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TACTivities:
Fostering Creativity Through Tactile Learning Activities

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Synopsis

As mathematics teachers, we hope our students will approach problems with a spirit of creativity. One way to both model and encourage this spirit — and, at the same time, to keep ourselves from getting bored — is through creative approaches to problem design. In this paper, we discuss *TACTivities*, mathematical activities with a tactile component, as a creative outlet for those of us who teach mathematics, and as a resource for stimulating creative thinking in our students. We use examples, such as our *derivative fridge magnets* TACTivity, to illustrate the main ideas. We emphasize that TACTivities can be engaging, to teachers and learners alike, at any level of mathematics, by including examples from different mathematics courses (calculus and mathematics for elementary teachers).

As an example, our derivative fridge magnets have moving pieces of words that look like small refrigerator magnets. These small pieces can be combined to make true mathematical statements, of the form d/dx (some function) $=$ some other function. There was creativity involved in the creation of these magnets, as the mathematics had to be challenging enough not to bore students yet have an easy entry for students to be successful. The students working with the magnets can use their creativity along with their mathematical knowledge while learning and/or reviewing a mathematical concept—in this case derivatives. We will expand on the creative side of the creation and implementation of TACTivities in this paper. Note that our definition of *tactile* only means moving pieces (usually pieces of paper), as this is different than work from others that involves tactile props such as pipe cleaners, yarn, Spirographs, building blocks, and so on. This other work is invaluable, and we use props like these ourselves at times, but we believe that our TACTivities add a different dimension to tactile learning.

Keywords: creativity, tactile learning, active learning

1. Introduction: What is a TACTivity?

In order to explain what a TACTivity is, we first explain how they came to be. Through the use of active learning in mathematics classrooms for the past 12 years, the author team has tried to make their classrooms both more engaging and more of a collaborative environment for their students. By doing this, they have found several benefits for their students, including an increased attention span in the mathematics classroom and an excitement for the learning of mathematics among students. Using Laursen and Rasmussen's [6] vision of four pillars—student collaboration and deep engagement, instructor inquiry and equitable practice—with inquiry-based learning, they are able to cover a great deal of mathematics material in a short period of time. One way they do this is by implementing activities, called TACTivities, that make learning active by design. These TACTivities were created to engage students as well as make drill-and-skill topics more minds-on and hands-on for students.

The relevance of creative thinking to mathematics and other STEM fields of study has been well documented (see, for example, [2, 3, 10]). Unfortunately, mathematics classrooms are typically considered formal, structured environments in which students learn mathematical content in a very algorithmic way. This leads students to think mathematics is about memorizing and following a recipe rather than engaging in mathematical creativity. We believe TACTivities are one way to teach students problem-solving skills versus simply memorizing formulas and patterns, thus enabling them to use mathematical creativity. And we hope that this ability to problem-solve, communicate, and collaborate will enable students to learn the interpersonal skills needed to survive future situations, be it a classroom or workplace.

The incorporation of TACTivities as a part of the mathematics classroom curriculum allows for students to see the creativity that many mathematicians witness on a regular basis. When using TACTivities, students must communicate, explain their thinking, think outside of the box, discuss and learn concepts in such a way that the conceptual understanding of mathematical ideas is developed without obstructing the learning of procedural skills. This pedagogy was coined "ambitious teaching" by Larsen, Glover, and Melhuish [5]. Ambitious teaching has also been shown to have an impact on student attitudes regarding the learning of mathematics.

TACTivities were born from consideration of how to incorporate ambitious teaching in mathematics classes. TACTivities are activities designed to be utilized in any level classroom, from preK-12 to higher education and beyond. TACTivities merge arts and science in a manner that requires students to think creatively while learning and/or practicing mathematics skills [4].

