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Paper 232

Nonlinear Finite Element Analysis of Strengthened Masonry Buildings subject to Seismic Action

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Abstract

Masonry structures are always used from the past until modern times but due to material degradation, imposed displacements, and structural alterations some members need strengthening to re-establish their performances. In this frame, fiber-reinforced polymer (FRP) composites in the form of bonded laminates applied to the external surface are an effective solution [1,2]. Despite research efforts in the last years, for the seismic analysis of the strengthened masonry system, there is still lack of numerical models, which have the advantages of accurate, high- efficiency and good-convergence [3,4]. In the first part of this paper, numerical approaches to model FRP strengthened masonry structures are discussed and in particular a material model suitable for micro-modelling of the interfacial behaviour FRP-masonry implemented in the Diana finite element (FE) program using a user subroutine is presented [5,6,7]. This micro-modelling approach based on interface elements is then used to develop and validate the global behaviour of a different type of FE that was implemented in the Opensees finite element framework. This new element is extremely effective for the seismic analysis of masonry buildings because of the significant advantage of drastically reducing the number of DOF of the FEM model [8,9,10]. Numerical results are validated by comparison with experimental results from tests performed at the University of Pavia and the Georgia Institute of technology. In particular, it shows a satisfactory degree of accuracy to analyse complex assemblages of masonry buildings including cyclic loads effects and FRP strengthening influence.

Keywords: masonry buildings, FRP, seismic analysis, Multi-Fan element, plasticity

1 Introduction

The current engineering practice for the seismic analysis of masonry buildings is moving away from simplified linear-elastic methods of analysis, and towards a more complex nonlinear-inelastic techniques. These procedures focus on the nonlinear behaviour of structural response and employ methods not previously emphasized in seismic codes. Up to now, in the design of buildings, the seismic effects and the effects of the other actions included in the seismic design situation, may be determined on the basis of four different methods: linear static procedures, mode superposition procedures, nonlinear static (pushover) procedures, nonlinear dynamic (time history) procedures. Limit analysis is often not sufficient for a full structural analysis under seismic loads, but it can be profitably used in order to obtain a simple and fast estimation of collapse loads.

Non-linear analyses should be properly substantiated with respect to the seismic input, the constitutive model used, the method of interpreting the results of the analysis and the requirements to be met. The mathematical model used for elastic analysis shall be extended to include the strength of structural elements and their post-elastic behaviour. As a minimum, bilinear force – deformation envelopes should be used at the element level. In masonry buildings, the elastic stiffness relation should correspond to cracked sections. Zero post-yield stiffness may be assumed. If strength degradation is expected, e.g. for masonry walls or for brittle elements, it has to be included in the envelope. Unless otherwise specified, element properties should be based on mean values of the properties of the materials. Gravity loads shall be applied to appropriate elements of the mathematical model. The seismic action shall be applied in both positive and negative directions and the maximum seismic effects shall be used.

2 Micro-modelling approach

The micro-modelling strategy for masonry, in which the units are discretized with continuum elements and the joints are discretized with interface elements is a very powerful tool to understand the behaviour of masonry. Lourenço [3] developed a constitutive model for the monotonic behaviour of interface elements within the incremental theory of plasticity, including all the modern concepts used in computational plasticity, such as the implicit return mapping and consistent tangent operators. The existing interface model was successfully used to simulate the interfacial behaviour of FRP-masonry joints in direct shear bond tests showing that a good agreement with experimental and analytical results can be achieved [5]. But when the model is applied to simulate the strengthening effects of FRP strips bonded to curved masonry elements some differences exist between numerical results and experimental evidence probably because the bond mechanism of these substrates requires a different description. Aiming to improve the existing interface model, a new multi-linear hardening law was proposed for shear and tension modes of failure and implemented in Diana 8 as a user subroutine [6]. Uncoupled behaviour for tension and shear mode is considered for the masonry-FRP interface, instead coupled behaviour for the masonry joints. The existing monotonic constitutive interface model is defined by a convex composite yield criterion, composed by three individual yield functions, where softening behaviour has been included for all modes according Equation (1):

Tensile criterion:
$f_t(\sigma,\kappa_1)=\sigma - \bar{\sigma}_1(\kappa_1)$
Shear criterion:
$f_s(\sigma,\kappa_2)=|\tau|+\sigma\tan\varphi-\bar{\sigma}(\kappa_2)$ (Eq.1)
Compressive criterion:
$f_c(\sigma,\kappa_3)=(\boldsymbol{\sigma}^T\mathbf{P}\boldsymbol{\sigma})^{1/2}-\bar{\sigma}_3(\kappa_3)$

Associated flow rules were assumed for tensile and compressive modes and a non-associated plastic potential was adopted for the shear mode with dilatancy angle ψ and cohesion c. Here, ϕ represents the friction angle and P is a projection diagonal matrix, based on material parameters (Cnn, Css, Cn). $\bar{\sigma}_1$, $\bar{\sigma}_2$ and $\bar{\sigma}_3$ are the isotropic effective stresses of each of the adopted yield functions, ruled by the scalar internal variables κ_1, κ_2 and κ_3. Figure 1 schematically represents the three individual yield surfaces in the stress space.

