

# Equilateral hyperbolic moiré zone plates with variable focus obtained by rotations

**Zbigniew Jaroszewicz**

*Institute of Applied Optics, Kamionkowska 18, 03-805 Warsaw, Poland and  
National Institute of Telecommunications, Szachowa 1, 04-894 Warsaw, Poland  
[mntzjaroszewicz@post.pl](mailto:mntzjaroszewicz@post.pl)*

**Andrzej Kołodziejczyk**

*Faculty of Physics, Warsaw University of Technology, Koszykowa 75, 00-662 Warsaw, Poland*

**Alejandro Mira and Rodrigo Henao**

*Facultad de Ciencias, Universidad de Antioquia, A.A. 1226 Medellín, Colombia*

**Salvador Bará**

*Area de Óptica, Facultade de Física, Universidade de Santiago de Compostela, 15706 Santiago de Compostela, Galicia, Spain*

**Abstract:** We present equilateral hyperbolic zone plates with variable focal length, which are formed as moiré patterns by a mutual rotation of two identical basic grids. Among others, all principal zone plates, except of the spherical one, can be used as these basic transmittances. Three most important advantages of the proposed moiré zone plates are: a constant aperture of the created element during the mutual movement of basic grids, lack of aberrations due to their undesired mutual lateral displacements and high diffraction efficiency of the binary phase version. To obtain clearer moiré fringe pattern, a radial carrier frequency can be added additionally to the transmittances of basic grids. The destructive interference between both arms of the focal cross of the equilateral hyperbolic moiré zone plate can be obtained by a constant phase shift introduced in the transmittances of the basic grids. Potential applications of discussed elements are indicated, including the most promising one in the three-point alignment technique.

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## 1. Introduction

The paper deals with equilateral hyperbolic zone plates (EHZP's) of variable focal length, which are formed as a moiré pattern by a superposition of two basic grids and their subsequent rotation. These ZP's turn out to be interesting because of their unusual optical properties and resulting thus promising potential applications.

The first mention of varifocal optical elements, which can be obtained by a mutual displacement of its two parts belongs to Alvarez [1] and to Lohmann [2], who proposed to shift laterally two superposed refractive elements with cubic surfaces and to obtain in this way a spherical lens with variable focal length. Next, Lohmann and Paris presented varifocal

spherical zone plates being moiré patterns created by translations and rotations of the basic grids, as well as cylindrical ZP's formed by translations [3].

Further improvement of the spherical moiré ZP's obtained by displacement relied on adding a linear carrier frequency to the basic grids described by curves of third degree, what allowed to make their period more uniform and to obtain thereby a clearer moiré fringe pattern [4], [5]. The moiré diffractive optical elements have small diffraction efficiency, equal to  $1/\pi^4$  ( $\approx 1.03\%$ ) in the case of binary amplitude basic grids and  $16/\pi^4$  ( $\approx 16.43\%$ ) in the case of binary phase basic grids. An increase to the values reaching up to 100% by superposition of two conjugated kinoforms was first proposed [6] and then realized experimentally [7], [8]. The principle of tunable moiré patterns was extended onto other wavefront shapes, leading to variable aberration generators with possible application in aberration control [9-11].

Another application of moiré diffractive optical elements is an alignment method known as the three-point technique, where a focusing element of small optical power is attached to the measured point of the system that has to be aligned and repeats its transversal displacements. In consequence the optical axis changes its direction and the image of the point object moves on the receiving device accordingly [12]. In contrary to the alignment technique using the autocollimating telescope, the three-point technique is insensitive against possible angular misalignments. The relative accuracies of measurement reaching the order of  $10^{-6}$  over a range of up to 80 m in the open air were reported using this method [5]. Since elements with very long focal length are required and they are hardly obtainable in the refractive version, ZP's became the best choice, because then they are even easier to manufacture.

One of main and most successful applications of the three-point method turned out to be the alignment of linacs. The Stanford Linear Accelerator (SLAC) was aligned in this way, both during its construction [13], as well as after the 1989 earthquake [14]. Since the whole set-up worked within the vacuum line, the relative accuracy higher than  $10^{-7}$  over a distance longer than 3 km was achieved. In this particular case an element composed of two crossed linear ZP's with constant focal length was used. Nowadays the three-point method becomes a standard alignment technique for accelerators [15-17]. Another recent application of this technique is the monitoring system of the world's greatest dam of Three Gorges [18].

