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Mathematical model for maintenance planning of machine tools

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Abstract

Preventive maintenance planning of machine tools may be a complex task for tools with multiple components. For manufacturing processes with high setup and downtime costs, components replacement should be combined to avoid too many production stoppages and therefore reduce costs. The combination that minimizes costs should take into account the lifetime distribution and the age of each component at the replacement time. Replacing too soon may imply a high number of replacements for a given component, while replacing too late may imply a high number of failures that lead to shutdowns, increasing costs. In this paper, a tool is seen as a series system, which means that whenever a component fails, a corrective action is needed and at least the failed component has to be replaced. In the literature, some of the models and heuristics for maintenance planning of series systems consider that a minimal repair is made when a component fails, while other models propose static approaches, i.e., the same combination and the same interval is used over time regardless of the ages of the components involved. This paper aims to propose a dynamic approach and presents a mathematical model to determine both the next time to perform a preventive maintenance task and the components that should be replaced in order to minimize the total cost. The model also intends to determine the components to be replaced preventively when unexpected events occur (such as the failure of a tool component or the machine, the shortage of raw material, etc.) or during planned stoppages (such as the end of a production order, machine preventive maintenance task, etc.).

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1. Introduction

Effective maintenance policies can influence the productivity and profitability of a manufacturing system [1]. Preventive maintenance strategies are followed to avoid the negative impact of failures such as: production and quality loss and safety issues. Therefore, maintenance planning and scheduling are required to define what, how, when and by whom the activities will be performed. In addition, maintenance planning should be combined with production planning to minimize the consumption of productive times [2].

Machine tools are important elements of equipment since its malfunction affects the quality of the product. When machine tools are composed of wear components, e.g., for producing metalworking products, the replacement of components is part of the process. Thus, methodologies or models to support decision making regarding the planning of maintenance activities on tools are needed to avoid excessive downtime and production loss.

The age replacement and the block replacement policies are the most commonly used preventive replacement policies [3] for single unit. For the age-based replacement policy, the preventive replacement of a component occurs at a defined age of the part. For the block replacement policy, the replacement occurs at regular time intervals, regardless of the age, or when the failure occurs [4]. The block replacement policy is easier to manage, since planned replacements occur at regular intervals and so are readily scheduled. On the other hand, age replacement seems more effective since under this policy planned replacement considers the age of the component [3]. Based on these policies, several maintenance models for components replacement were defined aimed at reducing maintenance costs or downtime.

The maintenance of machine tools is a complex problem since a tool can be seen as a multi-unit series system and, therefore the failure of a unit or a component leads to production interruption, which implies several costs: production loss, delays in deliveries, among others. In the literature, some models can be found for maintenance planning of multi-unit series systems. The grouping of components to be replaced at the same time is presented as a solution to reduce the number of stops [5-7]. Performing preventive maintenance activities during the same downtime can generate significant economic benefits [5].

For performing preventive replacements on a series system, Talukder and Knapp [5] proposed a heuristic method based on the block replacement policy. The heuristic method aims at grouping non-identical equipment into blocks using group technology (GT) concepts and similarity coefficient to generate groups of similar equipment whose times to failure are assumed to follow a Weibull distribution [5].

The preventive maintenance models are classified in the literature, according to the planning approach, as stationary models or dynamic models [9]. Stationary models are characterized by the use of static rules in the long-term and usually assume an infinite planning horizon [9]. The model proposed in [5] is such an example. On the other hand, dynamic models can take into account short-term information in order to set and adjust the planning of maintenance activities. These models generate dynamic decisions that can change over the horizon [9].

