Can the standard EBIT-based structural model replicate credit ratings? An empirical study on S&P500 non-financial firms

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Abstract

The objective of this thesis is to analyze whether the default measures obtained through the standard EBIT-based structural model are comparable to those obtained through credit ratings. This study covers all non-financial companies present on the S&P500 throughout the 2004-2018 period. Credit risk measures coming from the two approaches were found to be broadly comparable. Nevertheless, it was found that on average the structural model under predicts the credit-ratings probability of default by 0,68 p.p. and over predicts the distance to default by 0,57 standard deviations. This under prediction of credit risk was observed across all sectors, though with different degrees of intensity depending on the economic sector. The underprediction was found in all years of study except the financial crisis period.
This dissertation proceeded by analysing the relation between the model and rating agencies default measures. The two estimates show a relatively strong correlation, notably 44% in the case of the probability of default and 52% in the case of the distance to default. The relation between the distances to default measures has been further studied through panel data regressions both on levels (with and without firm fixed effects) and on time differences. Under all approaches the coefficient for the model distance to default measure was found to be relatively small but significant at all the usual confidence levels. This result suggests that the structural model tends to overreact on all new information, while rating agencies act more smoothly.

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Resumo

Esta tese tem como objetivo comparar a probabilidade de insolvência obtida através de um modelo estrutural baseado no EBIT da empresa e o resultante das classificações das agências de *rating*. Este estudo cobre todas as instituições não financeiras, inteiramente presentes no S&P500 durante o período de 2004 a 2018. As duas medidas de risco de crédito são grosso modo comparáveis. Contudo, concluiu-se que, em média, o modelo estrutural subestima as probabilidades de insolvência atribuídas pelas agências em 0.68 p.p. e sobrestima a distância à insolvência em 0.57 desvios-padrão. Esta subestimação do risco de crédito foi observada ao longo de todos os setores, ainda que com diferentes graus de intensidade. A subestimação ocorreu em todos os anos, com exceção do período da crise financeira. Esta dissertação analisou também a relação temporal entre o modelo e as medidas de insolvência provenientes de instituições de classificações de crédito. As duas estimativas mostram uma correlação relativamente forte, nomeadamente 44% para probabilidade de insolvência e de 52% para a distância à insolvência. A relação entre as medidas de distância à insolvência foi analisada através de regressões com dados em painel, em níveis (com e sem efeitos fixos da empresa) e em diferenças temporais. Em todas as abordagens, o coeficiente para o modelo da medida distência à insolvência mostrou-se relativamente pequeno, mas significativo a todos os níveis de confiança, sugerindo que o modelo estrutural tende a exagerar toda a informação nova, em contraponto com as agências de classificação de crédito que agem de forma mais gradual.

Título: Consegue o modelo estrutural padrão baseado no resultado operacional replicar as classificações de crédito das agências de rating? Um estudo empírico nas instituições não-financeiras do S&P500.
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1. Introduction

Throughout time, being able to assess whether a company is on the verge of defaulting on its obligations has always been of utmost importance, especially for those in the financial industry who hold large amounts of credit. Business reality changes however every day leading to changes in corporates creditworthiness. Most of the time it does so in a relatively smooth way as result of a sequence of mild shocks. However, these corporations are from time to time subject to massive shocks and drastic changes in corporate creditworthiness due to major global events, most recently seen due to the pandemic. As a consequence of the continuously changing market conditions in which these firms operate, credit risk analysis is essential to be able to separate the defaulters form the non-defaulters.

There are multiple ways in which corporate credit risk can be analyzed. In 1942 Charles Lewis Merwin (1942) constructed the first accounting-based credit risk model. His model was based on a set of ratios that he found to be predicting default. This approach has been developed further and is now mostly known due to the Altman Z-score (1968). Contrary to the earlier univariate approaches, Altman selected five ratios that he found to be most useful for default prediction. These accounting-based models are usually credited to be accurate in the short term but often fail to be able to predict well for longer horizons.

Since the before mentioned seminal works, several other approaches have been proposed. Most of them are mostly data-driven and lack a theoretical structure behind them. Structural credit risk models are a notable exception. This approach was firstly suggested by Black & Scholes (1973) and Merton (1974). Contrary to previous models one could finally value debt and equity using all observable variables and perform credit risk analysis based on a theoretical structure. When proposed, the so-called Merton-model (1974) became the go to model for credit risk analysis. Here default could be predicted as the likelihood of debt being valued less than nominal value. Following a number of critics to the model strict assumptions, multiple contributions have been made giving rise to a large body of literature known as structural credit risk models. As opposed to the accounting-based models, the theoretical framework behind structural models allowed them to use forward looking market data on stock prices contributing to its performance. However, these models still get outperformed by the accounting-based models for the shortest horizons.

Default probability can also be analyzed through a company's credit rating. The credit rating industry is dominated by three major players, notably, Standard & Poor's, Moody's and Fitch Group. While these companies use different scores, the underlying purpose is the same: measuring credit risk. Given that rating agencies take into consideration a great deal of information when assessing firms and issuing their ratings, they are seen as the benchmark in terms of credit risk analysis. This dissertation will study whether the bankruptcy measures estimated by the standard EBIT-based structural model described in section 3 of Goldstein, Ju and Leland (2001) are comparable to those of rating agencies. The scope of study is all non-financial companies present on the S&P500 throughout the 2004-2018 period.

2. Literature review

Structural models of contingent claims pricing and credit risk started from the work of Fischer Black & Myron Scholes (1973) and Robert Merton (1974). Following their seminal option pricing model, Black & Scholes (1973) suggested to view the equity of a given company as a European call option on the company's assets with strike equal to nominal debt. Debt on the other hand could be valued as a risk-free bond less the value of a put option on the company's assets. The probability of default could therefore be estimated as the probability of the put option ending up in the money. (Black & Scholes, 1973) (Merton, 1974)

In 1976, shortly after the issuance of the Black-Scholes-Merton framework, Fischer Black and John C. Cox (1976) introduced an adaptation of the previously mentioned framework, answering some of the model's harshest critics. Black and Cox assumed a first passage time model. This setting considered the possibility of the firm defaulting on its debt prior to maturity. Intuitively, this was introduced as a lower exogenous barrier on firm value, which when crossed equaled default. The introduction of this possibility was motivated by covenant clauses written on security indentures. Contrary to the Black-Scholes-Merton model, company behavior now mattered prior to debt maturity, which in turn increases debt value. Allowing for these new inclusions also made Black and Cox able to closer replicate the credit spreads observed in historical data. (Black & Cox, 1976)

Close to two decades later, in 1994 Hayne E Leland (1994) developed the Black and Cox model (1976) further. By introducing most importantly bankruptcy cost and corporate income taxes, Leland developed a model to determine the optimal capital structure compatible with the trade-off theory of optimal capital structure. Leland studied the implications of bankruptcy costs and taxes for the firm optimal capital structure when considering both an endogenous- and positive net worth default barrier. The endogenous barrier introduced by Leland assumes that coupons on issued debt is in fact solely repaid by the company's shareholders. Consequently, bankruptcy occurs when shareholders no longer find value in keeping the company alive. Shareholders decision on when to close the company depend on the model parameters. The lower the coupon rate and the higher the volatility, the interest rate and the tax rate, the lower the barrier is, implying that shareholders wait for longer to see whether the company recovers. (Leland, 1994)

A year later, in 1995 Francis A. Longstaff and Eduardo S. Schwartz (1995) further evolved the model introduced by Black and Cox (1976). Longstaff and Schwartz implemented several improvements to the model, among them, addressing the constant interest rate assumption made in the Black-Scholes-Merton framework. Contrary to the models which had previously introduced floating interest rates, Longstaff and Schwartz was indeed the first that were able to offer a closed-form solution on a first passage time setting. By introducing the possibility of floating interest rates Longstaff and Schwartz were able to construct a model for valuing corporate bonds which included both interest rate risk and default risk. This inclusion led to Longstaff and Schwartz being able to draw clearer connections between the probability of default, interest rate fluctuations and credit spreads. (Longstaff & Schwartz, 1995)

Further developing Leland's earlier works (Leland, 1994), Hayne E. Leland and Klaus B. Toft (1996) addressed the assumption of infinite life debt in their paper "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads". Contrary to the original assumption, Leland and Toft introduced debt rollover. As a result, in order to maintain the static debt structure proposed in the model, the company has to continuously replace debt that is maturing. Due to this dynamic, the shareholders of the company are subject to rollover risk. By rolling over under bad conditions replacing debt will be more costly, and incurred losses has to be covered by shareholders. Similar to Leland (1994), the default barrier is solely defined by the company's shareholders strategic behavior. For the same values of coupon expenses, the introduction of rollover risk leads however shareholders to abandon the company sooner. In addition, while in Leland's model distress costs were irrelevant to determine the barrier once the coupon rate was set, in Leland and Toft propose that distress costs are still relevant due to the rollover. Leland and Toft found their model able to better mimic the historical default and credit spreads, compared to what Leland had been able to achieve previously. (Leland & Toft, 1996)

While previous models used a diffusion process to characterize the evolution of company value, Chunsheng Zhou (2001) introduced the jump-diffusion process in 2001. In diffusion models, the asset value moves proportionally to time. So, it cannot change significantly in a short time span. In contrast, jump-diffusion models allow the company value to jump suddenly. This certain trait is something that is often due to unexpected wide market movements but can also be seen when companies offer completely unexpected new information to the market. Previous models were unable to capture this characteristic. Due to the possibility of jumping below the bankruptcy threshold, company value at default may vary. (Zhou, 2001)

In 2001 Robert Goldstein, Nengjiu Ju and Hayne Leland (2001) made another major contribution to the literature on optimal capital structure. While Leland's original paper sees the company as a combination of assets, tax benefits and distress costs (Leland, 1994), they consider the company as a claim on a project continuously generating earnings. Shareholders, debt holders, the government and distress costs are seen as claimants on this perpetual project. Notably, Goldstein, Ju and Leland (2001) related the project value with the company's capacity to generate earnings, while in Leland's model (1994) the asset value was taken to be completely exogenous to the model. Similar to Leland (1994), the barrier is set by shareholders as the result of an optimal stopping time problem. The main difference between the barrier proposed by Leland (1994) and the one used in this model, is that the firm has a continuous payout that can be used to pay the coupons. While the firm is solvent, earnings are split between the shareholders, debt holders and the government. In the case of default, the project value after distress costs is divided between debt holders and the government. In the second part of the paper, they consider that the firm has the opportunity to issue further debt later depending on how the project earnings evolve. As a result of this option, it is optimal for the firm to issue considerably less debt in the starting stages, which is often close in line with what is observed in real life. This option also increased shareholder interest in the project, making them hold on to the company for longer without declaring bankruptcy. As a result of the lower initial debt and the increases in shareholder interest, the barrier value is considerably lower than in a static model with the same amount of coupon. (Goldstein, Ju, & Leland, 2001)

Previously cited literature focus on either corporate finance or bond pricing questions. By purely focusing on improving the estimation of the probability of default, Jeff Bohn and Peter Crosbie (2003) introduce a pragmatic reinterpretation of the Black-Scholes-Merton framework in 2003. In his seminal article Merton (1974) does not discuss how to calibrate the model neither its performance in default prediction. Benefiting from Moody's extensive database, Bohn and Crosbie contribute on this side. In order to apply Merton-model (1974) in practice one needs to estimate the asset value, the asset return volatility and the default barrier. In Merton's original model (1974) the default barrier is the book value of debt, as this is what the firm is obligated to repay. These authors consider however that the default barrier should be somewhere between total- and short-term debt. When estimating the default trigger Bohn and Crosbie gives a lower weight to the long-term debt as repayment is often far in the future. Short-term debt is weighed higher, as it has to be serviced within a short horizon. Once the default barrier is set, they use the observed market capitalization of firms to estimate the asset value and the asset return volatility through the utilization of an iterative algorithm. In order to further improve the estimated asset volatility, they combine the asset volatility from the iterative approach with the averages of those values obtained for firms with the same size, operate in the same industry or operate in the same country. Once the model is calibrated, they computed the distance to default (DD) for each given company. The DD measures the distance between the expected asset value on the maturity date and the default barrier in terms of the number of standard deviations of the firm asset return. They claim that this measure is a powerful determinant of a firm creditworthiness but that the normal distribution is unable to translate it correctly into default probabilities. So, as a final step they directly infer the probability of a firm defaulting within a year by assessing the default ratios of firms in their database having the same distance-to-default. As this distribution is based on private information their results are not reproducible. However, it is known that Moody's distribution tends to give higher probabilities than the Normal distribution for high DD values and lower probabilities than the Normal for low DD values. The approach to probability of default estimation outlined by Crosbie and Bohn in combination with the Vasieck-Kealhofer model is known as Moody's MKMV Expected Default Frequency (MKMV EDF). MKVM EDF is a methodology developed by Moody's in order to estimate the likelihood of bankruptcy for shorter horizons. (Crosbie & Bohn, 2003)

In 2007 Alexander S. Reisz and Claudia Perlich (2007) introduced another adaptation of first-passage time model to the literature. Similar to Black and Cox (1976), Reisz and Perlich model the default barrier as an exponential function, rather than a flat barrier. Reisz and Perlich furthermore introduce an early default barrier in order for debt holders to extract value if some prespecified event would occur. Reisz and Perlich were able to outperform both the Black-Scholes-Merton framework and the KVM approaches when estimating bankruptcies for longer time horizons. However, according to the authors, for the 1-year forward estimations of bankruptcy the accounting-based models remain superior. (Reisz & Perlich, 2007)

Depending on the model, the point of default varies drastically. In order to understand when firms actually default, Sergei A. Davydenko (2012, November) conducted a purely empirical study of endogenous-barrier models in 2012. Contrary to the zero net worth barrier applied by many models, Davydenko's empirical findings show that asset value at the time of default often is significantly lower. They found values from 30% at the 5th percentile to 122% at the 95th percentile. The company's average asset value is 66% at the time of default. These values are compatible with many insolvent firms being able to keep operating and avoid bankruptcy for long time. Davydenko additionally studied the relevance of the different endogenous barrier determinants in structural models. By analyzing empirically observed barriers of default, he concluded that asset-volatility and bankruptcy costs were the only determinants which consistently had any clear effects on the default barrier. He went on to claim that the large variation in default boundaries is the main driver for structural models' difficulty in predicting default. He concludes that diffusion models with endogenous default barriers seem to be unable to replicate the empirical results, as there seems to be multiple unobservable variables that determine the point of default. (Davydenko, 2012, November)

3. Model

3.1 EBIT-Based Model of Dynamic Capital Structure

My choice of model in this thesis is the EBIT-based model proposed by Goldstein, Ju & Leland *(2001)*. The authors introduced two version of the model, one with the option to issue further debt in the future and one without this option. In this thesis the model is considered without this option. Even though the EBIT-based model is originally constructed to decide on the optimal capital structure, with minor adjustments, it is well-suited to estimate measures of default.

Goldstein, Ju & Leland (2001) considers a company to hold a perpetual project, producing a payout flow. The distinct characteristics of this payout flow is given by the process

Equation 1

$$\frac{d\delta}{\delta} = \mu_p dt + \sigma dz$$

where μ_p and σ both are constants. Here, μ_p is the drift of the project, whereas σ is the volatility of this projects returns. dz is a variation of the Brownian Motion or Wiener Process, which is a stochastic process in continuous-time. This Brownian motion has stationary increments, and a continuous path which means that the process is unable to jump between levels. Here the shocks to the log of δ are normally distributed, which is known in the literature as the Geometric Brownian motion (GMB).

By utilizing the risk neutral approach, the authors are able to value the project by discounting all future cash flows at the risk-free rate. Consequently, the value of the project under the risk neutral approach is

Equation 2

$$V(t) = E_t^Q(\int_t^\infty ds\delta_s e^{-rs}) = \frac{\delta_t}{r-\mu},$$

$$= \frac{\delta_t}{r + \theta\sigma - \mu_p}$$

where $\mu = (\mu_p - \theta\sigma)$ is the risk-neutral drift of the project and r is the risk-free rate. Both μ and r are assumed to be constants. Here θ is defined as the market price of risk. The same expression can be obtained by discounting δ_t at the rate μ_a, where $\mu_a = r + \theta\sigma$.
The projects risk neutral drift is simply the drift of the project less the volatility adjusted market price of risk.

Through applying Ito's lemma to the previously defined project value function, one is able to derive the specific dynamics of the project value as

Equation 3

$$\frac{dV}{V} = \mu_p dt + \sigma dz^p$$

Goldstein, Ju & Leland continue by defining the payout ratio of the company as

Equation 4

$$k = \frac{\delta_t}{V}$$

By substituting the payout ratio into Eq. (2), it can be shown that the payout ratio is equal to the difference between the risk-free rate and the previously define risk neutral drift of the project.

Equation 5

$$k = \frac{\delta_t}{V} = r - \mu$$

Subsequently the risk-neutral drift of the project is equal to the risk-free rate less the payout ratio

Equation 6

$$\mu = r - k$$

By rewriting Eq. (3), it follows that the dynamics of the project value can be described as

Equation 7

$$\frac{dV}{V} = (r - k)dt + \sigma dW_t^Q$$

Goldstein, Ju & Leland furthermore presupposes the company to take on debt in order to obtain the optimal capital structure. It is considered that the company issues a perpetual bond, with a constant coupon C. The firm must pay this coupon independently of the project payout. As there is no cash buffer in the model, whenever the project payout is not enough to pay the coupon, the difference must be paid by the shareholder. As further explained below, it is considered that the firm is liquidated whenever the project level reaches a certain level. The level of liquidation is known as the default barrier V_B. This level is chosen by the shareholder.

Under the before mentioned assumptions, any claim to the project has to satisfy the following partial differential equation (PDE):

Equation 8

$$(r-k)VF_v + \frac{\sigma^2}{2}V^2F_{VV} + F_t + P = rF$$

Here P is a general claim to the payout flow and will vary in accordance to what security one wants to price. A claim to the all δ prior to bankruptcy would result in $P = \delta$, whereas a claim to all C prior to default would result in $P = C$.

For all claims I am interested in, F_t=0 because their value is time independent. In this case, the partial differential equation becomes a second order ordinary differential equation (ODE):

Equation 9

$$0 = (r-k)VF_v + \frac{\sigma^2}{2}V^2F_{VV} + P - rF$$

It is well known in that the solution to this type of equation can be found by summing the general solution to the homogenous equation and a particular solution[1].

The general solution to the homogeneous equation is given by

Equation 10

$$F_{GS} = A_1V^{-y} + A_2V^{-x},$$

Where

Equation 11

$$x = \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right) + 2r\sigma^2}\right],$$

[1] The homogenous equation is same equation, with constants equal to zero. As a result, all that is not multiplied with F or its derivatives disappears.

Equation 12

$$y = \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right) + 2r\sigma^2}\right],$$

are constant. A_1 and A_2 are constants that are determined through boundary conditions specific to the claim that one wishes to price. The homogenous equation F_{GS} is not considering any intertemporal cash flows.

The particular solution depends on the specific claim one wants to price.

