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THE OPTIMALITY OF CONTINUOUS COVER VERSUS ROTATION FORESTRY IN THE PRESENCE OF FIRE RISK

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Abstract

This paper investigates the optimal harvesting decisions of a fire-exposed forest stand. An economic model is used to determine analytical continuous-time solutions for the optimal schedule of thinnings and clearcuts. We offer new results by proposing that, while thinnings should be anticipated in the presence of fire hazard, forest rotations do not necessarily need to be shortened and may, under certain conditions, be lengthened. In fact, for sufficiently high levels of risk, clearcut forestry can be suboptimal, leading to the adoption of a continuous cover regime.

Keywords: Fire hazard; Optimal rotation; Optimal thinning; Salvage harvesting.

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1 Introduction

When assessing the vulnerability of economic sectors to future climate scenarios, forestry is projected to be among the most heavily impacted activities. Inducted by global climate change, the incidence of extreme events such as wildfires is predicted to intensify (FAO, 2018). As one may expect, more devastating and frequent destructive events can be prohibitive for long term investments such as forestry. Therefore, for the exploitation of forests to be economically sustainable in the future, the adaptation to new fire regimes requires more sophisticated fire-resilient forestry practices. In this context, the integration of fire hazard in economic optimization models is of utmost importance.

The first studies to incorporate destruction by fire adopted discrete-time economic models to analyze the optimal rotation of forest stands (Martell, 1980; Routledge, 1980). In his seminal work, Reed (1984) was the first to introduce fire risk in a continuous-time Faustmann setting by proposing a stand survival model to optimize the rotation of an even-aged forest stand. Assuming that fires occur exogenously, Reed illustrated that the effect of fire hazard on the optimal clearcut age is equivalent to adding a premium to the discount rate in the Faustmann formula. Under this condition, increased probability of destruction should lead to shorter optimal rotations.

Many authors have extended Reed's model to analyze different issues. Englin et al. (2000) studied the effects of hazard on the rotation of a stand when non-timber forest amenities were taken into account. Reed (1987) considered the implications of endogenous fire hazard on the joint determination of optimal rotation and fire protection practices. Reed and Apaloo (1991) examined the effect of fire hazard on the optimal harvesting when the possibility of commercial thinning is introduced but salvaged timber has no value. Taking thinnings into account as a protection measure and not for commercial purposes, Thorsen and Helles (1998) evaluated the optimal thinnings and clearcut in a single-rotation forest subject to the risk of destruction by windthrow. Amacher et al. (2005) studied the thinning timing and intensity as a fuel treatment measure that positively impacts the salvaged timber price. In general, most of these studies suggest the anticipation of clearcuts when exposed to risk of catastrophic destruction. Being the exception to this tendency Amacher et al. (2005) that showed that, under certain fuel treatment conditions, it may be optimal to postpone clearcuts.

Many of these authors stressed the relative complexity that is involved in finding analytical solutions to such problems. As a result, the literature on the economics of fire-exposed forests has focused on numerical simulations and the analytical properties of the optimal solutions remains open for research debate. Additionally, literature on fire-exposed forestry usually disregards thinnings as an instrument to anticipate timber revenues, but rather as a fuel management practice. However, the possibility to anticipate revenues in a forest stand susceptible to involuntary destruction can be decisive. Furthermore, by restricting research to even-aged stands, these studies may unintentionally promote suboptimal management of naturally regenerating forest stands.

Forestry economics has predominantly been focused on the decision about when to clearcut a stand of trees. With the introduction of forest bioeconomic models that account for partial harvests (thinnings) into the growth dynamics, the issue became not only to determine the economically optimal harvest timing but also its intensity. One of these, proposed by Kilkki and Väisänen (1969), defined a discrete-time setting in which stand volume growth is given by a function of stand age, total volume and the rate of harvested timber. This model was later adapted to continuous time by Clark (1976) in order to derive singular-path solutions for the optimal thinning and to determine optimal rotations. The properties of the optimal thinning and clearcut schedules under Clark's model can be explored by making use of optimal control techniques (Cawrse et al., 1984; Betters et al., 1991). This specifications, however, have been limited to even-aged stand modeling. In order to study uneven-aged stands in a Faustmann setting, Tahvonen (2016) respecified Clark's model to take into account natural regeneration. By adding this feature, the possibility of continuous cover forestry solutions is taken into consideration. Assmuth and Tahvonen (2018) extended the work of Tahvonen (2016) to include carbon pricing into the optimal harvesting of uneven-aged stands.

Building upon these previous works, I developed a model to investigate the effects of fire hazard on the optimal harvesting decisions of a naturally regenerating stand. To the best of our knowledge, this is the first study to provide analytical results for the thinning and rotation schedules of a fire-exposed uneven-aged stand in a Faustmann setting. By taking salvage harvesting into account as well as the possibility to collect revenues through commercial thinning, this study adds to the economic literature of forestry under risk of destruction by suggesting that, when exposed to higher hazard, the forest manager may optimally postpone clearcuts and anticipate revenues through thinnings. Ultimately, for certain levels of fire hazard it may even become optimal to never clearcut the stand and thereby rely exclusively on thinnings and salvage harvests as sources of income. In this respect, this study emphasizes the conditions under which continuous cover forestry is optimal.

