Nuclear models on a lattice

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1. INTRODUCTION

We present the first results of a quantum field approach to nuclear interaction models obtained by lattice techniques. Since Yukawa pioneer work [1], these interactions are all based on one-boson exchange (OBE) Lagrangians. They constitute the starting point for building the NN potentials [2,3,4] which, inserted in Schrodinger-like equations, provides an "ab-initio" description of light nuclei up to $A \sim 10$ [5].

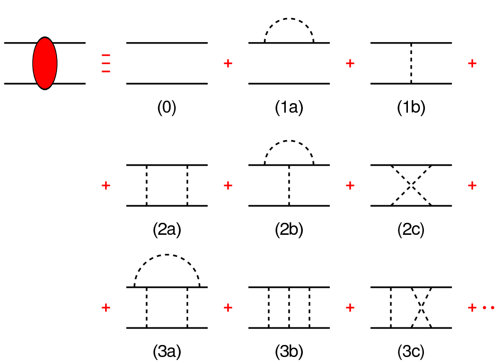

Figure 1. Perturbative expansion of the NN amplitude: ladder approximation corresponds to the (1b), (2a), (3b), . . . terms.

The potential approach takes however into account only a small, though infinite, fraction of diagrams of the perturbative series – the ladder sum displayed in figure 1. This represents a severe restriction of the interaction, specially taking into account the large values of the coupling constants involved. Chiral inspired NN models [6,7,8], which can be formally distinguisehd from the OBE ones, suffer from the same restrictions.

Our aim is to incorporate the full content of the OBE Lagrangians as it follows from a Quantum Field Theory (QFT) treatement of the interaction. Lattice techniques[10], based on a discrete Feynman path integral formulation of QFT, provide nowadays a genuine way to solve non perturbatively such a problem. A similar study was undertaken in [9] in the frame of a purely scalar $\phi^2\chi$ model.

The interest of this approach is manifold. On one hand it allows a comparison – coupling by coupling – with the results of the ladder approximation in different potential models. On the other hand it could provide a relativistic description of nuclear ground states in terms of the traditional degrees of freedom – mesons and nucleons – with no other restriction than those arising from the structureless character they are assumed to have. Of particular interest is to investigate the possibility of obtaining the effect due to the exchange of heavy mesons – e.g. σ, ρ – in terms of interacting pions alone.

In this contribution we will focus on the renormalization effects for fermion mass and coupling constant in case of scalar (S) and pseudoscalar (Ps) interaction lagrangian densities

$$\mathcal{L}(x) = g_0 \bar{\Psi}(x)\Gamma\Phi(x)\Psi(x) \qquad \Gamma = 1, i\gamma_5 \qquad (1)$$

driven by the bare coupling constant g_0. The scalar coupling with an additional $\lambda\phi^4$ term – Yukawa model – has been investigated in the framework of the Higgs mechanism [11].

2. THE MODEL

The Quantum Field Theory is solved in a discrete space-time lattice of volume $V = L^4$ and lattice spacing a. All dimensional quantities are redefined in terms of a which disappears from the formalism. Its value can be determined only after identifying an arbitrary computed mass to its physical value.

In terms of the dimensionless variables, the euclidean partition functions is given by

$$Z = \int [d\bar{\psi}][d\psi][d\phi] \exp\{-S_{KG} + S_F + S_{int}\}$$

where

$$S_{KG} = \frac{1}{2} \sum_x \left\{ m_0^2 \phi_x^2 + \sum_\mu (\phi_{x+\mu} - \phi_x)^2 \right\}$$

describes a real meson field with bare mass m_0. The fermionic and meson-fermion coupling action is written in the form

$$S_F + S_{int} = \sum_{xy} \bar{\psi}_x D_{xy} \psi_y$$

in which

$$\begin{aligned} D_{xy} &= (1 + g\Gamma\phi_x)\delta_{xy} \\ &- \kappa \sum_{\mu=1}^{4} (1-\gamma_\mu)\delta_{x,y-\hat{\mu}} + (1+\gamma_\mu)\delta_{x,y+\hat{\mu}} \end{aligned}$$

is the Dirac-Wilson operator.

