# STRUCTURAL EVALUATION OF ROLL QUALITY AND IN-ROLL STRESS ANALYSIS USING A NOVEL ON-LINE MEASUREMENT TECHNIQUE

by

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## Abstract

In a previous paper (Jaafar *et al.*, 1999) we reported on the rudimentary development of a new technique for the on-line measurement of a roll's coefficient of restitution (Cr) as it is being wound, and enunciated the theoretical underpinnings behind the development. In this paper, the Cr sensor has been used to evaluate the radial and tangential roll behavior as it is being built. Based on the experimental findings, numerical simulations are proposed for modeling, using energy-based formulations, radial modulus and tangential stress as a function of roll radius. The simulations take into account the additive effect of winding operations, and corrects for the use of such idealized set-ups as the stack experiment, first proposed by Pfeiffer (1966), by incorporating increasing number of layers.

In addition to basic structural assessment of roll quality in real time, a set of experiments have been devised to garner a fundamental understanding of the in-roll stress variations, based on which new insight into the constitutive relations is presented.

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## Introduction

The mechanics of winding has rigorously been treated by a number of researchers over the past four decades. All theoretical examinations have followed one of two approaches: i. Energy-based approaches, or ii. Force-based approaches. Moreover, roll structure formulations have undergone three stages of development. Altmann (1968) introduced an anisotropic model describing the relationship between internal stresses and wound-in tension assuming linear stress-strain behavior. Pfeiffer (1979; 1987) extended the analysis to deal with non-linear material response in the radial direction, following an energy-balance formulation. Independently, Hakiel (1987) improved Altmann's formulations to deal with general nonlinearity, purely based on a force-balance approach. All of the above were valid to center- or surface-wound set-ups. Core effects were first incorporated into roll structure analysis formulations in Pfeiffer's work (1977). Subsequently, core compliance was introduced by Yagoda (1980) as an inner boundary condition on center-wound rolls.

Altmann's early work (Altmann, 1968) provided a general solution for an anisotropic linear elastic roll material while using a non-linear constitutive relation to find the radial hoop stresses for consecutive wraps. The nonlinear approach is to account for interlayer contacts, material discontinuities and anisotropic interlayer wraps. He solved a second order differential equation for the anisotropic linear elastic materials in a center-wound roll and introduced an outer wrap described by a parabolic stress-strain relation. Boutaghou and Chase (1991) later showed that Maxwell's equations on elastic energy function reduce the number of independent terms in Altmann's elastic constants. General properties of center-wound rolls were also associated with anisotropic linear elastic media. The constitutive relations for stacked sheets were examined by Pfeiffer (1979), who suggested empirical forms for stress versus strain. A general nonlinear power-series constitutive law was alternatively used by Hakiel (1987); the first term of Hakiel's series is related to one of Pfeiffer's material constants. Hakiel incorporated nonlinear material properties into his expressions of the basic mechanics and numerical solutions of wound-roll stresses. Good *et al.* (1992) used Hakiel's model to compare his results of interlayer pressure measurements obtained using pull-tabs. They found that the model typically predicted stresses that were twice as large as their measured values. However, they managed to bring predicted and measured values closer together by modifying the outer hoop-stress boundary condition to relax relative to the outer-layer tensile stresses by their model of wound-on tension loss. More recently, Benson (1995) showed that for large strains, nonlinear constitutive laws that accounted for hoop-stress relaxation would give agreement between the predicted and measured interlayer pressure without additional assumptions about wound-on-tension losses.

In this paper we report on how an automated, precision-impact method (Jaafar *et al.*, 1999) to measure springiness of paper rolls from the core to the outside diameter and across the winder is used to obtain accurate information on roll quality. The sensor is based on the concept of an elastic collision of a steel object being freely dropped onto the surface of a roll, as it's being wound, from a known height in the direction normal to the contact surfaces and rebound to a measured elevation. The ratio of speed of separation to speed of approach in an elastic collision is termed the coefficient of restitution, Cr. In real-time scale of impact, several events take place in close succession. The contact event occurs when the tip of the Cr sensor makes a contact with the roll. This contact is followed by a period of compression, during which the kinetic energy at the collision point is converted into (stored) potential energy when deforming the roll (top) surface layers. At this juncture (velocity=0), maximum compression occurs; then follows a period of restitution, in which the stored energy is converted back into kinetic energy as the deformed portion regains its original shape. When the Cr tool has regained enough vertical velocity to lose contact with the roll, termination occurs and the collision is considered over. It is worthwhile noting, that during an impact, some of the kinetic energy is lost irreversibly: Some energy is dissipated into noise, internal atomic friction of the layers during compression and heat. It is possible for some materials to achieve termination before all the internal potential energy is completely converted to kinetic energy. The factor that accounts for all of the losses in kinetic energy is the coefficient of restitution, which is a measure of the elasticity of the collision.

