CONTACT MECHANICAL APPROACH TO THE WINDING NIP

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ABSTRACT

A contact mechanical model for the winding nip, consisting of the wound roll, winding drum and the intervening sheet, is presented. The roll and drum are modeled as linear, orthotropic, homogeneous cylinders with a rigid core. The elastic solutions for the cylinders are derived analytically in a series form. The sheet is modeled as a linear and orthotropic material as well. An approximate elastic solution for the sheet is obtained by assuming an internal stress distribution compatible with the boundary conditions (thin sheet approximation). The governing contact mechanical equations are presented and the appropriate form of the wound-on-condition of the sheet is presented.

NOMENCLATURE

a	nip half width
a_{nm}, b_{nm}	coefficients to be determined by the boundary conditions
A_{rr}, $A_{r\theta}$, $A_{\theta\theta}$, $G_{r\theta}$	elastic constants of the cylinder
$c_{11}, c_{12}, c_{22}, c_{66}$	compliance coefficients of the sheet
h	half of the sheet (or web) thickness
h_j	normal sheet surface stress at x_j
M_1, M_2	driving torques of the roll and drum
P	vertical compressive load on the cylinders
p, q	normal and tangential surface tractions on the cylinder
q_j^+, q_j^-	tangential upper and lower sheet tractions at x_j
r	radial polar coordinate
R	cylinder outer radius
R_0	core radius

T_i	incoming web tension
T_0	outgoing web tension
u, v	horizontal and vertical displacements of the sheet
v^+, v^-	vertical upper and lower sheet surface displacements
v_1, v_2	radial surface displacements of the roll and drum
v_r, v_θ	radial and tangential displacements of the cylinder
V^+, V^-	upper and lower sheet surface speeds
V_1, V_2	roll and drum surface speeds
x, y	cartesian coordinates
x_j	nip discretization point
δ_0	vertical approach of the roll and drum centers
Δx	nip discretization step
ε_0	horizontal displacement of the nip center
$\varepsilon^+, \varepsilon^-$	horizontal strains of upper and lower sheet surfaces
μ_+, μ_-	friction coefficients in the upper and lower contacts
σ^+, σ^-	normal stresses at the upper and lower sheet surfaces
σ_r, σ_θ	radial and tangential normal stresses of the cylinder
σ_x, σ_y	horizontal and vertical normal stresses of the sheet
τ^+, τ^-	shear stresses at the upper and lower sheet surfaces
$\tau_{r\theta}$	tangential shear stress of the cylinder
τ_{xy}	horizontal shear stress of the sheet

INTRODUCTION

The winding devices of the modern paper industry exclusively include a nip. Despite restricting air entrainment into the roll, a nip provides two additional external loads – normal and tangential – to control the wound roll structure. A winding nip typically generates a high stress concentration in the nip area and, hence, a possibility for web and roll defects. It is also well known that in the nip area layer-to-layer slippage may occur in the wound roll. This sets demands for the winder control system to keep the nip loads in a range so that the required roll structure is achieved and, on the other hand, that roll defects due to nip induced stresses are avoided. Currently, the appropriate range of the winding parameters is sought mainly by trial and error tests.

Despite that there exists a vast amount of literature on rolling contact of two parallel cylinders, a rigorous theory applicable to the winding nip is still lacking. Bentall & Johnson [1] have studied the rolling contact of two cylinders with an elastic strip going through the nip. They restricted their treatment to isotropic materials, identical cylinders and essentially to free rolling conditions. Also, a half-space approximation for the cylinders was used and, hence, the theory is not suitable for a drum with a thin elastic cover. Tervonen [2] has extended the treatment to linear, viscoelastic cylinders and tractive rolling. His model includes covered cylinders but is also restricted to isotropic materials. In neither of these papers the winding application is considered. Soong & Li [3,4] have considered the rolling contact of two cylinders with linear, elastic and isotropic layers bonded to a hard core and driving an elastic thin sheet with extensional stiffness. Welp & *al.* [5] have discussed the aspects of modeling the winding process and presented an idealized model for isotropic materials. They restricted the treatment to a rigid winding drum and based their treatment in one-dimensional web theory. In the web – wound roll contact the condition of total slip was assumed and, hence, only the web-winding drum contact was studied. All previously mentioned studies lack the proper nip exit condition for the web wound onto the roll.

