SIMPLE AND PRACTICAL OPTIMIZATION APPROACH BASED TO SOLVE A TRUCK LOAD AND DELIVERY PROBLEM AT LONG HAUL DISTANCES WITH HETEROGENOUS PRODUCTS

ABSTRACT

This paper proposes an optimization based approach for modeling and solving the logistic processes of deliveries scheduling and product accommodation during loading with a heterogeneous fleet of vehicles. The approach focuses on the case of products with "low density values" (low cost per unit weight) and high heterogeneous volume and weight, and with traveling large distances to different zones, in which transportation costs constitute a very important proportion of total logistic costs.

The proposed approach consists of a two-phase strategy: The first phase uses a "Cutting Stock Problem" formulation to define utilization areas inside trucks assigned to each product family. This task is achieved by minimizing the number of required vehicles and long-haul transportation costs as a function of the vehicle size, considering a set of predefined solutions for feasible and efficient loading (patterns), obtained as a result of the accumulated experience. The second phase consists of Bin Packing Problem version with a known number of bins, which were previously determined in the first phase of the approach. In this phase, different orders from a set of customers are assigned to each truck by obeying the predefined utilization areas per product category obtained in the first phase while minimizing the number of visits of each truck.

The results show that the proposed model addresses the analyzed problem in an efficient manner, which is reflected in reasonable resolution times and costs from a practical implementation perspective. Additionally, it is observed that long-haul delivery costs and vehicle utilization tend to improve with the increase of the utilized number of patterns even when the execution time is incremented.

Keywords: loading vehicles, delivery optimization, shipping scheduling, integer programming.

1. INTRODUCTION

In the current highly competitive business environment, obtaining comparative advantages is essential to staying competitive in the marketplace, particularly as a consequence of price competition. One of the main opportunities for significant cost reduction and increase the company´s efficiency is found in the logistic area, which is responsible of locating the correct products at the adequate location, at the right moment and desired condition by yielding the highest company profit. One of the main challenges of logistics area is to design and operate a supply chain so that total systemwide cost are minimized, while systemwide levels of service are maintained (Simchi-Levi, et al., 2008).

In this context, the current research is aimed to reduce the logistic cost by specifically optimizing loading processes, accommodation and delivery of trucks. In order to achieve this challenge, a practical two-phase optimization based approach is proposed for addressing truck loading issues in cases of low value density products, highly heterogeneous and incompatible with each other that are not possible to palletize.

The addressed problem in this research is pertinent for the case of companies that manages a range of products with the aforementioned characteristics. A company receives orders from multiple clients and needs to solve the problem of accommodating products in bins (trucks) and programming the truck dispatch to meet the demand while minimizing costs.

Many companies outsource transportation services to logistics partners that usually have a heterogeneous fleet of vehicles. The freight for a certain destination may have a fixed component that depends on the type of utilized vehicle and a variable component according to the number of served clients. Therefore, the freight does not depend significantly on the accommodated quantity that is finally shipped in the vehicle, so that the company´s interest is to take advantage of its maximum capacity.

The majority of the company customers are located in distant cities of the production plant and distribution center, as shown in Figure 1. Consequently, in terms of costs, transportation among cities has a major relevance than routing within each zone. Additionally, orders tend to be large, which entails that a same vehicle serves a small number of clients.

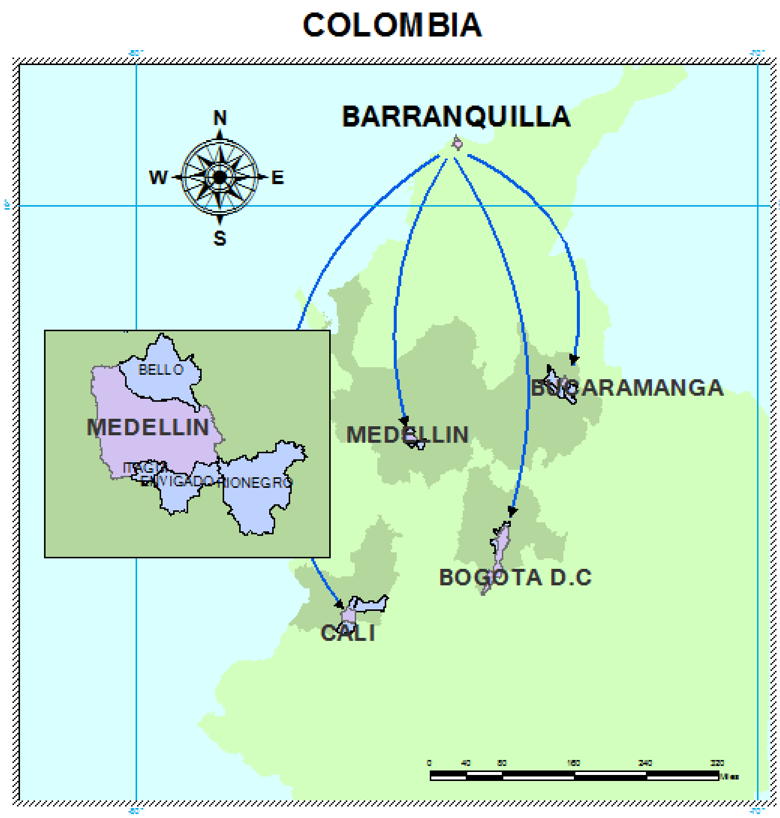


Figure 1 Distances between zones versus interzone distances for the Colombian case

In order to solve the real world complex problem presented in this study, the proposed approach takes advantages of the company experience with historical achieved dispatching records. The scheduling process of dispatching vehicles and product accommodation in vehicles has been performed manually and is time-consuming. These records (which define pseudo-efficient patterns of loading trucks) are useful for the approach presented in this research.

When analyzing the described situation, considering the nature of the transported products and the payment scheme to carriers, a further improvement in truck capacity is a key for cost reduction, i.e., achieve an improved product accommodation inside trucks.

Due to the high heterogeneity and incompatibility among different handled products and seeking to take advantage of historical records of dispatches carried out by the company (which establish loading patterns of feasible trucks), a resolution approach was employed where these records are considered as a database of possible solutions. Thus, the optimization model will search the best fit to the demanded items for the clients.

Summarizing, the objective of this research is to present a practical approach useful to support the decision-making process, loading planning, and dispatching of trucks of a company.

The remainder of the manuscript is organized as follows. Section 2 discusses approaches that are usually developed to solve problems that are similar to the case analyzed in this study, in addition being support for building and understanding the proposed approach. Section 3 presents the proposed optimization models. Section 4 presents an example that illustrates a model for a small scale case. Subsequently, an applied case of a Colombian company from the laminated steel sector is presented in Section 5. Finally, conclusions and final remarks are presented in Section 6.

2. BACKGROUND

The vehicle loading problem that gave rise to the current research has been addressed usually as a Container Loading Problem (CLP) with different variations. CLP may be interpreted as a geometric allocation problem, in which the three dimensions of small items must be assigned to a container. Typical models optimize the area (taking advantage of the unfilled container space) by fulfilling two basic restrictions: i) all objects must be completely inside of the container, and ii) the larger objects cannot superimpose one of smaller size (Bortfeldt & Wäscher, 2012).

The CPL problem may be addressed as a minimization or a maximization problem. In the first case, it is assumed that there are sufficient containers available for loading a certain quantity of items, so that the minimum number of containers is employed. Regarding the second case, a restriction exists that provides a limited quantity of containers, in order to load only a part of the items that seeks that the loaded item value should be the maximum possible (Bortfeldt & Wäscher, 2012). This study addresses the problem as a minimization case.

A set of variations of CLP exist in the literature depending on the problem conditions. For the minimization problem case, it is feasible to establish a categorization, as presented in Table 1 (Wäscher, et al., 2007).