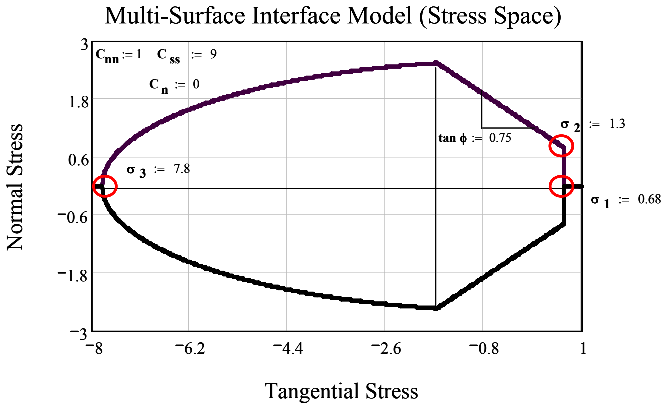


Figure. 1 Multi-surface interface model (stress space)

The consistent tangent stiffness matrix is evaluated according general formulation of multi-surface plasticity, see also Lourenço [3], according Equation (2):

$$\mathbf{D}^{ep}=\left.\frac{d\sigma_{n+1}}{d\varepsilon_{n+1}}\right|_{n+1}=\mathbf{H}-\frac{\mathbf{H}\frac{\partial g}{\partial\sigma}\gamma^T\mathbf{H}}{h+\gamma^T\mathbf{H}\frac{\partial g}{\partial\sigma}}\;; \quad \text{(Eq. 2)}$$

and therefore is a function of the hardening modulus h depending from the shape of the hardening law active.

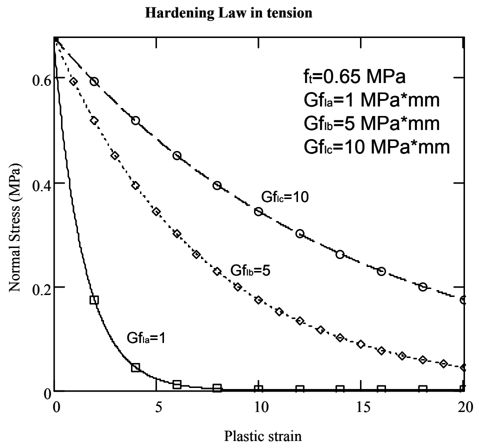


Figure. 2: yield value $\bar{\sigma}_1$ as a function of the fracture energy

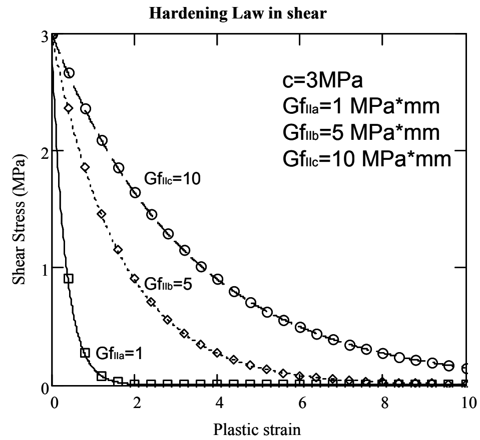


Figure. 3: yield value $\bar{\sigma}_2$ as a function of the fracture energy

A multi-linear law according the new implementation is reported in Figure 4. This new multi-linear hardening/softening law can be used to describe in a more general way, the behaviour of the FRP-masonry joint, allowing to define $\bar{\sigma}_2$ as a function of 10 parameters (including the cohesion and the fracture energy of the interface) that can easily be determined based on experimental results [6]. For the cap mode, the expression of h_3 and $\bar{\sigma}_3$, see Figure 5, are more complex and not given here for briefness, see Lourenço [3].

