THEORY AND APPLICATION OF DRAW CONTROL FOR ELASTIC WEBS WITH NIPPED PULL ROLLERS

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ABSTRACT

Draw control is defined as changing or maintaining web tension by controlling surface speeds of consecutive driven rollers. Even though draw control is commonly used in web handling applications, there remains some misunderstanding of the relationship between machine draw and the resulting web elongation and tension. Fundamentals of draw control and a new analytical solution will be presented.

Draw control is commonly used downstream of a constant tension zone. The strain at any point in the process is dependent on the upstream tension and the added web strain from the machine draw. Recent experimental studies of draw control have revealed a previously unknown influence of nip roller load, similar to pack rollers at winding. Models for nipped pull roller slip conditions, and nip induced draw and tension will be presented.

NOMENCLATURE

Var.		Description	Units
A	=	Cross sectional area of a web	mm^2
C	=	Constant of integration	--
D	=	Draw between two rollers	dimensionless
E	=	Elastic modulus	Pa
F	=	Force applied to a sliding block	N
H	=	Thickness of a web segment	m
L	=	Length of a web segment	m

M	=	Mass of material in a tension zone	kg
N	=	Normal load of nip roller	N
T	=	Tension in span or zone	N
t	=	Time	s
V	=	Surface velocity of a web or roller	m/s
W	=	Web width	m
ε	=	Strain	dimensionless
θ	=	Wrap angle of web around roller	degrees
μ	=	Coefficient of friction between two materials	dimensionless
ρ	=	Density of web material	kg/m^2
σ	=	Stress	Pa

Sub and superscripts

i	=	zone in a multi-zone web line
0	=	unstrained or initial conditions

DRAW CONTROL FUNDAMENTALS

The application of a tensile stress, σ, will cause a material to elongate in the direction of the applied force. The magnitude of this elongation relative to the unstressed dimension is called the strain, ε, and is defined by

$$\varepsilon = \frac{(L - L_0)}{L_0} , \tag{1}$$

For an elastic material, there is a linear relationship between the applied stress and the resultant strain given by

$$\sigma = \varepsilon E . \tag{2}$$

Stress and strain are key parameters in the two primary web handling techniques: tension and draw control.

Tension control is a force based system for loading a web during handling. Tension control commonly uses open-loop torque control, or closed-loop tensioning with load cell or dancer roller feedback. Stress is the independent variable in tension control systems. The web's strain is the dependent variable and will change under any web modulus, thickness, or width changes, as well as any variations in the applied stress.

Draw control is a speed or elongation based system for loading a web during handling. Draw control commonly uses the roller surface speed to control the speed of the web. The surface speed of a roller is a function of its diameter and angular velocity (rpm). Strain is the independent parameter in draw control systems. The web's stress is the dependent variable and will change under any web modulus, thickness, or width changes.

These control strategies are applied to the web in sections through web line. These web line sub-section are called zones and typically includes two pull rollers and the web path connecting them. Any two web loading or torque generating devices may be at either end of a zone including; unwinds, rewinds, and driven rollers. Motors, clutches, and brakes are commonly used.

Draw versus Strain

Strain is controlled using draw, sometimes causing these two terms to be confused for each other. Strain is a web property defined as the elongation change from an unloaded condition, as shown in Eqn. 1. Draw is an equipment property defined as the relative surface speed of one roller relative to another. Draw and strain percents are equal only if the web is in an untensioned, unstrained condition at the baseline speed. As will be shown, in many draw systems the actual web strain cannot be determined by roller surface speed alone.

Some of the confusion between draw and strain comes from relating draw control web handling to tensile-elongation testing. A tensile-elongation test is used to determine a web's modulus by drawing a sample and measuring the force required to obtain a particular level of strain. Since the sample starts in an unstrained state, the web strain is directly related to the draw. In this example, draw is defined in term of relative lengths rather than speeds. This comparison also leads to the misconception that draw is always required to maintain strain and that a negative draw automatically causes a web to go into compression.

Typical Draw Applications

Tension control is common in handling high modulus webs like paper and polyester. The high modulus means almost all typical web handling tensions result in strains of less than two percent. Low tension create extremely low elongation, making draw control a difficult option, often exceeding motor speed control capabilities. However, the extension forces are large even compared to roller drag and frictional losses, making tension control the best choice.

Draw control or speed based control are used in many niches of web handling. Low modulus materials with large strains (over 2 percent) are obvious candidates for draw control. Draw control is used to drive several rollers from a single motor. Multiple driven rollers, essentially several draw zones, are commonly used for low modulus webs since idler roller drag and inertia are a significant portion of tension requirements. Even web handling systems based largely on tension control may have draw control areas like two rollers in an "S" wrapped pull roller or the main drive section of a slitter/rewinder. Draw control is also common in small and medium sized multi-station printing systems.

ANALYTICAL SOLUTION FOR DRAW CONTROL USING UNNIPPED PULL ROLLERS

Draw in a web handling system can be understood by examining a two pull roller system (see Fig. 1). The first pull roller is set at a velocity of V_1, while the second pull roller has a surface velocity of V_2. This system has a draw defined as,

$$D = \frac{(V_2 - V_1)}{V_1}, \tag{3}$$

but the web strain is unknown.

Calculating the strain as a function of draw requires a knowledge of both the web's time zero strain and the upstream entering web strain. The time zero strain is based on the machine's static condition after threading. Often the web's weight is the only loading after threading; therefore, the web strain starts near zero. The web strain will not make a discontinuous leap to the draw percentage the instant the machine starts; rather, as the two pull roller draw system begins, the difference between the initial and steady-state strains will decrease exponentially (assuming a steady state entering strain). This steady state strain will be dependent on not only the two roller draw, but also the upstream strain level. This transient strain condition will be presented using a new analytical solution.

