DEFORMATION OF HIGHLY COMPRESSED WOUND ROLLS

by

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ABSTRACT

This study concerns the loss of lap tension and interlayer pressure in a wound roll due to the compression of the web. Compression of the web thickness also affects the amount of material that may be wound into a roll of a given diameter. Stress predictions are made using a new, nonlinear wound roll model developed by the Mechanics of Flexible Structures Project at the University of Rochester. Comparison to experimental data available in the literature is excellent. We find that for some materials such as polyethylene terephthalate (PET) the effect of web compressibility is relatively insignificant. For other materials, like paper, the effect is important.

INTRODUCTION

One of the complicating factors in predicting the internal stresses in a wound roll is the nonlinear nature of the interlayer compression. Initial strains are typically accompanied by slowly rising stresses which is a reflection of such inelastic phenomena as fiber crushing and air expulsion. At higher strains the stack can become quite compacted and stiff. The stress-strain law is more purely elastic, and the tangent modulus is much steeper. Sometimes the transition from one behavior to the other is gradual, and in other cases it is abrupt.

Existing wound roll models are very sophisticated in employing a nonlinear constitutive law for the interlayer compression. The best known and most widely used model is by Hakiel (1987). A second source of nonlinearity, however, has been far less studied. Specifically, there is a *geometric* nonlinearity that arises when interlayer compression is large. Depending on the material and the winding conditions, one may create strains on the order of 15%. This raises questions whether geometrically linear models can fully capture the behavior of the wound roll. A recent paper by Good, Pfeiffer and Giachetto (1993) addresses this issue.

A new model has been developed by the Mechanics of Flexible Structures Project at the University of Rochester (Benson, 1995) that fully couples both forms (constitutive and geometrical) of interlayer nonlinearity. To demonstrate the accuracy of the model we have made comparison to experimental data found in two recent papers by Good, Pfeiffer and Giachetto (1992), and Good and Xu (1993). The first paper presents interlayer pressure experiments for bond paper and newsprint. The second paper presents interlayer pressure data for polyethylene terephthalate (PET).

In the process of comparing against experiment, we will also compare the present model to the Hakiel model (1987). The Hakiel model accounts for nonlinearity in the interlayer stress-strain law, but assumes that web thickness changes are small enough to be neglected. We will see that new model and the Hakiel model give nearly identical results for the PET web, but give significantly different predictions when applied to a bond paper webs. We will assess the cause of the discrepancy (or lack of it), and give guidance as to when a model accounting for web thickness changes needs to be used.

ANALYSIS

Figure 1 shows a free-body-diagram for a segment of a lap within the roll. The interlayer pressure is P, the lap tension is T, the radius of the lap is r, and the number of the lap is i. The pressure is measured positive in compression and the tension (force per unit lap width) is measured positive in extension. Integer values of the lap number mark the centers of laps, while half-integers mark the interfaces between laps. It will be assumed that all effects are constant in the circumferential and axial directions of the roll. In other words, only radial variation will be considered. Similar to analyses of the past, the formation of the roll will be modeled as a succession of hoops being shrink-fit onto each other. A linear elastic constitutive model will be used for the lap extension, and a Pfeiffer model (1979) will be used for the interlayer compression.

Following Benson (1995), equations (1) - (7), below, govern the equilibrium of the wound roll. In what follows, a is the radius of the core, E_c is the effective elastic modulus of the core, E is the elastic modulus for stretching of the web in the running direction, T is the tension at which a lap enter onto the roll, $\bar{h}$ is the thickness of the web before being compressed, I is the number of the outermost lap, $\bar{r}$ is the radius of the outermost lap at the time when a new tensioned hoop is shrink-fit onto it, R is the "relaxation radius" to which the new hoop would like to shrink in order to relieve the tension, and α and β are constants used in the Pfeiffer constitutive model.

Radial equilibrium

$$r\frac{dP}{di} + hP + T = 0 \qquad (1)$$

Displacement compatibility

$$\frac{dr}{di} = h \qquad (2)$$

Boundary condition at the core

$$P(\tfrac{1}{2}) = E_c\left[1 - \frac{r(\tfrac{1}{2})}{a}\right] \tag{3}$$

Boundary condition at the outer lap

$$P(I+\tfrac{1}{2}) = 0 \tag{4}$$

Relaxation radius

$$R = \bar{r}\left[1 + \frac{\bar{T}}{E\bar{h}}\right]^{-1} \tag{5}$$

Tension in the lap

$$T = E\bar{h}\left[\frac{r}{R} - 1\right] \tag{6}$$

Interlayer pressure

$$P = \alpha\left[\left(\frac{\bar{h}}{h}\right)^{\beta} - 1\right] \tag{7}$$

This system of equations must be reevaluated every time a new lap is added to the roll. A quasilinear, finite-difference numerical scheme for their solution is presented by Benson (1995).