TACTivities were designed to be completed collaboratively in groups of two to four students at flat tables, but they can also be completed individually in any space. The only criterion we have in our definition of TACTivity is that there be moving pieces (often pieces of paper). TACTivities are most often completed with no written instructions, and the instructor saying very little other than helpful hints or posing directed questions to the students about their thinking process. When presented with a bag of movable pieces, students are expected to align or combine the pieces in such a way that a mathematical outcome is determined (in other words, how do these pieces fit together or align mathematically). The students self-check while moving pieces and collaborating with their group. The students are engaging with mathematics concepts via the nature of the particular TACTivity and while working, they are talking aloud to each other about mathematics. This talking provides the instructor with the knowledge of what the students are thinking as they must "think aloud" to work with other students. From this, the instructor can facilitate the situation, providing hints and tips—but not solutions—if and when students get stuck. Or, the instructor can use the time to guide students by asking them questions that will further their thinking about the mathematics [4].

TACTivities are creative in and of themselves. Because of the lack of formal instructions, students are forced to look at the movable pieces "outside the box" as there is literally no box with guidelines provided. The pieces might look familiar as a popular board game or other recognizable object, but the interaction of the pieces comes from how they "fit" together to make sense to the students. This is why the communication piece is so critical when doing TACTivities in groups. As students collaborate to try to figure out the TACTivity, they will use all knowledge, whether mathematical in nature or not, to determine the TACTivity goals [4]. Thus, one advantage of our TACTivities over props is that ours look more like "real" mathematics. If we really want to teach our students mathematics, at some point we have to get them to appreciate mathematical language, notation, and formalism. Our TACTivities do this, but in a way that can perhaps be more engaging than using pencil and paper alone to do mathematics.

Much of the literature on creativity and problem solving talks about "getting outside your comfort zone" (cf. [9, 12]). To us, this does not mean doing anything radical or silly—or particularly uncomfortable! It might just mean applying techniques not traditional to the field. In the field of mathematics, moving things around with one's hands is not a traditional learning approach. By taking a fresh approach, we engage our minds in new ways, that may prove fruitful. We believe this idea is central to what makes TACTivities valuable.

2. Examples of TACTivities

To make the above discussions more concrete, we present some examples of TACTivities that we have designed and implemented in the classroom.

2.1. Four fours

This TACTivity is a tactile take on the classic *four fours* activity [1], where students are asked to construct as many whole numbers as possible, using only basic operations and four instances of the number four. Each TACTivity set consists of three placemats, printed with the numbers to be used for forming correct equalities, and a number of cut-out operation symbols (including parentheses) to align on the placemats, and thereby produce these equalities. See Figure 1.

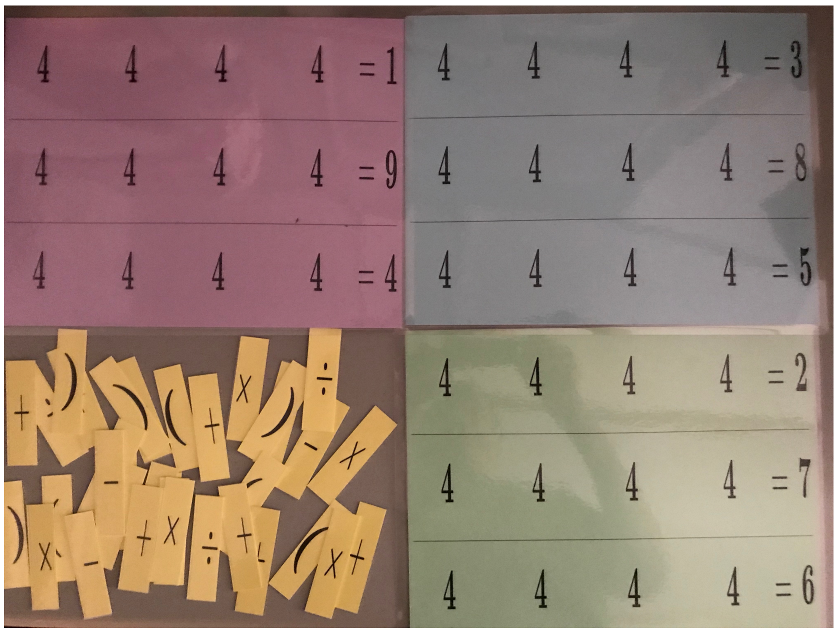

Figure 1: The four fours TACTivity.