## Understanding Constitutive Stress-Strain Behavior in Wound Rolls

Pfeiffer (1966) was first to recognize the nonlinear behavior of the radial modulus in wound rolls. He designed an experiment whereby a layered stack of paper was used to emulate roll winding, and measured the resulting strain due to known applied load. Consequently, a constitutive relation for the compressive behavior of fibrous materials was developed, which, thereafter, became the basis for all relevant work in this field.

Pfeiffer's original two-parameter relation was later modified (Pfeiffer and Hamad, 1991) to include a third term:

$$P= -K_1 + -K_1 e^{K_2 \varepsilon}+K_3\varepsilon \qquad \text{(Eq. 1)}$$

The three parameters $K_1$, $K_2$ and $K_3$ are inherent parameters: the first and third are pressure relation multipliers, the second is the springiness factor. $P$ and $\varepsilon$ are stress and strain, respectively.

Our compression testing set-up comprised a standard Zwick (model Z-2.5) compression test machine equipped with a maximum capacity of 2 kN load cell, which was used to produce compression strain versus stiffness plots. The Zwick machine was connected to a computer loaded with TESTXPERT software. Stacks of 50, 100, 200, 300, 400 and 500 layers of newsprint were examined. Each layer was 13cm x 11cm and 67 micron thick. Figure 1 indicates how the stack thickness is linearly maintained as the number of layers is increased.

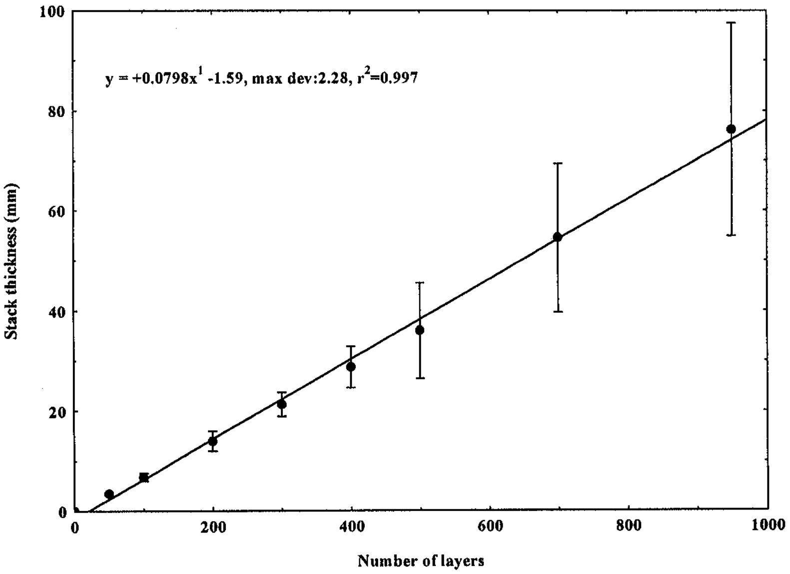

Figure 1: Stack thickness varies linearly with number of layers.

The typical stress vs. strain response for 50-500 layered stacks is depicted in Figure 2. To predict the response for a different stack thickness, say 950 layers, we produced a plot for compressive strain versus stack thickness (Figure 3). Figure 3 was constructed by selecting the thickness and corresponding strain coordinate at arbitrarily-chosen stress levels; an empirical relation was thus produced, from which the stress vs. strain behavior for the desired stack thickness, 950 layers in this case, may be predicted. Figure 4 depicts the predicted result for 950 layers in comparison to experimental results; the prediction fits well with the general pattern of the data. The data from Figure 4 was then input into Eq. 1 to produce the corresponding $K_1$, $K_2$ and $K_3$ parameter, which are plotted in Figure 5. It is evident from this figure that $K_1$ and $K_3$ vary significantly, while $K_2$ tends to change less rapidly with stack thickness. The changes become more pronounced for small-size stacks.

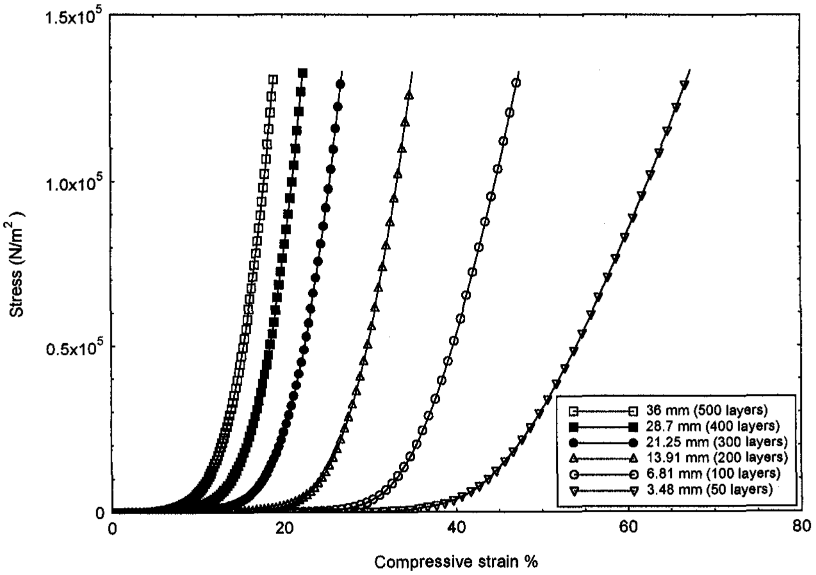

Figure 2: Compressive stress versus strain behavior for newsprint stacks of varying number of layers.