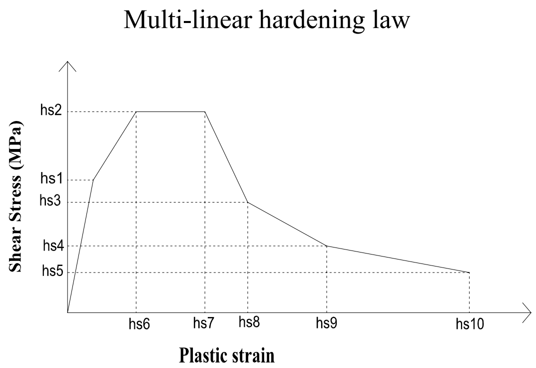


Figure. 4: New hardening/softening law for $\bar{\sigma}_2(\kappa_2)$

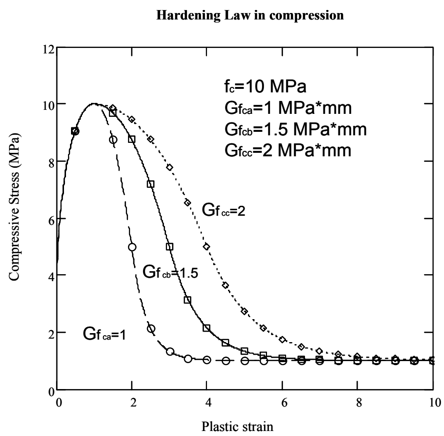
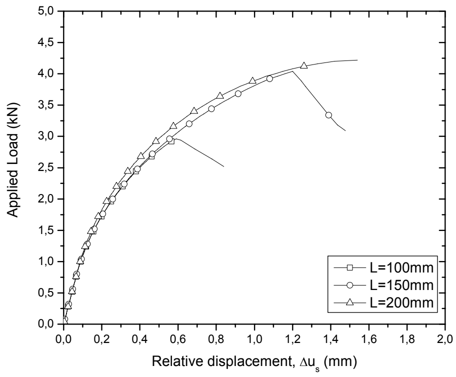


Figure. 5: yield value $\bar{\sigma}_3$ as a function of the fracture energy

The results obtained to simulate bond tests on plain and curved substrates are provided in the following, see Figure 6 and Figure 7.



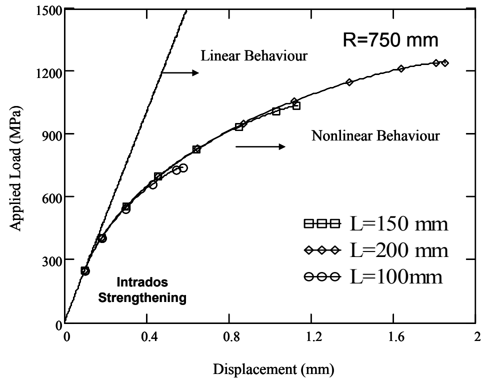


Figure 6: Numerical load displacement diagrams to predict bond strength of plain FRP-Masonry joints as a function of the bond length

Figure 7: Numerical load displacement diagrams to predict bond strength of curved FRP-Masonry joints as a function of the bond length

3 Macro-modelling approach

The monotonic Multi-Fan element was originally developed by Braga and Liberatore [8,9,10]. Each masonry panel in the structure can be accurately modelled by a single element. Panel here is taken to represent a rectangular part of the wall with free lateral edges. It assumes that the stress field of the panel follows a Multi-Fan pattern, see Figure 8.The material behaviour is assumed linear elastic in compression and non-reaction in tension. In addition, it is assumed that: the upper and lower faces of the panel are rigid, and there is no interaction in the circumferential direction between the infinitesimal fans, see Figure 9.

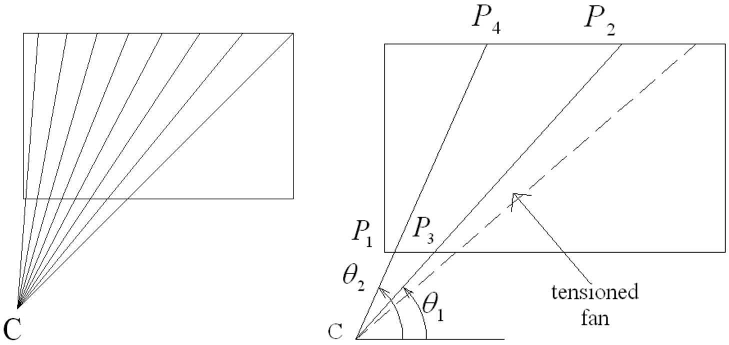


Figure 8: Multifan stress field

The unknowns about the element are the displacement of the first end second cross-section denoted by u_1, v_1, $Ø_1$ and u_2, v_2 and $Ø_2$, see Figure 9.

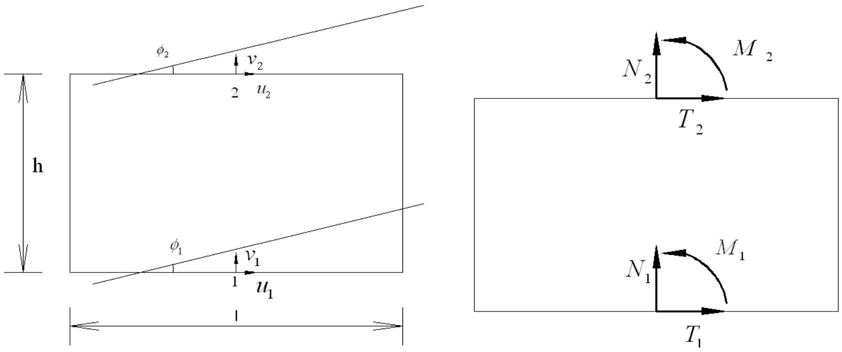


Figure 9: Unknown displacements and forces

The constitutive relationships for a radial compression stress field are according Equation (3):

