A robust solving strategy for the vehicle routing problem with multiple depots and multiple objectives

by

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1. CHAPTER I. PROJECT DESCRIPTION

Globalization is a clear trend in global economy nowadays, and it is observed in the free trade agreements between American, Latin-American, European and Asian economies. This particular trend affects directly logistics operations which are the pillar for this global economy.

Transportation operations are of a particular interest for logistic networks design because they are the most energy consuming operations, and hence the most representative in cost; moreover, transportation operations are the most important logistic operations to guarantee sustainable development because its efficiency may have a direct impact on fuel consumption and hence on environmental impact.

This Project aims to develop optimization-based solving strategies, for decision support in logistic distribution networks planning, on a multi-criteria environment, as a response to the competiveness problem which many economies are involved in, that are indeed influenced by logistics operations and strategies. The general objective of this project is to solve this distribution problem by the design of robust solving strategies supported on a strict literature review, which permits identify knowledge gaps, and then serve as a basis for optimization-based decision support systems development.

This document is organized in four chapters; in Chapter I the project description is presented, including problem statement, research framework, objectives and methodology. Chapter II, presents a complete literature review and analysis; then, knowledge gaps identification and the desired level of robustness is explained and defined. Chapter III presents the specific problem mathematical definition and the models that support the solving strategy. Chapter IV presents experimental results and the solving strategy performance evaluation. Finally conclusions and lines for future work are defined.

1.1. PROBLEM STATEMENT

On a global basis, freight transportation, and especially physical distribution of goods, is a very critical operation in decision making to reduce logistics cost impact, and frequently this area is very sensitive to incur on management inefficiencies that lead to increase costs due to inadequate resource utilization. In Colombia particularly, productivity of freight transportation and hence its competitiveness are low compared to other countries, including Latin-American countries with similar economies and economy growth; this is supported by World Bank reports[1] where Colombia's results, measured by the Logistics Performance Index – LPI, have not been satisfactory, standing at rank 82 of 150 countries, and below of Latin-American and Caribbean's average.

Logistics and hence transportation industry (which represents 40% to 50% of logistics costs) importance is sustained on European Economic Community EEC reports, that show, up to the year 2005, that logistics industry represented 13,8% of the Gross Domestic

[1] World Bank, Connecting to Compete, 2007.

Product GDB on a global scale, and its operation costs represents 10% to 15% of the total cost of a final good[2].

Research and development has proved the potential of the logistics industry to support and endorse economic growth; on the other hand, the global economy trend towards globalization has driven even more research and development efforts on the different areas of logistics and hence, transportation and distribution operations. Moreover, globalization trend have turned global markets even more dynamic and demanding, with more effectiveness and shorter response times; these demands have encouraged the design and development of transportation strategies such as *cross-docking* and *merge-in-transit* operations, that seek economies of scale in freight cost and better response times, but that require a strict synchrony and efficiency in logistic operations, especially in distribution operations. In addition, freight transportation operations demand great energy consumption, specifically fossil fuels consumption, which is of global concern and an imperative issue in sustainable development.

The main causes for low productivity and hence, low competitiveness in transportation industry are: excessive operational costs and resource sub-utilization; in turn, cost overruns may be caused by several reasons, among which may be considered, high fuel prices, multimodal transport inefficiency (or inexistence) and the lack of adequate logistic infrastructure such as: efficient ports, efficient roads with an appropriate free flow velocity, navigable waterways, intermodal transfer facilities; however, these issues require direct government intervention.

Transportation industry productivity (particularly in distribution), is extremely sensitive to resource allocation and scheduling, having a direct impact on operational costs and resource utilization; inefficient resource allocation and scheduling causes operational cost overruns and resource sub-utilization, causing a direct negative impact on this industry productivity. Resource allocation and scheduling inefficiencies are in turn, caused by an inadequate shipment consolidation and inefficient route sequencing.

A key element to enforce an effective integration and execution of supply processes, production and distribution of goods are Information Technologies that support the consolidation, guarding, manipulation and distribution of information to support strategic, tactic and operative decisions[3]. In the industrial development field, IT have become a source of competitive advantage and therefore, supply chain has not been immune to its positive impact, which has permitted the development of dynamic logistic operations on a global scale, where high quality and timely information is the best tool.

According this problematic approach, the following research question arises: How, by robust solving strategies design and implementation for the vehicle routing problem in transportation industry, the total operational cost and resource utilization can be improved?

1.2. RESEARCH FRAMEWORK

[2] La logística del transporte de mercancías en Europa – La clave para la movilidad sostenible. Comunicación de la Comisión de las Comunidades Europeas al Consejo, al Parlamento Europeo, al Comité Económico y Social Europeo y al Comité de las Regiones, 2006.

[3] BALLOU, RONALD. Business Logistics management. Prentice Hall, USA , 2004.

As pointed out in the problem statement, improvement in transportation operations, achieved by operational costs reduction and balanced resource utilization, lead to a productivity improvement that has a positive impact in logistics operations competitiveness supported by specific competitive advantages based on the information value; this is reached by advanced decision making.

Advanced decision making is supported on a variety of processes based on technical and scientific processes, such as Business Intelligence and Decision Science; this is because advanced decision making involves knowing information opportunely, this implicates data mining processes, data warehousing, and the development of robust Information Systems; on the other hand, this Information Systems must support an intelligent decision process which are based on advanced System Modeling, this type of systems are known as Decision Support Systems (DSS).

One of the tools for advance decision making supported by decision support systems are Operational Research techniques. Operational Research is a sub-field of applied mathematics, and it is an important approach for distribution operations optimization that in Operations Research is known as Vehicle Routing Problems (VRP). This framework presents a theoretical framework in which Operational Research approaches and methods for distribution operations optimization are presented.

1.2.1. Integer Linear Programming

In Operational Research, optimization is one of many fields, which broadly consists of maximizing or minimizing a mathematical function, often called objective function, that is subject to a set of constraints expressed as mathematical equalities or inequalities; that is, choosing the best feasible scenario to compute the best value of a given objective. Among linear optimization problems, two types of problems can be defined: continuous programs or linear programs (continuous variables) and integer programs (integer variables). Integer programs have been matter of a lot of research due to its complexity, and in many cases, due to its combinatorial nature which adds even more complexity. Integer Programs can be classified into:

- Binary Programs (BP): Problems that are defined only by binary variables 0 – 1.
- Integer Programs (IP): Problems that are defined only by integer variables.
- Mixed Integer Programs (MIP): Problems that are defined by integer and continuous variables.

Mixed Integer Linear Programing has been applied to many engineering topics or fields, such as:

- Production Planning and Scheduling
- Supply Chain and Distribution Networks design and optimization
- Telecommunication Networks design and optimization
- Transportation Engineering
- Marketing Engineering

1.2.2. Vehicle Routing Problems

Vehicle routing problems are discrete problems by nature; this is because the decisions made take integer and or binary values, e.g.: number of vehicles used (1,2,3,…,n); vehicle sequencing, that is the decision of allocate one destination before or after another destination (binary variables). By the above, Vehicle Routing Problems mathematical modeling is developed by Mixed Integer Linear Programing (MILP).

Vehicle Routing Problems (VRP) modeling takes first place in 1956, when Flood develop a mathematical formulation for the Traveling Salesman Problem *(Flood, 1956)*, this problem consists of a selling agent who has to visit *n* customers in any order at minimum total traveling cost. This problem's formulation is relatively simple, although, it took much attention because its complexity to optimally solve: there are *(n-1)!* possible combinations to solve this problem. To conclude, by complete enumeration, it can only be expected to solve very small problems; for only 100 customers, there are 9.33×10^{157} possible combinations.

Due to research on the Traveling Salesman Problem (TSP), many variants of the problem are formulated; this is where the Vehicle Routing Problem (VRP) formulations are published. There are many variants of the VRP, which can be summarized in:

– *Vehicle Routing Problem with Time Windows (VRPTW)*

This problem adds time windows constraints to the original VRP, consisting of *m* vehicles of capacity *Q*, dispatching to *n* customers with d_i demand which has to be satisfied in a (l_i, u_i) lapse.

– *Vehicle Routing Problem with Multiple Depots (MDVRP)*

This problem consists of *m* vehicles dispatching to *n* customers with d_i demand that can be attended from a set of *D* depots.

– *Vehicle Routing Problem with Pick-up and Delivery (VRPPD)*

In this problem customers can receive or send goods that can be simultaneously attended.

A complete description of these problems is found in *(Toth & Vigo, 2002)*.

1.2.3. Solution Methods

There are two possible approaches to solve complex problems: exact methods and approximate methods. As explained before, Mixed Integer Programs, like VRP's and its different variants, due to its combinatorial nature, require a dramatically great computational effort to be solved; according to this, if method's choice and implementation is naïve or inconvenient, a lot of computational time can be taken to solve a relatively small instance of the problem; that is why often, global optimum can be sacrificed to find a good solution in a dramatically shorter time.

– *Exact Methods*

There are several exact methods to solve MILP problems and of course VRP's which are a subset of MILP problems. Among these methods are: Branch and Bound methods; Branch and Cut methods, which are a variation of Branch and Bound, where several cutting plane procedures are applied to the generated tree of the Branch and Bound algorithm; and Column Generation algorithms.

Many works using exact methods have been published; *Laporte et al. (1986)*, use relaxation methods to solve a symmetric and an asymmetric VRP, reducing significantly the computational complexity *($O|V|^3$ and $O|V|^2|E|$)*, respectively, where V is the set of vehicles and E is the set of customers; although, the warrantied bound are relatively poor (9% and 30%, respectively). Moreover, the approach published by *Fischetti et al. (1994)*, proposed, based on branch and bound algorithms (such as Laporte), two additive procedures to compute lower bounds. The two procedures are considered additive because calculations for i^{th} iteration use the bounds for the $(i-1)^{th}$ iteration, and they prove that the sum of the two bounds found are lower bounds for the VRP. The results outperform results obtained by *Laporte et al. (1986)*, but at a high computational cost ($O(n^4)$ in both cases).

Agarwal et al. (1989), Desrochers et al. (1992) and *Bixby et al. (1997)* developed several techniques to find new routes for the VRP, this procedures are used to adapt *Dantzig & Wolfe (1960)* decomposition method to enumerate several feasible routes and the solve the respective set covering problem; although, a global optimal solution cannot be ensured because the set covering optimal solution is not necessary the optimal solution for the original VRP.

Finally, *Desrochers et al. (1992)* solved the VRP using a Branch and Price algorithm, in which each at node from the search tree, new columns are generated and added to the problem. This method ensures a global optimum.

– *Approximate Methods*

Among the approximation methods to solve the VRP are the Heuristic methods and the Meta-heuristic methods; the difference between both lies in the computation of new solutions, which is deterministic for the Heuristic methods and probabilistic for the Meta-heuristics. There are several Heuristic algorithms that can be classified in: algorithms based on savings procedures, and insertion algorithms. The following works can be highlighted: *Clarke & Wright (1964); Desrochers & Verhoog (1989); Wark & Holt (1994)*; *Mole & Jameson (1976); Christofides, Mingozzi & Toth (1979); Solomon (1987).* Among the Meta-heuristic approaches are Tabu Search algorithms, Simulated Annealing, Genetic Algorithms, Scatter Search procedures, Ant System algorithms, etc.

1.3. OBJECTIVES

1.3.1. General Objective

Design a robust solving strategy for the multiple objectives and multiple depot vehicle routing problem minimizing operational costs and load imbalance.

1.3.2. Specific Objectives

- Identify robustness gaps for solving strategies for the multiple objectives and multiple depots vehicle routing problem.
- Define the desired level of robustness for the proposed solving method.
- Design a multi-objective solution strategy for the vehicle routing problem with multiple depots.

- Validate the solution method and the computational tool performance, comparing efficiency (execution times) and effectiveness (solutions quality) solving known instances using exact methods.
- Calibrate execution parameters using formal experimentation techniques.
- Define implementation outlines for a Decision Support System development for local distribution operations.

1.4. METHODOLOGY

The objective of this research is to design and develop a robust solution strategy for the vehicle routing problem with multiple depots and multiple objectives as a response to the problematic of distribution operations which may have a strong impact on the retail price of goods. This problem is tackled using operational research techniques and applied discrete mathematics such as integer linear programming (MILP) and artificial intelligence methods (meta-heuristics) for the design of robust algorithms for combinatorial optimization problems, in this case, a vehicle routing problem.

Several experiments were run to evaluate the solving strategy performance; experiments were designed based on statistical theory and the instances used for the experiments were simulated. Other benchmark instances (Solomon's) were adapted for this particular problem, this benchmark instances were used to compare the results obtained with the proposed algorithm with exact results obtained with an MILP model.

The project was developed in three main phases:

- ***Phase 1: Definition of level of robustness for the solving strategy for the MDVRP:*** A complete literature review was made in this phase, to find gaps in the development of robust solving strategies for the MDVRP until 2012; then, based on this review the desired level of robustness was determined.
- ***Phase 2: Solving strategy design for the MDVRP and informatics tool development and validation:*** In this phase, the models and algorithms for the design and development of the informatics tool based on the solving strategy were design and adapted; then a performance validation was made based on the comparison with other method performance. The performance evaluation was made based on execution time and solution quality.
- ***Phase 3: Adjustments for the solving strategy and support tool and parameter calibration:*** In this phase necessary adjustments were made to guarantee the solving strategy robustness; formal experiments were made to calibrate all the execution parameters for the support tool.

2. CHAPTER II. LITERATURE REVIEW

In this chapter a complete literature review is provided; the literature review was made specifically for Vehicle Routing Problem with Multiple Depots (MDVRP). This review is presented as follows: first, the definition of the literature review and its scope is presented; then development works for the MDVRP with single objective are listed and detailed; subsequently, the development works for the MDVRP with multiple objectives are presented; finally an analysis of the literature and the desired level of robustness for solving strategies for the MDVRP is presented.

2.1. LITERATURE REVIEW DEFINITION AND SCOPE

The literature review is based on the work made by Montoya-Torres et al. (2012), which is a co-authored work, in this paper a complete literature review for the *Multiple Depots Vehicle Routing Problem* (MDVRP) is presented and discussed, and then a taxonomic classification was made to put "in the eye of the beholder" the current development in this field and to easily identify knowledge gaps. The motivation of a literature review exclusively for the MDVRP rather than VRP lies on that the MDVRP is more challenging and sophisticated than the single-depot VRP. The variant with multiple depots appears first in the literature on the works of Kulkarni and Bhave (1985), Laporte et al. (1988) and Carpaneto et al. (1989). Since then, considerable amount of research has been published (see Table 1) in the form of journal paper, conference paper, research/technical report, thesis or book.

To the best of our knowledge, despite the great amount of research papers published, there is no rigorous literature survey exclusively devoted to the vehicle routing problem with multiple depots. A short overview of academic works was proposed by Liu et al. (2011), but only presenting the most representative research papers. From the 58 cited references in their paper, only 23 of them explicitly refer to the MDVRP. Besides, these authors focus on the problem definition, solution methods (dividing them into exact algorithms, heuristics and meta-heuristics) and mention some problem variants. In fact, no actual systematic review was presented in that paper. Most of the published works focus on the single objective problem, while a few consider the multi-objective case. In this literature review, an analysis of both single and multiple objective problems is provided.

Table 1. Number and types of publications on MDVRP

Type of publication	Total
Journal	96
Conference	26
Thesis	7
Technical report	4
Book / book chapter	9
Total	**142**

Taken from Montoya-Torres et al. (2012)

An ambitious search was conducted using the library databases covering most of the major journals, such as *European Journal of Operational Research, Operations Research, Networks, Management Science, Computers & Operations Research, Journal of the Operational Research Society, Annals of Operations Research, Journal of Heuristics, IIE Transactions, International Journal of Production Economics, Computers & Industrial Engineering, Mathematical and Computer Modelling, Annals of Discrete Mathematics, Expert Systems with Applications, Transportation Science, Transportation Research Part C, Omega, IEEE Transactions on Automation Science and Engineering, RAIRO – Operations Research, Applied Artificial Intelligence, Applied Artificial Intelligence, Lecture Notes in Computer Science, 4OR: A Quarterly Journal of Operations Research,* etc. Some conference papers are also included in this review. In addition, the websites of leading research groups and the principal authors of major publications were also searched for further information about their research projects (PhD projects and sponsored projects) and publications. Working papers, theses and research reports were intentionally excluded, that were not available online on the Internet because they are very difficult to obtain.

The initial collection of references was screened first for their relevance and their significance for the purpose of this review. Only some representative publications were selected to be explained in detail within the text of this manuscript, which are authored by leading researchers or groups. These selected authors and research groups have, in fact, published a long list of research papers and reports in the field. A collection of over 115 representative publications are short-listed in this review (see Table 18 and Table 19 in Appendix). The short-listed publications are then examined in more detail. The analysis of methodological issues and problem variants are presented in detail in numeral (2.4).

2.2. THE SINGLE OBJECTIVE MDVRP

For the total of papers considered in this review, 89.3% considers only one optimization objective. The list of reviewed papers is presented in Table 18 in the Appendix. This section is divided into three main parts, each one corresponding to the type of solution procedure employed: exact method, heuristic or meta-heuristic. After the first works were published in the decade of 1980, more than a hundred of papers have studied the classical version of the MDVRP and its variants, some of them inspired from real-life applications.

2.2.1. Exact Methods

In the decade of the 1970's, some works already mentioned some problems related to distribution of goods with multiple depots. However, to the best of our knowledge, the first paper presenting formal models or procedures to find optimal solution for the multi-depot vehicle routing problem are those of Laporte et al. (1984) who formulated the symmetric MDVRP as integer linear programs with three constraints. These authors then proposed a branch-and-bound algorithm using a LP relaxation. The works of Kulkarni and Bhave (1985), Laporte et al. (1988) and Carpaneto et al. (1989) can also be considered as part of the pioneer works on exact methods for the MDVRP. The mathematical formulation proposed by Kulkarni and Bhave (1985) was later revised by Laporte (1989). More recently, Baldacci and Mingozzi (2009) proposed mathematical formulations for solving several classes of vehicle routing problems including the MDVRP, while Nieto Isaza et al. (2012) presented an integer linear program for solving the heterogeneous fleet MDVRP with time windows. Dondo et al. (2003) proposed a mixed-integer linear programming (MILP) model to minimize routing cost in the HFMDVRP, in which heterogeneous fleet of vehicles are available. The variant with pickup and deliveries and heterogeneous fleet was modeled by Dondo et al. (2008) using MILP model: this model is able to solve only small sized instances, and hence a local search improvement algorithm was then proposed by the authors for medium to large sized instances. This approach was later employed by Dondo and Cerdá (2009) to solve the HFMDVRPTW. The work of Kek et al. (2008) proposes a mixed-integer linear programming model and a branch-and-bound procedure for the MDVRP with fixed fleet and pickup and delivery. The objective function is the minimization of the total cost of routes. Cornillier et al. (2012) presented a MILP model for the problem in which heterogeneous fleet of vehicles is available and with maximization of total net revenue as objective function, while maximum and minimum demands constraints are given.

2.2.2. Heuristics

Because the NP-hardness of the MDVRP, several heuristic algorithms have been proposed in the literature. This section summarizes some of the most relevant works concerning different variants of the problem. The first works were published in the 1990's, in order to solve the capacitated version. Min et al. (1992) studied the version of the MDVRP with backhauling and proposed a heuristic procedure based on problem decomposition. Hadjiconstantinou and Baldacci (1998) considered a real-life problem taken from a utility company that provides preventive maintenance services to a set of customers using a fleet of depot-based mobile gangs. Their problem consists on determining the boundaries of the geographic areas served by each depot, the list of customers visited each day and the routes followed by the gangs. The objective is to provide improved customer service at minimum operating cost subject to constraints on frequency of visits, service time requirements, customer preferences for visiting on particular days and other routing constraints. This situation was solved using the periodic variant: MDPVRP for which a five-level heuristic was proposed: first and second levels solve the problem of determining the service areas and service days (the periodic VRP); third level solves the VRP for each day; fourth level solves a TSP for each route, and fifth level seeks the optimization of routes.

Salhi and Sari (1997) proposed the so-called "multi-level composite heuristic". This heuristic found as good solutions as those known at that time in the literature but using only 5 to 10% of their computing time. The heuristic was also tested on the problem with heterogeneous fleet. Salhi and Nagy (1999) proposed an insertion-based heuristic in order to minimize routing cost. Later, these authors (Nagy and Salhi 2005) also studied the problem with pickup and deliveries (MDVRPPD). Their approach avoids the concept of insertion and proposes a method that treats pickups and deliveries in an integrated manner. The procedure first finds a solution to the VRP, then it modifies this solution to make it feasible for the VRPPD and it finally ensures that it is also feasible for the multi-depot case. Jin et al. (2004) modeled the MDVRP as a binary programming problem and presented two solving methodologies: two-stage and one-stage approaches. The two-stage approach decomposes the MDVRP into two independent subproblems: assignment and routing, and solves it separately. In contrast, the proposed one-stage algorithm integrates the assignment with the routing. Their experimental results showed that the one-stage algorithm outperforms the published two-stage methods.

The HFMDVRP, in which heterogeneous fleet of vehicles is available have captured the attention of researchers since the work presented by Salhi and Sari (1997). Irmich (2000) proposed a set covering heuristic coupled with column generation and branch-and-price algorithm for cost minimization for the heterogeneous fleet and pickup and delivery MDVRP. Dondo and Cerdá (2007) proposed a MILP model as well as a three-stage heuristic. Before, a preprocessing stage for node clustering is performed and a more compact cluster-based MILP problem formulation is developed. Many other papers have been appeared in literature on or before 2007 and solution approaches have primarily been focused on meta-heuristic algorithms. Hence, this will be discussed more in detail in the next Section.

Concerning the periodic MDVRP, few works appear in literature with heuristic algorithm as solution approach. We have identified only the works of Hadjiconstantinou and Baldacci (1998), Vianna et al. (1999), Yang and Chu (2000), Maischberger and Cordeau (2011), and Maya et al. (2012). The problem with time windows was studied by Chiu et al. (2006) who presented a two-phase heuristic method. In contrast with other works in literature, these authors considered the waiting time as objective function. Results indicate that the waiting time has a significant impact on the total distribution time and the number of vehicles used when solving test problems with narrow time windows. The authors also considered a real-life case study of a logistics company in Taiwan.