In Figure 1, the simple two pull roller system shows the basic elements of a draw zone. The web velocity in upstream zone 1 is assumed to be equal to the surface speed of the first pull roller. Likewise, the web velocity in the draw zone 2 is assumed to be equal to the surface velocity of the second pull roller.

In order to derive an equation for predicting the strain in zone 2, a mass balance will be performed for this zone (as presented by Reid, Shelton, and Shin, Ref. 1). The mass of an arbitrary section of web is given by

$$M_0 = \rho_0 (L_0 W_0 H_0), \tag{4}$$

where the subscript, 0, indicates the value of a parameter in the unstrained state. If this section of web is now strained, the mass of the segment will be given by

$$M = \rho (LWH). \tag{5}$$

The relationship between the strained and unstrained dimensions can be determined by applying Eqn. 1. That is,

$$L = L_0(1 + \varepsilon_L), \tag{6}$$

$$W = W_0(1 + \varepsilon_W), \text{ and} \tag{7}$$

$$H = H_0(1 + \varepsilon_H). \tag{8}$$

Setting the strained and unstrained masses equal to each other yields

$$M_0 = \rho_0 (L_0 W_0 H_0) = \rho(LWH) = M. \tag{9}$$

If the strain is small, the density of the material will remain constant. The dimensions in the strained state can be expanded using Eqns. 6-8, and terms common to both sides can be canceled. Thus, Eqn. 9 reduces to

$$1 = (1+\varepsilon_L)(1+\varepsilon_W)(1+\varepsilon_H), \text{ or} \tag{10}$$

$$\frac{1}{(1+\varepsilon_L)} = (1+\varepsilon_W)(1+\varepsilon_H). \tag{11}$$

The expression for the conservation of mass can be written as

$$\frac{dM}{dt} = \frac{d[\rho LWH]}{dt} = 0 \quad . \tag{12}$$

Substituting Eqns. 6-8 and 11, Eqn. 12 can be reduced to

$$\frac{dM}{dt} = \rho_0 W_0 H_0 \frac{d\left(\frac{L}{1+\varepsilon_L}\right)}{dt} = 0 \quad . \tag{13}$$

Upon further reduction, this expression can be written as

$$\frac{d\left(\frac{L}{1+\varepsilon_L}\right)}{dt} = \frac{dL_0}{dt} = 0 \quad ; \tag{14}$$

therefore, the conservation of mass is equivalent to the conservation of unstrained web length. Following the unstrained material is the only way to determine the strain state created by draw conditions. The web transported in and out of the draw zone must be converted to its unstrained length to maintain the mass balance.

The rate at which strained web enters zone 2 is equal to V_1, while the rate at which web exits zone 2 is V_2. Noting that a velocity can be written as dL/dt, Eqn. 6 can be used to relate these velocities to the rate at which unstrained web length is transferred into and out of zone 2. That is

$$V_i = \frac{dL_i}{dt} = \frac{d\left(\frac{L_{0_i}}{1+\varepsilon_{L_i}}\right)}{dt} \quad . \tag{15}$$

Thus, a balance on the unstrained length of web in zone 2 can be written as

$$L_{0_2}(t+\Delta t) = L_{0_2}(t) + \frac{\overline{V}_1(t)\Delta t}{1+\overline{\varepsilon}_{L_1}(t)} - \frac{\overline{V}_2(t)\Delta t}{1+\overline{\varepsilon}_{L_2}(t)}, \tag{16}$$

where the overbar indicates the average value for a variable over the time Δt. Equation 16 states the new amount of unstrained material in zone two is determined from three parts; the original unstrained material at time zero, the entering unstrained material, and the exiting unstrained material. Rearranging this expression, and taking the limit as Δt approaches zero leads to

$$\frac{dL_{0_2}}{dt} = \frac{V_1(t)}{1+\varepsilon_{L_1}} - \frac{V_2(t)}{1+\varepsilon_{L_2}}. \tag{17}$$

Using Eqn. 6 to expand L_0, and multiplying the second term on the right hand side by L_2/L_2, Eqn. 17 can be rewritten as

$$\frac{d\left(\frac{L_2(t)}{1+\varepsilon_{L_2}(t)}\right)}{dt} + \left(\frac{V_2(t)}{L_2(t)}\right)\left(\frac{L_2(t)}{1+\varepsilon_{L_2}(t)}\right) = \frac{V_1(t)}{1+\varepsilon_{L_1}}. \tag{18}$$

This differential equation can be solved using an integrating factor defined by

$$f(t) = e^{\left(\int^t \frac{V_2(x)}{L_2(x)}dx\right)}, \tag{19}$$

where x is a dummy variable for the integration. The solution to Eqn. 18 is then given by

$$\frac{L_2(t)}{1+\varepsilon_{L_2}(t)} = \frac{1}{f(t)}\left(\int^t f(x)\frac{V_1(x)}{1+\varepsilon_{L_1}(x)}dx + C\right), \tag{20}$$

where C is a constant of integration. This expression can be solved provided the functional forms of L_2, V_1,V_2 and ε_1 are known. For the simple case where they are all constants, Eqn. 20 becomes

$$\frac{L_2}{1+\varepsilon_{L_2}(t)} = \frac{L_2}{V_2}\left(\frac{V_1}{1+\varepsilon_1}\right) + Ce^{-V_2 t/L_2}. \tag{21}$$