Table 1 CLP minimizing categorization according to the problem conditions *(Wäscher, et al., 2007).*

Container characteristics \ Item or product classification		Low heterogeneous	Highly heterogeneous
All fixed sizes	**Identical**	*Single Stock-Size Cutting Stock Problem (SSSCSP)*	*Single Bin-Size Bin Packing Problem (SBSBPP)*
	Low heterogeneous	*Multiple Stock-Size Cutting Stock Problem (MSSCSP)*	*Multiple Bin-Size Bin Packing Problem (MBSBPP):*
	Highly heterogeneous	*Residual Cutting Stock Problem (RCSP):*	*Residual Bin Packing Problem (RBPP)*

In addition to the aforementioned, the majority of the cases includes restrictions of vehicle capacity, loading mechanisms, orientation, positioning, stacking, stability, and compatibility of items or objects (Bortfeldt & Wäscher, 2012), involve an increase in complexity of the CLP solution for both the proposed scenario model and the resolution model. This leads to the frequent need of implementing heuristic or approximated methods, as the two procedures employed sequentially by Hassler & Talbot (1990). The first procedure is the stacking of items or objects, and the second procedure is the location of piles of objects. At the same time, Portmann (1990) proposes an approximation that fills the container with items in a bottom-up manner, while Gehring et al (1990) generate vertical layers in the container filling layer by layer. The methods that present the best results in the optimization process are metaheuristics such as GA genetic algorithms. An application is the study performed by Gehring & Bortfeldt (1997), who analyzed the loading case of highly heterogeneous products, and Sheng, et al. (2014), who analyzed a heuristic binary tree search method (HBTS) for the 3D-CLP, and includes full support constraint, orientation constraint and guillotine cutting constraint in the algorithm. The algorithm includes several steps, like the case of current research.

In more recent investigations they have tried different approaches seeking greater computational efficiency, i.e., less runtime. Araya & Riff (2014) present a beam search approach, and makes a comparison of results with the state of art, concerning runtimes and computational optimization. Similarly, Wei, et al. (2015) compares the results with the state of art in terms of both solution quality and computation time, introducing an approach that combines a prototype column generation method with a goal-driven strategy to CLP.

The complementary problem included in this study considers the Vehicle Routing Problem (VRP), which is well-known in the literature since in Dantzig & Ramser (1959) addressed it with a linear programming formulation. Subsequently, Clarke y Wright developed a known heuristics that improves the obtained results with an approach of Dantzig-Ramser. This marked the beginning of the development of a large number of studies proposing solutions to the VRP (Maffioli, 2003). Mere optimization methods such as linear programming and "branch and bound" processes are employed to solve small and medium size instances with relatively simple restrictions, whereas heuristics and metaheuristics obtain close to optimal solutions for problems with medium to large size problems (De la Cruz, et al., 2013).

However, since the problem addressed in this study is related to long distance vehicle dispatching, transportation costs between the factory and different zones are an important aspect to consider. A major cost reduction is obtained when reducing the number of vehicles to employ, and routing becomes less significant within each zone or city for very short distances when compared to routing between cities. Thus, routing is not included in the process, and transportation is limited between dispatching center and zones. Once clients are assigned to each vehicle, the routing problem is similar to the travel salesman problem.

In summary, this study involves a highly complex problem as a result of the inclusion of a set of compatibility restrictions between products in the model, and the simultaneous solution of vehicle loading problems, efficient assignment of items to a heterogeneous fleet, and dispatching to serve the client demand. A two phase strategy is proposed to address this problem. The first phase is the derivative of the Cutting Stock Problem (CSP), and the second phase is the application of the Bin Packing Problem (BPP).

2.1 Cutting Stock Problem

The Cutting Stock Problem (CSP) is a linear programming model that consists of dividing or cutting raw material in smaller size to fulfill client orders. The way an element is cut is known as cut pattern, and each pattern contributes to satisfy part of customer demands for each product size including a certain amount of material scrap (Reinertsen & Vossen, 2010).

This problem often has applications to a variety of manufacturing industries such as paper, cardboard, textile, timber, metallurgic, which require laminated cuts or coils of raw material to produce pieces of certain dimensions to minimize material loss (Carrascosa, et al., 1997).

The first formulation of the CSP was presented by Kantorovich in 1939. However, the major advancement in the resolution of this type of problems was performed by Gilmore & Gomory (1961) and Gilmore & Gomory (1963), who employed a cut pattern generation technique to minimize the loss of material using linear programming (Haessler & Sweeney, 1991).

Starting from the classical cutting stock model of Gilmore and Gomory (1961), Furini, et al. (2012) proposed a two-step approach. The first is a column generation based heuristic algorithm, and the second step is a mixed integer linear programming model (MILP). The paper presents the evaluation of the proposed approach in different computational experiments and compare the results the state of the art.

Dyckhoff (1990) discussed the close existing relationship between the Cutting Stock Problem and the Packing Problem; both are logical structure as two groups of basic data, the stock (large items) and the list or order book (small items), and both realizes patterns being geometric combinations of small items assigned to large objects (Dyckhoff, 1990). An extension of the model proposed by Dyckhoff (1990) for the one-dimensional cutting stock problem, was propose by Silva, et al. (2010) for the two-dimensional case. The relevance of this article is to propose an integer programming model like the case of current research.

CSP may be adapted considering the finished products as raw material, which are not cut physically, but divided in different containers with different loading capacities of the product and representing a pattern. The losses are represented as not employed truck loading capacity. Therefore, when minimizing unused capacity and maintaining a constant served demand, a reduction in the number of containers is obtained as a result (required vehicles for dispatching products). Figure 2 presents a description of this approximation with predefined loading patterns used efficiently to organize dispatching ordered items. The CSP planned shipments, taking as input a list of orders, with different demand for each product; and a list of load patterns, with different capacity for each product.

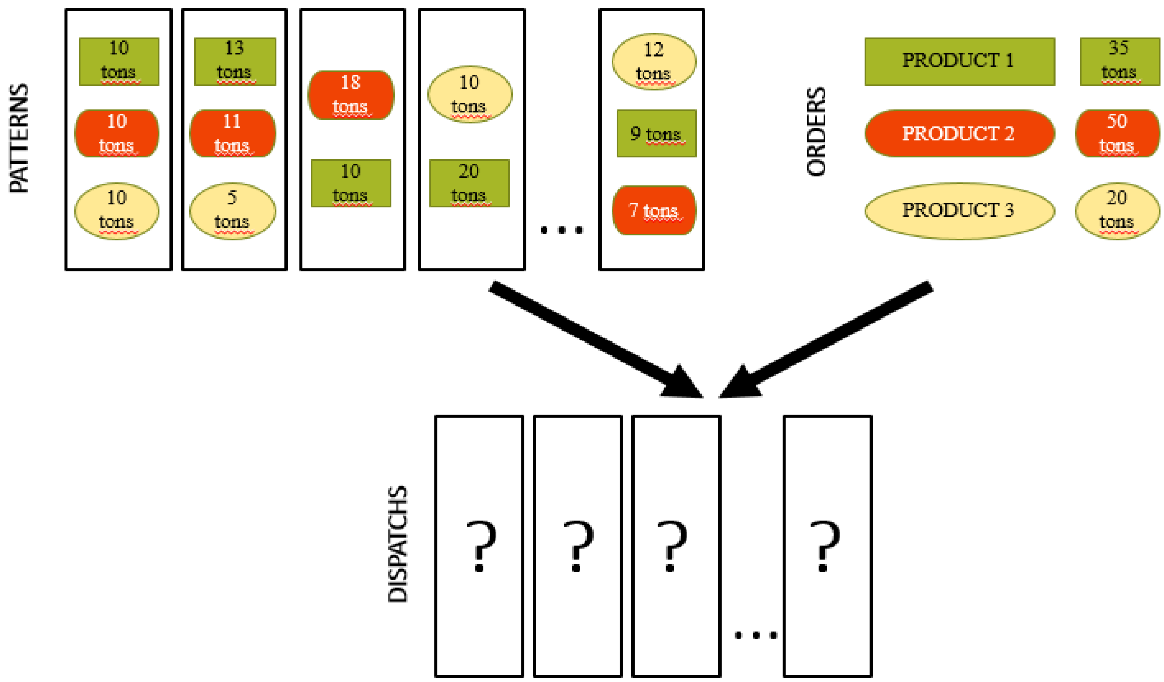


Figure 2 CSP schema

2.2 Bin Packing Problem

The BPP is considered one of the most fundamental and most current research topics in combinatorial optimization. In the classic BPP, a series of containers with limited capacity and a set of elements with known weight are presented, whose activity is to assign articles to a set of containers without exceeding their capacities and minimizing the number of required containers (Casazza & Ceselli, 2014).