Tsirimpas et al. (2007) considered the case of a single vehicle with limited capacity, multiple-products and multiple depot returns. Another characteristic of their problem is that the sequence of visits to customer is predefined. They developed a suitable dynamic programming algorithm for the determination of the optimal routing policy. For the MDSDVRP which consists on the combination of the MDVRP and the Split Delivery VRP (SDVRP). The work of Gulczynski et al. (2011) developed an integer programming-based heuristic. The objective of this study was to determine the reduction in traveled distance that can be achieved by allowing split deliveries among vehicles based at the same depot and vehicles based at different depots. The multi-depot capacitated vehicle routing problem with split delivery (MDCVRPSD) is studied by Liu et al. (2010). They proposed a mathematical programming model and its corresponding graph theory model, with the objective of minimizing empty vehicle movements. Also, a two-phase greedy algorithm was presented in order to solve practical large-scale problem instances. In the first phase, a set of directed cycles is created to fulfill the transportation orders. In the second phase, chains that are composed of cycles are generated. A set of local search strategies is also put forward to improve the initial results.

2.2.3. Meta-heuristics

As for other NP-hard combinatorial optimization problems, meta-heuristic procedures have been employed by several researchers for efficiently solving the single-objective MDVRP. The first meta-heuristic was proposed in the work of Renaud et al. (1996a) who studies the MDVRP with the constraints of vehicle capacities and maximum duration of routes (e.g. the time of a route cannot exceed the maximum working time of the vehicle). The objective to be optimized is the total operational cost. These authors proposed a Tabu Search algorithm for which the initial solution is built using the Improved Petal heuristic of Renaud et al. (1996b). Experiments were carried out using classical instances of Christofides and Eilon (1969) and Gillett and Johnson (1976). Later, Cordeau et al. (1997) proposed a Tabu Search algorithm with the initial solution being generated randomly for the MDVRP that can also be used to solve the periodic VRP (PVRP), while Tüzün and Burke (1999) proposed a Tabu Search procedure for minimizing the total cost of the routing. Cordeau et al. (2001) also proposed a TS procedure with the objective of minimizing the number of vehicles. An approximation to real industrial practice was studied by Parthanadee and Logendran (2006). In their problem, depots operate independently and solely within their own territories. The distributors may hence satisfy customers' requests by delivering products from other depots that hold more supplies. They proposed a mixed-integer linear programming model for the multi-product, multi-depot periodic distribution problem and presented three Tabu Search heuristics with different long-term memory applications. Results revealed that interdependent operations provide significant savings in costs over independent operations among depots, especially for large-size problems.

The first genetic algorithms were proposed by Filipec et al. (1997) for the problem of minimizing total travel distance, by Salhi et al. (1998) and by Skok et al. (2000, 2001). An evolutionary algorithm coupled with local search heuristic was proposed by Vianna et al. (1999) in order to minimize the total cost. Thangiah and Salhi (2001) proposed the use of genetic algorithms to define clusters of clients and then routes are found by solving a traveling salesman problem (TSP) using and insertion heuristic. This approach is called Genetic Clustering (GenClust). Solutions are finally optimized using the post-optimization procedure of Salhi and Sari (1997). Recently, Yücenur and Demirel (2011a) proposed geometric shape based genetic clustering algorithm for the classical MDVRP. The procedure is compared with the nearest neighbor algorithm. Their experiments showed that their algorithm provides a better clustering performance in terms of the distance of each customer to each depot in clusters, in a considerably less computational time.

In the survey by Gendreau et al. (2008) focused on the application of meta-heuristics for solving various variants of the VRP, a short revision of the multi-depot problem is presented. The equivalence between the MDVRP and the PVRP is also analyzed. Among the meta-heuristics presented therein, we can highlight the use of Genetic Algorithms (Filipec et al. 2000), Simulated Annealing (Lim and Zhu 2006) of the case of fixed vehicle fleet, and Tabu Search (Angeleli and Speranza 2002). Other works proposing meta-heuristics can be found in (Chao et al. 1993 and Chen et al. 2000).

The most studied variant of the problem has been the capacitated MDVRP. Among the meta-heuristics proposed in literature, we can highlight the Simulated Annealing algorithms of Wu et al. (2002) and Lim and Zhu (2006), the Variable Neighborhood Search procedure proposed by Polacek et al. (2005, 2008), Tabu Search algorithms from Lim and Wang (2005), Aras et al. (2011) and Maischberger and Cordeau (2011). Genetic Algorithms has been proposed as well for this problem variant, as illustrated in the works of Bae et al. (2006), Vidal et al. (2010). All of these works seek for the minimization of total route distance or cost, except the work of Aras et al. (2011) in which the objective is the maximization of vehicle utilization rate. It is to note here that the work of Aras et al. (2011) is inspired from a particular case of reverse logistics problem in which the aim is to collect cores from an enterprise's dealers. The problem is called the selective MDVRP with price. In addition to the Tabu Search procedure, the authors also formulated two mixed-integer linear programming (MILP) models.

Other meta-heuristics, such as GRASP, are presented in the works of Villegas et al. (2010) and Maya et al. (2012), respectively minimizing route cost and distance. Özyurt and Aksen (2007) solved the problem of depot location and vehicle routing using a hybrid approach based on Lagrangian relaxation (LR) and Tabu Search (TS). These procedures improve the best solutions found for the set of instances proposed by Tüzün and Burke (1999). A case study taken from waste collection system involving 202 localities in the city of Viseu, Portugal, is presented by Matos and Oliveira (2004). An Ant Colony Optimization (ACO) algorithm is proposed and compared with other procedures from the literature.

The great amount of heuristics algorithms proposed for the problem variant with heterogeneous fleet (HFMDVRP) has been focused on the design of meta-heuristics algorithms. We can highlight the works of Jeon et al. (2007), who proposed a hybrid genetic algorithm that minimizes the total distance traveled, and that of Filsberg et al. (2009) who considered a Tabu Search procedure. Simulated Annealing (SA) has been employed as well. Wu et al. (2002) coupled SA with Tabu Search to solve the heterogeneous fleet case of the integrated location-routing problem. In their problem, location of depots, routes of vehicles and client assignment problems are solved simultaneously. The multi-depot heterogeneous vehicle routing problem with time windows (MDHVRPTW) was studied by Dondo and Cerdá (2009), who proposed a MILP and a Local Search Improvement Algorithm that explores the neighborhood in order to find the lowest cost feasible solution.

Other research papers have also been very interested on the analysis of the problem with time windows (MDVRPTW). This variant is studied in 25% of the single-objective focused papers considered in this review. The first meta-heuristics reported in literature was the Tabu Search procedure of Cordeau et al. (2001) in which the objective function is the minimization of the number of vehicles. Polacek et al. (2005) proposed a Variable Neighborhood Search (VNS) algorithm for the MDVRP with time windows and with fixed distribution of vehicles. This problem was also studied by Lim and Wang (2005) with the characteristic of having exactly one vehicle at each depot. Jin et al. (2005), Yang (2008), Ghoseiri and Ghannadpour (2010) and Samanta and Jha (2011) proposed Genetic Algorithms, Pisinger and Ropke (2007) presented an Adaptive Large Neighborhood Search (ALNS) procedure with minimization of routing cost. Ting and Chen (2009) presented a hybrid algorithm that combines multiple ant colony systems (ACS) and Simulated Annealing (SA). Zarandi et al. (2011) presented a SA procedure to minimize routing cost, while Wang et al. (2011) coupled SA with a modified Variable Neighborhood Search algorithm, and a clustering algorithm is used to allocate customers in the initial solution construction phase. A branch-and-cut-and-price algorithm for the multi-depot heterogeneous vehicle routing problem with time windows (MDHVRPTW) was recently proposed by Bettinelli et al. (2011). Computational experiments showed that this procedure is competitive in comparison with local search heuristics.

The variants with split delivery (MDVRPSD) or with pickup & delivery (MDVRPPD) have been considered by very few authors in the scientific literature. The work of Wasner and Zapfel (2004) presents an application to postal, parcel and piece goods service provider in

Austria. The model employed is the MDVRPPD (MDVRP with pickup and deliveries) with the objective of determining the number and location of depots and hubs. Also, the client assignment problem is addressed. These authors develop a local search heuristic. As a real-life problem is solved, additional features are included in the algorithm in order to take into account the topography of the country (which is characterized by mountains) by considering maximum route length. The decision support system allowed the solution of large-sized instances with various millions of variables and constraints. The paper of Pisinger and Ropke (2007) studied the MDVRPPD, together with the variants of time windows and vehicle capacity constraint. These authors proposed an Adaptive Large Neighborhood Search procedure in order to minimize total routing cost. Flisberg et al. (2009) also considered heterogeneous fleet of vehicles and time windows constraints, in addition to pickups and split deliveries: their case-study is taken from a forest company in Sweden. Schmid et al. (2010) studied a realistic case inspired from companies in the concrete industry, and presented a mixed integer linear program (MILP) and a Variable Neighborhood Search (VNS) procedure to minimize routing cost for the variant with split deliveries. Mirabi et al. (2010) addressed the problem of minimizing the delivery time of vehicles. They compared three hybrid heuristics, each one combining elements from both constructive heuristic search and improvement techniques. The improvement techniques are deterministic, stochastic and simulated annealing (SA) methods.

Crevier et al. (2007) considered a MDVRP in which there are intermediate depots along vehicles' routes where they may be replenished. This problem was inspired from a real-life application at the city of Montreal, Canada. A heuristic combining adaptive memory, tabu search and integer programming was proposed. The model allows the assignment of vehicles to routes that may begin and finish at the same depot or that connect two depots to increase the capacity of vehicles to deliver goods. Zhen and Zhang (2009) considered a similar problem and proposed a heuristic combining the adaptive memory principle, a Tabu Search method for the solution of subproblems, and integer programming. Another variant of the MDVRP appears in the work of Zarandi et al. (2011). These authors studied the fuzzy version of the Capacitated Location-Routing Problem (CLRP) with multiple depots in which the location of depots have to be defined as well as the routes of vehicles. Fuzzy travel times between nodes and time window to meet the demand of each customer are considered. A simulation-embedded Simulated Annealing (SA) procedure was proposed. The framework was tested using standard data sets.

A good manner of improving the performance of meta-heuristics is to generate good initial solutions. Ho et al. (2008) proposed the use of the well-known Clarke & Wright Savings (C&WS) algorithm (Clarke and Wright 1964) to generate initial solutions, as commonly used for other vehicle routing problems (Juan et al. 2009). Once the solution is generated, the Nearest Neighbor (NN) heuristic is employed to improve such solution. In comparison with the random generation of initial solutions, their experiments showed that this hybrid C&WS + NN approach produces better results regarding total delivery time. Li and Liu (2008) considered the multi-depot open vehicle routing problem with replenishment during the execution of routes. They proposed a model and an Ant Colony Optimization resolution procedure.

Vidal et al. (2010, 2011) proposed a general framework to solve a family of vehicle routing problems, including the multi-depot VRP, the periodic VRP and the multi-depot periodic

VRP with capacitated vehicles and constrained route duration. Their meta-heuristic combines the exploration breadth of population-based evolutionary search, the aggressive improvement capabilities of neighborhood search based procedures and advanced population diversity management strategies. These authors improved the best known solutions and even obtained optimal values for these three problem cases.

2.3. THE MDVRP WITH MULTIPLE OBJECTIVES

An important characteristic of real-life logistics problems found in enterprises is that decision-makers, very often, have to simultaneously deal with multiple objectives. These objectives are sometimes contradictory (e.g., minimizing number of vehicles and maximizing service level). In the literature, there are very few papers on the MDVRP that consider multiple objectives (MOMDVRP): a bit less than 11% of the papers reviewed here.

The work of Lin and Kwok (2005) studies a realistic particular case of the MDVRP, named as location-routing problem (LRP) with multiple uses of vehicles. In this problem, decisions on location of depots, vehicle routing and assignment of routes to vehicles are considered simultaneously. Tabu search and simulated annealing procedures are designed and tested on both random-generated and real data. The objectives are the minimization of total operational cost and the balance on vehicle load. Both simultaneous and sequential procedures for the assignment of routes to vehicles are tested. Results show that the simultaneous versions have advantage over the sequential versions in problems where routes are capacity-constrained, but not in the time dimension. The simultaneous versions are also more effective in generating non-dominated solutions than the sequential versions.

A real-life transportation problem consisting on moving empty or laden containers is studied by Tan et al. (2006). They called the problem as the truck and trailer vehicle routing problem (TTVRP), but in fact it corresponds to a variant of the MDVRP: the solution to the TTVRP consists of finding a complete routing schedule for serving the jobs with minimum routing distance and number of trucks, subject to time windows and availability of trailers. These authors solved the multi-objective case using a hybrid multi-objective evolutionary algorithm (HMOEA) with specialized genetic operators, variable-length representation and local search heuristic. Lau et al. (2009) considered the multi-objective problem in which the travel time is not a constraint but an objective function to be optimized in the model together with the total traveled distance. The proposed solution procedure is a hybrid meta-heuristic named fuzzy logic guided non-dominated sorting genetic algorithm (FL-NSGA2). The procedure uses fuzzy logic to dynamically adjust the probabilities of mutation and crossover. The algorithm is compared with the well-known algorithms non-dominated sorting genetic algorithms 2 (NSGA2), strength Pareto evolutionary algorithm 2 (SPEA2) with and without fuzzy logic and the micro-genetic algorithm (MICROGA) with and without fuzzy logic. Experiments showed that the proposed FL-NSGA2 procedure outperformed the other procedures. This technique was also used by Lau et al. (2010) to solve the problem in which the cost due to the total traveling distance and the cost due to the total traveling time are minimized. In their work, several search methods, branch-and-bound, standard GA (i.e., without the guide of fuzzy logic), simulated annealing, and tabu search procedure are adopted to compare with FLGA in randomly generated data sets. Results of their experiments show that FLGA outperforms the other methods. Ombuki-

Berman and Hanshar (2009) and Weise et al. (2009) also proposed a genetic algorithm. The first authors considered the objectives of minimizing the total cost and the number of vehicles, while the latter authors considered the total distance, the idle capacity of vehicles and the number of externalized deliveries. A Simulated Annealing (SA) procedure was presented by Hasanpour et al. (2009) for minimizing transportation costs and maximizing probability of delivery to customers.

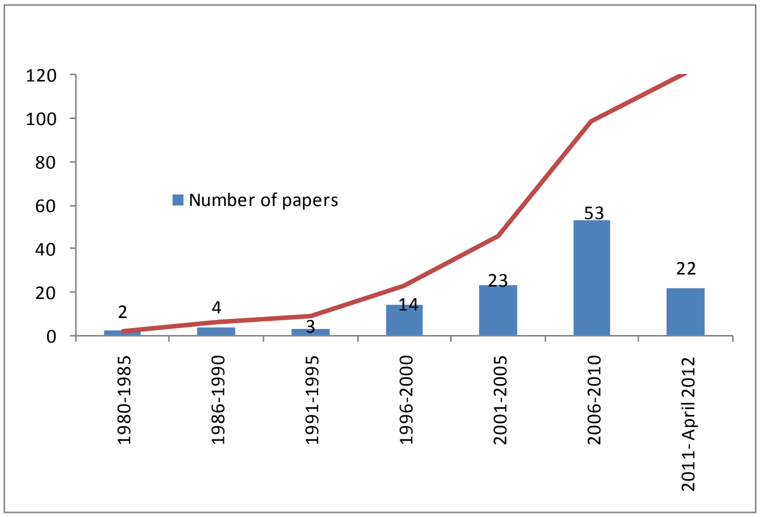
Weise et al. (2010) presented the use of evolutionary computation for a real-life problem inspired from a joint enterprise-academia research project. Results of the implementation are compared against the traditional routing structure employed by the enterprises associated with the research. The multi-objective MDVRP with time windows and split delivery is studied by Dharmapriya and Siyambalapitiya (2010). The objectives to be optimized were defined to be the total transportation cost, the total distance traveled, full use of vehicle capacity and load balancing. The problem is solved using Tabu Search, Simulated Annealing and a Greedy algorithm. Tavakkoli-Moghaddam et al. (2010) studied the multi-objective problem in which depot location and routes of vehicles have to be defined simultaneously. This problem is known in the literature as the Multi-Depot Location-Routing Problem. Traditionally, this problem is solved sequentially: first, the location of depots is addressed and then the routing of vehicles is approached. These authors proposed a scatter search algorithm that seeks to maximize total demand served and to minimize the total operational cost (cost of opening depots and variable delivery costs). Computational experiments showed that the proposed multi-objective scatter search (MOSS) algorithm outperformed an Elite Tabu Search (ETS) procedure.

Jiang and Ding (2010) minimized the distribution cost, the customer dissatisfaction and the changes of routes in a disruption measurement model and an immune algorithm. The procedure is tested using a simple example. Ghoseiri and Ghannadpour (2010) considered the problem of simultaneously optimizing total fleet size and total distance deviation by using a genetic algorithm coupled with goal programming. Finally, Venkatasubbaiah et al. (2011) proposed the use of a Fuzzy Goal Programming Method to solve the multi-objective problem. A fuzzy max-min operator is also proposed to improve the effectiveness of the procedure. The algorithm is tested on simple transportation problems from literature, and compared with previous works, while Li and Liu (2011) proposed a genetic algorithm so as to minimize the number of vehicles and the total travel distance.

2.4. ANALYSIS OF LITERATURE

As pointed out before, since the first publication on multi-depot vehicle routing problem appeared in 1984, more than 120 papers have been published in the scientific literature this problem and its variants. Between the middle of the 1980's (when the first works on the MDVRP were published) and the end of the twentieth century, very few papers were proposed in the literature (see Figure 1): only 18 publications (excluding those published in 2000), which gives an average of 1.12 papers per year. Between 2000 and 2005, there was an increase in the number of publication on MDVRP with an average of 4.7 publications per year. This gives a total of 46 publications until 2005. The most impressive growing on the number of papers published is observed between 2006 and April 2012) with a total 75 publications, including 4 papers already appeared in the first three months of 2012.

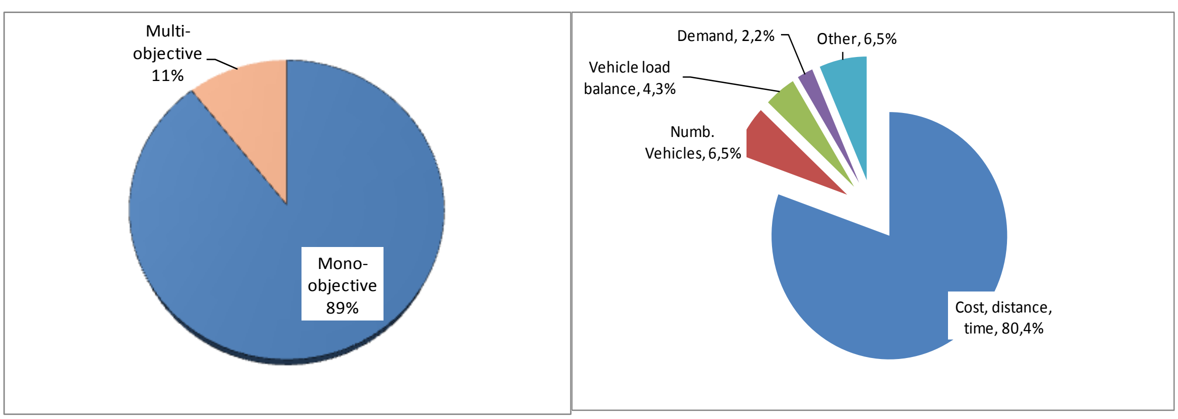
Figure 1. Distribution of published papers per year for the MDVRP



Taken from Montoya-Torres et al. (2012)

As can be seen, there is a clearly increasing trend which shows the growing interest in this field. It is reasonable to expect that in the coming years the MDVRP will receive an ever large amount of attention. There are however some remarks to be made. As shown in Figure 2, most of the works have been focused on the minimization of cost, distance or time. The papers dealing with vehicle load balancing are in fact papers that seek to optimize multiple objectives simultaneously (very often cost and vehicle load). As presented in previous sections of this review (see also Figure 2), majority of published works deals with the single objective problem. While this problem is of theoretical interest, very often decision-makers are faced to optimize multiple (contradictory) objectives. Very few published works deals with multi-objective problem. Hence, this gap in current research could be overlapped by proposing efficient and effective solution approaches for multi-objective environments.

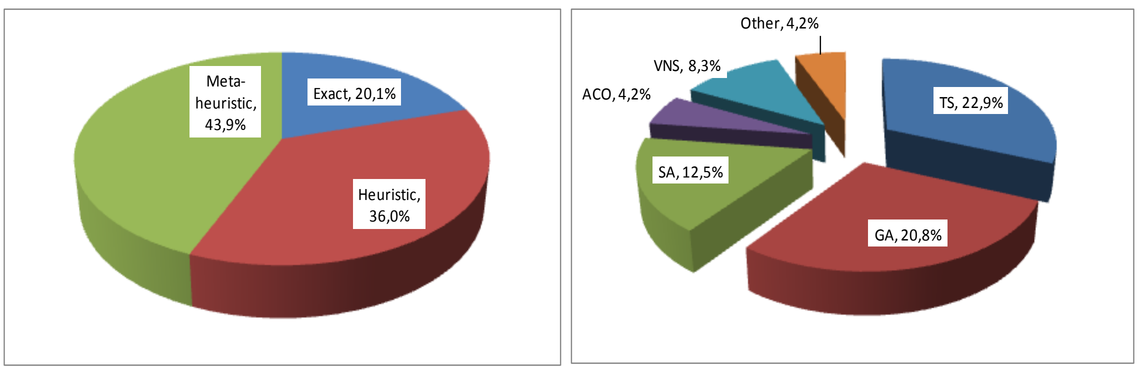
Figure 2. Distribution of objective functions



Taken from Montoya-Torres et al. (2012)

It is also interesting to study the different methodologies and techniques that the authors apply in the reviewed literature. Figure 3 shows two pie charts with this distribution: it first classifies the approaches as exact, heuristics and meta-heuristics algorithms, while the second pie presents a distribution of the different approximate algorithms employed in the reviewed literature. It can be observed that exact algorithms (branch and bound, mathematical programming) are employed in 20.1% of reviewed papers. We have to consider that these techniques have proven to be useful for simplified combinatorial optimization problems, specific settings and/or small instances. Hence, a larger focus is needed on approaches able to solve larger instances. Because of the NP-completeness of the MDVRP, approximate heuristics have also been proposed. Under the category of heuristic algorithms, a classification of many different algorithms and ad-hoc methods that are specific and do not contain a well known meta-heuristic template has been drawn. This represents 36% of the reviewed papers. For the other 43.9% a meta-heuristic algorithm is proposed. Among the available procedures, Tabu Search (TS), genetic algorithms (GA) and simulated annealing (SA) have been the most employed in the reviewed literature (22.9% uses TS, 20.8% uses GA and 12.5% uses SA). The other meta-heuristics have less employed: ant colony optimization (4.2%) and variable neighborhood search (8.3%). Clearly, there is a large opportunity for research here. Meta-heuristics have long ago established themselves as state-of-the-art methodologies for the vast majority of vehicle routing problems.