The value of the constant is determined from the boundary condition

$$\varepsilon_{L_2} = \varepsilon_{L_2}(0) \; at \; t = 0. \tag{22}$$

Thus, Eqn. 21 becomes

$$\frac{1}{1+\varepsilon_{L_2}(t)} = \frac{V_1}{V_2}\left(\frac{1}{1+\varepsilon_{L_1}}\right) + \left[\frac{1}{1+\varepsilon_{L_2}(0)} - \frac{V_1}{V_2}\left(\frac{1}{1+\varepsilon_{L_1}}\right)\right] e^{-V_2 t/L_2}. \tag{23}$$

Equation 23 is a new analytical solution to the draw zone transient conditions. The right side of Eqn. 23 has two parts, a steady state and a transient component. There are no small strain assumptions in this equation, so it can be applied to all constant density drawing process. In Figure 2, the initial draw was zero and the entering web's strain was one percent. The rate of change from the initial to steady state condition is determined by the time constant, V_2/L_2. For higher speeds and shorter zones the transition will be quick. Lower speeds and longer zones will cause a longer transition period.

At steady-state (as t approaches infinity) reduces Eqn. 23 to

$$\frac{V_1}{1+\varepsilon_{L_1}} = \frac{V_2}{1+\varepsilon_{L_2}}. \tag{24}$$

For values of strain less than one, the denominators can be replaced by their binomial expansion. Keeping only the first order terms, Eqn. 24 is reduced to its more familiar form, as shown by Reid, Shelton, and Shin (1),

$$V_1(1-\varepsilon_{L_1}) = V_2(1-\varepsilon_{L_2}). \tag{25}$$

For values of strain less than 0.1, Eqn. 25 can be used rather than Eqn. 24 with an error of less than one percent.

Equation 24 can be used to demonstrate that a negative draw does not necessarily result in compression. Draw was defined by Eqn. 3. This expression can be rearranged to obtain

$$D = \frac{V_2}{V_1} - 1. \tag{26}$$

Equation 24 can be rearranged to obtain

$$\varepsilon_{L_2} = \left(\frac{V_2}{V_1} - 1\right) + \varepsilon_{L_1}\left(\frac{V_2}{V_1}\right), \tag{27}$$

an expression for the strain in zone 2. The definition for draw given in Eqn. 6 can be used reduce Eqn. 27 to

$$\varepsilon_{L_2} = D + \varepsilon_{L_1}\left(\frac{V_2}{V_1}\right). \tag{28}$$

Therefore, even if there is a negative draw in the system($V_2 < V_1$), the strain in zone 2 will be positive provided the incoming strain is sufficiently great. That is,

$$\varepsilon_{L_2} > 0 \textit{ if } \varepsilon_{L_1} > ABS\left[D\left(\frac{V_1}{V_2}\right)\right]. \tag{29}$$

SLIP CRITERIA FOR NIPPED PULL ROLLERS

The slip criteria for a pull roller is a balance of tensions offset by frictional forces. Most web handling applications are designed assuming no roller slip. For idler rollers, this assumes small drag and inertial loads relative to friction. On pull rollers, this assumes sufficient frictional forces to offset any positive or negative tension changes. No slip or good traction conditions enable predictable web tracking and tensioning, independent of small frictional changes. Marginal or poor traction conditions are often behind many seemingly random web problems such as wrinkling, scratching, and lateral position error.

As long as friction is above the critical slip condition, the web to roller friction is considered to have no significant effect on web tension. However, when friction is low, the web and roller will slip relative to each other and the high tension side will not be isolated from low tension side. The amount of actual slip in percent relative surface speed is sometimes dramatic, but may also be minute enough to be undetectable with standard web speed measurement methods. A visible indication of slip was sometimes noted when the web to shuttered in the draw zone due to the vibration of the slip-stick condition at the nip rollers. The primary slip indicator was the change in draw zone tensions.

Friction is often measured using a sliding block test. The ratio of the applied force, F, required to slide a block of weight, N, is the coefficient of friction. That is,

$$\mu = \frac{F}{N}. \tag{30}$$

This is similar to the maximum frictional force created by the normal load of a nip roller.

In web handling we are interested in the friction between a web and a cylindrical roller. The web to roller friction is an important web process variable contributing to guiding, wrinkling, winding, as well as tensioning. In web handling, the frictional forces of an nipped and wrapped pull roller is determined by combining two basic equations. The first equation describes the maximum frictional force created by a nip roller.

$$T_A - T_B = \mu N \tag{31}$$

$$\mu_{CRIT} = \frac{T_A - T_B}{N}. \tag{32}$$

If the coefficient of friction is greater than μ_{CRIT}, then no slip will occur. This equation assumes the nip has a line contact with no significant wrap angle.

The second equation is the analytically derived and describes the relationship between a web and an unnipped roller. This equation, often referred to as the belt equation, is

$$\frac{T_{high}}{T_{low}} = e^{\mu\theta}, \text{ or} \tag{33}$$

$$\mu_{CRIT} = \frac{\ln(T_{high}/T_{low})}{\theta}. \tag{34}$$

If the coefficient of friction is greater than μ_{CRIT}, then no slip will occur.

