IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received October 18, 2019, accepted October 31, 2019, date of publication November 18, 2019, date of current version December 4, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2953185

An Integer Linear Programming Model Including Time, Cost, Quality, and Safety

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ABSTRACT Time, cost, quality and safety, are the four critical elements that contribute to project success. Traditionally, literature has focused on analysing only time and cost. Subsequently, other multi-objective optimization methods developed to optimize time, cost, quality or safety have been developed. New types of contracting methods designed by governments, included maximizing construction quality while minimizing its time and cost. Recently, due to the fact that the construction industry suffers from more accidents of greater severity than other industrial sectors, safety has become one of the four critical elements that contribute to project success. The project scheduling literature largely concentrates on the generation of a precedence and resource feasible schedule that optimizes the scheduling objective and that should serve as a baseline schedule for executing the project. However, these models do not allow to analyse alternative work plans that consider the trade-offs between time, cost, quality, and safety. In this paper, an integer linear programming problem is applied to a decision-CPM network in order to obtain an overall optimum including time, cost, quality and safety in a road building project. Using this type of model, the effects of alternative methods of performing the tasks can be considered and a greater degree of interaction between the planning and scheduling phases of a project is obtained.

INDEX TERMS Time, cost, quality, safety, decision CPM network.

I. INTRODUCTION

Planning and defining the scope of a project means defining what the project will deliver and what it will not deliver in order to deliver projects successfully [1]. Different factors can be considered major scope items for project management and the most common causes of project failure in the construction industry, i.e., financial and budgetary factors, labor productivity, materials availability, resource constraints, etc. Time, cost, quality and safety, with mutual influence, are some of these factors whose importance in construction engineering projects has been widely recognized by the construction industry. This can be reflected as "to take the necessary actions to ensure that construction sites are not the cause of immediate danger to the public or workers who must be able to work without suffering harm, attaining the acceptable levels of quality, on time, and at a minimum cost."

However, there is large discrepancy between the perception of private and public clients, contractors, owners, and consultants about what constitutes their expectations of project time, cost, quality, and safety. The emphasis on project planning and scheduling has been on managing the relationship between time and cost, with an implicit assumption of a fixed level of quality that is seldom explicitly examined [2]. Bennett and Grice [3] argue that clients of the construction industry are primarily concerned with quality, time, and cost, despite the fact that the majority of construction projects are procured on the basis of only time and cost. According to Fellow [4], private clients rank time as the most important criteria, while cost is considered the least important. On the other hand, public clients place more importance on quality and secondly on cost, while time is the least important. This is why most selection procedures employed by public clients during project tender assume that all eligible tenderers are capable of achieving the minimum quality standard defined by the clients's requirement [5]. In a survey conducted in South Africa among clients, architects, project managers, engineers and general contractors [6], clients may well be prepared to sacrifice construction time for improved quality and rate quality as more important than project time, whereas contractors, project managers engineers and architects rate time as the most important criteria.

The associate editor coordinating the review of this manuscript and approving it for publication was Vincenzo Piuri.

In many project situations there are alternative approaches for completing each task, each having its own time, cost, quality, and safety considerations. Differences can arise due to different bids offered by competing tenders to complete specific tasks. Even different bids by the same tender could imply different time, cost, quality and safety levels. In order to complete their projects successfully, project managers need to learn and obtain the necessary skills for solving problems in these complex environments such as the construction and engineering industry where organizations are in constant state of action to improve their methods of managing projects [7], [8]. In such situations, project managers must evaluate these alternative options for accomplishing project activities and decide on their levels for each task that best achieves the project's objectives. In this context, Sunindijo and Zou [9] investigated what constituted project management personnel's skill and how this skill can be developed and applied in the construction industry.

Despite the fact that quality and safety are acknowledged to be two important components of construction project management, its relationship with time and cost has received limited attention in planning and scheduling. Traditionally, literature has focused on analysing time and cost as the two most important criteria used for determining project performance. This led to a number of models that can be classified according to their optimisation objectives into models that attempt to [10]: (1) Minimize project time and/or improve resource utilization [11]–[15]; (2) Minimize time and cost for non repetitive construction using time-cost trade-off analysis [16]–[21]; (3): Minimize time and/or cost for repetitive construction [22]–[28].