Figure 3. Distribution of employed solution techniques, TS = tabu search, GA = genetic algorithm, SA = simulated annealing, ACO = ant colony optimization, VNS = variable neighborhood search



Taken from Montoya-Torres et al. (2012)

Analysis of literature showed that for the Vehicle Routing Problem with Multiple Depots and Multiple Objectives (MO-MDVRP), meta-heuristics are the most feasible and adequate alternative; however, it is observed that population-based algorithms such as Genetic or Evolutionary Algorithms and hybridization of these outperform multi-objective versions of path-based or single point meta-heuristics such as Simulated Annealing and Tabu Search, this is shown in Lau et al. (2010), in this work experiments showed that a Fuzzy Logic Genetic Algorithm clearly outperforms Tabu Search and Simulated Annealing.

As a final conclusion, scientific research shows a clear trend for Population-Based algorithms to solve the MO-MDVRP; this is explained because of the effectiveness of these approaches to explore a wide solution space, since it starts on a set of initial solutions; moreover, these techniques are proven to be more effective to widely explore a solution space finding good areas; moreover, population-based meta-heuristics are less difficult to adapt to variety of multi-objective combinatorial problems as stated in Blum & Roli (2003).

2.5. DEFINITION OF LEVEL OF ROBUSTNESS FOR SOLVING STRATEGIES FOR THE MDVRP

According to the literature review and analysis, solving strategies for the multi-depot vehicle routing problems, have to be efficient and effective decision-making tools in a multi-objective environment such as physical distribution of goods, which is essential in logistics systems. Efficiency and effectiveness are defined as the capability to solve a very complex problem in a reasonable time ensuring at least very good solutions. In addition, research works on combinatorial optimization problems show the importance of hybridization of meta-heuristics as pointed out in Blum & Roli (2003).

On the other hand, the literature review has shown that future lines for research must consider multi-criterion environments and non-traditional objective functions. Another aspect to consider is that the most realistic approach to solve these particularly complex problems is an approximate method, where Genetic Algorithms and Tabu Search have been the most popular; this draws a future line for research to explore other techniques like Grasp, Scatter Search, Ant Colony and hybrid procedures which are of a particular interest.

Performance analysis for the MO-MDVRP solving approaches (reviewed works presented in section 2.3) is focused on execution times, objectives comparison, convergence metrics, and Pareto Frontier generation capabilities; up to this date a robustness analysis including execution times, quality of solutions and the capability to generate equally expected quality of solutions independently of problem shape and size has not been made. Robustness will be measured in three dimensions: Efficiency of the solving strategy, measured as the capability to find solutions in reasonable times (related to the decision making horizon); Capability of the solving strategy to find good solutions, supported on the quality of the solutions found compared to exact results; and the capability to improve solutions independently of the problem structure i.e. problem size (number of clients and number of depots) and topology.

This research work is oriented to develop solving strategies for the MDVRP considering knowledge gaps supported by the literature review and analysis. The proposed solving strategy aims to provide robust solutions for a multi-objective vehicle routing problem with multiple depots; this research proposes a robustness analysis including execution times analysis (efficiency), quality of solutions (Pareto Frontier closeness) and an experimental analysis to evaluate the capability to generate equally expected quality of solutions independently of problem shape and size.

The proposed strategy tackles optimization on a multiple criteria environment considering non-financial functions like load balance in addition to a critical objective such as total operational cost. Moreover, the proposed solving strategy is based on a hybrid procedure based on Scatter Search which has not been widely used for the MDVRP.

3. CHAPTER III. THE SOLVING STRATEGY FOR THE MULTIPLE DEPOT VEHICLE ROUTING PROBLEM WITH MULTIPLE OBJECTIVES. MO-MDVRP

In this chapter, the design of the solution strategy is presented. This chapter includes the MDVRP formulation, as a single objective problem (classic formulation in section 3.1) and the specific formulation (minimization of total cost and load imbalance in section 3.2). The proposed MILP formulation is presented (sub-section 3.2.2); and then, the proposed solving algorithm design is developed (in section 3.3).

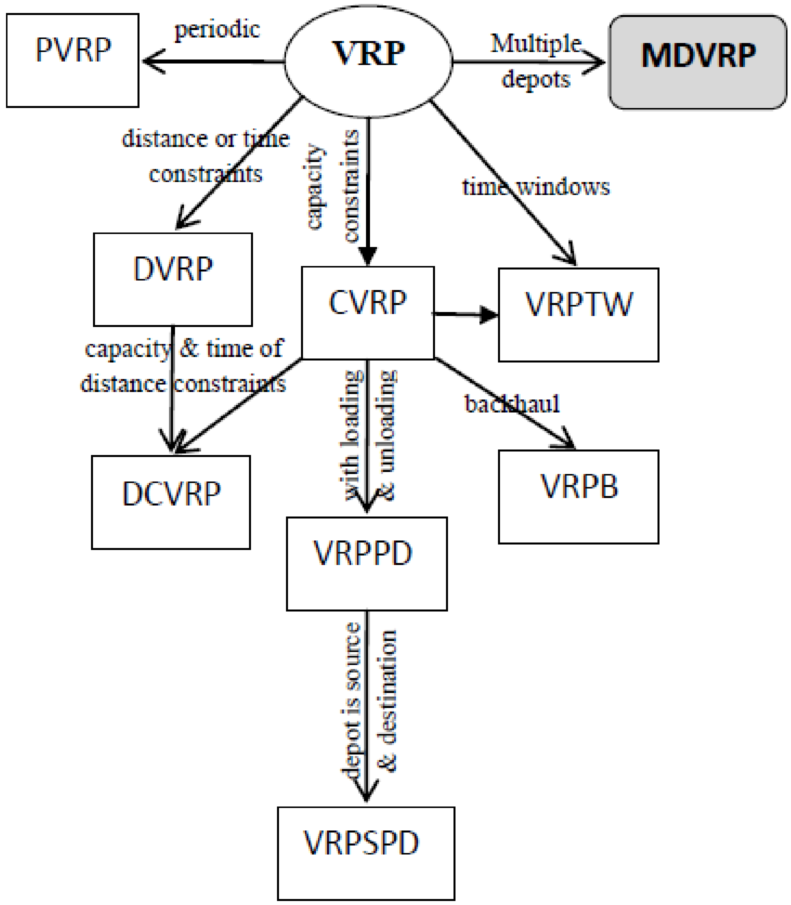
3.1. PROBLEM FORMULATION: *Vehicle Routing Problem (VRP) vs Vehicle Routing Problem with Multiple Depots (MDVRP)*

Formally, the classical Vehicle Routing Problem (VRP) is represented by a directed graph $G(E,V)$, where $V = \{0,1,\ldots n\}$ represents the set of nodes and E is the set of arcs. The depot is noted to be node $j = 0$, and clients are nodes $j = 1, 2, \ldots, n$, each one with demand $d_j > 0$. Each arc represents a route from node i to node j. The weight of each arc $C_{ij} > 0$ corresponds to the cost (time or even distance) of going from node i to node j. If $C_{ij} = C_{ji}$ then we are facing the symmetric VRP, otherwise the problem is asymmetric. From the complexity point of view, the classical VRP is known to NP-hard since it generalizes the Travelling Salesman Problem (TSP) and the Bin Packing Problem (BPP) which are both well-known NP-hard problems (Garey and Johnson 1979). A review of mathematical formulations for the classical VRP can be found in the work of Laporte (1992).

In the literature, lots of surveys have been presented analyzing published works on either the classical version (Bodin 1975, Bodin and Golden 1981, Laporte 1992, Desrochers et al. 1990, Maffioli 2002, Liong et al. 2008, Eksioglu et al. 2009) or its different variants: the capacitated VRP (Laporte and Nobert 1987, Gendreau et al. 2002, Toth and Vigo 2002, Laporte and Semet 2002, Cordeau et al. 2007, Baldacci et al. 2010), the VRP with heterogeneous fleet of vehicles (Baldacci et al. 2008, 2007, 2010), VRP with time windows (VRPTW), pickup and deliveries and periodic VRP (Solomon and Desrosiers 1988), dynamic VRP (DVRP) (Psaraftis 1995), Periodic VRP (PVRP) (Mourgaya and Vanderbeck 2006), VRP with multiple trips (VPRMT) (Şen and Bülbül 2008), Split Delivery vehicle routing problem (SDVRP) (Archetti and Speranza 2008).

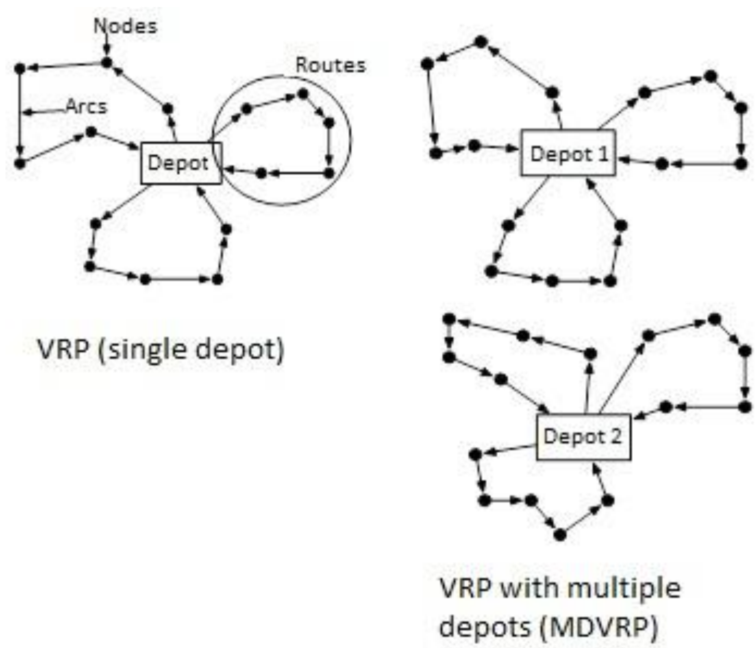
All of these works consider only one depot. Figure 4 presents a hierarchy of VRP variants. One of these variants considers a well-known (Crevier et al. 2007) more realistic situation in which the distributions of goods is done from several depots to final clients. This particular distribution network can be solved as multiple individual single depot VRP's, if and only if clients are evidently clustered around each depot; otherwise a multi-depot-based approach has to be used where clients are to be served from any of the depots using the available fleet of vehicles. This work considers the variant of the vehicle routing problem known as Multiple Depots Vehicle Routing Problem (MDVRP) in which more than one depot is considered (see Figure 5).

Figure 4. Variants of VRP problems



Taken from Montoya-Torres et al. (2012)

Figure 5. Comparison between VRP vs MDVRP



Taken from Montoya-Torres et al. (2012)

According to Reneaud et al. (1996a), the MDVRP can be formally described as follows. Let G=(V,E) be a graph, where V is the set of nodes and E is the set of arcs or edges connecting each pair of nodes. The set V is further partitioned into two subsets: V_c={ v_1, v_2,…, v_N} which is the set of customers to be served; and V_d={v_{N+1}, v_{N+2},…, v_M} which is the set of depots. Each customer $v_i \in V_c$ has a nonnegative demand d_i. Each arc belong to the set E has associated a cost, distance or travel time c_{ij}. There are a total of K vehicles, each one with capacity P_k. The problem consists on determining a set of vehicle routes in such a way that: (i) each vehicle route starts and ends at the same depot, (ii) each customer is serviced exactly once by a vehicle, (iii) the total demand of each route does not exceed the vehicle capacity, and (iv) the total cost of the distribution is minimized. According to the mathematical model of Kulkarni and Bhave (1985), the MDVRP can be formulated as follows.

(1)

Subject to:

(2)

(3)

(4)

(5)

(6)

(7)

(8)

For
and (9)

(10)

In this formulation, Constraints (2) and (3) ensure that each customer is served by one and only one vehicle. Route continuity is represented by Constraints (4). The sets of constraints (5) and (6) are the vehicle capacity and total route cost constraints. Vehicle availability is verified by Constraints (7) and (8) and subtour elimination is provided by Constraints (9). In this formulation, it is assumed that total demand at each node is either less than or at the most equal to the capacity of each vehicle.

3.2. THE MULTI-OBJECTIVE MDVRP: COST AND LOAD BALANCE

According to the problem statement and literature analysis, not only financial objective functions e.g.: Total Cost must be considered to improve transportation logistics, specifically distribution of goods; for this reason, another objective function is considered to improve resource utilization: Load Balance, measured as the allocated load range, as defined in Lin and Kwok (2005), i.e.: the difference between the maximum load assigned and the minimum load allocated to the vehicles. To deal with this problem, a meta-heuristic approach is proposed, based on Scatter Search algorithms, this Scatter Search algorithm is hybridized with several heuristics, SPEA and SPEA2 procedures, and evolutionary concepts. To help performance evaluation for the Multi-objective Scatter Search Hybrid Algorithm, a multi-objective MILP mathematical model was developed; an alternative formulation that avoids sub-tour elimination constraints is used, based on the Dondo & Cerdá (2007) formulation.

3.2.1. Mathematical Formulation for the MO-MDVRP

Sets

Set of nodes
Set of vehicles
Set of depots

Parameters

Vehicle capacity

Fix cost per use of vehicle

Travel cost for the arc $i - j$

Travel cost for the arc $i - p$

Demand for node *i*

Variables

Binary variable which denotes that vehicle *v* is assigned to depot *p*

Binary variable which denotes that node *i* is assigned to vehicle *v*.

Binary variable which denotes that node *i* is visited before node *j* or after node *j*

Route cost up to node *i*

Total route cost for vehicle *v*

Total load assigned to vehicle *v*

(1) *Objective Functions*

Subject to:

(2) *Client to vehicle assignments*

(3) *Vehicle to depot assignments*

(4) *Route cost up to client* ***i***

(5) *Vehicle sequencing constraints*

(6) *Total route cost for vehicle*

(7) *Vehicle load variable definition*

(8) *Vehicle capacity constraints*

3.2.1.1. Objective Functions

Two objective functions F1 and F2 are defined for the MO-MDVRP problem. F1 is defined as the total cost for the routing problem, consisting of fix costs per use of each vehicle and a variable cost depending on client assignments and sequencing for each route. The objective function F2 is defined as a load balancing equation (vehicle assigned load range), measured as the difference between the maximum load assigned to a vehicle and the minimum load assigned.

3.2.1.2. Problem Decision Variables

As explained in Dondo & Cerdá (2007), three types of binary variables are used: , this set of variables determines whether vehicle *v* *V* is allocated to depot *p* *P*; , which denotes if client *i* *I* is assigned to vehicle *v* *V*; and sequencing variables , which determines whether client *i* *I* is visited before node *j* (or after node *j* (. Note that variables are meaningful only if nodes (*i*,*j)* *I* are on the same route ; only one sequencing variable is defined for each pair of nodes (*i*,*j)*, this means that sequencing variables consider the relative order of each pair of nodes (*i*,*j)* in the set *I* (set of nodes is an ordered set) such that the variables are defined if the relative position of element (node) *i* is lower than the relative position of element *j*. As defined before, variables and define routing cost up to node *i* *I* and total routing cost for vehicle *v* *V*, respectively. For the MO-MDVRP mathematical formulation, a positive variable which defines the total load allocated to vehicle *v* was introduced, this variable is necessary to calculate the load assignment range for the routing plan.

3.2.1.3. Problem Constraints

Constraints (2) are used to assign client nodes to vehicles; it forces to attend one client node using only one vehicle. Constraints (3) allocate each vehicle to at least one depot; respective vehicle binary variables sum, must be maximum 1; it means that if the respective *depot – vehicle* binary variables sum is 1, the vehicle is allocated to one depot, and if its sum is 0, the vehicle is not used. Constraints (4) state that the routing cost up to node *i* ( ), must be at least the traveling cost from the depot to the respective node ( ), only if the client *i* is allocated to depot *p*, that is when the client node is assigned to vehicle *v* and the respective vehicle *v* is allocated to depot *p* (= = 1). Equations (5) are two sets of constraints defined for each vehicle and every combination (not permutation) of 2 nodes, that is, a constraint is declared for every vehicle *v* and every pair of client nodes (*i*,*j*), such that the relative position of node *i* in set *I* is lower than the relative position of node *j* in the same set (ord(*i*) < ord(*j*)); these constraints ensure that the routing cost up to node *j* ( ) is at least the traveling cost from the depot to the node *i* ( ) plus the travel cost for the arc *i*-*j* ( ) if the node *i* is a predecessor (not necessarily a direct predecessor) of node *j* in the sequence of the vehicle *v* (= 1); on the other hand another constraint is declared for the node *j* being a predecessor of node *i* (= 0); both constraints are valid only if both nodes *i* and *j* are in the same route i.e. = = 1. Note that these two sets of constraints are disjunctive, to deal with this type of constraints a "big M" ( ) approach is used. Constraints (6) are used to compute total routing cost for each vehicle *v* ( ), which must be greater than or equal to the routing cost up to node *i* ( ) plus the traveling cost from the depot to the node *i* (   ), for every node *i*; this constraint is valid if the client *i* is allocated to depot *p*, that is when the client node is assigned to vehicle *v* and the respective vehicle *v* is allocated to depot *p* (=   = 1). Equations (7) and (8) are used to compute the assigned load to each vehicle, which is the sum of allocated clients' demands to its respective vehicle.

3.2.2. A New Mixed Integer Linear Programing Formulation for the MO-MDVRP

The mathematical formulation for the MO-MDVRP presented in numeral (3.2.1.) cannot be solved because the balance function F2 defined in numeral *(1)* is a non-smooth function with discontinuous derivatives, which are forbidden for mixed integer linear programs (MILP); and the MDVRP formulation involves binary variables, which are forbidden for nonlinear programs with discontinuous derivatives (DNLP). According to this statement, the MO-MDVRP minimizing total cost and the assigned load range for all vehicles (load balance) is quite challenging, not only for its mathematical complexity, but also because there is no algorithm to solve exactly this problem due to the load balance function structure. To deal with this restriction an equivalent MILP formulation was developed; this new formulation avoids the use of the *maximum* and *minimum* functions to define the load balance function measured as the load range assigned to the vehicles (
).

The new MILP formulation introduces two continuous variables *R* and *Lmax*, where the variable *R* defines the load balance measured as the load range (the new objective function F2 is the variable R), and the variable *Lmax* defines the maximum load allocated to one vehicle. Moreover, two sets of equations are included to compute *R* and *Lmax* variables; each set of equations has |*V*| individual equations which are defined as follows in numerals *(9b)* and *(10b)*. The new MILP complete formulation is presented below.

(9b) Maximum load variable definition

(10b) Load range variable definition

Sets

Set of nodes
Set of vehicles
Set of depots

Parameters

Vehicle capacity
Fix cost per use of vehicle
Travel cost for the arc *i* – *j*
Travel cost for the arc *i* – *p*
Demand for node *i*

Variables

Binary variable which denotes that vehicle *v* is assigned to depot *p*

Binary variable which denotes that node *i* is assigned to vehicle *v*.

Binary variable which denotes that node *i* is visited before node *j* or after node *j*

Route cost up to node *i*

Total route cost for vehicle *v*

Total load assigned to vehicle *v*

Lmax Maximum load allocated to one vehicle **(*)**

R Load range **(*)**

(1) *Objective Functions*

(*)

Subject to:

(2) *Client to vehicle assignments*

(3) *Vehicle to depot assignments*

(4) *Route cost up to client* ***i***

(5) *Vehicle sequencing constraints*

(6) *Total route cost for vehicle*

(7) *Vehicle load variable definition*

(8) *Vehicle capacity constraints*

(9) *Maximum load variable definition* **(*)**

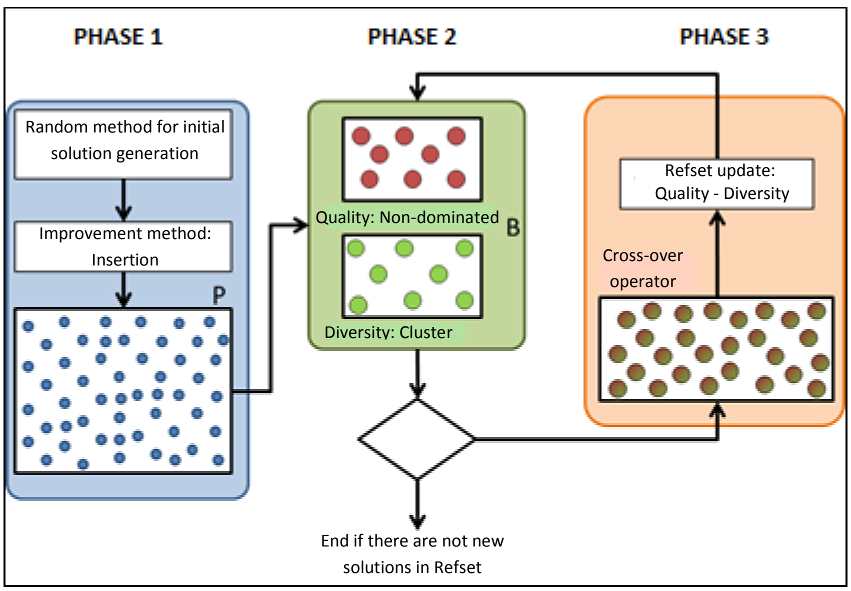
(10) *Load range variable definition* **(*)**

(*) Indicates: New statement.