RESULTS

We will study three wound rolls defined in the Specifications Table at the end of the paper: (1) a 23.4 micron PET web, (2) an 89 micron bond paper web, and (3) a 71 micron newsprint. Figure 2 shows the interlayer stress-strain graphs corresponding to these three cases. It will be seen that the PET stack compacts after a relatively small amount of strain, and then follows a near-linear elastic constitutive curve. The two paper stacks are more compressible, and show a strong degree of nonlinearity over the entire range of interest.

PET Results

We first examine the PET web. (First column of the Specifications Table.) Specifications were chosen to match a system studied by Good and Xu (1993). Figure 3 shows the interlayer pressure. The solid curve was computed by the present model, and the dashed curve was computed by the Hakiel model. The data was taken from Good and Xu (1993). It is seen that both models correlate well with the data. Similarly, Figure 4 shows a lap tension comparison between the Benson and Hakiel models. Figure 5 shows the build-up of core pressure as the radius of the roll grows. Figure 6 shows the core pressure as a function of the winding stress. (The data of the Specifications Table is kept the same, except for the winding stress which takes on different values from 1 MPa to 5 MPa.)

From these four graphs we can see that, as applied to the PET web, there is little appreciable difference between the Benson and Hakiel models. They both predict essentially the same internal loads, and both compare well to the available experimental data. Differences are also small in the prediction of the wound-in length. For this PET example the Benson model predicts a total web length of 429 meters, and the Hakiel model predicts a length of 415 meters.

Bond Paper Results

We next examine the bond paper web. (Second column of the Specifications Table.) Specifications were chosen to match a system studied by Good, Pfeiffer and Giachetto (1992). Figure 7 shows interlayer pressure results. A significant difference now arises between the Benson and Hakiel models. The Benson model gives substantially lower pressures, particularly at the core. The correlation to the data is better in the Benson model. Here, and again later in Figures 11 and 12, this improved correlation address the principal concern of the Good, Pfeiffer and Giachetto paper, which was the discrepancy between experimental data and existing computer models.

Figure 8 compares the two models for lap tension. The Benson model predicts lower lap tensions at all radial locations. It is important to note that the outer lap tension in the Benson model is not equal to the winding tension (the winding stress times the web thickness). That is because model allows for all laps, including the outermost one, to compress inward on the roll, thus relieving some of the hoop stress. This is the effect that Good, Pfeiffer and Giachettto described as "tension loss." A small tension loss, accumulated over many added laps, is what leads to the drop in interlayer pressure seen in Figure 7. By contrast, there is a very small tension loss for the PET example in Figure 4, and a correspondingly small pressure loss in Figure 3.

Figure 9 shows the build-up of the core pressure as the roll grows larger. The asymptotic values are substantially different between the geometrically linear and nonlinear models. Respectively, the Hakiel models predicts a core pressure of about 314 kPa while the Benson model predicts a core pressure of about 187 kPa. Figure 10, for the core pressure as a function of the winding stress, exhibits a similar relationship between the two models.

Figure 11 shows additional experiments from the Good, Pfeiffer and Giachetto paper for interlayer pressure. The top curve is the same as in Figure 7, and the lower curves are for smaller winding stresses. The correlation of the present model to this data is also good.

In terms of wound-in length, the Benson model predicts a length of 233 meters, and the Hakiel model predicts a length of 210 meters. Clearly, for this bond paper example, and accounting of web compressibility is important both for the prediction of internal stresses and for predicting the amount of material in the roll.

Newsprint Results

Figure 12 and 13 both show interlayer pressure data for the newsprint example. (Third column in the Specifications Table.) Data appearing in Figure 12 is taken from Good, Pfeiffer and Giachetto (1993). The correlation between the model and the data is excellent. By a different algorithm, Good, Pfeiffer and Giachetto also made an accounting of tension loss, and also achieved excellent correlation with the data appearing (here) in Figure 12.