These two equations can be combined to describe the slip condition of a roller which is both nipped and wrapped. There are actually two different slip conditions which result. The first occurs when the nip side tension is low relative to the other tension (shown below and detailed in Fig. 6).

$$\mu_{CRIT} = \frac{T_{HIGH} - T_{LOW}e^{\mu\theta}}{N}, \tag{35}$$

The other case is when the nip side tension is high relative to the other tension (shown below and detailed in Fig. 7).

$$\mu_{CRIT} = \frac{\frac{T_{HIGH}}{e^{\mu\theta}} - T_{LOW}}{N}. \tag{36}$$

In both cases the coefficient of friction, μ, must be greater than μ_{CRIT} to prevent slip.

Slip on the exit pull roller creates a draw zone tension not only based on unwind tension and nip induced tension, but also on the rewind tension. If the rewind tension was too low relative to the draw zone, the draw zone tension would drop below a level predicted by nip induced tension theory (see Fig. 9). If the rewind tension was significantly higher than the draw zone, the draw zone tension would be greater than predicted by nip induced tension theory (see Fig. 10).

ANALYTICAL SOLUTION FOR DRAW CONTROL CONSIDERING NIP INDUCED TENSION

The tensioning effect of nip rollers at winding has been understood for many years. The theory and experimental verification of nip induced tension at winding was documented by Good, Wu, and Fikes (2). In their paper, nip induced tension is described in terms of stress.

$$\sigma_{Winding} = \sigma_{Web\ Handling} + \sigma_{Nip\ Induced\ Tension} \tag{37}$$

The web handling stress is relative to the tension prior to entering the winding nip. The winding stress is the tension in the web after the nip and equal to the sum of the web handling and nip induced tension. The nip induced tension is

$$\sigma_{NIT} = \frac{\mu N}{tW} \tag{38}$$

determined from nip load, N, web thickness, t, web width, W, and the coefficient of friction between the outer two layers of the winding roll, μ. This theory of nip induced tension describes the added tension as a friction controlled phenomenon. The web enters the nip point under the web handling tension level. The normal load of the nip attempts to elongate the web in the machine direction, but this elongation is controlled by the frictional bond between the outer two layers of the wound roll. The web cannot elongate unless is overcomes the frictional forces. Also, the nip induced tension are controlled by the layer-to-layer friction. Therefore, the minimum and maximum nip induced tension is friction limited.

Since in our experiments involved laminated products, it was more convenient to describe nip induced tension in terms of tensile load and units of Newtons (or lbf). The load based nip induced tension relationship is

$$T_{Winding} = T_{Web\ Handling} + T_{Nip\ Induced\ Tension} \tag{39}$$

where the nip induced load is simply

$$T_{Nip\ Induced\ Tension} = \mu N \tag{40}$$

We found nip induced tension was not unique to winding in the presence of a nip roller, but also occurred under specific draw control conditions. This effect is not seen in common tension control systems since both sides of the nipped pull roller are controlled by tension or torque loops. However, draw control is sensitive to nip induced tension because the nip may change the draw zone's entering or exiting conditions.

We found the tension in a draw zone was based on three factors; the initial unstrained material in the zone, the rate of unstrained material entering the zone, and the rate of unstrained material exiting the zone. The nip induced tension may change either the entering or exiting conditions. The nip effects will be greatest when the web is in contact with the nip roller before or as it contacts the driven roller. With this arrangement, the web entering the zone based on the driven roller surface speed is not under the previous zones tension, but at a somewhat higher tension because of the added nip induced tension.

If the web is in contact with the driven roller for a large angle of wrap before the nip roller, then the nip induced tension will be reduced or eliminated. In our experiments the exiting nip contacted the second pull roller after a large wrap angle (greater than 90 degrees). The large wrap before the nip eliminated any effect of the nip changing the unstrained material leaving the draw zone.

Coefficient of Friction

The magnitude of the nip induced tension and the slip criteria are both depend on the web to roller coefficient of friction. The appropriate value to use is the kinetic coefficient of friction of the material sliding against the driven rollers. In order to determine these values, the belt equation was used. If the tension on both sides of the roller and the wrap angle are known, the coefficient of friction can be calculated from Eqn. 34.

An Instron was calibrated by hanging a weight and setting the readout to one; thus, all subsequent readings gave T_2/T_1 directly. The calibration weight was attached to a web of the material of interest. This web was then hung over the driven roller such that the wrap angle was 1.57 radians (90 degrees). The web was threaded to the upper jaw of the Instron (see Fig. 3). This jaw moved at a rate of 0.1 inch/minute, and the resulting load was recorded for one minute. The average was calculated and this value for T_2/T_1, along with the known wrap angle, was then used to determine the coefficient of friction of the web materials relative to the driven rollers. The driven roller surface was a special sputtered metal release coating. A summary of these values is given in the Table 1.

Material Contacting Driven Roller	**Kinetic Coefficient of Friction, μ**
clay coated paper	0.41
super-calandered Kraft paper	0.30
uniaxially oriented polypropylene	0.24
biaxially oriented polypropylene	0.22

Table 1. Coefficients of friction for the webs relative to the driven rollers.

Nip Induced Tension

A simple web line was used to investigate the nip induced tension in a draw zone (shown in Figure 4). Our experimental rewinder has three zones; a tension controlled unwind, a central draw zone, and a tension controlled rewind. The unwind and rewind tension zones use speed controlled motor with dancer roller feedback. The standard threadup of this rewinder goes directly from the first to second pull roller in an "S" wrap. For our trials, the web was routed over a tension sensing idler roller between the two driven rollers.

The web path between the nipped rollers is the draw zone in this system. The tension in this zone was monitored with a tension roller mounted between the pull rollers. Since both nipped rollers are the same diameter and are driven at the same rotational speed, their surface speeds are identical.. Thus, the draw in this zone is zero, and from Eqn. 8, the strain of the web in the draw zone should be equal to the incoming strain. Therefore, if nip induced tension effects are neglected, the tension of the web in the draw zone should equal the unwind tension, as controlled by the unwind dancer.