Subsequently, government started using new types of contracting methods designed to achieve multiple objectives, including minimizing construction time and cost while maximizing its quality [29]–[32]. Wanberg *et al.* [33] reviewed the use of agile methods to ensure the quick delivery of software products with minimal cost and user satisfaction. Hallowell [34] developed a discrete stochastic time-cost-quality trade-off model using a simulation-based integer linear programming approach. Pinto and Slevin [35] develop two hybrid metaheuristic algorithms for proactive and reactive project scheduling in order to minimize contractor's cash flow gap under random activity duration. Babu and Suresh [36] proposed a framework to study the trade-off among time, cost and quality using three interrelated linear programming models. The authors extended the standard time-cost trade-off analysis by assuming that quality, which was measured as the arithmetic or geometric mean of the quality of the activities, depends only on time and is independent of cost for a given time. Icmeli-Tukel and Rom [37] presented two mixed integer formulations for modeling and solving resource constrained project scheduling problems with the objective of maximizing quality, which was measured by the amount of time and money spent on reworking activities that did not satisfy specifications. Khang and Myint [38] attempted to apply the method presented by Babu and Suresh [36] to a cement factory construction project highlighting the managerial insights gained, as well as pointing out the problems and difficulties faced. The need for a more holistic measurement of performance quality suggested by Khang and Myint [38] and a more realistic model to describe the relationship between the quality of individual activities and the budget and time allowed presented in Babu and Suresh [36] lead Liberatore and Pollack-Johnson [2] to model quality at the task level using the continuous nonlinear form of the bivariate normal function of both time and cost. The authors incorporated the quality function into a mathematical programming model that enables project managers to evaluate the nonlinear trade-offs between quality, time, and cost. Paquin *et al.* [39] assessed project quality by decomposing client satisfaction into a hierarchical structure of quality dimensions that are measured and aggregated using a multicriteria approach. Pollack-Johnson and Liberatore [40] developed a mathematical programming model to maximize overall project quality by determining optimal discrete options defined in terms of time, cost, and quality combinations for specific tasks. Tareghian and Taheri [41] presented a time-cost-quality optimization model on the basis of three interrelated integer programming models. Afshar *et al.* [42] developed a meta-heuristic multi-colony ant algorithm for the optimization of time, cost, and quality and Rahimi and Iranmanesh [43] used a multi-objective method with Particle Swarm Optimization. Tiwari *et al.* [44] presented a multi-project multi-mode scheduling problem to maximize the project makespan under quality constraints. San Cristóbal [45] developed an integer programming model which enabled meeting quality output standards and meeting time and budget objectives respectively. Zhang and Xing [46] incorporated a fuzzy multi-attribute utility methodology with constrained fuzzy arithmetic operations and particle swarm optimization to evaluate each portfolio of time, cost, and quality. Pour *et al.* [47] presented a meta-heuristic algorithm to optimize total cost and time subject to quality constraints. Mokhtari *et al.* [48] applied an ant colony system to the stochastic discrete time-cost trade-off problem solved as a non-linear zero-one problem and Sonmez and Bettemir [49] developed a hybrid strategy using genetic algorithms, simulated annealing, and quantum annealing techniques. Fallah-Mehdipour *et al.* [50] applied two evolutionary algorithms, a multi-objective particle swarm optimization and a nondominated sorting genetic algorithm to solve two construction management problems. Kim *et al.* [51] presented a mixed integer linear programming model to minimize the direct cost and potential quality loss cost for excessive crashing activities. Mungle *et al.* [52] developed a fuzzy clustering-based genetic algorithm to solve the time-cost-quality trade-off problem and Tavana *et al.* [53] presented a new multi-objective multi-model for solving discrete time-cost-quality trade-off problem with preemption and generalized precedence relations. Fang and Chao [54] developed a non-linear stochastic programming model based on a multi-mode resource constrained project scheduling problem to minimize total quality cost. Heravi and Faeghi [55] presented a group decision making framework to seek the optimal resource utilization, considering time, cost and quality simultaneously. The framework incorporated Monte Carlo simulation for stochastic measurements of time and cost, a fuzzy simple additive weighting system for stochastic estimation of quality and the Borda-ordered weighted averaging method for the group decision-making process. Zhang *et al.* [56] developed an integrated optimization model on the basis of improved time-cost and quality-time models taking reward and punishment into consideration. The authors combine an immune genetic algorithm with a construction factor particle swarm optimization. Al-Haji and Sayers [57] identified whether project management as a discipline helps deliver the key project objectives of time, cost and quality. Meng [58] developed an input-process output-model to explore the relationship among early warning problem solving and project performance in terms of time, cost, and quality and Monghasemi *et al.* [59] presented a multi-objective genetic algorithm with NSGA-II procedures in forming the Pareto sets.