The four fours TACTivity is ideal for students working in groups of three. It has a design feature that we often try to build into our TACTivities: it can be given to students *cold*, without any explanation or instructions from the teacher. Just give each group a set and let them figure it out!

This TACTivity is accessible to students at nearly all grade levels, and serves as a good ice-breaker, or introduction to notions of active, collaborative learning. We have found it particularly appropriate for discussing TACTivities with non-mathematical audiences. (Even for students in calculus and beyond, though, a bit of an algebra refresher is often helpful.) It is also great for mathematics for elementary school teacher courses and for professional development for teachers.

2.2. Derivative fridge magnets

This TACTivity involves a large assortment of different calculus expressions, symbols, etc., printed out individually on magnets or cards. Students are asked to work with their groups to use the magnets to construct as many correct derivative sentences as possible in the allotted time.

Figure 2 below shows a complete set of magnets, arranged by the authors into as many sentences, with as few magnets left over, as we could manage. (Extras are shown in the top and bottom left hand corners.) Of course, student solutions will vary widely.

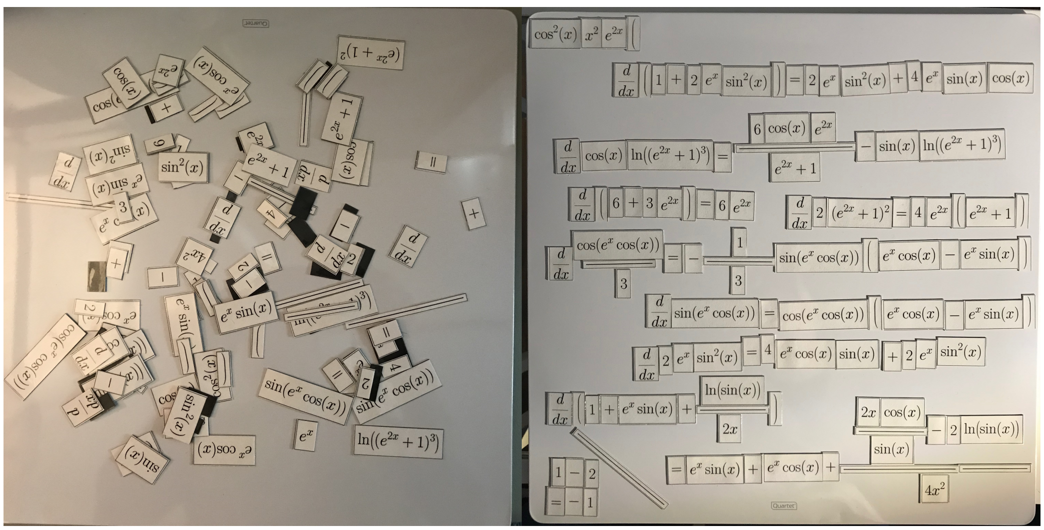

Figure 2: Derivative fridge magnets, out of the box (left) and a sample solution (right).

3. Mathematical creativity in designing TACTivities

Doing mathematics is a creative process. Perhaps more to the point, so is the designing of mathematical tasks that are intriguing, illuminating, challenging yet accessible, and within the zones of proximal development [14] of the students for whom these tasks are being designed.

For example, those who teach first semester calculus have perhaps experienced the experimentation, the trial and error, the guessing and checking, the creative problem solving that goes into designing *good* curve-sketching problems. By good, we mean: the function to be sketched has sufficiently many interesting calculus features like local extrema and inflection points, yet the coordinates of these features, and of others like x-intercepts, are not too messy, or difficult to determine, or unevenly spaced. It is something of an art to construct such functions. If one wants additional interesting features like saddle points or asymptotes, then one must exercise one's creative energies even more vigorously.

Curve sketching, as a calculus topic, has largely fallen out of favor, no doubt because of graphing calculators and other technologies that quickly generate graphs. Fortunately, opportunities for creative task design are everywhere in mathematics. This is particularly true of TACTivity design, as we now describe.

