10th IMC
10th International Masonry Conference
G. Milani, A. Taliercio and S. Garrity (eds.)
Milan, Italy, July 9-11, 2018

FINITE ELEMENT MODELLING OF MASONRY CROSS VAULTS: CONSIDERATIONS ON BLOCK INTERLOCKING AND INTERFACE PROPERTIES

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Keywords: cross vaults, masonry vaults, FEM; block interlocking; interface elements.

Abstract. *Masonry cross vaults had been used for centuries in the roofing of European buildings, palaces and churches, representing nowadays an integral part of national cultural heritage. In this regard, a sound knowledge of the structural response of cross vaults under vertical and horizontal loads is fundamental for planning accurate and compatible conservation programs. Whereas a certain consensus on the static behavior of the cross vault under gravitational loads has been reached, still more efforts are requested for assessing its seismic capacity.*

In the present work, the finite element approach has been implemented with a particular attention to the block interlocking and interface elements. On the one hand, an appreciable accommodation between the real block arrangement and computational effort is shown. On the other hand, modelling the blocks with rigid-infinitely resistant elements leads the interface as the only source of physical nonlinearities. Paralleling recent works on the seismic behavior of masonry arches, the influence of the mechanical parameters of the interface elements is discussed. Comparisons between numerical and experimental results available in literature are presented in terms of ultimate strength capacity and failure mechanisms.

1 INTRODUCTION

Masonry cross vaults represent one of the most fascinating structural topology of the European built cultural heritage. Their beauty and and technological perfection are the evidence of an ancient expertise that reached such a level of complexity that still impresses the modern observer. For centuries this experience and the "instructions" followed by the masons of that time had been jealously collected in the so-called rules of thumbs, which has guaranteed the long-lasting history of this kind of vaults [1]. Nevertheless, the rules of thumb addressed only dead loads. Conversely to what stated in *Naturalis Historia* (around 79 AD) by Pliny the Elder, who described small pozzolana concrete vaults as the safest place in case of earthquake, the high seismic vulnerability of masonry vaults is still evident, sometimes with incalculable loss in terms of cultural heritage.

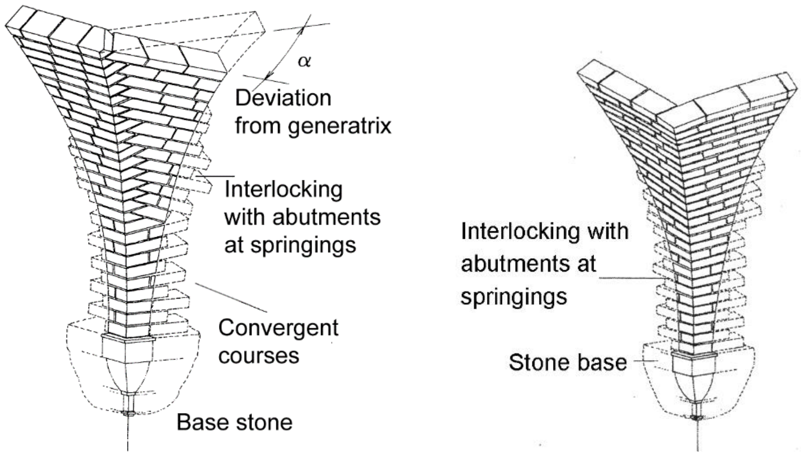
In this regard, the present work is aimed to investigate the seismic behavior of masonry groin vault, that is, the simplest form of cross vaults obtained by the intersection at right angle of two barrel vaults by means of a Finite Element Model (FEM). In particular, within a larger research program, the numerical model adopted in [2], [3] has been extended and validated against experimental evidences. According to the available literature, the experimental tests performed by Rossi and Co-workers [4]–[6] on 1:5 scaled groin vault have been considered. Thanks to a very detailed specimen, where the single brick is also scaled, it was possible to validate the numerical response of the vault modelled with an approximate block arrangement, a delicate issue from the structural and geometrical point of view.

According to the available literature (e.g. [7], [8]), in fact, the block arrangement plays an important role in the capacity of masonry vaulted structures. In case compound vaults are concerned (e.g. cross and cloister vaults), rather than for the webs, the influence of interlocking is more crucial for the groins, which represent the only connection between the shells. In this regard, considering the structural behavior of cloister vaults under gravitational loads, Tomasoni [7] stated that the block arrangement parallel to the springings may facilitate the occurrence of the typical diagonal cracks due to the alignment of the mortar joints along the groins.