In recent years, safety has become one of the most important but least considered objectives in the construction industry. The labor-intensive construction industry suffers from more accidents of greater severity than other industrial sectors and that is why is regarded as one of the most unsafe industrial sectors worldwide [60], [61]. Despite the fact that time, cost, quality and safety, are the four critical elements that contribute to project success, projects are often rated successful because they have come in on or near budget and scheduled and achieved an acceptable level of performance. Unfortunately, in many cases these key elements may be in conflict. Project quality and safety may be affected by project crashing. Hallowell [34] found that suboptimal safety investments will yield higher injury rates. Several studies have shown that an increase in schedule pressure decreased safety performance [62], [63].

The project scheduling literature largely concentrates on the generation of a precedence and resource feasible schedule that optimizes the scheduling objective and that should serve as a baseline schedule for executing the project [64]. However, these models do not allow to analyse alternative work plans that consider the trade-offs between time, cost, quality, and safety. In this paper an integer linear programming problem is applied to a road building project using a decision-CPM (critical path method) network. This type of network allows a greater degree of interaction between the planning and scheduling phases of a project and an overall optimum including time, cost, quality and safety can be obtained. Using the model presented in this paper, several different alternatives are available to perform during the scheduling phase and if one work plan fails to meet the project's goal, alternative work plans can be considered. The paper is organized as follows. In the next section, the decision-CPM network, the interactions between the planning and scheduling phases, and the required integer linear programming model to obtain the optimum including time, cost, quality, and safety are shown. In section 3, in order to show the usefulness of the model, it is applied to a real road-building project. Finally, there is a concluding section with the main findings of the paper.

II. DECISION CPM NETWORK

Despite CPM is considered to be a technique for planning and scheduling of projects, there is no interaction between these two phases of the CPM analysis unless the technique of the job crashing is used [65]. A much degree of interaction between the planning and scheduling phases is essential to re-evaluate decisions previously considered optimal during the planning phase and that may be changed during the scheduling phase. If, during the execution of a project, an overall optimum is to be obtained and there are a number of competing methods of performing some of the tasks, each method having its own time, cost, quality and safety considerations. The effects of these alternative methods of performing the tasks can be considered and decisions previously optimal may be changed during the execution of the project. Crownston and Thompson [66] called this problem, the decision-CPM problem.

Let $J = \{S_1, S_2, S_3, \ldots\}$ be a set of job sets that must be done to complete the project. Some of the job sets are unit sets $S = \{S_{i1}\}$ and other sets have several members, $S_i = \{S_{i1}, S_{i2}, S_{i3}, \ldots\}$. If all job sets are unit sets, then all of the jobs in the project are independent and the project reduces to the ordinary project of the usual CPM variety. If one or more of the job sets have more than one member, then for each such set a decision must be made as to which job of the set is to be done. Once such decision is made for each job set, the result is an ordinary CPM project.