The Good, Pfeiffer and Giachetto model began to exhibit numerical instability at lower winding stresses. Figure 13 is included to demonstrate that the present model can deliver stable results even for very small winding stresses. Whether or not an actual roll can be wound at such small stresses is debatable. On the basis of experiments, Good, Pfeiffer and Giachetto argue that the roll begins to exhibit gross slippage near the onset of numerical instability in their model, thus rendering the stability problem moot.

CONCLUSIONS

A new wound roll model that couples nonlinear stress-strain behavior with large, nonlinear thickness changes has proven effective in matching experimental data available in the literature. For the case of the PET web, the match to experiment, while good, was no better than existing models. For the case of the bond paper web the new model gave a significant improvement in matching the data.

The basis for this can be found in Figure 2. At comparable stresses, the bond paper web experiences roughly 4 times the strain of the PET web. This is perhaps intuitive considering the fibrous composition of the paper and the dense composition of the PET. A model that uses linear kinematics would likely give acceptable answers for strains on the order of 3%, but start to break down if strains exceeded 10%. Likewise, the added complexity of a kinematically nonlinear model would be unnecessary for systems expected to be in a small strain regime, but would become increasingly more important as strains grew larger.

ACKNOWLEDGEMENT

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Hakiel, Z., 1987, "Nonlinear Model for Wound Roll Stresses," *Journal of the Technical Association of the Paper and Pulp Industries (TAPPI)*, Vol. 70, No. 5, pp. 113-117.

Pfeiffer, J.D. 1979, "Prediction of Roll Defects from Roll Structure Formulas," *Journal of the Technical Association of the Paper and Pulp Industries (TAPPI)*, Vol. 62, No. 10, pp. 83-88.

Specification	**PET**	**Bond Paper**	**Newsprint**
Core radius, a	44.5 mm	44.5 mm	44.5 mm
Outer radius, b	71.2 mm	89.0 mm	133.5 mm
Core modulus, E_c	33.1 GPa	12.4 GPa	33.1 GPa
Web thickness, $\bar{h}$	23.4 μm	89.0 μm	71.0 μm
Winding stress, $T/\bar{h}$	3.45 MPa	5.17 MPa	5.17 MPa
Web elastic modulus, E	4.15 GPa	4.13 GPa	3.37 GPa
Interlayer modulus, α	0.5181 kPa	1.10 kPa	3.70 kPa
Interlayer springiness, β	179.1	45.0	45.3

