

Lattice measurement of α_s with a realistic charm quark

B. Blossier^{a,*}, Ph. Boucaud^a, M. Brinet^b, F. De Soto^c, X. Du^b, V. Morénas^d, O. Pène^a, K. Petrov^e,
J. Rodríguez-Quintero^f

^aLaboratoire de Physique Théorique, Université de Paris XI, Bâtiment 210, 91405 Orsay Cedex, France

^bLaboratoire de Physique Subatomique et de Cosmologie, CNRS/IN2P3/UJF; 53, avenue des Martyrs, 38026 Grenoble, France

^cDpto. Sistemas Físicos, Químicos y Naturales, Univ. Pablo de Olavide, 41013 Sevilla, Spain

^dLaboratoire de Physique Corpusculaire, Université Blaise Pascal, CNRS/IN2P3 63177 Aubière Cedex, France

^eLaboratoire de l'Accélérateur Linéaire – IN2P3/CNRS, Centre Scientifique d'Orsay, Bâtiment 200 – BP 34, 91898 Orsay Cedex, France

^fDpto. Física Aplicada, Fac. Ciencias Experimentales; Universidad de Huelva, 21071 Huelva; Spain

Abstract

We report on an estimate of α_s , renormalised in the \overline{MS} scheme at the τ and Z^0 mass scales, by means of lattice QCD. Our major improvement compared to previous lattice calculations is that, for the first time, no perturbative treatment at the charm threshold has been required since we have used statistical samples of gluon fields built by incorporating the vacuum polarisation effects of u/d , s and c sea quarks. Extracting α_s in the Taylor scheme from the lattice measurement of the ghost-ghost-gluon vertex, we obtain $\alpha_s^{\overline{MS}}(m_Z^2) = 0.1200(14)$ and $\alpha_s^{\overline{MS}}(m_\tau^2) = 0.339(13)$.

1. Introduction

The recent announcement by ATLAS and CMS of their observation at 5σ significance of a new particle with a mass around 125 GeV [1], interpreted as the Brout-Englert-Higgs (BEH) boson, makes even more crucial than before a satisfying control on theoretical inputs of analytical expression of the Higgs decay channels. Indeed, the era of precise Higgs physics (measurement of the couplings, ...) will certainly open soon: assessing the sensitivity of forthcoming detectors will be a key ingredient. There are different modes of Higgs boson production: however the gluon-gluon fusion is by far the dominant process, as shown in Fig. 1. Over the uncertainty $\Delta\sigma_{gg \rightarrow H \rightarrow X}^{\text{th}}$ of 20 - 25 % claimed at LHC ($\sqrt{s} = 7$ TeV), about 4 % come from the uncertainty $\delta\alpha_s$ on $\alpha_s(m_{Z^0}^2)$ [2]. A complementary approach of the α_s measurement from the analysis of Deep Inelastic Scattering data, physics of jets, τ decay and $e^+e^- \rightarrow$ hadrons [3] is its computation by numerical simulations. In the following section we will report on the work performed by the ETM Collaboration to measure α_s from $N_f = 2 + 1 + 1$ gauge configurations [4].

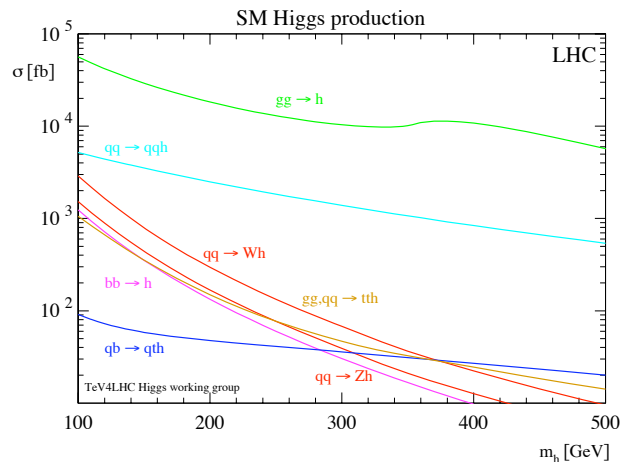


Figure 1: Prediction of the Standard Model BEH boson production in function of its mass for different production channels at LHC

2. α_s from numerical simulations

In the past years tremendous progresses have been made by the lattice community to perform simulations that are closer to the physical point. It means including more and more quark species in the sea, u/d quarks ($N_f = 2$), then the strange ($N_f = 2 + 1$) and even the charm ($N_f = 2 + 1 + 1$) since a couple of years. Pion masses ~ 250 MeV are now common and several collaborations are even able to simulate a real pion, either in a small volume (PACS-CS Collaboration) [5] or using a quark