The authors proceed by introducing $P_B(V)$. They define this as a claim that pays $1 contingent on the company value reaching the default barrier. As $P_B(V)$ is in line with F_{GS} (i.e. it is not subject to any intertemporal cash flows), its solution is the solution to the homogeneous equation. Thus,

Equation 13

$$P_B(V) = A_1 V^{-y} + A_2 V^{-x},$$

They proceed by considering these boundary conditions

Equation 14

$$\lim_{V \to \infty} P_B(V) = 0, \lim_{V \to V_B} P_B(V) = 1,$$

As company value goes towards infinity, the claim that pays $1 contingent on the company value reaching the default barrier becomes zero. If the company value on the other hand goes towards the default barrier, the value of this claim becomes one.

Taking these conditions into account, one obtains

Equation 15

$$P_B(V) = \left(\frac{V}{V_B}\right)^{-x}$$

The second claim that the authors consider is the claim to all the intertemporal cashflows of the project. As long as the firm does not default, shareholders, the government and debtholders will divide the payout of the project among them. These claimants are receiving the payout flow through dividends, taxes and coupon payments.

The combined value of these claims to the payout flow of the company is defined as V_{solv}. Here P will be replaced by $(k * V)$ in the ODE (Eq. 9). A particular solution to this ODE is thus V. V_{solv} is thus given by

Equation 16

$$V_{solv} = V + A_1 V^{-y} + A_2 V^{-x}$$

Again, A1 and A2 can be found by imposing boundary conditions. In the case that V goes to infinity, V_{solv} goes towards V, On the contrary in the case that V is equal to V_B, V_{solv} becomes zero. This allow us to determine A_1 and A_2.

Consequently V_{solv} can be written as

Equation 17

$$V_{solv} = V - V_B P_B(V)$$

Following the same approach as for V_{solv}, the value of the claim to the interest payments is given by

Equation 18

$$V_{int} = \frac{C}{r}[1 - P_B(V)]$$

The value of the claims of the shareholders, the government and debtholders are

Equation 19

$$E_{solv}(V) = (1 - \tau_{eff})(V_{solv} - V_{int}),$$

Equation 20

$$G_{solv}(V) = \tau_{eff}(V_{solv} - V_{int}) + \tau_{int} V_{int},$$

Equation 21

$$D_{solv}(V) = (1 - \tau_i)V_{int},$$

Here τ_{eff} is the effective tax rate and τ_i is the tax on interest.

In order to find the optimal default barrier V_B, the authors use the smooth-pasting condition

Equation 22

$$\left.\frac{\partial E}{\partial V}\right|_{V=V_B} = 0.$$

By solving the smooth-pasting condition they are able to find the optimal V_B^*

Equation 23

$$V_B^* = \lambda \frac{C^*}{r},$$

where

Equation 24

$$\lambda = \left(\frac{x}{x+1}\right),$$

and C* is the optimal amount of debt, which is found in the paper by maximizing the sum of equity and debt value.

Goldstein, Ju & Leland did not provide any formula to compute the default probability. The first passage time probability of a GBM is nevertheless standard in the literature. Following for instance Forte & Lovreta (2012), this can be computed as

Equation 25

$$P_{nd}(\sigma,\mu_v) = \Phi\left[\frac{\left(\mu_a - k - \frac{\sigma^2}{2}\right)T - ln\left(\frac{V_B}{V_1}\right)}{\sigma\sqrt{T}}\right] - e^{\frac{2}{\sigma^2}\left(\mu_a - k - \frac{\sigma^2}{2}\right)ln\left(\frac{V_B}{V_1}\right)}\Phi\left[\frac{\left(\mu_a - k - \frac{\sigma^2}{2}\right)T + ln\left(\frac{V_B}{V_1}\right)}{\sigma\sqrt{T}}\right],$$

As the formula in Eq. (25) computes the probability of survival, probability of default is found by subtracting one.

The second measure of default estimated in the model is the distance to default. Which was computed in the following manner

Equation 26

$$DD = -\frac{\log\left(\frac{V}{V_B}\right) + \left(\mu_a - k - \left(\frac{\sigma}{2}\right)\right) * T}{\sigma * T}$$

4. Calibration strategy

Prior to estimating the default measures, I calibrated the asset value, asset volatility (σ_a), payout ratio (k) and the market price of risk (θ). Asset value, σ_a and k are set through the iterative scheme presented in Section 4.1 and further detailed in Section 5.3. θ is set as explained in Section 4.2 and further detailed in Section 5.4. The remaining parameters are calibrated through accounting or market data. The EBIT and equity are set as explained in Section 5.2. The β, EQRP and σ_e are set as explained in Section 5.4.

4.1 Iterative approach

In order to calibrate the Goldstein, Ju & Leland model (2001), I have utilized the iterative approach proposed by Vassalou & Xing (2004). This approach is frequently referred in Moody's KMV technical documentations[2]. This iterative approach was originally suggested to calibrate the Merton-model (1974), since one only has to give values to asset and sigma.

In words, their approach works as follows.

1. Compute the volatility of equity based on daily observation for the last year and use this value as a starting point for σ_a.
2. For each day of trading, estimate the value of assets through the equity valuation formula from Black & Scholes (1973) by using the real and observed value of equity and the "estimated" σ_a.
3. Recompute σ_a as the standard deviation of the newly estimated daily asset values.
4. Repeat this process until σ_a coming from two repeated iterations converge.

[2] (Crosbie & Bohn, 2003)

Vassalou & Xing set their tolerance level of convergence to be 0,0001. Following the estimation of σ_a, they could proceed to obtain the value of assets.

Following the same procedure as my fellow student Lukas Weisel (2020), I extended this approach also to calibrate k.

1. Based on the iterative approach of Vassalou & Xing (2004), the tolerance level of convergence was set to 0,0001.
2. The initial estimate for k to be used in the iterative approach was set to 5 %.
3. Proceed to use the iterative approach of Vassalou & Xing on the model of Goldstein, Ju & Leland (2001) to estimate the project value and σ_a .
4. Estimate the value of k. Here k is equal to the average of EBIT divided by the asset values obtained through the iterative approach.
5. Repeat this process until the value of k obtained from two consecutive iterations is below the tolerance level of convergence of 0,0001.

Following the estimations of both k and σ_a, a time series for the value of assets can be attained. In this dissertation both k and σ_a are assumed to be constant for the whole time series.

4.2 Market price of risk - θ

In order to compute the actual probability of default, one must use measure P as opposed to Q. In order to do this, the market price of risk has to be obtained. While there are many approaches that can be followed in order to construct the market price of risk, for simplicity I have chosen to utilize the capital asset pricing model.

The capital asset pricing model is frequently used to find the required rate of return one should demand when investing in risky assets, mostly used for stocks.

The capital asset model states that the required rate of return by an investor investing in stocks/equity is given by

Equation 27

$$r_i = rf + \beta_i * (r_m - rf)$$

Here r_i is the required rate of return, rf is the risk-free interest rate, β_i is the beta of the stock and r_m is the return of the market portfolio. $(r_m - rf)$ is therefore the excess return above the risk-free rate or the equity risk premium (EQRP) obtained by an investor, when investing in the market portfolio. β_i is defined as a measure of the systematic risk of a company. $\beta_i * (r_m - rf)$ therefore, expresses the required return above the risk-free rate expected by an investor, based on the systematic risk of a stock relative to the market portfolio.

Goldstein, Ju & Leland (2001) do not give an expression to compute the expected return on stocks/equity. However, it is well known that the expected return on stocks/equity under this model can be written as

Equation 28

$$\mu_e = rf + \theta * \sigma_e$$

Here μ_e is the expected return on stocks/equity, θ is the market price of risk and σ_e is the volatility of that particular stock/equity. This expression follows from the application of Ito's lemma to the equity valuation formula. One should note however that under this model, σ_e is not a constant but rather a function of the project value and project returns volatility σ. When comparing to Eq. (27) it is clear that $(r_i - rf)$ from Eq. (27) and $(\mu_e - rf)$ from Eq. (28) expression is the same thing i.e. the expected return from the stock/equity above the risk-free rate. Consequently $\beta_i * (r_m - rf)$ or $\beta_i * (EQRP)$ from Eq. (27) and $(\theta * \sigma_e)$ from Eq. (28) has to be the same.

Replacing $(\mu_e - rf)$ with $\beta_i * (EQRP)$ in Eq. (28) the market price of risk can be obtained as

Equation 29

$$\theta = \frac{\beta_i * (EQRP)}{\sigma_e}$$

The market price of risk is the usually referred to as the Sharpe ratio. Which is constructed as the risk adjusted equity risk premium (stocks return above risk free rate), divided by the particular stocks/equity volatility. The estimation of β, σ_e and $EQRP$ is found in Section 5.4.1, 5.4.2 and 5.4.3, respectively.

5. Data

5.1 Company selection

Prior to doing any estimations or treatments of data I first had to construct the portfolio of companies. Through accessing Compustat - Capital IQ in the Wharton Research Data Service (WRDS) database I was able to obtain the list of index constituents on the S&P500 for the 2004-2018 period. This selection of companies was narrowed down by excluding 251 companies that are not present on the S&P500 throughout the entire period covered by this dissertation, which spans from 2004 to 2018. As I wanted to assess how default measures varied across time, while assuring that the companies were of comparable size, I wanted to keep the same sample of companies across the entire timespan.

Furthermore, I excluded 44 financials companies (i.e. those belonging to the S&P Economic Sector Code – 800). The initial round of company selection resulted in a sample of 206 companies across 10 sectors[3].

The second and final round of data cleaning was performed post acquiring the accounting data for the initial sample. Since the model assumes a Geometric Brownian motion, negative EBIT violates the model assumptions. Nevertheless, the estimation method used is able to overcome this as long as the average EBIT is positive. This resulted with 5 companies with an average negative EBIT being removed. Furthermore, 42 firms with zero interest expense for one or multiple periods were excluded. As the model relies on interest cost to estimate the default barrier and thus all measures of default, this firms would simply never default.

After the two rounds of data cleaning, the final sample consisted of 159 companies across 9 sectors as outlined in Table 1.

Table 1 - Company overview

S&P Economic Sector Code	600	700	905	925	935
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
Amount of companies	6	19	18	23	11
S&P Economic Sector Code	940	970	976	978	All
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	-
Amount of companies	15	13	30	24	159

The detailed description of all firms is found in Appendix 1-2.

[3] The S&P 500 is mostly composed of very large companies. This exclusion of companies entering post 2004 might have exacerbated the selection bias.

5.2 Parameters calibrated through accounting and market data

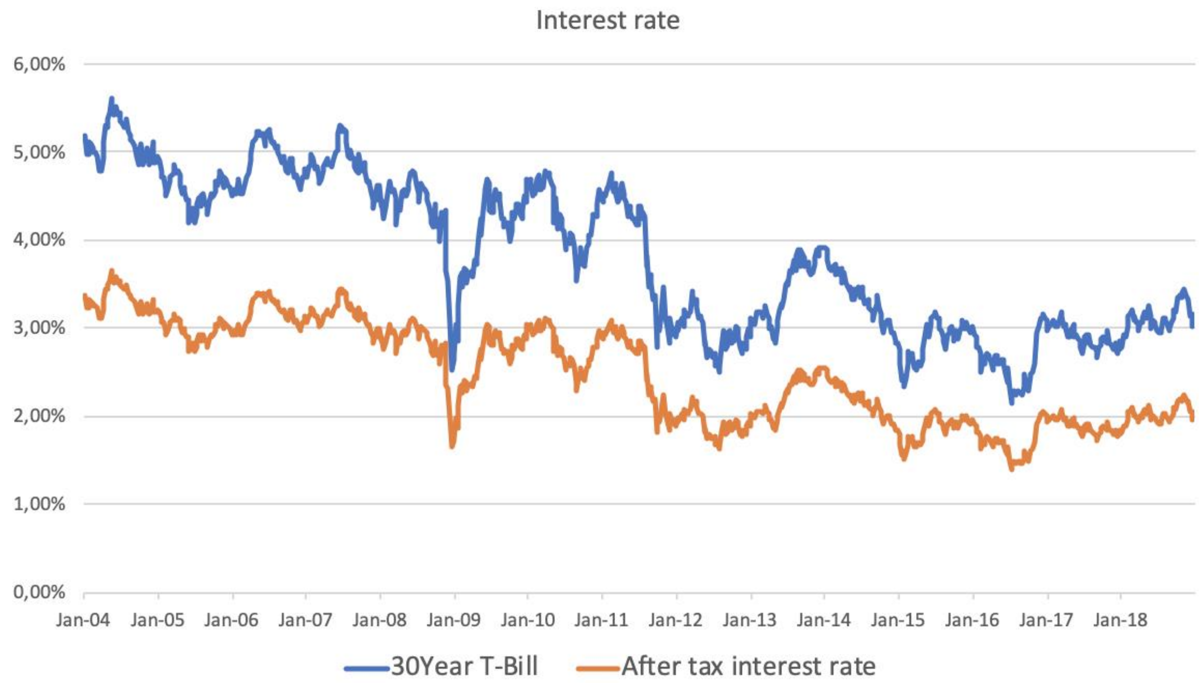
5.2.1 EBIT and interest cost

The company specific EBIT and interest cost were retrieved from DataStream. Due to both EBIT and interest costs being accounting data, these variables were downloaded as yearly values. In order to adapt these variables to fit the weekly increments of the model, I used interpolation. The interpolation process consisted of two steps. The initial step was to construct start of the year values for each year as the yearly EBIT or interest costs divided by 52. Finally, I computed all weekly values in between by constructing a straight line between the start of the year values, known as linear interpolation.

5.2.2 Interest rate

The interest rate for the 2004-2018 period was approximated using the time series of the 30-Year US Treasury Bill. This long-term interest rate is used since it reduces the number of cases where a negative value for the risk-free rate could occur. The time series was downloaded in weekly increments from DataStream. The interest rate used in the model is the after-tax interest rate, found through applying the tax rate on interest. While the tax rate on interest has varied slightly from 2004-2018, it is desirable to have a constant rate across time. Based on the values observed in the period of study, I have assumed it to be constant at 35%. The interest rate before and after applying interest rate tax is presented in Figure 1.

Figure 1 - Interest rate



Here a major drop in the interest rate is observed in the end of 2008 during the financial crisis. Other significant drops are seen at the end of 2011 during the EU sovereign debt crisis and during the oil crisis in 2014-2016.

5.2.3 The effective tax rate on corporate profits

In addition to the tax rate on interest, the model uses the effective tax rate. In order to compute this, I had to know both the tax rate on dividends and the corporate tax rate. Both the corporate- and dividend tax rate has varied across the timespan of the analysis. However, I wanted to keep both of these variables' constant throughout the calculations. Considering the different values for both variables and timespan these values were present, it was chosen to use the same tax rate of 20% for both the corporate- and the dividend tax rate.

The effective tax rate was therefore computed in the following manner as

$$(1 - Efftax) = (1 - Corptax) * (1 - Divtax)$$

Resulting in an effective tax rate of 36%.

5.2.4 Market value of equity

Equity is not a parameter, but an input that is used to find the market value of the project. Company specific equity was as with previously mentioned variables retrieved from DataStream. As market value of equity is a flow measure, the values were downloaded in weekly increments. The market value of equity is the product of the number of shares outstanding and the price per share. As the number of shares rarely change, variations are caused by the change in the share price. In order to compare the evolution of the market cap of the portfolio with the S&P500, I downloaded the weekly time series of the S&P500 from DataStream.

The combined market value of all firms can be seen in Figure 2

Figure 2 - Market capitalization

*Values in millions	Correlation: 0,989

As seen from both the plot and the correlation between the series, the portefolio of 159 companies follow the dynamics S&P500 closely. These results are a little suprising, considering the S&P500 continuously has firms exiting and entering the index and that the entire financial sector is excluded. As observe in the Figure 2 above, market capitalization is drastically reduced during the financial crisis of 2008-2009. Furthermore one can also see the significant drops in market capitalication caused by the sovereign debt cirisis within the EU in 2011 and the end of the oil crisis in 2014-2016.

5.3 Parameters calibrated through the iterative algorithm

5.3.1 Asset volatility - $\boldsymbol{\sigma_a}$

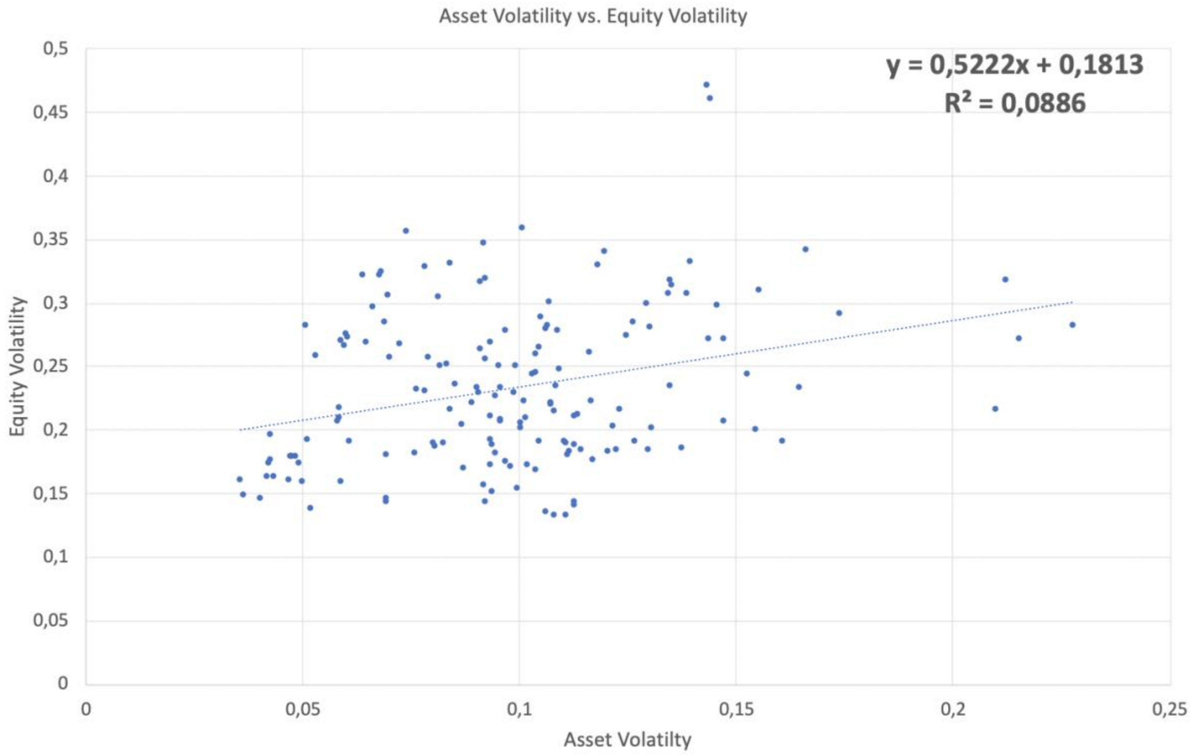
Asset volatility is one of two variables obtained from the iterative approach outlined in Section 4.1. The asset volatility used for estimations in the model, is extracted when two repeated iterations converge. When extracted, σ_a is equal to the product of the standard deviation of the log changes in asset values and the square root of 52.
The average asset volatility across the different sectors is found in Table 2. It is clear that average asset volatility for the utilities and the consumer staples sector are considerably lower than the other sectors (Table 2). In contrast, the health care and technology sector show the highest average asset volatility.