3.3. THE HYBRID MULTI-OBJECTIVE SCATTER SEARCH ALGORITHM

The proposed methodology to solve the MDVRP is based in the problem decomposition method; a cluster first and route second approach is adapted and then optimized by a hybrid Scatter Search algorithm, local improvement and solution polishing heuristics. As mentioned in Tavakkoli-Moghaddam et al. (2010), Scatter Search (SS) is a robust solution method for solving combinatorial problems such as binary problems, assignment problems, and scheduling and routing problems; moreover, due to its population-based approach, it is quite useful and simple to work on multi-objective optimization problems; in addition, as concluded in the literature review (see Figure 3), SS procedures have not been widely used for the MO – MDVRP, which makes SS an interesting alternative to develop a solving strategy for the MDVRP.

Figure 6. General structure for the scatter search algorithm



Taken from López & Nieto (2012)

The General Structure for the Scatter Search-based algorithm is defined by a three phase procedure. As shown in Figure 6, in *phase 1*, a solution generation method is applied to build **P** diverse feasible solutions for the MDVRP problem, the solution method consists of client to depot assignment, route generation and a local improvement procedure based on a sequential insertion heuristic; in *phase 2*, a reference set **B** is selected from the initial set of **P** solutions generated in phase 1, where **|B|/2** solutions are selected by a quality criterion and the remaining **|B|/2** are selected by a diversity criterion. In *phase 3*, a hybrid evolutionary procedure is executed updating the reference set by the replacement of the solutions with new solutions generated by a cross-over and improvement procedure, if any new solution outperforms at least one of the solutions in the reference set, the evolutionary procedure is executed until no new solution is found to update the reference set **B**. The general MOSS procedure is shown in Figure 7.

Figure 7. Hybrid MOSS General Structure

Hybrid MOSS Procedure

- Generate diverse random solutions (P)
- Apply solution improvement procedure to (P)
- Build a reference set of solutions RefSet (B), with b1 quality (multi-objective) solutions and b2 most diverse solutions of (P) set

While (∃ solution in (B))

- Apply selection rule for RefSet solution combination

While (∃ solution in (B) that has not been combined)
- Apply combination method to generate new solutions (B')
- Apply correction method and improvement method to new solutions (B')

Let **x** be and improved new solution

If ($F(f_1(\mathbf{x}), f_2(\mathbf{x})) < F(f_1(\mathbf{x}^B), f_2(\mathbf{x}^B))$ *and* ($\mathbf{x} \notin$ RefSet))
If **x** dominates $\mathbf{x}^B$, then **x** replaces $\mathbf{x}^B$
End If

End While

End While

End Procedure

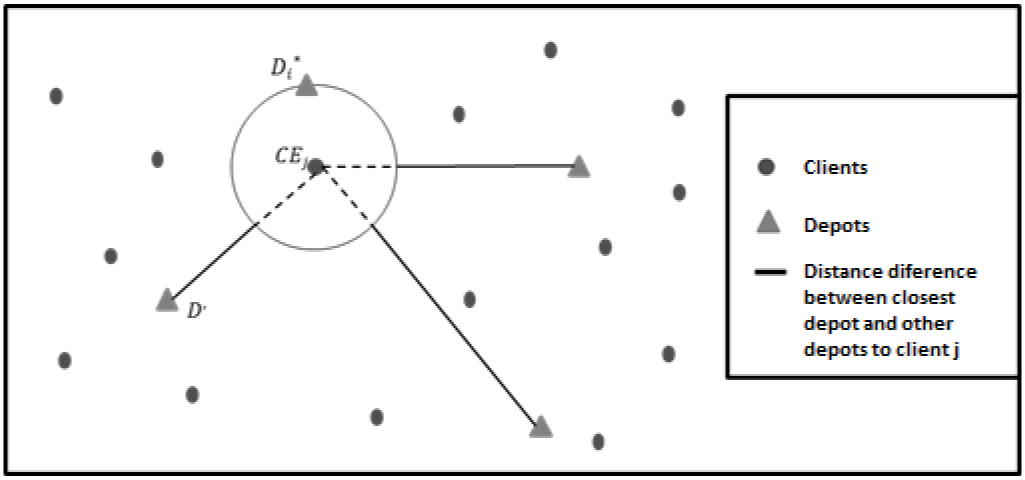
3.3.1. Diverse Solution Generation Method

As presented in López & Nieto (2012), the diverse solution generation method, is based on the *cluster first - route second* method, which, adapted to the MDVRP, consists of client-to-depot assignment, client-to-vehicle assignment, and client sequencing. The client-to-depot assignment procedure is based on a *Simplified Assignment of Highest Urgency* method, Giosa et al. (1998), in which clients are assigned, prioritizing by highest urgency criterion, to its closest depot with unfulfilled capacity; the urgencies for a client ***j*** **(μ_j)** and its closest and second closest depots **D*** and **D'**, respectively, are calculated as follows:

$$\mu_j = d(j,D') - d(j,D^*) \qquad \textbf{(Eq 3.3.1.)}$$

Where, *d(j,D')* is the assignment cost of client *j* to the second closest depot; and *d(j,D*)* is the assignment cost of client *j* to its closest depot.

Figure 8. Client-depot assignment by simplified assignment urgency procedure



Taken from López & Nieto (2012).

Figure 8 illustrates the urgency computation for a certain client.

The client-to-vehicle assignment is executed by a random procedure; the objective of this randomized procedure is to generate multiple solutions dispersed in a wide solution region. Client sequencing is done by a sequential insertion based on the Mole & Jameson (1976) algorithm; the purpose of this procedure is to locally improve the randomized solutions, such that better solutions may enter the evolutionary procedure.

3.3.2. Reference Set of Solutions

Given the **P** set of initial solutions, the scatter search procedure will apply systematic combinatorial procedures to a relatively small subset of solutions; this set is called **Reference Set**, from now on: **Ref-Set B** (a major difference with evolutionary algorithms such as genetic algorithms, SPEA, SPEAII, etc.). The Ref-Set **B** is selected from the **P** set based on two criteria: Quality solutions and diverse solutions, each subset with **|B/2|** solutions.

3.3.2.1. Quality Solutions Selection for Ref-Set B

Since the problem to solve is a multi-objective optimization problem, quality for a given solution must be evaluated carefully; this is because in this case two different and in many cases conflictive objectives must be improved. To deal with this situation a multi-criteria measure was calculated, based on non-domination criteria using the fitness calculation defined for SPEA2 algorithms, Zitzler et al. (2001). The **|B|/2** solutions with lower (better) fitness are selected to enter to the Ref-Set; note that in most cases the quality solutions will be the non-dominated solutions of the **P** set (initial solutions).

3.3.2.2. *Diverse Solutions Selection for Ref-Set B*

The idea of selecting the most diverse solutions is to enter **|B|/2** elements from the **P** set, distributed all across the solution space; this is, to select the most distanced elements in **P**. To pick this subset of elements in the solution space, a multi-objective clustering procedure was implemented. The clustering procedure computes **|B|/2** different clusters; then, the element with the lowest Euclidean distance to the centroid of the cluster is selected. This selection procedure based on clustering is defined in Figure 9.

Figure 9. Clustering procedure for diverse solution selection

Clustering Procedure

- Set all solutions in **P** to an individual cluster

While (number of clusters > |B|/2)

- Compute distance between all clusters (for cluster with more than 1 element, mean distance is used)
- Merge closest pair of clusters

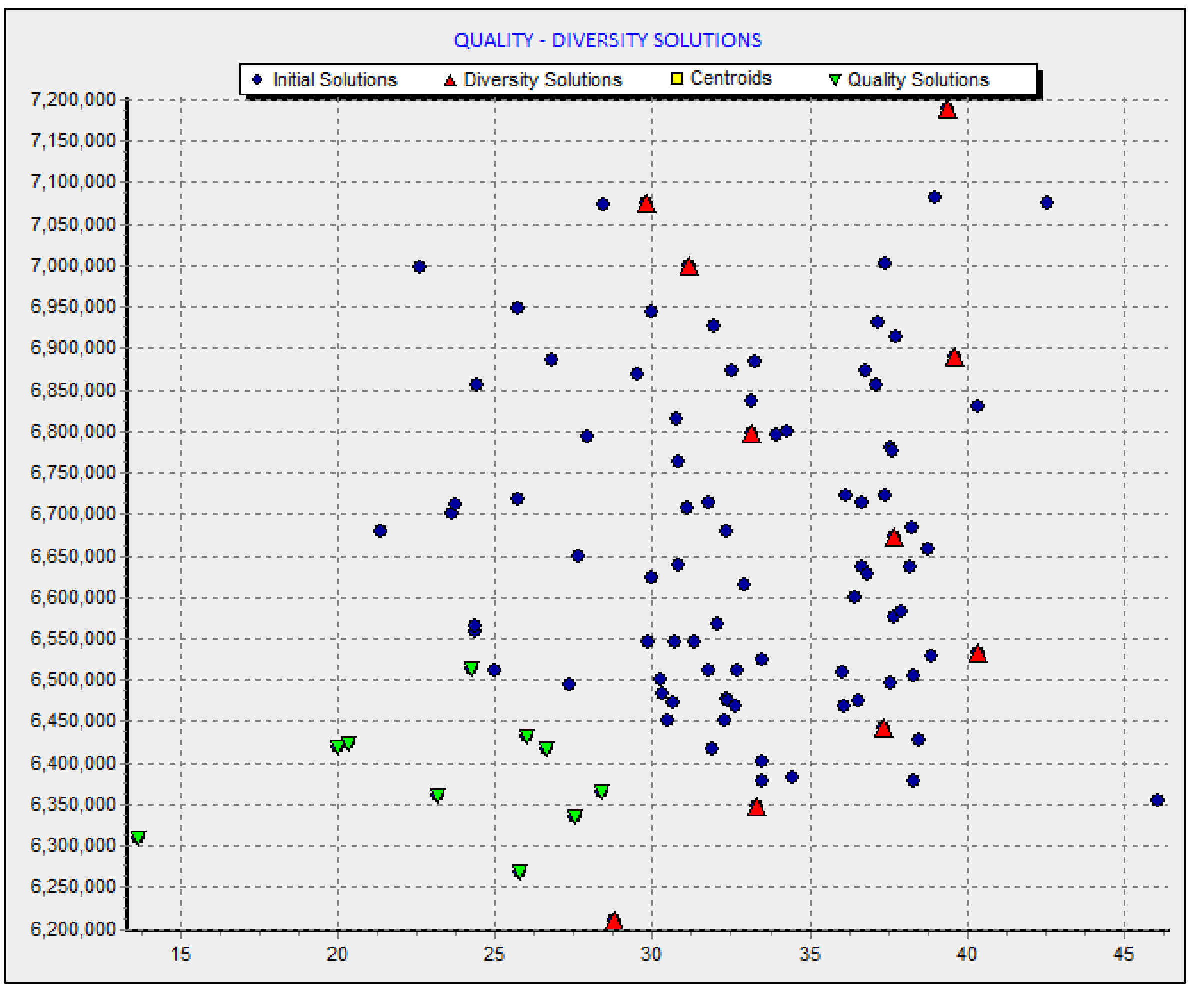
End While

- Calculate centroid for each cluster
- Select the closest element (solution) to the centroid of each cluster
- Include selected elements in the Ref-Set **B**

End Procedure

Figure 10 shows a screenshot taken from the router prototype that illustrates Ref-Set for an instance of 300 customers and a set **P** of 100 solutions.

Figure 10. Initial solution space



Taken form López & Nieto (2012)

As shown in the figure above, quality solutions have a trend to the min-min (y axis represent total cost, and x axis represent load balance) zone, and diversity solutions are dispersed across the solutions space.

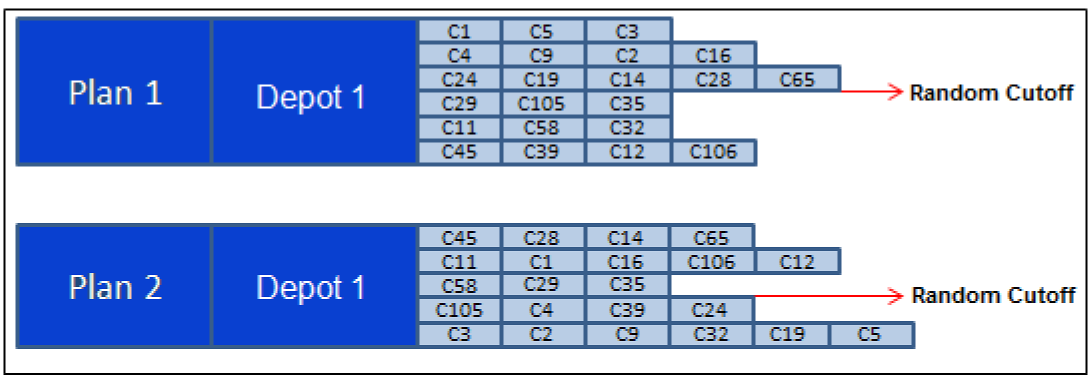
3.3.3. Cross-over Procedure

Cross-over operator allows the MOSS algorithm to explore and intensify solution space to find new solutions which are potentially better or non-dominated solutions. To explore the solution space in a wide range, a random based cross-over operator was used and then a multi-criteria (total cost and load balance) correction heuristic (will be explained after) was designed to improve Ref-Set actualization. Cross-over procedure takes every solution in the **B** set and applies the cross-over operator to a pair of solutions determined by a sub-set generation procedure designed to improve diversification for the MOSS algorithm.

3.3.3.1. Cross-over operator

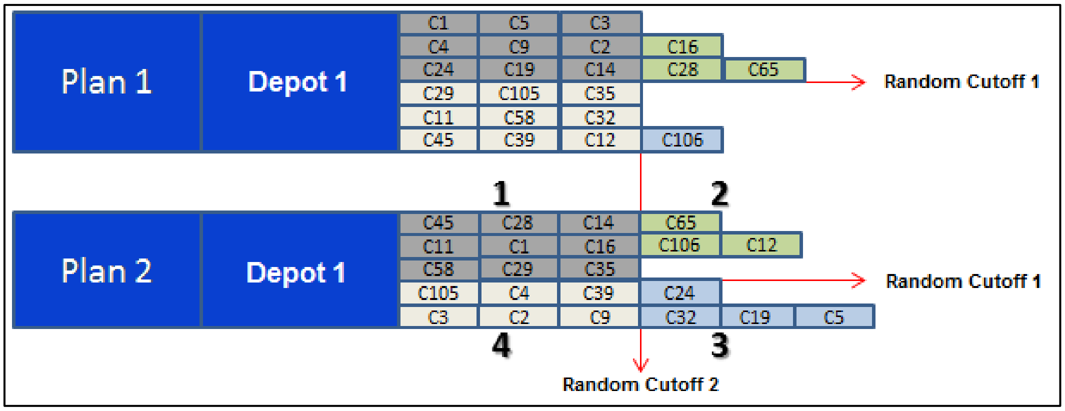
Cross-over operator is based on a random partitioning procedure (widely used in Genetic Algorithms) to generate solutions in a wide range and to avoid early local optimum convergence. Cross-over is applied within and between depots, and the method can be described as follows: Two cutoff points are defined, a cutoff point between routes and a cutoff point within clients; these points split each solution into four parts. The first cutoff point is randomly (uniform probability) determined; the cutoff point is generated based on the instance with the lowest number of routes; the cutoff point within clients is based on the route with the highest number of clients. Cross-over operator generates four new solutions. Figure 11, Figure 12 and Figure 13, outline the cross-over operator.

Figure 11. Cutoff point between routes for cross-over operator



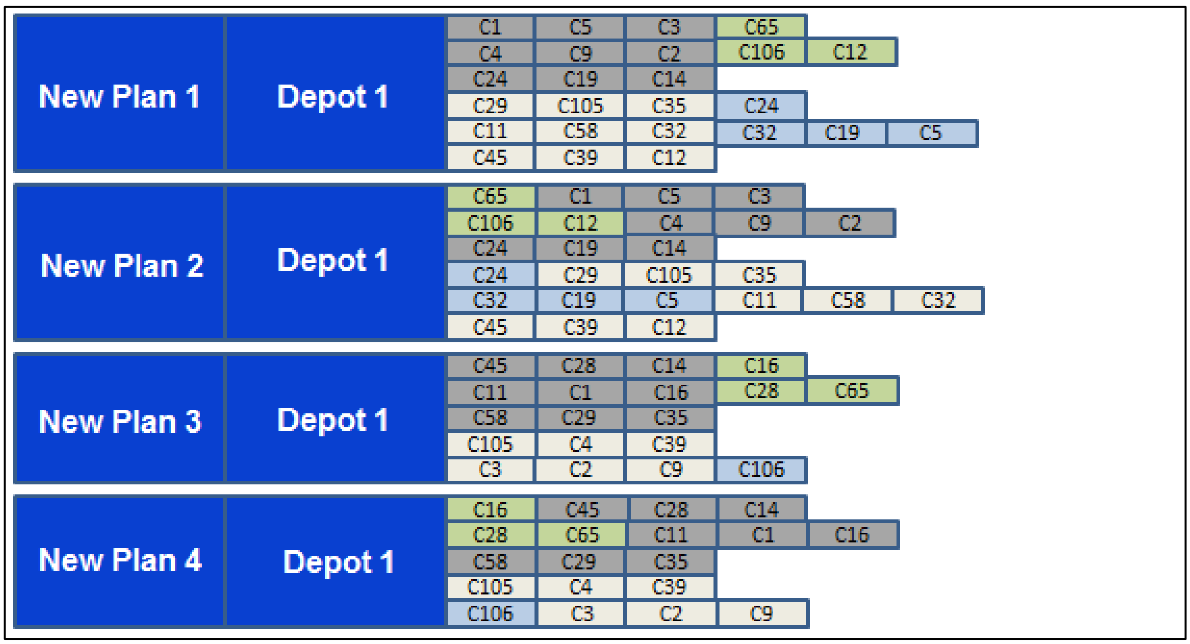
Taken from López & Nieto (2012)

Figure 12. Second cutoff point within clients



Taken from López & Nieto (2012)

Figure 13. Resulting solutions applying cross-over operator for two plans



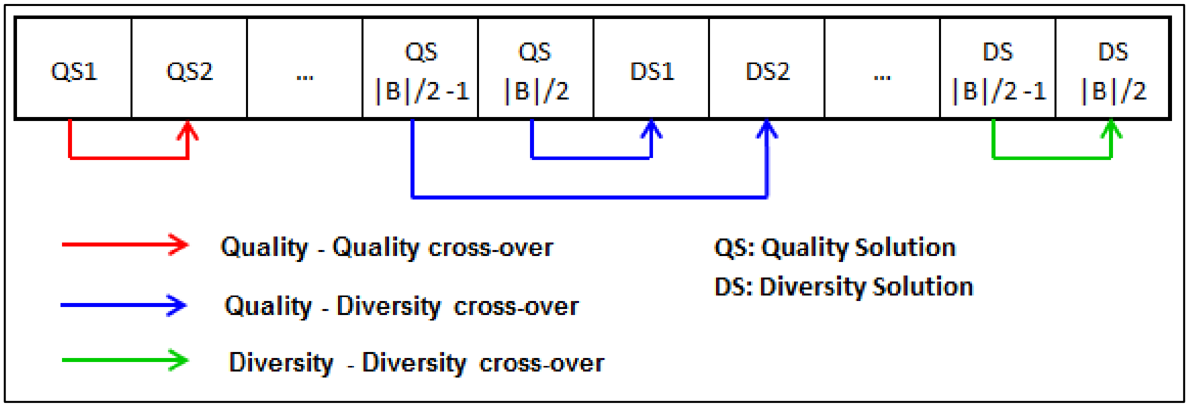
Taken from López & Nieto (2012)

3.3.3.2. Subset generation method

As described in Martí & Laguna (2003), one of the differences between Genetic Algorithms and Scatter Search Algorithms is the way in which combinatorial procedures are made; Scatter Search uses a systematic method to cross solutions in Ref-Set instead of crossing solutions based on random procedures; this difference makes Scatter Search more exhaustive on a relatively small sub-set of the solution space.

Sub-set generation method was designed to ensure that the best (quality) solutions and most diverse (diversity) solutions are crossed within and between them; to do this, Ref-Set was divided into four parts, where the first two parts are quality solutions, and the last two parts of the Ref-Set are diversity solutions. The Sub-set generation procedure builds pairs of solutions of quality (quality – quality), pairs of solutions of diversity (diversity – diversity), and quality and diversity pairs of solutions (quality – diversity); then, cross-over operator is applied to every generated pair of solutions. Figure 14 illustrates this method.

Figure 14. Sub-set generation method for the cross-over procedure



3.3.4. Optimization Procedure

3.3.4.1. Correction and improvement heuristics

The cross-over operator was designed taking certain concepts used and proved for Genetic Algorithms and its multi-objective modifications such as SPEA and SPEA2. This cross-over operator, as mentioned before, splits each solution from the selected set (selected pair of solutions to apply cross-over operator by sub-set generation method) into four parts by a random method, and then four new solutions are generated combining every part of each split solution; this type of cross-over operator and the sub-set generation method was adapted to explore widely the solution space; however, cross-over does not ensure that the new generated solutions are feasible; to correct every infeasible solution, a correction and multi-criteria (cost and load balance) improvement heuristic was designed.

The bi-criteria improvement and correction heuristic identifies every infeasible solution, considering capacity constraint violation for each route (vehicle) and repeated client assignment. If the infeasibility lies on repeated clients, every repeated client is removed from the route and placed into a temporal list; on the other hand, if the identified feasibility problem is capacity constraint violation, a predetermined number (as many as required to satisfy capacity constraint) of clients of lowest demand are removed and placed in the temporal list to satisfy capacity constraint.

Given the temporal list (list of removed clients), an allocation method is applied; this method allocates clients from the temporal list using a bi-criterion parameter, this parameter is calculated for every (client – route) pair, and its goal is to allocate clients to the closest route with maximum vehicle utilization.