The model depends on 3 parameters: the "hopping" parameter κ, related to the fermion bare mass M_0 by

$$\kappa = \frac{1}{2M_0 + 8} ,$$

the lattice coupling constant g related to the interaction Lagrangian (1) by

$$g_0 = \frac{g}{2\kappa} \tag{2}$$

and the bare meson mass m_0.

Our first task is to investigate how the parameter set (M_0, g_0, m_0) – or alternatively (κ, g, m_0) – maps into the renormalized values (M_R, g_R, m_R). This task is considerably simplified in the "quenched" approximation, which consists in neglecting all virtual nucleon-antinucleon pairs originated from the meson field $\phi \to \bar{\psi}\psi$. Because of the heaviness of the nucleon mass this is a good approximation for the problem at hand and has been adopted all along this work. In this case, the meson field is trivially renormalized and one then has $m_R = m_0$.

3. RENORMALIZED FERMION MASS

In lattice calculations, the fermion propagator

$$G_{\alpha\beta}(x-y) = < 0|\psi_\alpha(x)\bar{\psi}_\beta(y)|0 >$$

is obtained by averaging over all the meson field configurations – generated by Montecarlo techniques – the inverse of the Dirac operator

$$G_{\alpha\beta}(x-y) = \frac{1}{N_\phi} \sum_\phi D^{-1}_{\alpha x,\beta y}[\phi]$$

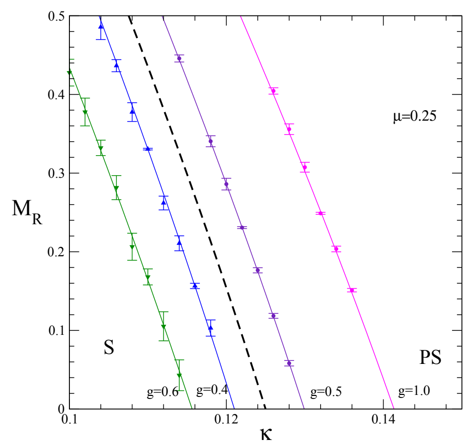

Figure 2. $M_R(\kappa)$ dependence for several values of scalar and pseudoscalar coupling. Dashed line corresponds to the free ($g = 0$) case.

The renormalized fermion mass M_R is extracted from the time-slice correlator

$$C_{\alpha\beta}(t) = \sum_{\vec{x}} G_{\alpha\beta}(x).$$

The trajectories $M_R(\kappa)$ are displayed in figure 2 for several values of g and $m_0 = 0.25$. They are monotonous functions of κ and vanish at the critical values $\kappa_c(g, m_0)$. In the region $M_R << 1$ they are well fitted with

$$M_R(\kappa, g, m_0) = \frac{Z_m(g, m_0)}{2} \left[\frac{1}{\kappa} - \frac{1}{\kappa_c(g, m_0)} \right] \tag{3}$$

Coefficients $Z_m(g, m_0)$ and $\kappa_c(g, m_0)$ can be calculated in perturbation theory and provide a test of numerical calculations. The dotted line corresponds to the free ($g = 0$) case for which $Z_m = 1$ and $\kappa_c^0 \equiv \kappa_c(g = 0) = 1/8$.

As one can see, S and Ps trajectories lie in both sides of the $g = 0$ one with respectively $\kappa_c < \kappa_c^0$ and $\kappa_c > \kappa_c^0$. Keeping the leading order $Z_m \approx 1$ in (3) one has

$$M_R - M_0 = \frac{1}{2}\left(\frac{1}{\kappa_c^0} - \frac{1}{\kappa_c}\right)$$

This indicates that the renormalized nucleon mass is made lighter by a scalar coupling and heavier by a pseudoscalar one.