$$\varepsilon_r = \frac{1}{E}[\sigma_r - \mu\sigma_\theta] \qquad \varepsilon_\theta = \frac{1}{E}[\sigma_\theta - \mu\sigma_r] \qquad \gamma_{r\theta} = \frac{1}{G}\tau_{r\theta} = \frac{2(1+\mu)}{E}\tau_{r\theta} \quad \textbf{if} \quad \sigma_\theta = 0 \longrightarrow \quad \varepsilon_r = \frac{1}{E}\sigma_r \qquad \varepsilon_\theta \geq \frac{1}{E}(-\mu\sigma_r)$$

(Eq.3)

where, the first equation holds only for the fans under compression and fans in tension are eliminated. The second equation presents the cracking condition. The generalized forces acting at the end cross-sections have the expressions in Equation (4):

$$N_K = t\int_{\theta_1}^{\theta_2} f_{rk} r_k \left|\sin\theta\right| d\theta$$

$$T_K = t\int_{\theta_1}^{\theta_2} f_{rk} r_k \cos\theta \cdot sign(\sin\theta) d\theta$$

$$M_K = t\int_{\theta_1}^{\theta_2} f_{rk} r_k x_k \cdot \left|\sin\theta\right| d\theta$$

(Eq.4)

while the total complementary energy (TCE) for a prescribed displacements at the end cross-sections is according Equation (5):

$$TCE = \frac{1}{2} t \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} \frac{\sigma_r^2}{E} r \cdot dr d\theta - t\int_{\theta_1}^{\theta_2} \left(f_{r1} s_1 r_1 + f_{r2} s_2 r_2\right) d\theta = -\frac{1}{2} t \int_{\theta_1}^{\theta_2} E r_0 (s_1 - s_2) d\theta$$

(Eq.5)

According to this formulation the macro finite element is implemented in the finite element open source program Opensees.

4 Comparison macro-modelling and micro–modelling results

The micro-modelling approach described at the point 2 based on interface elements is used to provide a validation to the macro-modelling implementation. In particular two case studies were employed according to Lourenço 1997 [3]. The geometric configuration of the wall (with and without openings) and the load pattern is described in Figure 10, [3].

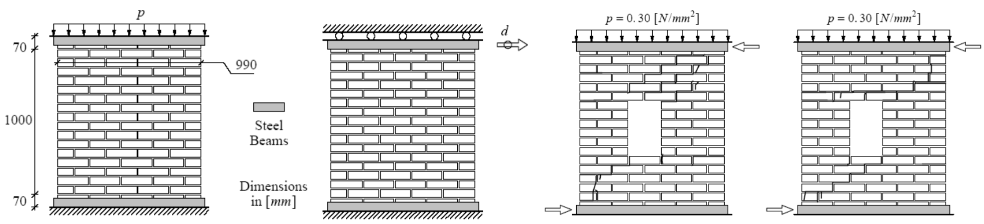


Figure 10: Test setup and geometrical configuration

The strategy used in the micro-modelling analysis is to represent separately the bricks, the mortar joints and also the interface. In particular interface elements are created in the centre line of a brick to allow openings due to compression failure of the brick, see Figure 11. Only 8 elements are used in the macro-modelling mesh instead in the micro-modelling around 1000 elements are employed, see Figure 12.

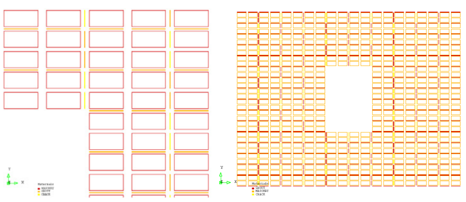


Figure 11: Mesh strategy used in the finite element micro modelling approach

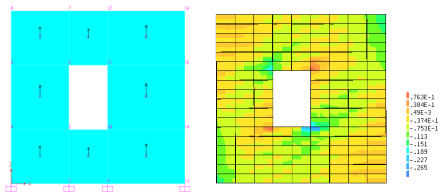


Figure 12: Comparison between macro-modelling and micro-modelling mesh configuration

In the following pictures, the results obtained with the different approaches are provided, see Figure 13. It is clear that the micro-modeling approach provides more accurate results (even if it requires a huge computational effort) also in terms of failure mechanism, see Figure 14. At the same time the macro-modelling approach gives reasonable results and therefore can be used to assess the safety of real masonry buildings.