A client–to–route proximity is defined as the distance (Euclidean distance) between the client and the centroid of the route (calculated as the mean coordinates of every client in the route); and the load measure is defined as the difference between vehicle's idle capacity and client's demand. The heuristic allocates the client which minimizes the bi-criterion parameter (if it is feasible) and stops until every client is allocated to one route. The bi-criterion parameter matrix $\boldsymbol{\lambda(i,j)}$ where the subscript $\boldsymbol{i}$ is the unassigned client and $\boldsymbol{j}$ is every existing route is defined as follows.

$$\lambda(i,j) = (IC_j - D_i) + d(i,k_j) \qquad \textbf{Eq(3.3.2)}$$

Where $\lambda(i,j)$ is the bi-criterion parameter matrix; IC_j is the idle capacity of route $\boldsymbol{j}$; D_i is client's demand; and $d(i,k_j)$ is the distance from the client $\boldsymbol{i}$ to the centroid of route $\boldsymbol{j}$ (k_j). In Figure 15 the Correction and improvement heuristic is detailed.

Figure 15. Correction and improvement heuristic procedure

Correction and improvement heuristic procedure

- Identify infeasible routes with repeated clients
- Set repeated clients in the temporal list
- Identify infeasible routes with violation of capacity constraint

While (Total allocated load > Vehicle capacity)
Set lowest demand client in the temporal list
End while

While (Temporal list ≠ ϕ)
- Compute $\lambda(i,j)$ for every client in temporal list
- Allocate client i with minimum $\lambda(i,j)$ to route j

End while
End Procedure

3.3.4.2. Reference set actualization

As explained in section *3.3.3.2.* the sub-set generation method splits Ref-set into 4 parts to ensure cross-over within and between quality and diversity solution; for this reason, Ref-set actualization method has to be updated respecting relative order by quality and diversity criterions, that is, half set is ordered by quality and the other half by diversity.

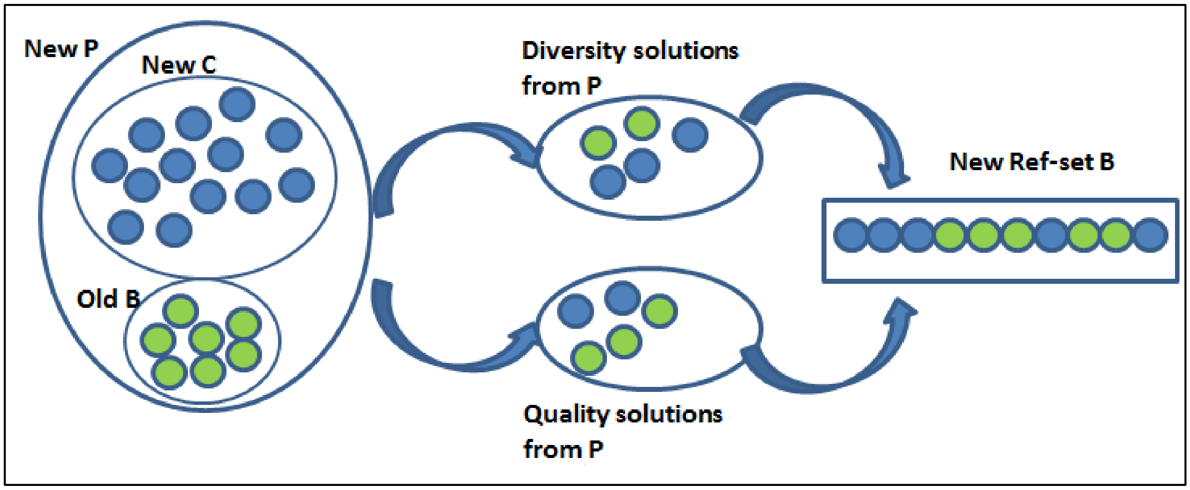
Figure 16. Ref-set actualization procedure

Ref-set actualization procedure

- Clear **P**
- Set solutions from **B'** to **P**
- Set solutions from **C** to **P**
- Select **|B|/2** diversity solutions from **P** Applying clustering procedure with **|B|/2** clusters and set selected solutions in **B**
- Sort solutions in **P** by quality (applying quality criterion defined in numeral *3.3.2.1*)
- Set **|B|/2** quality solutions (applying quality criterion defined in numeral *3.3.2.1*) from **P** to **B**

End procedure

Figure 17. Ref-set actualization scheme



Ref-set actualization method takes solutions of previous iteration Ref-set (**B’**) and new generated solutions by cross-over operator (**C**) to update initial population (**P = B’ + C**); new Ref-set **B** is updated with **|B|** solutions of highest quality from updated **P** set which are placed and ordered in one half by diversity, and in the other half by quality. Ref-set actualization is described in Figure 16 and Figure 17.

4. CHAPTER IV. EXPERIMENTATION AND RESULTS SOLVING THE MULTIPLE DEPOT VEHICLE ROUTING PROBLEM WITH MULTIPLE OBJECTIVES. MO-MDVRP

This chapter presents experimentation and results phase to evaluate performance for the proposed solving strategy for the MO-MDVRP. The performance evaluation will consider the robustness of the proposed method (solving strategy), as explained in section 2.5, this is: reasonable execution times, quality of the solutions and improvement capability. First, MILP model results for small known instances are shown: Solomon's instances R101, C101 and RC101 with 10 and 20 nodes; next, parameter calibration experiments for the proposed hybrid scatter search algorithm are discussed and presented; as a final point, a comparison between exact results, approximate results obtained with the proposed H-MOSS and a performance evaluation is carried out.

4.1. EXACT RESULTS

Exact results were computed using the MILP formulation for the MO-MDVRP minimizing total cost and allocated load range (load balance as defined in Lin and Kwok (2005)). The model was encoded using standard optimization software with classical multi-objective techniques. We used an ε - restriction technique to draw a set of non - dominated solutions or *Pareto Frontier*; the ε - restriction method takes only one objective function to optimize and declare the other objective function as a "less than or equal to" constraint with an ε value on the right side. To obtain the Pareto Frontier a set E of different ε values is defined and $|\mathrm{E}|$ models are run to obtain different non – dominated solutions. The mathematical model was encoded using GAMS® and solved with CPLEX®; several instances based on the randomized, clustered and randomized-clustered Solomon instances of 10 and 20 nodes were solved. The experiments were run using an Intel I7 CPU at 2.20 GHz with 8 GB of RAM for the 10 nodes instances and 2 parallel Intel XEON processors at 2.20 GHz for the 20 nodes instances.

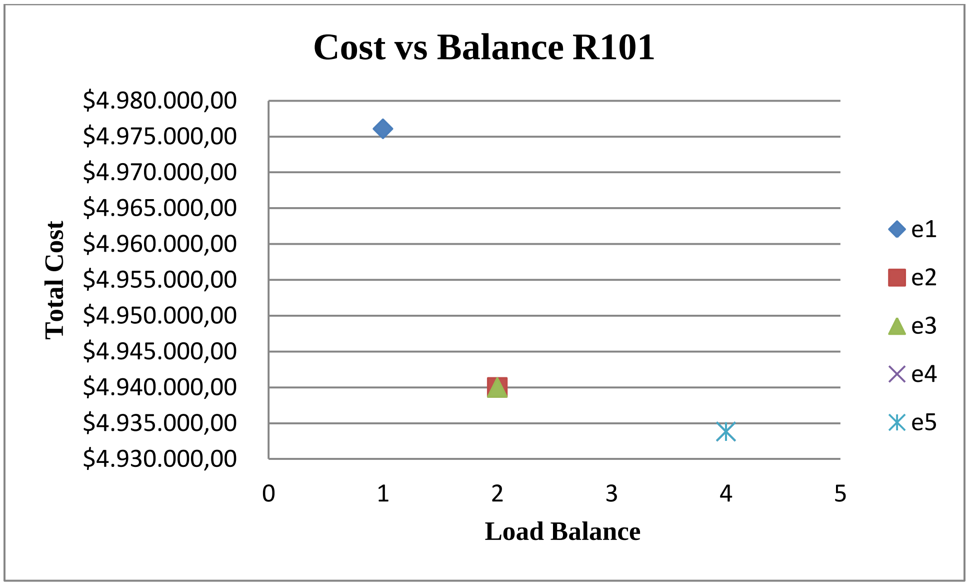
4.1.1. R101 – 10 Nodes

For this instance, 3 non dominated solutions were found for a 3 vehicle plan; each vehicle at 50 units of capacity. Computational results with an optimality gap of 0.0% are shown in Table 2 and Figure 18.

Table 2. Computational Results for R101 – 10 Nodes

Pareto Optimal Solution	Cost	Balance	CPU Time (Seconds)	# Iterations
ε=1	$ 4.976.101,64	1	23,85	511.109
ε=2	$ 4.940.007,94	2	5,41	148.638
ε=3	$ 4.940.007,94	2	6,49	183.770
ε=4	$ 4.933.853,28	4	22,20	636.772
ε=5	$ 4.933.853,28	4	21,84	585.979

Figure 18. Pareto Frontier for R101 – 10 Nodes



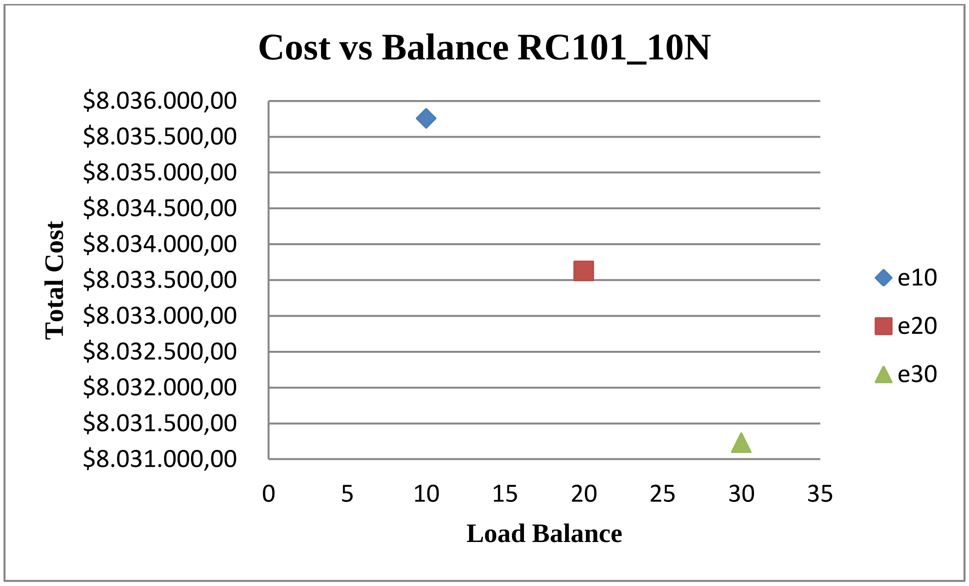
4.1.2. RC101 – 10 Nodes

For this instance, 3 non dominated solutions were found for a 5 vehicle plan; each vehicle at 50 units of capacity. Computational results with an optimality gap of 0.0% are shown in Table 3 and Figure 19.

Table 3. Computational Results for RC101 – 10 Nodes

Pareto Optimal Solution	Cost	Balance	CPU Time (Seconds)	# Iterations
ε=10	$ 8.035.758,29	10	9,87	208.356
ε=20	$ 8.033.629,33	20	7,78	153.114
ε=30	$ 8.031.233,17	30	27,58	761.869

Figure 19. Pareto Frontier for RC101 – 10 Nodes



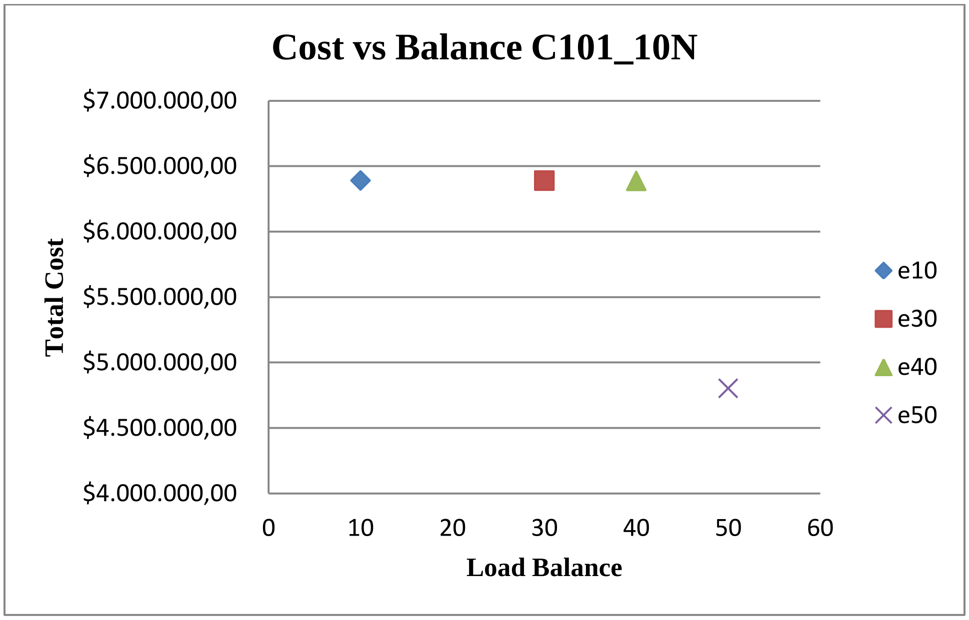
4.1.3. C101 – 10 Nodes

For this instance 4 non dominated solutions were found for a 4 vehicle plan; each vehicle at 50 units of capacity. Computational results with an optimality gap of 0.0% are shown in Table 4 and Figure 20.

Table 4. Computational Results for C101 – 10 Nodes

Pareto Optimal Solution	Cost	Balance	CPU Time (Seconds)	# Iterations
ε=10	$ 6.392.047,35	10	28,33	664.415
ε=30	$ 6.391.043,33	30	292,78	5.838.335
ε=40	$ 6.389.094,76	40	82,09	1.757.830
ε=50	$ 4.804.100,79	50	51,25	985.406

Figure 20. Pareto Frontier for C101 – 10 Nodes



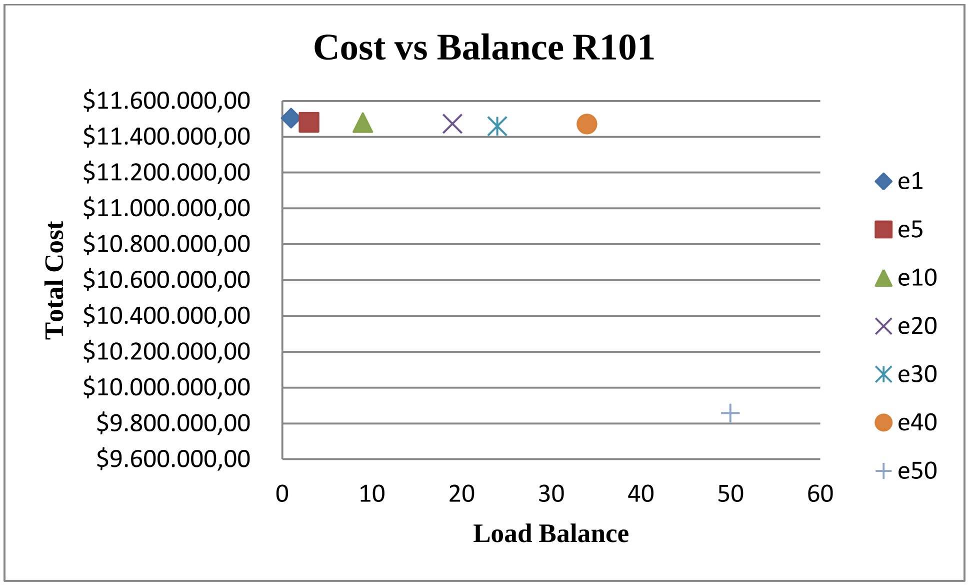
4.1.4. R101 – 20 Nodes

Results of instance R101 with 20 nodes were calculated with an optimality gap of 3%. For this instance 7 models were run, but only 6 non dominated solutions were found for a 7 vehicle plan. Computational results are shown in Table 5 and Figure 21.

Table 5. Computational Results for R101 – 20 Nodes

Pareto Optimal Solution		Cost	Balance	CPU Time (Seconds)	# Iterations
ε=1	$	11.504.754,12	1	7.200,02	44.184.206
ε=5	$	11.481.312,21	3	3.431,07	26.559.897
ε=10	$	11.479.747,00	9	2.795,46	11.479.747
ε=20	$	11.473.437,74	19	603,68	4.190.244
ε=30	$	11.459.877,51	24	453,48	2.093.711
ε=40	$	11.471.563,48	34	1.721,5	8.478.105
ε=50	$	9.858.492,63	50	2.714,57	18.538.061

Figure 21. Pareto Frontier for R101 – 20 Nodes



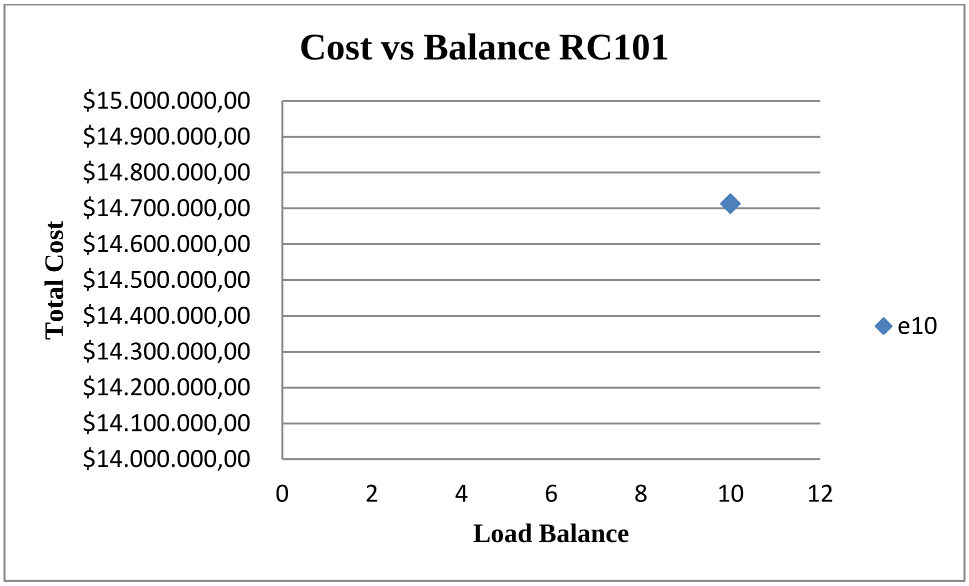
4.1.5. RC101 – 20 Nodes

Results of instance RC101 with 20 nodes were calculated with an optimality gap of 3%. For this instance only 1 feasible solution was found for a 9 vehicle plan; initial ε – parameter was set to 10, (Load Range value for optimum cost: *R=10*), then no other feasible Pareto solutions were found. Computational results are shown in Table 6 and Figure 22.

Table 6. Computational Results for RC101 – 20 Nodes

Pareto Optimal Solution	Cost	Balance	CPU Time (Seconds)	# Iterations
ε=10	$ 14.713.573,19	10	34,43	78.999

Figure 22. Pareto Frontier for RC101 – 20 Nodes



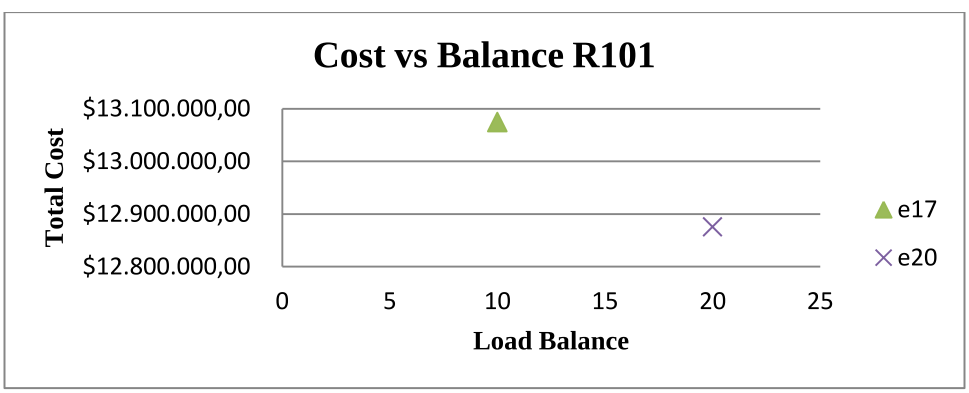
4.1.6. C101 – 20 Nodes

Results of instance C101 with 20 nodes were calculated with an optimality gap of 3%. For this instance 2 solutions were found for an 8 vehicle plan; initial ε – parameter was set to 20, (Load Range value for optimum cost: *R=20*), then, ε – parameter was set to 17 but the next feasible value for this instance was *R=10* (Load Range), no other feasible Pareto solutions were found. Computational results are shown in Table 7 and Figure 23.

Table 7. Computational Results for C101 – 20 Nodes

Pareto Optimal Solution	Cost	Balance	CPU Time (Seconds)	# Iterations
ε=17	$ 13.074.572,31	10	0,58	452
ε=20	$ 12.875.650,98	20	448,75	2.623.809

Figure 23. Pareto Frontier for C101 – 20 Nodes



4.2. EXPERIMENTATION WITH THE HYBRID MULTI-OBJECTIVE SCATTER SEARCH ALGORITHM FOR THE MDVRP

Several tests were carried out to validate the proposed H-MOSS performance for the MDVRP. To improve the procedure performance, formal experiments were designed to calibrate execution parameters for the H-MOSS procedure. All experiments were performed in the same computer with an Intel Core i7 processor at 2.20 GHz.

Experiments were designed to discard non-significant factors (execution parameters and problem size parameters) for each response variable. Two response variables, one for each objective, were computed to measure changes in the H-MOSS performance; response variables were defined as the percent reduction of maximum cost and load imbalance in the initial ref-set compared to the lowest cost and load imbalance in the ref-set of the last generation (iteration). This is defined in Eq.(4.2.1) and Eq.(4.2.2).

Eq.(4.2.1)

Eq.(4.2.1)

Where **%Δ*C*** and **%Δ*R*** are the cost and load imbalance percent reduction, respectively; and are the maximum cost and load imbalance in the Ref-Set of the first iteration, respectively; and and are the minimum cost and load imbalance in the Ref-Set of the last iteration.

Execution parameters for the H-MOSS were considered as design factors, and problem size variables (number of clients and number of depots) were considered as block variables, factors and its levels are shown in Table 8.