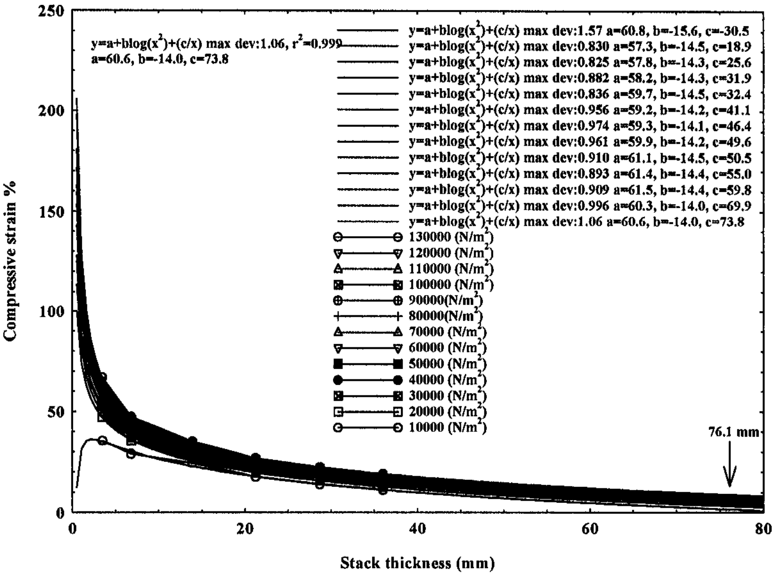

Figure 3: Compressive strain versus stack thickness – concomitant to Figure 2.

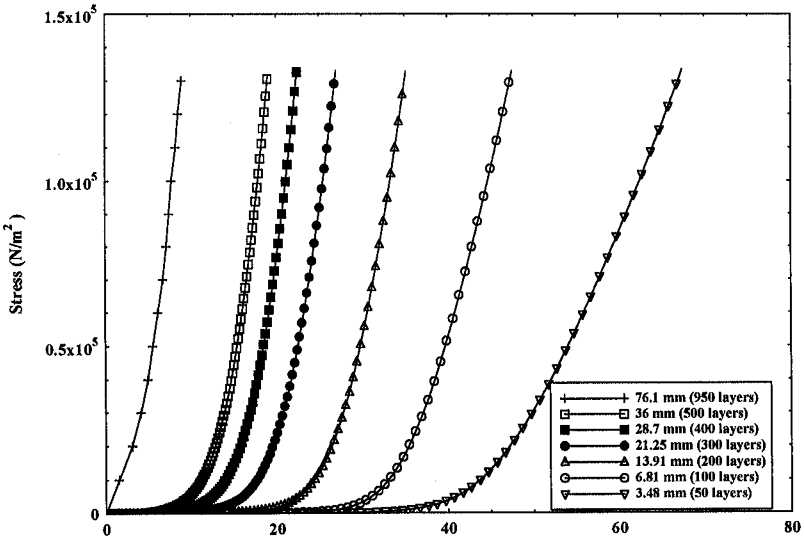

Figure 4: Compressive stress-strain stack behavior including predictions for 950 layers (first curve).

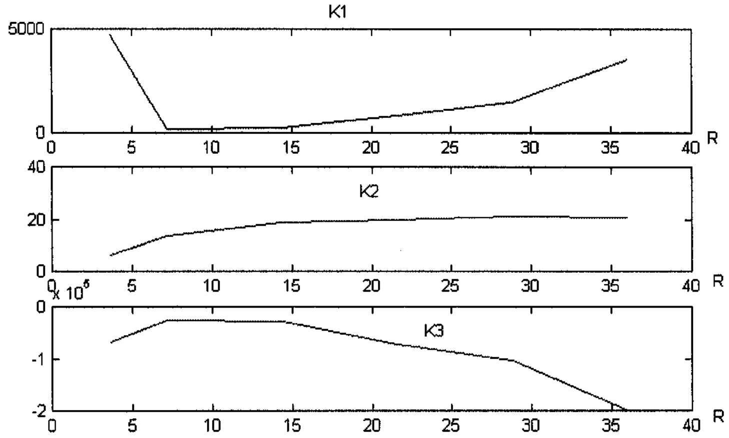

Figure 5: $K_1$, $K_2$ and $K_3$ variations with increasing number of layers, or roll radius.