The parameter space of physical interest is determined by the region $M_R(\kappa, g, m_0) > 0$, i.e. the values $\kappa \in [0, \kappa_c(g, m_0)]$. This region is represented in figure 3 for S and Ps couplings and a fixed meson mass $m_0 = 0.25$. Dotted lines denote the perturbative results to g^2 order. Notice that they are symmetric with respect to $k_c = 1/8$.

In the scalar case the parameter space is a compact domain limited by a critical lattice coupling constant $g_s^c \lesssim 1$. The precise determination of the $g_s^c \simeq 1$ value is made difficult by the appearence of negative eigenvalues in the Dirac-Wilson operator. They start appearing for $g \simeq 0.6$ and provoke the failure of all the algorithms we used in its inversion.

On the contrary the Ps coupling has a large parameter space, in principle infinite. The spectral properties of the corresponding Dirac-Wilson operator are very different and the negative eigenvalues disappear for large enough g values.

Notice that g_0 is related to g by (2) and has sensibly larger values than those appearing in figure 3, specially for the S coupling where $\kappa < \kappa_c^0 = 0.125$. On the other hand these values are not yet renormalizaed and have no physical content.

4. RENORMALIZED COUPLING

We have used the MOM renormalization scheme [12] where the renormalized coupling constant at a scale μ is defined as

$$g_R(\mu) = \frac{G^{(3)}_{R,\mu}(p^2 = \mu^2)}{S_{R,\mu}(p^2 = \mu^2)\; S_{R,\mu}(p^2 = \mu^2)\; \Delta_{R,\mu}(0)}.$$

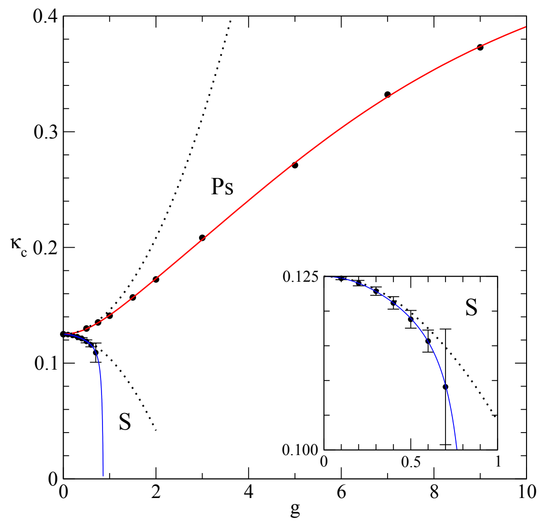

Figure 3. Lattice parameter space for S and Ps couplings. In the S case – detailed in the zoom – there is a critical coupling constant $g_s^c \simeq 1$.

$S_{R,\mu}(p)$ and $\Delta_{R,\mu}(p)$ are respectively the fermion and meson renormalized propagators and $G^{(3)}_{R,\mu}(p)$ the 3-point Green function represented in Figure 4 and defined as

$$G^{(3)}_{R,\mu}(p) = Z_\psi(\mu) Z_\phi^{1/2}(\mu) < \psi(p)\tilde{\phi}(0)\bar{\psi}(-p) >$$

with Z_ψ the fermion field renormalization constant. In the quenched approximation one has $Z_\phi = 1$

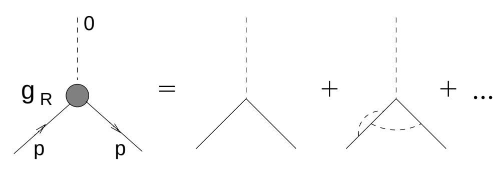

Figure 4. Renormalized coupling constant

Our results concern the momentum dependence of the coupling constant and the ratio g_R/g_0. The first point is illustrated in Figure 5 for the scalar coupling. No structure is seen up to $\frac{\mu}{m_0} \sim 12$, corresponding to $p \sim 6$ GeV, although the "triviality" of the theory imposes the existence of a Landau pole at very large momenta. This result was found to be independent of the bare coupling g_0 and the mass ratio M_R/m_0. The same behaviour has been observed with Ps coupling for moderate g_0 values.