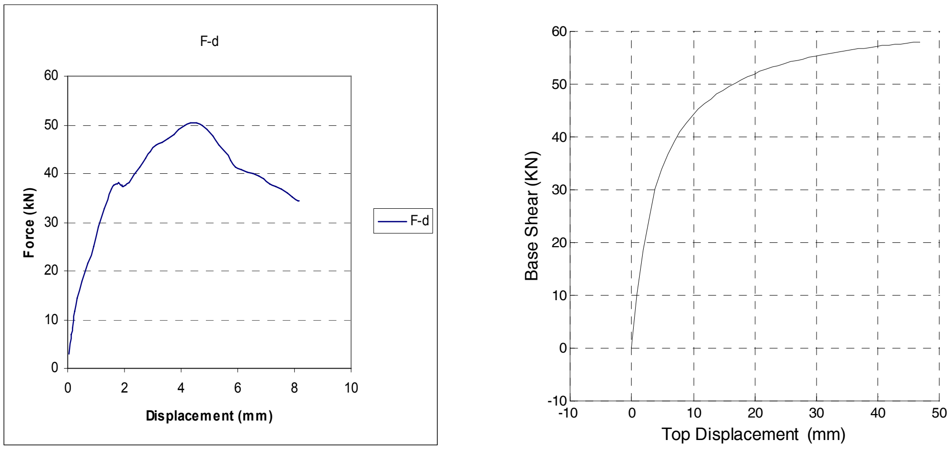


Figure 13: Comparison of the results obtained with the micro-modelling (left) and macro-modelling (right) in terms of pushover curves

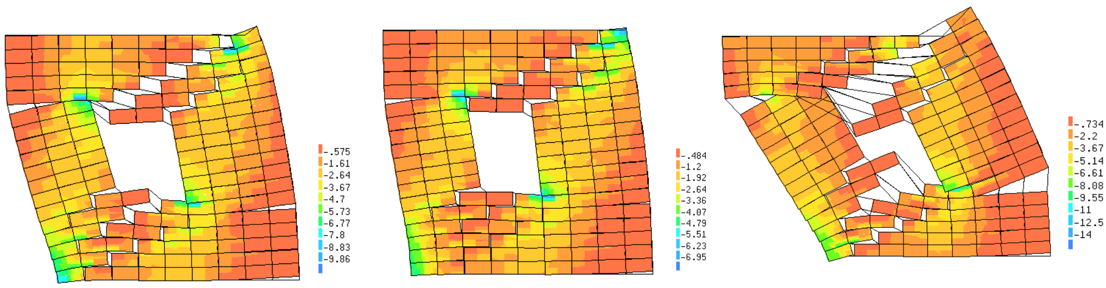


Figure 14: Failure mechanisms obtained using the micro modelling approach

5 Theory of the cyclic Multi-Fan element

After the monotonic version of the model is developed and validated some modifications are introduced to allow numerical modelling of the cyclic behavior of masonry buildings under horizontal seismic forces. The general idea is to make an element system; which means connect the monotonic Multi-Fan element with some additional springs, both in the shear direction and the rotational direction. So the deformation of the global Multi-Fan element can be separated into two parts: the elastic deformation part and the plastic deformation part. The plastic deformation will take place in the additional spring. Then the monotonic Multi-Fan element will cooperate with the additional springs to generate the cyclic hoops. The original Multi-Fan Element is a four nodes element where each node has two degrees of freedom and consists of a sub-structure including a 2-nodes element. The update Multifan Element instead includes a zero-length sping in shear and one in bending, see Figure 15.

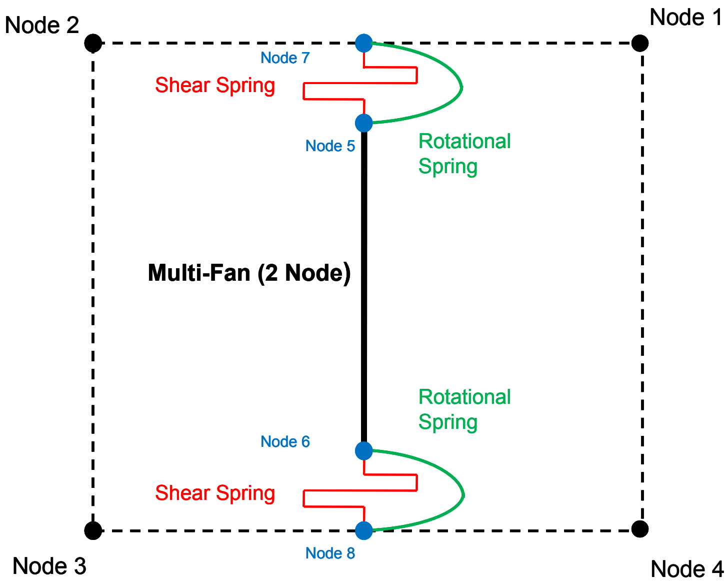
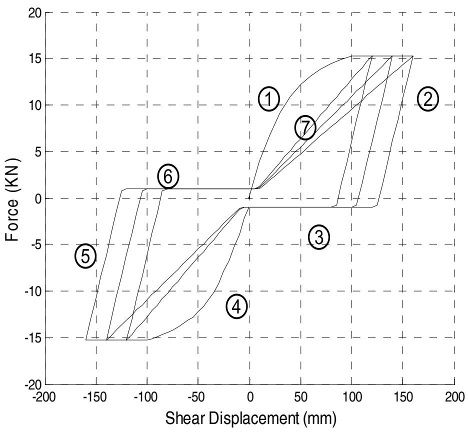