## Roll Winding Conceptualization: Experiments and Simulations Based on the Coefficient of Restitution Approach

### Experimental

The Cr sensor design arrangement consists of a 100 mm long stroke displacement transducer or LVDT held by a U-shaped frame when mounted on top of a roll using a stand arrangement. This ensures that the LVDT's shaft can fall freely and vertically on a roll being wound in the same location at each cycle. The movement of the shaft is automated using an electrical motor to repeat the impact cycles. Two wheels support the frame with the movement of the roll as it is being wound. Another wheel is located on the tip of the LVDT to minimize friction upon impact. The LVDT is linked to a computer whereby data acquisition is carried out at the rate of 1 kHz using LabView software.

Two main sets of experiments were performed in this study, in which the Cr sensor was utilized to obtain and analyze data. The stack experiment, discussed in the previous section, was adapted to study the relation between the roll's coefficient of restitution and the radial compressive stiffness; another experiment employed a specially designed rig to examine the relations between tangential stresses versus roll radius and the roll's coefficient of restitution versus applied tension.

An Avery dial balance (250 kg maximum weight) was used to investigate the behavior of the coefficient of restitution versus the radial stiffness of a stack of paper. Figure 6 shows that in conjunction with the balance, two G-clamps were used to uniformly compress a 3 mm steel sheet and a stack (35cm x 35cm) of 1000 layers. The Cr sensor, positioned above a small circular hole in the center of the steel sheet, impacts the paper stack.

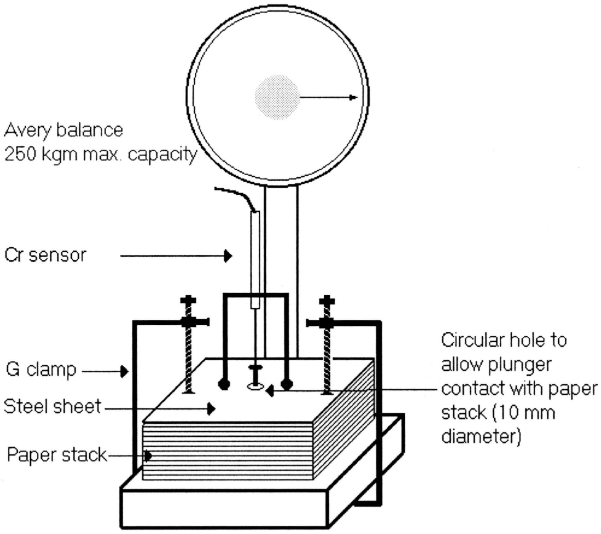

Figure 6: Schematic illustration of the Avery dial balance used to examine the relationship between the stack's coefficient of restitution and its radial stiffness.

A rig, schematically shown in Figure 7, was assembled to correlate the in-roll tangential stresses ($\sigma_\theta$) and roll springiness as a function of roll radius. The rig replicate a wound roll whereby tangential stresses can be measured using rigging balances and can be altered using straining screws. Each staining screw could be secured to a stack of 200 layers. This set-up is used in two manners: (a) Applying a known constant load to each stack, and measuring the resulting tangential stresses in each once all the stacks are laid in place (emulating winding under constant tension); and (b) Consecutively measuring the Cr value and radius for each stack, to which a known constant load is applied. To obtain a series of data points, the procedure is repeated for a number of constant loads. The thickness and width of each layer is 67 micron and 35 cm, respectively; the length, however, would vary depending on the position within the stack.

Newsprint paper was used for all experiments; the average basis weight of the samples was 45.5 g/m$^2$. Relevant strength data were: tensile strength (MD) = 2.257 kN/m and in the cross direction = 0.75 kN/m.

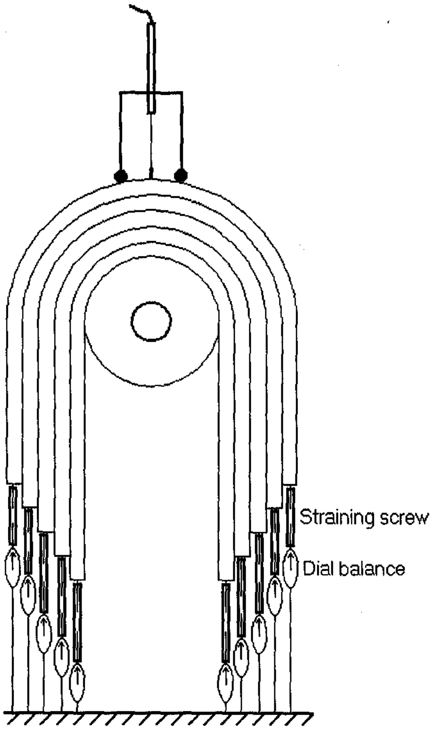

Figure 7: Schematic illustration of the assembled rig to measure simultaneously three roll parameters: Cr, r, applied tension and/or $\sigma_\theta$.