2 An economic model for a fire-exposed forest stand

2.1 The model

Let the stand volume be represented by $x(t)$ and the volume harvested through thinning operations by $h(t)$. Volume is expected to grow according to the differential equation specified by Clark (1976):

$$\dot{x}(t) = g(t)f[x(t)] - h(t). \tag{1}$$

Where $g(t)$ and $f[x(t)]$ represent age- and volume-dependent growth, respectively. Suppose:

$$g(0) > 0, g'(t)|_{t>0} < 0, \lim_{t\to\infty} g(t) \to \tilde{g} > 0, \tag{Ass. 1}$$

$$f(0) \geq 0, f(\bar{x}) = 0; f''(x) < 0; f'(\hat{x}) = 0, 0 < \hat{x} < \bar{x}. \tag{Ass. 2}$$

Following Tahvonen (2016), this forest growth model imposes the existence of an uneven-aged forest by setting that $g(t) \to \tilde{g} > 0$ as stand age tends to infinity. Hence, with natural regeneration, undisturbed stand growth ($g(t)f[x(t)]$) is assumed to remain strictly positive even for very old stands unless volume reaches the site maximum capacity ($\bar{x}$) where $f(\bar{x}) = 0$.

The occurrence of fires is assumed to be exogenous to the stand characteristics (Reed, 1984) and, in this way, it is characterized by a time-independent Poisson process with an average number of fires per year, denoted by the constant parameter λ. The time until stand destruction is therefore a random variable denoted by X that follows an exponential distribution. Thus:

$$\begin{aligned}
&\rho(t) = \Pr(X = t) = \lambda e^{-\lambda t}, \\
&F(t) = \Pr(X < t) = \textstyle\int_0^t \rho(\tau) d\tau = 1 - e^{-\lambda t}, \\
&S(t) = \Pr(X \geq t) = 1 - F(t) = e^{-\lambda t}, \\
&\Pr(X = t | X \geq t) = \tfrac{\rho(t)}{S(t)} = \lambda.
\end{aligned} \tag{Ass. 3}$$

Where $\rho(t)$ denotes the p.d.f. of the time between successive fires (X), $F(t)$ denotes the c.d.f. and $S(t)$ represents the survival function. Hence, the so-called "hazard function", i.e., the probability of a fire event at time t conditional on not having yet burned until t is constant and corresponds to the average yearly rate of fire occurrence λ.

Let P represent the stumpage price per unit of volume net of harvesting costs which is assumed to be the same either it came from a full (clearcut) or a partial harvest (thinning). After a fire has occurred, the burnt stand volume is immediately sold. However, the commercial uses for the salvaged timber are limited and usually depend on various factors such as the fire severity or the stand characteristics. For simplicity, assume that the salvage price net of harvesting costs, P_{fire}, is a proportion of P and is endogenous to stand volume, thus:

$$0 \leq P_{fire}(x) \leq P, \forall x \in [0, \bar{x}] \tag{Ass. 4}$$

This study analyzes three different salvage harvesting scenarios: constant salvage price ($P'_{fire}(x) = 0$), increasing salvage price ($P'_{fire}(x) > 0$) and decreasing salvage price ($P'_{fire}(x) < 0$).

The re-establishment cost after destruction is represented by c and is assumed to be equal whether the stand was destroyed by a fire or a clearcut. The instantaneous rate of discounting is denoted by δ and the clearcut age is represented by T. The net present value of one rotation depends on whether the cause of stand destruction is a fire ($X < T$) or a clearcut harvest ($X = T$) and is given by:

$$\begin{cases} \int_0^X Ph(t)e^{-\delta t}dt + \{P_{fire}[x(X)]x(X) - c\}\, e^{-\delta X} & , \quad X < T \\ \int_0^T Ph(t)e^{-\delta t}dt + [Px(T) - c]\, e^{-\delta T} & , \quad X = T \end{cases} \tag{2}$$

Since at the time of clearcut a voluntary stand destruction is imposed, the distribution of the random variable X described in (Ass. 3) becomes truncated at $t = T$. The expected present value of net revenues earned over one cycle, π, is obtained by integrating the discounted net revenues with respect to the distribution of time until stand destruction.

$$\begin{aligned} \pi = \; & \int_0^T \rho(X)(\int_0^X Ph(t)e^{-\delta t}dt + \{P_{fire}[x(X)]x(X) - c\}\, e^{-\delta X})dX \\ & + S(T)\{\int_0^T Ph(t)e^{-\delta t}dt + [Px(T) - c]\, e^{-\delta T}\}. \end{aligned} \tag{3}$$