Table 2 - Average asset volatility

Asset volatility					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
σ_a	8,47%	4,79%	12,47%	9,57%	10,82%
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	-
σ_a	12,62%	9,74%	10,60%	3,17%	-

When comparing the asset volatilities with the empirical equity volatilities (Table 5), there is a much lower level clearly observed for all firms.
I furthermore assessed if there is any connection linking the individual company's asset- and equity volatility (Section 5.4.2) by running a regression between the two series as seen in Figure 3.

Figure 3 - Asset volatility vs equity volatility



Even though the coefficient of determination is low, it indicates that some of the variation in the equity volatility series can be explained by the variation in the asset volatility series. This relation is weaker than expected, which shows that more business risk does not necessarily mean more equity risk, as firms with lower business risk may be more levered.

5.3.2 Payout ratio – k

The company payout ratio similarly to asset volatility is found through the iterative approach (Section 4.1). The payout ratio at the sector level is found in Table 3.

Table 3 - Payout ratio

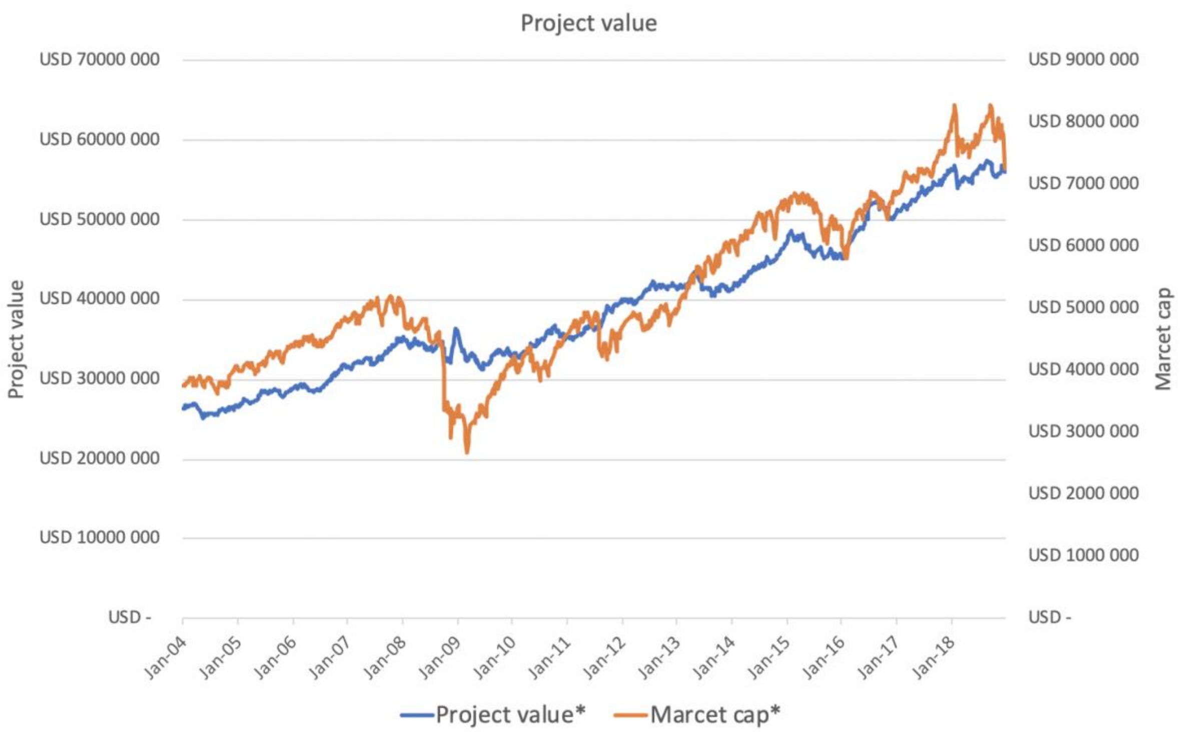
Payout ratio					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
k	3,47%	5,28%	2,77%	3,42%	3,35%
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	-
k	2,93%	3,72%	3,27%	3,17%	-

Considering Table 3 it is apparent that the utilities sector has a much higher payout than the other sectors. Since the utilities sectors on average is very stable and mature, the high payout ratio was not surprising. On the other side of the spectrum a similar payout ratio is seen across the remaining eight sectors, with technology and health care being slightly lower. Due to the necessity of continuous research into new technologies and new pharmaceutical products, as well as the emphasis on growth for the technological companies these payout ratios are representable.

5.3.3 Project value – V

The project value is found through the iterative approach (Section 4.1) following the estimations of σ_a and k. The project value is the one implicit in the market cap (i.e. equity value) conditional on the model (Section 3) and the calibrated value of all its parameters. The combined project value of all firms can be seen in Figure 4. It is clear that the project- and equity value follow each other, as indicated by the lines in Figure 4. However, one can observe that the variations in the equity value are amplified when compared to the project value. This is due to equity being a leveraged claim.

Figure 4 - Project value



*Values in millions	Correlation: 92,16%

5.4 From measure Q to measure P

5.4.1 Beta – β

The company specific beta-coefficients were also acquired through DataStream. As for the market value of equity, beta was downloaded in weekly increments. In order to avoiding that changes in the PD computations was a result of changes in beta value, the company specific beta used for estimation was the average of all company beta values.

The average beta for each industry can be seen in Table 4 below.

Table 4 - Beta

Beta					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
Beta	0,94	0,44	0,71	1,12	1,13
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	Total
Beta	1,37	1,04	1,12	0,55	0,92

There are significant differences between the average beta of each industry (Table 4). Due to utilities being a necessity, its demand stays relatively stable regardless of market condition. Hence, the low beta value is expected. The same characteristics can be seen for both consumer staples and health care. On the contrary it is apparent that the technology sector is considered riskier than the other sectors. This result was also expected due to the extreme volatility that usually describes technological companies.

5.4.2 Empirical equity volatility - $\boldsymbol{\sigma_e}$

Empirical equity volatility is mainly used in the calibration of the market price of risk.

The equity volatility was computed in the following manner. Initially I estimated the non-annualized empirical equity volatility as the standard deviation of the log changes in equity. As a second step I recomputed a new series with the log changes in equity. In the third step I constructed upper and lower limits for the series of log changes in equity as plus/minus three standard deviations. Furthermore, I removed all values outside these boundaries. Finally, I computed the volatility of equity as the product of the standard deviation of this time series and the square root of 52. This process follows the approach done by my fellow student Lukas (2020). As σ_e is used in the calibration of the model, this will help make results comparable.

The average equity volatility in the different sectors is seen in Table 5.

Table 5 - Average empirical equity volatility

Equity volatility					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
σ_e	24,36%	17,97%	21,65%	23,94%	30,62%
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	-
σ_e	26,67%	26,78%	25,43%	17,83%	-

It is apparent that the utilities and consumer staples sector has the lowest average equity volatility (Table 5). As previously mention in Section 5.4.1, these sectors tend to be very stable, therefore these results are not surprising.

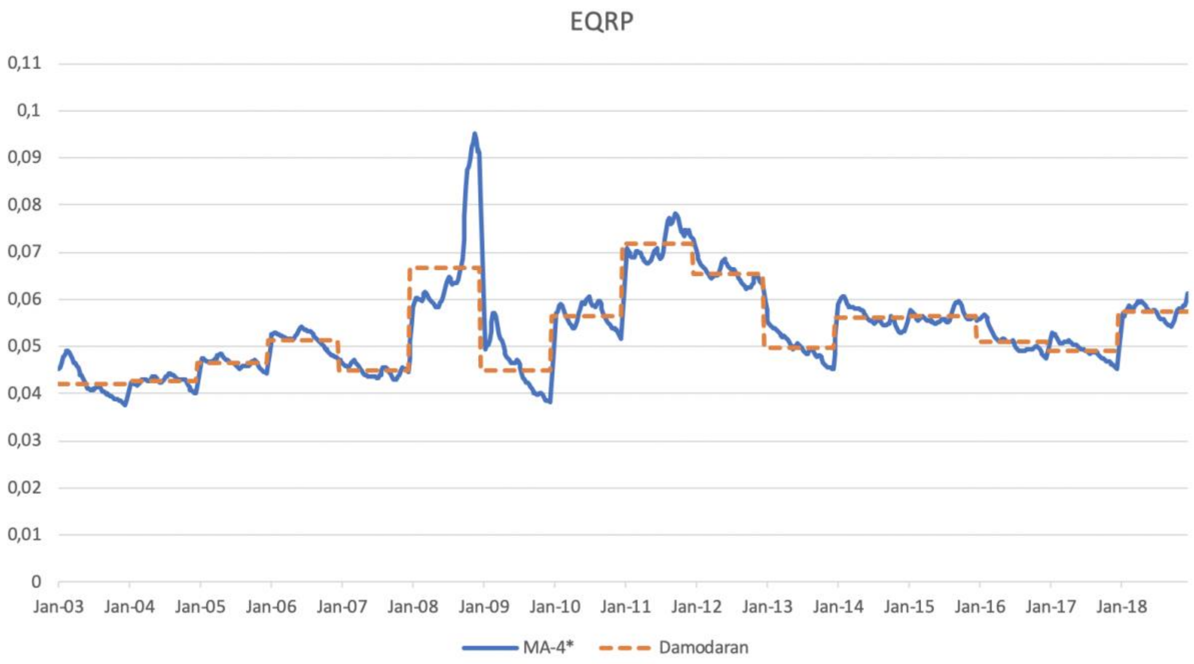
Additionally, I computed the model implied equity volatility through the application of Ito's lemma. The regression between the empirical equity volatility and the model implied equity volatility resulted in a R-Squared of 0,99, which suggest that the model is well calibrated. The section on model implied equity volatility and the regression can be found in Appendix 9.

5.4.3 Equity risk premium (EQRP)

The equity risk premium for the 2003-2018 period was downloaded from the web page of professor Aswath Damodaran. Professor Damodaran is an often-cited source when it comes to valuation and is therefore also used in this thesis. In his website, Professor Damodaran present several estimates of the equity risk premium for the U.S. market. In this dissertation, two estimates are used. The first measure Implied Premium (FCFE) refers to implied premium based on the free cash flow to equity, which is seen as dividends to stockholders. The second measure Implied Premium (FCFE with sustainable Payout) on the other hand assumes that free cash flow to equity will decrease over time to a more sustainable level. By combining the Implied Premium (FCFE) (Appendix 8) and Implied Premium (FCFE with sustainable Payout) (Appendix 8), I obtained a new yearly series of EQRP values (Appendix 8). Due to the yearly nature of the EQRP many of the detailed changes within the year is non observable. In order to solve the previously mentioned problem, I proceeded by doing the following. I downloaded a weekly time series of the S&P500 from DataStream for the period 2003-2018. Constructed weekly ratios by dividing each weekly observation by the average value of the S&P500 that year. Created weekly EQRP values by dividing the combined EQRP for that year with the weekly ratios. Finally, the weekly EQRP for the 2004-2018 period was computed as the 4-week moving average of the constructed timeseries.

The constructed EQRP timeseries and the combined yearly EQRP from Damodaran can be seen in Figure 5 below.

Figure 5 - Equity risk premium



When referring to the Figure above it is obvious the combined EQRP from Damodaran has changes only at year end, whereas the constructed EQRP has a lot more detail. From the MA-4* it is apparent that EQRP peaks at the end of 2008 during the financial crisis, before it drops drastically in the first months of 2009.

5.4.4 Market price of risk – $\boldsymbol{\theta}$

As previously outlined in Section 4.2, θ is found as the product of β and EQRP divided by the empirical equity volatility. Since β and σ_e are set constant across time, all variation in θ is caused by changes in the EQRP.

The time series of θ in all sectors is seen in Figure 6 (panel A and B).

Figure 6 - Market price of risk

Panel A

Panel B

Due to the constant β and σ_e the dynamics of the time series is equal to those of the EQRP (Figure 5).

The average θ in the different sectors is found in Table 6.

Table 6 - Average θ

Average θ					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
θ	18,62%	13,41%	14,46%	21,25%	17,19%
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	-
θ	23,70%	17,70%	20,07%	13,78%	-

As can be observed in both Figure 6 and Table 6 there is quite big differences between the different sectors. Since the EQRP is the same for all sectors, the variation is driven by the differences in β and σ_e. Since the average σ_e across sectors is within relatively close proximity of each other, the differences in θ are caused by the large span of the beta-coefficient.

5.5 Robustness check

5.5.1 State variable – $\boldsymbol{\delta}$

In the EBIT-based model (2001), Goldstein, Ju & Leland assume that the dynamics of the model state variable (EBIT) follow a Geometric Brownian Motion (GBM). Consequently, the state variable has a Log-normal distribution. In order to assess whether the EBIT for each individual company has a Log-normal distribution, I have tested the log changes of EBIT using the Shapiro-Wilk test.

The Shapiro-Wilk test tests the null hypothesis that the log changes in EBIT are normally distributed. Following most scientific papers, I have assumed an alpha of 0,05. This means that if a series of log changes in EBIT obtains a p-value of less than 0,05 one rejects the null hypothesis of normal distribution. If the p-value is above 0,05 one cannot reject the null hypothesis of normal distribution.

P-values from the Shapiro-Wilks test for all firms can be found in Appendix 3-5. The number of companies with p-values below 0,05 within each sector is found in Table 7.

Table 7 - State variable

State variable					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
Rejected/Total	0/6	9/19	6/18	7/23	5/11
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	-
Rejected/Total	6/15	4/13	12/30	13/24	62/159

The Shapiro-Wilk test is rejected for 62 out of 159 firms (Table 7), which is approximately 39%. Consumer staples is the sector with the largest rejection rate, whereas transportation surprisingly has no rejections. Additionally, a 40% rejection rate can be observed for the consumer cyclicals sector. The name of the sector itself suggest that the GMB may not fit well. The GBM states that the past is irrelevant for prediction, only the present matters. In contrast, a cyclical sector means that it from time to time returns to something. As a result, it is normal that the GBM tends to be more rejected in this type of sector.

6. Results

This chapter is divided into two parts. The first part presents the 5-year distance to default (i.e. the expected distance in 5 years between the market value of the business and the default barrier normalized by business risk) and the probability of default that is produced by the model (i.e. the likelihood of the asset process hitting the default barrier at least once in the next 5 years). In the second part, these results are compared with the ones implied by credit ratings.

6.1 Model estimated default measures

6.1.1 Distance to default (DD)

The distance to default is a widely used credit risk metric since Merton (1974) first proposed its ground-breaking model. Summarized, this metric measures the expected distance in T years between the market value of the assets (or business) and the default barrier normalized by business risk. It thus synthetizes in a single measure three key corporate characteristics: market leverage, expected dynamics and risk. In this thesis as for PD I have considered a 5-year horizon for the DD estimations. The average value per sector can be seen in Table 8. The average aggregate DD is computed directly on firms and has a mean value 2,92 (Table 8). With the exception of the utilities sector, and at a lesser extent, the basic materials sector, all sectors have an average DD close to 3. The lowest DD found on a sector level is 1,37 for the utilities sector in May 2009, whereas the health care sector has the highest DD of 4,54 in December 2004.

Table 8 - Average DD

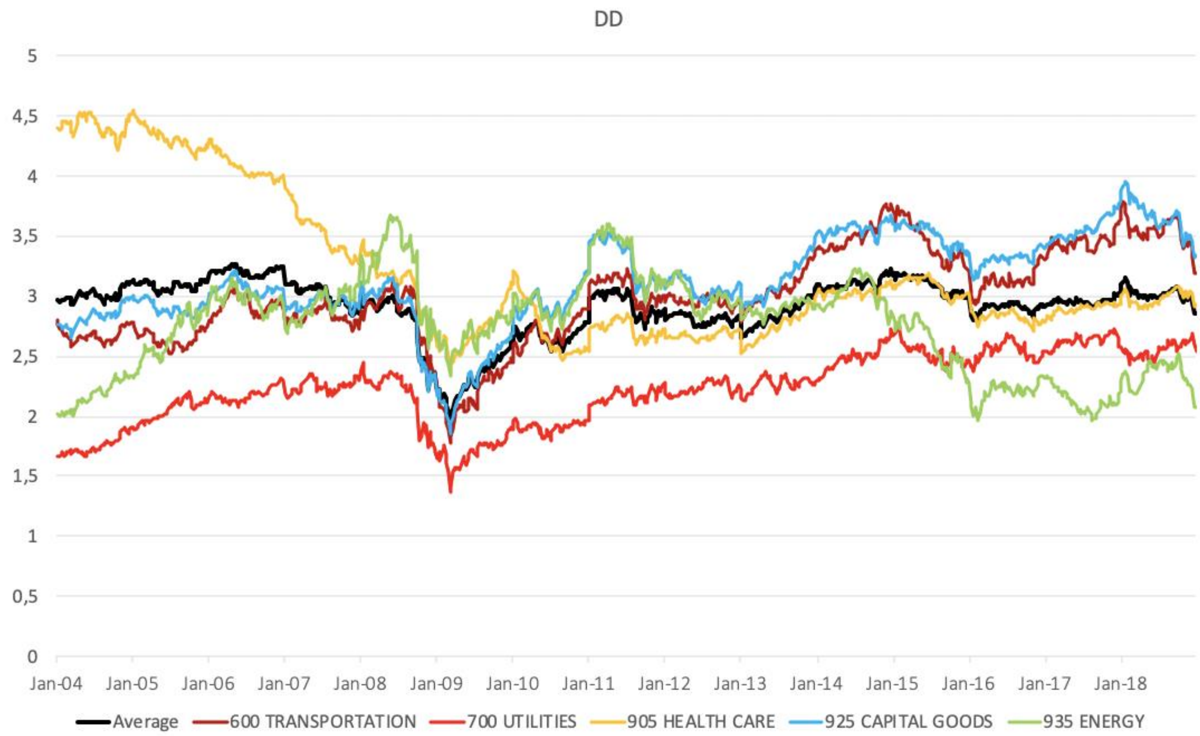
DD					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
Average	3,00	2,23	3,19	3,13	2,74
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	Total
Average	3,21	2,61	3,02	2,97	2,92

The time-series of the average DD by sector is presented in Figure 7 (panel A and B). The aggregate average DD ranges from 1,93 in May 2009 to 3,27 in May 2006[4]. One can observe that all sectors show a high level of co-movement (Figure 7). For instance, it is noticeable that all sectors have a significant drop in DD during the financial crisis. While there is quite a large spread between the sectors prior to this event, post 2009 this gap becomes a lot tighter. This global pattern is mixed with some sector specific events. As an example, during the oil crisis the energy-, technology- and transportation sectors show a larger decline, while the other sectors are less effected.