Results and Discussion

Figure 8 represents the response of a paper stack's coefficient of restitution versus radial compressive stiffness (compressive stress/change in stack height). For small strains, the stack exhibits viscoelastic behavior. As the air trapped between the layers, which provides cushioning to the stack, escapes, the stiffness increases, as does the stack's coefficient of restitution. This would then plateaus just above a Cr value of 0.55.

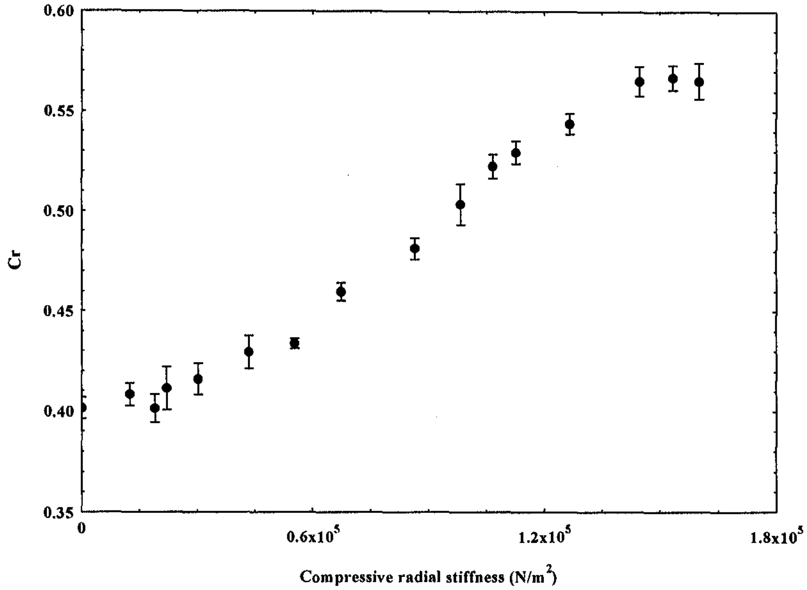

Figure 8: Coefficient of restitution for a stack of paper versus compressive radial stiffness.

Tangential stress variations resulting from the cumulative effect of winding wraps of material under constant tension was experimentally simulated using the set-up of Figure 7. Once a stack is added, the tangential load of the one(s) underneath is recorded from the dial balance; this is repeated for a number of applied loads. The resulting response of tangential stress versus radius for five 200-layer stacks, to each of which a 100, 70, 50 and 30 kg load is consecutively applied, is depicted in Figure 9.

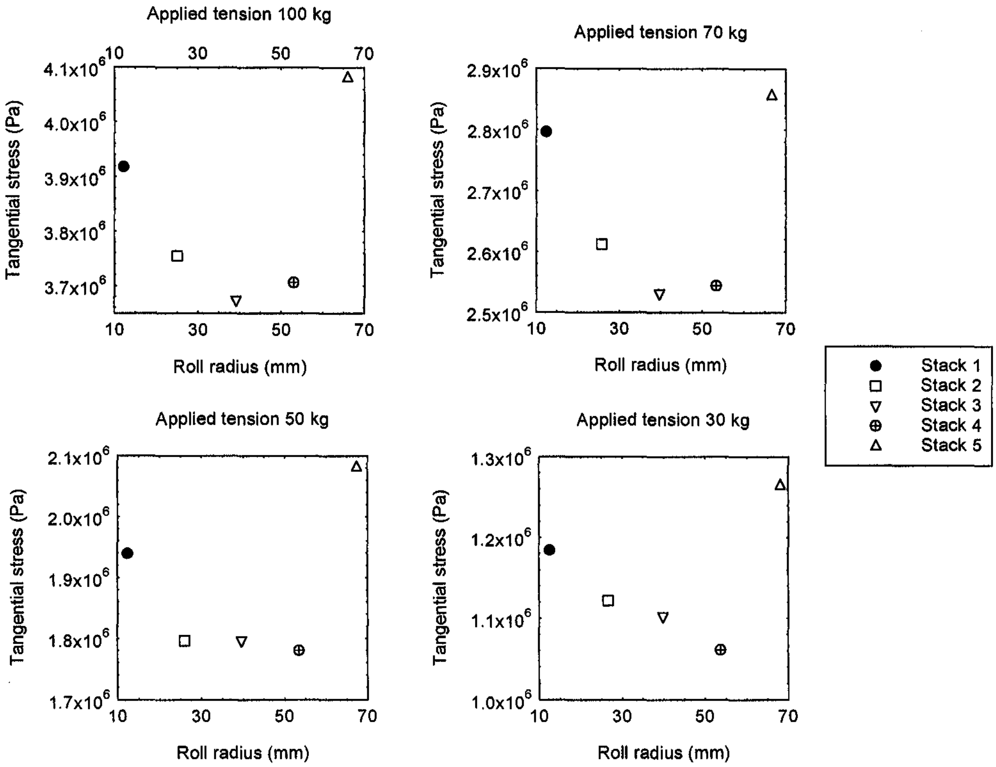

Figure 9: Tangential stress variation with respect to radius for a 5-stack set-up using the rig of Figure 7.