In this paper a contact mechanical model for the winding nip is presented. The model consists of the wound roll, winding drum and the sheet in the nip. The wound roll and winding drum are modeled as linear, orthotropic, homogeneous cylinders with a rigid core. The elastic solutions are derived analytically in a series form. The sheet is also modeled as a linear and orthotropic material. An approximate elastic solution for the sheet is obtained by assuming an internal stress distribution compatible with the boundary conditions and by integrating the orthotropic constitutive equations (thin sheet approximation). The appropriate form of the wound-on-condition of the web is presented.

FUNDAMENTAL ELASTIC SOLUTIONS

Let us consider a linear, orthotropic cylinder of radius R (see Fig. 1) with a rigid core of radius R_0 and loaded by the radial and tangential surface load distributions p and q, respectively. Hence, the boundary conditions in the polar coordinate system attached to the center of the core are

$$\begin{aligned} v_r(R_0,\theta) &= 0 \quad , \quad \sigma_r(R,\theta) = -p(\theta) \\ v_\theta(R_0,\theta) &= 0 \quad , \quad \tau_{r\theta}(R,\theta) = q(\theta) \ , \end{aligned} \tag{1}$$

where v_r and v_θ denote the radial and tangential displacements, and σ_r and $\tau_{r\theta}$ the radial normal and tangential shearing stresses, respectively. The linear constitutive equations of the orthotropic cylinder can be written as

$$\begin{aligned} \sigma_r &= A_{rr}\frac{\partial v_r}{\partial r} + A_{r\theta}\frac{1}{r}\left(v_r + \frac{\partial v_\theta}{\partial \theta}\right), \\ \sigma_\theta &= A_{r\theta}\frac{\partial v_r}{\partial r} + A_{\theta\theta}\frac{1}{r}\left(v_r + \frac{\partial v_\theta}{\partial \theta}\right), \\ \tau_{r\theta} &= G_{r\theta}\left(\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right), \end{aligned} \tag{2}$$

where the coefficients A_{rr}, $A_{r\theta}$, $A_{\theta\theta}$ and $G_{r\theta}$ are the *orthotropic elastic constants* of the material. Substituting the above equations into the equations of equilibrium

$$\begin{aligned} &\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \ , \\ &\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + 2\frac{\tau_{r\theta}}{r} = 0 \end{aligned} \tag{3}$$

a coupled system of partial differential equations for the displacements v_r and v_θ is obtained. Due to the periodicity 2π of the solutions with respect to θ, the displacements admit the Fourier series presentations

$$v_r(r,\theta) = u_0(r) + \sum_{n=1}^{\infty}\left[u_n^s(r)\sin n\theta + u_n^c(r)\cos n\theta\right],$$
$$v_\theta(r,\theta) = v_0(r) + \sum_{n=1}^{\infty}\left[v_n^s(r)\sin n\theta + v_n^c(r)\cos n\theta\right]. \tag{4}$$

Substituting expressions (4) into the equilibrium equations, written in terms of the displacements, we obtain an ordinary differential equation system for the Fourier coefficients u_0, u_n^s, u_n^c, v_0, v_n^s and v_n^c. It can be shown that the solutions may be written in the form

$$u_0(r) = \sum_{m=1}^{2} b_{0m}\phi_{0m}(r), \quad v_0(r) = \sum_{m=1}^{2} a_{0m}\varphi_{0m}(r),$$
$$u_n^s(r) = \sum_{m=1}^{4} a_{nm}\phi_{nm}(r), \quad v_n^c(r) = \sum_{m=1}^{4} a_{nm}\varphi_{nm}(r), \tag{5}$$
$$u_n^c(r) = \sum_{m=1}^{4} b_{nm}\phi_{nm}(r), \quad v_n^s(r) = \sum_{m=1}^{4} -b_{nm}\varphi_{nm}(r).$$