The land expectation value is given by the expected present value of net revenues earned over an infinite number of cycles:

$$J = \sum_{n=1}^{\infty} E[e^{-\delta(X_1+X_2+\ldots X_{n-1})}]\pi = \sum_{n=1}^{\infty}\prod_{i=1}^{n-1} E(e^{-\delta X_i})\pi = \frac{\pi}{1 - E(e^{-\delta X_i})}. \tag{4}$$

Where X_i corresponds to the i-th stand destruction. From the distribution of X, $E(e^{-\delta X_i})$ can be expressed as $\int_0^T \rho(X)e^{-\delta X}dX + S(T)e^{-\delta T}$ which is equal to $1 - \delta\int_0^T S(t)e^{-\delta t}dt$ after integration by parts. Thus, the land expectation value, J, can be simplified to:

$$J = \frac{\pi(T)}{\delta\int_0^T S(t)e^{-\delta t}dt}. \tag{5}$$

Thus, the optimization problem can be stated as:

$$\max_{\{h(t),T\}} J = \frac{\pi(T)}{\delta \int_0^T S(t)e^{-\delta t}d} =$$
$$= \frac{\int_0^T \rho(X)(\int_0^X Ph(t)e^{-\delta t}dt + \left\{P_{fire}[x(X)]x(X)-c\right\}e^{-\delta X})dX + S(T)\left\{\int_0^T Ph(t)e^{-\delta t}dt + [Px(T)-c]\,e^{-\delta T}\right\}}{\delta \int_0^T S(t)e^{-\delta t}dt} \tag{6}$$

subject to (1), $x(0) = 0$, $h \in [0, h_{MAX}]$, $T \geq 0$ and $x(T) \geq 0$.

2.2 Necessary conditions

The optimal harvesting problem can be solved in two steps. First, we solve for the optimal thinning schedule for a given T. Then, given the optimal thinning function, the optimal rotation, T, can be derived by maximizing the land expectation value. Choosing a finite T implies clearcut forestry while infinite T implies continuous cover forestry.

After some simplification (Appendix A), π can be expressed as:

$$\begin{aligned} \pi = \;& -c + S(T)\{P - P_{fire}[x(T)]\}x(T)e^{-\delta T} + \int_0^T S(t)(P - P_{fire}[x(t)] \\ & -P'_{fire}[x(t)]x(t)\}h(t) + \{P_{fire}[x(t)] + P'_{fire}[x(t)]x(t)\}g(t)f[x(t)] \\ & -\delta\{P_{fire}[x(t)]x(t) - c\})e^{-\delta t}dt. \end{aligned} \tag{7}$$

The problem of optimizing thinning can now be handled as an optimal control problem with a state variable $x(t)$ and a control variable $h(t)$. The current-value costate variable is defined by $\varphi(t)$ and the current-value Hamiltonian is given by $H = S[(P - P_{fire} - P'_{fire}x)h + (P_{fire} + P'_{fire}x)gf - \delta(P_{fire}x - c)] + \varphi(gf - h)$. Further consider $\psi(T) = -c + S(T)\{P - P_{fire}[x(T)]\}x(T)e^{-\delta T}$ as the scrap value function. The Pontryagin Maximum Principle sets the necessary conditions for an optimality candidate $[x^*(t), h^*(t)]$ (Sydsæter et al., 2005). After some simplification (Appendix B), the conditions read as:

$$
\begin{aligned}
&S(P - P_{fire} - P'_{fire}x) - \varphi < 0 \quad \Rightarrow \quad h = 0, \\
&S(P - P_{fire} - P'_{fire}x) - \varphi = 0 \quad \Rightarrow \quad h \in [0, h_{MAX}], \\
&S(P - P_{fire} - P'_{fire}x) - \varphi > 0 \quad \Rightarrow \quad h = h_{MAX},
\end{aligned} \tag{8}
$$

$$
\varphi' = \varphi(\delta - gf') - S[P''_{fire}\dot{x}x + P'_{fire}x(gf' - \delta) + 2P'_{fire}\dot{x} + P_{fire}(gf' - \delta)], \tag{9}
$$

$$
[\varphi(T) - S(T)\{P - P_{fire}[x(T)] - P'_{fire}[x(T)]x(T)\}]x(T) = 0. \tag{10}
$$

Once the optimal thinning schedule is determined, the optimal rotation length, T, should maximize the land expectation value. Hence, by differentiating J with respect to T (Appendix B), the condition for the optimal rotation is given by:

$$
y(T) \equiv Pg(T)f[x(T)] - \lambda\{P - P_{fire}[x(T)]\}x(T) - \delta[Px(T) - c + J] = 0. \tag{11}
$$

2.3 Sufficient conditions

For the optimal control candidate to solve the problem, the sufficiency theorem of Arrow (Sydsæter et al., 2005, p. 331) states that the maximized current-value Hamiltonian should be concave in x for every $t \in [0, T]$ in an end constrained optimal control problem. When P_{fire} is constant ($P'_{fire} = 0$), $\frac{\partial^2 H}{\partial x^2} = (SP_{fire} + \varphi)gf''$. From conditions (8) and (10), the optimality candidate satisfies $S(P - P_{fire} - P'_{fire}x) - \varphi \leq 0$ for every $t \in [0, T]$, which implies that $SP_{fire} + \varphi \geq SP \geq 0$. Since $gf'' < 0$, then $\frac{\partial^2 H}{\partial x^2} \leq 0$. However, for volume-dependent salvage prices, it is impossible to guarantee the sufficiency of the solution in advance and, therefore, the concavity of the maximized Hamiltonian should be evaluated case-by-case.