The directive to design a TACTivity is much broader than just to design a curve sketching problem (for example). Hence, there are numerous possibilities for creative inspiration in TACTivity design. Just about anything can serve as a TACTivity muse.

One of the authors of the present work was early for a TACTivity brainstorming session with another author, and stopped in a local bookstore to pass some time. From a perusal of the Toys and Games section of this bookstore by an educator preoccupied with tactile mathematics activities, the idea for the *derivative fridge magnets* TACTivity was born.

Of course, it is a long road to go from inspiration to implementation. But we are always up for a mathematical road trip. And we find the scenery particularly stimulating when TACTivities are the destination.

Because the idea of derivative fridge magnets is quite broad on the surface, there was still plenty of room for brainstorming and decision-making. What notation should be used? Which functions should be involved? What differentiation rules should be tested? What kinds of algebraic operations will arise; what kinds of operation symbols, and how many of each, will we need? How many magnets, total, is enough? What kinds of tricks can we employ to encourage creative thinking in our students? (For example: we included a couple of magnets that read $\ln((e^{2x}+1)^3)$, and one magnet that read $(e^{2x}+1)$, but none that read $(e^{2x}+1)^3$ alone. Thus, if students were to differentiate the former quantity, they would need to either (i) recognize that $\ln((e^{2x}+1)^3) = 3\ln(e^{2x}+1)$ *before* differentiating, and go from there, or (ii) simplify the quantity $(e^{2x}+1)^2/(e^{2x}+1)^3$ that arises (from the chain rule) *after* differentiating.) In the future, one could include other magnets, so that students could demonstrate alternate methods of differentiating by providing multiple solutions.

The use of tricks has been supported in the mathematics literature by Savic *et al.* [11], in their presentation of the Creativity-in-Progress Rubric (CPR) on proving. By using a conventional tool or trick, or creating a new one, the student is demonstrating creativity in their problem-solving methods. We encourage the use of tools or tricks when working with TACTivities.

Incidentally, our derivative fridge magnets were, in fact, actual magnets; we printed them on magnetic inkjet paper. This is by no means necessary; the TACTivity should work just as well when printed on heavy paper or cardstock. In fact, we encourage the field test of a TACTivity to be done on regular paper. Then one can laminate them or make them into magnets. Rather than being necessary, the magnetic feature added an extra element of fun, both for us and for our students.

The fridge magnets were well received. We used them again, later in the semester, as part of exam review. But this time we took a different tack: instead of requesting that students build as many derivative sentences as possible, we asked that each group build a single sentence that uses five different rules: the sum, constant multiple, product, quotient, and chain rules. (A sample student solution appears in Figure 3 below. The chain rule and constant multiple rule implementations here are minimal, but they are not wrong.) In this way, we were able to exercise our creativity not only in designing the overall task, but in determining multiple ways to use it.

TACTivities provide, for mathematics teachers, limitless opportunity for creative problem design. And when creative problem design fosters creative problem solving, everyone wins.

4. Creativity for students in a mathematics classroom

While much of the creativity inherent in TACTivities lies in their design, benefiting mathematicians and mathematics educators, students who are completing the TACTivities also turn on their creative side. In this section, we discuss some of the ways we have witnessed students being creative while completing TACTivities.