Figure 15: Cyclic multifan element

The update Multi-Fan Element still has four nodes where each node has two degrees of freedom. The nodes displacements of the springs are reduced using condensation techniques so if the nodes of the global multi-fan element are nodes 1, 2, 3, 4, and the nodes of the sub-structure are nodes 5, 6, 7, 8 where each node has three degrees of freedom, the static condensation is done to get the stiffness matrix of the system as a function of only the nodes 1,2,3,4. Moreover, using the springs some failures modes are also introduced according Italian code for seismic evaluation of masonry buildings. In particular, the following failure modes are considered: shear failure due to joint sliding, shear failure due to traction and shear failure due to bending.

The full displacement-force path resulting from the above cyclic Multi-Fan element is shown in Figure 16, where:

1. Loading branch, (including the yielding branch if it happens);
2. Unloading branch (unloading from the loading or yielding branch);
3. Friction Force branch(to model the closing of the crack);
4. Loading in another direction;
5. Unloading in another direction;
6. Friction Force branch in another direction;
7. Reloading, which goes along a straight line pointing to the last unloading point.



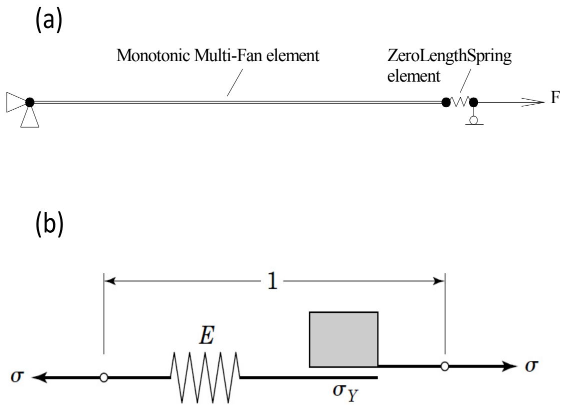


Figure 16: Displacement-force path of the MF element and schematic representation

6 Verification of the numerical model

The cyclic Multi-Fan element is then used to simulate the brick masonry walls of the building prototypes experimented at the Department of Structural Mechanics of the University of Pavia [11] and of the Georgia Institute of technology [12]. It shows a satisfactory degree of accuracy to analyze complex assemblages in 3D, under cyclic loads and in case of strengthening. The geometric description of the walls are shown in Figure 17 and Figure 18.

The finite element models are both made up of 36 nodes and 21 elements. Each panel in the structure is modelled through a single MF element. In case of FRP application, truss element can be added to the multi-fan element to predict strengthening effects. The displacement history is performed by imposing the horizontal displacement w_1 and w_2 to the nodes at the floor levels of floor 1 and floor 2 (with free vertical displacements). The cyclic response of the window wall B2, in terms of the base shear versus the imposed second floor displacement w_2, is plotted in Figure 19. The comparison between experimental results and the numerical simulation shows that the Multi-Fan element can capture the cyclic behaviour of the masonry structure.