The model of Zhou et al. [8] is based on dynamic programming to minimize the short-term cost savings and integrates imperfect effect into maintenance actions. The time to perform a preventive maintenance on a system unit is defined by a reliability threshold. The model considers that, at downtime due to preventive maintenance, an opportunity arises to perform preventive maintenance to other system units. This concept is known in the literature as opportunistic maintenance. Opportunistic maintenance stems from the fact that there is economic dependence between units of a system. In the case of a series system, in which the failure of any unit causes the system to stop, it is possible to take advantage of the system downtime to perform preventive actions to other units [10-11]. However, the model does not consider downtimes due to failures as an opportunity to perform preventive maintenance since it considers minimal repairs. In addition, the solution is obtained through numerical simulation applying the model to a system with a limited number of units (three units). According to the authors, the model is not applicable to a higher number of units.

Chalabi et al. [13] also considers minimal repairs at failures. A preventive maintenance grouping strategy for multi-component series systems that uses a particle swarm optimization algorithm is presented. The proposed approach for dynamic grouping of machines preventive maintenance activities has two objectives: improvement of the system availability and minimization of preventive maintenance cost.

Downtimes due to failures are considered as replacement opportunities for other system components in Laggouné et al. [11]. The possible times for preventive replacements are multiple of the shortest replacement time (or age) of

the components under study and are defined according to the age replacement policy and the model of Barlow and Hunter [12]. However, the approach is stationary since the obtained planning is not updated based on new information.

The approach presented by Dekker et al. [7] uses age replacement policy and considers that failure repairs can be combined with preventive maintenance. The proposed heuristic is based on Markov decision theory and dynamic programming. However, the model does not consider systems inactivity due to events external to the system as opportunities to perform preventive maintenance. On the other hand, the models of Do Van et al. [14] and Wildeman and Dekker [15] do consider external events for opportunistic maintenance (example: due to production / commercial planning). However, they are based on the block replacement policy.

The block replacement policy reveals weaknesses, since it leads to the replacement of components without considering its remaining lifetime. In this way, age-based replacement policy presents a better response to multi-component systems, avoiding the replacement of components with short lifetimes [16].

This paper aims to propose a dynamic approach and presents a mathematical model, based on age replacement policy, to determine the components to be preventively replaced at downtimes and the next time to perform a preventive maintenance action and respective components to be replaced, in order to minimize the total cost. For this purpose, in addition to the downtimes due to components failure, the model considers system inactivity due to external events as opportunities to perform preventive maintenance. These external events can be planned (e.g. equipment maintenance) or unplanned (e.g. shortage of raw material). Thus, the model intends to determine the components to be replaced preventively when unexpected events occur (such as the failure of a tool component or the machine, the shortage of raw material, etc.) or during planned stoppages (such as the end of a production order, machine preventive maintenance task, etc.).

The model was developed to support the maintenance planning of machine tools of a metalworking company that produces metal parts. The production system consists of stamping machines and associated tools, responsible for operations of forming, cutting, folding, among others.

This article is organized in six sections. Section 2 presents the notations and assumptions used. Section 3 presents the mathematical model, with an explanation of the objective function and constraints. In section 4, an application example is presented. The last section presents the conclusions and points out future works.

2. Assumptions and notation

The proposed model intends to group components to be replaced in the same period based on their ages at the decision time and their optimal replacement ages, which are defined considering the age replacement policy and the model proposed by [12]. The considered times for preventive replacements grouping are the optimal replacement times (or ages) of each considered tool component. Preventive replacements can also occur at downtimes due to tool failures or due to events that are external to the tool, referred as external events. If the external event is unexpected, the preventive replacement of components will be considered in a similar way to the failure occurrence of the tool. If the external event is expected and planned, the scheduled time for its occurrence will be considered by the model as a possible time for a group preventive replacement. Since the tool wear depends on the number of produced units, the time unit is measured in the number of parts produced. Nevertheless, the respective variable is treated as continuous.