For an axi-symmetric roll, the relation between tangential and radial stresses, for an element in equilibrium, is:

$$\frac{dP}{dr}+\frac{P-T}{r}=0 \qquad \text{(Eq. 2)}$$

The tangential stress profile may be simulated using a finite-difference formulation as follows:

$$\left[\frac{dP}{dr}\right]i=\frac{P(i+1)-P(i-1)}{2\Delta r} \qquad \text{(Eq. 3)}$$

where *i* denotes the specific stack of layers. Re-organizing, Eq. 3 would become:

$$P(i-1) = \frac{2\Delta r}{r(i)}(T(i) - P(i)) + P(i+1) \qquad \text{(Eq. 4)}$$

The last equation is thus used recursively to compute the tangential stress or pressure profile. The two necessary boundary conditions are given by:

$P(N+1) = 0$ and

$$P(N) = \frac{\Delta r}{r(N)} T(N)$$

where $N$ is the number of stacks of layers. The strain applied to a corresponding number of stacks may be computed using:

$$\in(i) = \frac{r(i) - r_c}{hM} \qquad \text{(Eq. 5)}$$

where $r_c$ denotes the core radius, $h$ the layer thickness, and $M$ the number of layers at $r(i)$.

Figures 10 and 11 illustrate the resulting simulations for tangential stress and strain, respectively, as a function of roll radius and applied tension.

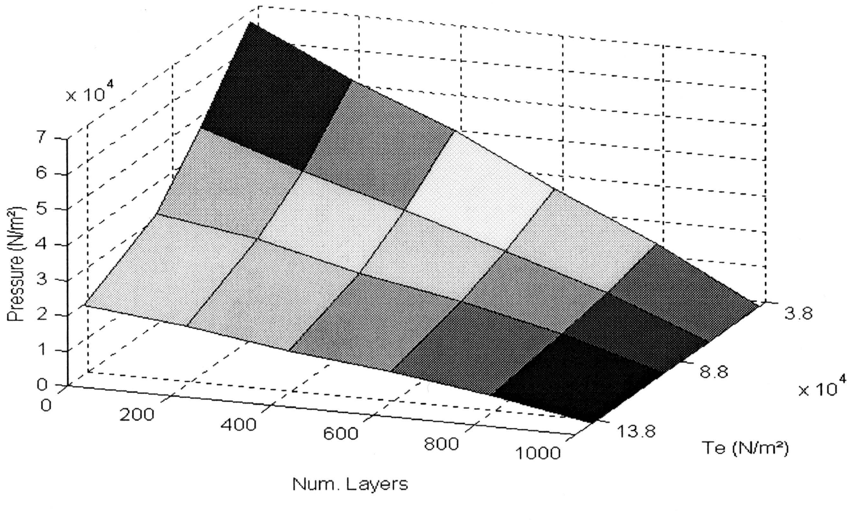

Figure 10: Finite-difference simulations of the tangential stress as a function of the number of layers and the applied tensile stress, Te.

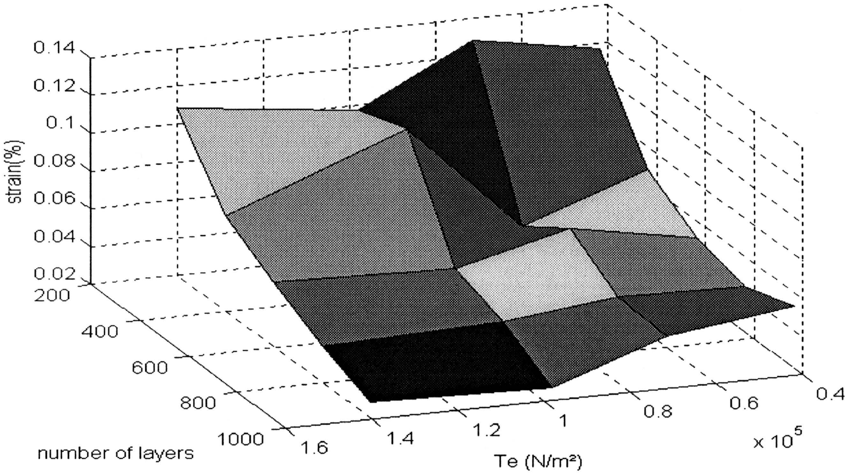

Figure 11: Finite-difference simulations of the strain as a function of the number of layers and applied tensile stress, Te.