However, in order to apply the three-point method in varying conditions, a diffractive optical element with long and variable focus would be required, as it was proposed for the spherical moiré ZP obtained by translations in earlier works [4], [5]. A focal cross composed of two perpendicular arms, similar to that of two crossed cylindrical ZP's used for linacs alignment, is created by the EHZP [19], [20]. Therefore the principal aim of this paper is to find basic grids forming such moiré patterns by their mutual rotations and to discuss their properties.

## 2. Determination of the basic grid

### 2.1 General solution

Looking for a solution of our problem, we will make use of the general method described in Refs. [21-23], which allows to find basic grids producing given moiré pattern for a particular kind of mutual infinitesimal displacement. The corresponding relationship reads:

$$\nabla\Phi(\vec{r})\Delta\vec{r} = \Psi(\vec{r}), \quad (1)$$

where  $\Phi(\vec{r})$  stands for the basic grid curves,  $\Psi(\vec{r})$  for the moiré pattern curves and  $\Delta\vec{r}$  is the displacement vector (in general, in our case it can be composed both from rotations as well as from displacements). For the pure rotations the above equation reduces to the following simple formula:

$$\frac{\partial\Phi(r, \theta)}{\partial\theta}\Delta\theta = \Psi(r, \theta), \quad (2)$$

where  $\Delta\theta$  is the angle of basic grids mutual rotation and  $r, \theta$  are the polar coordinates.

The set of curves of an EHZP, e.g., determining the borders of the zones, which we want to obtain as the moiré pattern, is given by the following equation [19], [20]:

$$\Psi(r, \theta) = r^2 \sin(2\theta) / 2\lambda f_M = n, \quad (3)$$

where  $f_M$  is the focal length of the moiré ZP, which we want to create,  $\lambda$  is the wavelength of the illuminating beam, and  $n$  is an integer indexing the curves. Substituting Eq. (3) into Eq. (2), we obtain the general expression for the basic grid:

$$\Phi(r, \theta) = -r^2 \cos^2(\theta) / 2\lambda f_{BG} + f(r) + c = m, \quad (4)$$

where  $f_{BG} = f_M \Delta\theta$  is the focal length of the cylindrical ZP present in the solution for the basic grid,  $f(r)$  is an arbitrary function depending on radial coordinate only,  $c$  is a constant, and  $m$  is the integer indexing the curves. As it can be seen, the found solution is periodic in respect to full rotations by  $2\pi$  radians and therefore the whole aperture will be occupied by the desired moiré pattern, whereas in the case of all moiré ZP's produced by translation the width of the element must to decrease during the lateral shifting of its parts. In turn, in the case of the spherical moiré ZP produced by rotations its focal spot will be disturbed, because the sector of angular width equal to the rotation angle  $\Delta\theta$  within the aperture of the moiré element, where the coarse part of one grid overlaps with the dense part of the second one, is occupied by an undesired ZP pattern of different focal length [21].

Now the validity of the found solution should be checked for arbitrary rotations. Let us to superpose two basic grids given by Eq. (4) and rotated by angles  $\theta_0$  and  $-\theta_0$  (i.e.,  $\Delta\theta = 2\theta_0$ ), correspondingly. In order to find the resulting moiré pattern, we will make use of the indicial equation for the moiré beats [24], [25]:

$$m_1 - m_2 = n, \quad (5)$$

where  $m_1$  and  $m_2$  are integers indexing the curves of the basic grids. The final result for the obtained moiré pattern, after substitution of Eq. (4) into Eq. (5) is given by:

$$\Psi(r, \theta) = r^2 \sin(2\theta) / 2\lambda f_M = n, \text{ where } f_M = f_{BG} / \sin(2\theta_0) \quad (6)$$

and confirms that the basic grids given by Eq. (4), which were found for infinitesimal rotations, can be applied in the case of arbitrary rotations too. In continuation we will consider in more detail some special cases of the general solution.