The following assumptions are considered:

- Tool components failure probabilities are mutually independent;
- Components replacement time is neglected, since it is considered relatively small compared to the time for tool disassembly and subsequent assembly on the machine;
- The costs considered are the acquisition cost of the components (c_i) and the maintenance cost, which includes the costs associated with labor time and production downtime. In case of tool failure, the maintenance cost also includes costs associated with the failure, such as costs of defects production, failure propagation and maintenance waiting time;

- The preventive maintenance cost (C_p) is assumed to be lower than the corrective maintenance cost (C_f), since the later implies further costs associated with the failure. During external events, preventive maintenance cost (C_p^*) is even lower, since the downtime cost is not considered as a preventive maintenance cost;
- The lifetime of each component is assumed to follow a Weibull distribution;
- Upon replacement, the as good as new state is considered for the component since it is replaced by a new one from the same population (with the same reliability function).

The model considers that preventive replacements are defined based on the reliability function of the components. Potential monitored data about the components reliability is not considered as well as possible stochastic dependence between components. The model also assumes that new components to be introduced into the system are always available. In practice, this situation usually leads to the repair of the current components which originates a higher downtime and lower components reliability.

Notation

i	component index, $i \in \{1, 2, \dots, I\}$
$C_i(t)$	average cost per time unit for individual preventive replacement of component i at age t
$F_i(\cdot)$	failure cumulative distribution function of component i
$R_i(\cdot)$	reliability function of component i
t_i	age of component i at the beginning of the planning horizon
t_i^*	optimal individual replacement age of component i
c_i	acquisition cost of component i
C_f	corrective maintenance cost
C_p	preventive maintenance cost
C_p^*	preventive maintenance cost, during an external event
T_i^*	time left until the optimal replacement age of component i at the beginning of the planning horizon
k	external event index, $k \in \{1, 2, \dots, K\}$
EE_k	time left until the external event k at the beginning of the planning horizon
j	time index, in ascending order of the time left until each time, $j \in \{1, 2, \dots, J\}$, $J \leq I + K + 1$
T_j	time left until time j
E_j	binary variable that defines if time j is an external event, $E_j \in \{0, 1\}$
H_{ij}	cost of moving the replacement of component i to the time j
Y_j	binary variable that defines if a preventive action occurs at the time j , $Y_j \in \{0, 1\}$
X_{ij}	binary variable that defines if the component i preventive replacement occurs at the time j , $X_{ij} \in \{0, 1\}$
Z	binary variable that defines if the tool is failed at the beginning of the planning horizon, $Z \in \{0, 1\}$

3. Mathematical model

The mathematical model is integrated in an algorithm that will be run at different decision times (Fig. 1):

- (1) When a component fails and a corrective action is needed. The technician identifies the failed component and then the algorithm runs to define other components to be replaced preventively during this downtime, the next preventive action time and the respective components to be replaced. Afterward, the maintenance action is initiated;
- (2) At the time of a previously scheduled preventive action. Since the group of components to be preventively replaced was previously defined, the preventive action is initiated without running the algorithm. However, at the end of the intervention the algorithm runs to define the next preventive actions;
- (3) When an unexpected external event occurs or there is new information about planned external events. The algorithm runs and, according to the algorithm results, the action may be immediately initiated or not.