Regarding the sufficiency of the optimal rotation length candidate, since $y'(T)|_{y(T)=0} < 0$ (Appendix B), any finite candidate T that solves $y(T) = 0$ is unique and therefore the global optimum.

3 Results

3.1 Optimal thinning

Differentiating $S(P - P_{fire} - P'_{fire}x) - \varphi = 0$ with respect to t leads to:

$$S'(P - P_{fire}) - SP'_{fire}\dot{x} - S'P'_{fire}x - SP''_{fire}\dot{x}x - SP'_{fire}\dot{x} - \varphi' = 0. \tag{12}$$

Substituting equation (9), it can be simplified to:

$$\varphi = \frac{S'(P - P_{fire} - P'_{fire}x)}{\delta - gf'} - SP'_{fire}x - SP_{fire}. \tag{13}$$

Substituting back in equation (8) and simplifying yields:

$$Pgf' - \lambda(P - P_{fire}) + \lambda P'_{fire}x = P\delta. \tag{14}$$

Differentiating (14) with respect to time gives $Pg'f' + Pgf''\dot{x} + \lambda P''_{fire}\dot{x}x + 2\lambda P'_{fire}\dot{x} = 0$. Applying equation (1) and simplifying yields:

$$h = \frac{Pg'f'}{Pgf'' + \lambda(P''_{fire}x + 2P'_{fire})} + gf. \tag{15}$$

Hence, the differential equation (1) defining stand growth can be respecified as:

$$\dot{x} = \begin{cases} gf & , \quad Pgf' - \lambda(P - P_{fire}) + \lambda P'_{fire}x - P\delta > 0 \\ -\frac{Pg'f'}{Pgf'' + \lambda(P''_{fire}x + 2P'_{fire})} & , \quad \bar{P}gf' - \lambda(P - P_{fire}) + \lambda P'_{fire}x - P\delta = 0 \end{cases}. \tag{16}$$

The optimal thinning schedule is defined by condition (14). From an economic standpoint, this equation works as an arbitrage condition between investing one additional cubic meter *in situ* or investing elsewhere in the economy. Investing the value of one cubic meter of harvested timber elsewhere in the economy returns the interest rate on the unitary stumpage price ($P\delta$). While the expected returns from keeping that cubic meter on the stand correspond to the value growth increment due to that extra cubic meter (Pgf'), net of the expected loss of value of that cubic meter to fires ($\lambda(P - P_{fire})$), plus the change in the expected value of the standing trees. ($\lambda P'_{fire} x$).

The time at which undisturbed growth is interrupted by thinnings is denoted by t_1. Before t_1, $Pgf' - \lambda(P - P_{fire}) + \lambda P'_{fire} x > P\delta$, i.e., leaving one cubic meter on delivers higher expected returns than the interest returns and, in that sense, the stand should grow undisturbed. After t_1, thinning operations should harvest as much volume as to guarantee that marginal expected net benefits equal the marginal opportunity cost from leaving one cubic meter *in situ*.

When there is no risk of fire, Tahvonen (2016) proves that the optimal thinning condition is given by $Pgf' = P\delta$. Thus, the result of Tahvonen is the particular case of condition (14) with $\lambda = 0$.

The effect of fire hazard on the optimal thinning is given by $\frac{\partial\left[Pgf' - \lambda(P - P_{fire}) + \lambda P'_{fire} x\right]}{\partial \lambda} = -(P - P_{fire}) + P'_{fire} x$. Since the properties of the thinning schedule can vary significantly with the characteristics of the salvage price function, these results are discussed under different salvage price scenarios.

Proposition 1 *With $P'_{fire} = 0$, the introduction of fire hazard leads to the anticipation of optimal thinning when compared to no-risk scenario if $P_{fire} < P$. For $P_{fire} = P$, the optimal thinning schedule is the same.*

With a constant salvage price the condition for optimal thinning (14) can be written as $Pgf' - \lambda(P - P_{fire}) = P\delta$. Intuitively, when the risk of destruction by fire is introduced, the expected returns from leaving one additional cubic meter on the stand decrease. Therefore, a rational landowner should optimally anticipate thinning revenues in order to mitigate the exposure to destruction. The extent of this anticipation is a result of the magnitude of potential destruction and, hence, depends on the salvage price. Eventually, if the salvage price equals the stumpage price, no losses result from fire and there should be no anticipation. However, at this point this result is only valid under a constant salvage price.