An appreciable accommodation between the complexity of the real arrangement (as follows from ancient construction manuals) and the computational effort is here proposed. The blocks are modelled with rigid-infinitely resistant elements, leaving the interfaces as the only source of physical nonlinearities. Adopting a Coulomb friction interface, with cohesion, tensile strength and dilatancy set to zero, the discussion is limited to the influence of interface normal and tangential stiffness. Comparisons between numerical and experimental results are presented in terms of ultimate strength capacity and failure mechanisms according to two typologies of tests, namely in-plane mechanism and tilting test.

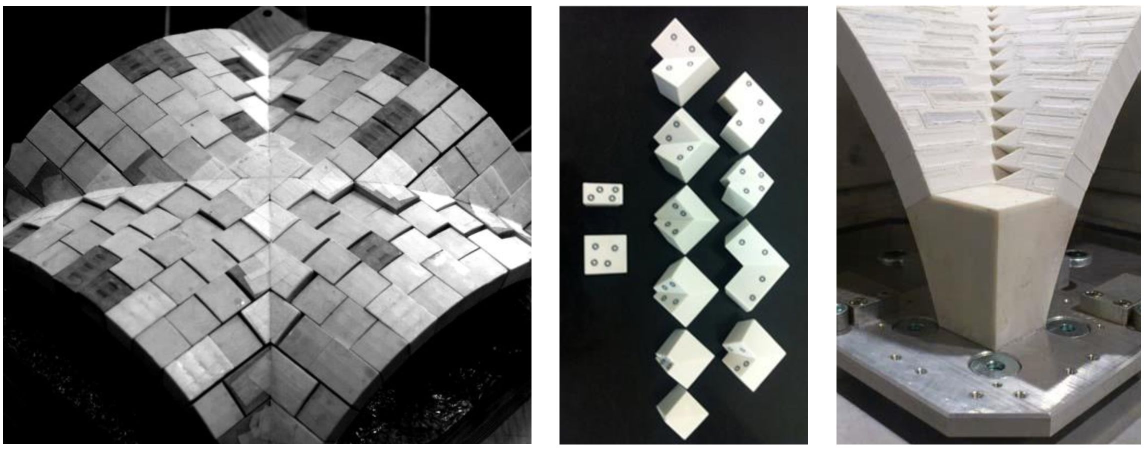
2 BLOCK ARRANGEMENT

In order to guarantee a good interlocking between bricks or stone blocks at the groins, in the antiquity accurate expertise was requested in the field of stereotomy [9]–[11]. In particular, three arrangements were mainly suggested: orthogonal, parallel and oblique (herringbone bond) with respect to the generatrix of the web. The last pattern supposes courses oriented perpendicularly to the plane of the groins and connected in the middle of the webs [12], [13]. More recently, Giovannetti [14] presented a detailed study on this topic with the description of the brick disposition and the necessary cut for the elements shaping the groin. Two cases are examined, namely, arrangement along the generatrix of the webs and herringbone bond, reported in Figure 1a-b, respectively.



a) b)

Figure 1: Groin vault: a) arrangement parallel to the generatrix and b) herringbone bond arrangement [14].



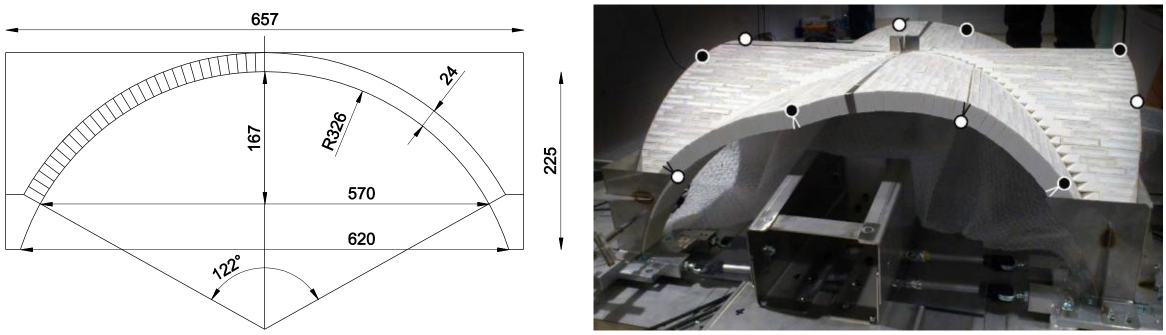
a) b) c)

Figure 2: Experimental model for cross vault testing: a) [15]; b) [16]; c) [4].