Consider a job set $S_{ij} = \left\{S_{i1}, S_{i2}, \ldots, S_{ik(i)}\right\}$ and its associated $k(i)$ variables $d_{i1}, d_{i2}, \ldots, d_{ik(i)}$ with constraints given by.

$$d_{ij} = \begin{cases} 1 & \text{if job } j \text{ is to be performed} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

If exactly one of the jobs must be performed then, the mutually exclusive interdependence condition is expressed by

$$\sum_{j=1}^{k(i)} d_{ij} = 1 \tag{2}$$

In addition to the relations described above there will be precedence relations between the jobs of a decision project. Let $S_{ij} \leq S_{mn}$ denote a relation between two pairs of jobs S_{ij} and S_{mn} and is read S_{ij} is an immediate predecessor of S_{mn}, indicating that all immediate predecessors of a job must be completed before that job can be started. If one of the jobs in a job set is decided to be done, then all immediate predecessor relations that the job satisfies must hold in the final graph. If we decide not to do that job, then none of its immediate predecessor relations hold and that job must be removed together with all edges that impinge on it from the decision project graph to obtain the final project graph. If on any path, two jobs are separated by a job which could be eliminated, and if it is desired to maintain a technological ordering of two jobs when one job must be completed before the other one can start, a dummy immediate predecessor relation must be established between them.

We associate with each job, S_{ij}, a time t_{ij}, and a cost c_{ij}. Also, a reward payment or "r" Euros per day for each day the project is under the required due date D, and a penalty payment "p", for each day beyond D, are assumed. Then, the integer programming problem of selecting the best project graph and finding its critical path is formulated.

$$\text{Min } C = \sum_{i=1}^{h}\sum_{j=1}^{k} d_{ij}c_{ij} - rw_F^- + pw_F^+ \tag{3}$$

$$\text{s. t. } w_F - w_F^+ + w_F^- - D = 0 \tag{4}$$

$$w_i + t_i \leq w_m \tag{5}$$

$$-\,M\left(1 - d_{ij}\right) + w_{ij} + t_{ij} \leq w_m \tag{6}$$

$$\sum_{j=1}^{k(i)} d_{ij} = 1 \tag{7}$$

$$0 \leq d_{ij} \leq 1, \quad \text{integer} \tag{8}$$

where w_F is the early start time of finish the last job in the project; w_F^- is the number of days that the project is delivered before D (accelerated) and w_F^+ is the number of days that the project is delivered after D (delayed), w_i is the early start time of job S_i. The first term in (3) calculates the costs of all the decision jobs that are to be performed and the second term, $rw_F^- + pw_F^+$, is explained by constraint (4). If the project is delivered on time, then $w_F = D$ and there is neither reward nor penalty ($w_F^+ = 0$ and $w_F^- = 0$). If $w_F > D$, then the project is not completed until after the due date so that $w_F^+ = w_F - D$, and a penalty of pw_F^+ is included in the objective function. If $w_F < D$, then the project is completed before the due date so that $w_F^- = D - w_F$, and a reward of rw_F^- is included in the objective function. Constraint (5) indicates that if job sets S_i and S_m are unit sets, then S_i is to be performed before S_m. If S_m is a unit-job set and S_{ij} is from a multi-job set, constraint (6) says that job S_{ij} is to be performed before S_m. Since M is a large enough number the inequality is restrictive only if $d_{ij} = 1$. If S_{ij} is not performed (i.e., $d_{ij} = 0$), the inequality does not constrain the variables. Thus all paths though the jobs which are not performed will be broken.