Table 8. Design factors and block variables for performance experiments design

		Low	**High**
Design Factors	# Iterations	30	80
	Initial Population Size	50	100
	Ref-Set Size	8	20
Block Variables	# Clients	50	300
	# Depots	2	8

As the block variables have two levels, four different blocks appear (combination of each level of each block variable); problem size variables were considered as blocks variables to evaluate the robustness of the H_MOSS procedure. By the above, a 2^3 with four blocks experiment was designed. In Table 9 the design of the experiment and its results are shown.

Table 9. Design of Experiment for Robustness Evaluation and Parameter Calibration

Experiment	Blocks	# Iterations	Initial population size	Ref_Set Size	%ΔC	%ΔR	Execution Time
1	1	80	100	8	25	73	17,34
2	1	30	50	20	32	69	15,09
3	1	80	50	8	17	33	16,50
4	1	80	50	20	27	47	20,46
5	1	80	100	20	25	94	33,95
6	1	30	50	8	7	26	6,19
7	1	30	100	8	15	45	8,63
8	1	30	100	20	23	39	13,32
9	3	80	50	20	33	84	93,13
10	3	30	100	20	21	68	40,87
11	3	80	100	20	38	61	166,68
12	3	30	50	8	23	61	13,37
13	3	30	50	20	17	72	80,58
14	3	80	50	8	27	44	37,13
15	3	30	100	8	14	64	16,40
16	3	80	100	8	16	67	66,47
17	2	80	50	20	13	71	102,23
18	2	30	50	20	19	39	48,91
19	2	80	100	20	22	63	123,95
20	2	30	100	8	17	26	20,75
21	2	30	50	8	3	11	15,69
22	2	80	50	8	11	48	28,53
23	2	30	100	20	21	49	38,85
24	2	80	100	8	6	35	30,76
25	4	80	100	20	28	35	230,18
26	4	80	50	8	22	42	85,29
27	4	30	100	8	9	58	32,50
28	4	30	50	20	23	75	92,71
29	4	80	100	8	24	37	56,84
30	4	30	50	8	13	39	30,14
31	4	80	50	20	33	57	216,20
32	4	30	100	20	24	39	98,35

A total of 32 instances with different execution parameters were solved in a randomized order; different sizes for the instances were also included as blocks to discard robustness loss related to problem size.

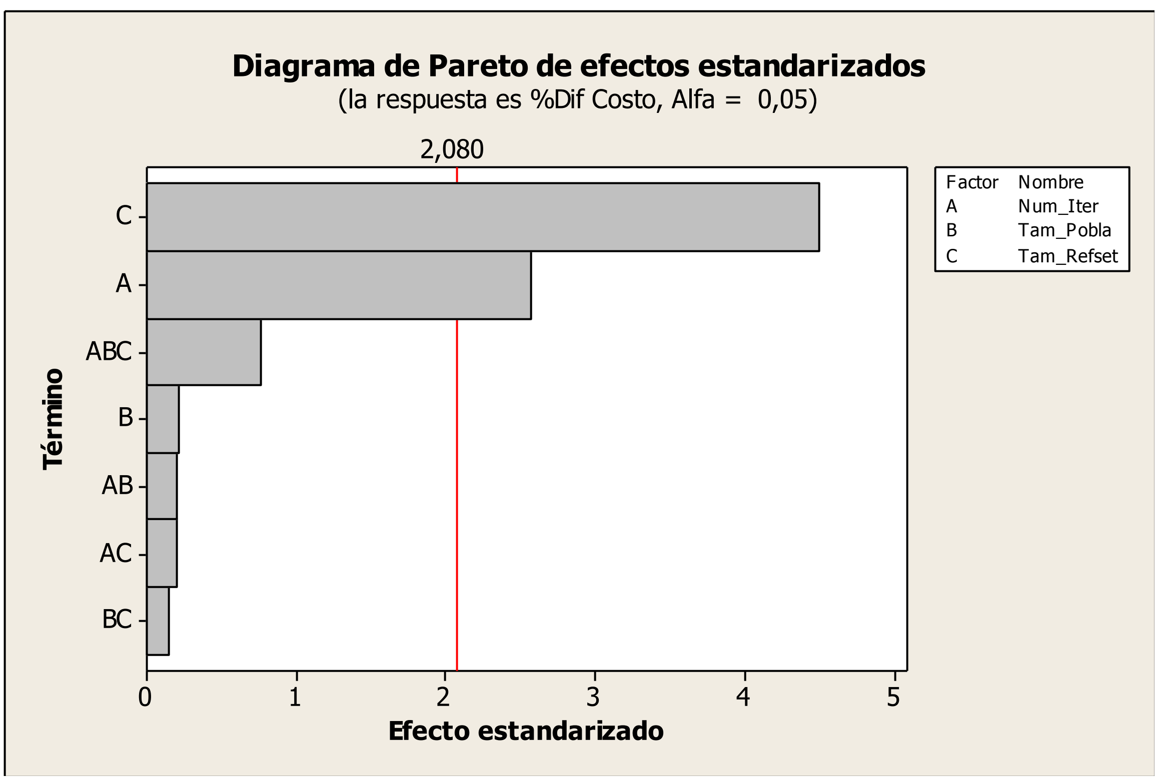
4.2.1. Performance Analysis and Parameter Calibration

Performance analysis in this stage is oriented to evaluate the H-MOSS robustness; this is analyzed evaluating the significance of the problem structure (topology and size of the problem) for the response variables related to each objective (total cost and load imbalance), these response variables measure each objective improvement capability; on the other hand, the experiment is oriented to calibrate execution parameters for the H-MOSS.

Execution parameters (ref-set size, initial population size and maximum number of iterations) are defined as main effects to analyze best results (calibration) and problem size parameters are set as blocks so that problem size effects may be extracted from execution parameters effects. As two different objectives are optimized, the experiments results have to be analyzed separately.

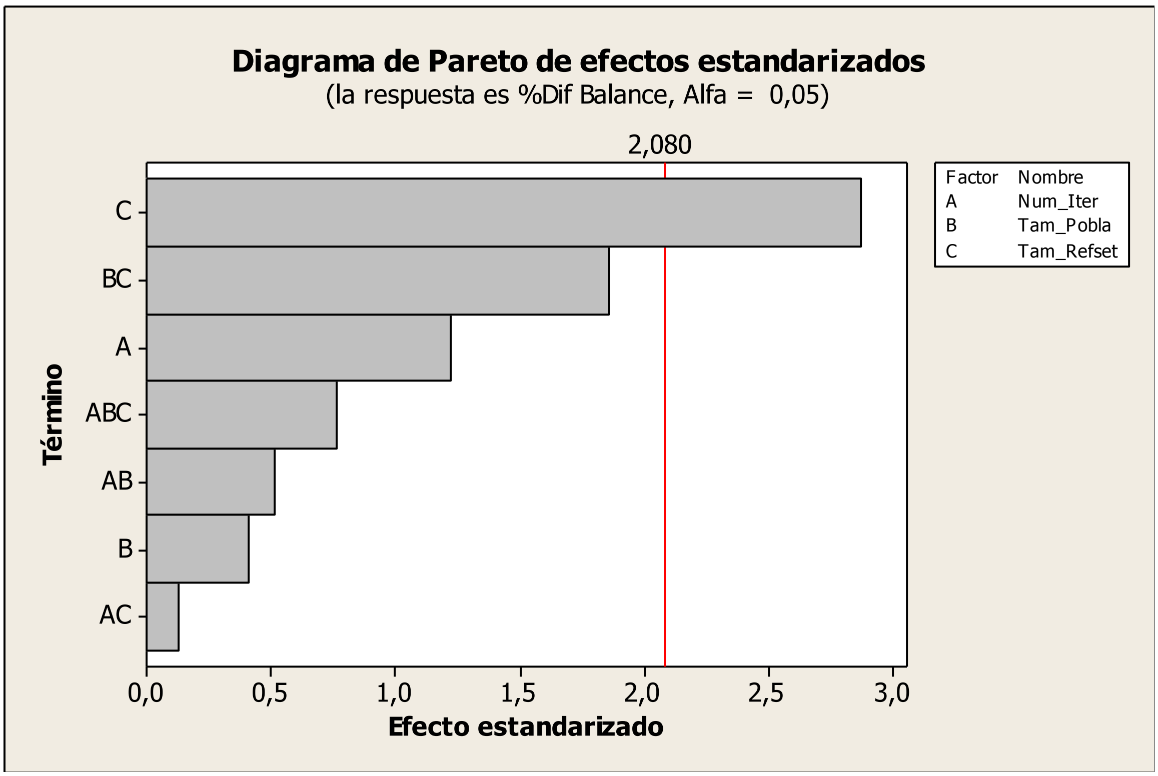
Experiment results for the response variable related to cost improvement (**%Δ*C***) showed that the maximum percent reduction was **38%** setting **Ref-set size** to **20** and **80 iterations**, with and average percent improvement of **20%**. Experiments included instances of different size which showed that the significant execution parameters are **Ref-set size** and **Number of iterations**. Normality, homoscedasticity and independence tests were carried out showing positive results (see Figure 36 in the Appendix). Figure 24 shows significance of the factors where factor **A** symbolizes **Number of iterations,** factor **B** represents **Initial population size** and factor **C** represents **Ref-set size**. Blocks were significant for this response variable, due to this, a parallel analysis was made to evaluate performance for instances of different size.

Figure 24. Pareto Chart for Standardized Effects (%ΔC)



Analysis of the response variable related to the load imbalance (%ΔR) showed that blocks are not significant (level of significance of 0.95), this means that robustness of the method is not compromised by the size of the problem. Moreover, the experiment showed that maximum percent reduction was **94%** setting **Ref-set size** to **20** and **80 iterations**, with and average percent improvement of **50%.** Experiments showed that the only significant execution parameter for this variable is the **Ref-set size** (Factor C, see Figure 25). Normality, homoscedasticity and independence tests were carried out showing positive results (see Figure 37 in the Appendix).

Figure 25. Pareto Chart for Standardized Effects (%ΔR)



Experiments related to the response variable %Δ*C* showed that the blocks (problem size parameters) were significant for a significance level of 0.95 (P-value = 0.019); to evaluate robustness of the solving strategy related to the problem size, a parallel analysis was executed setting the block variables and the significant (supported on the experiment results) execution parameters as main factors.

The parallel analysis showed that at a higher number of clients, the probability of cost improvement (measured by the response variable %Δ*C*) is higher; on the other hand, the experiment showed that for instances with a higher number of clients more iterations are needed to improve the response variable %Δ*C*, this is explained by the significance of **Number of iterations** and **Number of clients** interaction, this is shown in Figure 27. This shows that there is no evidence of robustness loss for instances of higher number of clients. Figure 26 shows main effects charts for %Δ*C*.

Figure 26. Main effects charts for %ΔC (Parallel analysis)

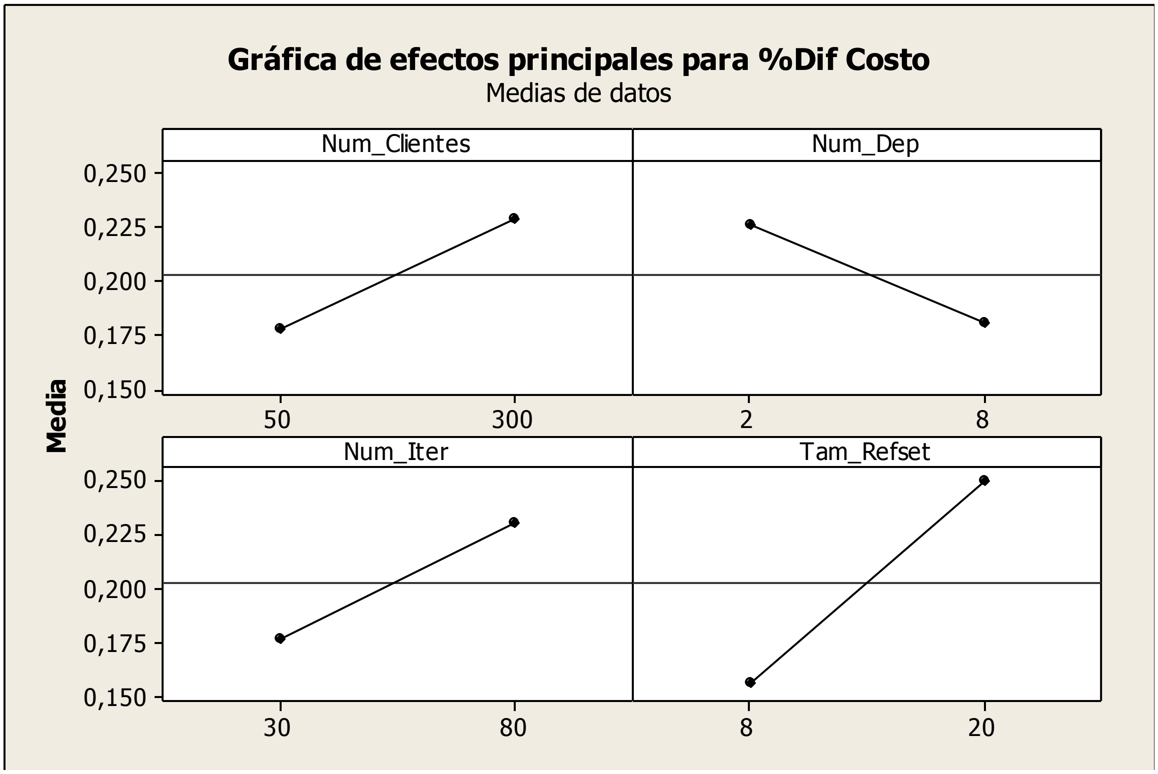
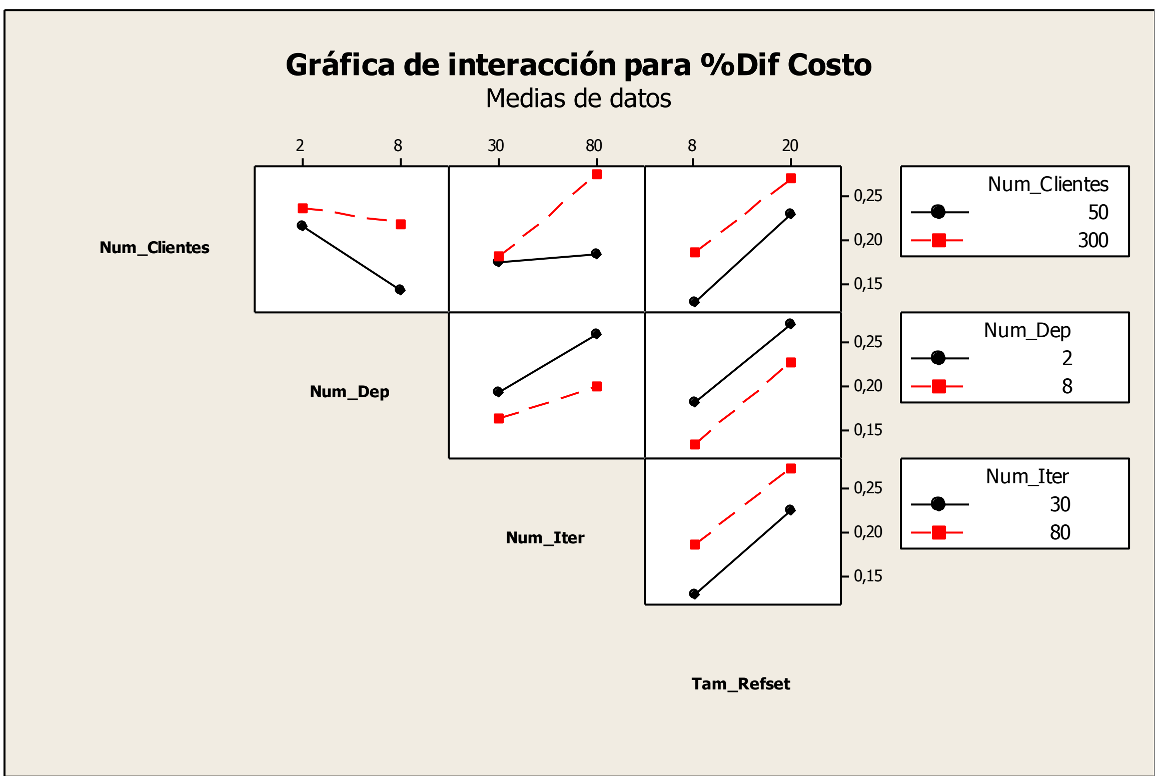


Figure 27. Interaction charts for %ΔC (Parallel analysis)



On the other hand, the parallel analysis shows that an increase of number of depots has a slight impact on the level of robustness for the solving strategy (see Figure 26).

Supported on the significant factors and their effects (see main effects charts in Figure 38 and Figure 39 in the Appendix) parameters are set to **Number of iterations** at **80** and **Ref-set size** at **20;** due to the **Initial population size** low significance, this parameter is set to **50** to reduce execution time. This is supported on cube charts for the response variables %ΔC and %ΔR (see Figure 28 and Figure 29).

Figure 28. Cube chart for %ΔC

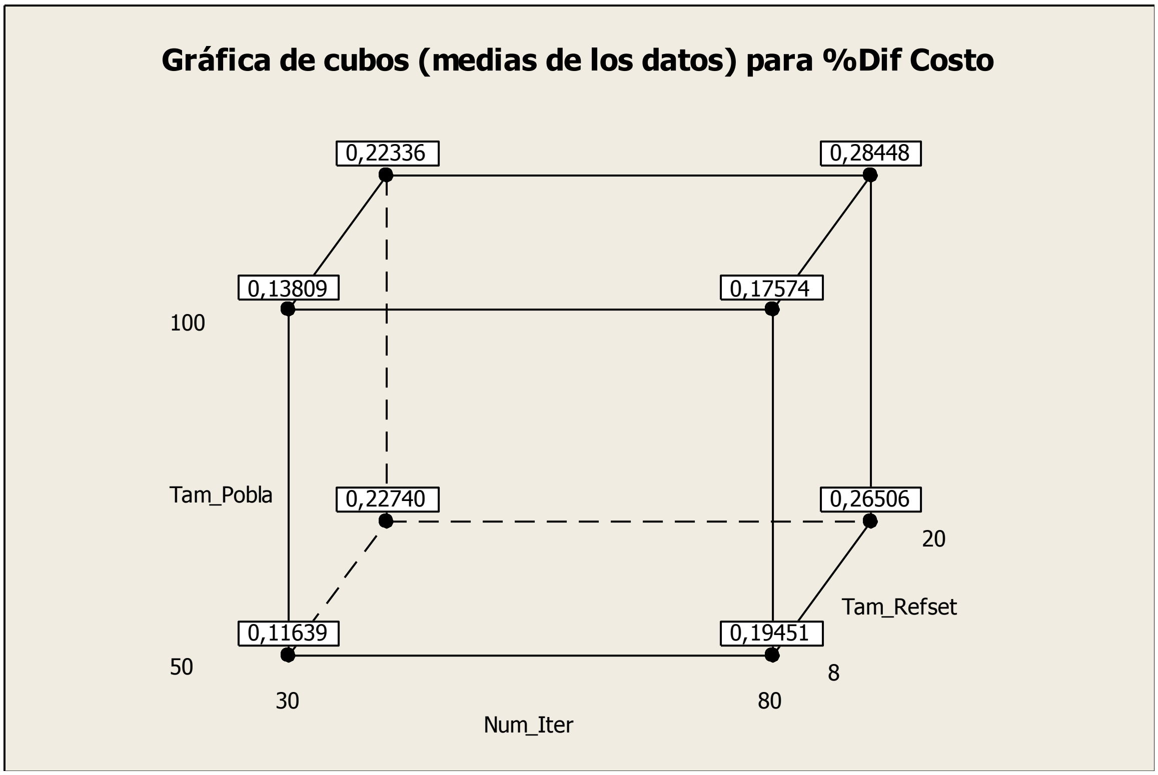
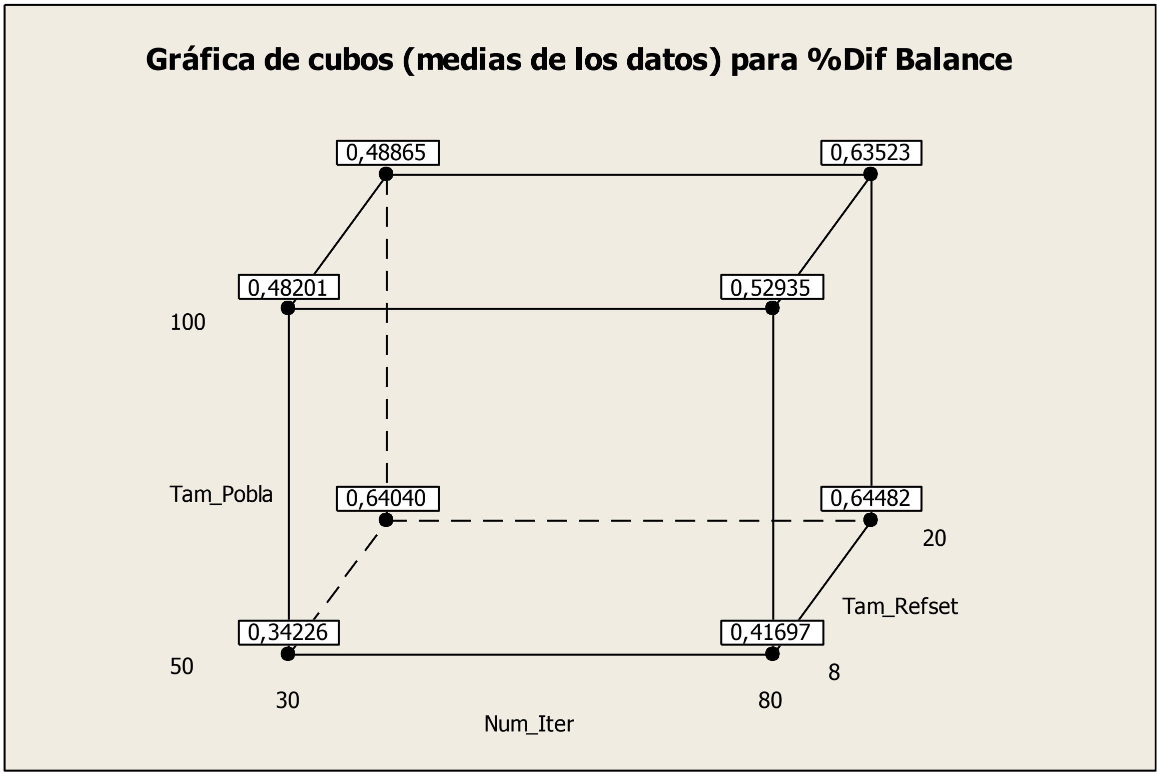


Figure 29. Cube chart for %ΔR



Robustness of the solving strategy is very acceptable, although on instances with a high number of depots a slight robustness loss is observed measured by cost reduction capability; there is no proof for robustness loss for load imbalance reduction capability; this is proved by the low significance of block variables (problem size) on the response variable %ΔR. Moreover, robustness of the solving strategy measured as execution time is proven by the execution time results, where the highest execution time was **230.18 seconds (less than 4 minutes)** for a relatively large instance (300 clients and 8 depots).