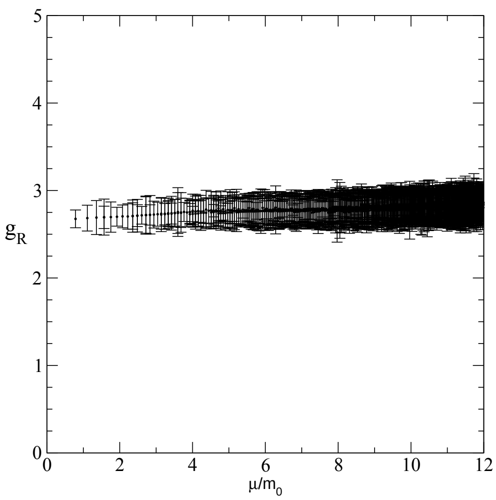

Figure 5. Momentum dependence of the scalar coupling constant obtained with $M_R/m_0 = 2$.

As the renormalized coupling constants exhibit very small dependence on the momentum, we have computed g_R as a function of g_0 at $p = 0$. Results corresponding to $m_0 = 0.25$ are displayed in Figures 6 and 7. Here again both couplings

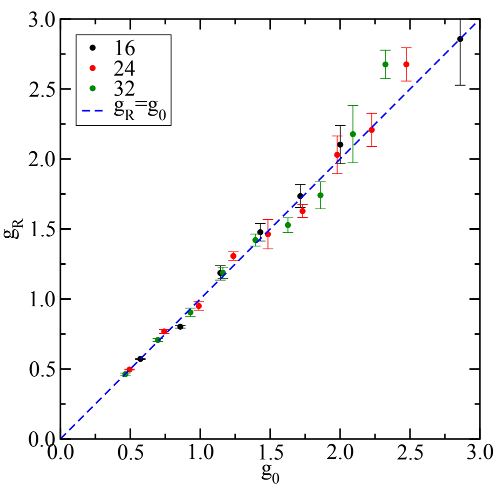

Figure 6. Renormalized coupling constant versus bare values g_0 for the scalar case.

manifest very different behaviours. While in the S case one has $g_R = g_0$ in all the accesible range ($g_0 = \frac{g}{2\kappa} \lesssim 3$), the pseudoscalar coupling constant g_R strongly deviates from its bare value. The different scales on both figures are justified by the effective strength of these interactions. In

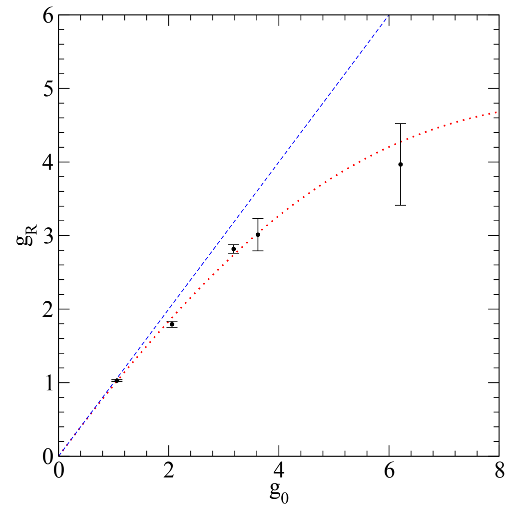

Figure 7. Renormalized coupling constant versus bare values g_0 for the pseudoscalar.

the non relativistic limit, they lead to the same potential provided one has $g_s \equiv \frac{1}{4}\left(\frac{m_0}{M}\right)^2 g_{ps}$.

In conclusion, we have obtained the physical parameters (M_R, g_R) of the OBE – scalar and pseudoscalar models – in terms of the bare quantities appearing in the Lagrangians. Work is in progress to obtain the dynamical properties like binding energies of multifermion systems.

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