Specifications Table

EQUILIBRIUM OF LOADS

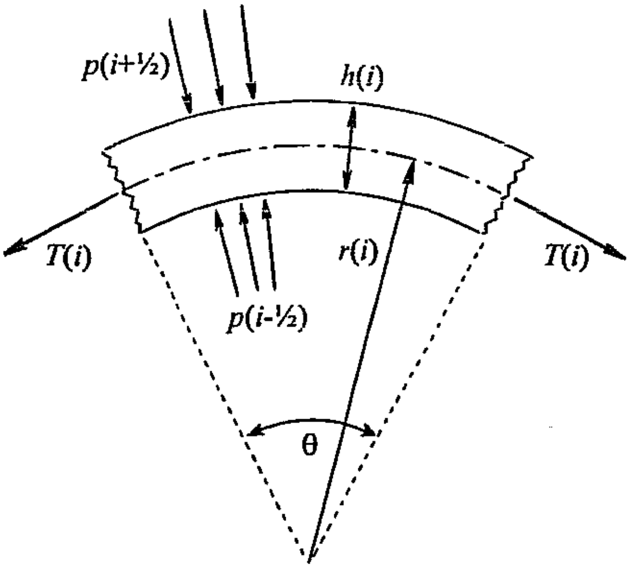

Figure 1: Geometry

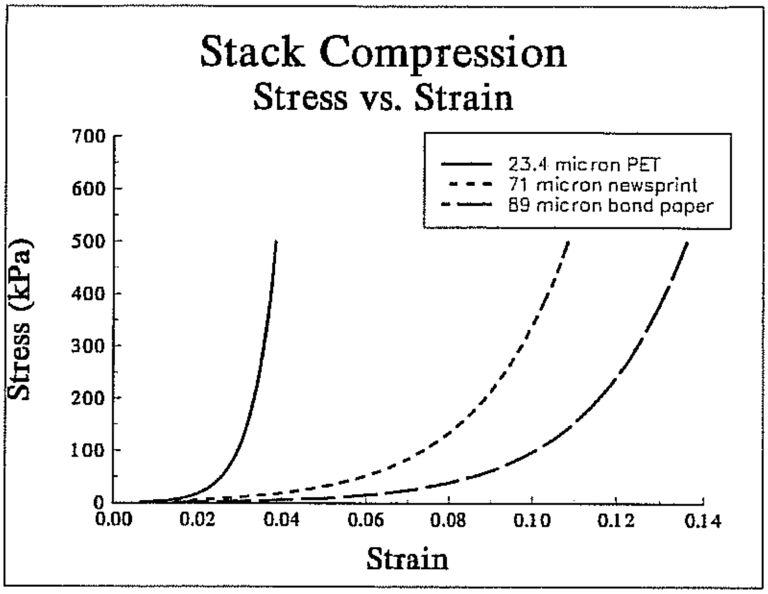

Figure 2: Interlayer Stress-Strain

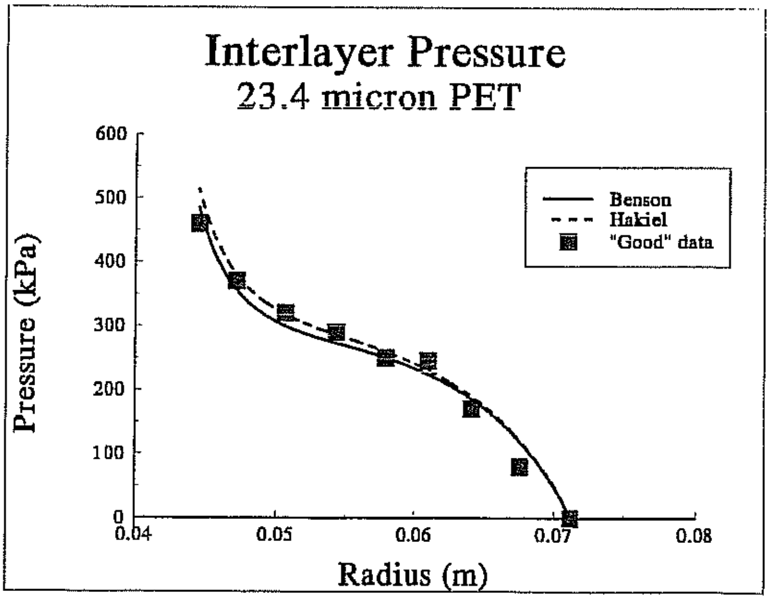

Figure 3: Interlayer Pressure, PET

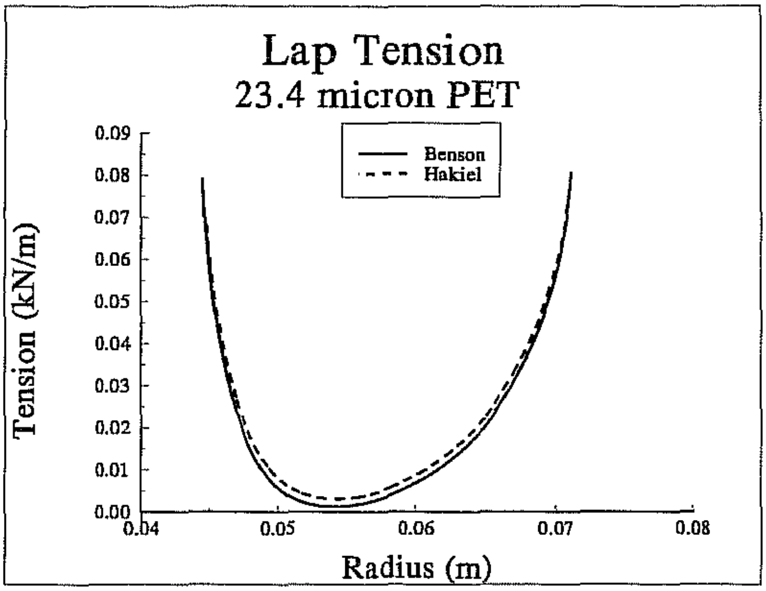

Figure 4: Lap Tension, PET

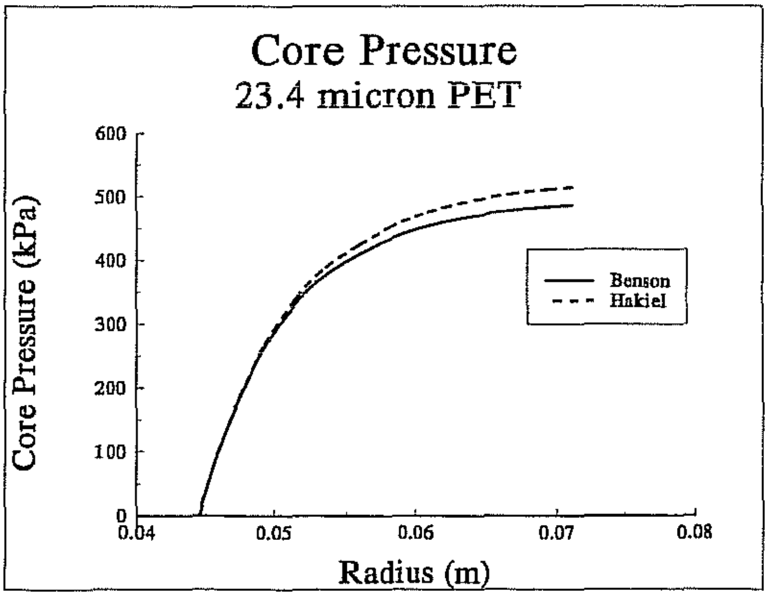

Figure 5: Core Pressure, PET

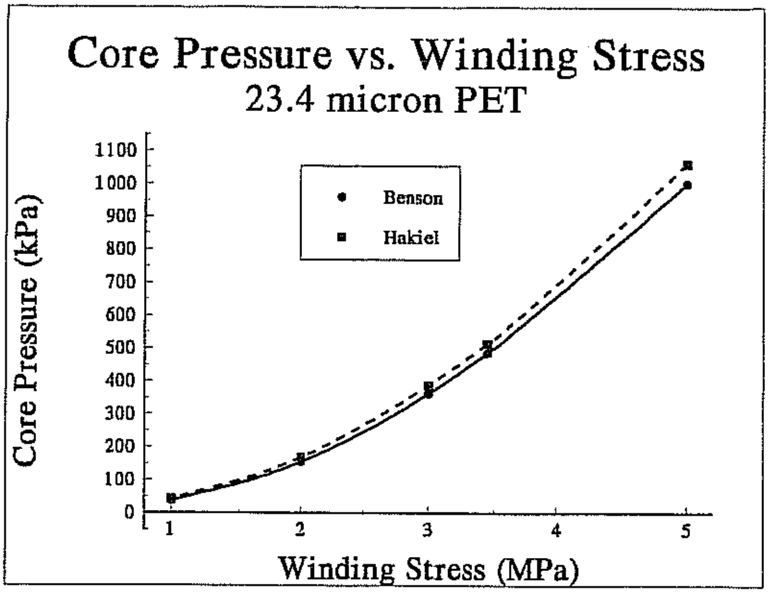

Figure 6: Core Pressure Versus Winding Stress, PET

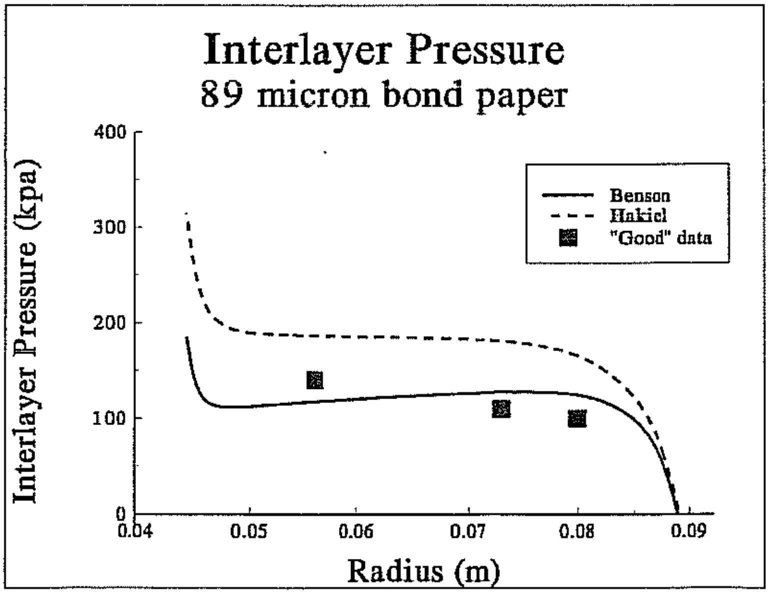

Figure 7: Interlayer Pressure, Bond Paper

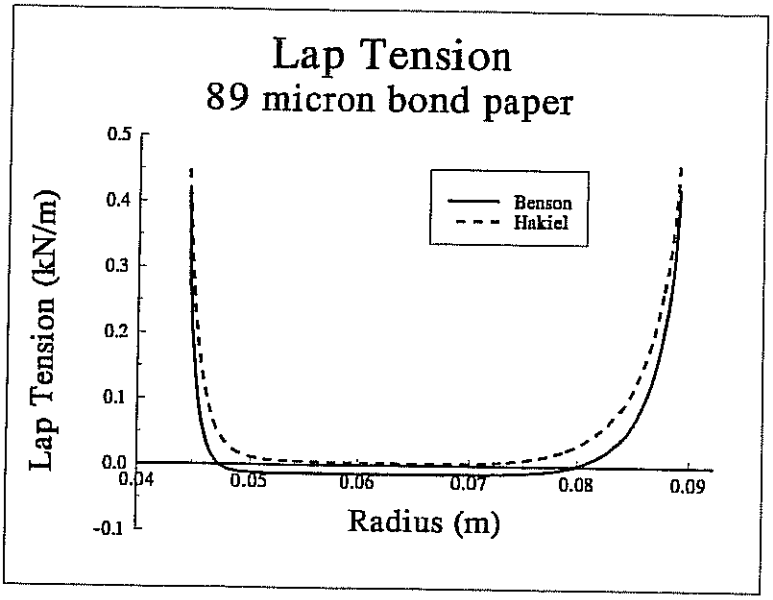

Figure 8: Lap Tension, Bond Paper

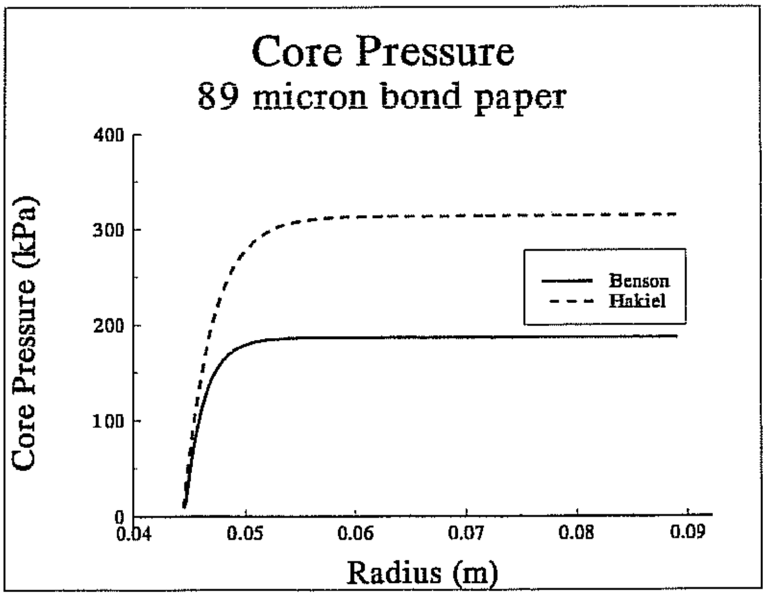

Figure 9: Core Pressure, Bond Paper

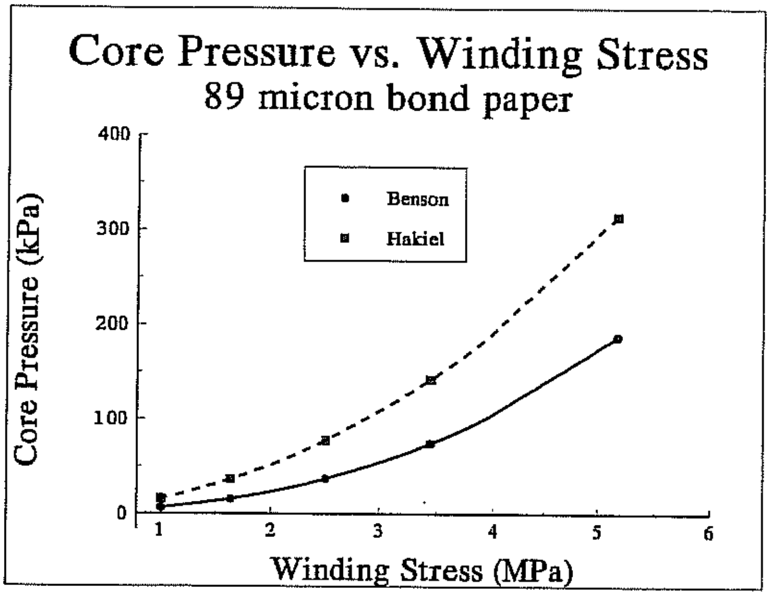

Figure 10: Core Pressure Versus Winding Stress, Bond Paper

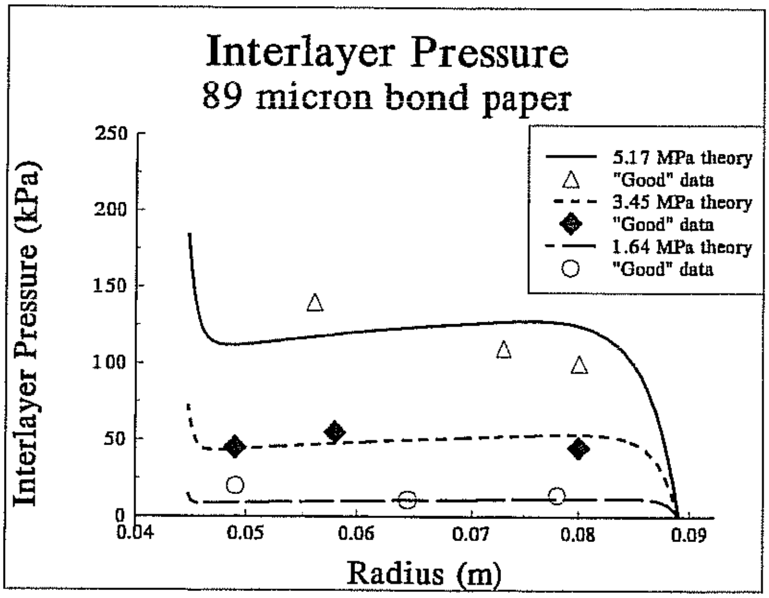

Figure 11: Interlayer Pressure (Different Winding Stresses), Bond Paper

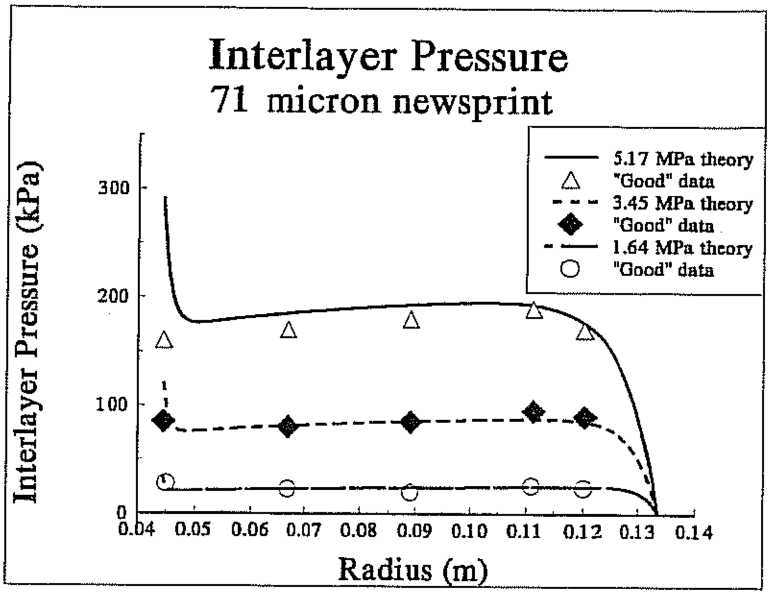

Figure 12: Interlayer Pressure (Larger Winding Stresses), Newsprint