The coefficients a_{nm} and b_{nm} are determined by the boundary conditions (1) and the functions ϕ_{nm} and φ_{nm} are given by

$$\phi_{13} = \ln r \quad , \quad \phi_{14} = 1 \quad ,$$
$$\varphi_{01} = r \quad , \quad \varphi_{02} = r^{-1} \quad , \quad \varphi_{13} = \ln r + \frac{A_{r\theta} + G_{r\theta}}{A_{\theta\theta} + G_{r\theta}} \quad , \quad \varphi_{14} = 1 \quad , \tag{6}$$
$$\phi_{nm} = r^{\lambda_{nm}} \quad , \quad \varphi_{nm} = \eta_{nm} r^{\lambda_{nm}} \quad \text{otherwise} ,$$

where the characteristic roots are defined by (n=2,3,..)

$$\lambda_{01} = -\lambda_{02} = \sqrt{\frac{A_{\theta\theta}}{A_{rr}}} \quad , \quad \lambda_{11} = -\lambda_{12} = \sqrt{\frac{\mu_1}{A_{rr}G_{r\theta}}} \quad ,$$
$$\lambda_{n1} = -\lambda_{n2} = \sqrt{\frac{\mu_n + \sqrt{\mu_n^2 - \kappa_n}}{2A_{rr}G_{r\theta}}} \quad , \quad \lambda_{n3} = -\lambda_{n4} = \sqrt{\frac{\mu_n - \sqrt{\mu_n^2 - \kappa_n}}{2A_{rr}G_{r\theta}}} \tag{7}$$

and the constant coefficients by

$$\eta_{nm} = \frac{A_{rr}\lambda_{nm}^2 - A_{\theta\theta} - n^2 G_{r\theta}}{(A_{r\theta}+G_{r\theta})n\lambda_{nm} - (A_{\theta\theta}+G_{r\theta})n} \ ,$$

$$\mu_n = A_{\theta\theta}G_{r\theta} - A_{r\theta}(A_{r\theta}+2G_{r\theta})n^2 + A_{rr}\left(G_{r\theta}+A_{\theta\theta}n^2\right) \ , \qquad (8)$$

$$\kappa_n = 4A_{rr}A_{\theta\theta}G_{r\theta}^2\left(n^2-1\right)^2 .$$

Let us consider next the elastic sheet in the nip. An analytical solution for the sheet can be obtained by using the method of homogeneous solutions and biorthogonality properties of the 2D elasticity problem formulated in terms of the stress function and generalized biharmonic equation [6], by using the method of separation of variables leading to a general series representation [7,8], or by using a sixth degree polynomial for the stress function accounting for linearly changing boundary loads [8]. These solutions would be valid for any sheet thickness. However, due to the complexity of these methods, an alternative approximate solution, utilizing the thinness of the sheet, is considered here. Consider the sheet of Fig. 2 loaded by the normal stresses $\sigma^+(x)$ and $\sigma^-(x)$ and the shear stresses $\tau^+(x)$ and $\tau^-(x)$ at the top and bottom surfaces, respectively. The web tension at the left cross-section of the sheet is T_i. Within the thin sheet approximation one can assume that $\sigma^+(x) = \sigma^-(x) \equiv \sigma^\pm(x)$. We postulate the following stress distribution, compatible with the boundary conditions, inside the sheet

$$\sigma_x = \frac{1}{2h}\left\{T_i - \int_{-a}^{x}\left[\tau^+(\xi) - \tau^-(\xi)\right]d\xi\right\} ,$$