In order to explore the robustness of this finding, consider now that the salvage price is a function of stand volume. Recall that the optimal thinning condition, $Pgf' - \lambda(P - P_{fire}) + \lambda P'_{fire}x = P\delta$, now has to account for the effect of thinning on the expected value of the standing trees ($\lambda P'_{fire}x$).

Proposition 2 *If $P'_{fire} < 0$, thinnings should be anticipated when compared to the no-risk scenario. If, instead, $P'_{fire} > 0$, the effect of hazard on t_1 is ambiguous.*

If $P'_{fire} > 0$ and under certain circumstances, the introduction of fire hazard may lead to a delayed t_1, compared to the no-risk scenario of Tahvonen (2016). In cases of sufficiently elastic and large salvage price, $P'_{fire}x > (P - P_{fire}) \Rightarrow \frac{\partial t_1}{\partial \lambda} > 0$, and thus optimal thinning is postponed. When letting the stand grow brings substantial gains to the salvage value of the standing trees, one can afford expose the asset to destruction by postponing thinning.

Accordingly, it can be concluded that, if $P'_{fire} \leq 0$, thinnings should be unambiguously anticipated when fire hazard increases, with the exception of the borderline case of $P'_{fire} = 0$ and $P_{fire} = P$.

3.2 Optimal rotation

Eq. (11) sets the necessary condition for a finite optimal rotation age, T. Making use of equation (1) it can be written as:

$$y(T) \equiv P[h(T) + \dot{x}(T)] - \lambda\{P - P_{fire}[x(T)]\}x(T) - \delta[Px(T) - c + J] = 0 \tag{17}$$

If $y(T)$ never reaches zero for any finite rotation period, the remaining candidate is $T = \infty$ (continuous cover forestry).

From an economic point of view, as long as $y(T) > 0$, or equivalently, $Ph(T) + P\dot{x}(T) - \lambda(P - P_{fire})x(T) > \delta[Px(T) - c] + \delta J$, the marginal expected benefits from postponing clearcut exceed the marginal opportunity costs. Consequently, the rotation should be extended. In other words, it is optimal not to clearcut as long as the thinning revenues

(Ph) net of the change in total stand stumpage value $(P\dot{x})$ minus the expected loss of total timber value to fires $(\lambda(P - P_{fire})x)$ pay for the interest returns from the clearcut net revenues $(\delta(Px - c))$ plus the site rental (δJ).

Once again, in the reference scenario of zero probability of fire occurrence, the optimal rotation condition (17) is given by the condition derived by Tahvonen (2016), i.e. $P[h(T) + \dot{x}(T)] - \delta[Px(T) - c + J] = 0$. With the introduction of fire hazard, however, the *in situ* returns have to account for the expected loss of value to fires of the standing trees $(\lambda(P - P_{fire})x)$.

The marginal effect of fire hazard on the optimal rotation age can be algebraically simplified to $\frac{\partial y(T)}{\partial \lambda} = -(P - P_{fire})x(T) - \delta \frac{\partial J}{\partial \lambda}$ (Appendix C).

Proposition 3 *Assuming that $P_{fire} = P$, by $\frac{\partial y(T)}{\partial \lambda} = -\delta \frac{\partial J}{\partial \lambda} > 0$, the optimal rotation should be unequivocally postponed when fire hazard is introduced.*

The intuition for delaying the clearcut results from the fact that, with $P_{fire} = P$, fire occurrence causes no devaluation to the standing trees but it leads, in fact, to losses in land value since it imposes involuntary clearcuts. That is, the marginal expected benefit from postponing clearcut is unchanged but the marginal opportunity cost decreases (lower site rental). Hence, under the hypothesis that $P_{fire} = P$, the introduction of fire hazard should have no impact on the thinnings (Proposition 1) but optimal rotations should be longer.

Proposition 4 *When $P_{fire} < P$, the effect of fire hazard on the optimal rotation length is ambiguous.*

This ambiguity results from the fact that $(P - P_{fire}) > 0$ and $\frac{\partial J}{\partial \lambda} < 0$. On the one hand, a higher risk increases the expected loss of value of the standing trees $(\lambda(P - P_{fire})x(T))$, motivating shorter rotations to avoid exposure. On the other hand, however, a higher fire hazard leads to a lower opportunity cost of the investment held on the site (δJ), inducing longer rotations. As long as $(P - P_{fire})x(T) < -\delta \frac{\partial J}{\partial \lambda}$, a higher λ leads to longer optimal rotations. Thus, when the total standing timber value lost in the case of fire is smaller than the decrease in the site rental, a higher hazard of fire postpones optimal rotation.

By excluding the possibility of thinning the stand ($h = 0$) and assuming $P_{fire} = 0$, Reed (1984) sets the condition for the optimal rotation as $P\dot{x}(T) - \lambda Px(T) = \delta[Px(T) - c] + \delta J$. Therefore, the time of clearcut should be such that the expected marginal increase in stumpage value of the standing trees ($P\dot{x}(T) - \lambda Px(T)$) equals the sum of the opportunity cost of the investment held in the current stand ($\delta[Px(T) - c]$) and in the site (δJ). This is no more than a particular case of the optimal rotation condition (17) with $h = 0$ and $P_{fire} = 0$. However, in a no-salvage scenario, this model establishes that, by adopting thinnings, expected *in situ* returns from delaying clearcut ($P(h + \dot{x}) - \lambda Px$) have to account for the existing thinning revenues (Ph).