Further insight into assessing the development of the in-roll stresses during winding may be garnered from Figure 12, which depicts the results of performing the stack experiment (in conjunction with the set-up of Figure 7) using the Cr sensor. For a specific applied tension, the first stack (near the core) experiences an increase in Cr as a result of laying other stacks on top. This increase corresponds to a direct increase in hardness, which has embedded in it the material's internal properties. As such the coefficient of restitution, Cr, is an inherent property of the roll.

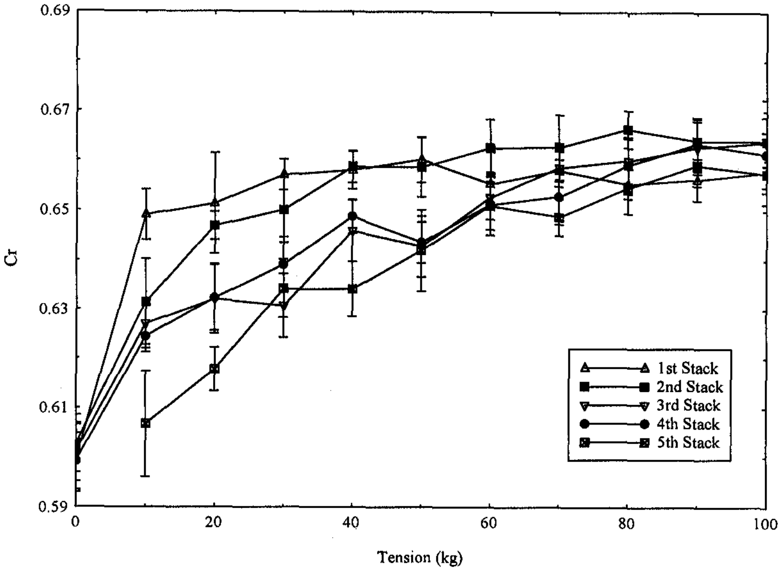

Figure 12: Variation of the coefficient of restitution with respect to applied tension.

## Conclusions

This paper has endeavored to present a comprehensive and novel insight into the mechanisms of roll deformation based on quantitative experimental data. It has been shown that the roll's coefficient of restitution, an inherent material property, ought to be considered as a primary parameter when attempting to characterize roll hardness or perform roll structural analysis.

Two principal experimental designs were used; both to simulate the cumulative addition of wraps as winding takes place and how this process influences the material response as a result. The experimental investigations were adapted to record the Cr values for linear stacks of sheets whose (overall) thickness may be varied, and for stacks adapted to a specially designed rig. The compressive stress-strain curves for the material in question (newsprint) were analyzed vis-à-vis the characteristic $K_1$, $K_2$ and $K_3$ parameters obtained from the modified Pfeiffer equation. Finite-difference simulations, based on the energy-balance approach and taking into account varying numbers of layers, were developed for predicting the radial modulus (and concomitant in-roll stresses) in wound rolls. The dependence of the $K_1$, $K_2$ and $K_3$ parameters on the number of layers (which translates to roll radius) has been discussed; the parameters, moreover, should take into account the contribution of the increase in stack height (roll radius) and their properties (surface contact, air entrapment, etc.).

## Acknowledgement

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*Structural Evaluation of Roll Quality and In-roll Stress Analysis Using a Novel On-line Measurement Technique*

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| Name & Affiliation | Question |
|---|---|
| D. Pfeiffer – JDP Innovations | Dr. Hamad, as your former Master's thesis supervisor, I would like to strongly question some of your data; particularly figure 2 with the varying heights of stacks of paper and where you're coming to the conclusion that the modulus of paper tends to get very low when the number of sheets gets very high and this is totally in deference to the way we think of things that the modulus is a paper property and not dependent on the paper height. But you're showing that when you add a large number of sheets that the slope of a curve becomes very low. When you have a small number of sheets the slope of the curve becomes very high and this is in contradiction to the definition of modulus that I'm familiar with and it is in contradiction to the results that I've seen measuring very tall stacks of paper, some as high as 18" in height. Would you comment on that? |
| **Name & Affiliation** | **Answer** |
| W. Hamad – International Paper | To answer your question; this experiment was repeated for three materials, for a large number of stack heights. The stack modulus varies because of air escaping between the layers. For small stack heights, the modulus is actually lower than for taller stacks. Above 100 sheets, it is more or less the same. We also need to remember that we're not merely testing paper, but stacks emulating, presumably, rolls. As you apply pressure, the effective compressive modulus in the radial direction actually does change as you change the stack thickness. By all means, I can provide you with all the results and would be happy for you to critique it as you wish. |
| **Name & Affiliation** | **Comment** |
| D. Pfeiffer – JDP Innovations | There may be something wrong with your definition of strain. You are not using per unit strain or common definition of strain because you are showing in the slope of a pressure curve a 5-1 change in the value of the modulus. This is like telling us the cross-grain modulus of soft pine depends on how much pine we're talking about. Whereas in the handbooks, it's always printed as a constant and similarly here, 500 sheets, 100 sheets, the modulus should be the same. |

