of days-off and breaks, and the possibility of workers change shifts between them. It would also be interesting to implement the suggested solution method in an organization, in order to analyze some performance measures, like attendance and reactivity percentage.

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## Appendices

DATA OF COMPUTATIONAL EXPERIMENTS

In this appendix, several figures of the data files used to test the proposed model can be consulted.

Note that the values that are in the figures referring to the preferences are not directly used in the model. They are used to calculate the associated penalizations taking into account the penalization's system proposed in Chapter 3.

| workers | preference | workers | preference | workers | preference |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $00: 00: 00$ | 10 | $05: 00: 00$ | 20 | $10: 00: 00$ |
| 1 | $00: 30: 00$ | 11 | $05: 30: 00$ | 21 | $10: 30: 00$ |
| 2 | $01: 00: 00$ | 12 | $06: 00: 00$ | 22 | $11: 00: 00$ |
| 3 | $01: 30: 00$ | 13 | $06: 30: 00$ | 23 | $11: 30: 00$ |
| 4 | $02: 00: 00$ | 14 | $07: 00: 00$ | 24 | $12: 00: 00$ |
| 5 | $02: 30: 00$ | 15 | $07: 30: 00$ | 25 | $12: 30: 00$ |
| 6 | $03: 00: 00$ | 16 | $08: 00: 00$ | 26 | $13: 00: 00$ |
| 7 | $03: 30: 00$ | 17 | $08: 30: 00$ | 27 | $13: 30: 00$ |
| 8 | $04: 00: 00$ | 18 | $09: 00: 00$ | 28 | $14: 00: 00$ |
| 9 | $04: 30: 00$ | 19 | $09: 30: 00$ | 29 | $14: 30: 00$ |

Figure 6: Workforce's time preferences - pref0

| workers | preference | workers | preference | workers | preference | workers | preference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $00: 00: 00$ | 13 | $06: 30: 00$ | 26 | $13: 00: 00$ | 39 | $19: 30: 00$ |
| 1 | $00: 30: 00$ | 14 | $07: 00: 00$ | 27 | $13: 30: 00$ | 40 | $20: 00: 00$ |
| 2 | $01: 00: 00$ | 15 | $07: 30: 00$ | 28 | $14: 00: 00$ | 41 | $20: 30: 00$ |
| 3 | $01: 30: 00$ | 16 | $08: 00: 00$ | 29 | $14: 30: 00$ | $21: 00: 00$ |  |
| 4 | $02: 00: 00$ | 17 | $08: 30: 00$ | 30 | $15: 00: 00$ | $21: 30: 00$ |  |
| 5 | $02: 30: 00$ | 18 | $09: 00: 00$ | 31 | $15: 30: 00$ | 42 | 44 |
| 6 | $03: 00: 00$ | 19 | $09: 30: 00$ | 32 | $16: 00: 00$ | 45 | $22: 30: 00: 00$ |
| 7 | $03: 30: 00$ | 20 | $10: 00: 00$ | 33 | $16: 30: 00$ | 46 |  |
| 8 | $04: 00: 00$ | 21 | $10: 30: 00$ | 34 | $17: 00: 00$ | 46 | $23: 00: 00$ |
| 9 | $04: 30: 00$ | 22 | $11: 00: 00$ | 35 | $17: 30: 00$ | 47 | $23: 30: 00$ |
| 10 | $05: 00: 00$ | 23 | $11: 30: 00$ | 36 | $18: 00: 00$ | $00: 00: 00$ |  |
| 11 | $05: 30: 00$ | 24 | $12: 00: 00$ | 37 | $18: 30: 00$ | $00: 30: 00$ |  |
| 12 | $06: 00: 00$ | 25 | $12: 30: 00$ | 38 | $19: 00: 00$ | 49 |  |