A specific case of BPP identified as Variable Cost and Size Bin Packing Problem (*VCSBPP*), employs different types of containers that vary with capacity and price (Baldi, et al., 2012). Haouari & Serairi (2009) also study the VCSBPP and raised six heuristics solution; in which employed: exact solution, column generation and genetic algorithm methods. Their computational study, carried out on a large variety of problem instances, provides strong evidence of the efficacy of the set covering heuristic. However, Haouari & Serairi (2009) research is limited to one-dimensional problems (1D-BPP), which have lower complexity than the problem approached in this investigation. Similarly, Fleszar & Hindi (2002) discussed the 1D-BPP, and they present several new heuristics. The most effective algorithm turned out to be one based on running one of the former to provide an initial solution for the latter, based on the minimal bin slack (MBS) heuristic of Gupta and Ho.

Lodi, et al. (2002) surveyed recent advances obtained for the two-dimensional bin packing problem. The review by Lodi, et al. is important for its emphatic analysis on the exact solutions, and indicates that one of the major advances in this topic was Martello & Vigo (1998). Martello & Vigo pose an exact algorithm that adopts a nested branching scheme, and permits the exact solution of the problems involving up to 120 pieces.

The proposed model in this study considers a VCSBPP since containers, represented as utilized trucks to serve clients, constitute a heterogeneous fleet. There are different types of trucks with different capacities and fleet costs and each must accommodate different ordered products. However, the number of containers is constant and it is known in advance, which is obtained as a result from the CSP model in the first phase. Thus, the assignment is sought to minimize delivery costs.

3. PROBLEM DESCRIPTION AND MODELING APPROACH

3.1 Problem Description

The problem to solve is to optimize logistic processes of deliveries scheduling and product accommodation during loading. In the case of products with “low density values” (low cost per unit weight) and high heterogeneous volume and weight, and with traveling large distances to different zones, in which transportation costs constitute a high proportion of total logistic costs.

The modeling approach for loading and dispatching of vehicles comprises three sequential components. A preprocessing process for clusterization of clients to be served is performed in the initial phase. Next, in a first optimization stage, a model is applied to identify the type and minimum number of required vehicles to satisfy a given demand. Finally, in the last phase, a second optimization model is employed to load these vehicles while minimizing the associated transportation costs.

3.2 Preprocessing: Client Clusterization

A clusterization procedure is developed prior to the implementation of the proposed optimization models, which analyzes each client as a destination node forming a node network by associating groups according to their geographic proximity. Therefore, depending of the desired aggregation level, clients are grouped at the city, metropolitan area or region level to define distribution zones. This strategy seeks to consolidate the load and find an improved solution to the truck loading problem in the following models.

3.3 Optimization Phase 1. Loading Pattern Selection Problem (LPSP)

The first model is of the cutting stock type, and performs the selection of the type and quantity of required patterns to dispatch, in order to satisfy multiproduct and multizone demand, while minimizing transportation costs, and heterogeneous fleet size (number of vehicles). A feasible configuration of loading and filling of a certain type of truck is defined a pattern that indicates the loading quantity for each family of products. A pattern involves the type of vehicle loaded with a set of products. The patterns may consist of a single family or several families of different products. The latter are obtained from the vast documented experience of the company on dispatched product accommodation, or from adaptations or combinations of existing patterns.

A set of patterns $p=1,2,\ldots,P$; a set of family of products $f=1, 2,\ldots,F$; a set of zones $z=1,2,\ldots,Z$; and the shipping costs (C_{pz}) from the warehouse to zone z of pattern p are defined. The integer decision variable X_{pz} represents the amount of loaded vehicles according to pattern p sent to zone z while r_{fp} is the amount of product f in pattern p. Finally, d_{fz} is the demand of product f in zone z.

The optimization problem is described as follows:

$$Min \quad \sum_{z=1}^{P} C_{pz} \cdot X_{pz} \qquad (1)$$

subject to:

$$\sum_{p=1}^{P} X_{pz} \cdot r_{fp} \geq d_{fz} \qquad \forall f = 1,\ldots,F;\ z = 1,\ldots,Z \qquad (2)$$

$$X_{pz} \in \mathbb{Z}^{+} \qquad \forall p = 1,\ldots,P;\ z = 1,\ldots,Z \qquad (3)$$

The objective function optimizes the fixed costs associated to the vehicles sent to the distribution zones for all possible patterns. The set of restrictions (2) warrants that the demand is satisfied for each family of products and for each defined zone in the clusterization process. Finally, Restrictions (3) limit the decision variable to the set of non-negative integers since it deals with number of vehicles. The vehicle capacity restriction is unnecessary because patterns are feasible.

3.4 Optimization Phase 2: Orders Allocation Problem (OAP)

The second Bin Packing type model is responsible of allocating orders to be served by each vehicle-pattern defined by the model in the previous phase, and of minimizing the number of visits performed by the set of vehicles X_{pz}, which is known from the previous model and represents the number of selected vehicles for each loading pattern p assigned to serve every zone z. The problem in this optimization phase consists of minimizing the total number of visits to clients, which determine the transportation cost structure. This problem must be solved for one of the zones since it defines the distribution at the client level.

In this case, the set of vehicles $v= 1,2,\ldots,V$, of clients $c= 1,2,\ldots,C$, and of families of products $f=1,2,\ldots,F$ are specified. The decision variable Y_{vc} has a value of 1 in the case that the client c is served by vehicle v and 0 otherwise, where a client may be served by more than one vehicle. In addition, decision variable X_{vcf} indicates the proportion of the demand of client c for the family f that is assigned to vehicle v. Variable t_{cf} has a value of 1 if the demand of family f exists on the part of the client c, 0 otherwise.

In the proposed model, q_{cf} is the demand of the product family f for client c, whereas Q_{vf} is the amount of product f that can be loaded on vehicle v, as maximum value (capacity) The latter variable is essential in terms of the relationship between the first and second model since it is determined from the optimization of the first model, where the employed patterns are defined. For each specific selected pattern, the assigned or reserved space is known for each family of products.

One of the assumptions of the second model is the order division is possible, and more than one vehicle may be dispatched. In other words, a client may be served through various patterns (vehicles). Consequently, a portion of the order of each client that is allocated to each vehicle is modeled continuously.