$$\sigma_y = \sigma^\pm(x) + (y^2 - h^2)g_1(x) , \qquad (9)$$

$$\tau_{xy} = \frac{1}{2}\left[\tau^+(x) + \tau^-(x)\right] + \frac{y}{2h}\left[\tau^+(x) - \tau^-(x)\right] ,$$

where $2h$ is the thickness of the sheet and $g_1(x)$ an integration "constant" to be determined when integrating the horizontal and vertical displacements from the constitutive equations

$$\frac{\partial u}{\partial x} = c_{11}\sigma_x + c_{12}\sigma_y ,$$

$$\frac{\partial v}{\partial y} = c_{12}\sigma_x + c_{22}\sigma_y , \qquad (10)$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = c_{66}\tau_{xy} \quad .$$

The stresses (9) satisfy the equation of horizontal equilibrium and the boundary conditions. The equation of vertical equilibrium, however, is not fulfilled exactly. Models of this type ('extensional strip models') have been used by several authors [4,5]. By integrating equations (10) and neglecting terms of order $0(h^2)$ we obtain

$$u(x,y) = \frac{c_{11}}{2h}(x+a)T_i - \frac{c_{11}}{2h}\int_{-a}^{x}(x-\xi)\left[\tau^+(\xi)-\tau^-(\xi)\right]d\xi$$

$$+c_{12}\int_{-a}^{x}\sigma^{\pm}(\xi)d\xi + Cy + u(-a,0) \quad , \tag{11}$$

$$v(x,y) = \frac{c_{12}}{2h}\left\{T_i - \int_{-a}^{x}\left[\tau^+(\xi)-\tau^-(\xi)\right]d\xi\right\} + \frac{c_{66}}{2}\int_{0}^{x}\left[\tau^+(\xi)+\tau^-(\xi)\right]d\xi$$

$$+c_{22}\sigma^{\pm}(x)y - Cx + v(0,0)$$

for the horizontal and vertical displacements of the sheet.

FORMULATION OF THE CONTACT PROBLEM

The radial displacements of the upper and lower cylinder (roll and drum) surfaces v_1 and v_2, and the vertical displacements of the top and bottom surfaces of the sheet v^+ and v^-, respectively, must satisfy the *indentation equation*

$$v_1 + v_2 + v^+ - v^- + \delta_0 = \frac{1}{2R}(x^2 - 2\varepsilon_0 x) \tag{12}$$

within the contact area. In equation (12) the coordinate x is measured along the center-line of the deformed sheet, δ_0 is the vertical approach of the cylinder centers, ε_0 the horizontal displacement of the center of the nip, and $1/R = 1/R_1 + 1/R_2$ the relative curvature of the cylinders.

In the zones of stick the velocities of the contacting surfaces must be equal. Using small displacement approximation the necessary *stick condition* between the paper roll and the sheet can be written as

$$\varepsilon^+ - \varepsilon_1 + \xi_1 = 0 \, . \tag{13}$$

Similarly, the stick condition within the contact area of the winding drum and the sheet can be written as

$$\varepsilon^- - \varepsilon_2 + \xi_2 = 0 \, . \tag{14}$$

Here $\varepsilon^+ = \partial u(x,h)/\partial x$ and $\varepsilon^- = \partial u(x,-h)/\partial x$ are the horizontal strains of the top and bottom surfaces of the sheet, and ε_1 and ε_2 the circumferential surface strains of the roll and drum, respectively. Note that due to equation (11) the condition $\varepsilon^+ = \varepsilon^-$ is valid. The constants ξ_1 and ξ_2 are the *creep ratios*, i.e., fractional differences between the speeds of the contacting bodies far away from the contact area. Within the slip areas we assume *Amonton's law* of friction, i.e.,

$$\begin{aligned} \operatorname{sgn}(V^+ - V_1)\,\tau^+ &= \mu_+\sigma^+ \, , \\ \operatorname{sgn}(V^- - V_2)\,\tau^- &= -\mu_-\sigma^- \, , \end{aligned} \tag{15}$$

where V^+ and V^- are the local speeds of the upper and lower sheet surfaces, V_1 and V_2 the local speeds of the surfaces of the upper and lower cylinders, and μ_+ and μ_- the friction coefficients in the upper and lower contacts, respectively.