Figure 7: Workforce's time preferences - pref1

| workers | preference | workers | preference | workers | preference | workers | preference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $00: 00: 00$ | 19 | $09: 30: 00$ | 38 | $19: 00: 00$ | 57 | $04: 30: 00$ |
| 1 | $00: 30: 00$ | 20 | $10: 00: 00$ | 39 | $19: 30: 00$ | 58 | $05: 00: 00$ |
| 2 | $01: 00: 00$ | 21 | $10: 30: 00$ | 40 | $20: 00: 00$ | 59 | $05: 30: 00$ |
| 3 | $01: 30: 00$ | 22 | $11: 00: 00$ | 41 | $20: 30: 00$ | 60 | $06: 00: 00$ |
| 4 | $02: 00: 00$ | 23 | $11: 30: 00$ | 42 | $21: 00: 00$ | 61 | $06: 30: 00$ |
| 5 | $02: 30: 00$ | 24 | $12: 00: 00$ | 43 | $21: 30: 00$ | 62 | $07: 00: 00$ |
| 6 | $03: 00: 00$ | 25 | $12: 30: 00$ | 44 | $22: 00: 00$ | 63 | $07: 30: 00$ |
| 7 | $03: 30: 00$ | 26 | $13: 00: 00$ | 45 | $22: 30: 00$ | 64 | $08: 00: 00$ |
| 8 | $04: 00: 00$ | 27 | $13: 30: 00$ | 46 | $23: 00: 00$ | 65 | $08: 30: 00$ |
| 9 | $04: 30: 00$ | 28 | $14: 00: 00$ | 47 | $23: 30: 00$ | 66 | $09: 00: 00$ |
| 10 | $05: 00: 00$ | 29 | $14: 30: 00$ | 48 | $00: 00: 00$ | 67 | $09: 30: 00$ |
| 11 | $05: 30: 00$ | 30 | $15: 00: 00$ | 49 | $00: 30: 00$ | 68 | $10: 00: 00$ |
| 12 | $06: 00: 00$ | 31 | $15: 30: 00$ | 50 | $01: 00: 00$ | 69 | $10: 30: 00$ |
| 13 | $06: 30: 00$ | 32 | $16: 00: 00$ | 51 | $01: 30: 00$ | 70 | $11: 00: 00$ |
| 14 | $07: 00: 00$ | 33 | $16: 30: 00$ | 52 | $02: 00: 00$ | 71 | $11: 30: 00$ |
| 15 | $07: 30: 00$ | 34 | $17: 00: 00$ | 53 | $02: 30: 00$ | 72 | $12: 00: 00$ |
| 16 | $08: 00: 00$ | 35 | $17: 30: 00$ | 54 | $03: 00: 00$ | 73 | $12: 30: 00$ |
| 17 | $08: 30: 00$ | 36 | $18: 00: 00$ | 55 | $03: 30: 00$ | 74 | $13: 00: 00$ |
| 18 | $09: 00: 00$ | 37 | $18: 30: 00$ | 56 | $04: 00: 00$ |  |  |