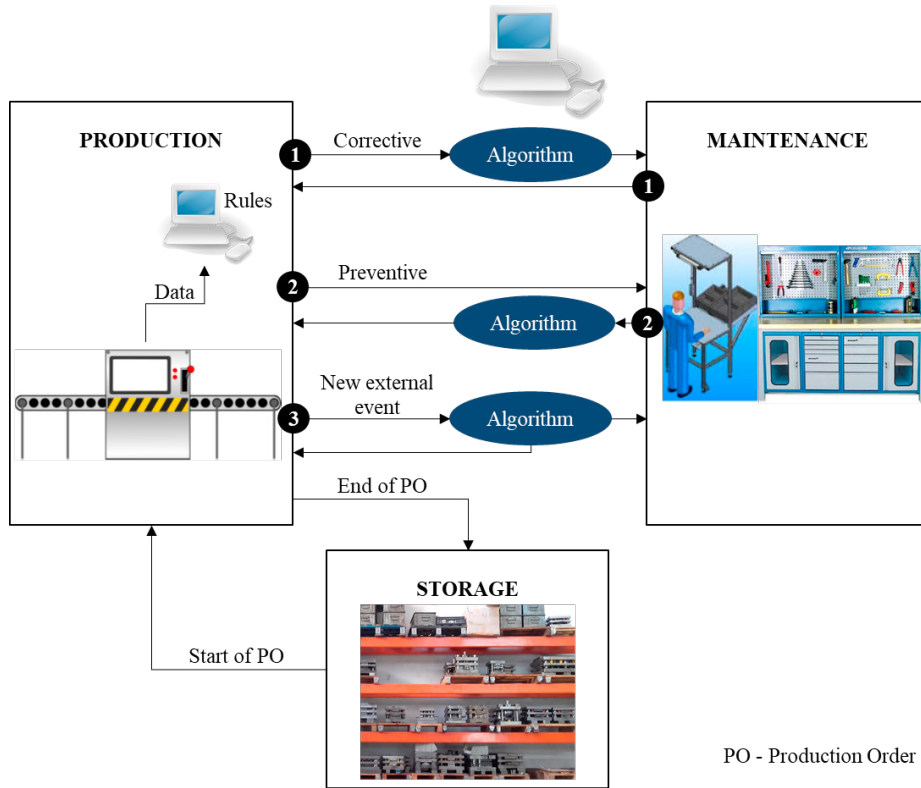


Fig. 1. Model dynamics

In the first step of the algorithm, the optimal ages t_i^* to preventively replace each component, individually, are calculated according to the age based replacement model in Eq. 1, by minimizing the maintenance cost per unit of time $C_i(t)$.

$$C_i(t) = \frac{(c_i + C_p)R_i(t) + (c_i + C_f)F_i(t)}{\int_0^{\tau} R_i(\tau) d\tau} \tag{1}$$

As can be seen in Eq. 1, the costs associated with a preventive or corrective maintenance action of a component, besides the respective maintenance cost (C_p or C_f) also include the acquisition cost of that component (c_i).

Then, based on the optimal replacement ages and using the objective function that consists in maximizing the G function in Eq. 2, the components to be grouped and respective replacement times are defined. This function, explained in detail below, represents the gain of grouping components for preventive replacement compared to the solution that consists in replacing each component at their optimal individual replacement age. The function adds the cost savings due to grouping components together, since the preventive maintenance cost C_p is shared among components, and the increase in cost of shifting the components replacement times from the optimal individual replacement age, to allow groupings.

$$G = \sum_{j=1}^J Y_j \left(\sum_{i=1}^I X_{ij} - 1 \right) C_p + \sum_{j=1}^J Y_j E_j (C_p - C_p^*) + ZY_1 C_p - \sum_{j=1}^J \sum_{i=1}^I X_{ij} H_{ij} \tag{2}$$

The decision variables are X_{ij} , while the variables Y_j are given by:

$$Y_j = \min \left(1, \sum_{i=1}^I X_{ij} \right) \tag{3}$$

Each component is preventively replaced once, condition which is given by the following restriction:

$$\sum_{j=1}^J X_{ij} = 1 \tag{4}$$

The algorithm considers J times, at which preventive actions can take place. These times include the times when each component reaches its optimal individual replacement ages and can also include the decision time and the expected external event times.

The function G (Eq. 2) is composed of four terms, described below:

- (1) For each time j , if there is a scheduled preventive action, there is a cost saving of $(\sum X_{ij} - 1)C_p$, since instead of individually replacing $\sum X_{ij}$ components at different times, which implies a total preventive maintenance cost of $\sum X_{ij}C_p$, the components will be replaced together at the same time, which implies only a preventive maintenance cost of C_p ;
- (2) If there is a scheduled preventive action at an external event time, the preventive maintenance cost is C_p^* instead of C_p , so there is a cost saving of $(C_p - C_p^*)$;
- (3) If the decision time was triggered by a failure and a preventive action is also performed, no preventive maintenance cost is incurred, so C_p is saved;
- (4) For each time j and each component i , there is an increase in cost of shifting the replacement age of component i to time j . The increase in cost is obtained by integrating the function presented in Eq. 1 between the optimal individual replacement age and the new replacement age, as given in Eq. 5.