4.2.2. Optimality gap evaluation

Results obtained with the H-MOSS were compared to exact results obtained with MILP mathematical model. Results for Solomon's instances R101, RC101 and C101 with 10 nodes and 20 nodes are presented below.

4.2.2.1. 10 Nodes Instances

Table 10. Results for Solomon's R101 with 10 nodes

R101	MILP			H-MOSS		
Pareto Optimal Solution	**Cost**	**Balance**	**CPU Time (Seconds)**	**Cost**	**Balance**	**CPU Time (Seconds)**
1	$ 4.976.101,64	1	23,85	$ 4.939.101,00	4	10,605
2	$ 4.940.007,94	2	5,41			
3	$ 4.940.007,94	2	6,49			

4	$ 4.933.853,28	4	22,2
5	$ 4.933.853,28	4	21,84

Table 11. Results for Solomon's RC101 with 10 nodes

RC101	MILP			H-MOSS		
Pareto Optimal Solution	Cost	Balance	CPU Time (Seconds)	Cost	Balance	CPU Time (Seconds)
1	$ 8.035.758,29	10	9,87	$ 8.035.244,00	20	10,639
2	$ 8.033.629,33	20	7,78			
3	$ 8.031.233,17	30	27,58			

Table 12. Results for Solomon's C101 with 10 nodes

C101	MILP			H-MOSS		
Pareto Optimal Solution	Cost	Balance	CPU Time (Seconds)	Cost	Balance	CPU Time (Seconds)
1	$ 6.392.047,35	10	28,33	$ 6.393.041,00	10	9,235
2	$ 6.391.043,33	30	292,78			
3	$ 6.389.094,76	40	82,09			
4	$ 4.804.100,79	50	51,25			

Figure 30. MILP and H-MOSS Pareto frontier comparison for 10 Nodes Solomon's R101

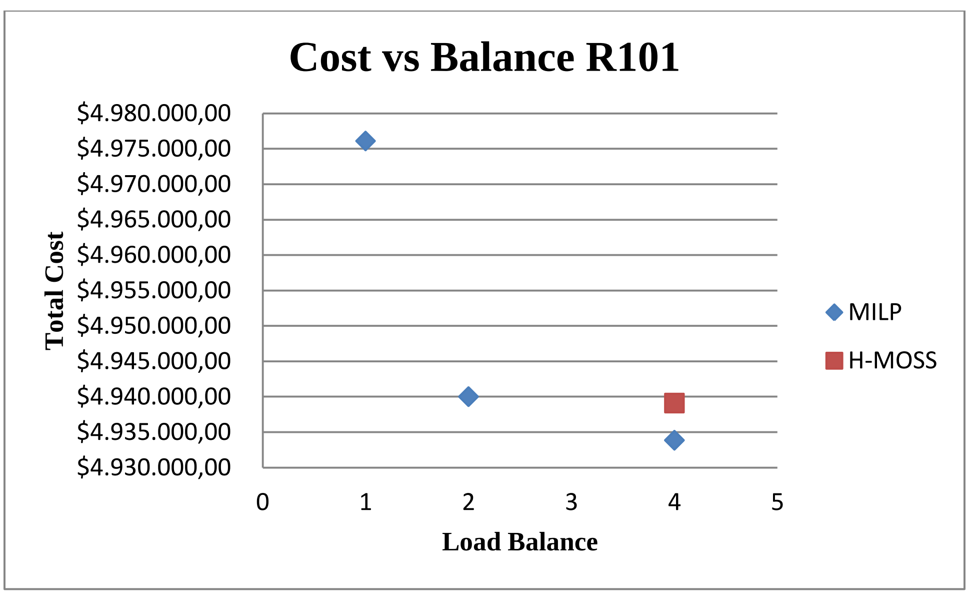


Figure 31. MILP and H-MOSS Pareto frontier comparison for 10 Nodes Solomon's RC101

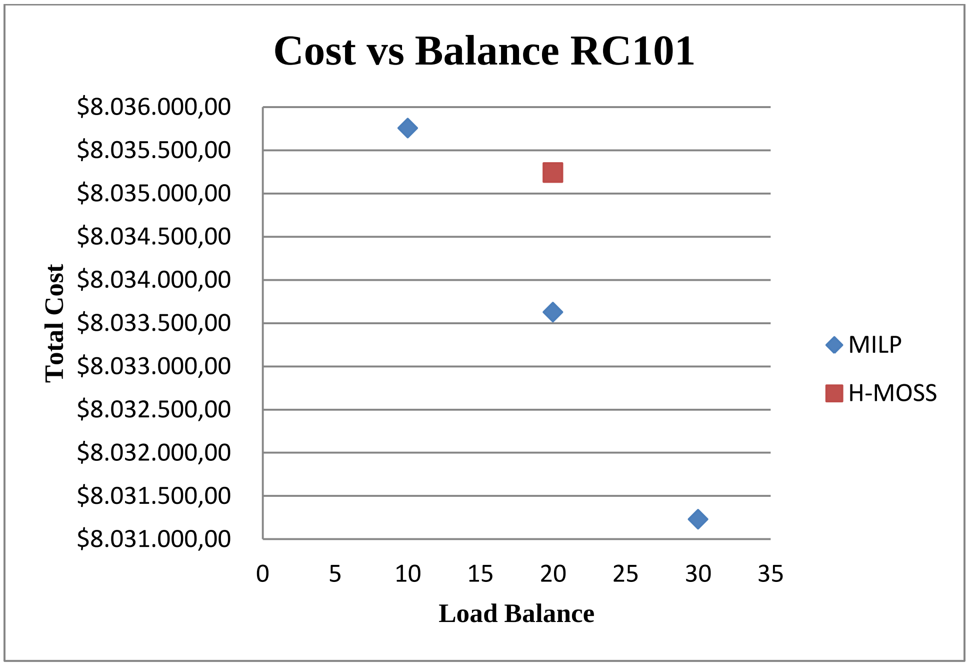
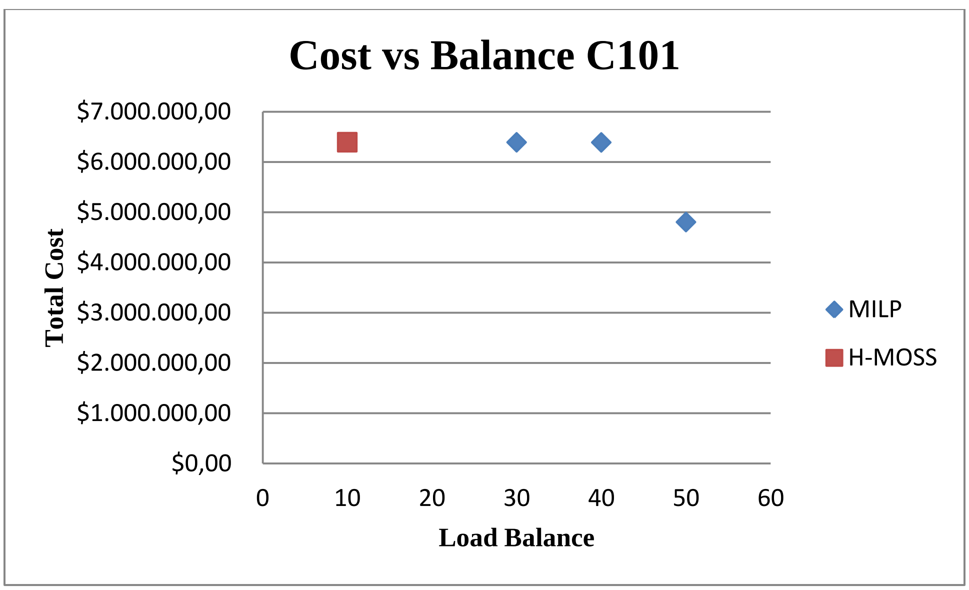


Figure 32. MILP and H-MOSS Pareto frontier comparison for 10 Nodes Solomon's C101



4.2.2.2. 20 Nodes Instances

Table 13. Results for Solomon's R101 with 20 nodes

R101	MILP			H-MOSS		
Pareto Optimal Solution	Cost	Balance	CPU Time (Seconds)	Cost	Balance	CPU Time (Seconds)
1	$ 11.504.754,12	1	7.200,02	$ 9.872.108,00	13	13,85

2	$ 11.481.312,21	3	3.431,07	$ 9.863.646,00	13	13,85
3	$ 11.479.747,00	9	2.795,46			
4	$ 11.473.437,74	19	603,68			
5	$ 11.459.877,51	24	453,48			
6	$ 11.471.563,48	34	1.721,50			
7	$ 9.858.492,63	50	2.714,57			

Table 14. Results for Solomon's RC101 with 20 nodes

RC101	**MILP**			**H-MOSS**		
Pareto Optimal Solution	**Cost**	**Balance**	**CPU Time (Seconds)**	**Cost**	**Balance**	**CPU Time (Seconds)**
1	$ 14.713.573,19	10	34,43	$ 14.835.273,23	10	15,303

Table 15. Results for Solomon's C101 with 20 nodes

C101	**MILP**			**H-MOSS**		
Pareto Optimal Solution	**Cost**	**Balance**	**CPU Time (Seconds)**	**Cost**	**Balance**	**CPU Time (Seconds)**
ε=17	$ 13.074.572,31	10	0,58	$ 12.872.995,00	30	13,605
ε=20	$ 12.875.650,98	20	448,75			

Figure 33. MILP and H-MOSS Pareto frontier comparison for 20 Nodes Solomon's R101

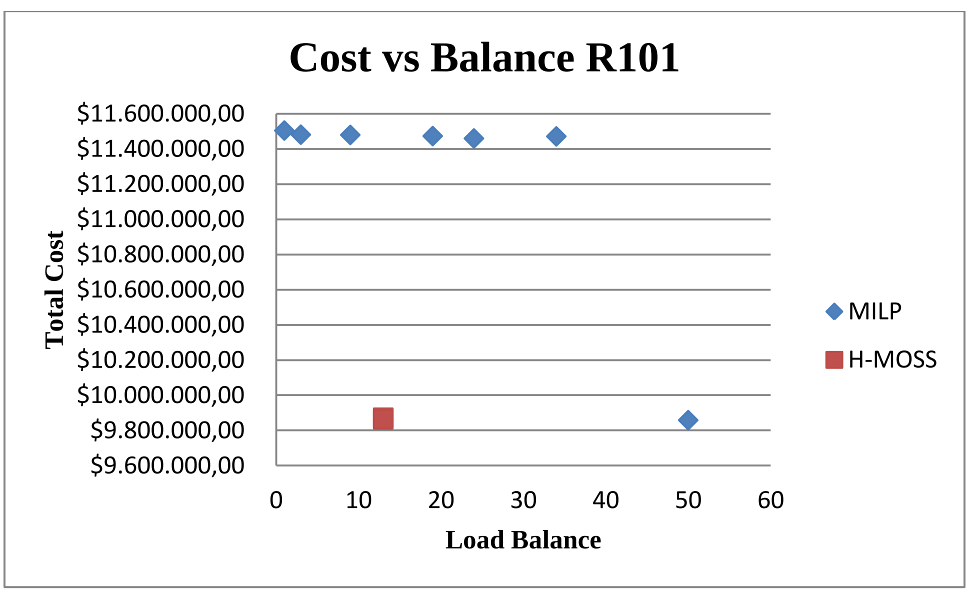


Figure 34. MILP and H-MOSS Pareto frontier comparison for 20 Nodes Solomon's RC101

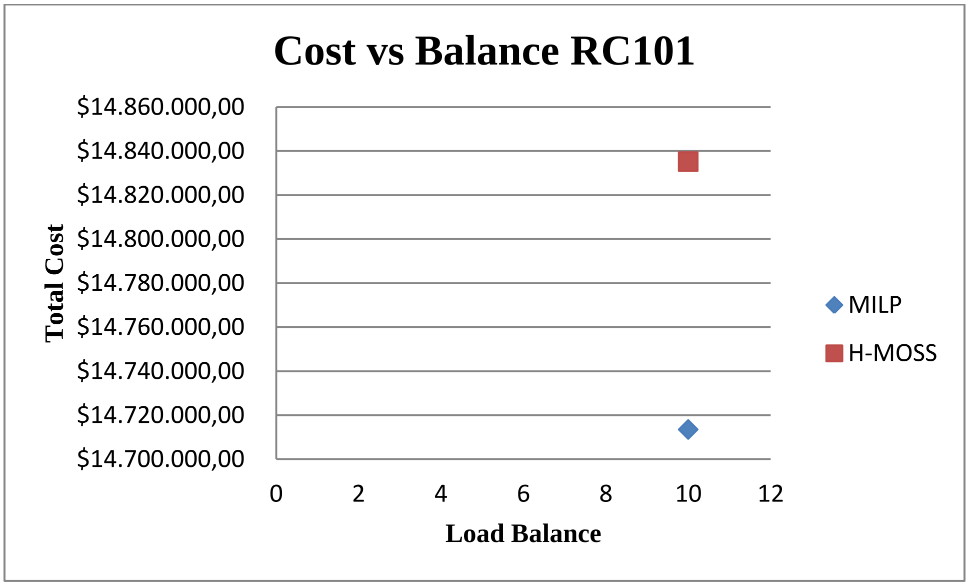


Figure 35. MILP and H-MOSS Pareto frontier comparison for 20 Nodes Solomon's C101

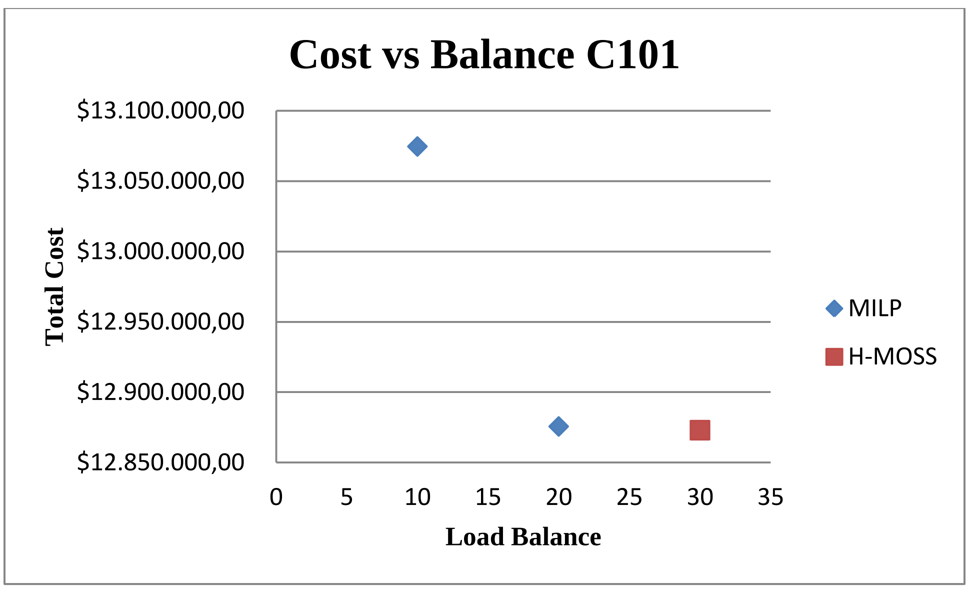


Table 16. Execution time comparison for 10 nodes instances

Instance	MILP average CPU time (Seconds)	H_MOSS average CPU time (Seconds)	% CPU time increment
R101	15,958	10,605	50%
RC101	15,07666667	10,639	42%
C101	113,6125	9,235	1130%

Table 17. Execution time comparison for 20 nodes instances

Instance	MILP average CPU time (Seconds)	H_MOSS average CPU time (Seconds)	% CPU time increment
R101	2702,825714	13,853	19411%
RC101	34,43	15,303	125%
C101	224,665	13,605	1551%

In general terms, results show a good performance finding in most cases non-dominated solutions; dominated solutions found as best solutions by the H-MOSS algorithm are clearly close to the Pareto Frontier found by the MILP model. Table 16 and Table 17 show the percent increment of execution time for the exact method.

5. IMPLEMENTATION OUTLINES

As pointed out in the problem statement transportation is the most expensive process in logistics chain; furthermore, distribution operations are even more expensive than massive transportation. Information technologies may be a key element to increase transportation efficiency optimizing data processing and analysis and the decision making processes, to achieve this, the Intelligent Transportation System (ITS) development is an important field. Operations Research is an important theoretical framework to develop robust applications to automate and optimize decisions for transportation planning, however, due to the increasing dynamism in logistic operations, including physical distribution of goods, classical optimization approaches may not be complete for real life implementation; this is why in many cases optimization approaches may consider dynamic variables in the system such as:

- Dynamic demand information
- Dynamic service requests
- Dynamic traffic information
- Dynamic vehicle availability

This type of distribution problems are called Dynamic Vehicle Routing Problems (DVRP). Up to this date DVRP's have not been widely studied in the operations research field, probably because the difficulty of its nature; nevertheless, many technological advances have increased the interest in this type of problem, and also have made information technologies based on optimization approaches more feasible for real implementation. A full review for DVRP's is presented in (Pillac et al. 2013).

Physical distribution operations are different depending of the type of product, service, network configuration, strategic transportation operations (e.g. cross docking) and so on; this is why Vehicle Routing Problems have been classified in many variants: *With Time Windows*, *Heterogeneous Fleet*, *Multiple Depots*, *Capacitated*, *Stochastic*, *Dynamic*, etc. Information technologies for distribution systems must consider as many constraints and special characteristics as may be possible. This leads to a general view of VRP's a general VRP considering many special characteristics and constraints, this general problem is called Multiple Attributes Vehicle Routing Problems (MAVRP), a full survey and synthesis for this family of problems is presented in (Vidal et al. 2013). Real life implementation of information technologies including basic algorithms and its support technology must consider this two situations: Dynamic Information and Multiple Attributes.

The proposed (initial) solving strategy serve as a basis for the development of Dynamic and Multiple Attribute algorithms; also continuing its development will serve as a basis for the development of robust Decision Support Systems (DSS) for distribution operations; this DSS would support operative planning, providing optimized routing plans; however, the implementation of the solution strategy has to be preceded of other processes to provide a successful tool. These processes are summarized 4 phases.

5.1. IMPLEMENTATION PHASE 1: System Characterization.

A complete system characterization is proposed to properly design a DSS for the distribution operations; this is necessary to do requirements elicitation in a proper way, considering variables that may affect not only the optimization algorithm performance but software usability itself. The system characterization would be made considering two different dimensions: Distribution system characterization and Information technology characterization.

- *Distribution System Characterization*: A complete analysis for the distribution system is proposed, including: installed infrastructure, human resources and system restrictions related to infrastructure constraints, product constraints, geographical constraints, customer constraints.
- *Information Technology Characterization*: Current information technology and its processes must be analyzed; a characterization of current information systems such as ERP (Enterprise Resource Planning), TMS (Transportation Management System), WMS (Warehouse Management System), OMS (Order Management System), must be considered to properly link up the proposed DSS.

5.2. IMPLEMENTATION PHASE 2: Data Mining

The DSS is based on optimization algorithms (solving strategy) which have to be loaded with geographical, traffic, technical and financial information. To collect geographical information a Geographical Information System (GIS) is required, the GIS will aid to compute important geographical information needed to compute distance matrix, for example. Traffic information collection may be more difficult, because in-field data such as traffic counts to compute free flow velocity that is needed to estimate time travels, must be collected; this collection process must be designed and will probably need support technology and data mining and data warehousing techniques. Technical and financial information may be collected from other information systems, so system interfaces must be developed.

5.3. IMPLEMENTATION PHASE 3: Software Design and Development

In phase 3 software design must be done considering the system characterization and requirements elicitation done in phase 1; the design sub-phase must take into account software architecture aspects such as infrastructure needed to support the system. In the development sub-phase, adjustments to the general algorithm (solving strategy) must be taken into account to consider all constraints defined in phase 1. Define phase must consider the system users and their roles.

5.4. IMPLEMENTATION PHASE 4: Validation and Final Implementation

In this phase validation must be done solving simulated instances and solving instances with historic data. A process performance comparison must be done, comparing real historic data of executed plans and solutions provided by the DSS. In this phase software design validation must be done, and some modifications must be required. In final implementation all system users and administrators must be included, it is important to validate their roles, information access and properties. Final implementation must have a separate budget and must consider all the human resource aspects including resistance to change, training and professional profiles.

6. CONCLUSIONS

Vehicle routing problems are typical problems in distribution network operations planning and they are one of the most difficult problems to solve efficiently because of its combinatorial nature; on the other hand, vehicle routing problems with multiple depots are a more realistic approach to solve real life distribution operations, and they are even more challenging for optimization; moreover, logistics problems are indeed multi-objective and real life solutions must consider this condition. An extensive literature review was conducted and the literature analysis showed that most of the scientific development in this field (vehicle routing problems with multiple depots) is not oriented for multi-objective problems, with only 13 publications (11%); this showed an interesting line for research. Literature analysis also showed that the preponderant solution strategies are meta-heuristics, being genetic algorithms and tabu search the most popular techniques.

Literature review and analysis showed that a strict robustness analysis has not been made considering the method capability to find solutions with an equal expected quality, independently of the problem topology or size, up to this date the performance analysis has considered execution times quality of the solutions and (for multi-objective cases) Pareto Frontier distribution. A robustness analysis in three dimensions was proposed: Quality of solutions, execution times and the capability to generate equally expected quality of solutions independently of problem shape and size.