*Speaker

Email address: benoit.blossier@th.u-psud.fr
(B. Blossier)

inverting the discretised Faddeev-Popov operator. However, as the $O(4)$ symmetry is broken on the lattice to the $H(4)$ group, getting α_T from the dressing functions G and F is not straightforward: $O(a^2 p^2)$ and $H(4)$ invariants artifacts, the so-called hypercubic artifacts, have to be properly taken into account [15]:

$$\alpha_T^{\text{Latt}}\left(a^2 p^2, a^2 \frac{p^{[4]}}{p^2}, \dots\right) = \widehat{\alpha}_T(a^2 p^2) + \frac{\partial \alpha_T^{\text{Latt}}}{\partial \left(a^2 \frac{p^{[4]}}{p^2}\right)} \Big|_{a^2 \frac{p^{[4]}}{p^2}=0} a^2 \frac{p^{[4]}}{p^2} + \dots,$$

$p^{[4]} = \sum_i p_i^4$. We have shown in Fig. 3 that a "fishbone" structure, that are those hypercubic artifacts, is clearly present in F but curable, as also seen on the plot. The remaining cut-off effects are removed by

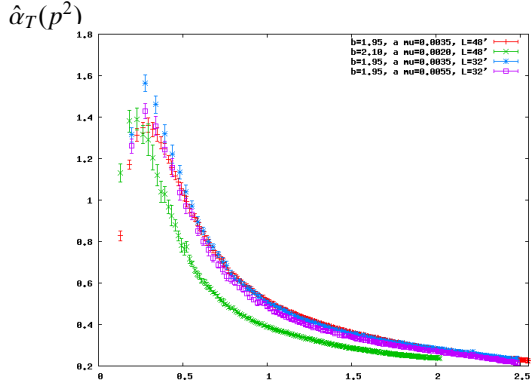


Figure 4: Strong coupling constant obtained after the elimination of the dominant hypercubic artifacts

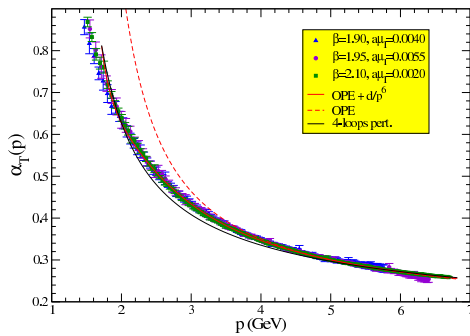


Figure 5: Raw data of $\alpha_T(p^2)$ compared to a purely perturbative running and OPE with power corrections

fitting $\widehat{\alpha}_T(p^2)$ according to the formula $\widehat{\alpha}_T(a^2 p^2) = \alpha_T(p^2) + c_{a2p2} a^2 p^2 + O(a^4)$. Fig. 4 illustrates the benefit, in term of statistical error on α_T , to do the calculation in Taylor scheme, as we pointed earlier in the

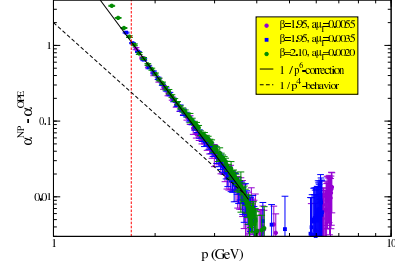


Figure 6: Subtraction of $\alpha_T(p^2)$ from the perturbative running compared to generic power corrections.

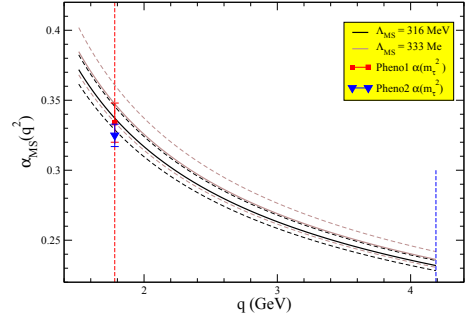


Figure 7: Comparison of $\alpha_s(m_\tau)$ measured on the lattice with estimates based on phenomenological analyses of τ decay data [18], [17].

text.