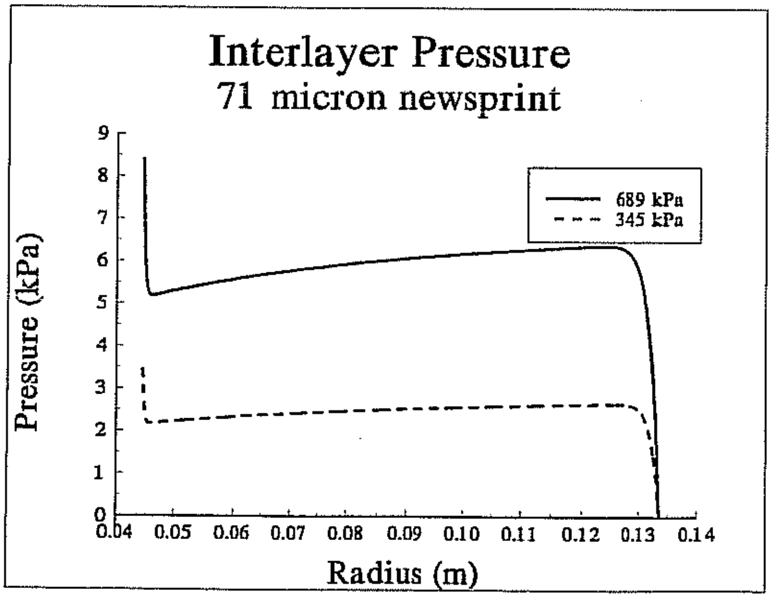

Figure 13: Interlayer Pressure (Smaller Winding Stresses), Newsprint

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Question - I was wondering, in the Hakiel Model, it is possible to modify it with the radial information under certain conditions. Have you done that?

Answer - We have not done that. We have produced an entirely new formulation. I know Keith and Dr. Pfeiffer have done and have predicted very good results accounting for the tension losses which is really just accounting for the thickness changes. They've also produced similar good results.

Comment - A point of clarification here. There were actually two models. The Pfeiffer model, as you know, is energy-based and Zig's model was based on elasticity, for instance, and they were both altered to produce a tension-loss code, if you will, and that's why you see in the literature (Losses in Wound-In-Tension in Centerwound Rolls, Applied Mechanics Division Vol. 149, ASME 1992) Pfieffer and Hakiel's models altered for tension loss due to deformation.