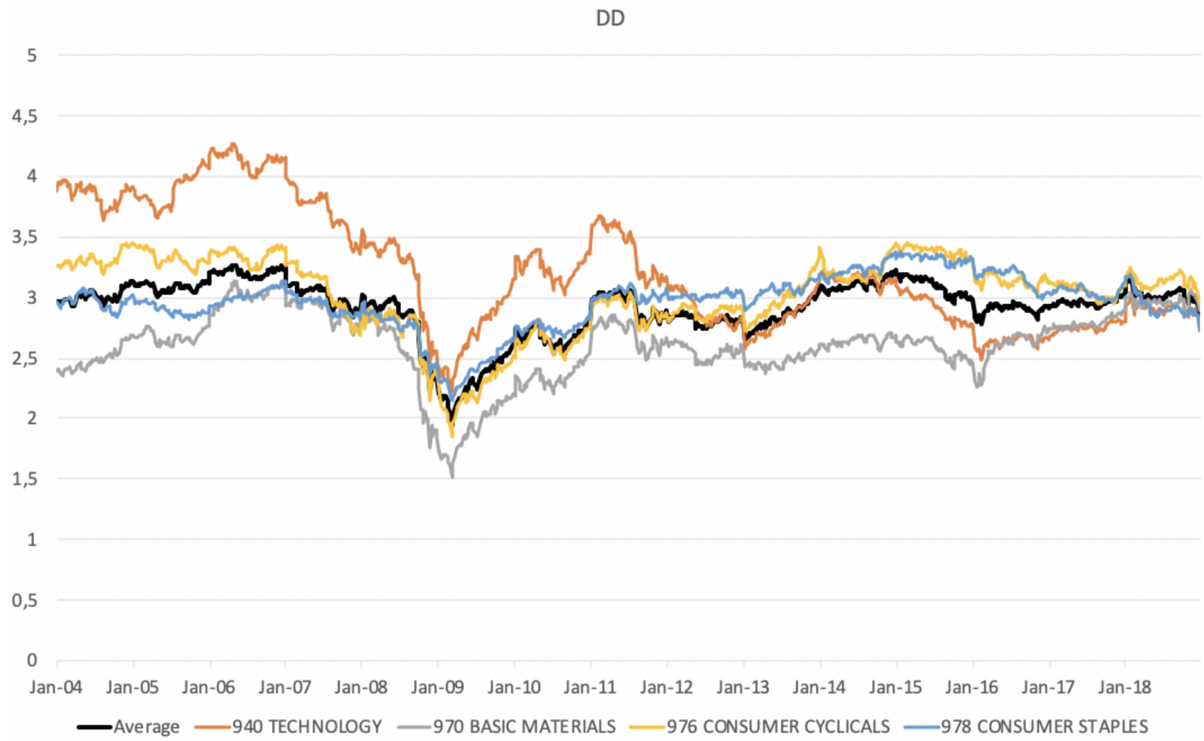
[4] Individual sector ranges: Transportation(1,76:3,78), Utilities(1,37:2,74), Health care(2,35:4,54), Capital good(1,84:3,95), Energy(1,96:3,66), Technology(2,22:4,27), Basic materials(1,52:3,14), Consumer cyclicals(1,85:3,45), Consumer staples(2,14:3,37)

Figure 7 - DD

Panel A



Panel B



The dynamic of the DD can be better understood by looking at business risk (i.e. σ_a), the drift term (μ_p) and the ratio of the market value of assets to the barrier. The first measure was already discussed in Section (5.3.1). If a company's business risk increases all else equal, the distance to default decreases. In Section 5.3.1 we have seen that the consumer staples and utilities sector had a level of business risk below all other sectors. In contrast, the technology sector showed a level of business risk above all others. The measure P drift and the ratio between the market value of assets and the default barrier are presented in Table 9 and 10, respectively.

The project drift is the return on assets (Appendix 10) less the companies payout ratio (Section 5.3.2). This is also referred to as the company's expected growth rate. Looking at Table 9, it can be noted that there is big difference between the sectors. For the utilities sector the growth rate is actually negative, whereas the technology sector has a growth rate well above the remining sectors. The remaining sectors can be divided in two groups. Here the transportation, basic materials and consumer staples sector range between a lower range of 0,41% to 0,60%. Whereas the health care, capital goods. energy and consumer cyclicals sector ranges between 0,97% to 1,46%. Furthermore, it should be noted that the technology sector is the only sector with a drift above mean inflation level over the past years

Table 9 - Project drift

Project drift					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
μ_p	0,55%	-2,21%	1,46%	1,02%	0,97%
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	-
μ_p	2,44%	0,41%	1,25%	0,60%	-

The default barrier to asset ratio measures how far the asset value is from the barrier without scaling by the project volatility. It can be thus seen as a market-based leverage measure. Due to the fact that a company defaults if the value of assets is equal to or less than the barrier, the ratio must lie between zero and one. If the barrier to asset ratio of a company increases, so does the probability of default, while the DD decreases. The barrier to asset ratio for all sectors is presented in Table 10. On average most sectors have a barrier to asset ratio close to each other. The utilities sector has a significantly higher barrier to asset ratio signalling a high degree of financial leverage.

Table 10 - Barrier/Assets

Barrier/Asset					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
Average	0,58	0,71	0,46	0,55	0,54
Max	0,71	0,79	0,55	0,69	0,66
Min	0,48	0,64	0,34	0,46	0,46
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	-
Average	0,48	0,59	0,53	0,54	-
Max	0,59	0,71	0,66	0,63	-
Min	0,39	0,53	0,46	0,47	-

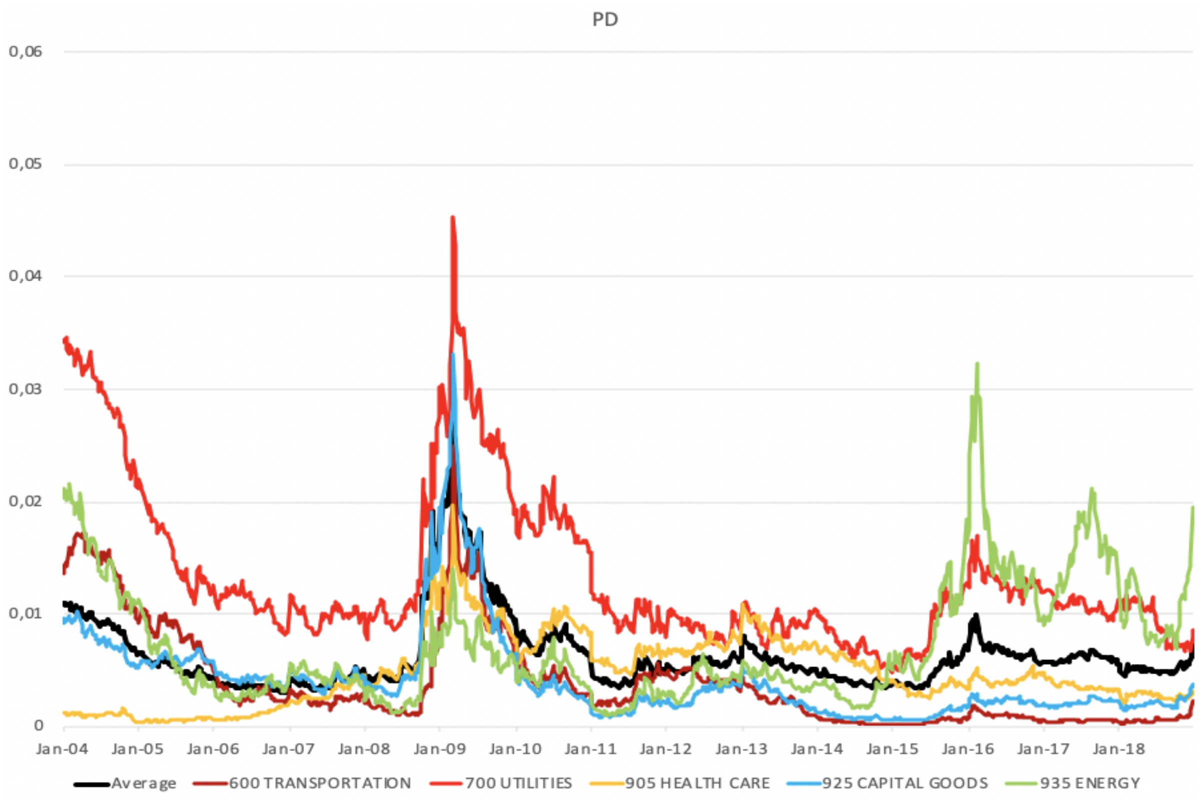
The three indicators just presented help us understand the DD results. In particular, regarding the utilities sector, it can be concluded that though it has lower business risk than the other sectors, it also shows a higher degree of leverage and the lowest drift. As a result, it has the lowest DD over time. The technology sector shows the opposite characteristics. With the highest business risk, the highest drift and the lowest leverage it has the highest DD across time.

6.1.2 Probability of default (PD)

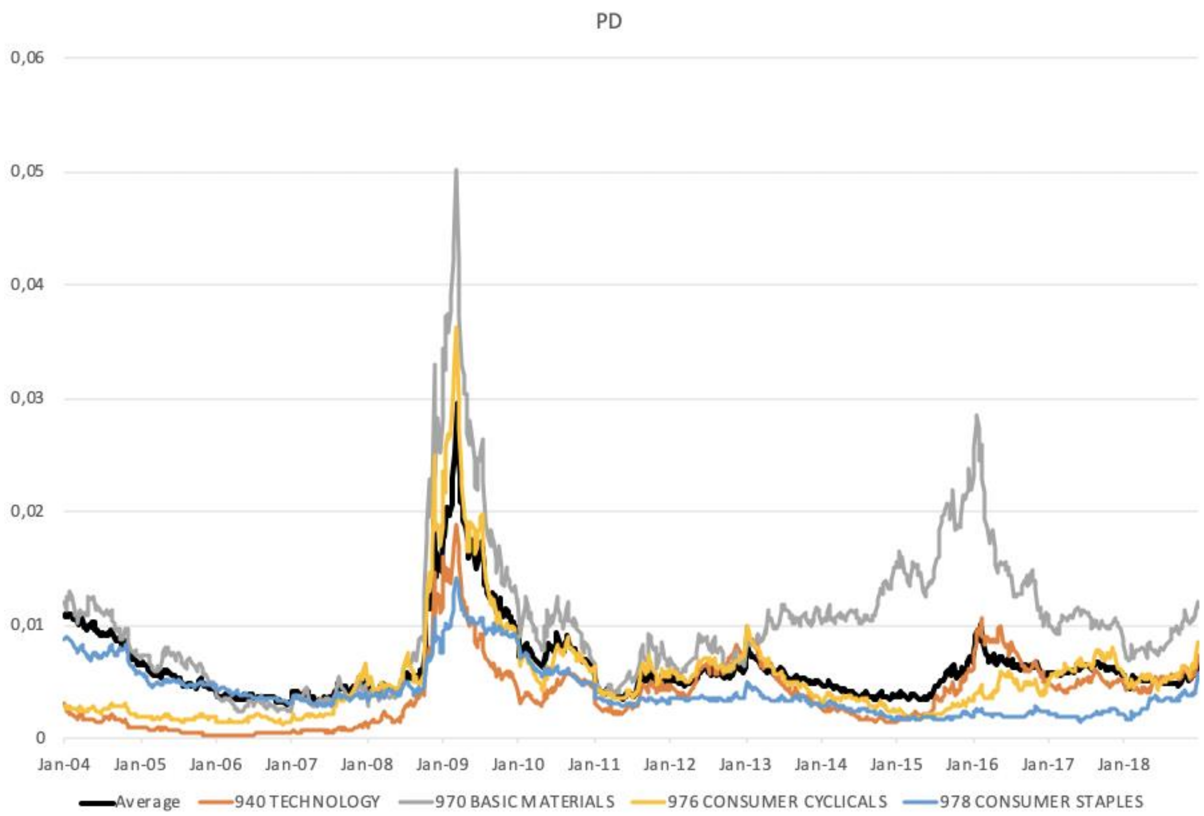
The most used credit risk measure is the probability of default (PD). This is presented in Figure 8 (panel A and B). In this dissertation as for the DD, the horizon considered is 5 years. The probabilities presented are cumulative probabilities.

Figure 8 – PD

Panel A



Panel B



The average probability of default of the entire portfolio is 0,63%, ranging between 0,30% in December 2006 and 2,95% in March 2009. This relatively tight range hides however considerable variation at the sector (Table 12) and especially at the firm level (Table 11). As expected, and in line with the DD, the utilities and basic materials sectors are the ones with the highest PD. In contrast, the transportation sector shows the lowest PD.

Table 11 – PD, sector ranges at individual firm level

PD	Sector ranges at individual firm level				
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
Max	5,45%	18,12%	8,15%	17,19%	10,08%
Min	0,00%	0,00%	0,00%	0,00%	0,00%
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	Total
Max	10,44%	17,47%	14,72%	6,83%	18,12%
Min	0,00%	0,00%	0,00%	0,00%	0,00%

Table 12 - Average PD

PD					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
Average	0,39%	1,34%	0,48%	0,42%	0,72%
Max	2,49%	4,53%	1,97%	3,32%	3,23%
Min	0,01%	0,48%	0,04%	0,05%	0,10%
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	Total
Average	0,50%	1,03%	0,52%	0,42%	0,63%
Max	1,89%	5,01%	3,62%	1,42%	2,95%
Min	0,02%	0,23%	0,13%	0,15%	0,30%

On the time dimension, it is apparent that all sectors spent the period from 2004 until the start of 2008 recovering from the financial turmoil associated with the dotcom bubble (Figure 8). Throughout this period the PD for the utilities sectors stands out from the remaining sectors. This sector shows a non-negligible PD whereas most other sectors show close to no probability of default. During the financial crisis of 2008-2009 there are significant observable difference in how the different sectors PD evolve. The basic materials, consumer cyclicals and utilities sectors have a much larger increase in the PD, than the remaining sectors. The utilities and basic materials sectors have PDs of 4,5% and 5% at the peak of the crisis, respectively. For all sectors, PDs decrease towards post crisis levels within 2011. For the 2015-2016 oil crisis there are clear differences between how individual sector PDs react. As expected, the PD for the energy sector peaks at 3,2%, significantly higher than for the others. With the exception of the basic materials, at a lesser extent the technology sector, whose PDs also increased during this period, the PD for all other sectors remained more or less stable. Lastly, the energy sector has a clear peak at the end of 2018, just in line with the steep drop in oil prices.

6.2 Credit rating

A credit rating is a letter issued by a credit rating agency, which represents the probability of the debt not being repaid. The world leaders in credit rating are Moody's, S&P and Fitch Group. These credit rating agencies take into account enormous amounts of both firm and macroeconomic information when issuing credit ratings. Consequently, their credit ratings are seen as the benchmark[5] when evaluating the probability of default of a given firm. The model here proposed is much easier to implement and is based solely on widely available information. An obvious question is thus how far credit ratings are from the credit risk measures. In this Section, and in order to further assess the default measures estimated by the model, I compare them to the credit rating-based measures. In section 6.2.1 this is done by looking at average values and through scatter plots. In section 6.2.2 the analysis is extended by running some panel data regressions.

6.2.1 A comparison between credit rating implied measures and model measures

In order to compare the measures, I started by gathering the credit ratings from both Moody's and Standard and Poor's for all companies. I proceeded by transforming the credit ratings from Moody's and Standard and Poor's into one single measure. This was done by transforming the individual measures into numerical values using a numerical scheme[6], averaging them and finally restating the obtained value in the usual S&P scale (Appendix 7). The average credit rating per sector, per year is found in Table 13. It is clear that most of my sample is made of investment grade corporates (Table 13). Taking all corporate-date pairs, one can conclude that 92% of all individual ratings are investment grade.

5 The fact that they are seen as the benchmark, does not mean that they are right.

6 1 to AAA, 2 to AA+, 3 to AA, 4 to AA-, 5 to A+, 6 to A, 7 to A-, 8 to BBB+, 9 to BBB, 10 to BBB-, 11 to BB+, 12 to BB, 13 to BB-, 14 to B+, 15 to B, 16 to B-, 17 to CCC+ and 18 to CCC.

Table 13 - Average credit ratings by year and sector of activity

Credit rating	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY	Total
2004	BBB+	BBB-	BBB+	BBB+	BBB+	BBB
2005	BBB+	BBB	A-	BBB+	BBB+	BBB
2006	BBB+	BBB	A-	BBB+	BBB+	BBB
2007	BBB	BBB	BBB+	BBB+	BBB+	BBB
2008	BBB	BBB	BBB+	BBB+	BBB+	BBB
2009	BBB	BBB	BBB+	BBB+	BBB+	BBB
2010	BBB	BBB	BBB+	BBB+	BBB+	BBB
2011	BBB	BBB	BBB+	BBB+	BBB+	BBB
2012	BBB	BBB	A-	BBB	BBB+	BBB
2013	BBB	BBB	A-	BBB+	BBB+	BBB
2014	BBB+	BBB	BBB+	BBB+	BBB+	BBB
2015	BBB+	BBB+	BBB+	BBB+	BBB+	BBB
2016	BBB+	BBB	BBB+	BBB+	BBB	BBB
2017	BBB+	BBB+	BBB+	BBB+	BBB	BBB
2018	BBB+	BBB+	BBB+	BBB+	BBB	BBB
Average	BBB+	BBB	BBB+	BBB+	BBB+	BBB
Credit rating	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	-	Total
2004	BB+	BBB	BBB-	BBB-	-	BBB
2005	BBB-	BBB	BBB-	BBB-	-	BBB
2006	BBB-	BBB	BBB	BBB-	-	BBB
2007	BBB	BBB	BBB	BBB-	-	BBB
2008	BBB-	BBB	BBB	BBB-	-	BBB
2009	BBB-	BBB	BBB	BBB-	-	BBB
2010	BBB-	BBB	BBB	BBB-	-	BBB
2011	BBB	BBB	BBB	BBB-	-	BBB
2012	BBB	BBB	BBB	BBB-	-	BBB
2013	BBB	BBB	BBB	BBB-	-	BBB
2014	BBB	BBB	BBB	BBB-	-	BBB
2015	BBB	BBB	BBB-	BBB	-	BBB
2016	BBB-	BBB	BBB-	BBB	-	BBB
2017	BBB-	BBB	BBB	BBB	-	BBB
2018	BBB	BBB	BBB	BBB	-	BBB
Average	BBB-	BBB	BBB	BBB-	-	BBB

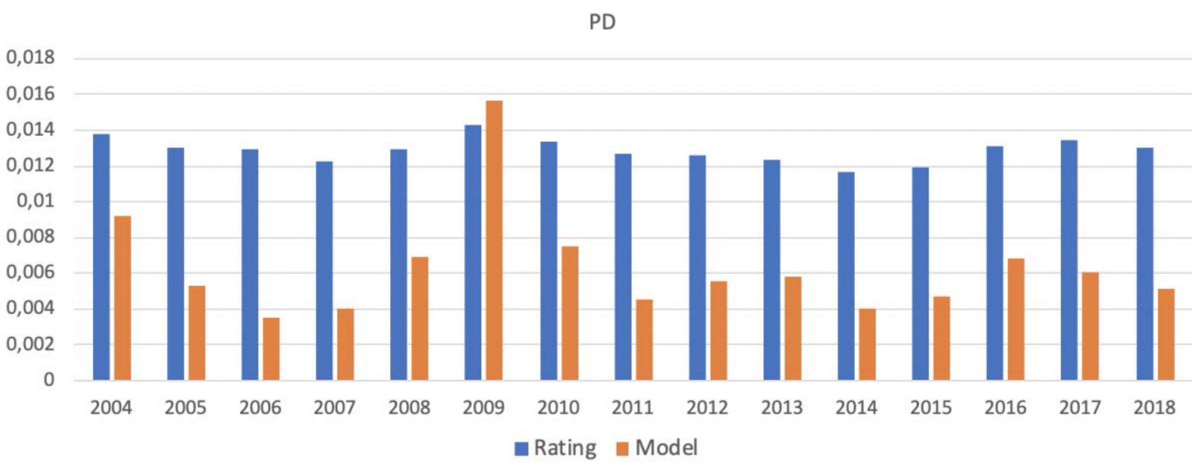
Lastly, I utilized the transition matrix issued by Standard and Poor's (Appendix 6) in order to transform the credit ratings into probabilities of default. This resulted in yearly probabilities of default for all companies. The average probability of default per sector can be seen in Table 14. This table shows that model PDs underestimate the credit rating PDs by 0,68 p.p. on average. This underestimation is computed as the p.p. difference between measures. Though this difference is large in relative terms (credit rating implied PDs are 0,68 p.p. higher), it can be deemed small in absolute terms. This type of difference is observed in most sectors, except the utilities sector, where the PDs are noticeably close. On the contrary one can see a very large spread between the estimates for the technology sector. Here, the probability of default implied by credit ratings are 1,76 p.p. higher than the ones coming from the model.