We then use the OPE formalism so that we can relate $\alpha_T(p^2)$ to $\alpha_T^{\text{pert}}(p^2)$ [16], including power corrections:

$$\alpha_T(p^2) = \alpha_T^{\text{pert}}(p^2) \left[1 + \frac{C(p, q_0) g_T^2(q_0^2) \langle A^2 \rangle q_0^2}{p^2} + \frac{d}{p^6} + \dots \right],$$

with q_0 fixed to 10 GeV and C a combination of a Wilson coefficient and a running of the gluonic operator $\langle A^2 \rangle$. Eventually α_T^{pert} is expressed at $N^3\text{LO}$ in function of Λ_T with $\frac{\Lambda_{\overline{\text{MS}}}}{\Lambda_T} \equiv \exp\left(-\frac{507 - 40N_f}{792 - 48N_f}\right)$. The parameters to be fitted are thus $a\Lambda_{\overline{\text{MS}}}$, $g_T^2 \langle A^2 \rangle$, d and the ratios of lattice spacings $a_\beta/a_{1.9}$ obtained by imposing that the various curves of α_T merge onto a universal one. We have shown in Fig.5 that the purely perturbative running formula does not match with the $\alpha_T(p^2)$ data, adding the $1/p^2$ fits nicely with them down to $p = 3.5$ GeV while including the $1/p^6$ term improves our ability to describe them further down to the τ mass scale. One could expect a $1/p^4$ power correction but Fig.6 indicates that the fit is meaningless: still we do not exclude that the corresponding Wilson coefficient would mimic an additional $1/p^2$ factor. Collecting in Tab. 1 $\Lambda_{\overline{\text{MS}}}$, $g^2 \langle A^2 \rangle$ and the d coefficient, with the lattice spacing $a_{1.9} = 0.08612(42)$ fm [13], we can run $\alpha_s^{\overline{\text{MS}}}$ up to the Z^0 mass scale or at the τ mass scale. For the latter we obtain $\alpha_s^{\overline{\text{MS}}}(m_\tau^2) = 0.337(8)$ and $\alpha_s^{\overline{\text{MS}}}(m_\tau^2) = 0.342(10)$ with our

2 estimates of $\Lambda_{\overline{\text{MS}}}$; combining both of them and adding in quadrature the errors we get $\alpha_s^{\overline{\text{MS}}}(m_\tau^2) = 0.339(13)$. It is in very good agreement with τ decay data analysed with dispersion relations [17], [18], as plotted in Fig.7. Running α_s up to the $\overline{\text{MS}}$ scheme b quark mass m_b , by using the β function and the $\Lambda_{\overline{\text{MS}}}^{N_f=4}$ parameter we have measured, we can match with the $N_f = 5$ theory: $\alpha_{\overline{\text{MS}}}^{N_f=5}(m_b^2) = \alpha_{\overline{\text{MS}}}^{N_f=4}(m_b^2) \left(1 + \sum_n c_n (\alpha_{\overline{\text{MS}}}^{N_f=4}(m_b))^n\right)$. Then a second running is applied up to the Z^0 mass scale. We obtain $\alpha_s^{\overline{\text{MS}}}(m_{Z^0}^2) = 0.1198(9)$ and $\alpha_s^{\overline{\text{MS}}}(m_{Z^0}^2) = 0.1203(11)$ with, again, our 2 estimates of $\Lambda_{\overline{\text{MS}}}$; combining both results and adding in quadrature the errors we get $\alpha_s^{\overline{\text{MS}}}(m_{Z^0}^2) = 0.1200(14)$. We have shown in Fig.8 a comparison between lattice results [8], [19] and [4], DIS data [20], the world average quoted by the Particle Data Group [21] (WA '12) and a world average realised by replacing the $N_f = 2 + 1$ lattice results by the $N_f = 2 + 1 + 1$ one (WA' '12), theoretically more reliable. In [3] one can find an almost exhaustive collection of results. A single, very precise, lattice value dominates strongly the weighted world average of $\alpha_s^{\overline{\text{MS}}}(m_{Z^0}^2)$; removing it enlarges its uncertainty. Our estimates is in the same ballpark as other approaches and is using a complementary framework.

Table 1: Fit parameters of $\alpha_T(p^2)$ analysed by means of OPE.

$\Lambda_{\overline{\text{MS}}}^{N_f=4}$ (MeV)	$g^2(q_0^2) \langle A^2 \rangle_{q_0^2}^R$ (GeV ²)	$d^{1/6}$ (GeV)	$\alpha(m_Z)$
316(13)	4.5(4)		0.1198(9)
324(17)	3.8(1.0)	1.72(3)	0.1203(11)

3. Conclusions

We have reported on the first measurement of α_s from lattice simulations taking into account the vacuum polarisation effects by charm quark in the so-called $N_f = 2 + 1 + 1$ theory. The main benefit of our set-up is that there is no perturbative treatment at the charm threshold. We have used the OPE formalism to analyse gluon and ghost propagators to extract the strong coupling in the MOM Taylor scheme. We have taken care of the hypercubic artifacts and included power corrections in the OPE, that cannot be neglected. An on-going project is to study whether other Green functions (3-gluon vertex, quark propagator,...) present the same feature: our extraction of $\Lambda_{\overline{\text{MS}}}$ presented here will help us to reduce the uncertainty on the fits of those Green functions.