Figure 8: Workforce's time preferences - pref2

| workers | preference | workers | preference | workers | preference | workers | preference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $00: 00: 00$ | 25 | $12: 30: 00$ | 50 | $01: 00: 00$ | 75 | $13: 30: 00$ |
| 1 | $00: 30: 00$ | 26 | $13: 00: 00$ | 51 | $01: 30: 00$ | 76 | $14: 00: 00$ |
| 2 | $01: 00: 00$ | 27 | $13: 30: 00$ | 52 | $02: 00: 00$ | 77 | $14: 30: 00$ |
| 3 | $01: 30: 00$ | 28 | $14: 00: 00$ | 53 | $02: 30: 00$ | 78 | $15: 00: 00$ |
| 4 | $02: 00: 00$ | 29 | $14: 30: 00$ | 54 | $03: 00: 00$ | 79 | $15: 30: 00$ |
| 5 | $02: 30: 00$ | 30 | $15: 00: 00$ | 55 | $03: 30: 00$ | 80 | $16: 00: 00$ |
| 6 | $03: 00: 00$ | 31 | $15: 30: 00$ | 56 | $04: 00: 00$ | 81 | $16: 30: 00$ |
| 7 | $03: 30: 00$ | 32 | $16: 00: 00$ | 57 | $04: 30: 00$ | 82 | $17: 00: 00$ |
| 8 | $04: 00: 00$ | 33 | $16: 30: 00$ | 58 | $05: 00: 00$ | 83 | $17: 30: 00$ |
| 9 | $04: 30: 00$ | 34 | $17: 00: 00$ | 59 | $05: 30: 00$ | 84 | $18: 00: 00$ |
| 10 | $05: 00: 00$ | 35 | $17: 30: 00$ | 60 | $06: 00: 00$ | 85 | $18: 30: 00$ |
| 11 | $05: 30: 00$ | 36 | $18: 00: 00$ | 61 | $06: 30: 00$ | 86 | $19: 00: 00$ |
| 12 | $06: 00: 00$ | 37 | $18: 30: 00$ | 62 | $07: 00: 00$ | 87 | $19: 30: 00$ |
| 13 | $06: 30: 00$ | 38 | $19: 00: 00$ | 63 | $07: 30: 00$ | 88 | $20: 00: 00$ |
| 14 | $07: 00: 00$ | 39 | $19: 30: 00$ | 64 | $08: 00: 00$ | 89 | $20: 30: 00$ |
| 15 | $07: 30: 00$ | 40 | $20: 00: 00$ | 65 | $08: 30: 00$ | 90 | $21: 00: 00$ |
| 16 | $08: 00: 00$ | 41 | $20: 30: 00$ | 66 | $09: 00: 00$ | 91 | $21: 30: 00$ |
| 17 | $08: 30: 00$ | 42 | $21: 00: 00$ | 67 | $09: 30: 00$ | 92 | $22: 00: 00$ |
| 18 | $09: 00: 00$ | 43 | $21: 30: 00$ | 68 | $10: 00: 00$ | 93 | $22: 30: 00$ |
| 19 | $09: 30: 00$ | 44 | $22: 00: 00$ | 69 | $10: 30: 00$ | 94 | $23: 00: 00$ |
| 20 | $10: 00: 00$ | 45 | $22: 30: 00$ | 70 | $11: 00: 00$ | 95 | $23: 30: 00$ |
| 21 | $10: 30: 00$ | 46 | $23: 00: 00$ | 71 | $11: 30: 00$ | 96 | $00: 00: 00$ |
| 22 | $11: 00: 00$ | 47 | $23: 30: 00$ | 72 | $12: 00: 00$ | 97 | $00: 30: 00$ |
| 23 | $11: 30: 00$ | 48 | $00: 00: 00$ | 73 | $12: 30: 00$ | 98 | $01: 00: 00$ |
| 24 | $12: 00: 00$ | 49 | $00: 30: 00$ | 74 | $13: 00: 00$ | 99 | $01: 30: 00$ |