The tension in the draw zone was measured for the following set of operating conditions. Three different laminate web structures were used as test materials. In each case, a three inch width web was used. One layer of the laminates was common to all three webs, the clay coated paper. Three other materials were used for the second layer: a super-calandered Kraft paper; a uniaxially oriented polypropylene, or a biaxially oriented polypropylene.

A preliminary series of experiments demonstrated that the draw zone tension was independent of the web speed for the range attainable on this web line; thus, the web speed was fixed at 31 mpm (100 fpm) for all subsequent experiments. A two level factorial design experiment was run with the unwind tension, and the two nip loads as the independent parameters. The levels of the parameters investigated are given in Table 2. Because the web wraps the nip rollers as it entered and exited the draw zone, the tension of the web increased the nip loads. This effect was accounted for in the air pressure loading of the nip rollers in order to maintain constant normal loads independent of the web tension.

Parameter	**Values Investigated**
web width	constant at 0.076 m (3 in.)
unwind tension	low = 13 N (3 lbf) high = 80 N (18 lbf)
first nip load	low = 44 N (10 lbf) high = 89 N (20 lbf)
second nip load	low = 44 N (10 lbf) high = 89 N (20 lbf)
rewind tension	13, 27, 40, 53, 67 and 89 N (3, 6, 9, 12, 15, and 18 lbf)

Table 2. Range of parameters investigated in the nip induced tension study.

The data from one set of experiments is shown in Figure 5. In this plot, the results from the entire factorial design are shown. The upper set of data points represents the data for the unwind tension of 89 N, while the lower set of data points is for an unwind tension of 13 N. For both sets of curves, four sets of data points are shown. These data are coded as follows. Squares represent data collected at the lower level of the first nip load, while diamonds are used for the upper level. The squares and diamonds are unfilled for the lower level of the second nip load while they are filled for the upper level.

A number of interesting features can be observed in Figure 5. First, the draw zone tension is not independent of the rewind tension. In this case, the draw zone tension drops off at low values of rewind tension, while it is constant at higher values. The lower limit for the rewind tension, above which the draw zone tension is independent of the rewind tension appears to vary with the unwind tension. This phenomenon was discussed in the previous section on slip past nipped pull rollers. Second, the draw zone tension levels off at a value greater than the unwind tension. This plateau value increases with increasing value of the first nip load but is independent of the value of the second nip load.

These observations were confirmed by performing an analysis of variance on the data from these trials. At the 99% confidence level, the key parameters affecting the plateau draw zone tension were: the unwind tension, the web material, and the first nip load.

The concept of a nip induced tension was introduced relative to the tensions developed in wound rolls. We are extending this idea to the case of nipped rollers controlling a draw zone.

For each experiment in the previously described factorial design, the predicted tension in the draw zone was calculated as

$$T_{pred} = T_{unwind} + T_{nip} , \tag{41}$$

where the nip induced tension is

$$T_{nip} = \mu N , \tag{42}$$

The predicted values of the draw zone tension are plotted versus the measured values in Figure 10. The line of 1:1 correspondence is also shown. The values of the coefficient of friction used to determine the nip induced tension were given in Table 1.

In order to see the effect of the nip induced tension, the data are replotted in Figure 11, assuming the coefficient of friction were zero; thus neglecting the nip induced tension effect. As can be seen by comparing these two plots, a significantly better fit to the data is obtained by including the effects of the nip induced tension.

CONCLUSIONS

The first vital concept to understanding nipped draw zones is that draw and strain are related, but not equivalent, terms. Strain is a material property determined by tracking the unstrained material using a mass balance approach. Draw is an equipment property which contributes, along with upstream tensions and nip loading, to web's strain in a draw zone.

A new analytical solution has been presented which eliminates small strain assumptions. This relationship has application outside of typical web handling strains, and can be applied to constant density, large strain web processes.

Since most web tension models assume no web to roller slip, the friction criteria for nipped and wrapped pull rollers was reviewed. Draw zone tension measurements verified the combined nip and wrap friction relationship.

Finally, the new application of nip induced tension was shown to apply not only to winding in the presence of a nip roller, but also to nip controlled draw zones. The draw zone nip induced tension effect is most significant if the web wraps the first pull roller nip, ensuring nip induced tension as the web first contacted the driven roller. Historic perspective on draw control is presented in Figure 12, where the unwind tension determines the draw strain (assuming zero percent draw). Nip induced tension adds to the unwind tension baseline, increasing web strain in the draw zone (see Fig. 13).

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2. Good, J. K., Wu, Z., Fikes, M. W. R., "Stresses Within Rolls Wound in the Presence of a Nip Roller," Proceedings of the First International Conference on Web Handling. Web Handling Research Center, Oklahoma State University, Vol. I, 1991.

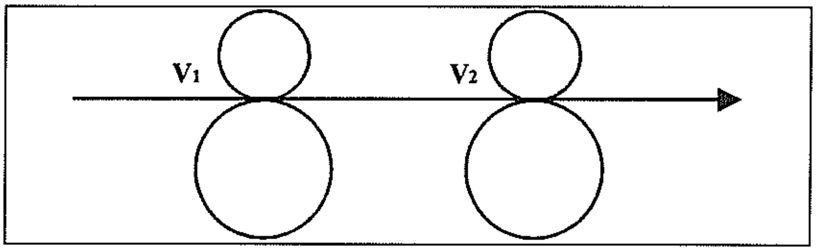


Figure 1 Schematic of a simple two roller draw zone.