Table 14 - PD comparison[7]

PD					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
Credit rating: Average	1,12%	1,49%	0,85%	1,03%	1,08%
Model: Average	0,73%	1,34%	0,48%	0,42%	0,72%
Difference	0,39%	0,15%	0,37%	0,61%	0,36%
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	Total
Credit rating: Average	2,26%	1,55%	1,31%	1,08%	1,31%
Model: Average	0,50%	1,03%	0,52%	0,42%	0,63%
Difference	1,76%	0,52%	0,79%	0,66%	0,68%

The average PD for all companies across the period 2004-2018 can be seen in Figure 9. The PDs obtained from credit ratings are very stable, with a peak in 2009 (Figure 9). This was already in part expected as all companies assessed in this thesis have stayed listed on the S&P500 throughout the entire period and are mostly investment grade credit ratings. Notice that for these ratings PDs tend to be very small. Apart from the PD coming from the model in 2009, model PDs are consistently lower than the ones implied by credit ratings.

Figure 9 - PD, rating vs. model



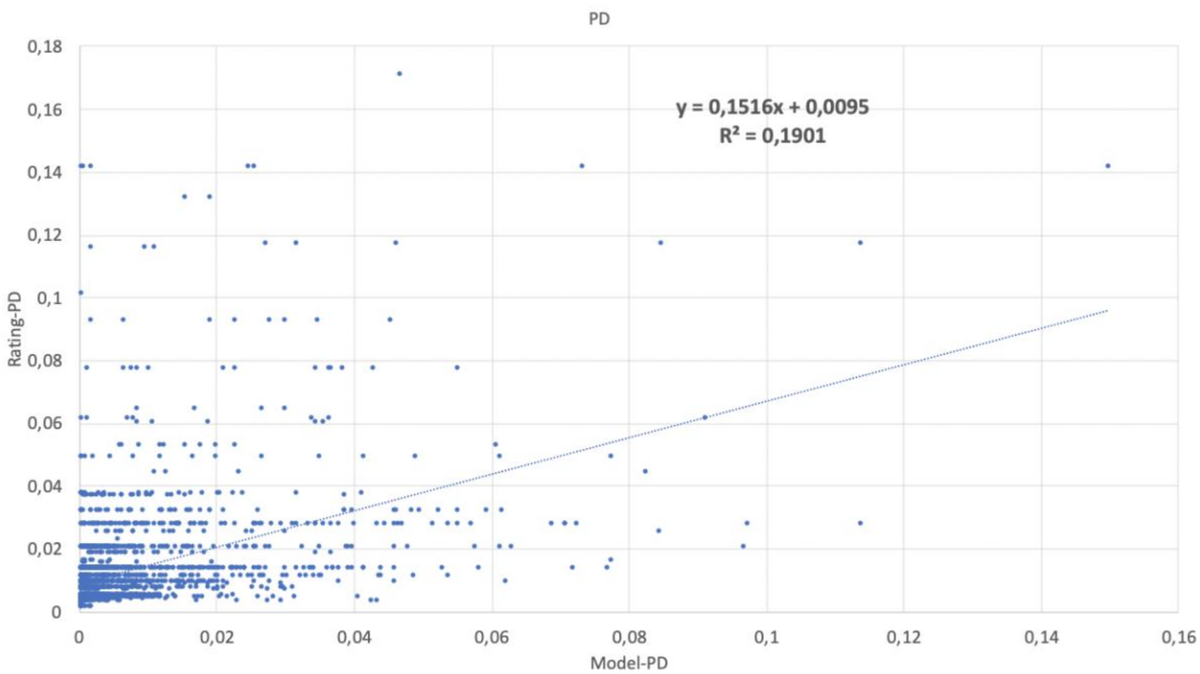
The probability of default is known to be very non-linearly related with fundamentals, which turns hard any econometric analysis built on traditional linear models. Given these non-linear relations, it is common to carry most of the analysis on an alternative setting and then restore it back. This is what occurs in a Logit or Probit credit risk regression-based models. In the latter case, this is done by using the Normal distribution. Interestingly, something similar occurs in Merton's model. In this case, the distance to default (i.e. the risk adjusted distance between the market values of assets and the default barriers) is computed and then translated into a PD by

[7] Difference = Average credit rating PD – Average model PD

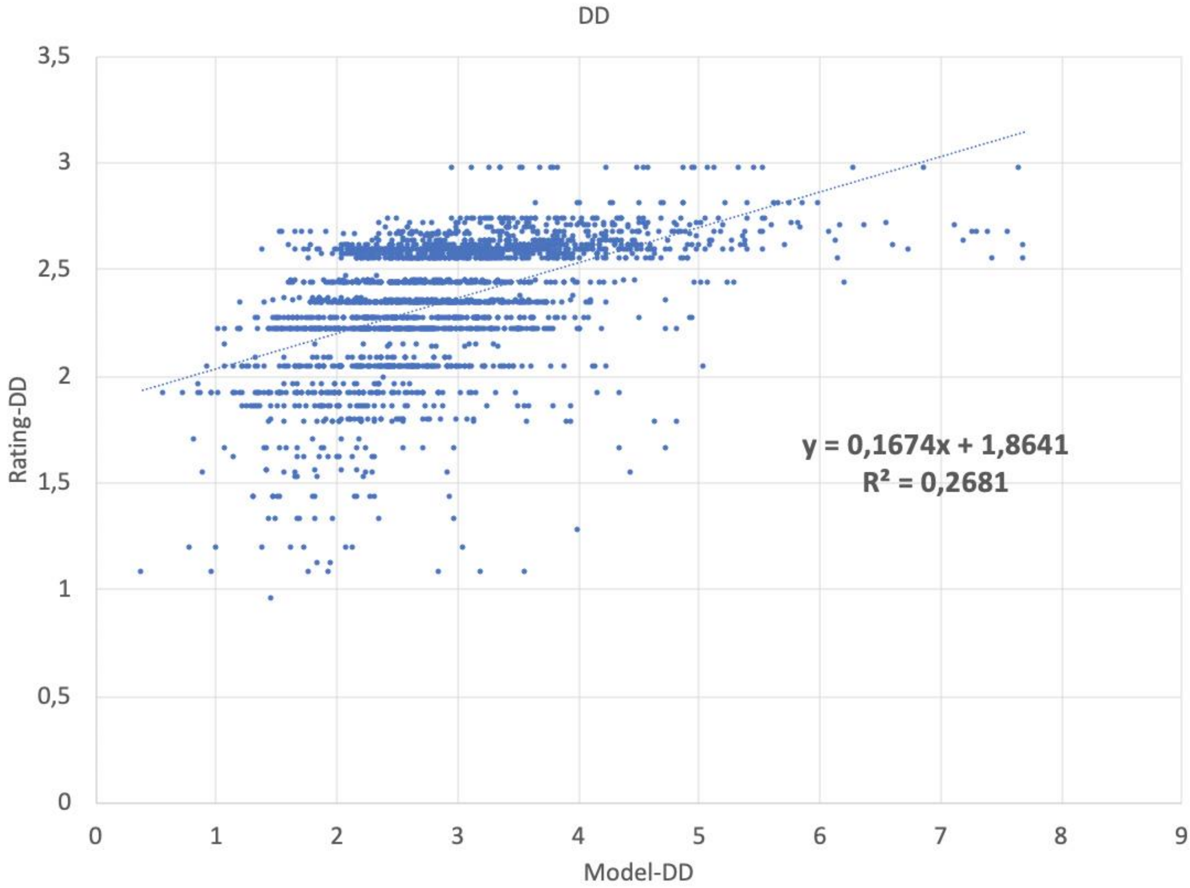
using the Normal distribution function. In the case of Merton's model, the distance to default is thus just the inverse normal of the PD. In the model here presented, due the possibility of hitting the barrier before the considered maturity, the inverse normal is not equal to the risk adjusted distance between the market values of assets and the default barriers, as previously defined. Nevertheless, it can still be seen as a more tractable credit risk indicator. Figure 10 (panel A and B) shows two scatter plots of the PD and the DD, respectively. The DDs were computed from the PDs at the individual company level by using NORM.INV in Microsoft Excel. From these figures, it can be seen that while in the case of the PD, a correlation of 44% was found, in the case of the DD, this figure increases to 52%.

Figure 10 - Scatter plot regressions

Panel A



Panel B



The aggregated DD for all sectors across the 2004-2018 period can be seen in Figure 11. Furthermore, the average DD per sector can be seen in Table 15. On average DD coming from the model is 0,57 standard deviations higher than the credit rating implied (Table 15). This is estimated as the difference in DD between the two measures. It is evident from Figure 11 that the DD implied by credit rating remains very stable over time. Even though there is a minor decline in the DD in 2009, it remains within 2,3-2,4 for the entire timeseries. The DD from the model has much larger variations. Under normal circumstances the DD coming from the model remains higher than the ones implied by credit ratings, whereas in 2009 during the financial crisis it is lower.

Figure 11 -DD, ratings vs. model

Table 15 - DD averages[8]

DD					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
Credit rating: Average	2,33	2,22	2,47	2,43	2,39
Model: Average	3,00	2,23	3,19	3,13	2,74
Difference	-0,67	-0,01	-0,72	-0,70	-0,35
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	Total
Credit rating: Average	2,22	2,28	2,35	2,40	2,35
Model: Average	3,21	2,61	3,02	2,97	2,92
Difference	-0,99	-0,33	-0,67	-0,57	-0,57

Throughout this section we have seen that the model is underestimating credit risk, but why? In the standard EBIT-based structural model company dynamics are assumed to follow a Geometric Brownian motion, which leads to a continuous path meaning that the process is unable to jump between levels in a short period of time. Related to this, the project return distribution does not have much probability in the tails. In my model, as in the original Merton-model, except for the consideration of the first passage time, the PD is basically the negative of the DD evaluated under the Normal Distribution. The drawback of this approach has been illustrated by Hamilton, Munves & Sun (2012) from Moody's Analytics. Since the Normal distribution does not have much probability in the tails, for higher values of DD, we end up with an underestimation of credit risk. Moody's compensates this by using their empirical mapping, which related observed default rates with their model distances do default. In the case of very low DDs, the use of the Normal distribution leads to an overestimation of credit risk.

[8] Difference = Average credit rating DD – Average model DD

In addition to the assumption of GBM, the EBIT-based model does not consider debt rollover. As a consequence of not rolling over debt, the companies are not subject to the potential liquidity risk. This can cause the probability of default to be underestimated.

6.2.2 An econometric analysis of the results

Section 6.1 has shown that the structural model proposed in this dissertation tends to underestimate credit risk as compared to credit rating agencies. Is this difference constant across all firms and sectors? Is this a problem just on the mean level? In this section these questions are addressed by carrying some panel regressions using the function "plm" in R. These regressions can be divided into two groups. First, the relation between the credit ratings implied DD and the model DD is studied. Then, the focus turns to the time variation in the DDs.

To start, I run a pooling model with and without intercept term. Here the credit rating implied DD is the dependent variable and model DD is the independent variable. The results from these regressions are found in Table 16. One can see that in the case with intercept I obtain a large intercept of 1,87 and a β of only 0,17 (Table 16). These are basically the same values presented in Figure 10 (Panel B). Both of these coefficients are significant at all significance levels. Additionally, an R-Squared of 0,26 is attained. I proceed by analysing the same regression while removing the intercept. Here we can notice a significant change in β, which is now 0,75. The mean error level is now positive at 0.177. While the value has changed, it is still significant at all significance levels. In addition, it is worth mentioning that, though the intercept was significant, taking it out does not change the R-Squared. Both R-Squared obtained from the regressions on levels tell me that the model DD is able to explain 26,05% of the variation in credit rating implied DD.

Table 16 - Panel regression, levels

Levels							
Model	Pooling			Model	Pooling, no intercept		
Coefficients	Estimate	Std. Error	P-value	Coefficients	Estimate	Std. Error	P-value
Intercept	1,8700693	0,0175333	< 2.2e-16	Intercept	-	-	-
Beta	0,1656341	0,0057173	< 2.2e-16	Beta	0,7458548	0,0042256	< 2.2e-16
R-Squared	0,26047			R-Squared	0,26047		

Next, a panel regression with firm fixed effects was run. The results of this fixed effect regression are found in Table 17. Here I obtain a β of only 0,07, significant at all significance levels. Additionally, an R-Squared of 12% is obtained. This value is nevertheless the add-on due to the covariate. When the firm fixed effects are taken into account, the R-Square increases to 83%. Most of the variation is thus explained by the firm fixed effects rather than the structural model distance to default.

Table 17 - Panels regression, levels and fixed effects

Model	Fixed Effects		
Coefficients	Estimate	Std.Error	P-value
Intercept	-	-	-
Beta	0,0692662	0,0040183	< 2e-16
R-Squared	0,11781		
R-Squared*	0,82605429		
* Taking into account the firm fixed effects			

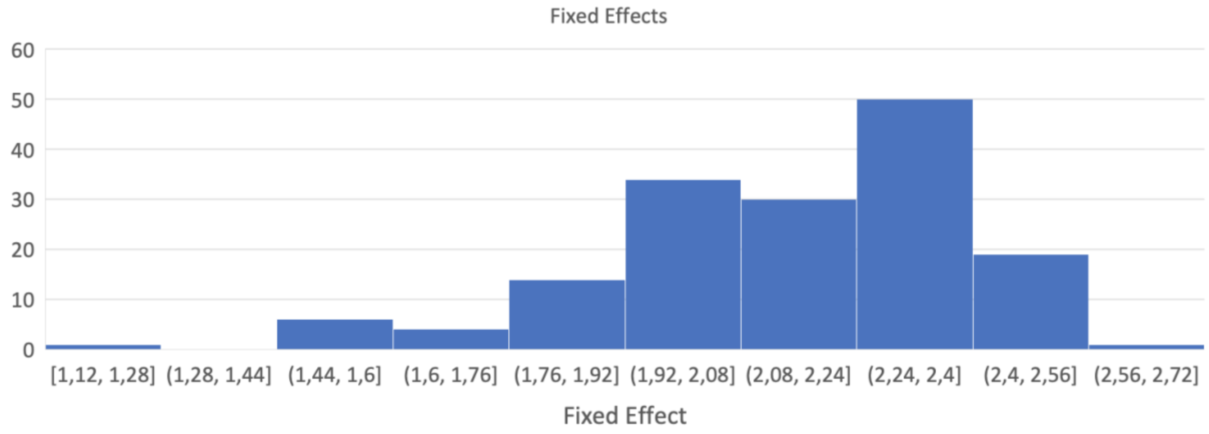
I proceeded by extracting the firm fixed effects using the function "fixef". The average, maximum and minimum fixed effect per sector can be seen in Table 18. Here it is noticeable that the average fixed effect is similar across sector. Sectors average fixed effect range from 2 in the technology sector to 2,25 in the health care sector. Interestingly, the average fixed effect is higher than the intercept computed in the pooling model (Table 16). Within each sector, the technology sector provides the widest range of 1,43, whereas the transportation sector noticeably has a much narrower spread than all other sectors.

Table 18 - Fixed effects

Fixed Effects					
Sector	Transportation	Utilities	Health care	Capital goods	Energy
Average	2,13	2,08	2,25	2,21	2,20
Max	2,44	2,35	2,49	2,45	2,45
Min	1,98	1,71	1,82	1,54	1,80
Range	0,46	0,64	0,67	0,90	0,66
Sector	Technology	Basic materials	Consumer cyclicals	Consumer staples	Total
Average	2,00	2,10	2,14	2,19	2,15
Max	2,56	2,42	2,68	2,48	2,68
Min	1,12	1,63	1,51	1,58	1,12
Range	1,43	0,79	1,17	0,89	1,56

I advanced by making a histogram of all fixed effects. This is presented in Figure 12. Here one can see that the fixed effects are relatively concentrated between 1,92 and 2,4. There is however some left skew. This left skewness is mainly caused by the technological sector.

Figure 12 - Fixed effects, histogram



Following the analysis on levels, I proceeded by analysing the effect of changes in the model DD on the changes in the credit rating implied DD at the individual firm level. Here changes in credit rating implied DD is the dependent variable and changes in model DD is the independent variable. The results of this regression are found in Table 19. If the difference between the model and the credit rating implied DD were just a question of levels, one should have a coefficient of 1. Instead, the regression results in a small negative intercept and a β of only 0,04 (Table 19). The intercept is significant at a 5% significance level, whereas β is significant at all significance levels. Furthermore I attain an R-squared of 0,05, which tells us that the changes in the model DD is able to explain only 5% of the variance in the changes in the credit rating implied DD. I proceeded by running the same regression, but now not allowing for an intercept. Notice that since the model is written in differences, an intercept implies a trend in the DD. The results of this regression are also found in Table 19, where it is apparent that the β and R-squared almost do not change.

Table 19 - Panel regression, changes

Changes							
Model	Pooling			Model	Pooling, no intercept		
Coefficients	Estimate	Std. Error	P-value	Coefficients	Estimate	Std. Error	P-value
Intercept	-0,0017792	0,0018042	0.03242	Intercept	-	-	-
Beta	0,0447128	0,0040372	< 2.2e-16	Beta	0,0447082	0,0040372	< 2.2e-16
R-Squared	0,05227			R-Squared	0,05227		

Both regressions in Table 19 provided very small β-values and an R-Squared slightly lower than expected. One possible explanation for this finding is the high persistence in credit ratings. In order to explore this hypotehsis I estimated the amount of periods where there is no change in credit ratings at the individual company level. The aggregated results of this can be seen in Table 20.

Table 20 - Credit rating, periodical changes

Periods of no rating change					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
Average	70%	73%	75%	82%	72%
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	Total
Average	74%	73%	71%	74%	74%

It can be noted that 74% of the time there are no changes in credit ratings (Table 20). Consequently the small size of β may be due to the fact that ratings do not change, which turns estimation particularly hard with a linear model.