Figure 9: Workforce's time preferences - pref3

| workers | preference | workers | preference | workers | preference | workers | preference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 00:00:00 | 38 | 19:00:00 | 76 | 14:00:00 | 114 | 09:00:00 |
| 1 | 00:30:00 | 39 | 19:30:00 | 77 | 14:30:00 | 115 | 09:30:00 |
| 2 | 01:00:00 | 40 | 20:00:00 | 78 | 15:00:00 | 116 | 10:00:00 |
| 3 | 01:30:00 | 41 | 20:30:00 | 79 | 15:30:00 | 117 | 10:30:00 |
| 4 | 02:00:00 | 42 | 21:00:00 | 80 | 16:00:00 | 118 | 11:00:00 |
| 5 | 02:30:00 | 43 | 21:30:00 | 81 | 16:30:00 | 119 | 11:30:00 |
| 6 | 03:00:00 | 44 | 22:00:00 | 82 | 17:00:00 | 120 | 12:00:00 |
| 7 | 03:30:00 | 45 | 22:30:00 | 83 | 17:30:00 | 121 | 12:30:00 |
| 8 | 04:00:00 | 46 | 23:00:00 | 84 | 18:00:00 | 122 | 13:00:00 |
| 9 | 04:30:00 | 47 | 23:30:00 | 85 | 18:30:00 | 123 | 13:30:00 |
| 10 | 05:00:00 | 48 | 00:00:00 | 86 | 19:00:00 | 124 | 14:00:00 |
| 11 | 05:30:00 | 49 | 00:30:00 | 87 | 19:30:00 | 125 | 14:30:00 |
| 12 | 06:00:00 | 50 | 01:00:00 | 88 | 20:00:00 | 126 | 15:00:00 |
| 13 | 06:30:00 | 51 | 01:30:00 | 89 | 20:30:00 | 127 | 15:30:00 |
| 14 | 07:00:00 | 52 | 02:00:00 | 90 | 21:00:00 | 128 | 16:00:00 |
| 15 | 07:30:00 | 53 | 02:30:00 | 91 | 21:30:00 | 129 | 16:30:00 |
| 16 | 08:00:00 | 54 | 03:00:00 | 92 | 22:00:00 | 130 | 17:00:00 |
| 17 | 08:30:00 | 55 | 03:30:00 | 93 | 22:30:00 | 131 | 17:30:00 |
| 18 | 09:00:00 | 56 | 04:00:00 | 94 | 23:00:00 | 132 | 18:00:00 |
| 19 | 09:30:00 | 57 | 04:30:00 | 95 | 23:30:00 | 133 | 18:30:00 |
| 20 | 10:00:00 | 58 | 05:00:00 | 96 | 00:00:00 | 134 | 19:00:00 |
| 21 | 10:30:00 | 59 | 05:30:00 | 97 | 00:30:00 | 135 | 19:30:00 |
| 22 | 11:00:00 | 60 | 06:00:00 | 98 | 01:00:00 | 136 | 20:00:00 |
| 23 | 11:30:00 | 61 | 06:30:00 | 99 | 01:30:00 | 137 | 20:30:00 |
| 24 | 12:00:00 | 62 | 07:00:00 | 100 | 02:00:00 | 138 | 21:00:00 |
| 25 | 12:30:00 | 63 | 07:30:00 | 101 | 02:30:00 | 139 | 21:30:00 |
| 26 | 13:00:00 | 64 | 08:00:00 | 102 | 03:00:00 | 140 | 22:00:00 |
| 27 | 13:30:00 | 65 | 08:30:00 | 103 | 03:30:00 | 141 | 22:30:00 |
| 28 | 14:00:00 | 66 | 09:00:00 | 104 | 04:00:00 | 142 | 23:00:00 |
| 29 | 14:30:00 | 67 | 09:30:00 | 105 | 04:30:00 | 143 | 23:30:00 |
| 30 | 15:00:00 | 68 | 10:00:00 | 106 | 05:00:00 | 144 | 00:00:00 |
| 31 | 15:30:00 | 69 | 10:30:00 | 107 | 05:30:00 | 145 | 00:30:00 |
| 32 | 16:00:00 | 70 | 11:00:00 | 108 | 06:00:00 | 146 | 01:00:00 |
| 33 | 16:30:00 | 71 | 11:30:00 | 109 | 06:30:00 | 147 | 01:30:00 |
| 34 | 17:00:00 | 72 | 12:00:00 | 110 | 07:00:00 | 148 | 02:00:00 |
| 35 | 17:30:00 | 73 | 12:30:00 | 111 | 07:30:00 | 149 | 02:30:00 |
| 36 | 18:00:00 | 74 | 13:00:00 | 112 | 08:00:00 |  |  |
| 37 | 18:30:00 | 75 | 13:30:00 | 113 | 08:30:00 |  |  |