In order to obtain the displacements and stresses at the contact we assume the following piecewise linear distributions of the normal and shear stresses on the sheet surfaces within the contact area $-a \le x \le a$

$$\sigma^{\pm}(x) = \sum_{j=1}^{M} h_j \psi_j(x) \ , \qquad \tau^{\pm}(x) = \sum_{j=1}^{M} q_j^{\pm} \psi_j(x) \ , \tag{16}$$

where h_j, q_j^+ and q_j^- are the unknown surface tractions at the points

$$x_j = -a + (j-1)\Delta x \quad , \quad j = 1,2,\dots,M \tag{17}$$

and the piecewise linear local basis functions are

$$\psi_j(x) = \begin{cases} (x - x_{j-1}) / \Delta x \ , \ x_{j-1} < x \le x_j \\ (x_{j+1} - x) / \Delta x \ , \ x_j < x \le x_{j+1} \\ 0 \qquad , \text{ elsewhere} , \end{cases} \tag{18}$$

where $\Delta x = 2a / (M-1)$. Utilizing the elastic solutions (4) and (11) we can write the vertical displacements and tangential strains at an arbitrary contact point x_i in the form

$$\begin{aligned} v_{1,i} &= \sum_{j=1}^{M} A_{ij,1} h_j + \sum_{j=1}^{M} B_{ij,1} q_j^+ \ , \ v_{2,i} = \sum_{j=1}^{M} A_{ij,2} h_j + \sum_{j=1}^{M} B_{ij,2} q_j^- , \\ \varepsilon_{1,i} &= \sum_{j=1}^{M} C_{ij,1} h_j + \sum_{j=1}^{M} D_{ij,1} q_j^+ \ , \ \varepsilon_{2,i} = \sum_{j=1}^{M} C_{ij,2} h_j + \sum_{j=1}^{M} D_{ij,2} q_j^- , \\ v_i^+ &= \sum_{j=1}^{M} A_{ij}^+ h_j + \sum_{j=1}^{M} B_{ij}^+ q_j^+ + \sum_{j=1}^{M} B_{ij}^- q_j^- + \tilde{B}_i T_i \ , \\ v_i^- &= \sum_{j=1}^{M} A_{ij}^- h_j + \sum_{j=1}^{M} B_{ij}^- q_j^+ + \sum_{j=1}^{M} B_{ij}^+ q_j^- - \tilde{B}_i T_i , \\ \varepsilon_i^+ &= \varepsilon_i^- = \sum_{j=1}^{M} C_{ij}^+ h_j + \sum_{j=1}^{M} D_{ij}^+ q_j^+ + \tilde{D}_i T_i \quad . \end{aligned} \tag{19}$$

The coefficients $A_{ij,k}$, $B_{ij,k}$, $C_{ij,k}$ and $D_{ij,k}$ (k =1,2) are determined via equations (1), (4), (5) and (16), and the coefficients $A_{ij}^{\pm}$, $B_{ij}^{\pm}$, $C_{ij}^{\pm}$, $D_{ij}^{\pm}$, $\tilde{B}_i$ and $\tilde{D}_i$ via equations (11) and (16). Note that the effect of the tension T_0 on the deformation of the roll is not accounted for in equations (19). A complete treatment of T_0 evidently calls for the use of an *Archimedean spirale* for the roll.

