Lattice measurement of α_s with a realistic charm quark

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Abstract

We report on an estimate of α_s , renormalised in the \overline{MS} scheme at the τ and Z^0 mass scales, by means of lattice QCD. Our major improvement compared to previous lattice calculations is that, for the first time, no perturbative treatment at the charm threshold has been required since we have used statistical samples of gluon fields built by incorporating the vacuum polarisation effects of u/d, s and c sea quarks. Extracting α_s in the Taylor scheme from the lattice measurement of the ghost-ghost-gluon vertex, we obtain $\alpha_s^{\overline{MS}}(m_Z^2) = 0.1200(14)$ and $\alpha_s^{\overline{MS}}(m_T^2) = 0.339(13)$.

1. Introduction

The recent announcement by ATLAS and CMS of their observation at 5 σ significance of a new particle with a mass around 125 GeV [1], interpreted as the Brout-Englert-Higgs (BEH) boson, makes even more crucial than before a satisfying control on theoretical inputs of analytical expression of the Higgs decay channels. Indeed, the era of precise Higgs physics (measurement of the couplings, ...) will certainly open soon: assessing the sensitivity of forthcoming detectors will be a key ingredient. There are different modes of Higgs boson production: however the gluon-gluon fusion is by far the dominant process, as shown in Fig. 1. Over the uncertainty $\Delta \sigma_{gg \to H \to X}^{\text{th}}$ of 20 - 25 % claimed at LHC $(\sqrt{s} = 7 \text{ TeV})$, about 4 % come from the uncertainty $\delta \alpha_s$ on $\alpha_s(m_{70}^2)$ [2]. A complementary approach of the α_s measurement from the analysis of Deep Inelastic Scattering data, physics of jets, τ decay and $e^+e^- \rightarrow$ hadrons [3] is its computation by numerical simulations. In the following section we will report on the work performed by the ETM Collaboration to measure α_s from $N_f = 2 + 1 + 1$ gauge configurations [4].



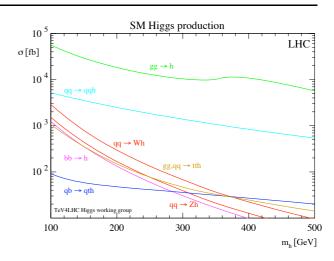


Figure 1: Prediction of the Standard Model BEH boson production in function of its mass for different production channels at LHC $\,$

2. α_s from numerical simulations

In the past years tremendous progresses have been made by the lattice community to perform simulations that are closer to the physical point. It means including more and more quark species in the sea, u/d quarks ($N_f = 2$), then the strange ($N_f = 2 + 1$) and even the charm ($N_f = 2 + 1 + 1$) since a couple of years. Pion masses $\sim 250 \text{ MeV}$ are now common and several collaborations are even able to simulate a real pion, either in a small volume (PACS-CS Collaboration) [5] or using a quark

regularisation with a rather aggressive cut-off of the UV regime (BMW Collaboration) [6]. Discretisation errors are kept under control by considering lattice spacings a smaller than 0.1 fm and lattice extensions L are such that $Lm_{\pi} \gtrsim 3.5$ to get rid of finite size effects. We have collected in Fig. 2 the simulation points performed by the lattice community using different quark and gluon regularisations.

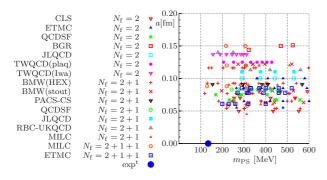


Figure 2: Simulation points obtained by the lattice community in the plane (a, m_π)

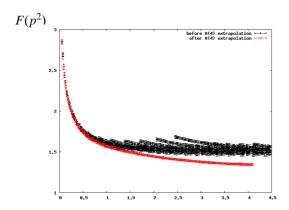


Figure 3: Raw and cured data of the ghost propagator dressing function

The more vacuum polarisation effects are incorporated in the Monte-Carlo sample, the more reliable any result on α_s is. Several methods are proposed in the literature to extract it: for instance analysing the static quark potential V(r) at short distance [7], comparing the moments of charmonium 2-pt correlation function with a perturbative formula after an extrapolation to the continuum limit [8], integrating the β function at discrete points in a finite volume renormalisation scheme, for instance in the Schrödinger Functional scheme [9] or fitting 3-gluons amputated Green functions in the framework of Operator Product Expansion (OPE) [10]. A last and particularly elegant approach consists in applying the OPE formulae to the ghost-ghost-gluon amputated

Green function [11], that we will discuss in more details.