As far as experimental tests are concerned, the strict respect of the traditional rules may request a significant effort and a simpler approach is usually employed. For the sake of clearness, the details of the blocks arrangements of three models [4], [15], [16] are reported in Figure 2. As it clearly visible, the two first research groups built the groin with unrealistic blocks (V-shaped), completely neglecting the real pattern and providing the model with higher stiffness and strength along the groins (not on the safe side). On the other hand, Rossi et al. proposed a simplified and more accurate approach based on a 1:5 scaled modern brick ($6 \times 12 \times 24$ cm^3). The physical model provides: 1) an appreciable block interlocking at the groin, 2) no distortion of the block shape (only plane slicing from the original parallelepiped shape), and 3) an overall block pattern simplification. The only drawback regards the gaps at the extrados that reduce in size approaching the vault crown.

3 DESCRIPTION OF EXPERIMENTAL TESTS

The model studied in the present work is the one experimentally tested by Rossi et al. [4]–[6]. The experimental tests were performed on a 1:5 scaled groin vault made by dry-joint 3D printed plastic blocks. The geometry of the vault was generated on a square bay by the inter-



a) b)

Figure 3: Overall dimension of the model: a) front view (measures in mm); b) picture of one of the tests.

section of two semi-circular barrel vaults with an inner radius of 0.326 m. All the geometrical quantities are reported in Figure 3. The blocks were made by a 3D prototyping technique called SLS (Selective Laser Sintering). The mean friction coefficient μ = 0.56 of the blocks was determined by testing 12 couples of blocks. The elastic modulus E = 120 MPa was measured by testing three assemblages of six blocks each under uniaxial compression. The density of the plastic material was ρ = 550 kg/m^3. Since this quite low value would have compromised the model stability under accidental actions, the weight of the model was increased by inserting a steel plate within each block. This technical measure allowed achieving an equivalent density of about 2700 kg/m^3. The mass of the whole structure was about 35.6 kg.

4 NUMERICAL MODEL

In order to replicate the results through numerical analyses based on rigid-infinitely resistant blocks and friction interface elements, great attention has been paid to the discretization of the vault. Despite the accuracy of the real specimen, from the computational point of view, meshing the real size block may have represented a significant increment of DOFs, i.e. more effort and time of running.

In this regard, the present numerical model was built considering a “double-block” composed by two physical blocks, that is, merging two blocks of 24 × 12 × 48 mm^3 each into a single one of 24 × 24 × 48 mm^3. Moreover, starting from stretcher bond (the simplest arrangement for masonry elements), the methodology adopted for the block pattern is sketched in Figure 4 and synthesized into three main steps:

- beginning one web with half brick for a geometrical shift of the courses (Figure 4a);
- slicing the bricks according to the plane of orthogonal courses of adjoin webs (Figure 4a);
- finishing the intrados surface (Figure 4b).

As it is possible to notice, the main drawback of this approach is still the gaps along the extrados of the groin, more pronounced close to the springings. However, comparing Figure 4b with Figure 2c, the gaps are better distributed and with an overall half number of blocks. Finally, it is worth noting that, in both experimental and numerical models, the shape of the blocks was slightly trapezoidal to geometrically compensate for the lack of mortar joints.

Regarding the boundary conditions, the lowest blocks were constrained, and the rest of the vault simply leaned against them through friction interfaces. Furthermore, even though Rossi et al. did not describe this aspect in detail, the vault corners were laterally constrained by steel plates (Figure 3a).

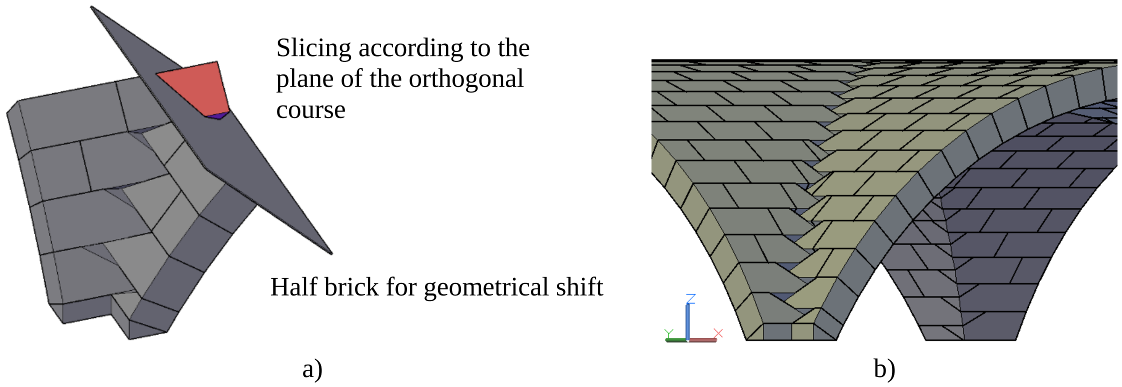


Figure 4: Block pattern adopted in the present study: a) methodology; b) extrados and intrados view.