In order to include the overall quality performance at the project level the quality function suggested by El-Rayes and Kandil [10] is selected, which enables the aggregation of the estimated quality for all considered activities to provide an overall quality (Q_T) using simple weighted approach. To estimate the overall safety performance (S_T) at the project level, a function similar to Eq. (9) is proposed:

$$Q_T = \sum_{i=1}^{h} Ws_i \sum_{j=1}^{k} Q_{ij} * d_{ij} \tag{9}$$

$$S_T = \sum_{i=1}^{h} Ws_i \sum_{j=1}^{k} F_{ij} * d_{ij} \tag{10}$$

where Ws_i is the weight of job set i compared to other sets in the project representing the importance and contribution of the quality and safety of this set to the overall quality and safety of the project; Q_{ij} and F_{ij} are the performance of quality and safety indicators of job j in set i; and d_{ij} is as defined in Eq. (1). At the job level, we adopt the quality function suggested by Liberatore and Pollack-Johnson [2] that assigns to each combination of time (t) and cost (c) for a job a corresponding quality value given by:

$$Quality\,(t, c) = K * e^{-\left[\left(\frac{t-\mu_t}{\sigma_t}\right)^2 + \left(\frac{c-\mu_c}{\sigma_c}\right)^2\right]} \tag{11}$$

Since safety can also be considered an increasing function of cost (time) holding time (cost) constant, a similar safety function to Eq. (11) is proposed at the job level:

$$Safety\,(t, c) = S * e^{-\left[\left(\frac{t-\mu_t}{\sigma_t}\right)^2 + \left(\frac{c-\mu_c}{\sigma_c}\right)^2\right]} \tag{12}$$

where K and S are the maximum quality and safety levels possible; μ_t and μ_c are the maximum time and cost values for a given job; and σ_t and σ_c are the standard deviation parameters that give a measure of how slowly quality and safety drop compared to the maximum values for time and cost respectively. These standard deviation values are calculated using the following formulas:

$$\sigma_t = \frac{t_0 - \mu_t}{\sqrt{-ln\left(q_0/K\right)}} \tag{13}$$

$$\sigma_c = \frac{c_0 - \mu_c}{\sqrt{-ln\left(q_0/K\right)}} \tag{14}$$

$$\sigma_t = \frac{t_0 - \mu_t}{\sqrt{-ln\left(s_0/S\right)}} \tag{15}$$

$$\sigma_c = \frac{c_0 - \mu_c}{\sqrt{-ln\left(s_0/S\right)}} \tag{16}$$

where t_0 and c_0 are the values of time and cost that would achieve a specified quality and safety value ($\mathrm{q}_0 q_0$ and s_0) when the other variable is at its maximum. In situations where n bids, alternative work plans or scenarios specifying levels of time, cost, quality and safety (t_j, c_j, q_j, s_j) have been received for a given job, the four parameters of the bivariate quality function can be determined using nonlinear least square estimation [2].

The quality and safety functions in Eqs. (11) and (12) are normalized so that the maximum time and cost values correspond to a quality and safety of 100 or some other constant indicating the maximum quality level possible. These functions are able to represent a nonlinear relationships (decreasing and convex) between time and cost for a given value of quality and safety. According to these functions, quality and safety are increasing functions of cost (time) holding

TABLE 1. Data of the project.

Task	S_{ij}	Time			Cost			Quality	Safety	Ws_i
		Days	μ_t	σ_t	€*10^6	μ_c	σ_c			
Demolitions	S_1	27	28	9	4.06	5	4.2	94	94	0.10
Transport of earths	S_{21}	36	38	6.3	3.74	4	2.5	89	88	0.10
	S_{22}	31	35	10	4.20	4.5	2.5	84	89	
Transversal drainage	S_3	25	28	12	0.35	0.7	0.9	81	65	0.15
Walls	S_{41}	29	33	8.4	5.67	7	3.5	69	74	0.20
	S_{42}	26	30	6.7	7.09	8.4	4.3	64	60	
Granular asphalt capes	S_5	39	42	7.4	4.79	5.6	2.8	78	76	0.15
Longitudinal drainage	S_6	20	21	9.7	0.35	0.6	0.7	87	90	0.20
System of road signs	S_7	3	14	8.9	0.13	0.5	0.9	83	84	0.10

time (cost) constant. Thus, if cost (time) is fixed, allocating more time (resources) to the jobs will increase quality and safety, or to maintain the same level of quality and safety and to reduce time, we will have to pay increasingly more money per unit, such as in standard project activity crashing [67].