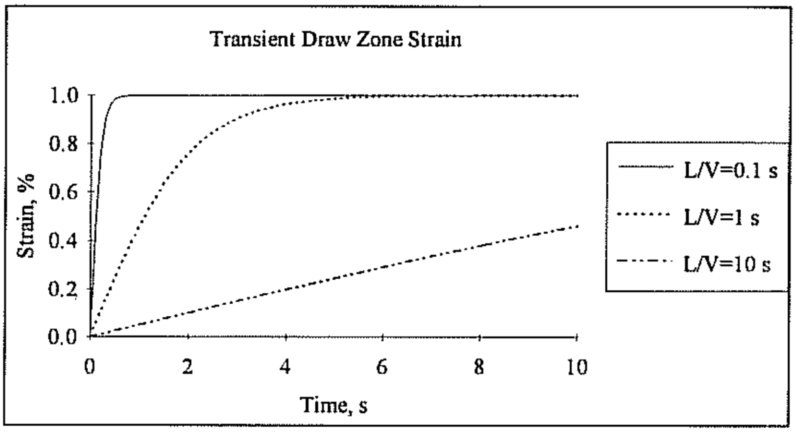


Figure 2 Draw zone tension in transition from initial to steady state conditions. Entering web strain is one percent. Draw zone condition is 1:1 or zero percent draw. Solutions are graphed for three time constants based on speed, V, and zone length, L.

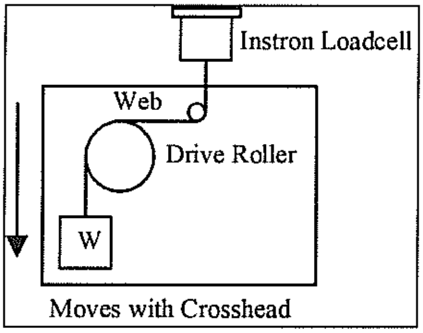


Figure 3 Schematic of the experimental apparatus used to measure web to roller coefficient of friction using the belt equation..

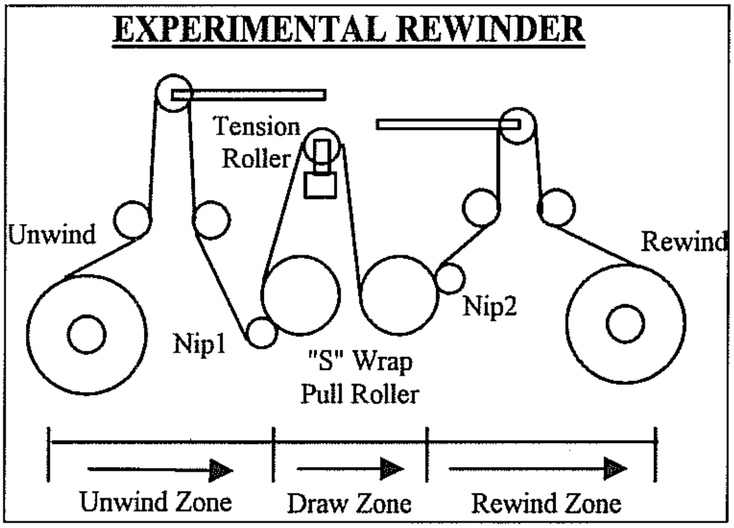


Figure 4 Schematic of experimental rewinder used in verification trials. The "S" wrap pull rollers are equal diameter and driven together making draw equal to zero percent or 1:1.

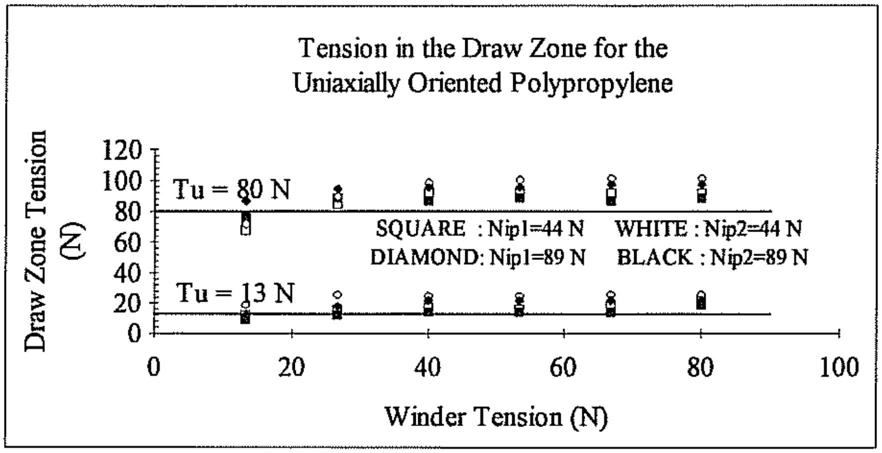
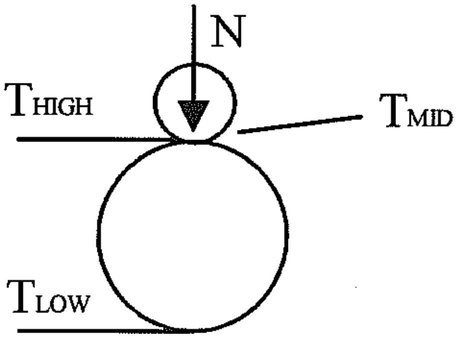


Figure 5 Draw zone tension versus rewind tension for various unwind tensions and nip pressures.