From the constitutive point of view, given the similarities of this specimen with the arch studied in [2], [3] (e.g. 3D printed blocks, dry joints, and overall dimensions) the same mechanical parameters have been chosen. In particular, the blocks were modelled by way of rigid-infinitely resistant elements with nonlinear friction interfaces. On the other hand, the interface stiffness values in the range 0.1 ÷ 1 MPa were considered the most suitable for the analyses.

5 IN-PLANE MECHANISM

This is a typical failure mechanism for vaults of church aisles. Compared to the external wall, the remarkable lower stiffness of the central nave colonnade produces a differential translation of the two opposite sides of the vaults. This basically means shear action in the plane of the vault. Figure 5 reports the scheme of the test together with the indication of the additional constrains added to reproduce the effect, the steel plates that cover the lower blocks of the specimen (Figure 3).

The first aim of the analysis was to investigate the influence of the interface stiffness. In general, no matter the interface stiffness, all the calculated capacity curves displayed an increasing monotonic trend achieving a maximum capacity equal to around 20% of the weight. This behavior is stressed in Figure 6, where the the capacity curves for three values of $K_t = K_n$ are reported. The curve with $K_t = K_n$. = 0.5 MPa/mm is the closest to the experimental results.

Similarly to what shown in [3], [17], the effect of a 10% reduction of the overall thickness of the vault (to account for slight variations in block size, rounded corners and imperfections of the manually assembled geometry) was considered with the results reported in Figure 7.

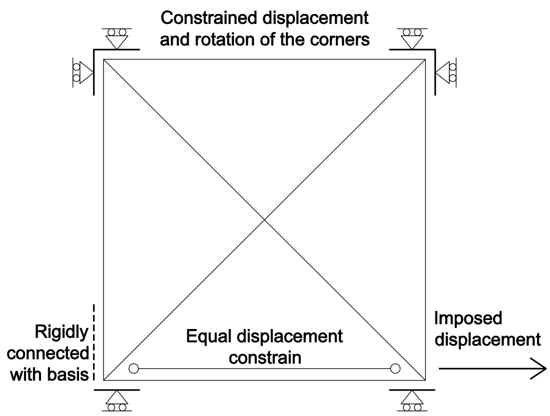


Figure 5: Layout of the numerical model for in-plane shear mechanism.

In particular, assuming $K_t = K_n$. = 0.5 MPa/mm, the curve provides a good agreement in terms of maximum strength (around 15% of the weight) and most of the capacity curve. In terms of failure mechanism, the results are in line with the experimental ones (Figure 8).

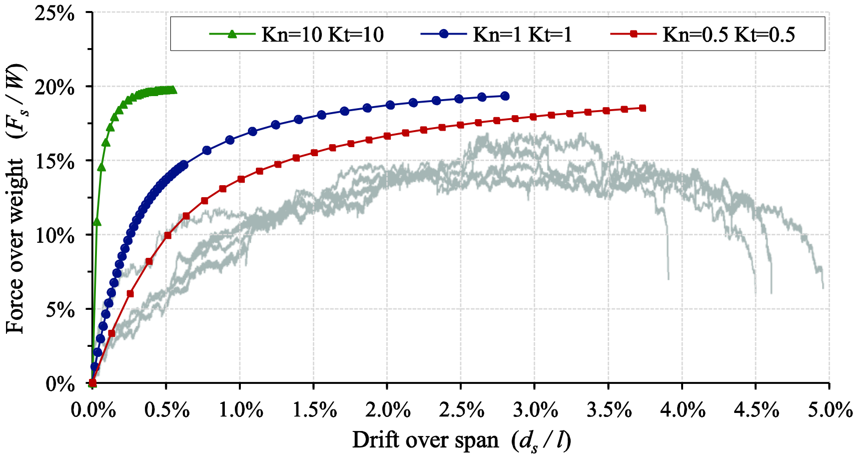


Figure 6: In-plane shear mechanism: numerical results considering $K_n = K_t = 0.5$, 1, 10 MPa/mm.