The optimization problem is presented as follows.

$$Min \quad \sum_{v=1}^{V}\sum_{c=1}^{C} Y_{vc} \qquad (4)$$

subject to :

$$Y_{vc} \geq X_{vcf} \qquad \forall v = 1,...,V;\ c = 1,...,C;\ f = 1,...,F \qquad (5)$$

$$\sum_{v=1}^{V} X_{vcf} = t_{cf} \qquad \forall j = 1,...,M;\ f = 1,...,F \qquad (6)$$

$$\sum_{c=1}^{C} X_{vcf} \cdot d_{cf} \leq Q_{vf} \qquad \forall i = 1,...,N;\ f = 1,...,F \qquad (7)$$

$$0 \leq X_{vcf} \leq 1 \qquad \forall v = 1,...,V;\ c = 1,...,C;\ f = 1,...,F \qquad (8)$$

$$Y_{vc} \in \{0,1\} \qquad \forall v = 1,...,V;\ c = 1,...,C \qquad (9)$$

The objective function minimizes the number of visits. The set of restrictions (5) avoid that variables X are different than zero for clients that are not visited; restrictions (6) guarantees the complete fulfillment of client demands, where parameters t_{cf} are defines according to equation (10).

$$t_{cf} = \begin{cases} 0 & q_{cf} = 0 \\ 1 & q_{cf} \geq 0 \end{cases} \qquad \forall c = 1,...,C;\ f = 1,...,F \qquad (10)$$

Restrictions (7) does not allow that the allocated capacity to product f of vehicle v is not exceeded. Finally, (8) and (9) state domain constraints for the model decision variables.

4. SAMPLE ILLUSTRATION

This section presents a small size scenario to illustrate the proposed problem and the solution process. The pattern selection model employs the following parameters:

Zone (Z) is the final destination of the dispatch, where a city or set of cities are served by the same vehicle due to their geographic proximity. These zones are defined after the clusterization process to group the clients. This example consists of four dispatching zones.

Family or product category (F) refers to the product classification based solely on the logistic management. Therefore, the products with equal or similar loading and dispatching management are located in the same family. In this case, four families or product categories are considered.

Loading patterns (P) are configurations of products to load. This is feasible from the accommodation point of view without exceeding the maximum capacity of the vehicle. Thus, each loading pattern is related to a specific vehicle type. This example has three types of vehicles: truck with 2-axle truck with a maximum loading capacity of 10 tons (C2), 3-axle truck with a maximum loading capacity of 18 tons (C3), and 6-axle truck with a maximum loading capacity of 35 tons (C6). This information complies with the allowable load for each type of vehicle. In this case, the maximum load is defined as established by the Colombian Ministry of Transportation (2009).

A pattern is defined as a possible pseudo-optimal solution to the problem of filling a truck with certain amount of products without exceeding the vehicle capacity. A set of patterns is obtained from the company experience on vehicle loading and dispatching. Therefore, the search universe of the model solutions is composed by a group of defined patterns. Accordingly, it is convenient to generate a number of feasible pseudo-optimal loading patterns as large as possible.

In the example, 40 patterns for each type of vehicle and family of products are presented in Table 2. Patterns P1C6 through P12C6 are loading patterns for truck C6 with a maximum capacity of 35 tons, which may be referred as multiproduct loadings due to logistic or geometric incompatibilities.

Table 2 Loading patterns of the example in Tons.

		LOADING PATTERN																			
		TRUCK C6												**TRUCK C3**				**TRUCK C2**			
		P1C6	**P2C6**	**P3C6**	**P4C6**	**P5C6**	**P6C6**	**P7C6**	**P8C6**	**P9C6**	**P10C6**	**P11C6**	**P12C6**	**P1C3**	**P2C3**	**P3C3**	**P4C3**	**P1C2**	**P2C2**	**P3C2**	**P4C2**
FAMILY OF PRODUCTS	**F1**	35	0	0	0	19	0	13	0	15	0	5	10	18	0	0	0	6	0	0	0
	F2	0	35	0	0	0	19	0	13	10	15	0	5	0	18	0	0	0	6	0	0
	F3	0	0	35	0	0	13	19	0	5	10	15	0	0	0	18	0	0	0	6	0
	F4	0	0	0	35	13	0	0	19	0	5	10	15	0	0	0	18	0	0	0	6

The variables included in the election pattern model are the following:

C_{pz}: Costs associated to the shipping of a loaded truck according to pattern p to zone z. Only freight costs were included in this scenario, thus, the cost is given by the freight value for the type of vehicle belonging to pattern p from the warehouse to the city (destination). Table 3 shows the costs for the example in dollars.

Table 3 Freight costs (US$)

		ZONES			
		BOGOTÁ	**CALI**	**CARTAGENA**	**MEDELLIN**
		Z1	**Z2**	**Z3**	**Z4**
TYPES OF VEHICLES	**TRUCK C6**	2222	2694	228	1640
	TRUCK C3	1431	1743	146	1062
	TRUCK C2	1183	1451	117	901

d_{fz} This parameter refers to the total demand of each product per zone to be satisfied with the current dispatching schedule. The amount of demands of the example is shown in Table 4.

r_{fp} This variable indicates the quantity of family of product f that may be loaded in pattern p. In other words, this variable is the capacity of the vehicle allocated to this family if the truck is loaded according to pattern p. The values of r_{fp} are presented in Table 2. Finally, X_{pz} is the decision variable of the pattern election model, which indicates the amount of loaded truck dispatched to zone z according to pattern p so that the cost is as minimum as possible while satisfying the demand. When referring to the number of patterns, this variable may only be an integer, and each pattern is related to a single type of vehicle. The number of vehicle type to dispatch is determined from the number of vehicles assigned to each pattern.

Table 4 Amount of demands in tons.

		ZONES			
		BOGOTÁ	CALI	CARTAGENA	MEDELLIN
		Z1	Z2	Z3	Z4
FAMILY OF PRODUCTS	F1	85	0	8.9	0
	F2	50	25	10	45
	F3	0	0	0	7.3
	F4	9.2	17	15	5

Table 5 presents the matrix of X_{pz} obtained as a result of applying the first model. Note that 11 vehicles are dispatched to all zones to satisfy the demand, 5 dispatched to Bogotá (Z1), 2 to Cali (Z2), 2 to Cartagena (Z3), and 2 to Medellin (Z4). From these vehicles, eight are 6-axle trucks, 2 are 3-axle trucks, and 1 is a 2-axle truck. Once the required patterns are known, the type of vehicle is identified since each pattern refers to a single type of vehicle. The total dispatching cost is equal to $ US18.381.

Table 5 Matrix for pattern selection model

		ZONES			
		BOGOTÁ	CALI	CARTAGENA	MEDELLIN
		Z1	Z2	Z3	Z4
PATTERNS	P1C6	2			
	P2C6	1			1
	P5C6	1			
	P8C6		1		
	P10C6				1
	P12C6			1	
	P2C3	1	1		
	P2C2			1	

The results of Model 1 are the input for Model 2 in the customer allocation. In this case, the matrix for pattern selection yields the list of vehicles to be assigned to different orders, as shown in Table 6.

Table 6: List of vehicles to be dispatched as a result of the pattern selection model

VEHICLE	PATTERN	DESTINATION ZONE
V1	P1C6	Z1
V2	P1C6	Z1
V3	P2C6	Z1
V4	P5C6	Z1
V5	P2C3	Z1
V6	P8C6	Z2
V7	P2C3	Z2
V8	P12C6	Z3
V9	P2C2	Z3
V10	P2C6	Z4
V11	P10C6	Z4

In addition to the results of the previous model, the selection model requires other input data such as demand only at the zone level, but discretized by client (See Table 7). Naturally, the demand must be the same employed in the selection model. The total demand per zone must be remain the same for all models. The demand for Family 1 (F1) towards Bogotá (Z1) has a total of 85 tons, which is equal to the sum of the demand of this family for each of the four clients in Bogotá.