Figure 10: Workforce's time preferences - pref4

| workers | preference | workers | preference | workers | preference |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 222769 | $17: 30: 00$ | 8966103 | $06: 30: 00$ | 4686900 | $07: 00: 00$ |
| 382098 | $07: 30: 00$ | 18827 | $09: 00: 00$ | 4789722 | $07: 00: 00$ |
| 664439 | $14: 30: 00$ | 140226 | $07: 30: 00$ | 5219301 | $07: 00: 00$ |
| 708052 | $07: 00: 00$ | 225210 | $14: 30: 00$ | 5714816 | $07: 00: 00$ |
| 731539 | $13: 00: 00$ | 259250 | $11: 00: 00$ | 5930835 | $06: 00: 00$ |
| 1564665 | $05: 30: 00$ | 411371 | $22: 00: 00$ | 6322749 | $11: 00: 00$ |
| 1857609 | $14: 30: 00$ | 422934 | $00: 00: 00$ | 6671903 | $00: 00: 00$ |
| 2051847 | $07: 00: 00$ | 1121846 | $07: 00: 00$ | 6791687 | $07: 00: 00$ |
| 2622619 | $07: 00: 00$ | 1277782 | $07: 30: 00$ | 6988118 | $07: 30: 00$ |
| 3674294 | $07: 30: 00$ | 1749854 | $00: 00: 00$ | 7114974 | $07: 30: 00$ |
| 4115166 | $10: 30: 00$ | 1929437 | $07: 30: 00$ | 7316137 | $06: 30: 00$ |
| 4898950 | $07: 00: 00$ | 2441231 | $10: 30: 00$ | 7755988 | $07: 30: 00$ |
| 4967951 | $05: 30: 00$ | 2719210 | $07: 00: 00$ | 8231292 | $14: 30: 00$ |
| 5120629 | $08: 00: 00$ | 3047348 | $07: 00: 00$ | 8277533 | $10: 00: 00$ |
| 5154436 | $07: 30: 00$ | 3138844 | $07: 30: 00$ | 8335596 | $06: 30: 00$ |
| 5631188 | $10: 30: 00$ | 3514380 | $06: 30: 00$ | 8645856 | $05: 00: 00$ |
| 6131668 | $10: 30: 00$ | 3653153 | $15: 30: 00$ | 8897259 | $15: 00: 00$ |
| 6425853 | $08: 00: 00$ | 4073382 | $07: 00: 00$ | 9131030 | $08: 00: 00$ |
| 6624260 | $05: 30: 00$ | 4289129 | $09: 30: 00$ | 9140308 | $06: 30: 00$ |
| 6652500 | $07: 00: 00$ | 4309314 | $11: 00: 00$ | 9383085 | $06: 30: 00$ |
| 7463623 | $14: 00: 00$ | 4480906 | $07: 00: 00$ | 9542304 | $06: 00: 00$ |
| 7550190 | $07: 30: 00$ | 4488427 | $13: 30: 00$ | 9610527 | $09: 30: 00$ |
| 7781665 | $07: 30: 00$ | 4572012 | $07: 00: 00$ | 9839940 | $07: 00: 00$ |
| 8572972 | $07: 00: 00$ | 4686534 | $07: 00: 00$ |  |  |

Figure 11: Workforce's time preferences - pref5

| workers | preference | workers | preference | workers | preference | workers | preference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 642378 | $00: 00: 00$ | 3983009 | $07: 00: 00$ | 1551752 | $14: 30: 00$ | 6294198 | $10: 00: 00$ |
| 3354966 | $07: 30: 00$ | 4236836 | $09: 30: 00$ | 2518092 | $10: 30: 00$ | 7503924 | $14: 00: 00$ |
| 4675319 | $06: 30: 00$ | 4307143 | $13: 30: 00$ | 4804938 | $07: 00: 00$ | 1546914 | $11: 00: 00$ |
| 5058082 | $12: 00: 00$ | 4731764 | $09: 00: 00$ | 7677260 | $15: 00: 00$ | 2404166 | $08: 00: 00$ |
| 5167679 | $10: 30: 00$ | 4847160 | $06: 30: 00$ | 9171231 | $10: 00: 00$ | 2882482 | $06: 00: 00$ |
| 6177357 | $09: 00: 00$ | 5089183 | $07: 00: 00$ | 9471462 | $06: 00: 00$ | 3043569 | $10: 30: 00$ |
| 7088180 | $09: 00: 00$ | 5213236 | $07: 00: 00$ | 1364660 | $10: 00: 00$ | 4592162 | $08: 00: 00$ |
| 9167432 | $00: 00: 00$ | 7035626 | $17: 30: 00$ | 1754091 | $16: 00: 00$ | 4635747 | $10: 00: 00$ |
| 9474974 | $00: 00: 00$ | 7890473 | $10: 00: 00$ | 2575962 | $14: 00: 00$ | 5986625 | $09: 00: 00$ |
| 1445582 | $14: 00: 00$ | 8311231 | $13: 30: 00$ | 3422590 | $18: 00: 00$ | 6132487 | $10: 00: 00$ |
| 2149990 | $07: 00: 00$ | 8574282 | $06: 00: 00$ | 3455714 | $22: 00: 00$ | 7126783 | $08: 00: 00$ |
| 2186519 | $14: 30: 00$ | 9062282 | $10: 00: 00$ | 5085051 | $00: 00: 00$ | 8590229 | $09: 00: 00$ |
| 2919661 | $08: 00: 00$ | 966402 | $10: 00: 00$ | 5766563 | $06: 30: 00$ | 9444274 | $07: 00: 00$ |
| 3437864 | $08: 00: 00$ | 1215150 | $10: 00: 00$ | 5969923 | $06: 30: 00$ |  |  |