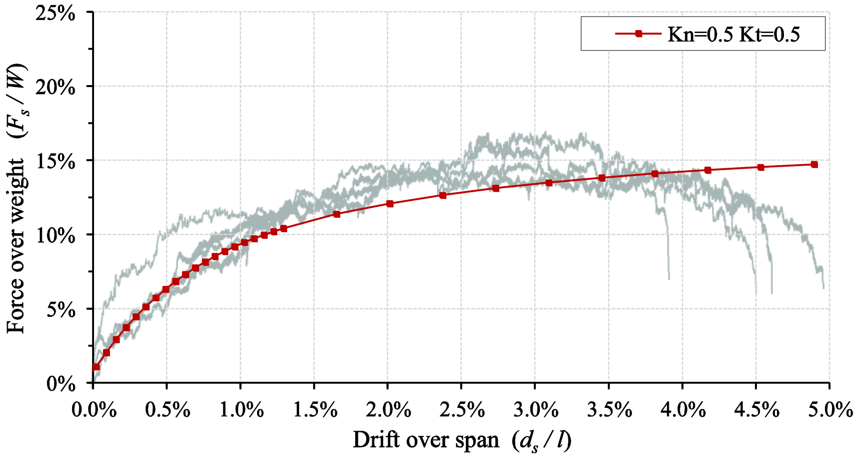


Figure 7: In-plane shear mechanism: capacity curve considering a 10% reduction of the thickness and $K_n = K_t = 0.5$ MPa/mm.

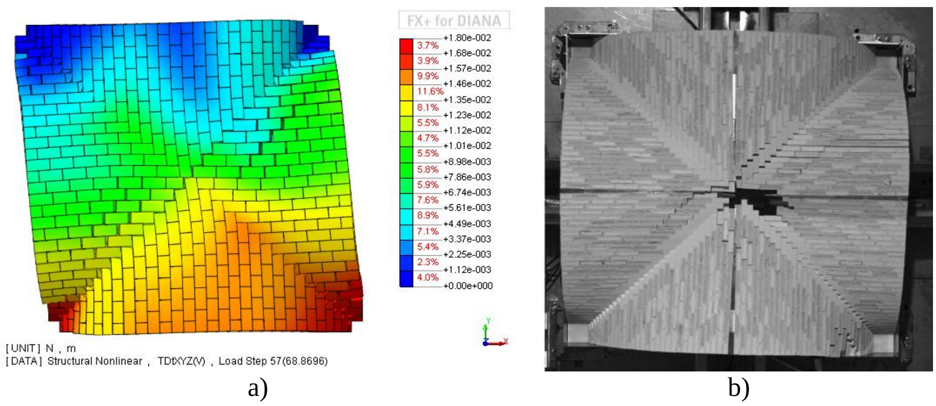


Figure 8: In-plane shear mechanism: a) deformed shape for the numerical model with $K_n = K_t = 1$ MPa/mm (graphic scale 4:1 with colors according to total x-y-z displacement); b) picture of the test.

6 TILTING TEST

Dealing with rigid blocks, the tilting test can be regarded as a first-order seismic assessment method to evaluate the collapse mechanism and the corresponding horizontal load multiplier. The tests are generally performed by means of a quasi-static rotation of the base platform until failure occurred. Moreover, since the seismic actions can hit the structure from any direction, the vault response was investigated considering six different seismic directions, from ϕ=0° (orthogonal to the web profile) to ϕ=45° (along the diagonal axis). As far as the numerical model is considered, an incremental horizontal load proportional to the mass (pushover analysis) was implemented. About the lateral plates, the boundary conditions of the numerical model have been changed according to each seismic direction.

Paralleling the study described in the previous section, the values of interface stiffness that best fitted the experimental results are $K_n = K_t = 1$ MPa/mm, and the results in terms of maximum strength are reported in Figure 9. The capacity is overestimated by the numerical model (up to 20% in case ϕ =45°) and, conversely to the experimental results, the capacity increases from 0° to 45°. The differences between the deformed shapes are shown in Figure 10, where an appreciable match is notable.