Question - I am coming from Comstom,Germany. More from the practical side. .have you thought about an ideal profile of winding tension in relationship to what you have found out about what kind of profile has to be done in relationship to winding and diameter, for example?

Answer - Well, the only thing that I can comment on is that our model would predict much more wound-in length than is given in the paper. When you account for the thickness changes, we would predict we would wind-in more materials for the same running case As far as an ideal tension profile, I can't really comment on that. Keith, would you like to comment on that?

Comment - What these models do for you is they give you accurate portrayals of how the radial stress rates vary throughout the wound roll. You'll see in the papers later on today , we've already heard Zig mention torque capacity for the roll. It's with these models that for the first time, we can really predict good radial pressure variations throughout the roll that you can make good predictions for what the torque capacity is and help you to figure out if these rolls are going to slip at whatever design tension.

Question - John, you're really basing the model on change in the thickness of the web as you wind it , right? The modifications that Keith has done to the Hakiel Model was to look at the change in the radius, essentially a strain-based model with a zero-pressure side of tension. Have you run your model holding H of I constant and run your model otherwise to see if it is really the thickness or if it is the change from a pressure-based to a strain-based to see if it makes a difference?

Answer - We haven't run it with H remaining constant because that's built in to the formulation. My expectation is that if we were to do that is that we would come up with exactly the same results that Zig comes up with in his original model. Does that help answer the question? The other thing I'll just add is that in the new formulation we can account for large deformations. We have defined things in terms of stretch ratios and new constitutive laws could be implemented into the model for materials like cigarette filters or rubber or things that could deform highly in the compressed role We could handle that a little bit better than in the past, because, we account for that thickness change. That's our biggest contribution.

Question - Have you measured the stress or tension in a wound roll? If yes, how did you manage it?

Answer - We did not measure the tension in a wound roll or do any experimental verifications of it. That work does exist in the literature and has been performed here at OSU by Keith Good where they have used a pull tab test to determine interlayer pressures. Once we know that our interlayer pressures in the model are correct, we can assume that the other variables which are all dependent on that, as well. If you're asking about the tensions in that model, we use a constant tension for all the cases I have shown you . In a lot of applications, people are winding at constant torque where they have a linear variation in tension throughout the winding process and all of these models are capable of handling those winding parameters.

Thank you.