The starting point is to consider the bare gluon and ghost propagators in Landau gauge:

$$(G^{(2)})_{\mu\nu}^{ab}(p^2, \Lambda) = \frac{G(p^2, \Lambda)}{p^2} \delta_{ab} \left(\delta_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{p^2} \right),$$

$$(F^{(2)})_{\mu\nu}^{ab}(p^2, \Lambda) = -\delta_{ab} \frac{F(p^2, \Lambda)}{p^2}.$$

Choosing a MOM scheme, the renormalised dressing functions G_R and F_R , defined by

$$\begin{split} G_R(p^2,\mu^2) &= \lim_{\Lambda\to\infty} Z_3^{-1}(\mu^2,\Lambda)\,G(p^2,\Lambda)\,,\\ F_R(p^2,\mu^2) &= \lim_{\Lambda\to\infty} \widetilde{Z}_3^{-1}(\mu^2,\Lambda)\,F(p^2,\Lambda)\,, \end{split}$$

read $G_R(\mu^2, \mu^2) = F_R(\mu^2, \mu^2) = 1$. The amputated ghost-gluon vertex is given by

The renormalised vertex is $\widetilde{\Gamma}_R = \widetilde{Z}_1 \Gamma$; with a MOM prescription it reads

$$\lim_{\Lambda \to \infty} \widetilde{Z}_1(\mu^2, \Lambda) \left(H_1(q, k; \Lambda) + H_2(q, k; \Lambda) \right) \Big|_{q^2 = \mu^2} = 1.$$

The renormalised strong coupling constant is given by $g_R(\mu^2) = \lim_{\Lambda \to \infty} g_0(\Lambda) \frac{Z_3^{1/2}(\mu^2, \Lambda)\widetilde{Z}_3(\mu^2, \Lambda)}{\widetilde{Z}_1(\mu^2, \Lambda)}$. In the case of a zero incoming ghost momentum k = 0, we are in a kinematical configuration where the non renormalisation theorem by Taylor [12] applies: $H_1(q, 0; \Lambda) + H_2(q, 0; \Lambda) = 1$ and then $\widetilde{Z}_1(\mu^2, \Lambda) = 1$. The renormalised coupling in the Taylor scheme reads finally

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda \to \infty} \frac{g_0^2(\Lambda)}{4\pi} G(\mu^2, \Lambda) F^2(\mu^2, \Lambda).$$

The main advantage of the MOM Taylor scheme is that there is no need to compute any 3-pt correlation function: it is enough to extract the dressing functions of gluon and ghost propagators.

We have analysed the $N_f = 2+1+1$ ensembles produced by the ETM Collaboration [13], with bare couplings $\beta = 2.1$, 1.95 and 1.9 that correspond to $a_{\beta=2.1} \sim 0.06$ fm, $a_{\beta=1.95} \sim 0.08$ fm and $a_{\beta=1.9} \sim 0.09$ fm, respectively. Pion masses are in the rang [250-325] MeV. Landau gauge is obtained by standard methods to minimise $A^{\mu}A_{\mu}$ [14] while the ghost propagator is computed by

inverting the discretised Faddeev-Popov operator. However, as the O(4) symmetry is broken on the lattice to the H(4) group, getting α_T from the dressing functions G and F is not straightforward: $O(a^2p^2)$ and H(4) invariants artifacts, the so-called hypercubic artifacts, have to be properly taken into account [15]:

$$\begin{array}{lcl} \alpha_T^{\rm Latt} \left(a^2 p^2, a^2 \frac{p^{[4]}}{p^2}, \ldots \right) & = & \widehat{\alpha}_T (a^2 p^2) \\ & + & \left. \frac{\partial \alpha_T^{\rm Latt}}{\partial \left(a^2 \frac{p^{[4]}}{p^2} \right)} \right|_{a^2 \frac{p^{[4]}}{p^2} = 0} a^2 \frac{p^{[4]}}{p^2} \end{array}$$