$$H_{ij} = \int_{\min(t_i^*, t_i + T_j)}^{\max(t_i^*, t_i + T_j)} C_i(t) dt \tag{5}$$

4. Application example

For the application example, *Matlab* was used to run the algorithm. The heuristic to obtain the solution was a genetic algorithm. For the application of the algorithm the following situation was considered: a tool with five components. The lifetime of each component was assumed to follow a Weibull distribution with scale parameter (η) and shape parameter (β). The parameters values, as well as the acquisition cost (c_i) and the age at the beginning of the planning horizon (t_i) of each component, are presented in Table 1. The preventive and corrective maintenance costs were considered to be $C_p=900$ and $C_f=1600$, respectively. For this example, the number of produced parts is measured in thousands of parts.

Table 1. Data of 5 tool components

i	c_i	η	β	t_i
1	40	150	2.0	100
2	40	200	3.0	120
3	40	400	2.0	10
4	40	350	2.1	10

5	40	100	2.2	10
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First, t_i^* are calculated by minimizing the cost function presented in Eq. 1. Then, the scheduled preventive times for each component are determined taking into account their current ages t_i and the time left until the optimal replacement age of component i , T_i^* . The results are shown in Table 2.

Table 2. Algorithm results for the considered data

i	t_i	t_i^*	T_i^*	Scheduled preventive action
1	100	196	96	96
2	120	180	60	96
3	10	521	511	414
4	10	424	414	414
5	10	114	104	96

The solution for the considered situation is: components 1, 2 and 5 are preventively replaced at the optimal individual replacement age of component 1, while components 3 and 4 are preventively replaced at the optimal individual replacement age of component 3. The value obtained for the objective function G was 1795 which corresponds to the gain obtained by grouping replacements compared to the solution that consists of replacing each component at the individual optimal replacement age. Only the first scheduled preventive action is considered, since the next ones are reevaluated in the next decision times, when the algorithm runs again.

Considering a planned external event that occurs after producing 80 thousand parts, during which preventive maintenance cost is $C_p^*=400$, the obtained solution changes as shown in Table 3.

Table 3. Algorithm results considering an external event

i	t_i	t_i^*	T_i^*	Scheduled preventive action
1	100	196	96	80
2	120	180	60	80
3	10	521	511	414
4	10	424	414	414
5	10	114	104	80

In this new situation, components 1, 2 and 5 are preventively replaced during the planned external event, instead of at the optimal individual replacement age of component 1. The value obtained for the objective function G is 1956, which is higher than the value obtained for the initial situation, which means that the cost savings are higher. This solution was expected since, during an external event, the associated downtime cost is not assigned to maintenance.

5. Conclusions and future works

This work addresses the problem of preventive maintenance planning of series systems of non-identical components. The planning is based on age replacement policy and the concept of opportunistic maintenance. The target of the study is the tools that are inserted into stamping machines of a metalworking company. An algorithm was defined to determine the tool components to be replaced preventively at scheduled times or whenever stops occur, leading to downtimes. The considered downtimes are of two types: the downtime to replace the failed components and the inactivity periods due to external events (e.g., the machine failure, the shortage of raw material,

the end of a production order). Since the components replacement time is relatively small compared to the time for tool disassembly and subsequent assembly on the machine, this time is neglected.

The components to be replaced in each preventive intervention are defined considering their ages and the individual replacement ages that minimize costs in the long run. The algorithm determines the gain obtained by reducing the number of stops for preventive replacement. The application example, which involved a tool composed of five components, revealed the effectiveness and efficiency of the algorithm since the solution was obtained in few seconds. The results show that cost savings are obtained when replacements are performed in group and when downtime due to external events is also used for that purpose.

As future work, the authors will integrate this algorithm into an information system that records data on failure times in order to support tool maintenance by providing information to the technicians.

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