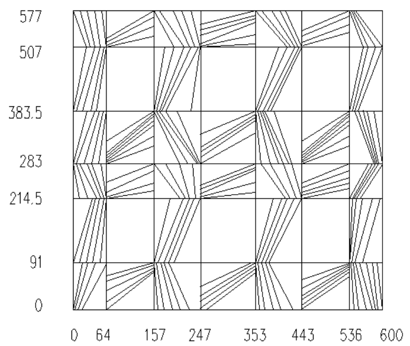


Figure 17a: Geometric description (cm) of wall B2

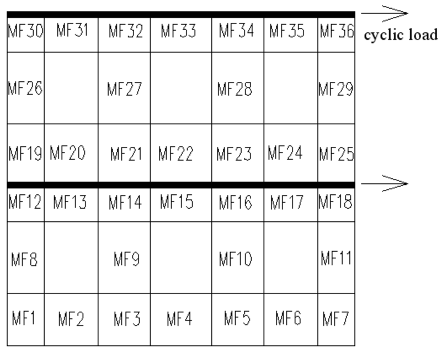


Figure 17b: Multi-Fan element model of wall B2

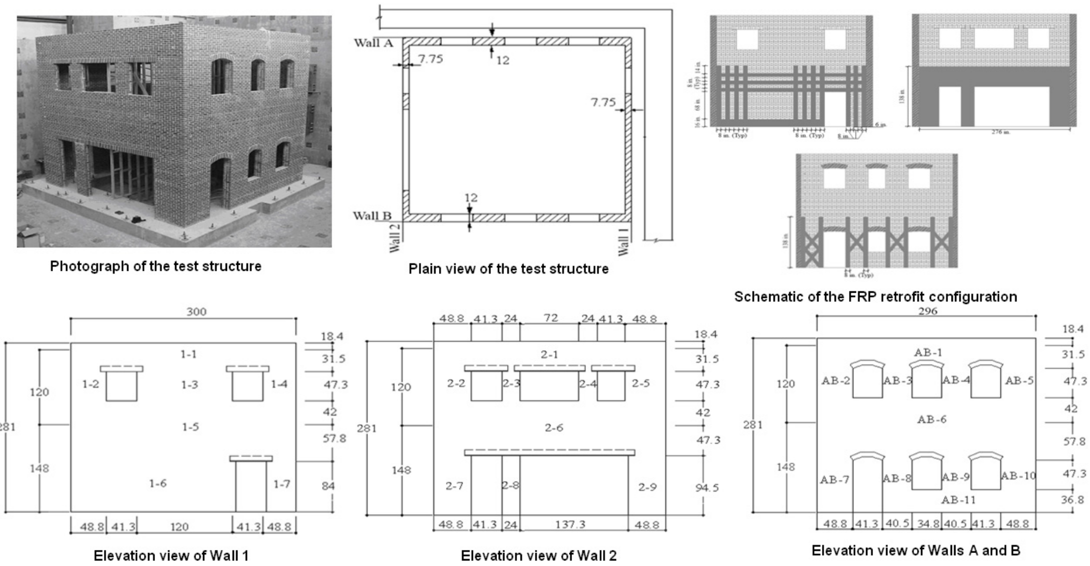


Figure 18: Geometry of the building prototype with indication of strengthened parts

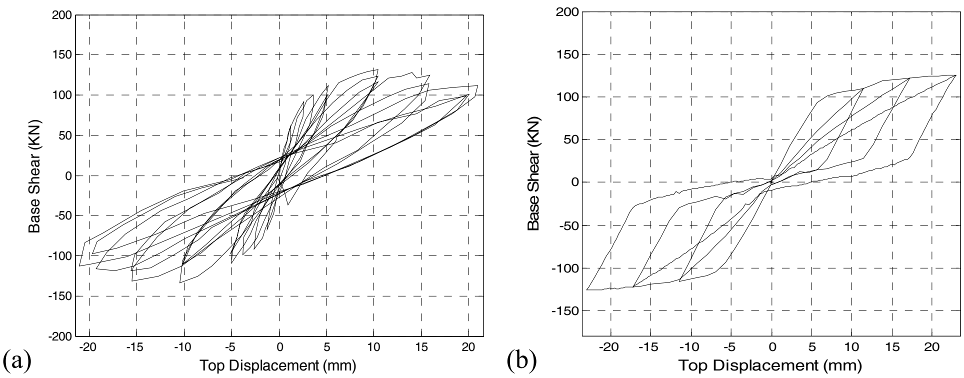


Figure 19: Cyclic response of door wall B2: a) experimental; b) MF numerical simulation

For the FRP strengthened prototype, a preliminary linear analysis is instead performed to understand possible crack distribution and identify piers and sprandel elements to be used in the macro-modelling, see Figure 20. Then the focus is on the wall B of the whole prototype and Figure 21 a, b, c, shows the comparison in terms of experimental and numerical results for the 4 cases: masonry modelled as a no tension material with infinite and finite compression strength prior to retrofit and after strengthening. Again a good estimation of the experimental behaviour can be achieved.