We can still utilize the total equilibrium conditions of the subsystems. It is easy to see from Fig. 2 that the horizontal equilibrium condition for the sheet and the torque balance conditions for the roll and drum lead to the equations

$$\begin{aligned} &\int_{-a}^{a} \tau^{+}(x)\,dx - \int_{-a}^{a} \tau^{-}(x)\,dx + T_0 = T_i \,, \\ &\int_{-a}^{a} \tau^{+}(x)\,dx + T_0 = \frac{M_1}{R_1} \,, \\ &\frac{M_1}{R_1} + \frac{M_2}{R_2} = T_i \,. \end{aligned} \tag{20}$$

In a real winding situation two of the quantities M_1, M_2 and T_i - let's say M_1 and T_i - are given. The vertical equilibrium conditions for the roll and drum give

$$\int_{-a}^{a} \sigma^{+}(x)\,dx = \int_{-a}^{a} \sigma^{-}(x)\,dx = -P \,. \tag{21}$$

If the compressive load P is given, equations (21) can be used as part of the solution procedure providing also a value for the nip contact width $2a$. If, on the other hand, the nip width is given the corresponding compressive load can be calculated by equations (21). The latter alternative is preferable since it leads to a simpler set of equations to be solved.

If the sheet goes through the nip with an externally set tension behind the nip, the tension T_0 is known. However, if the sheet is wound on the roll, the tension T_0 is not known a priori so that the number of unknowns is increased by one. The wound-on-condition, on the other hand, yields one more equation. Since the sheet behind the nip, after being stuck onto the roll surface, becomes part of the roll, one can readily conclude that the *wound-on-condition* takes the form

$$\xi_1 = 0 \,. \tag{22}$$

It has been pointed out in [5] that the web tension and curvature of the surfaces within the contact area give rise to an additional surface pressure = web tension/radius of curvature. Note that the radius of curvature contains information about the displacements and, hence, accounting for it would greatly complicate the solution process. Fortunately, the nip pressure due to curvature is typically about 5 % of the average nip pressure due to the external compressive load and, therefore, may be ignored in a first approximation.

The unknowns of the problem are $h_1, \ldots, h_M$, $q_1^{\pm}, \ldots, q_M^{\pm}$, δ_0, a, ε_0, ξ_1, ξ_2, and T_0. The equations at disposal are the indentation equations (12) and the stick/slip equations (13)-(15) written at discrete points within the contact area, the equilibrium equations (20)-(21) and the wound-on-condition (22). The appropriate solution process is a variation of the *Panagiotopoulos Process* [9]. For the sake of simplicity the contact half-width a is taken as a given quantity as explained above and, hence, the iteration of the contact area is excluded from the solution procedure. The problem is now solved by the following algorithm:

Step 0. Initiate with $q_i^+ = q_i^- = 0$.

Step 1. Calculate h_i $(i = 2, 3, \ldots, M-1)$, ε_0 and δ_0 from the indentation equation.

Step 2. Assume stick at all points x_i. With h_i obtained from step 1, calculate q_i^+, q_i^-, ξ_2 and T_0 from the stick equations and total equilibrium (20).

Step 3. If $\left|q_i^\pm\right| > -\mu_\pm h_i$, then index i is placed in $S_\pm$ (index set: area of slip in upper or lower contact).

If $\left|q_i^\pm\right| \le -\mu_\pm h_i$, then i is placed in $A_\pm$ (index set: area of adhesion in upper or lower contact).

Step 4. If i is in $S_\pm$, then $q_i^\pm = -\mathrm{sgn}(q_i^\pm)\mu_\pm h_i$. If i is in $A_\pm$, then the corresponding stick equation is used. In conjunction with the total equilibrium equations (20) these are linear equations. Solve them.

Step 5. If i is in $A_\pm$, as well as the just-found $\left|q_i^\pm\right| > -\mu_\pm h_i$, then i is placed in the area of slip $S_\pm$.

Step 6. If $A_\pm$ is changed in step 5, then go to step 4.