Figure 12: Workforce's time preferences - pref6


Figure 13: Workforce's requirements - req0

Figure 14: Workforce's requirements - req1


Figure 15: Workforce's requirements - req2


 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 |
| 3 | 2 | 3 | 3 |
| 3 | 2 | 3 | 4 |
| 4 | 4 | 3 | 4 |
| 4 | 4 | 3 | 5 |
| 5 | 4 | 3 | 5 |
| 5 | 4 | 3 | 5 |
| 7 | 4 | 3 | 6 |
| 7 | 4 | 4 | 7 |
| 8 | 4 | 4 | 7 |
| 9 | 4 | 5 | 8 |
| 10 | 4 | 6 | 9 |
| 10 | 4 | 7 | 10 |
| 11 | 4 | 7 | 10 |
| 12 | 4 | 8 | 10 |
| 12 | 4 | 9 | 11 |
| 11 | 3 | 9 | 11 |
| 10 | 2 | 8 | 11 |
| 10 | 2 | 7 | 10 |
| 9 | 2 | 7 | 9 |
| 8 | 0 | 7 | 8 |
| 8 | 0 | 8 | 8 |
| 7 | 0 | 9 | 7 |
| 7 | 0 | 9 | 7 |
| 6 | 0 | 9 | 7 |
| 5 | 0 | 8 | 6 |
| 4 | 0 | 8 | 5 |
| 3 | 0 | 8 | 5 |
| 2 | 0 | 7 | 4 |
| 2 | 0 | 5 | 3 |
| 1 | 0 | 5 | 2 |
| 0 | 0 | 5 | 2 |
| 0 | 0 | 4 | 2 |
| 0 | 0 | 3 | 1 |
| 0 | 0 | 3 | 1 |
| 0 | 0 | 3 | 0 |
| 0 | 0 | 3 | 0 |



Figure 16: Workforce's requirements - req3


Figure 17: Workforce's requirements - req4

Figure 18: Workforce's requirements - req5


Figure 19: Workforce's requirements - req6

Figure 20: Workforce's requirements - req7


Figure 21: Workforce's requirements - req8


Figure 22: Workforce's requirements - req9


Figure 23: Workforce's requirements - req10


Figure 24: Workforce's requirements - req $11_{\text {team } 1}$


Figure 25: Workforce's requirements - req $11_{\text {team } 2}$


Figure 26: Workforce's requirements - req $12_{\text {team } 1}$


Figure 27: Workforce's requirements - req $12_{\text {team } 2}$


Figure 28: Workforce's requirements - req $13_{\text {team } 1}$


Figure 29: Workforce's requirements - req $13_{\text {team } 2}$
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Figure 30: Workforce's requirements - req $13_{\text {team }}$


Figure 31: Workforce's requirements - req $13_{\text {team } 4}$

Figure 32: Workforce's requirements - req $13_{\text {team } 5}$