In order to adress this issue, a dummy variable was added to the regression of the effect of changes in the model DD on the changes in the credit rating implied DD at the individual firm level. This dummy variable took the value one when there were no changes in credit rating, and zero otherwise. In addition, I added an interaction term between the dummy variable and the model DD. The results of this regression can be found in Table 21.

From Table 21 one can see that I obtain a small estimate for the dummy variable, which is not deemed significant. However, a clearly negative and significant cross term Dummy:Beta was found. This result is in line with the expectations. In addition, the model β increased from 0,04(Table 19) to 0,15 (significant at all levels). Despite a clear increase, this is still a small figure. The R-Squared also increased significantly from 0,05 (Table 19) to 0,17. Hence, the inclusion of the dummy variable clearly provides to regression with greater explanatory power.

From all this it can be concluded that changes in the model DD and changes in the credit rating implied DD clearly covary. However, the changes in model DD have a significantly larger variation leading the β to be statistically different from one.

Table 21 - Panel regression, changes with dummy variable

Changes, with dummy variable			
Model	Pooling		
Coefficients	Estimate	Std.Error	P-value
Intercept	-0,006209	0,0033135	0,06113
Dummy	0,006208	0,0038486	0,10688
Beta	0,1476431	0,0068566	< 2e-16
Dummy:Beta	-0,1476431	0,0082104	< 2e-16
R-Squared	0,1736		

7. Conclusion

The main objective of this dissertation was to study whether the bankruptcy measures estimated by the standard EBIT-based structural model presented in section 3 of Goldstein, Ju and Leland (Goldstein, Ju, & Leland, 2001) are comparable to those produced by credit rating agencies. The scope of study was all non-financial companies present on the S&P500 throughout the 2004-2018 period.

Model and credit rating implied credit risk measures were found to be different, but broadly comparable. On average the probability of default coming from the structural model underestimated credit implied values by 0,68 p.p. Whereas, distance to default coming from the structural model was found to overestimate credit implied values by only 0,57 standard deviations. This underprediction of credit risk was prominent across all sectors. However, the degree of underprediction was found to be varying vastly. The clear underprediction of credit risk was found in all years of the study, except from the financial crisis period, where the probability of default coming from the structural model overestimated credit implied values by 0,14 p.p. This dissertation also discussed possible reasons for this underestimation, notably the lack of jumps and debt rollover.

Following the analysis of averages, the dissertation proceeded to analyse the relation between the model and credit rating probabilities of default and distance to default. Both credit risk measures were found to show a relatively strong correlation. The probability of default had a correlation between the two approaches of 44%, whereas the distance to default obtained a clearly higher correlation of 52%. The particular relation between the model and credit rating distance to default was studied further through econometric analysis.

The econometric analysis was done through multiple panel data regressions. These consisted of regressions on levels with and without firm fixed effects and on time differences. The average firm fixed effects was found to be similar across sectors. All regressions found the model distance to default measure to be significant at all usual confidence levels. However, the coefficient associated with the measure was found to be small for all approaches. This suggests that the structural model tends to overreact on all new information, while the credit rating agencies act more smoothly.

Nevertheless, there are a few limitations that have to be mentioned regarding this dissertation. The EBIT-based model (Goldstein, Ju, & Leland) does not allow for companies with negative EBIT. This results in the model not being able to asses many companies in the developing stages and other companies operating on a negative EBIT. The assumption of a Geometric Brownian motion does not allow for jumps, which is prominent in real life. Additionally, debt is considered to be perpetual, which is not always the case in real life.

In addition to limitations stemming from the model, there are also limitations as a result of the methods used in model calibration. During construction of the model I assumed constant tax rates across all periods, if this had been varying through time, results may have changed. I removed outliers when estimating equity volatility in order to make it comparable to previous studies. Using non altered values may have altered estimates slightly. Several of the variables have been constructed through interpolation. Different approaches were possible, which could lead to different results. Lastly, the removal of companies with negative EBIT on average and/or no interest costs may result in a skewed representation of both individual sectors and the total portfolio. This limitation is especially relevant in the technological sector.

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9. Appendix

Appendix 1 - Company overview pt.1

S&P Economic Sector Code	600	700	905	925	935
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
Amount of companies	6	19	18	23	11
Company name	CSX CORP	AMEREN CORP	ABBOTT LABORATORIES	3M CO	ANADARKO PETROLEUM CORP
Company name	FEDEX CORP	AMERICAN ELECTRIC POWER CO	AMGEN INC	AVERY DENNISON CORP	APACHE CORP
Company name	NORFOLK SOUTHERN CORP	CONSOLIDATED EDISON INC	ANTHEM INC	BALL CORP	CHEVRON CORP
Company name	SOUTHWEST AIRLINES	DOMINION ENERGY INC	BAXTER INTERNATIONAL INC	BOEING CO	CONOCOPHILLIPS
Company name	UNION PACIFIC CORP	DTE ENERGY CO	BECTON DICKINSON & CO	CATERPILLAR INC	DEVON ENERGY CORP
Company name	UNITED PARCEL SERVICE INC	DUKE ENERGY CORP	BIOGEN INC	CUMMINS INC	EOG RESOURCES INC
Company name		EDISON INTERNATIONAL	BOSTON SCIENTIFIC CORP	DANAHER CORP	HALLIBURTON CO
Company name		ENTERGY CORP	BRISTOL-MYERS SQUIBB CO	DEERE & CO	HESS CORP
Company name		EXELON CORP	CIGNA CORP	EATON CORP PLC	MARATHON OIL CORP
Company name		FIRSTENERGY CORP	HUMANA INC	EMERSON ELECTRIC CO	OCCIDENTAL PETROLEUM CORP
Company name		NEXTERA ENERGY INC	JOHNSON & JOHNSON	FLUOR CORP	SCHLUMBERGER LTD
Company name		NISOURCE INC	LILLY (ELI) & CO	GENERAL DYNAMICS CORP	
Company name		PINNACLE WEST CAPITAL CORP	MEDTRONIC PLC	HONEYWELL INTERNATIONAL INC	
Company name		PPL CORP	MERCK & CO	ILLINOIS TOOL WORKS	
Company name		PUBLIC SERVICE ENTRP GRP INC	PFIZER INC	JOHNSON CONTROLS INTL PLC	
Company name		SEMPRA ENERGY	QUEST DIAGNOSTICS INC	LOCKHEED MARTIN CORP	
Company name		SOUTHERN CO	STRYKER CORP	NORTHROP GRUMMAN CORP	
Company name		WILLIAMS COS INC	UNITEDHEALTH GROUP INC	PACCAR INC	
Company name		XCEL ENERGY INC		PARKER-HANNIFIN CORP	
Company name				ROCKWELL AUTOMATION	
Company name				SEALED AIR CORP	
Company name				TEXTRON INC	
Company name				WASTE MANAGEMENT INC	

Appendix 2 - Company overview pt.2

S&P Economic Sector Code	940	970	976	978
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES
Amount of companies	15	13	30	24
Company name	AGILENT TECHNOLOGIES INC	AIR PRODUCTS & CHEMICALS INC	AUTOZONE INC	ALTRIA GROUP INC
Company name	APPLIED MATERIALS INC	ARCHER-DANIELS-MIDLAND CO	BEST BUY CO INC	AMERISOURCEBERGEN CORP
Company name	CORNING INC	EASTMAN CHEMICAL CO	BLOCK H & R INC	BROWN FORMAN CORP
Company name	DOVER CORP	ECOLAB INC	CARNIVAL CORPORATION & PLC	CAMPBELL SOUP CO
Company name	EQUIFAX INC	FREEPORT-MCMORAN INC	CINTAS CORP	CARDINAL HEALTH INC
Company name	FISERV INC	INTL FLAVORS & FRAGRANCES	COSTCO WHOLESALE CORP	CLOROX CO/DE
Company name	GRAINGER (W W) INC	INTL PAPER CO	GENUINE PARTS CO	COCA-COLA CO
Company name	HP INC	LINDE PLC	HARLEY-DAVIDSON INC	COLGATE-PALMOLIVE CO
Company name	INTL BUSINESS MACHINES CORP	NEWMONT CORP	HASBRO INC	CONAGRA BRANDS INC
Company name	MICRON TECHNOLOGY INC	NUCOR CORP	HOME DEPOT INC	DARDEN RESTAURANTS INC
Company name	MOTOROLA SOLUTIONS INC	PPG INDUSTRIES INC	INTERPUBLIC GROUP OF COS	DISNEY (WALT) CO
Company name	NORTONLIFELOCK INC	VULCAN MATERIALS CO	KOHL'S CORP	GENERAL MILLS INC
Company name	PERKINELMER INC	WEYERHAEUSER CO	L BRANDS INC	HERSHEY CO
Company name	WATERS CORP		LEGGETT & PLATT INC	KELLOGG CO
Company name	XEROX HOLDINGS CORP		LOWE'S COS INC	KIMBERLY-CLARK CORP
Company name			MACY'S INC	KROGER CO
Company name			MARRIOTT INTL INC	MCCORMICK & CO INC
Company name			MASCO CORP	MCDONALD'S CORP
Company name			MATTEL INC	MCKESSON CORP
Company name			NIKE INC -CL B	MOLSON COORS BEVERAGE CO
Company name			NORDSTROM INC	PEPSICO INC
Company name			OMNICOM GROUP	PROCTER & GAMBLE CO
Company name			SHERWIN-WILLIAMS CO	SYSCO CORP
Company name			SNAP-ON INC	YUM BRANDS INC
Company name			STANLEY BLACK & DECKER INC	
Company name			TARGET CORP	
Company name			TIFFANY & CO	
Company name			TJX COS INC (THE)	
Company name			VF CORP	
Company name			WHIRLPOOL CORP	

Appendix 3 - Shapiro-Wilks pt.1

S&P Economic Sector Code	600		700		905	
Sector	TRANSPORTATION		UTILITIES		HEALTH CARE	
Amount of companies	6	Shapiro-Wilks pvalue	19	Shapiro-Wilks pvalue	18	Shapiro-Wilks pvalue
Company name	CSX CORP	0,188	AMEREN CORP	0,016	ABBOTT LABORATORIES	0,318
Company name	FEDEX CORP	0,821	AMERICAN ELECTRIC POWER CO	0,004	AMGEN INC	0,652
Company name	NORFOLK SOUTHERN CORP	0,083	CONSOLIDATED EDISON INC	0,422	ANTHEM INC	0,062
Company name	SOUTHWEST AIRLINES	0,716	DOMINION ENERGY INC	0,983	BAXTER INTERNATIONAL INC	0,048
Company name	UNION PACIFIC CORP	0,577	DTE ENERGY CO	0,025	BECTON DICKINSON & CO	0,183
Company name	UNITED PARCEL SERVICE INC	0,254	DUKE ENERGY CORP	0,567	BIOGEN INC	0,005
Company name			EDISON INTERNATIONAL	0,107	BOSTON SCIENTIFIC CORP	0,000
Company name			ENTERGY CORP	0,000	BRISTOL-MYERS SQUIBB CO	0,246
Company name			EXELON CORP	0,723	CIGNA CORP	0,006
Company name			FIRSTENERGY CORP	0,000	HUMANA INC	0,980
Company name			NEXTERA ENERGY INC	0,105	JOHNSON & JOHNSON	0,589
Company name			NISOURCE INC	0,000	LILLY (ELI) & CO	0,621
Company name			PINNACLE WEST CAPITAL CORP	0,004	MEDTRONIC PLC	0,518
Company name			PPL CORP	0,959	MERCK & CO	0,926
Company name			PUBLIC SERVICE ENTRP GRP INC	0,640	PFIZER INC	0,864
Company name			SEMPRA ENERGY	0,371	QUEST DIAGNOSTICS INC	0,169
Company name			SOUTHERN CO	0,015	STRYKER CORP	0,316
Company name			WILLIAMS COS INC	0,025	UNITEDHEALTH GROUP INC	0,001
Company name			XCEL ENERGY INC	0,107		

Appendix 4 - Shapiro-Wilks pt.2

S&P Economic Sector Code	925		935		940	
Sector	CAPITAL GOODS		ENERGY		TECHNOLOGY	
Amount of companies	23	Shapiro-Wilks pvalue	11	Shapiro-Wilks pvalue	15	Shapiro-Wilks pvalue
Company name	3M CO	0,693	ANADARKO PETROLEUM CORP	0,016	AGILENT TECHNOLOGIES INC	0,017
Company name	AVERY DENNISON CORP	0,246	APACHE CORP	0,003	APPLIED MATERIALS INC	0,031
Company name	BALL CORP	0,099	CHEVRON CORP	0,000	CORNING INC	0,682
Company name	BOEING CO	0,660	CONOCOPHILLIPS	0,165	DOVER CORP	0,949
Company name	CATERPILLAR INC	0,875	DEVON ENERGY CORP	0,000	EQUIFAX INC	0,000
Company name	CUMMINS INC	0,556	EOG RESOURCES INC	0,758	FISERV INC	0,983
Company name	DANAHER CORP	0,373	HALLIBURTON CO	0,363	GRAINGER (W W) INC	0,118
Company name	DEERE & CO	0,113	HESS CORP	0,152	HP INC	0,033
Company name	EATON CORP PLC	0,325	MARATHON OIL CORP	0,136	INTL BUSINESS MACHINES CORP	0,105
Company name	EMERSON ELECTRIC CO	0,005	OCCIDENTAL PETROLEUM CORP	0,044	MICRON TECHNOLOGY INC	0,002
Company name	FLUOR CORP	0,267	SCHLUMBERGER LTD	0,318	MOTOROLA SOLUTIONS INC	0,000
Company name	GENERAL DYNAMICS CORP	0,000			NORTONLIFELOCK INC	0,108
Company name	HONEYWELL INTERNATIONAL INC	0,979			PERKINELMER INC	0,290
Company name	ILLINOIS TOOL WORKS	0,186			WATERS CORP	0,803
Company name	JOHNSON CONTROLS INTL PLC	0,001			XEROX HOLDINGS CORP	0,119
Company name	LOCKHEED MARTIN CORP	0,934				
Company name	NORTHROP GRUMMAN CORP	0,433				
Company name	PACCAR INC	0,009				
Company name	PARKER-HANNIFIN CORP	0,481				
Company name	ROCKWELL AUTOMATION	0,033				
Company name	SEALED AIR CORP	0,139				
Company name	TEXTRON INC	0,013				
Company name	WASTE MANAGEMENT INC	0,383				

Appendix 5 - Shapiro-Wilks pt.3

S&P Economic Sector Code	970		976		978	
Sector	BASIC MATERIALS		CONSUMER CYCLICALS		CONSUMER STAPLES	
Amount of companies	13	Shapiro-Wilks pvalue	30	Shapiro-Wilks pvalue	24	Shapiro-Wilks pvalue
Company name	AIR PRODUCTS & CHEMICALS INC	0,383	AUTOZONE INC	0,034	ALTRIA GROUP INC	0,016
Company name	ARCHER-DANIELS-MIDLAND CO	0,880	BEST BUY CO INC	0,000	AMERISOURCEBERGEN CORP	0,035
Company name	EASTMAN CHEMICAL CO	0,113	BLOCK H & R INC	0,551	BROWN FORMAN CORP	0,049
Company name	ECOLAB INC	0,253	CARNIVAL CORPORATION & PLC	1,000	CAMPBELL SOUP CO	0,001
Company name	FREEPORT-MCMORAN INC	0,000	CINTAS CORP	0,010	CARDINAL HEALTH INC	0,000
Company name	INTL FLAVORS & FRAGRANCES	0,857	COSTCO WHOLESALE CORP	0,022	CLOROX CO/DE	0,003
Company name	INTL PAPER CO	0,047	GENUINE PARTS CO	0,479	COCA-COLA CO	0,210
Company name	LINDE PLC	0,013	HARLEY-DAVIDSON INC	0,004	COLGATE-PALMOLIVE CO	0,959
Company name	NEWMONT CORP	0,002	HASBRO INC	0,001	CONAGRA BRANDS INC	0,360
Company name	NUCOR CORP	0,445	HOME DEPOT INC	0,000	DARDEN RESTAURANTS INC	0,004
Company name	PPG INDUSTRIES INC	0,622	INTERPUBLIC GROUP OF COS	0,615	DISNEY (WALT) CO	0,644
Company name	VULCAN MATERIALS CO	0,389	KOHL'S CORP	0,705	GENERAL MILLS INC	0,190
Company name	WEYERHAEUSER CO	0,130	L BRANDS INC	0,713	HERSHEY CO	0,001
Company name			LEGGETT & PLATT INC	0,016	KELLOGG CO	0,020
Company name			LOWE'S COS INC	0,129	KIMBERLY-CLARK CORP	0,193
Company name			MACY'S INC	0,454	KROGER CO	0,236
Company name			MARRIOTT INTL INC	0,270	MCCORMICK & CO INC	0,568
Company name			MASCO CORP	0,000	MCDONALD'S CORP	0,022
Company name			MATTEL INC	0,200	MCKESSON CORP	0,000
Company name			NIKE INC -CL B	0,101	MOLSON COORS BEVERAGE CO	0,000
Company name			NORDSTROM INC	0,332	PEPSICO INC	0,045
Company name			OMNICOM GROUP	0,005	PROCTER & GAMBLE CO	0,333
Company name			SHERWIN-WILLIAMS CO	0,139	SYSCO CORP	0,986
Company name			SNAP-ON INC	0,059	YUM BRANDS INC	0,507
Company name			STANLEY BLACK & DECKER INC	0,039		
Company name			TARGET CORP	0,434		
Company name			TIFFANY & CO	0,211		
Company name			TJX COS INC (THE)	0,473		
Company name			VF CORP	0,961		
Company name			WHIRLPOOL CORP	0,205		

Appendix 6 - Conversion matrix[9]

Rating	PD-5-Year
AAA	0,15
AA+	0,32
AA	0,35
AA-	0,36
A+	0,43
A	0,48
A-	0,54
BBB+	0,97
BBB	1,36
BBB-	2,77
BB+	3,69
BB	6,17
BB-	9,27
B+	14,15
B	17,09
B-	25,43
CCC	46,06

[9] Data from S&P's 2018 annual corporate default study and rating transition report.

Appendix 7 - Numerical scheme[10]

Rating	Numerical Value
AAA	1
AA+	2
AA	3
AA-	4
A+	5
A	6
A-	7
BBB+	8
BBB	9
BBB-	10
BB+	11
BB	12
BB-	13
B+	14
B	15
B-	16
CCC	17

Appendix 8 - Damodaran equity risk premium

Year	Implied Premium (FCFE)	Implied Premium (FCFE with sustainable Payout)	Average Premium
2003	3,69%	4,74%	4,22%
2004	3,65%	4,86%	4,26%
2005	4,08%	5,22%	4,65%
2006	4,16%	6,12%	5,14%
2007	4,37%	4,59%	4,48%
2008	6,43%	6,92%	6,68%
2009	4,36%	4,64%	4,50%
2010	5,20%	6,09%	5,65%
2011	6,01%	8,34%	7,18%
2012	5,78%	7,30%	6,54%
2013	4,96%	4,99%	4,98%
2014	5,78%	5,48%	5,63%
2015	6,12%	5,16%	5,64%
2016	5,69%	4,50%	5,10%
2017	5,08%	4,75%	4,92%
2018	5,96%	5,55%	5,76%

[10] Source: Sajjad, Faiza. (2018). Credit Rating as a Mechanism for Capital Structure Optimization: Empirical Evidence from Panel Data Analysis. International Journal of Financial Studies. 6. 13. 10.3390/ijfs6010013.