Table 7 Discretized demand per client

ZONE	CLIENT	DEMAND			
		F1	F2	F3	F4
Z1	**C1**	20	50		
Z1	**C2**	50			
Z1	**C3**				9.2
Z1	**C4**	15			
Z2	**C5**		10		
Z2	**C6**		5		17
Z2	**C7**		10		
Z3	**C8**	8.9			
Z3	**C9**		10		15
Z4	**C10**		35		
Z4	**C11**		10		5
Z4	**C12**			7.3	

The allocation model assumes that the transportation cost takes into account only freight when serving a single client. If the truck must perform many visits, then an additional cost is applied that depends on the number of clients to visit. The model reduces the number of visits and minimizes the variable cost of each visited client since this additional cost varies with each case. Two matrices are presented as a result of the client allocation model. The first matrix indicates the number of visits or points that each dispatching vehicle must serve defined by the clients to be served. This second model seeks to minimize this variable. The total cost of dispatching operations is known through this matrix along with the MEP results, which is the global objective function of the proposed models.

Table 8 Number of clients served per truck as a result of the client allocation model

		VEHICLES										
		V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11
ZONES	Z1	2	1	1	2	1						
	Z2						2	2				
	Z3								2	1		
	Z4										1	2

OAP minimizes the summation of the values shown in Table 8. In other words, the generated costs by the dispatched products must be minimized. The second matrix of the OAP is the allocation (See Table 9), in which the quantity of each order is served for each dispatching vehicle (in the case that an order is served by more than one vehicle).

The loading schedule and vehicle dispatching is determined with the information from the allocation matrix since the pattern assigned to each vehicle is known by indicating the loading mechanism and configuration of the truck and the orders that are served by each vehicle. Note that vehicle 8 is a truck C6 with the city of Cartagena as the final destination, as shown in Table 6. This truck must be loaded with 8.9 tons from Family 1 for client 8, 5 tons from Family 2, and 10 from Family 4 for client 9. In addition, vehicle 8 must visit two clients in Medellin. The number of vehicles that will serve the same client is also determined. For example, vehicles 2 and 4 (truck C6) will serve client 2 located in Bogotá.

Table 9 Orders assigned to each vehicle in tons

ZONE	CLIENT	FAMILY	VEHICLE										
			V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11
Z1	**C1**	**F1**	20										
Z1	**C1**	**F2**			35		15						
Z1	**C2**	**F1**		35		15							
Z1	**C3**	**F4**				9.2							
Z1	**C4**	**F1**	15										
Z2	**C5**	**F2**						8	2				
Z2	**C6**	**F2**						5					

ZONE	CLIENT	FAMILY	VEHICLE										
			V1	**V2**	**V3**	**V4**	**V5**	**V6**	**V7**	**V8**	**V9**	**V10**	**V11**
Z2	**C6**	**F4**						17					
Z2	**C7**	**F2**							10				
Z3	**C8**	**F1**								8.9			
Z3	**C9**	**F2**								5	5		
Z3	**C9**	**F4**								15			
Z4	**C10**	**F2**										35	
Z4	**C11**	**F2**											10
Z4	**C11**	**F4**											5
Z4	**C12**	**F3**											7.3

5. A REAL-WORLD CASE APPLICATION

The models were evaluated with a real-world case with a larger size and complexity than in the example previously presented. The models were applied to a laminated steel company in Colombia, which provides a range of products that are classified in 115 families of products based on two criteria. The first criterion corresponds to the logistic management, where products are compatible with each other in the loading and seven logistic categories are established. The second criteria refer to the dimensions of the product (length and width). All products with the same logistic management and same dimensions are part of the same family of products since they use the same space as they are stacked on the base of the truck.

The company dispatches throughout the country and establishes 120 zones, as illustrated in Figure 3. The served zones, depicted in blue color, group close municipalities and are served by one vehicle.

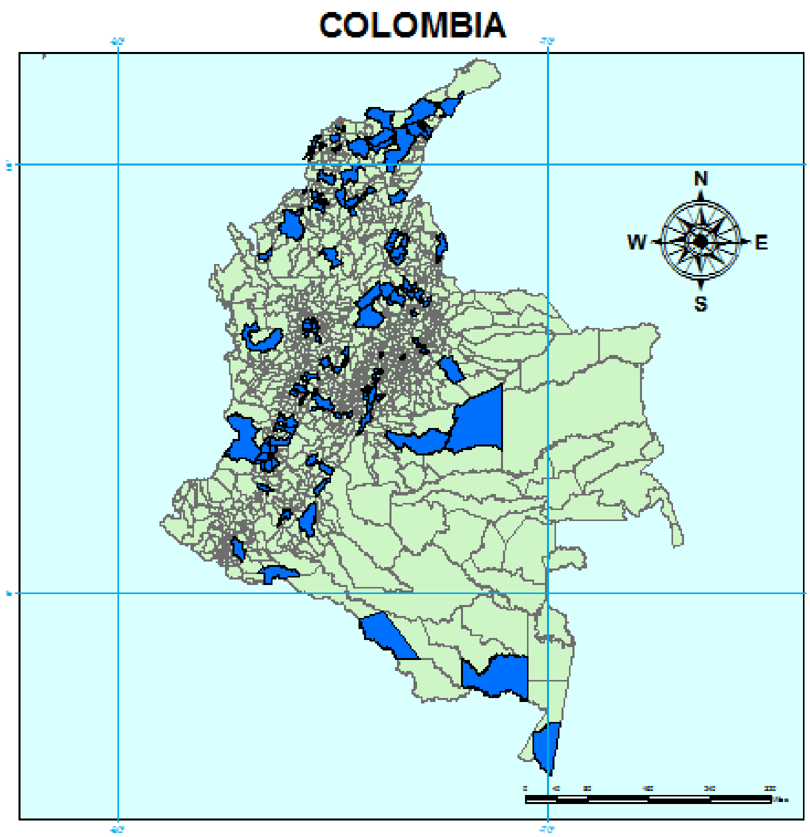


Figure 3 Dispatching zone for the Steel Company in Colombian. ACESCO S.A.

The company utilizes four types of vehicles with different capacities and dimensions. Given the experience of the company, a data base of approximately 4,500 patterns (with the four types of vehicles) was consolidated as a result of the dispatching information analysis from the recent years. Regarding the orders, the weekly demand with a list of approximately 1,100 orders from 150 clients was consolidated.

From the aforementioned information, the pattern selection models (LPSP) and order allocation (OAP) model were evaluated for different scenarios using the same pattern and order data base.

The sensitivity of the model was assessed for these scenarios as certain variables vary. The LPSP was estimated with variations in the number of patterns, zones, and families. The number of patterns and zones were varied for the OAP.

An experimental design was performed for the LPSP that consists of three levels for each variable with approximately 30%, 60%, and 100% of the original problem size. A total of 27 scenarios were evaluated as a combination of the three levels of each variable, as presented in Table 10.

Table 10 Scenarios for the LPSP.

SCENARIO	N° ZONES	N° PATTERNS	N° FAMILIES	SCNEAERIO	N° ZONES	N° PATTERNS	N° FAMILIES
1	120	4500	110	**15**	80	3000	40
2	120	4500	70	**16**	80	2000	110
3	120	4500	40	**17**	80	2000	70
4	120	3000	110	**18**	80	2000	40
5	120	3000	70	**19**	40	4500	110
6	120	3000	40	**20**	40	4500	70
7	120	2000	110	**21**	40	4500	40
8	120	2000	70	**22**	40	3000	110
9	120	2000	40	**23**	40	3000	70
10	80	4500	110	**24**	40	3000	40
11	80	4500	70	**25**	40	2000	110
12	80	4500	40	**26**	40	2000	70
13	80	3000	110	**27**	40	2000	40
14	80	3000	70				

It is important to highlight that a random elimination in the case of patterns and families was performed to reduce the number of variables from the scenarios, verifying that patterns with family of products were present in the respective scenario. With respect to the zones, no elimination took place and these were grouped according to geographic proximity until the number of zones was reduced to the indicated scenario. This indicates that demand of the problem was reduced for the scenarios with lower number of families. The demand remained constant for scenarios with a lower number of zones.