From the fact that $y'(T)|_{y(T)=0} < 0$, continuous cover forestry is optimal if $\lim_{T\to\infty} y(T) > 0$. Since $\dot{x} = 0$ at the steady-state, one should never clearcut the stand if:

$$P\tilde{h} - \lambda(P - \tilde{P}_{fire})\tilde{x} > \delta(P\tilde{x} - c) + \delta J. \tag{18}$$

Being $\tilde{h}$, $\tilde{x}$ and $\tilde{P}_{fire}$ the steady-state thinning, stand volume and salvage price, respectively. Hence, it is optimal to never clearcut the stand if the steady-state thinning revenue net of the expected loss of stand value to fires pays for the interest earned on the clearcut net revenues plus the site rental.

Rearranging condition (18), we get that $\frac{P\tilde{h} - \lambda(P - \tilde{P}_{fire})\tilde{x}}{\delta} > (P\tilde{x} - c) + J$. Then, in the presence of fire hazard, a continuous cover regime is economically optimal if the present value of perpetual sustainable thinnings net of expected losses to fires is higher than the net revenues from clearcutting the steady-state volume and selling the land.

4 Numerical results

To further investigate the implications of the model, a stand growth function (Tahvonen, 2016) fitted for the Norway spruce is used to provide numerical results. Undisturbed stand growth (m^3/ha) is assumed to follow:

$$g(t)f[x(t)] = \left(\frac{1.6}{1+0.04t^{1.2}}+1\right)0.065[x(t)+8]\left[1-\frac{x(t)+8}{378}\right].$$

Hence, age-dependent growth $g(t)$ is strictly positive and converging to $\tilde{g} = 0.065$, a level that allows for volume growth to depend exclusively on volume and equal $0.065f(x)$ at the steady-state. The growth-maximizing volume (at which $f'(\hat{x}) = 0$) is reached at $\hat{x} = 180$, implying that until a volume of 180 m^3, the density effect of volume-dependent growth dominates over the competition effect. For volumes higher than 180m^3, lower volume-dependent growth is expected, meaning that the effect of competition between trees (for sunlight, nutrients or water, for instance) starts dominating over the density effect until the stand reaches the site capacity at $\bar{x} = 370m^3$ where competition is so high that the stand ceases to grow. For comparison with the baseline no-risk scenario (Tahvonen, 2016), assume a stumpage price (P) of 40 EUR/m^3, re-establishment costs (c) of 1000 EUR/ha and a discount rate (δ) of 3%. Hence, in the absence of fire hazard, the reference optimality results are $t_1 = 25.54$ and $T = 109.47$.

Assuming a constant salvage price, Figures 1 and 2 depict the optimal stand volume over time for different levels of fire hazard and salvage price, respectively.

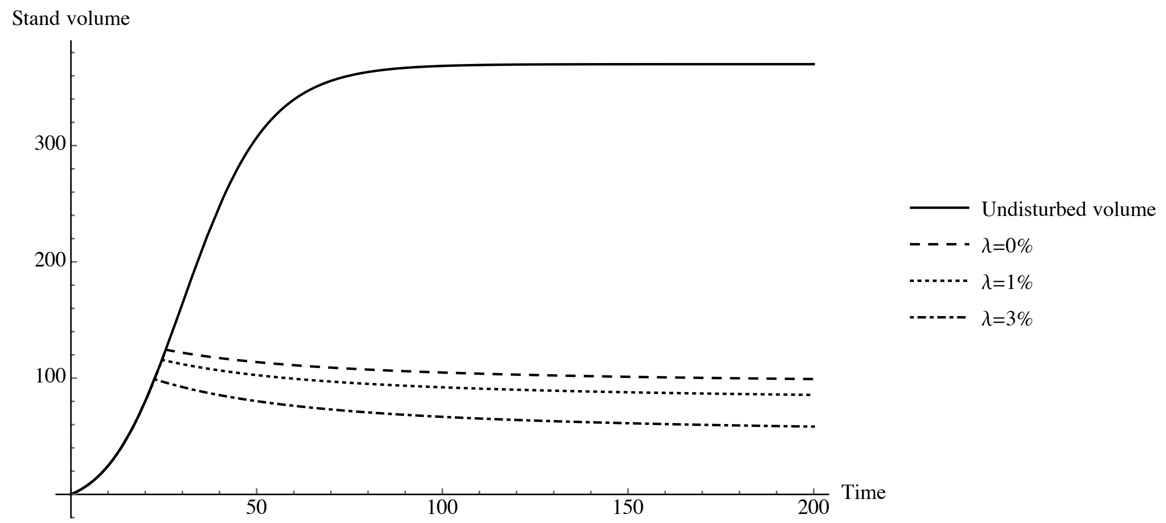

Figure 1 - The effect of fire hazard (λ) on the optimal stand volume (x).