This work presents the development of a new hybrid scatter search approach to solve the multi-objective vehicle routing problem with multiple depots (H-MOSS), minimizing total cost and load imbalance; moreover a new mixed integer linear programing mathematical model (MILP) was designed to aid for H-MOSS performance evaluation. The development of the mathematical model led to conclude that the multi-objective mathematical model minimizing total cost and load imbalance cannot be solved because of the discontinuous structure of the load imbalance function; however a new MILP formulation was proposed converting the load imbalance function into linear constraints to solve the problem as a MILP program. For the H-MOSS development, local improvement techniques based on classical heuristics, and multi-criteria heuristics were designed and adapted for this particular multi-objective problem; moreover, SPEA and SPEAII multi-objective evolutionary concepts were used to hybridize the MOSS procedure.

H-MOSS results showed, based on formal experimentation, that the proposed solving strategy is quite robust for different problem structure including problem topology and problem size. On the other hand, exact results obtained for small instances versus H-MOSS results comparisons showed that the non-dominated solutions found are indeed solutions from the exact Pareto Frontier in most of cases, or at least very close to the Pareto Frontier; although, the proposed hybrid procedure showed difficulty to find a set of non-dominated solutions for small instances (only 1 non-dominated solution was found in most of cases). Execution times for the H-MOSS procedure are very acceptable for large instances, for instances of 300 clients and 8 depots the highest CPU time did not exceed 4 minutes.

The aim of the development of robust solving strategies for distribution operations is to provide effective methods for decision making support. This solution strategy may serve as a basis model for Decision Support Systems (DSS) development. Implementation outlines were defined for this type of DSS. Implementation of optimization-based DSS is expected to have a positive impact in the efficiency (cost reduction and balanced resource utilization) of distribution operations, especially in local distribution. As indicated in the problem statement, transportation costs are 50% of total logistics costs which in turn represent up to 15% of products final price; according to that, the positive impact in distribution efficiency is expected to reduce product unit cost, which may lead to increase industrial competitiveness.

Future lines for research must consider knowledge gaps identified in the literature review analysis. These knowledge gaps can be classified in two:

- *Considerations for the problem structure towards real life implementation*: Literature analysis showed that most of the works consider mono-objective problems; this may be a problem in real life applications, since logistics problems, and especially transportation problems are multi-objective by nature; moreover, most of the works consider classic objective functions such as total cost or total distance traveled. In addition, applications designed to support operative decisions in distribution systems must consider changing information, such as travel times, traffic, and even topographic information. Future works must consider multi-objective problems, considering not only financial objective functions but objectives related to time travel, load balance, and objectives related to systems sustainability e.g.: carbon footprint, fuel consumption etc. Future works must consider also changing information, this is an important field of research, and future lines for research must be oriented to develop robust algorithms to solve dynamic vehicle routing problems.
- *Considerations to develop robust solution methods:* Future works must be oriented to provide robust solution strategies. To provide robust algorithms, robustness must be considered in three dimensions: Efficiency in execution times, optimality capabilities and independency for problem topology (size and shape). It has been shown that actually, the most adequate strategy to tackle this type of problem, with the aim to provide applications for real implementation, is to develop meta-heuristic algorithms. Future works have to consider the development of hybrid meta-heuristics, since it has been proved that for difficult problems the possibility to exploit characteristics of multiple types of heuristics and meta-heuristics lead to better solutions.

To continue this work, a performance comparison with other approximate (classic and hybrid) methods is proposed due to lack of benchmark instances specifically for the MDVRP minimizing total cost and load balance. An improvement of initial population is proposed using fuzzy choice for heuristics and/or implementation of various classic techniques; furthermore, an improvement of inter-depot-inter-route local search is proposed. Finally, sustainable development is a very important field, due to this, the inclusion of objective functions related to carbon footprint is proposed.

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APPENDIX

Table 18. Reviewed papers about the Single-Objective MDVRP

Year	Reference	Constraints							Solution method		
		Time Windows	Heterogeneous fleet	Capacitated	Periodic	Pick up & Delievery	Split delivery	Other	Exact	Heuristic	Meta-heuristic
1984	Laporte et al. (1984)								X		
1985	Kulkarni and Bhave (1985)			X					X		
1987	Laporte and Nobert (1987)								X		
1988	Laporte et al. (1988)			X					X		
1989	Laporte (1989)								X		
	Carpaneto et al. (1989)			X					X		
1992	Min et al. (1992)			X				X		X	
	Wilger and Maurer (1992)									X	
1993	Chao et al. (1993)									X	
1996	Renaud et al. (1996a)			X				X			X
1997	Filipec et al. (1997)			X							X
	Salhi and Sari (1997)		X							X	
	Cordeau et al. (1997)				X						X
1998	Salhi et al. (1998)										X
	Hadjiconstantinou and Baldacci (1998)				X					X	
1999	Vianna et al. (1999)				X					X	X
	Tüzün and Burke (1999)							X			X
	Salhi and Nagy (1999)							X		X	
2000	Irnich (2000)		X			X			X	X	
	Filipec et al. (2000)			X						X	X
	Skok et al. (2000)			X						X	X
	Yang and Chu (2000)				X					X	
2001	Thangiah and Salhi (2001)									X	X
	Skok et al. (2001)			X						X	X
	Cordeau et al. (2001)	X			X						X
	Chan et al. (2001)							X		X	
2002	Wu et al. (2002)		X	X							X
	Angelelli and Speranza (2002)				X			X			X
	Zhang et al. (2002)			X					X	X	
	Giosa et al. (2002)	X								X	
2003	Dondo et al. (2003)	X	X						X		
	Kazaz and Altinkemer (2003)							X	X	X	
2004	Matos and Oliveira (2004)				X						X
	Wasner and Zapfel (2004)					X				X	
	Jin et al. (2004)									X	
2005	Mingozzi (2005)				X				X		
	Nagy and Salhi (2005)					X				X	
	Polacek et al. (2005)	X		X							X
	Lim and Wang (2005)			X					X		X
	Baltz et al. (2005)									X	
	Songyan et al. (2005)										X
	Jin et al. (2005)	X									X
	Songyan and Akio (2005)										X

2006	Parthanadee and Logendran (2006)				X						X
	Yang et al. (2006)	X									X
	Chiu et al. (2006)	X								X	
	Lim and Zhu (2006)			X				X		X	X
2007	Dondo and Cerdá (2007)	X	X						X	X	
	Jeon et al. (2007)		X								X
	Crevier et al. (2007)								X		X
	Bae et al. (2007)		X	X							X
	Pisinger and Ropke (2007)	X		X		X					X
	Özyurt and Aksen (2007)								X		X
	Carlsson et al. (2007)									X	
	Hu et al. (2007)					X				X	
	Wang et al. (2007)										X
	Lou (2007)							X		X	X
	Tsirimpas et al. (2007)			X				X	X		
2008	Ho et al. (2008)									X	X
	Kek et al. (2008)			X		X			X		
	Dondo et al. (2008)	X	X			X			X	X	
	Polacek et al. (2008)	X		X							X
	Dai et al. (2008)									X	X
	Li and Liu (2008)										X
	Chen et al. (2008a)							X		X	
	Yang (2008)	X	X								X
	Chen et al. (2008b)	X	X	X							
	Goela and Gruhn (2008)	X	X	X		X	X	X		X	
2009	Flisberg et al. (2009)	X	X			X	X		X		X
	Dondo and Cerdá (2009)	X	X						X	X	
	Baldacci and Mingozzi (2009)								X		
	Ting and Chen (2009)	X									X
	Zhen and Zhang (2009)							X	X		X
2010	Schmid et al. (2010)		X				X		X		X
	Mirabi et al. (2010)									X	X
	Liu et al. (2010)			X			X			X	
	Vidal et al. (2010)		X	X	X						X
	Ma and Yuan (2010)										X
	Sombunthama and Kachitvichyanukulb (2010)	X				X				X	
	Sepehri and Kargari (2010)							X	X		
	Gajpal and Abad (2010)					X				X	
	Villegas et al. (2010)			X				X		X	X
	Garaix et al. (2010)			X				X	X	X	
	Ghoseiri and Ghannadpour (2010)	X							X	X	X
	Kansou and Yassine (2010)									X	X
2011	Gulczynski et al. (2011)						X		X	X	
	Bettinelli et al. (2011)	X	X	X					X	X	
	Yücenur and Demirel (2011a)										X
	Aras et al. (2011)			X							X
	Zarandi et al. (2011)	X		X							X
	Yang et al. (2011)							X	X		X
	Wang et al. (2011)	X							X	X	X
	Samanta and Jha (2011)	X						X			X
	Yücenur and Demirel (2011b)									X	X
	Yu et al. (2011)										X
	Lei et al. (2011)									X	
	Fard and Setak (2011)							X		X	X
	Zhang et al. (2011)							X			X

	Surekha and Sumathi (2011)									X	X
	Maischberger and Cordeau (2011)	X		X	X			X		X	X
2012	Maya et al. (2012)			X	X					X	X
	López Franco and Nieto Isaza (2012)			X						X	
	Nieto Isaza et al. (2012)	X	X						X		
	Cornillier et al. (2012)	X	X					X	X	X	

Table 19. Reviewed papers about the multi-objective MDVRP

Year	Reference	Constraints							Solution method		
		Time windows	Heterogeneous fleet	Capacitated	Periodic	Pick up & Delievery	Split delivery	Other	Exact	Heuristic	Meta-heuristic
2005	Lin and Kwok (2005)							X			X
2006	Tan et al. (2006)	X	X								X
2009	Lau et al. (2009)										X
	Hasanpour et al. (2009)							X			X
	Ombuki-Berman and Hanshar (2009)			X				X		X	X
2010	Dharmapriya and Siyambalapitiya (2010)	X					X				X
	Tavakkoli-Moghaddam et al. (2010)			X							X
	Jiang and Ding (2010)					X		X		X	
	Lau et al. (2010)								X		X
	Weise et al. (2010)	X		X	X	X		X		X	X
	Ghoseiri and Ghannadpour (2010)	X								X	X
2011	Venkatasubbaiah et al. (2011)									X	
	Li and Liu (2011)										X

Table 20. Variance analysis for %Δ*C*

```
Análisis de varianza para %Dif Costo (unidades codificadas)

Fuente                               GL   SC Sec.   SC Ajust.   CM Ajust.      F
Bloques                               3  0,042996  0,0429956  0,0143319   4,11
Efectos principales                   3  0,093708  0,0937081  0,0312360   8,95
  Num_Iter                            1  0,023014  0,0230141  0,0230141   6,60
  Tam_Pobla                           1  0,000168  0,0001678  0,0001678   0,05
  Tam_Refset                          1  0,070526  0,0705262  0,0705262  20,21
2-Interacciones de (No.) factores     3  0,000367  0,0003666  0,0001222   0,04
  Num_Iter*Tam_Pobla                  1  0,000145  0,0001447  0,0001447   0,04
  Num_Iter*Tam_Refset                 1  0,000144  0,0001443  0,0001443   0,04
  Tam_Pobla*Tam_Refset                1  0,000078  0,0000776  0,0000776   0,02
3-Interacciones de (No.) factores     1  0,002044  0,0020436  0,0020436   0,59
  Num_Iter*Tam_Pobla*Tam_Refset       1  0,002044  0,0020436  0,0020436   0,59
Error residual                       21  0,073269  0,0732691  0,0034890
Total                                31  0,212383

Fuente                                   P
Bloques                              0,019
Efectos principales                  0,001
  Num_Iter                           0,018
  Tam_Pobla                          0,829
```

Tam_Refset	0,000
2-Interacciones de (No.) factores	0,991
Num_Iter*Tam_Pobla	0,841
Num_Iter*Tam_Refset	0,841
Tam_Pobla*Tam_Refset	0,883
3-Interacciones de (No.) factores	0,453
Num_Iter*Tam_Pobla*Tam_Refset	0,453
Error residual	
Total	

Table 21. Variance analysis for %Δ*R*

Análisis de varianza para %Dif Balance (unidades codificadas)

Fuente	GL	SC Sec.	SC Ajust.	CM Ajust.	F
Bloques	3	0,22054	0,220544	0,073515	2,97
Efectos principales	3	0,24526	0,245256	0,081752	3,30
Num_Iter	1	0,03728	0,037279	0,037279	1,51
Tam_Pobla	1	0,00412	0,004122	0,004122	0,17
Tam_Refset	1	0,20385	0,203855	0,203855	8,23
2-Interacciones de (No.) factores	3	0,09249	0,092488	0,030829	1,24
Num_Iter*Tam_Pobla	1	0,00659	0,006588	0,006588	0,27
Num_Iter*Tam_Refset	1	0,00042	0,000419	0,000419	0,02
Tam_Pobla*Tam_Refset	1	0,08548	0,085481	0,085481	3,45
3-Interacciones de (No.) factores	1	0,01437	0,014367	0,014367	0,58
Num_Iter*Tam_Pobla*Tam_Refset	1	0,01437	0,014367	0,014367	0,58
Error residual	21	0,52007	0,520070	0,024765	
Total	31	1,09272			

Fuente	P
Bloques	0,055
Efectos principales	0,040
Num_Iter	0,233
Tam_Pobla	0,687
Tam_Refset	0,009
2-Interacciones de (No.) factores	0,319
Num_Iter*Tam_Pobla	0,611
Num_Iter*Tam_Refset	0,898
Tam_Pobla*Tam_Refset	0,077
3-Interacciones de (No.) factores	0,455
Num_Iter*Tam_Pobla*Tam_Refset	0,455
Error residual	
Total	

Figure 36. Residuals graphs for %Δ*C*

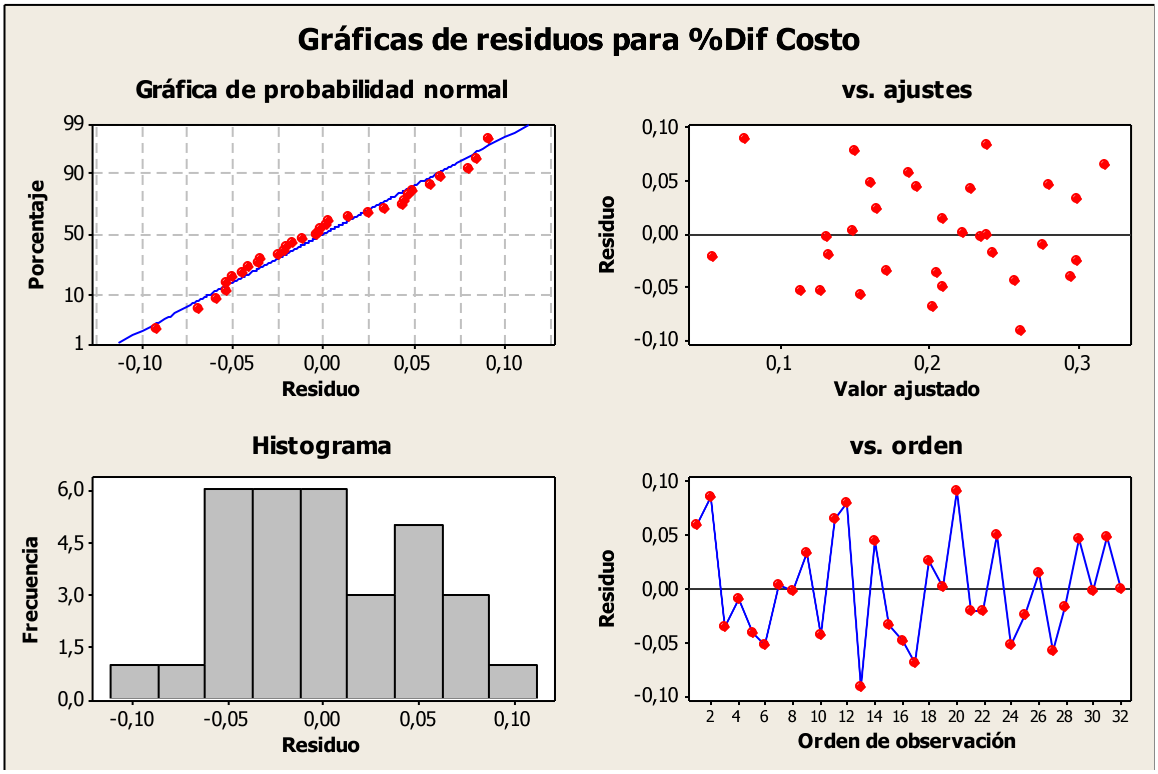

Figure 37. Residuals graphs for %ΔR

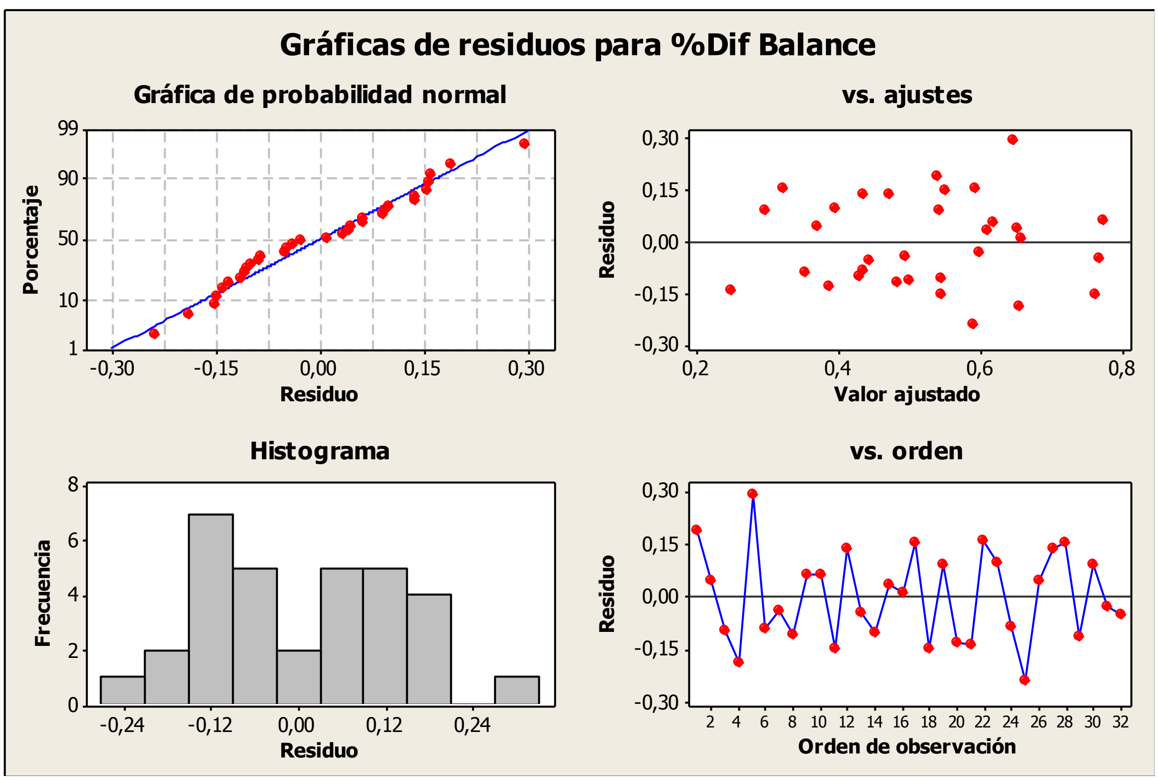

Figure 38. Main effect charts for %ΔC

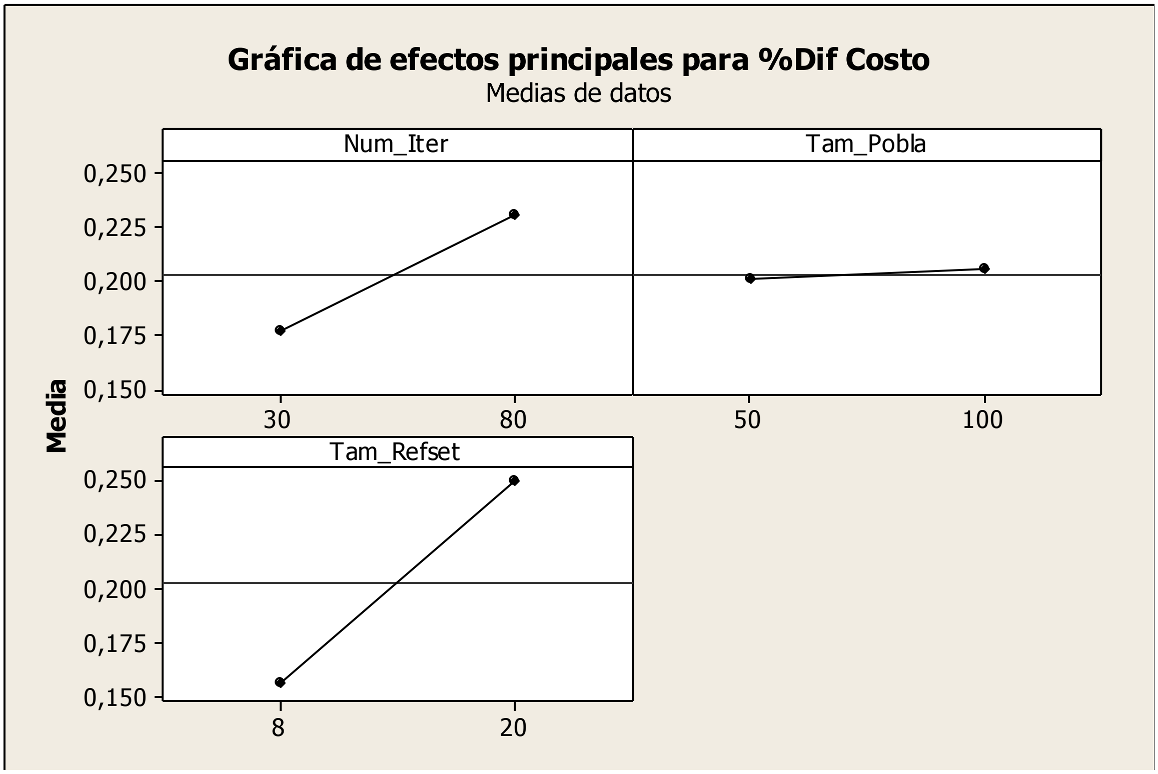

Figure 39. Main effect charts for %Δ*R*