 $p^{[4]} = \sum_i p_i^4$. We have shown in Fig. 3 that a "fishbone" structure, that are those hypercubic artifacts, is clearly present in F but curable, as also seen on the plot. The remaining cut-off effects are removed by

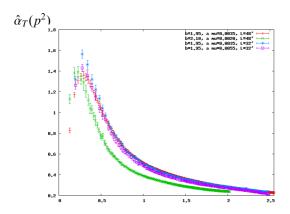


Figure 4: Strong coupling constant obtained after the elimination of the dominant hypercubic artifacts

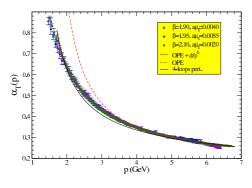


Figure 5: Raw data of $\alpha_T(p^2)$ compared to a purely perturbative running and OPE with power corrections

fitting $\hat{\alpha}_T(p^2)$ according to the formula $\widehat{\alpha}_T(a^2p^2) = \alpha_T(p^2) + c_{a2p2} a^2p^2 + O(a^4)$. Fig. 4 illustrates the benefit, in term of statistical error on α_T , to do the calculation in Taylor scheme, as we pointed earlier in the

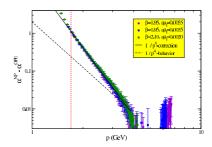


Figure 6: Subtraction of $\alpha_T(p^2)$ from the perturbative running compared to generic power corrections.

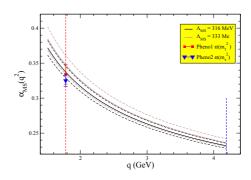


Figure 7: Comparison of $\alpha_s(m_\tau)$ measured on the lattice with estimates based on phenomenological analyses of τ decay data [18], [17].

text

We then use the OPE formalism so that we can relate $\alpha_T(p^2)$ to $\alpha_T^{\text{pert}}(p^2)$ [16], including power corrections:

$$\alpha_T(p^2) = \alpha_T^{\text{pert}}(p^2) \left[1 + \frac{C(p,q_0)g_T^2(q_0^2)\langle A^2 \rangle_{q_0^2}^R}{p^2} + \frac{d}{p^6} + \cdots \right],$$

with q_0 fixed to 10 GeV and C a combination of a Wilson coefficient and a running of the gluonic operator $\langle A^2 \rangle$. Eventually α_T^{pert} is expressed at N³LO in function

of
$$\Lambda_T$$
 with $\frac{\Lambda_{\overline{\rm MS}}}{\Lambda_T} \equiv \exp\left(-\frac{507 - 40N_f}{792 - 48N_f}\right)$. The parame-

ters to be fitted are thus $a\Lambda_{\overline{\rm MS}}$, $g_T^2\langle A^2\rangle$, d and the ratios of lattice spacings $a_{\beta}/a_{1.9}$ obtained by imposing that the various curves of α_T merge onto a universal one. We have shown in Fig.5 that the purely perturbative running formula does not match with the $\alpha_T(p^2)$ data, adding the $1/p^2$ fits nicely with them down to p = 3.5 GeVwhile including the $1/p^6$ term improves our ability to describe them further down to the τ mass scale. One could expect a $1/p^4$ power correction but Fig.6 indicates that the fit is meaningless: still we do not exclude that the corresponding Wilson coefficient would mimic an additional $1/p^2$ factor. Collecting in Tab. 1 $\Lambda_{\overline{MS}}$, $g^2\langle A^2\rangle$ and the d coefficient, with the <u>lattice</u> spacing $a_{1.9} = 0.08612(42)$ fm [13], we can run $\alpha_s^{\overline{\text{MS}}}$ up to the Z^0 mass scale or at the τ mass scale. For the latter we obtain $\alpha_s^{\overline{\rm MS}}(m_\tau^2) = 0.337(8) \text{ and } \alpha_s^{\overline{\rm MS}}(m_\tau^2) = 0.342(10) \text{ with our}$