K. Petrov acknowledges the support of "P2IO" Laboratory of Excellence.

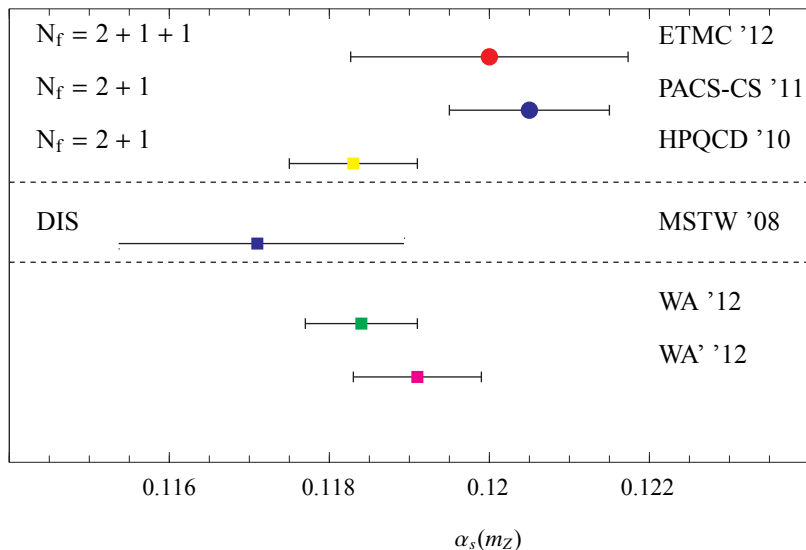


Figure 8: Collection of results on $\alpha_s(m_Z)$.

References

- [1] ATLAS Collaboration, [arXiv:1207.7214]; CMS Collaboration, [arXiv:1207.7235].
- [2] J. Baglio and A. Djouadi, JHEP **1103**, 055 (2011) [arXiv:1012.0530].
- [3] S. Bethke *et al*, [arXiv:1110.0016].
- [4] B. Blossier *et al*, Phys. Rev. D **85**, 034503 (2012) [arXiv:1110.5829 [hep-lat]]; Phys. Rev. Lett. **108**, 262002 (2012) [arXiv:1201.5770].
- [5] S. Aoki *et al*, Phys. Rev. D **81**, 074503 (2010) [arXiv:0911.2561].
- [6] S. Dürr *et al*, JHEP **1108**, 148 (2011) [arXiv:1011.2711].
- [7] A. X. El-Khadra *et al*, Phys. Rev. Lett. **69**, 729 (1992).
- [8] I. Allison *et al*, Phys. Rev. D **78**, 054513 (2008) [arXiv:0805.2999].
- [9] M. Lüscher *et al*, Nucl. Phys. B **389**, 247 (1993) [hep-lat/9207010]; Nucl. Phys. B **413**, 481 (1994) [hep-lat/9309005].
- [10] B. Alles *et al*, Nucl. Phys. B **502**, 325 (1997) [hep-lat/9605033]; Ph. Boucaud *et al*, JHEP **9810**, 017 (1998) [hep-lat/9810322]; Phys. Rev. D **63**, 114003 (2001) [hep-ph/0101302].
- [11] A. Sternbeck *et al* PoS **LAT2007**, 256 (2007) [arXiv:0710.2965]; Ph. Boucaud *et al*, Phys. Rev. D **79**, 014508 (2009) [arXiv:0811.2059].
- [12] J. Taylor, Nucl. Phys. B **33**, 436 (1971).
- [13] R. Baron *et al*, JHEP **1006**, 111 (2010) [arXiv:1004.5284]; PoS **LATTICE2010**, 123 (2010) [arXiv:1101.0518].
- [14] L. Giusti *et al*, Int. J. Mod. Phys. A **16**, 3487 (2001) [hep-lat/0104012].
- [15] D. Becirevic *et al*, Phys. Rev. D **60**, 094509 (1999) [hep-ph/9903364]; F. De Soto and C. Roiesnel, JHEP **0709**, 007 (2007) [arXiv:0705.3523].
- [16] B. Blossier *et al*, Phys. Rev. D **82**, 034510 (2010) [arXiv:1005.5290].
- [17] S. Narison, Phys. Lett. B **673**, 30 (2009) [arXiv:0901.3823].
- [18] A. Pich, [arXiv:1107.1123].
- [19] S. Aoki *et al*, JHEP **0910**, 053 (2009) [arXiv:0906.3906].
- [20] A. Martin *et al*, Eur. Phys. J. C **64**, 653 (2009) [arXiv:0905.3531].

[21] J. Beringer *et al*, Phys. Rev. **D 86**, 010001 (2012).