Six zones were initially evaluated for the OAP, and then these zones were reduced to two due to geographic proximity. Table 11 shows 16 scenarios that were estimated with 66% and 100% of the number of patterns. Note that the Metropolitan Area of Medellin includes the municipalities of Medellin, Bello, Envigado and Itagui, and the Metropolitan Area of Cali includes the municipalities of Cali and Yumbo.

Table 11 Scenarios for the OAP

SCENARIO	ZONE	NUMBER OF PATTERNS	NUMBER OF ZONES
1	METROPOLITAN AREA OF MEDELLIN	66%	66%

SCENARIO	ZONE	NUMBER OF PATTERNS	NUMBER OF ZONES
2	METROPOLITAN AREA OF MEDELLIN	100%	66%
3	METROPOLITAN AREA OF CALI	66%	66%
4	METROPOLITAN AREA OF CALI	100%	66%
5	MEDELLIN	66%	100%
6	MEDELLIN	100%	100%
7	BELLO (close to Medellín)	66%	100%
8	BELLO (close to Medellín)	100%	100%
9	ENVIGADO (close to Medellín)	66%	100%
10	ENVIGADO (close to Medellín)	100%	100%
11	ITAGUI (close to Medellín)	66%	100%
12	ITAGUI (close to Medellín)	100%	100%
13	CALI	66%	100%
14	CALI	100%	100%
15	YUMBO (close to Cali)	66%	100%
16	YUMBO (close to Cali)	100%	100%

Two parameters were employed to evaluate optimization models. The first indicates the dispatching costs for the LPSP and the total number of visits or deliveries to clients in each zone for the OAP. The second is an assessment parameter that indicates the time of execution of the model. A computer with a I7-3540M processor with 3 GHz, and RAM memory of 8 GB was employed in this evaluation. The problems were considered as *Mixed Integer Programming (MIP)* since the decision variable is the number of trucks in the first model, and the number of visit to clients in the second model. With respect to the optimization, the program is solved using the *Linear Programming LP Relaxation*. The CPLEX solver is part of the GAMS program and employs the Branch and Cut algorithm. This algorithm is a combination of cutting plane method and branch-and-bound algorithm, which solve the sequence of LP relaxation. The *cutting plane method* improves the relaxation of the problem to approximate to a MIP. The *branch-and-bound algorithm* performs with a sophisticated approach of dividing and conquering when solving the optimization problem (Mitchell, 2002).

The list with the conditions and results of the 27 evaluated scenarios with the first model, as shown in Table 12. This table presents the costs and execution times. Figures 4 and 5 illustrate graphs with the results.

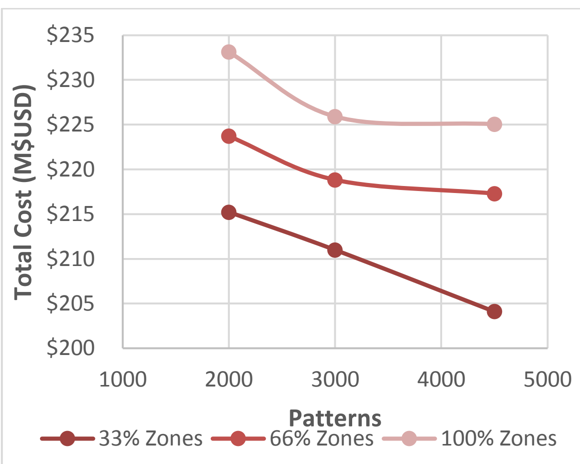
Table 12 Results of the sensitivity analysis for the Loading Pattern Selection Problem (LPST).

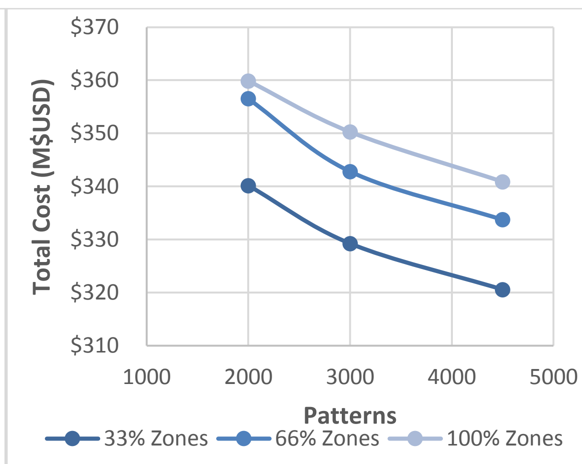
SCENARIO	ZONES	PATTERNS	FAMILIES	TOTAL COST ($COP)	TOTAL COST (1,000 $USD)	TIME (SEC)
1	120	4500	110	$ 823,718,000	$ 427.46	172
2	120	4500	70	$ 656,860,000	$ 340.87	42
3	120	4500	40	$ 433,684,000	$ 225.06	33
4	120	3000	110	$ 850,728,000	$ 441.48	97
5	120	3000	70	$ 674,930,000	$ 350.25	27
6	120	3000	40	$ 435,318,000	$ 225.90	30
7	120	2000	110	$ 879,334,000	$ 456.32	41
8	120	2000	70	$ 693,432,000	$ 359.85	21
9	120	2000	40	$ 449,200,000	$ 233.11	18
10	80	4500	110	$ 814,320,000	$ 422.58	955
11	80	4500	70	$ 643,070,000	$ 333.72	46
12	80	4500	40	$ 418,704,000	$ 217.28	33
13	80	3000	110	$ 837,896,000	$ 434.82	75
14	80	3000	70	$ 660,596,000	$ 342.81	36
15	80	3000	40	$ 421,660,000	$ 218.82	27
16	80	2000	110	$ 866,256,000	$ 449.54	106
17	80	2000	70	$ 686,986,000	$ 356.51	30
18	80	2000	40	$ 431,074,000	$ 223.70	23
19	40	4500	110	$ 776,972,000	$ 403.20	954
20	40	4500	70	$ 617,674,000	$ 320.54	33
21	40	4500	40	$ 393,306,000	$ 204.10	18
22	40	3000	110	$ 809,846,000	$ 420.26	317
23	40	3000	70	$ 634,412,000	$ 329.22	22
24	40	3000	40	$ 406,572,000	$ 210.99	15
25	40	2000	110	$ 830,888,000	$ 431.18	70
26	40	2000	70	$ 655,474,000	$ 340.15	18
27	40	2000	40	$414,716,000	$ 215.21	17

The dispatching cost includes the transportation cost paid by the carrier for each transported vehicle, i.e., a reduction in the number of vehicles is equivalent to a reduction in the cost required for serving the given demand. Figure 4 suggests that a better loading solution is obtained with a larger number of patterns in the database. Whereas, with respect to zoning, an improved solution was obtained with a small number of zones since destinations are more aggregated. However, it is important to highlight that any VRP formulation, in this case the OAP, for each zone is more complex as the number of zones is reduced, and then the solution to the second problem is more difficult to obtain. Finally, this routing cost may represent a relevant cost within the total dispatching cost; therefore, it must be balanced by setting the number of zones, so that the model assumptions are maintained; where routing is a secondary problem, with a smaller share of total costs.

With respect to the variation in the number of categories or families of products, inferior costs are inferred as less families are included in the problem, i.e., more alternatives to the solution of feasible loading as the characterization of the products are less disaggregated yielding an improved result in the model. However, this is subject to the fulfillment of the compatibility requirements. If at least two categories of products no coincide with loading compatibilities, then they may not be aggregated to a single category. A minimum number of families are advisable to satisfy the compatibility requirements.