III. APPLICATION

Quality and safety represent two important concerns influenced in large part by decisions made during the planning and design process of projects rather than during construction. It is during these preliminary stages that component configurations, materials specifications and functional performance are decided. Some designs or construction plans can be inherently difficult and dangerous to implement whereas others may reduce considerably the possibility of accidents. Accidents and failures in quality can result in very large direct costs, even with minor defects which require re-construction. Besides these direct costs, indirect costs of insurance, inspection, regulation, stop work order, workmen's compensation, insurance premiums, etc., should also be taken into account [64].

In this section, the integer linear programming problem is applied to a road building project in the mountain pass of La Braguia, in the north of Spain. Since different types of projects require different levels of control, monitoring, care, and compliance with quality and safety requirements, project managers should evaluate the different alternatives and assign and select the best one that best fits quality and safety requirements. According to these requirements, Table 1 shows the tasks to undertake, their time, cost, quality and safety associated on a 0-100 scale, the job set they belong to, and the weight of these sets representing their importance and contribution to the overall quality and safety of the project. As we can see, job sets S_1, S_3, S_5, S_6 and S_7 are unit sets because there is only one task in each set whereas job sets S_2 and S_4 are multiple sets since there are two different ways to perform these jobs. For example, job S_2 (Transport of earths) can be completed in 36 days (S_{21}) or in 31 days (S_{22}) with different cost, quality and safety requirements.

A graphical representation of the combined planning and scheduling problem is shown in the decision project graph of Fig. 1, where the circular "AND" nodes represent jobs that must be performed and the triangular "OR" nodes introduce the mutually exclusive job alternatives of a job set. In Fig. 1 the additional interdependence of a contingent relationship between jobs S$_{41}$ and S$_{22}$ ($S_{41} \geq S_{22}$) is included. We may include job S$_{22}$ if and only if we perform job S$_{41}$. Therefore, there are only three possible sets of decisions: $\{S_{21}, S_{42}\}$, $\{S_{21}, S_{41}\}$, and $\{S_{22}, S_{41}\}$. The project graphs resulting from each of these sets of decisions are shown in Figures 2, 3, and 4 respectively.

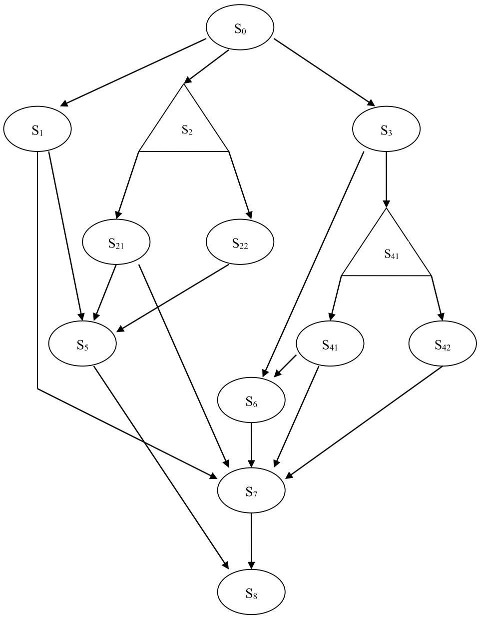

FIGURE 1. Decision-CPM network.

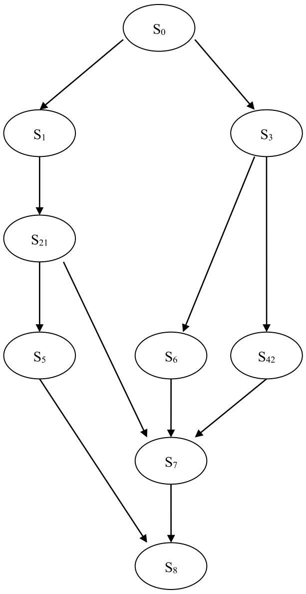

FIGURE 2. Decision set $\{S_{21}, S_{42}\}$.