NIPPED PULL ROLLER SLIP CRITERIA



$$T_{HIGH} = T_{MID} + \mu N$$

$$\frac{T_{MID}}{T_{LOW}} = e^{\mu\theta}$$

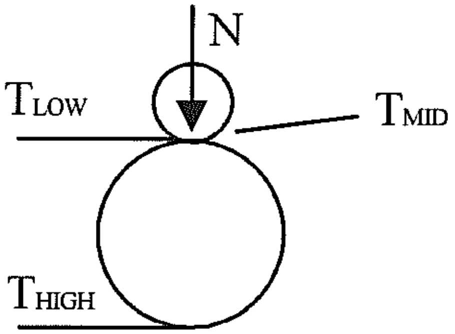
$$\therefore T_{HIGH} = T_{LOW} e^{\mu\theta} + \mu N$$

$$or$$

$$\mu_{CRIT} = \frac{T_{HIGH} - T_{LOW} e^{\mu\theta}}{N}$$

Figure 6 Slip condition schematic and equations for high tension on nip side of pull roller. This is the criteria used when rewind tension was low relative to draw zone tension

NIPPED PULL ROLLER SLIP CRITERIA



$$T_{LOW} = T_{MID} - \mu N$$

$$\frac{T_{HIGH}}{T_{MID}} = e^{\mu\theta}$$

$$\therefore T_{LOW} = \frac{T_{HIGH}}{e^{\mu\theta}} - \mu N$$

$$or$$

$$\mu_{CRIT} = \frac{\frac{T_{HIGH}}{e^{\mu\theta}} - T_{LOW}}{N}$$

Figure 7 Slip condition schematic and equations for low tension on nip side of pull roller. This is the criteria used when rewind tension was high relative to draw zone tension

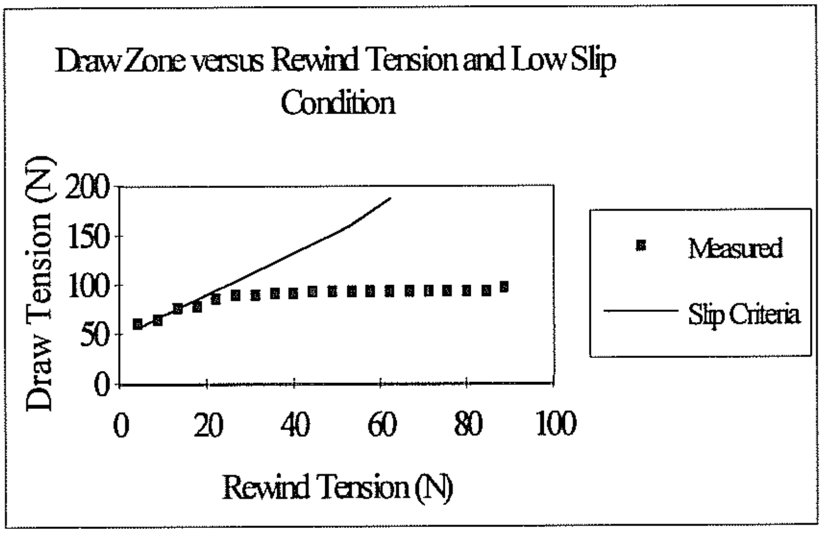


Figure 8 Draw zone and rewind tension showing low rewind tension slip condition. Unwind tension equals 89 N (18 lbf). Note draw zone tension decreases based on high to low exit nip slip conditions.

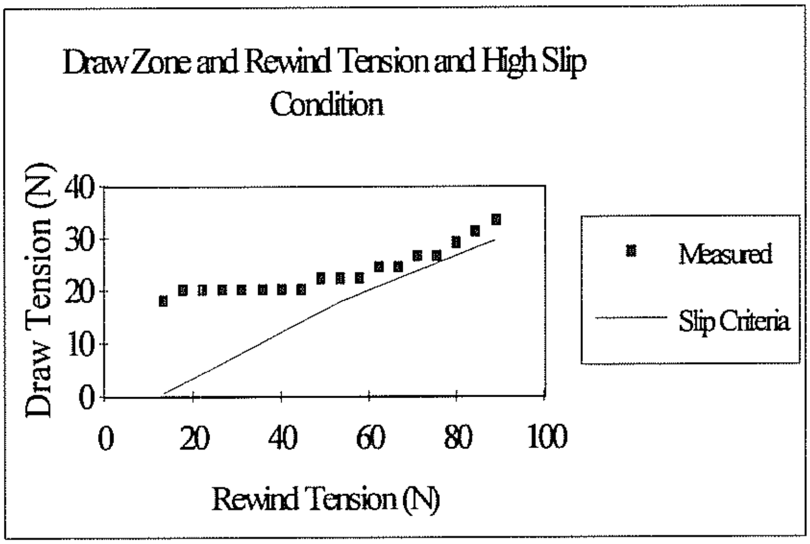


Figure 9 Draw zone and rewind tension showing high rewind tension slip condition. Unwind tension equals 13 N (3 lbf). Note draw zone tension increases based on low to high exit nip slip conditions.