Model implied equity volatility

The model implied equity volatility is found through the application of Ito's lemma

$$\frac{dPb}{dV} = -\frac{x}{V\left(\frac{V}{Vb}\right)^{x}}$$

$$\frac{dV_{solv}}{dV} = \frac{Vbx}{V\left(\frac{V}{Vb}\right)^{x}} + 1$$

$$\frac{dVint}{dV} = -\frac{Cx}{rV\left(\frac{V}{Vb}\right)^{x}}$$

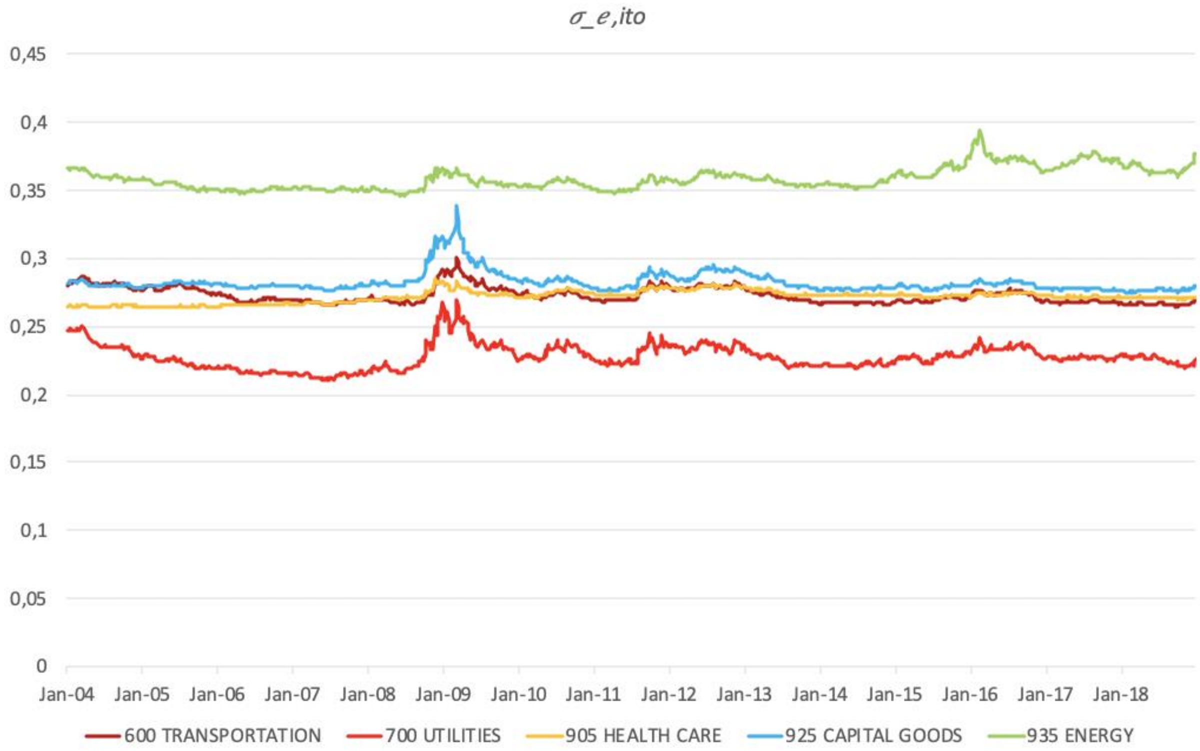
$$\frac{dEsolv}{dV} = (1 - t_{eff}) * (\frac{Vbx}{V\left(\frac{V}{Vb}\right)^{x}} + 1 + \frac{Cx}{rV\left(\frac{V}{Vb}\right)^{x}})$$

$$Sigma_e = \frac{dEsolv}{dV} * sig_a * \frac{V}{Equity}$$

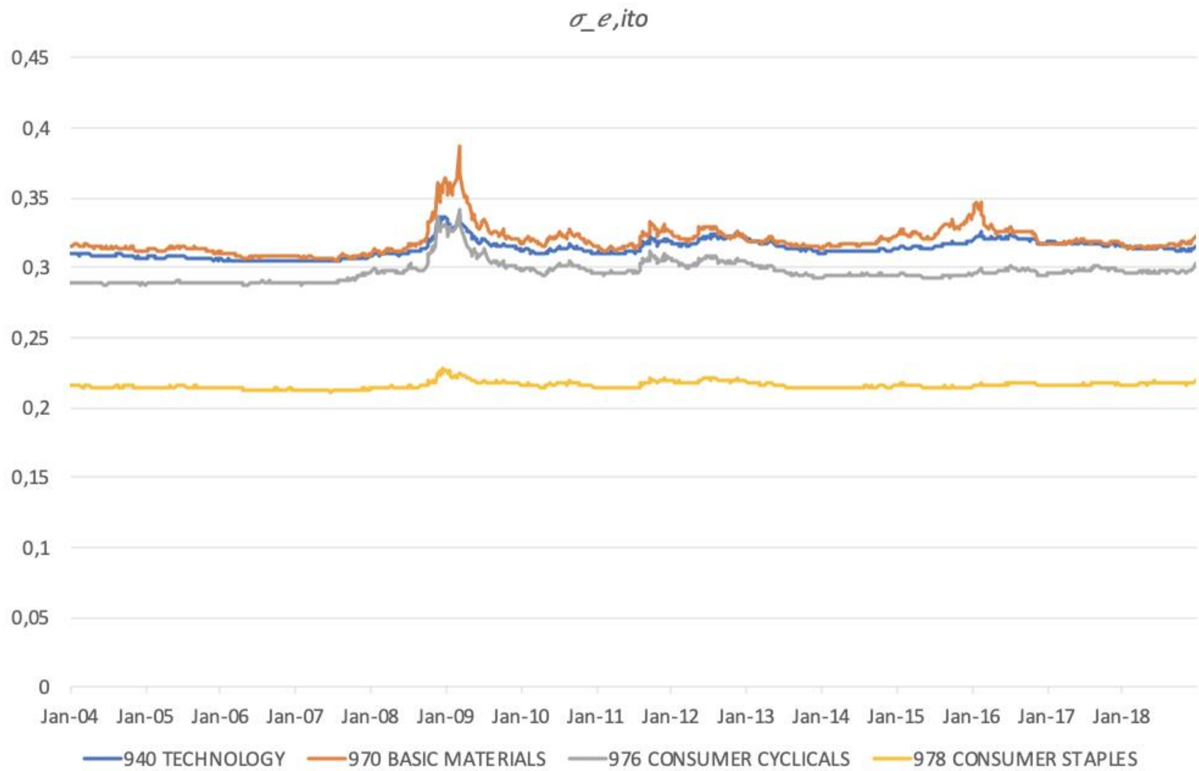
Figure 13(panel A and B) shows the timeseries of average model implied equity volatility per sector.

Figure 13 - Model implied equity volatility

Panel A



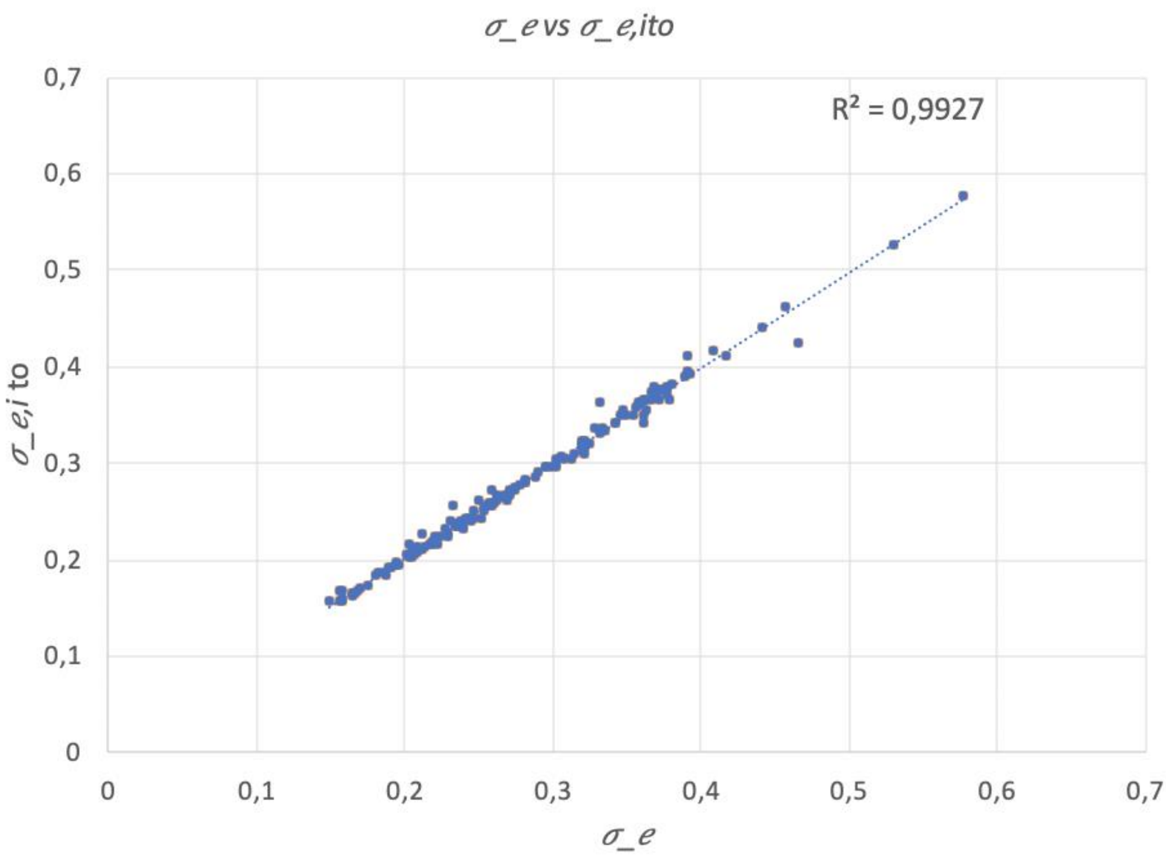
Panel B



In Figure 13 one can see that in times of crisis when the value of assets is closer to the barrier of default $\sigma_{e,ito}$ is high. Whereas in calmer times, $\sigma_{e,ito}$ is lower.

Even though the equity volatility varies across time, the average value should be very close to the one calculated based on log changes in equity. This comparison is shown in Figure 14.

Figure 14 - Empirical equity volatility vs. model implied equity volatility



As expected, the average equity volatility found through the application of Ito's lemma is able to explain close to all variation in the empirical equity volatility.

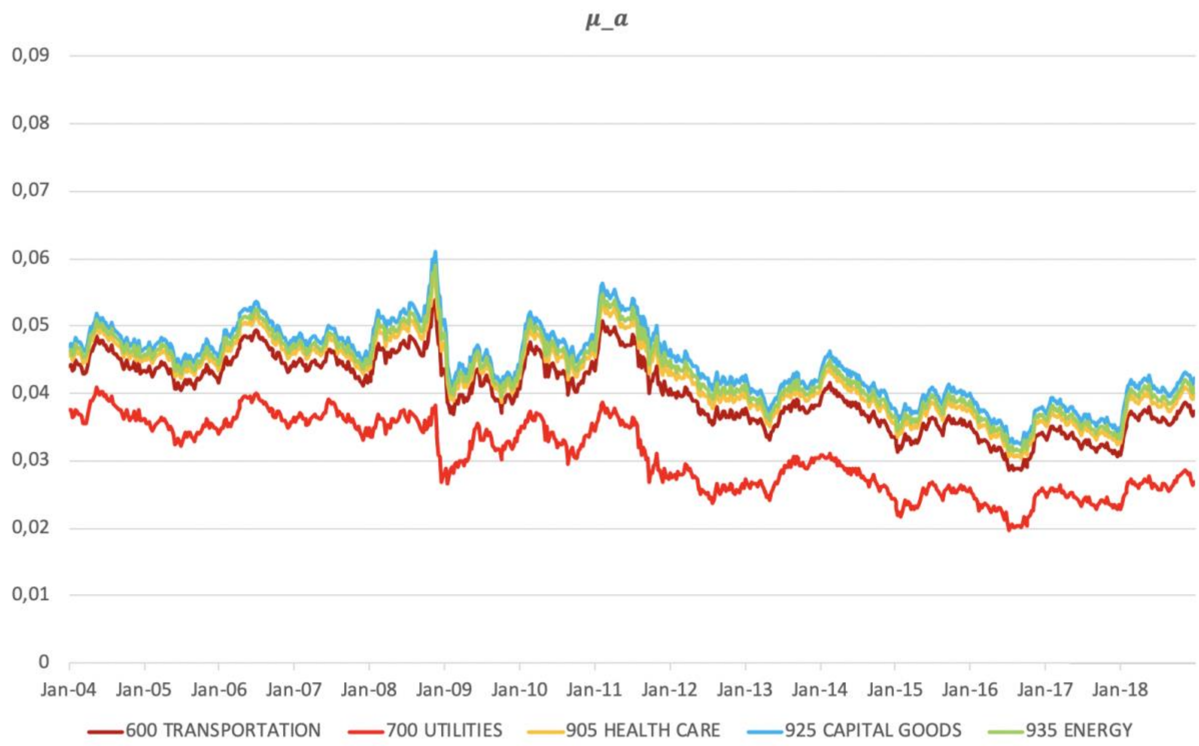
Appendix 10 - Return on assets

Return on assets - $\boldsymbol{\mu_a}$

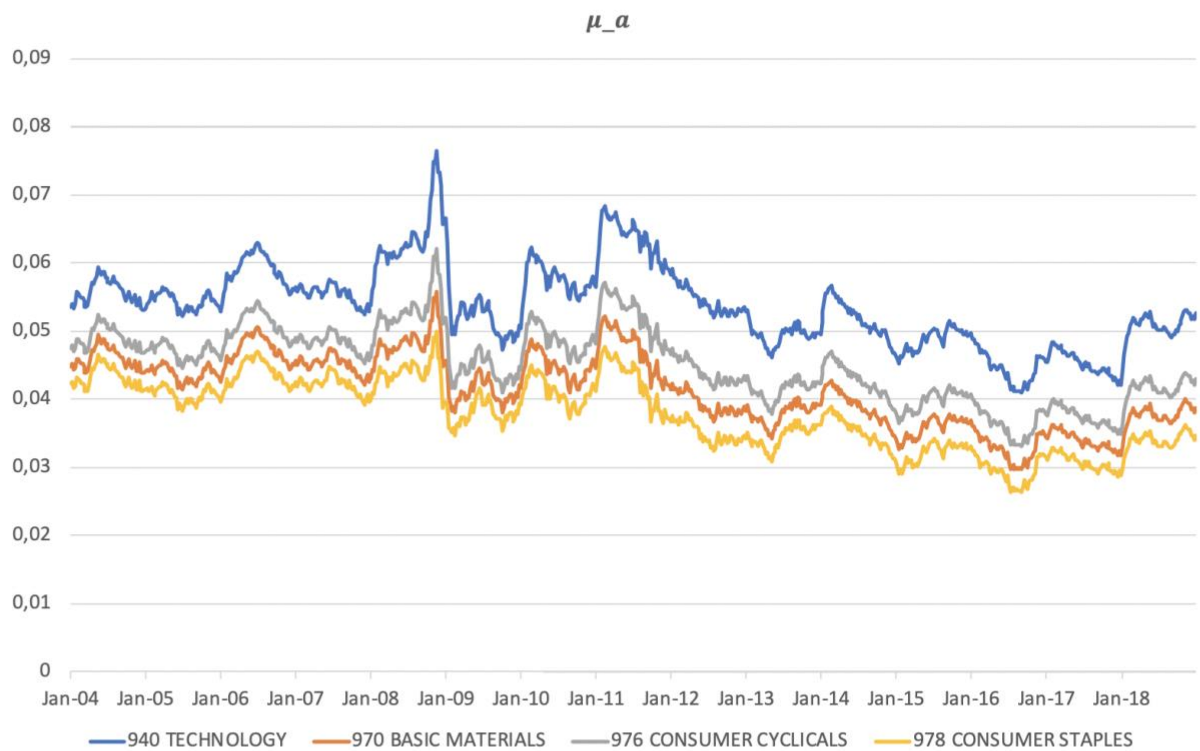
The return on assets is estimated as the risk-free rate plus the product of θ and σ_a. Since the risk-free rate is the same for all companies, the difference between the sectors is driven by θ and σ_a. This difference can easily be seen in Figure 15 (panel A and B)

Figure 15 - Return on assets

Panel A



Panel B



As the average asset volatility of the 9 sectors are close in promilitary to each other, the difference in levels are mainly cause by θ.

The average μ_a across the time series can be found in Table 22

Table 22 - Average return on assets

Return on assets					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
µ_a	4,01%	3,07%	4,23%	4,44%	4,32%
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	-
µ_a	5,38%	4,13%	4,51%	3,78%	-

Here one sees that the technology sector has a significantly higher average μ_a than the other sectors, whereas the utilities- and communication services sector has a markedly lower value. These results were expected in both the technology- and utilities sector.

All of these results are significantly lower than the return on equity, as the asset volatility is much lower than the empirical equity volatility. This issue is further analysed in Appendix 11.