In the construction industry, contracts usually include liquidated damages to be paid to the owner when the

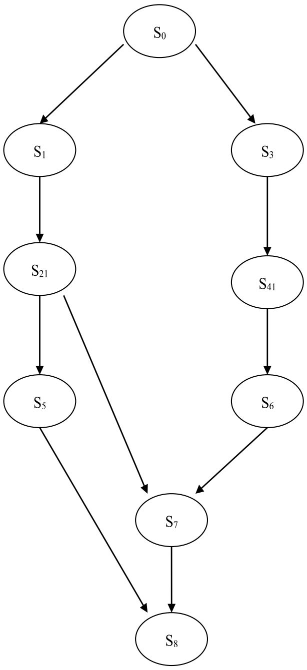

FIGURE 3. Decision set $\{S_{21}, S_{41}\}$.

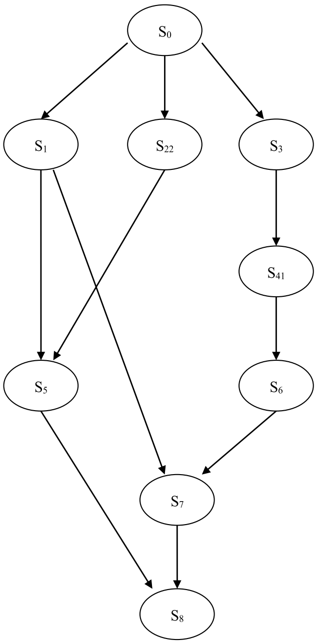

FIGURE 4. Decision set $\{S_{22}, S_{41}\}$.

contractor fails to meet the project completion date. Liquidated damages are not penalties, they are damages that represent the profits lost by the owner waiting for completion of late projects. In this paper, liquidated damages are not contemplated, but instead, incentives and disincentives are considered. Specifically, a reward payment of €10,000 per day for each day the project is under the required due date $D = 105$ days, and a penalty payment of €15,000, for each day beyond D. Regarding quality and safety, for analytical purposes and in order to show the usefulness of the model, the overall quality and safety achieved in the project must be equal to a level of 96 and 80 respectively. The linear programming problem of selecting the best project graph and finding its critical path minimizing total costs can be expressed as follows:

$$\begin{aligned}
&\text{Min: } C = 5.67d_{41} + 7.09d_{42} + 3.74d_{21} + 4.20d_{22}\\
&\qquad - 0,01w_F^- + 0,015w_F^+ + 9.68\\
&\text{Subject to: } t_1 + t_7 \le w_8\\
&\qquad t_1 + t_5 \le w_8\\
&\qquad t_3 + t_6 + t_7 \le w_8\\
&\qquad t_1 - M\,(1 - d_{21}) + t_5 \le w_8\\
&\qquad t_1 - M\,(1 - d_{22}) + t_7 \le w_8\\
&\qquad - M\,(1 - d_{22}) + t_5 \le w_8\\
&\qquad t_3 - M\,(1 - d_{42}) + t_7 \le w_8\\
&\qquad t_3 - M\,(1 - d_{41}) + t_7 \le w_8\\
&\qquad t_3 - M\,(1 - d_{41}) + t_6 + t_7 \le w_8\\
&\qquad 8.9d_{21} + 8.4d_{22} + 13.8d_{41} + 12.8d_{42} + 58.9 \ge Q\\
&\qquad 8.8d_{21} + 8.9d_{22} + 14.8d_{41} + 12d_{42} + 56.9 \ge S\\
&\qquad w_8 - w_F^+ + w_F^- = 105\\
&\qquad d_{22} \le d_{41}\\
&\qquad d_{21} + d_{22} = 1\\
&\qquad d_{41} + d_{42} = 1\\
&\qquad Q = 96\\
&\qquad S = 80\\
&\qquad t_1 = 27;\quad t_3 = 25;\ t_6 = 20;\ t_7 = 3\\
&\qquad w_8 = 78;\ 103;\ 1050 \le d_{ij} \le 1,\quad \text{integer}
\end{aligned}$$