## 2.2 Zone plates as the basic grids

Placing  $f(r) = ar^2 / 2\lambda f_{BG}$  we can obtain hyperbolic ( $0 < a < 1$ ), elliptic ( $a < 0, a > 1$ ) and linear ( $a = 0, a = 1$ ) zone plates as basic grids:

$$\Phi(x, y) = x^2 / 2\lambda f_x + y^2 / 2\lambda f_y = m, \text{ where } f_x = f_{BG} / (a - 1), \quad f_y = f_{BG} / a. \quad (7)$$

Some particular solutions of this kind were already published elsewhere, e.g., the EHZP moiré pattern created by elliptic ZP's was used for determination of the ellipticity distortion of the scanning beam lithography applied for the fabrication of spherical ZP's [26]. Such defect can appear due to slightly different magnification in perpendicular directions. Similar moiré pattern created by two EHZP's was also presented recently [27]. Let us mention that the wide range of different ZP's, which we can use as basic grids, allows us to choose the range and sensitivity of the focal length change in function of the rotation angle. The greatest range of the focal lengths and thus the smallest sensitivity gives the EHZP ( $a = 1/2$ ) used as the basic

grid [27], whereas in the case of elliptic ZP's with decreasing ellipticity (i.e., for  $a \rightarrow \pm\infty$ ), the range of the focal length change decreases to zero. An example of the EHZP moiré pattern created by two elliptical ZP's and the focus, being two perpendicularly crossed focal lines for the case when both basic grids are binary phase transmittances, are shown in Fig. 1. As it was already derived elsewhere [28] and can be seen in Fig. 1, length of the focal cross arms is twofold as long as width of the moiré EHZP aperture.

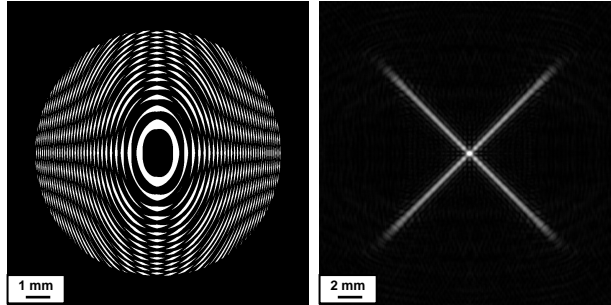


Fig. 1. A superposition of two binary amplitude elliptic ZP's with  $f_x=600$  mm and  $f_y=1500$  mm for  $\lambda=632.8$  nm rotated mutually by an angle of  $2\theta_0=0.075\pi$  rad creates the moiré EHZP oriented perpendicularly with  $f_M=4284$  mm (left); the focal pattern in the distance  $f_M=4284$  mm formed by the moiré EHZP made from the binary phase versions of the basic grids (right).

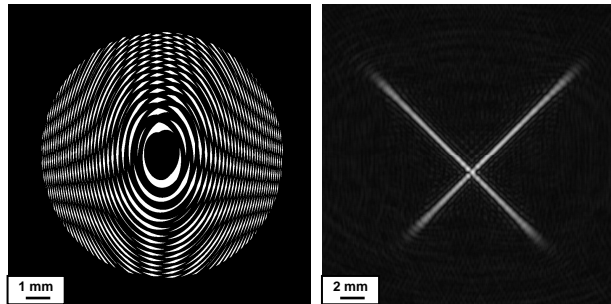


Fig. 2. A superposition of two binary amplitude elliptic ZP's with  $f_x=600$  mm and  $f_y=1500$  mm for  $\lambda=632.8$  nm, rotated mutually by an angle of  $2\theta_0=0.075\pi$  rad, one of them shifted from the centre, creates the displaced moiré EHZP oriented perpendicularly with  $f_M=4284$  mm (left); the focal pattern in the distance  $f_M=4284$  mm formed by the moiré EHZP made from the binary phase versions of the basic grids (right).

An interesting feature of ZP's used as the basic grids for creation of moiré ZP's by rotations is that any additional lateral shifting of both transmittances during the rotation does not introduce any aberration affecting the quality of the focal cross, whereas already known solutions for ZP's produced by translations [4], [5] or spherical ZP's obtained by rotations [3] were susceptible for this kind of aberration. Since phase functions of the basic grid ZP's are of second degree, any undesired shearing between them results in appearance of a linear carrier frequency, thus any misalignment of this kind results only in the displacement of the focal cross (Fig. 2).