Appendix 11 - Return on equity

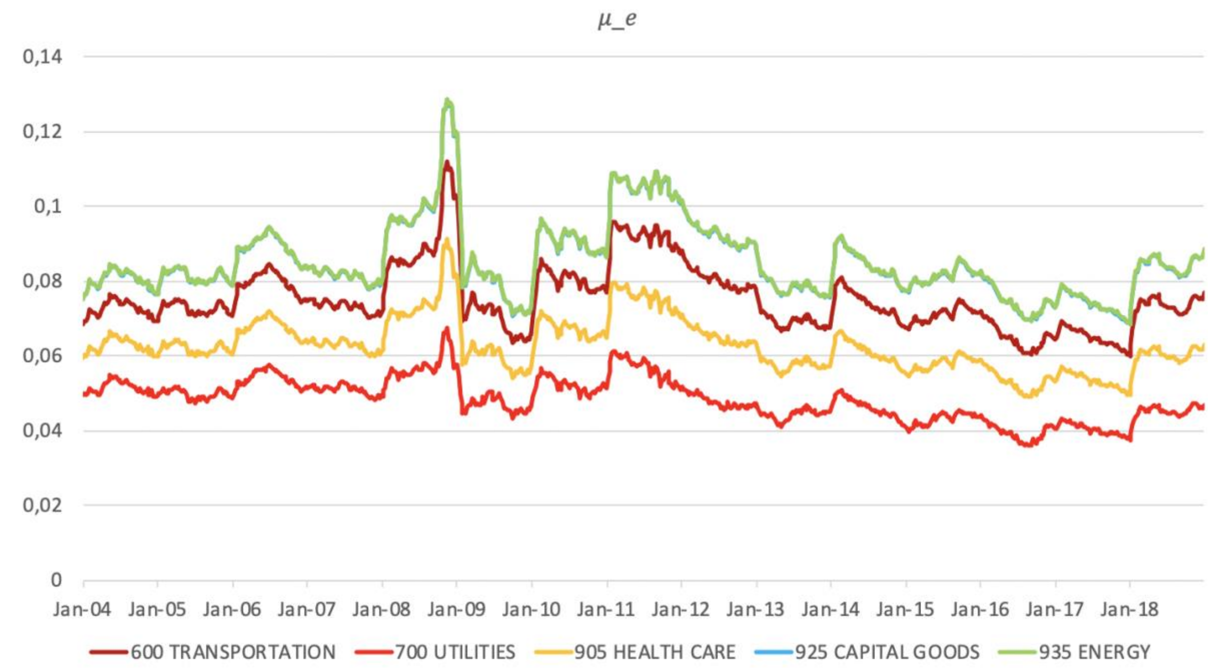
Return on equity - $\boldsymbol{\mu_e}$

The return on equity is computed according to Eq. (29), which simplifies to the expected return on equity in accordance to the capital asset pricing model. Here return on equity is the risk-free rate plus the product of β and EQRP. When computing μ_e it is important to remember that both risk-free rate and EQRP is a time series, whereas β is constant. Consequently, variations in μ_e is cause either by changes in the EQRP or the risk-free rate.

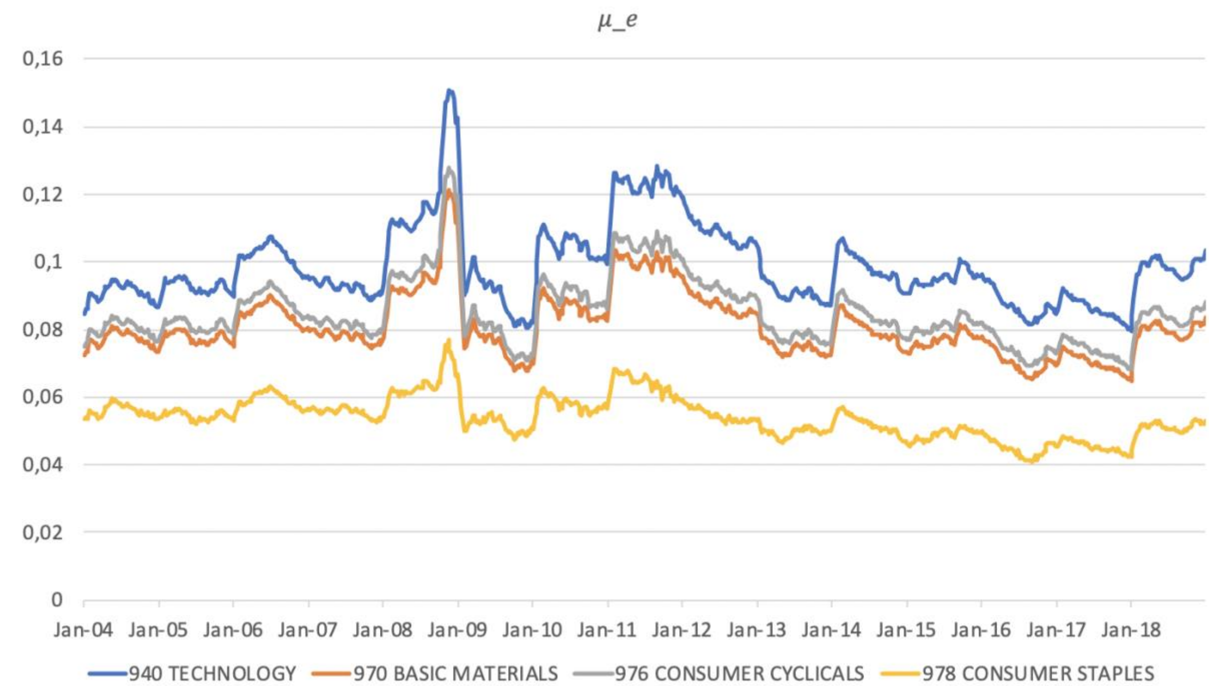
The evolution of μ_e for all sector can be seen in Figure 16(panel A and B). In Figure 16(panel A) the series of the capital goods- and energy sector lie on top of each other.

Figure 16 - Return on equity

Panel A



Panel B



Due to the constant β and a quite stable interest rate, all of the time series are dominated by the dynamics of the EQRP. As one can see the return on equity peaks at the end of 2008 during the financial crisis, in 2011 during the sovereign debt crisis in the EU and in 2014 during the oil crisis.

The average μ_e in the different sectors is found in Table 23.

Table 23 - Average return on equity

Return on equity					
Sector	TRANSPORTATION	UTILITIES	HEALTH CARE	CAPITAL GOODS	ENERGY
μ_e	7,53%	4,85%	6,28%	8,53%	8,54%
Sector	TECHNOLOGY	BASIC MATERIALS	CONSUMER CYCLICALS	CONSUMER STAPLES	-
μ_e	9,86%	8,10%	8,50%	5,40%	-

Since all sectors considered has the same EQRP and is subject to the same risk-free rate, the variations are cause singlehandedly by differences in β. As the technology sector is subject to more systematic risk, its return on equity is higher. Whereas the utilities sector due to its small β has a drastically lower return on equity.

Appendix 12 - Code

```
#GJL model functions

x_function <-function(rf, k, sig_a) {
  miu <- rf-k
  a <- sig_a^2/2
  b <- 2*rf*sig_a^2
  c <- (miu-a)^2
  d <- miu-a
  e <- d+sqrt(c+b)
  x <- e/sig_a^2
  return(x)
}
y_function <- function(rf, k, sig_a) {
  miu <- rf-k
  a <- sig_a^2/2
  b <- 2*rf*sig_a^2
  c <- (miu-a)^2
  d <- miu-a
  e <- d-sqrt(c+b)
```

```
  y <- e/sig_a^2
  return (y)
}
#Default Barrier Function
#found by invoking smooth pasting condition
v_b_function <- function( rf, k, sig_a, C) {
  l_d <- x_function(rf=rf, k=k, sig_a=sig_a) / (x_function(rf=rf, k=k, sig_a=sig_a)+1)
  V_b <- l_d*C*(1/rf)
  return (V_b)
}
#v_b_function(rf,k,sig_a,C=Intexp)

p_b_function <- function( v_a, rf, k, sig_a, C){
  R<- v_a/v_b_function(rf=rf, k=k, sig_a=sig_a, C=C)
  p_b <- R^(-1*x_function(rf=rf, k=k, sig_a=sig_a))
  return ((R>1)*p_b+(R<=1)*10^10)
  return (p_b)
}
#p_b_function(v_a,rf,k,sig_a,C)

v_int_function <- function( v_a, rf, k, sig_a, C ){
  v_int <- (1-p_b_function( v_a=v_a, rf=rf, k=k, sig_a=sig_a, C=C ))*C/rf
  return(v_int)
}

v_solv_function <- function(v_a, rf, k, sig_a, C){
  v_solv <- v_a - v_b_function( rf=rf, k=k, sig_a=sig_a, C=C )*p_b_function( v_a=v_a, rf=rf,
k=k, sig_a=sig_a, C=C)
  return(v_solv)
}

e_function <- function( v_a, rf, k, sig_a, C, TaxCorp, TaxDiv){
  Tx_eff <- (1-TaxCorp)*(1-TaxDiv)
```

```
  return(Tx_eff*(v_solv_function( v_a=v_a, rf=rf, k=k, sig_a=sig_a, C=C)-v_int_function (
v_a=v_a, rf=rf, k=k, sig_a=sig_a, C=C )))
}

#Data, CSX
#Data, FDX
#Data, NSC
#Data, LUV
#Data, UNP
#Data, UPS
xpto<-as.matrix(UPS)

Time   <- xpto[,0]
date    <- as.numeric(xpto[,1])
EBIT     <- as.numeric(xpto[,2])
Equity  <- as.numeric(xpto[,3])
Intexp     <- as.numeric(xpto[,4])
RF      <- as.numeric(xpto[,5])*0.01
rf      <- as.numeric(xpto[,5])*0.01

FindV <- function(x, k, sig_a, TimeM) {
  ModelEquity<-e_function(v_a=x, rf=RF[TimeM], k=k, sig_a, C=Intexp[TimeM],
TaxCorp=0.2, TaxDiv=0.2)
  #print(ModelEquity)
  return(Equity[TimeM]-(ModelEquity>0 & ModelEquity<x)*ModelEquity)
}

#Finds the project value that matches equity value
Vfunction <- function(k, sig_a){

BBsolve(par=v_b_function(rf=RF,k=k,sig_a=sig_a,C=Intexp)+100000,fn=FindV,k=k,sig_a=
sig_a,1:783)$par
}
#Vfunction(k,sig_a)
```

```
#Finds sig_a1
FindAssetVol <- function (k, Start_sig_a) {
  Error <- 10^10
  while (Error > 0.00001){
    RecoveryAssetVec_1 <- Vfunction(k, sig_a=Start_sig_a)
    log_ret <- diff(log(RecoveryAssetVec_1),lag=1)
    sig_a1 <-  sd(log_ret)*sqrt(52)
    Error<- abs(Start_sig_a-sig_a1)
    Start_sig_a <- sig_a1
  }
  return(sig_a1)
}

# K as average of  (EBIT)/AssetVector
Findk<- function(Start_k, sig_a){
  Error <- 10^10
  for (i in 1:783)
    while (Error>0.00001){
      RecoveryAssetVec_1 <- Vfunction(k=Start_k, sig_a)
      k_a1 <- (sum(EBIT[1:783]))/sum(RecoveryAssetVec_1)
      Error<- abs(Start_k-k_a1)
      Start_k <- k_a1
    }
  return(k_a1)
  return(RecoveryAssetVec_1)
}

FindEstimates<- function(Start_k, Start_sig){
  Error <- 10^10
  for (i in 1:783)
    while (Error>0.00001){
      sig_a1 <-FindAssetVol(k= Start_k , Start_sig_a= Start_sig )
      RecoveryAssetVec_1 <- Vfunction(k= Start_k,sig_a= Start_sig )
```

```
      k_a1 <- (sum(EBIT[1:783]))/sum(RecoveryAssetVec_1)
      Error<- abs(Start_k-k_a1)
      Start_k <- k_a1
    }
  return(sig_a1)

}

FindEstimates2<- function(Start_k, Start_sig){
  Error <- 10^10
  for (i in 1:783)
    while (Error>0.00001){
      sig_a1 <-FindAssetVol(k= Start_k , Start_sig_a= Start_sig )
      RecoveryAssetVec_1 <- Vfunction(k= Start_k,sig_a= Start_sig )
      k_a1 <- (sum(EBIT[1:783]))/sum(RecoveryAssetVec_1)    #changed formula do find
delta/assets aka k
      Error<- abs(Start_k-k_a1)
      Start_k <- k_a1
    }
  return(k_a1)
}

#run the iterative approach at once and safe the values in sig_a & k
Sig_K <-FindEstimates(Start_k=0.05,Start_sig=0.2)
sig_a <- Sig_K[1]

Sig_K1 <-FindEstimates2(Start_k=0.05,Start_sig=0.2)
k    <- Sig_K1[1]

# Define Market price risk
EQRP <- as.numeric(xpto[,6]) #in percentage
beta <- mean(as.numeric(xpto[,7]))
miu_e<- RF+beta*(EQRP)
```

```
UPS_miu_e<-miu_e

#cleaned empirical achieved standard deviation of Equity
#cleaning the data from outliers
#empirically computing the equity standard deviation
sig_e1      <- sd(diff(log(Equity[1:783])))
all_outliers<- diff(log(Equity[1:783]))
limits <- 3*sig_e1
all_outliers<- all_outliers[!(all_outliers> limits)]
all_outliers<- all_outliers[!(all_outliers< -limits)]
b           <- boxplot(all_outliers)
sig_e1      <-sd(all_outliers)*sqrt(52)

UPS_sig_e1<-sig_e1

Mk_Rsk <- function(EQRP,beta,sig_e1){
  Mk_Rsk    <- (beta*EQRP)/sig_e1
  return(Mk_Rsk)
}
Mk_Rsk <- Mk_Rsk(EQRP,beta,sig_e1)

UPS_mkrsk<-Mk_Rsk

# Sigma as standard deviation of log returns
Findmiu_a <- function(beta, EQRP, sig_a,Mk_Rsk){
  miu_a     <- RF+Mk_Rsk*sig_a
  return(miu_a)
}
miu_a <- Findmiu_a(beta, EQRP, sig_a,Mk_Rsk)

UPS_miu_a<-miu_a

#Find miu_d => drift of the project/process
miu_d_function <- function(beta,EQRP,sig_e1,sig_a,k,Mk_Rsk){
```

```
  miu_d     <- RF+Mk_Rsk*sig_a-k
  return(miu_d)
}
miu_d <- miu_d_function(beta, EQRP, sig_e1,sig_a,k,Mk_Rsk)
#returns miu_d's time series

UPS_miu_d<-miu_d

#Probability of default function
##scope of bankruptcy is DELTA_T

RecoveryAssetVec_1 <- Vfunction(k,sig_a)
Barrier<- 1:783
for(i in 1:783){
  Barrier[i]<-v_b_function(rf=RF[i], k, sig_a, C=Intexp[i])
}
V_b_Ratio <- Barrier / RecoveryAssetVec_1      #Barrier => v_b_function => optimal level
of default //// Recoveryassetvector => Asset value at time T
max(Barrier/RecoveryAssetVec_1)

# Time Series of PDs
#Gives distance to distress (DD) at each moment in time
# DD = how many standard deviations away from default
DD<-function(k, sig_a, miu_a, years, Time){
  Delta_t <- 1/52
  TimeT   <- 52*years
  a <- (miu_a - k - (sig_a^2/2))
  b <- TimeT*Delta_t
  c <- log(RecoveryAssetVec_1[Time]/v_b_function(rf=RF[Time], k=k, sig_a=sig_a,
C=Intexp[Time]))
  d <- sig_a*sqrt(b)
  e<- (c+a*b)/d
}
#Gives Probability of V being below V_B at time T (ignores first passage time)
```

```
AuxProbability<-function(k, sig_a, miu_a, years, Time){
  pnorm(-DD(k, sig_a, miu_a, years, Time))
}
#Gives the probability of defaulting in "years"-years at time "Time"
PDfunc <- function(k, sig_a, miu_a, years, Time)
{
  Delta_t <- 1/52
  TimeT   <- 52*years
  a <- (miu_a - k - (sig_a^2/2))
  b <- TimeT*Delta_t
  c <- log(v_b_function(rf=RF[Time], k=k, sig_a=sig_a,
C=Intexp[Time])/RecoveryAssetVec_1[Time])
  d <- sig_a*sqrt(b)
  e <- pnorm(((a*b)-c)/d)
  f <- exp((2/sig_a^2)*a*c)*pnorm(((a*b)+c)/d)
  g <- e-f
  return(1-g)
}

PD_Series <- 1:783
DD_Series <- 1:783
PD_Series_aux1 <- 1:783

#Computes output
for(i in 1:783){
  DD_Series[i]<-DD(k, sig_a, miu_a=miu_a[i], years=5,Time=i)
  PD_Series_aux1[i]<-AuxProbability(k, sig_a, miu_a[i], years=5,Time=i)
  PD_Series[i] <- PDfunc( k, sig_a, miu_a[i], years=5, Time =i )
}
UPSPD_Series<-PD_Series
UPSDD_Series<-DD_Series
UPSPD_Series_aux1<-PD_Series_aux1
UPS_V_b_Ratio<-V_b_Ratio
UPS_Sig<-sig_a
```

```
UPS_k<-k

#Creating all the necessary new functions in order to compute the timeseries of estimated
equity volatilities. All functions below is like their previously defined functions, but now the
derivative with respect to asset value.

# The derivative of the probability of default with respect to asset value.
p_b_deriv_function <- function( v_a, rf, k, sig_a, C){
  R<- v_a/v_b_function(rf=rf, k=k, sig_a=sig_a, C=C)
  a<- x_function(rf=rf, k=k, sig_a=sig_a)
  b<- (v_a*(R^(1*x_function(rf=rf, k=k, sig_a=sig_a))))
  c<- -(a/b)
  return ((R>1)*c+(R<=1)*10^10)
  return (c)
}

# The derivative of V_int with respect to asset value.
v_int_deriv_function <- function( v_a, rf, k, sig_a, C ){
  v_int_deriv <- (p_b_deriv_function( v_a=v_a, rf=rf, k=k, sig_a=sig_a, C=C ))*C/rf
  return(v_int_deriv)
}

# The derivative of V_solv default with respect to asset value.
v_solv_deriv_function <- function(v_a, rf, k, sig_a, C){
  a<-(-p_b_deriv_function( v_a=v_a, rf=rf, k=k, sig_a=sig_a, C=C ))
  b<- v_b_function(rf=rf, k=k, sig_a=sig_a, C=C)
  v_solv_deriv<- (a*b)+1
  return(v_solv_deriv)
}

# The derivative of equity with respect to asset value.
e_deriv_function <- function( v_a, rf, k, sig_a, C, TaxCorp, TaxDiv){
  Tx_eff <- (1-TaxCorp)*(1-TaxDiv)
```

```
  return(Tx_eff*(v_solv_deriv_function( v_a=v_a, rf=rf, k=k, sig_a=sig_a, C=C)-
v_int_deriv_function ( v_a=v_a, rf=rf, k=k, sig_a=sig_a, C=C )))
}

# Computing the time series of equity volatility. As the product of the derivative of equity
with respect to asset value, asset volatility and asset value over equity.

Sigma_E<- 1:783
for(i in 1:783){
  Sigma_E[i]<- e_deriv_function(v_a=RecoveryAssetVec_1[i], rf=RF[i], k=k, sig_a,
C=Intexp[i], TaxCorp=0.2, TaxDiv=0.2)*sig_a*(RecoveryAssetVec_1[i]/Equity[i])
}

#Firm specific output.
Output2<-do.call(rbind.data.frame,Map('c', PD_Series, DD_Series, V_b_Ratio,
RecoveryAssetVec_1, sig_a, k, sig_e1, Sigma_E, miu_a. miu_d, miu_e,Mk_Rsk ))
write.table(Output2, file = "UPS_specific Important.csv",  sep=";", dec = ",")

#Sector output of core data for easier treatment.
Output2<-do.call(rbind.data.frame,Map('c', CSXPD_Series, FDXPD_Series, NSCPD_Series,
LUVPD_Series, UNPPD_Series, UPSPD_Series, CSXDD_Series, FDXDD_Series,
NSCDD_Series, LUVDD_Series, UNPDD_Series, UPSDD_Series, CSX_V_b_Ratio,
FDX_V_b_Ratio, NSC_V_b_Ratio, LUV_V_b_Ratio, UNP_V_b_Ratio, UPS_V_b_Ratio))
write.table(Output2, file = "600 Important.csv",  sep=";", dec = ",")
```