$(P = 40, P_{fire} = 20, \delta = 0.03)$

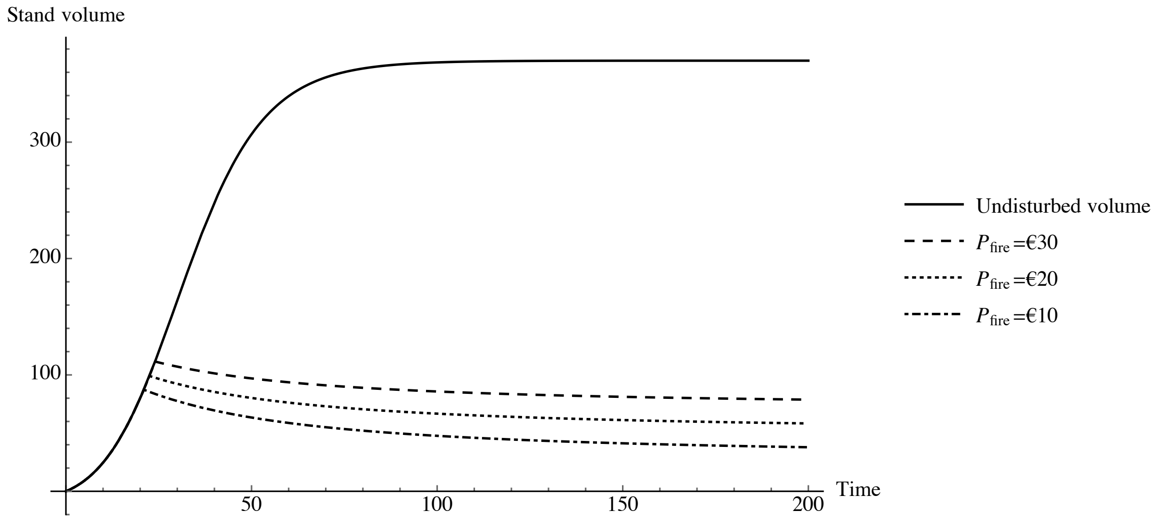

Figure 2 - The effect of the salvage price (P_{fire}) on the optimal stand volume (x).

$(P = 40, \lambda = 3\%, \delta = 0.03)$

Graphical interpretation of the numerical results validates our finding that higher levels of fire hazard should lead to the anticipation of optimal thinning (Figure 1) and that this anticipation should be mitigated under higher salvage prices (Figure 2). In all circumstances, unless $P_{fire} = P$, thinnings should start earlier than the no-risk scenario. Intuitively, when subject to the risk of destruction, the forest manager will anticipate thinning revenues and leave a smaller asset in the fire-exposed site.

Being the effect of fire on thinning clear, one of the questions that remains unsettled from the analytical results is the effect of fire hazard on the optimal rotation length. Numerical results summarized in Table 1 suggest that, in general, optimal rotation length should increase with fire hazard. Although a higher fire occurrence implies larger expected loss of value of the standing trees, by the anticipation of thinnings, the volume of standing trees being exposed to fire is lower. Additionally, with higher risk there is a significant reduction in the land value which implies a lower opportunity cost from leaving the trees *in situ* for a longer period. As a result, one may optimally adopt longer rotations.

Table 1 - Numerical results. ($P = 40, c = 1000, \delta = 0.03$)

λ	*Thinning*	P_{fire}	t_1	T	J	$E(X)$	
0%	No	-	-	35.15	€3,942.53	35.15	(1)
	Yes	-	25.54	109.47	€4,566.43	109.47	(2)
1%	No	€0	-	32.47	€2,890.09	27.73	(3)
	Yes	€0	23.51	106.34	€3,377.24	65.47	
	Yes	€10	24.01	117.17	€3,566.69	69.02	
	Yes	€20	24.52	131.73	€3,763.29	73.21	
	Yes	€30	25.03	152.21	€3,966.94	78.17	
	Yes	€40	25.54	183.03	€4,177.47	83.96	
2%	No	€0	-	30.09	€1,994.09	22.61	(3)
	Yes	€0	21.48	124.71	€2,405.11	45.87	
	Yes	€10	22.50	142.81	€2,705.22	47.13	
	Yes	€20	23.51	183.01	€3,028.16	48.71	
	Yes	€30	24.52	290.37	€3,372.93	49.85	
	Yes	€40	25.54	∞	€3,738.57	50.00	
3%	No	€0	-	27.98	€1,218.22	18.93	(3)
	Yes	€0	19.45	∞	€1,597.24	33.33	
	Yes	€10	20.98	391.01	€1,954.28	33.33	
	Yes	€20	22.50	497.67	€2,354.97	33.33	
	Yes	€30	24.01	∞	€2,795.98	33.33	
	Yes	€40	25.54	∞	€3,274.18	33.33	

(1) Faustmann rule.
(2) Tahvonen (2016).
(3) Reed (1984).