As a general conclusion, the model results are improved as the number of patterns increases. It is also advisable to aggregate the demand per families or zones, as long as the model assumptions and product compatibility are maintained.





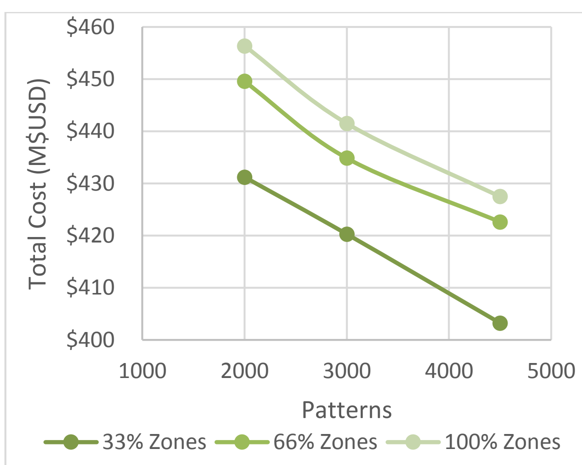


Figure 4 Results of the sensitivity analysis for the costs of the LPSP in 1,000 US$. a) 33% of the families. b) 66% of the families. c) 100% of the families.

In the second parameter, the execution time is increased as the problem size is incremented since a larger number of variables are considered such as for the number of families and patterns. The execution time for the number of zones depends on the alternatives and the quality of the feasible solutions. When the number of zones is higher, the level of disaggregation does not allow many solution alternatives, thus, the model converges in less time. On the contrary, when the number of zones is more aggregated, there is a large amount of possible solutions, whose evaluation delays the execution process in the optimal search. Notice that the problem may present a larger delay with an inferior number of variables than in a larger size problem, as depicted in Figure 5.

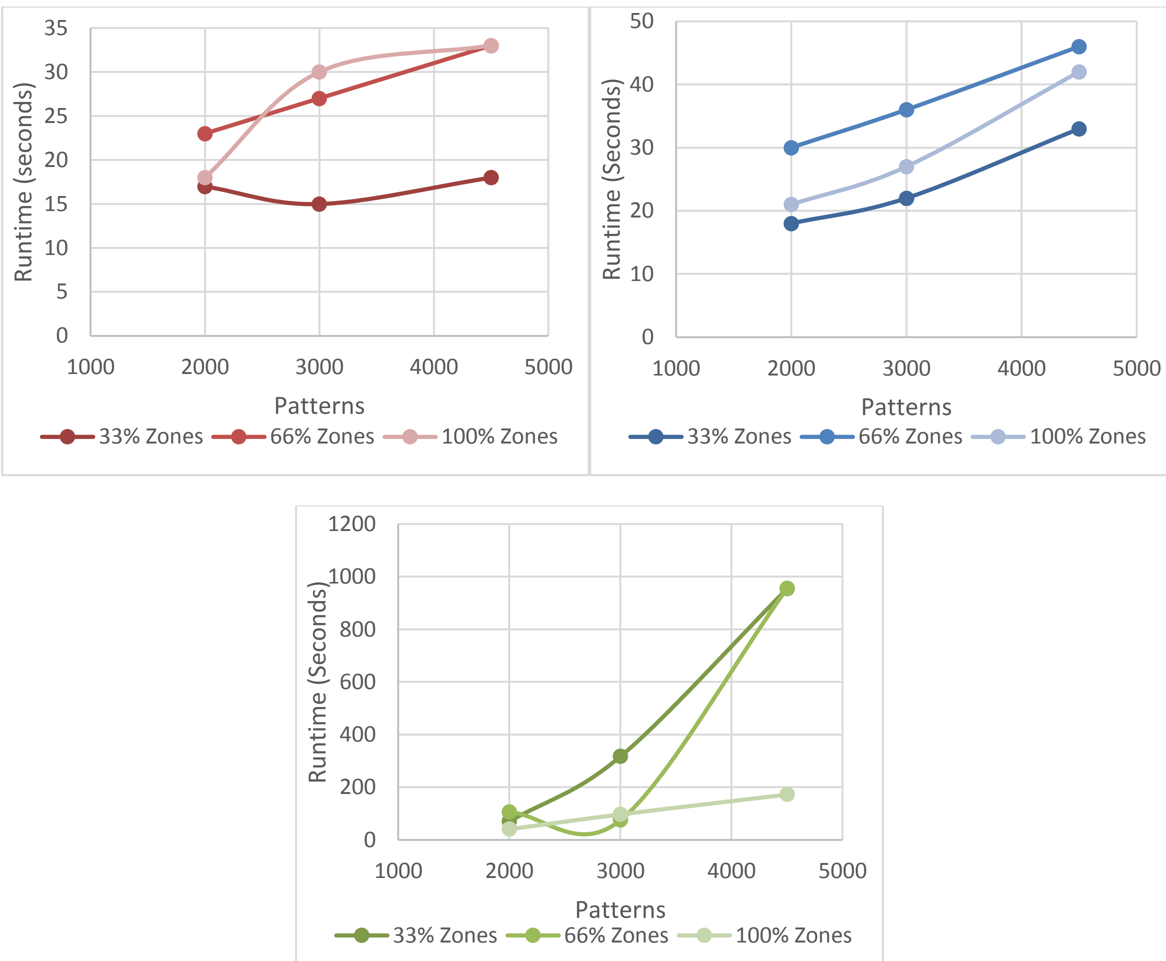


Figure 5 Result of the sensitivity analysis for the execution time of the LPSP. a) 33% of the families. b) 66% of the families. c) 100% of the families.

In the OAP model, the scenarios shown in Table 11 are evaluated, and their results are presented in Table 13. This table suggests that Model 1 yields improved results with a larger zone aggregation, requiring an inferior number of vehicles to serve the demand (aggregated case). Not as good solutions are obtained with a larger disaggregation of the zones since fewer loading consolidation possibilities are encountered.

Table 13 Result of the sensitivity analysis for the Orders Allocation Problem (OAP).

ZONA		PATTERNS	TOTAL VISITS	TOTAL VEHICLES	AVERAGE VISITAS PER VEHICLE
METROPOLITAN AREA OF MEDELLIN	AGGREGATED	66%	107	45	2.38
		100%	110	49	2.24
	DISAGGREGATED PER MUNICIPALITY	66%	96	48	2.00
		100%	103	54	1.91
METROPOLITAN AREA OF CALI	AGGREGATED	66%	64	20	3.20
		100%	62	20	3.10
	DISAGGREGATED PER MUNICIPALITY	66%	65	28	2.32
		100%	69	30	2.30

No clear tendency in the behavior was obtained for the number of visits or services to clients from the second model. In the Medellin scenario, the total number of visits was reduced as the municipalities were disaggregated. The converse was observed in the Cali case. The average number of visits per truck was reduced as the zones were aggregated because a greater utilization of the vehicle capacity through a higher loading consolidation is obtained. This requires a higher combination of orders from different clients in the same vehicle yielding more visits.

The execution time of the OAP was similar to the different analyzed scenarios varying between 48 and 70 seconds. Thus, the aggregated situation is preferred in terms of time. The delivery to different aggregated municipalities are optimized in a single scenario, while for the same process, the total time is the sum of each of the separate municipalities.