2 estimates of $\Lambda_{\overline{MS}}$; combining both of them and adding in quadrature the errors we get $\alpha_s^{\overline{\rm MS}}(m_\tau^2) = 0.339(13)$. It is in very good agreement with τ decay data analysed with dispersion relations [17], [18], as plotted in Fig. 7. Running α_s up to the $\overline{\text{MS}}$ scheme b quark mass m_b , by using the β function and the $\Lambda_{\overline{MS}}^{N_f=4}$ parameter we have measured, we can match with the $N_f=5$ theory: $\alpha \frac{N_f=5}{MS}(m_b^2) = \alpha \frac{N_f=4}{MS}(m_b^2) \left(1 + \sum_n c_{n0}(\alpha \frac{N_f=4}{MS}(m_b))^n\right)$. Then a second running is applied up to the \mathbb{Z}^0 mass scale. We obtain $\alpha_s^{\overline{\rm MS}}(m_{Z^0}^2) = 0.1198(9)$ and $\alpha_s^{\overline{\rm MS}}(m_{Z^0}^2) = 0.1203(11)$ with, again, our 2 estimates of $\Lambda_{\overline{\rm MS}}$; combining both results and adding in quadrature the errors we get $\alpha_s^{\overline{\rm MS}}(m_{70}^2) = 0.1200(14)$. We have shown in Fig.8 a comparison between lattice results [8], [19] and [4], DIS data [20], the world average quoted by the Particle Data Group [21] (WA '12) and a world average realised by replacing the $N_f = 2 + 1$ lattice results by the $N_f = 2 + 1 + 1$ one (WA' '12), theoretically more reliable. In [3] one can find an almost exhaustive collection of results. A single, very precise, lattice value dominates strongly the weighted world average of $\alpha_s^{\overline{\rm MS}}(m_{Z^0}^2)$; removing it enlarges its uncertainty. Our estimates is in the same ballpark as other approaches and is using a complementary framework.

Table 1: Fit parameters of $\alpha_T(p^2)$ analysed by means of OPE.

$\Lambda_{\overline{\rm MS}}^{N_f=4} ({ m MeV})$	$g^2(q_0^2)\langle A^2\rangle_{q_0^2}^R (\text{GeV}^2)$	$d^{1/6}$ (GeV)	$\alpha(m_Z)$
316(13)	4.5(4)		0.1198(9)
324(17)	3.8(1.0)	1.72(3)	0.1203(11)

3. Conclusions

We have reported on the first measurement of α_s from lattice simulations taking into account the vacuum polarisation effects by charm quark in the so-called $N_f = 2+1+1$ theory. The main benefit of our set-up is that there is no perturbative treatment at the charm threshold. We have used the OPE formalism to analyse gluon and ghost propagators to extract the strong coupling in the MOM Taylor scheme. We have taken care of the hypercubic artifacts and included power corrections in the OPE, that cannot be neglected. An on-going project is to study whether other Green functions (3-gluon vertex, quark propagator,...) present the same feature: our extraction of $\Lambda_{\overline{\rm MS}}$ presented here will help us to reduce the uncertainty on the fits of those Green functions.

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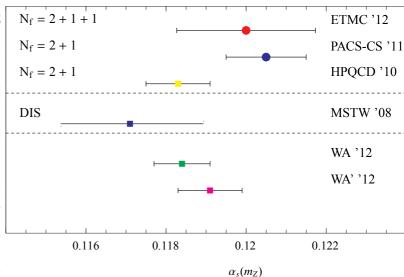


Figure 8: Collection of results on $\alpha_s(m_Z)$.

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