| Name & Affiliation | Answer |
|---|---|
| W. Hamad – International Paper | You refer to pine, do you refer to the modulus of a single fiber or a sheet? |
| **Name & Affiliation** | **Question** |
| David Pfeiffer – JDP Innovations | The cross-grain modulus of a soft pine block, a piece of wood. |
| **Name & Affiliation** | **Answer** |
| W. Y. Hamad – International Paper | I don't agree with your assessment. First, it's a percent strain. Second, there is variation in the radial modulus of single fibers, or, subsequently and strictly speaking, wood chips or blocks. There are recent results from research done at STFI, Stockholm, that confirms this. I mean, it has been put forward that the value is constant; but literally speaking, it isn't. And I'll be happy to discuss that further with you if you'd like. |
| **Name & Affiliation** | **Question** |
| C. Bronkhorst – Weyerhaeuser | Perhaps you can clarify the way in which you define the initial thickness of your stacks. As you know, even for single sheets of paper, thickness is an ill-defined property. |
| **Name & Affiliation** | **Answer** |
| W. Hamad – International Paper | You're right. The average sheet thickness, or the caliper of each sheet is taken as an average of 10 readings across the surface of the sheet. This is repeated randomly for a total of 100 sheets. The total stack thickness is based on the number of sheets multiplied by an average caliper. |
| **Name & Affiliation** | **Question** |
| C. Bronkhorst – Weyerhaeuser | I have a concern with that. As you know, a sheet of paper has a high surface roughness. As you stack sheets together the emphasis of that roughness on apparent thickness will be mitigated because the surface roughness of adjoining sheets will intertwine. I therefore think it's necessary to have a way to define the thickness of your stack. Did you actually, for your stack, use that to define the initial thickness of the stack? |
| **Name & Affiliation** | **Answer** |
| W. Y. Hamad – International Paper | Yes we did. With regard to your first comment, I totally agree. The roughness will make a difference. And the degree of roughness, whether the sheet is calendared, etc., could have 5-15% difference in stack thickness. We've tried afterwards to account for that; this is not discussed here. Our work has accounted for this by actually developing an image analysis approach to measuring roughness on each sheet and then calculating an average caliper for each sheet. This was something we could do easily once we have developed the image analysis but this is not something that was discussed here. |
| **Name & Affiliation** | **Question** |
| Curt Bronkhorst – | Could you define the actual load area versus the sample |

| | |
|---|---|
| Weyerhaeuser | size area? |
| **Name & Affiliation** | **Answer** |
| W. Y. Hamad – International Paper | In this experiment? |
| **Name & Affiliation** | **Question** |
| Curt Bronkhorst – Weyerhaeuser | That's right. |
| **Name & Affiliation** | **Answer** |
| W. Y. Hamad – International Paper | We used a standard Zwick compression device, the area is 13cm x 11cm, if I'm not mistaken, and the load is applied uniformly. You have a set of two G-clamps that are depressing a rigid steel surface on top of the paper stack. Uniform pressure is applied in this way. In another experiment, we have a disk of 3mm diameter and we assume that the significance of this is minor. |
| **Name & Affiliation** | **Question** |
| R. Lucas – GL&V | I was intrigued by your coefficient of restitution test and I am trying to relate this to a practical application such as rolls of paper rolling down a ramp and hitting a roll stop or things of that sort. In such an application, you have a somewhat uniformly applied load across a roll as it hits a stop. Here you have a sort of a triaxial stress field resulting from the impulse of your bouncing piece. How did you choose this to be an indicator of a physical property? It's an unusual application of how the stresses result from an impulse and I would have expected those numbers to be, at least in my suggestion of a roll bumping into a roll stop or some practical thing like that, to be much closer to .3 instead of .6. |
| **Name & Affiliation** | **Answer** |
| W. Y. Hamad – International Paper | Why did we choose such a tool to measure a material property in the first place? The practical implications of this can be many. The main intention there is to try to obtain a measure of online hardness. Hardness can be defined different ways; people could use various definitions that are an indirect measure of a property of the material. The coefficient of restitution is a true material property. You can measure it for a specific material and that would be a correct physical measure. This is what the approach is trying to do. We are trying to use the physical principle of coefficient of restitution to measure a property that will be directly related to hardness as the roll is being wound or unwound |
| **Name & Affiliation** | **Comment** |
| R. Lucas – GL&V | You are trying to use this measurement as an indication of roll hardness. Long before Dave Pfeiffer developed the Rho meter, there were people performing a variety of bounce tests to infer wound roll tightness and they were fraught with frustration because of very unpredictable |

| | |
|---|---|
| | results. |
| **Name & Affiliation** | **Answer** |
| W. Y. Hamad –<br>International Paper | Yes, I'm familiar with what's available in the literature in that regard. I think we've produced something that yields a predictable result. There are various reasons for that in how the experiment is conducted. Some of this is detailed in our 1999 IWEB paper. |