Step 7. If i is in $S_\pm$, and $q_i^\pm$ and the relative speed between the surfaces have the wrong sign with respect to each other, then i is placed in $A_\pm$.

Step 8. If $S_\pm$ is changed in step 7, then go to step 4.

Step 9. If the difference between the just calculated and the previous shear stresses is larger than the required tolerance, go to step 1. Else stop.

CONCLUSIONS

Starting from first principles we have presented the equations for the contact mechanical approach for the winding nip consisting of the wound roll, winding drum and the sheet obeying the orthotropical material law. In the next stage the model should be implemented into a computer for nip simulations. The aim of these calculations is to find out the state of the sheet in the nip as well as the wound-on-tension needed to evaluate the stresses in the roll. The sheet model used can be progressively improved, starting from the approximate model presented in this paper and proceeding towards the more complete models of the sixth degree polynomial stress function and a general series representation.

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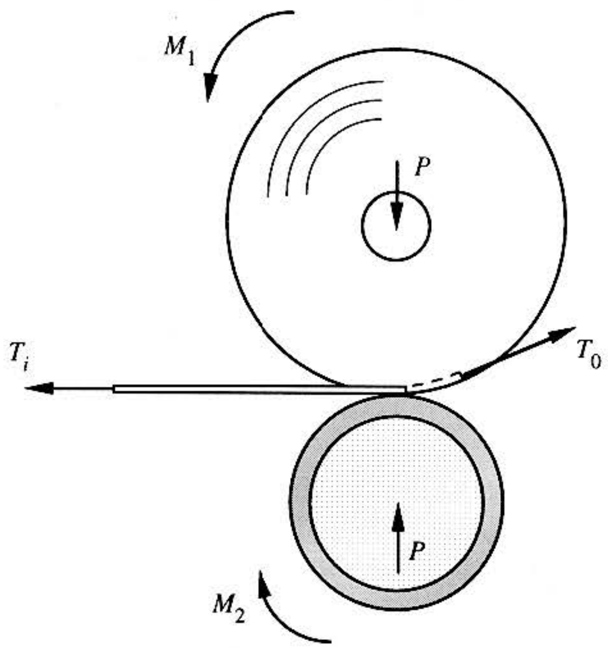

Fig. 1 Winding configuration

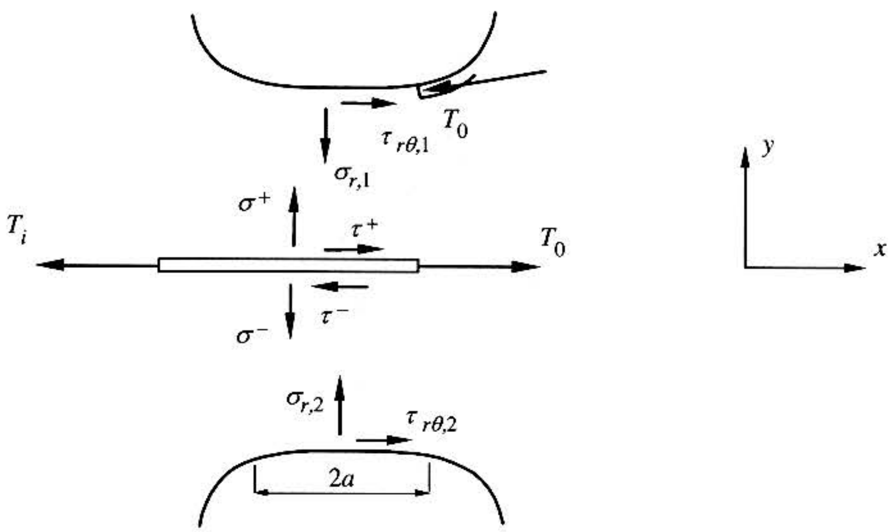

Fig. 2 Notation of the stresses within the contact area