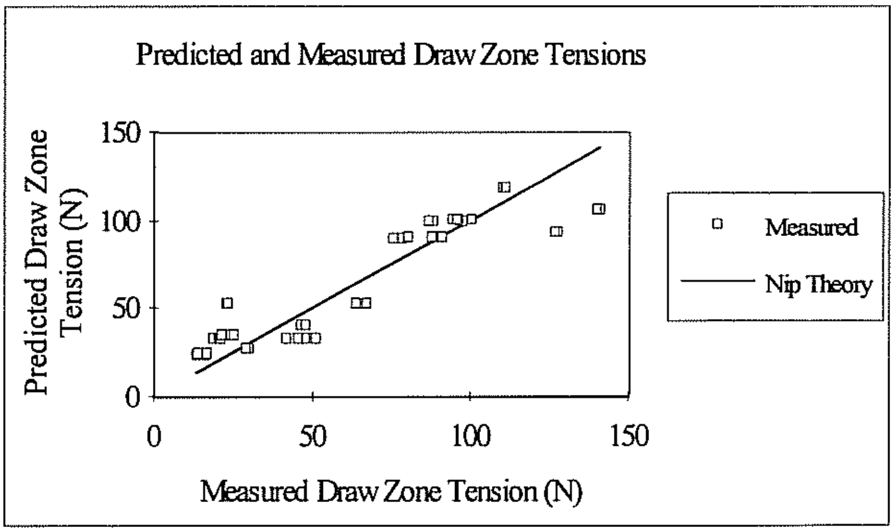


Figure 10 Predicted and measured draw zone tensions according to nip induced tension theory.

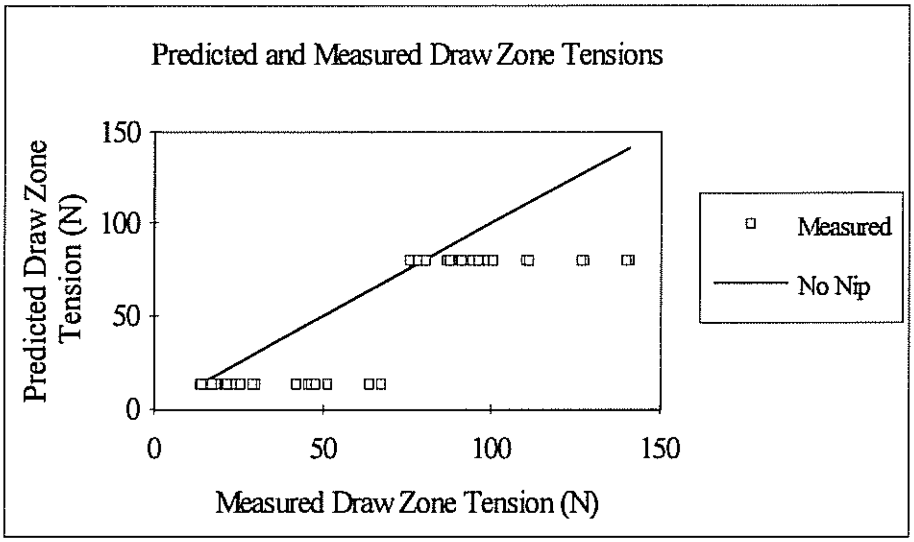


Figure 11 Predicted and measured draw zone tensions with no consideration for nip induced tensioning.

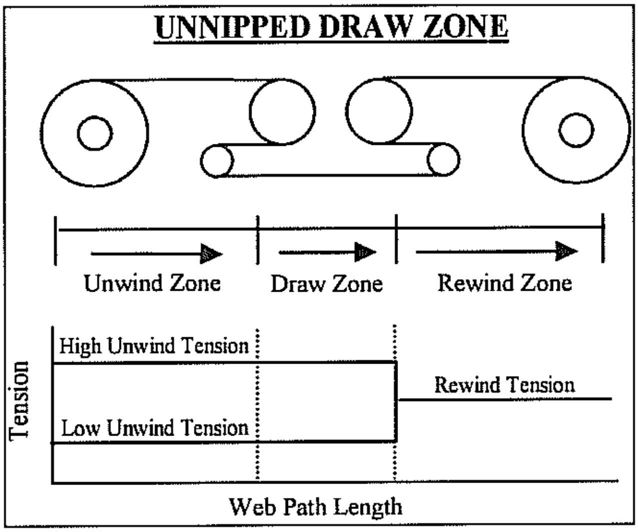


Figure 12 Schematic of intermediate draw zone and steady state tension conditions through the web line. Note: unwind tension controls the draw zone tension.

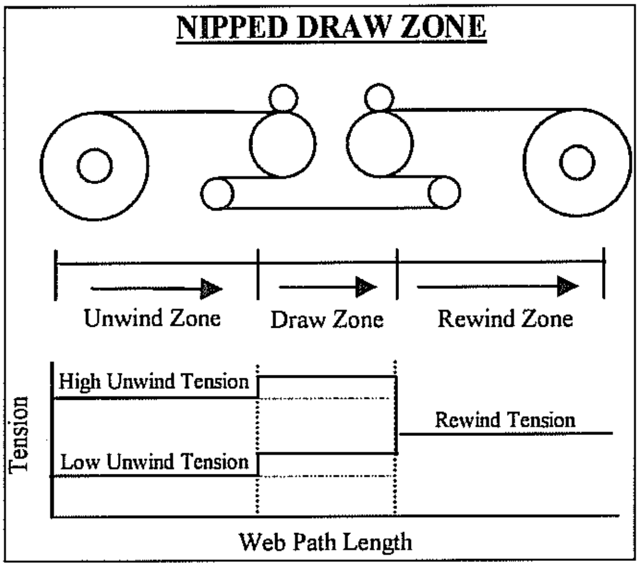


Figure 13 Schematic of intermediate draw zone and steady state tension conditions including the nip induced tensioning. Note: unwind tension controls the baseline of the draw zone tension and the entering nip load is additive.