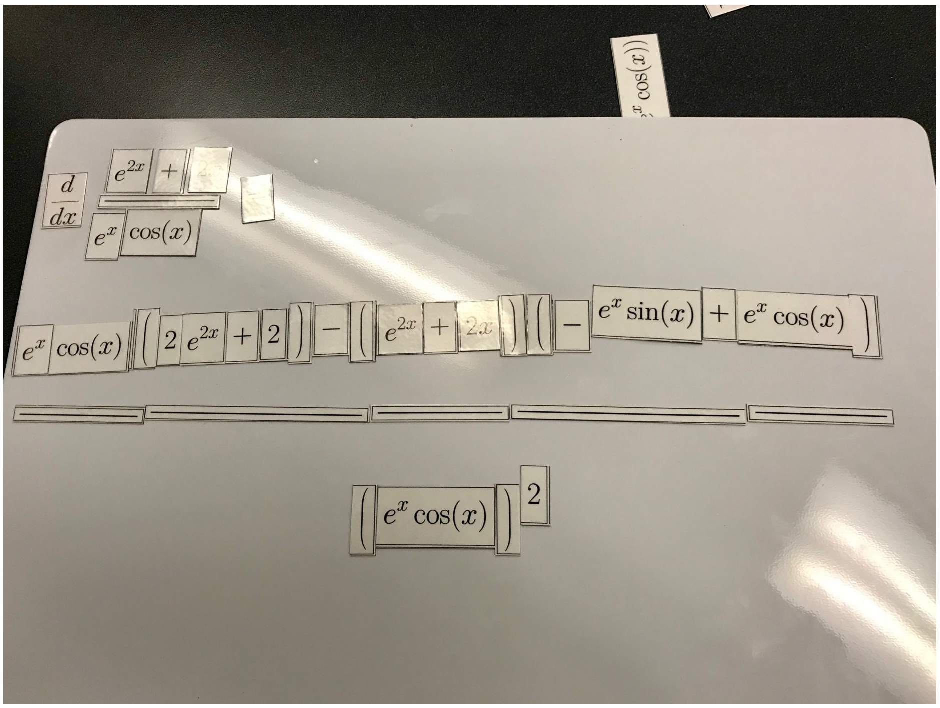

Figure 3: Student work with the "derivative fridge magnets" TACTivity. (The two magnets obscured by glare in the upper left read "$2x$" and "=" respectively.)

Many TACTivities model childhood games like dominoes or matching games. There is a lot of creativity that goes into creating those games, but there is something about playing a game that brings out the creative/playful side of a person as well. It is this playfulness that we have witnessed in students of all ages when engaged with TACTivities. Instead of being stressed about doing mathematics, students are playfully engrossed in learning creatively while completing TACTivities.

Since many of the TACTivities are passed out in plastic bags without directions, students must think creatively in determining how to arrange the pieces they are given. The first time students are provided these open-ended tasks, they may feel slightly uncomfortable. They are accustomed to following a recipe of rules in a mathematics classroom. It is not the norm in the mathematics classroom to let students be creative in how they complete a task. To help with these feelings of being uncomfortable, we encourage the instructor to tell students: "just try something." (Additionally, we find that the discomfort of approaching mathematics differently is often mitigated by the playfulness inherent in the TACTivities.) In play mode, it is expected that there will be uncertainty, risk taking [11], and time spent on thinking about what to do next. Such practices are not typically fostered in K-12 mathematics education, but as Siraman [13] has indicated, allowing for uncertainty can lead to perseverance, and thus creativity, in problem solving approaches similar to those utilized by professional mathematicians. People are more comfortable with uncertainty and creativity in play than in traditional mathematics learning. And uncertainty in K-12 mathematics can "maximize mathematical creativity," along with other principles [13, page 28].

We also encourage students to work together and talk to each other. When students are talking, we praise the noise and thank them for working hard. You could model what to do, but we think this takes away from some of the natural creativity that occurs when the students are encouraged to play with the mathematics. Unexpectedly, we also sometimes come up with ways we did not even consider in which to use the TACTivity by letting the students work without direction.

In any case, it does not take long for the students to determine a path to a solution when they see a bag on their desks/tables. After just one time doing TACTivities, students are willing to open the bag to find out what their pieces entail on that day. Some TACTivities have a similar structure, so these are easy for students to figure out and immediately turn to mathematics to solve the TACTivity. Others (fridge magnets versus four fours) require deeper thinking due to unexpected components and prompt the students to think creatively about the rules of the new game. Some TACTivities do have more structure, but many of them encourage the creativity of playing a game in which the students must determine the rules.

Or they can make up their own rules! We have seen this, for example, with the fridge magnets TACTivity. Student used whole-number magnets as exponents, where we had only intended their use as multipliers. One group needed an extra x, so they rotated a plus sign forty-five degrees. Another group used a minus sign as a fraction bar.