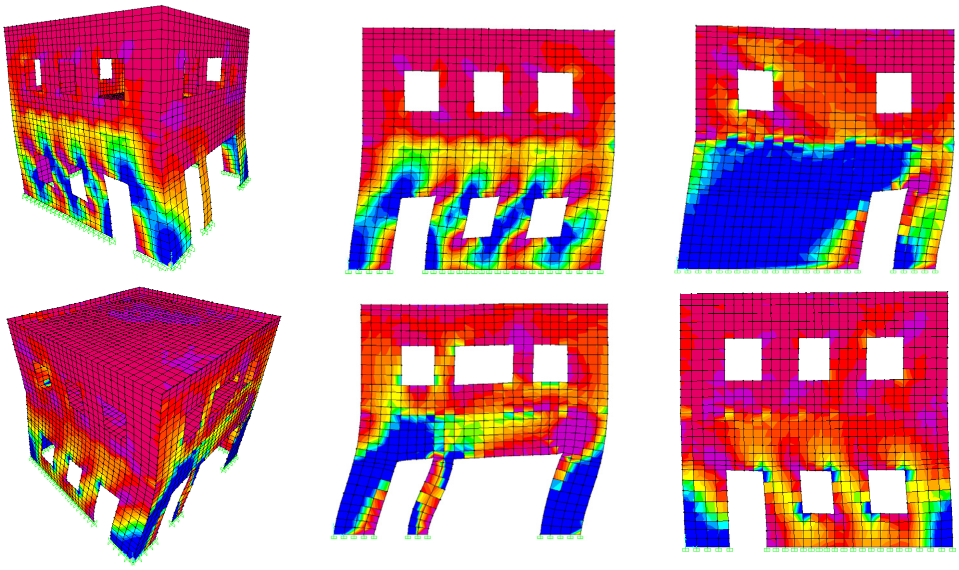


Figure 20: Principal stress distribution

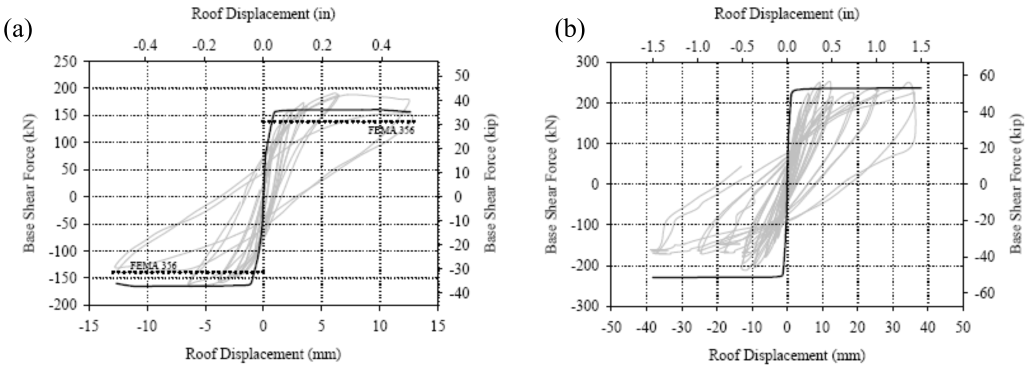


Figure 21: Experimental base shear versus roof displacement response of wall B prior (a) and after to retrofit (b)

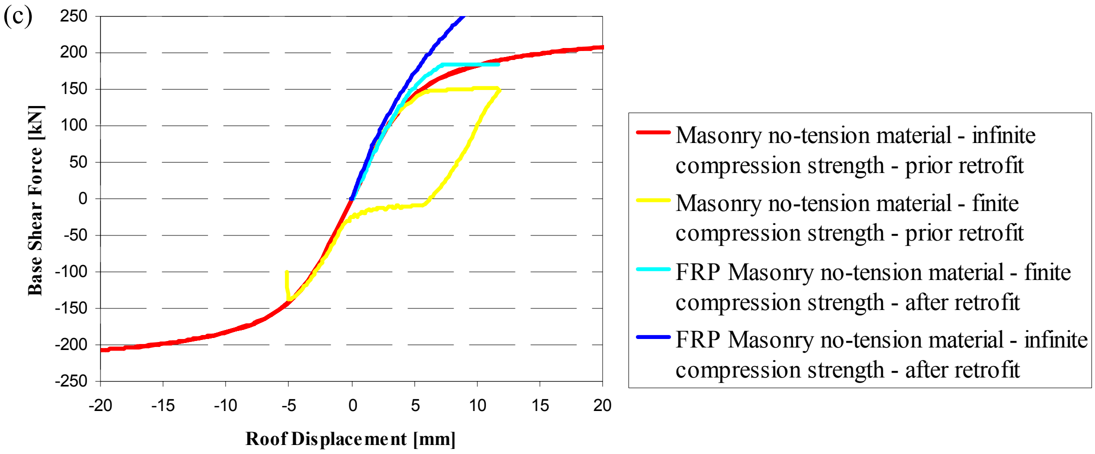


Figure 21c: Numerical base shear versus roof displacement response of Wall B prior and after to retrofit

7 Conclusions

The primary contributions are the development of a material model for the analysis of the FRP-masonry interface and of a suitable finite element for analysis of masonry buildings under seismic actions. The micro-modelling strategy is used to validate the macro-modelling approach and both the results are compared to experimental tests on small scale walls and big scale prototypes of buildings. The material model proposed and implemented in the finite element program Diana 8 is very useful to model the FRP-masonry interface : both for planar and curved substrates and allows to obtain the global full shear force-displacement path and also to simulate the stress distribution at the interface. The multi-Fan element proposed is instead extremely effective for the seismic analysis of masonry buildings and has been implemented in the Object-Oriented Nonlinear Dynamic Analysis program-OpenSees. Then the Zero-Length Spring has been added to the Multi-Fan element system to model shear and bending failure and the cyclic behaviour has been included. The multifan element developed can be used to analyze the building prototypes experimented at the Department of Structural Mechanics of the University of Pavia and at the Georgia institute of technology. Finally, it is shown as a satisfactory degree of accuracy at the global level, keeping an efficient computational time for the analysis can be achieved when are analyzed complex assemblages in 3D even in the case of cyclic loads or when strengthening techniques are considered.

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