Given a daily penalty of €15,000, a daily premium of €10,000, and a quality and safety level requirement of 96 and 80 respectively, Table 2 shows how the total project cost will change with the due date established and the work plan selected. The lowest cost in Table 2 corresponds to the work plan $\{S_{21}, S_{41}\}$ (Fig. 3) and a due date D = 105. In this case, the project can be delivered 2 days before and an early premium of €20,000 (2∗€10,000) is obtained. Thus, the total cost is €19.07 million (19.09 – 0.02). Selecting the work plan $\{S_{21}, S_{42}\}$ (Fig. 2) and a due date D = 90, the project cannot be delivered as planned and the overtime penalty is €225,000 (15 days at a cost of $15,000 per day). Thus, the total cost is €20,735 million (20,510 + 0.225), the highest value of Table 2. Several other possible combinations are shown in the Table, all of them meeting the quality and safety requirements. If one work plan fails to meet the project's goal, alternative work plans can be considered.

IV. CONCLUSION

Construction projects are conducted with a complex technology, open-air work environment, and tight constraints of time, cost, quality and safety. Under these circumstances, during the planning and scheduling phases of construction

TABLE 2. Total cost (€$*10^6$) of the project with given decision sets and due date.

	Decision set		
Due date	$\{S_{21}, S_{42}\}$	$\{S_{21}, S_{41}\}$	$\{S_{22}, S_{41}\}$
105	19.07	19.28	20.51
103	19.09	19.30	20.54
90	19.28	19.43	20.73

engineering projects, project managers need to narrow down potential alternatives and decide on an optimal solution. These decisions have an influential role in the overall cost and performance of the project and in turn can lead to significant savings.

Linear programming is a mathematical technique used to solve complex problems in many real-world situations affecting the construction industry by making some simplifying assumptions. It is used to allocate limited resources such as capital, materials, financial, etc., on the basis of a given criterion of optimality. Managers, provided with the required skills, are those who, during the model formulation phase, convert the available resources into mathematical expressions that represent the relationship among decision variables, resources, and constraints. The model presented in this paper illustrates some of the possible richness of the problem formulation that is possible within the decision CPM framework such as the possibility of making decisions during the execution of the project with the knowledge of scheduling information and due date, consider the effects of alternative methods of performing the tasks, etc. From the lowest cost of €19.07 million and a due date of 105 days to the highest cost of €20,735 million and a due date of 90 days, a number of alternative combinations that can be used by managers to meet time, cost, quality, and safety requirements are shown. The model presented here can help project managers to select work plans at the task level and to better understand how the decisions made in the planning and scheduling phases of an ongoing project affect overall project performance. The planning process should include the identifications of alternative work plans that consider the trade-offs between time, cost, quality, and safety.

During the scheduling phase, if there is a major cost overrun or time delay that makes achieving the optimal desired level of quality and/or safety virtually impossible, using the model presented in this paper, the remaining tasks could have their time and/or cost allocations updated such that a significant increase in quality and/or safety can occur with minimal effect on time and cost. Similarly, time and/or cost can be appreciably improved if quality and safety allocations for the remaining tasks are updated.

It is well recognised that time, cost, quality and safety, are critical elements that contribute to project success. However, in many cases these elements may be in conflict and project quality and safety may be affected by construction time and cost. If, during the execution of a project an overall optimum is to be obtained and there are a number of competing methods of performing some of the tasks, each method having its own time, cost, quality and safety considerations, a much greater degree of interaction between the planning and scheduling phases is essential. Since time, cost, quality and safety are becoming a great concern to judge whether a project is successful, future research should focus on how to quantify quality and safety and how to balance these four critical factors within the project scope.

ACKNOWLEDGMENT

Prof. José R. San Cristóbal thanks Prof. Emma Diaz from the University of Cantabria for her comments to the final version of this paper and two anonymous referees for the critical reviewing of the manuscript and their valuable suggestions.

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