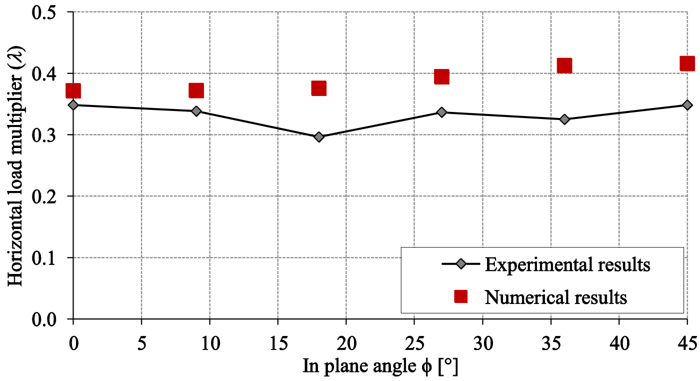


Figure 9: Horizontal load multiplier of the vault according to the seismic direction ϕ: experimental and numerical results ($K_n = K_t = 1$ MPa/mm).

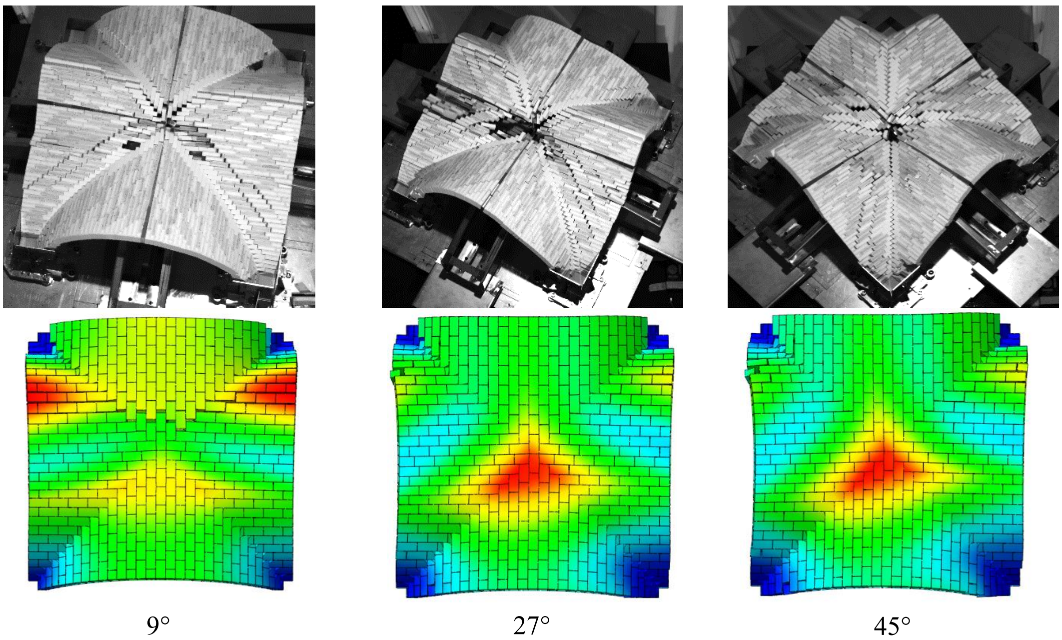


Figure 10: Comparison between the experimental and numerical failure mechanism according to different seismic directions with $K_n = K_t = 1$ MPa/mm (azimuth view).

7 CONCLUSIONS

The present work described the numerical analyses performed according to the results (available in literature) of a recent experimental campaign on a scaled groin vault. The tests regarded two different experimental configurations, namely in-plane shear distortion and tilting test, whose results have been used to validate FE model built with an approximate block arrangement and to investigate the influences of the mechanical parameters of the interface elements. The assumptions and the main results are briefly reviewed.

The FE model was implemented adopting a moderately different block pattern and dimensions with the respect to the experimental ones. The motivation of this choice is the sensible reduction of DOFs and of the amount of interface elements (the only source of physical non-linearities), as well as the overall simplicity of the pattern adopted. In terms of catching the failure mechanism, no significant differences were notable between the experimental and numerical results, with an overall good matching of the crack pattern. The interface stiffness has been seen as not influential on the failure mechanisms, being, obviously, more pronounced for low values of stiffness.

Regarding the capacity of the vault, the model calibration confirmed the results of literature about the values of normal and tangential stiffness in the range 0.1-1 MPa/mm as the most suitable for these analyses. However, it is worth noting that the present work addressed the study of a scaled vaulted structure build with plastic blocks and with an overall low level of stress (if compared with real structures). Being these aspects crucial in the definition of the interface stiffness, an experimental campaign concerning different scale and mass density is rather desirable.

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