By providing a novel, unfamiliar way of doing mathematics, TACTivities engage students' minds in different ways, and thereby encourage creativity in their problem-solving approaches.

5. Implementation

Many university calculus courses used to feature, and many still do feature, one-day-a-week *recitations*. These recitations often entail an instructor, or graduate teaching assistant, standing by the blackboard and answering questions. Many, if not all, of these questions will be of the "How do you do homework problem x in section y?" variety. Attendance at these recitations, unless required (and even sometimes when required), can be spotty.

At our institutions, we have replaced recitations with what we call *tutorials*, or *project days*, which entail student work in small groups instead of the kind of Q&A described above. We use TACTivities for much of this small group work. Attendance is required, and essentially constitutes the student's grade for the work. That is, students are given full credit if they show up and participate actively in the problem solving. It then becomes the instructor's, or the teaching assistant's, job to facilitate the interactions, to guide the students in their discovery (rather than just divulging answers) where necessary, to make sure everyone is engaged, and to see that, as much as possible, each group completes and understands the assignment.

Students and facilitators have found this model to be more productive and enjoyable than the old one. It does have a few disadvantages: it means that the instructor for lecture days, if different from the one who leads the tutorials, misses out on the TACTivity fun; and of course, it doesn't work in courses that do not have designated recitation days. Both of these issues can be mitigated, though, by the implementation of shorter TACTivities during what would otherwise be lecture periods. These can be done instead of working through examples at the board, for instance.

Still, setting aside time for something new will always mean less time for something old. We view this fact as an opportunity to reconsider which old material is really essential. In our experiences, the need to make room for TACTivities in our syllabi has, in fact, helped us streamline our courses.

6. Conclusion

Creativity is not often thought of as a tool for teaching in a traditional mathematics classroom, but its addition will spark the curiosity and engagement of both students and instructors. As many have been stating in recent mathematical teaching and learning conversations, we do not have to keep teaching in the traditional way. We can reinvent teaching and learning and the foundations of how and why people learn. As educators, we should be innovative in our reimagination of classrooms. Obviously, we want our students to succeed and we believe creativity is one aspect that will help that success. In an era of computer simulated activities, sometimes you should step back and put activities into the students' hands, literally, thus engaging them and reminding them that their own creativity can lead them to mathematical solutions when problem solving.

In recent years, several TACTivities have been utilized in Calculus I and II classrooms by one of the authors. Not only has the use of these tools provided opportunities for students to engage more fully in the class, but the impact on student learning has been evidenced. Students in different calculus sections were tracked as they enrolled in further mathematics courses. It was found that the students who were in classes taught by the instructor who used active learning (including the use of TACTivities) performed better than other students in successive math courses. Although the immediate cause and effect of the TACTivities cannot be identified, it is clear that students in these courses learned the necessary material and were well-equipped to move into further courses. Further details on the construction, implementation, and success of TACTivities can be found in [4].

Although the examples of creativity we provided were from mathematicians/mathematics educators and mathematics students, these TACTivities have also been used in mathematics methods courses and in professional development workshops for teachers. In these cases, teachers/future teachers attempt a TACTivity or two. They are then asked to come up with their own TACTivity. We essentially move them from the role of the student learning/doing mathematics to the role of mathematician creating the mathematical activities. We also encourage these audiences to think up new mathematical games or puzzles and share with others. It is in these creative work sessions that new TACTivities are created. Hence, the possibilities for mathematical creativity are endless with TACTivities. Commonly, we find ourselves doing daily activities and unexpectedly a new TACTivity is inspired. Whether it is perusing a bookstore, looking at manipulatives for other educational purposes, game night with the family, or flying across the country, TACTivities can emerge from any number of places and purposes. With a little creativity and modification, you have a new mathematics TACTivity.

We hope we have clearly described some of the ways in which our TACTivities have inspired us in the teaching of mathematics, and have sparked your creative energies. We encourage you all to create some of your own and share with others (including us)!

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