Although such change of the focal cross position will overlap with displacements of the measured system, nevertheless, in some situations it can be at least partly eliminated using correctly chosen basic ZP's (e.g., the linear ZP is not sensitive against displacements along its axis of symmetry) and basing on *a priori* knowledge about the kind of expected movements which have to be measured. An example can be one-dimensional displacement sensing, what is quite frequent situation, e.g., in the case, when bending of bridges is measured.

### 2.3 Basic grid with radial carrier frequency

Let us recall that in the case of moiré ZP's obtained by translations, the basic grids are cubic curves [3], thus their lines are sparse in the center, what makes the moiré fringes less clear in this region, because their period becomes comparable to the period of the basic grids. This limitation can be avoided by adding a linear carrier frequency, say, in the direction OX [4], [5], what makes the period of the basic lines more uniform. Although in our case the basic grids, being ZP's, i.e., a set of second degree curves, are less affected by this problem, however their period also can be made more uniform by adding a radial linear frequency. The equation for the basic grid will be then equal to:

$$\Phi(r, \theta) = -r^2 \cos^2(\theta) / 2\lambda f_{BG} + ar^2 / 2\lambda f_{BG} + \alpha r = m, \text{ where} \quad (8)$$

$\alpha$  is the radial direction cosine of the linear carrier frequency. In Fig. 3 both the basic grid being a cylindrical zone plate with linear radial frequency and the resulting pattern of moiré EHZP are shown. The price for such solution is that now the undesired displacements of the basic grids not only displace the focal cross, but also introduce aberrations, because the basic grid is no longer a pure ZP transmittance (Fig. 4).

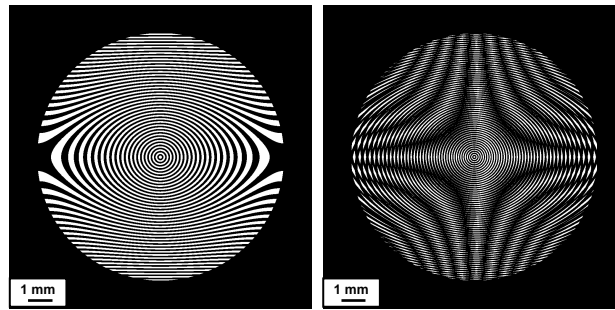


Fig. 3. The binary amplitude cylindrical ZP with  $f_X = f_{BG} = 1000$  mm for  $\lambda = 632.8$  nm and radial carrier frequency  $\alpha = -0.005$  (left); the moiré EHZP oriented perpendicularly with  $f_M = 4284$  mm created by superposition of two such binary amplitude cylindrical ZP's with radial carrier frequency  $\alpha = -0.0075$  rotated mutually by an angle of  $2\theta_0 = 0.075\pi$  rad (right).

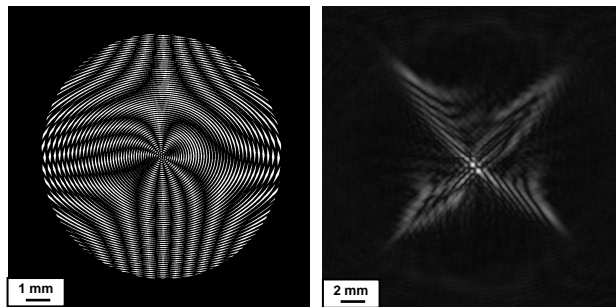


Fig. 4. A superposition of two binary amplitude cylindrical ZP's shown in Fig. 3, rotated mutually by an angle of  $2\theta_0 = 0.075\pi$  rad, one of them shifted from the centre, creates the aberrated moiré EHZP oriented perpendicularly with  $f_M = 4284$  mm (left); the focal pattern in the distance  $f_M = 4284$  mm formed by the moiré EHZP made from the binary phase versions of the basic grids (right).

#### 2.4 Basic grid with constant phase shift

The accuracy of alignment in obvious manner depends on the precision with which the centre of the focal pattern can be determined. A focal pattern distribution with zero irradiance in its centre was claimed to be superior over the ordinary ones thanks to smaller characteristic dimensions and owing to the fact that displacements from the desired position can be measured then by an infinite proportional change of the irradiance [29]. A focal pattern of this kind in the case of the EHZP's can be achieved by introducing a phase shift of  $\pi$  radians between both arms of the focal cross, what results in destructive interference in their crossing point. Such phase difference will appear, if the EHZP with the phase function given equal to:

$$\Psi(x, y) = \pi(x^2 - y^2) / \lambda f_M + \pi/2, \quad (9)$$

will be applied [30].