6. CONCLUSIONS

The proposed model of this study solves a complex logistic problem in a practical manner, consisting on loading heterogeneous products with compatibility problems and heterogeneous dispatching vehicles in two optimization phases. In the first phase, the Loading Pattern Selection Problem (LPSP) maximizes the utilization of vehicles as the freight cost is minimized and demand is satisfied. In the second phase, the Orders Allocation Problem (OAP) minimizes the number of allocated clients to each vehicle while achieving a fewer number of visits or stops in the route. The mixed integer linear programming models for each optimization phase provide exact solutions in a reasonable amount of time. The proposed approach is useful in the case of shipping products of low density value (low price per ton) between cities, where a maximum utilization of the truck capacity is achieved, particularly considering that the freight is negotiated per shipped truck than per ton.

The proposed methodology takes advantage of the company experiences represented by a database for conforming patterns, which constitutes the search space of solutions for the LPSP, and thus, avoiding another problem for this type of scenario such as the compatibility between products during loading. Given that only verified pattern database is employed, the incompatibility problem is avoided since the loading possibilities of true vehicles with feasible accommodations.

The approach limitation includes the MEP dependency of the available pattern database (search space of solutions). Hence, the size and quality of the set of patterns presents a high incidence in the quality of the results. Another aspect to consider is the implication of the order consolidation in the quality of the results for both the LPSP and the OAP since improved loading and routing solutions are obtained as the demand is consolidated (i.e., more orders are accumulated when scheduling dispatches), and a larger reduction in the costs.

The proposed models were analyzed for different scenarios using a Colombian steel company. The evaluation was based on two parameters: model solution and execution time of the model. For the LPSP, the solution is the total number of vehicle type required for transporting a given demand. If this number is multiplied by the freight of each vehicle, then the transportation cost of merchandise from the distribution center to the clients. The solution for the AOP is the number of visits given by the number of clients assigned to a vehicle.

From the results the first model, it is concluded that as the pattern database is increased and zones are aggregated, the solution is improved. However, if zones are reduced, then the routing error is more important since the model assigns distant customers to a same vehicle although a better load consolidation and utilization of vehicle capacity is reached. This yields an inferior number of required vehicles for transporting the same demand of products.

The solution for the LPSP model is improved by reducing the number of vehicles, however the AOP model is forced to assign more clients to a same vehicle, and thus, increase the number of visits. An improvement in the solutions of the first model entails worsening the solution of the second model.

The proposed approach prioritizes the utilization of the dispatching trucks in a nationwide transportation context, also known as long haul trucking. The urban routing process is less important in the case analyzed. This is reasonable since it deals with few clients per vehicle, representing simple routing problems to be addressed. Additionally, the proposed approach becomes relevant in the case of highly heterogeneous and incompatible products, justifying the focus of the problem on optimizing long haul trucking transportation cost process.

Future research can includes routing restrictions such as the maximum number of visits per clients, or a given client is not served by more than a certain number of vehicles. In order to ensure feasible solutions, it is possible to include slack to the vehicle capacity for different patterns.

Finally, another future research may be the use of dynamic and variable zones, which depend on the orders to ship. Further research includes advancing in the utilization of solution methods and heuristics to reduce runtime in the problem resolution.

REFERENCES

Araya, I. & Riff, M.-C., 2014. A beam search approach to the container loading problem. *Computers & Operations Research,* Volume 43, pp. 100-107.

Baldi, M. M., Crainic, T. G., Perboli, G. & Tadei, R., 2012. The generalized bin packing problem. *Transportation Research Part E: Logistics and Transportation Review,* pp. 1205-1220.

Bortfeldt, A. & Wäscher, G., 2012. *Otto Von Guericke Universität Magdeburg.* [Online] Available at: www.fww.ovgu.de/fww_media/femm/femm_2012/2012_07-EGOTEC-etunm0nikj3ps5sl84hou5b0slqqn8b6.pdf

Carrascosa, C. et al., 1997. *Implementación de un Sistema de Optimización para el Corte de Materia Prima Mediante un Algoritmo Anytime.* Málaga, F. Martín, F. Triguero, pp. 45-52.

Casazza, M. & Ceselli, A., 2014. Mathematical programming algorithms for bin packing problems with item fragmentation. *Computers and Operations Research,* pp. 1-11.

Dantzig, G. & Ramser, J., 1959. The Truck Dispatching Problem. *Management Science,* 6(1), pp. 80-91.

De la Cruz, J., Paternina Arboleda, C., Cantillo, V. & Montoya Torres, J., 2013. A two-pheromone trail ant colony system - tabu searchapproach for the heterogeneousvehicle routing problem with time windows and multiple products. *Journal of Heuristics,* 19(2), pp. 233-242.

Dyckhoff, H., 1990. A typology of cutting and packing problem. *European Journal of Operational Research*, 44(2), pp. 145-159.

Fleszar, K. & Hindi, K. S., 2002. New heuristics for one-dimensional bin-packing. *Computers & Operations Research,* Volume 29, pp. 821-839.

Furini, F. et al., 2012. A column generation heuristic for the two-dimensional two-staged guillotine cutting stock problem with multiple stock size. *European Journal of Operational Research,* 218(1), pp. 251-260.

Gehring, H. & Bortfeldt, A., 1997. A Genetic Algorithm for Solving the Container Loading Problem. *International Transactions in Operational Research,* pp. 401-418.

Gilmore, P. C. & Gomory, R. E., 1961. A linear programing approach to the cutting stock problem. *Operations Research*, 9(6), pp. 848-859.

Gilmore, P. C. & Gomory, R. E., 1963. A linear programing approach to the cutting stock problem Part II. *Operations Research*, 11(6), pp. 863-888.

Haessler, R. W. & Sweeney, P. E., 1991. Cutting stock problems and solution procedures. *European Journal of Operational Research*, pp. 141-150.

Haouari, M. & Serairi, M., 2009. Heuristics for the variable sized bin-packing problem. *Computers & Operations Research,* Volume 36, pp. 2877-2884.

Kang, J. & Park, S., 2003. Algorithms for the variable sized bin packing problem. *European Journal of Operational Research,* Volume 147, pp. 365-372.

Lodi, A., Martello, S. & Vigo, D., 2002. Recent advances on two-dimensional bin packing. *Discrete Applied Mathematics,* Volume 123, pp. 379-396.

Maffioli, F., 2003. The vehicle routing prblem: A book review. *Journal of the Belgian, French and Italian Operations Research Societies,* pp. 149-153.

Malaguti, E., Medina Durán, R. & Toth, P., 2014. Approaches to real world two-dimensional cutting problems. *Omega,* Volume 47, pp. 99-115.

Martello, S. & Vigo, D., 1998. Exac Solution of the Two-Dimensional Finitive Bin Packing Problem. *Management Science,* 44(3), pp. 388-399.

Reinertsen, H. & Vossen, T., 2010. The one-dimensional cutting stock problem with due dates. *European Journal of Operation Research,* Volume 201, pp. 701-711.

Sheng, L., Wei, T., Zhiyuan, X. & Xiwei, L., 2014. A tree search algorithm for the container loading problem. *Computers & Industrial Engineering,* Volume 75, pp. 20-30.

Silva, E., Alvelos, F. & Valerio de Carvalho, J., 2010. An integer programming model for two- and three-stage two-dimensional cutting stock. *European Journal of Operational Research,* Volume 205, pp. 699-708.

Simchi-Levi, D., Kaminsky, P. & Simchi-Levi, E., 2008. *Designing and Managing the Supply Chain. Concepts, Strategies and Case Studies.* Third ed. New York: McGraw-Hill.

Wäscher, G., Haußner, H. & Schumann, H., 2007. An improved typology of cutting and packing problems. *European Journal of Operational Research,* pp. 1109-1130.

Wei, L., Zhu, W. & Lim, A., 2015. A goal-driven prototype column generation strategy for the multiple container loading cost minimization problem. *European Journal of Operational Research*, 241(1), pp. 39-49.