In turn, the moiré pattern of this shape can be created by any of two basic grids expressed by Eq. (4) with an additional condition that

$$c_1 - c_2 = \pi/2, \text{ where} \quad (10)$$

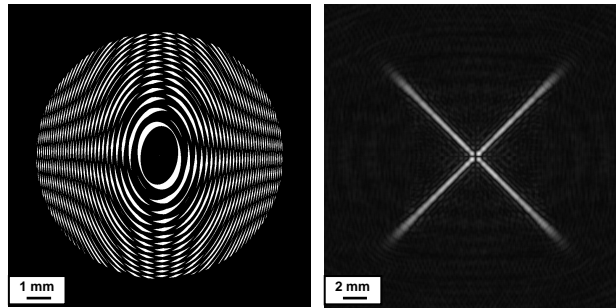


Fig. 5. A superposition of two binary amplitude elliptic ZP's with  $f_x=600$  mm and  $f_y=1500$  mm for  $\lambda=632.8$  nm rotated mutually by an angle of  $2\theta_0 = 0.075\pi$  rad, one of them with phase shift equal to  $\pi/2$ , creates the moiré EHZP oriented perpendicularly with  $f_M=4284$  mm and with phase shift equal to  $\pi/2$  (left); the focal pattern in the distance  $f_M=4284$  mm formed by the moiré EHZP with the initial phase shift equal to  $\pi/2$  made from the binary phase versions of the basic grids (right).

$c_1$  and  $c_2$  are constant. The corresponding basic grids, the resulting moiré pattern, and the obtained focal pattern are shown in Fig. 5. All moiré patterns and focal distributions shown in Figs. 1-5 are a result of numerical simulation based on the convolution approach [31].

#### 2.5 Diffraction efficiency of moiré ZP's

Another quantity of interest is the diffraction efficiency of the proposed moiré element. The diffraction efficiency of moiré ZP's can reach 100%, when both basic grids are conjugate kinoforms [6-9]. However, the focal pattern of the EHZP consists of two perpendicular lines, which are created by two conjugate diffraction orders, therefore the kinoform version of the EHZP creates only one focal line. On the other hand, using two binary phase EHZP's as basic grids, one can collect light in two perpendicular focal lines of equal intensity, useful for two-dimensional displacement sensing. The diffraction efficiencies of the first diffraction order and its conjugate of the moiré EHZP are formed by orders (1,-1) and (-1,1) of the product of basic grids transmittances, i.e., to squared diffraction efficiency of the basic grid's first diffraction order [6-8], hence the total amount of energy collected in the focal cross is equal to

$$\eta_{TOT} = \eta_{+1} + \eta_{-1} = 2\left(\frac{2}{\pi}\right)^4 = 32.85\%, \quad (11)$$

The phase functions of remaining two orders (1, 1) and (-1,-1), which have the same high efficiency, are adding and therefore they usually exhibit much shorter focal length and do not disturb the focal pattern of interest.

### **3. Conclusions**

The moiré pattern in the form of EHZP was analyzed. Its focal length can be controlled by mutual rotation of transparencies forming the mentioned moiré pattern and by a proper choice of the basic grid's particular form. It was shown that hyperbolic, cylindrical and elliptic ZP's can be used as the basic grids for the EHZP moiré pattern creation. It was shown also that the found solution, in contradiction to earlier ones, exhibits constant aperture during the mutual displacement of the basic grids. Moreover, another interesting feature of the newly found solution is that any residual lateral displacement of the basic grids does not introduce any additional aberration into the focal pattern except of its displacement, thus leaving the focal shape unaffected. An additional linear radial frequency allows to increase clarity of moiré fringes by making their period more uniform in the center of the transmittances. As far as the diffraction efficiency of the moiré ZP is concerned, relatively high diffraction efficiency can be achieved by an application of a simple binary phase version of the basic grids. Because of these reasons we dare to express an opinion that the moiré EHZP obtained by rotation is maybe the best candidate among other ZP's moiré patterns for successful application in the three-point alignment method.

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