However, the possibility of conversion to shorter rotations is not excluded, as it is the case if $P_{fire} = 0$ and $\lambda = 1\%$ ($T = 106.34$) when compared to the baseline scenario

($T = 109.47$). Intuitively, if the salvaged timber has no commercial value, the expected losses from exposing the stand for a longer period can be such that it becomes optimal to clearcut the stand earlier. In contrast, with higher salvage prices and $\lambda = 1\%$, rotations are longer than in the no-risk case. This result is illustrative of the importance of taking the value of salvage harvests into account and the implications it has on the economically optimal decisions.

Figure 3 illustrates the effect of fire hazard on the optimal rotation length. For comparison with scenarios with no-thinnings scenarios, we estimated the optimal rotation solutions of this specific stand under the clearcut rule of Reed (1984).

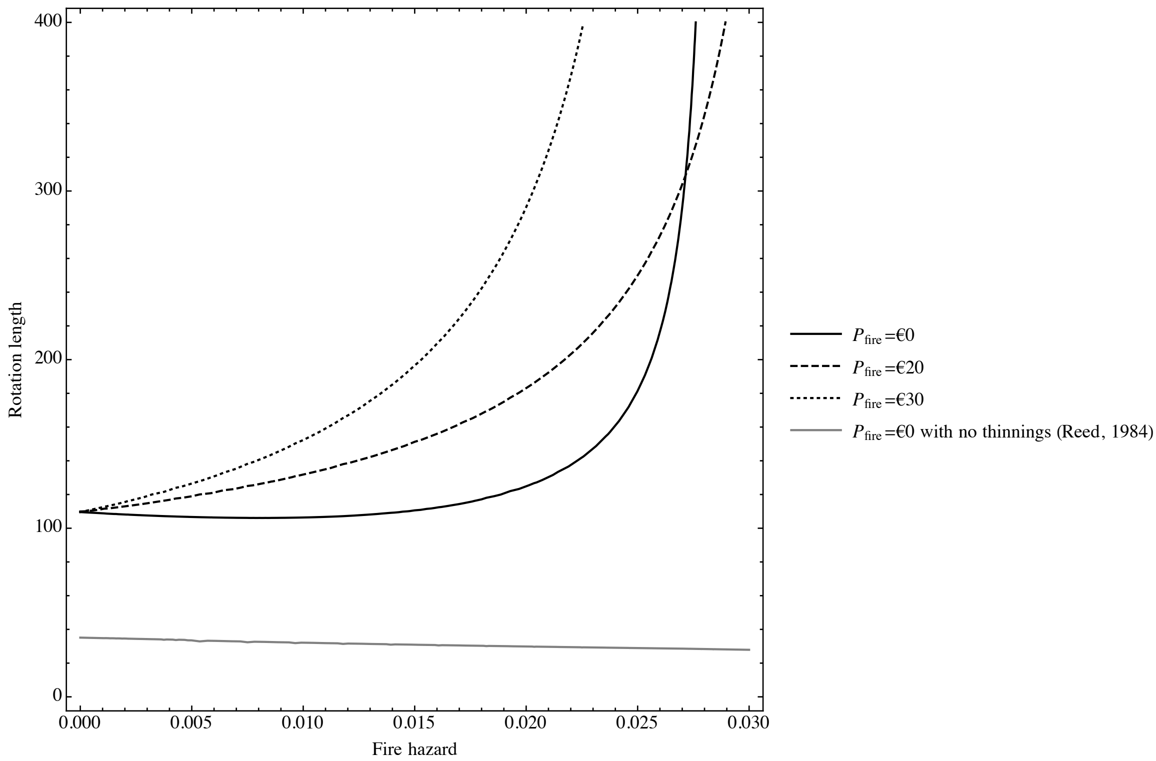

Figure 3 - The effect of fire hazard (λ) on the optimal rotation length (T).

$(P = 40, c = 1000, \delta = 0.03)$

Comparing results, the introduction of the possibility of thinning the stand leads to longer optimal rotations. This result is driven by the fact that, after t_1, the stand starts to yield thinning revenues and volume decreases ever after. Hence, since volume is kept at lower levels, clearcutting and interrupting thinning revenues becomes relatively less attractive and optimal rotations are lengthened. Regarding the economic impact from adopting thinnings, by comparing with the results under Reed's specification, there can be gains in terms of land expectation value from 15% (with $\lambda = 1\%$) up to 30% ($\lambda = 3\%$).

Furthermore, Reed's model implied that increases in fire hazard should unambiguously shorten optimal rotations. By taking thinnings into account, however, fire hazard can influence the optimal clearcut decision in different ways. The primary direct effect of fire hazard - the increased exposure of the asset to destruction - should motivate shorter rotations. But there is an important indirect effect though, that is generated by thinnings and therefore has not been addressed by Reed. With increased fire probability, optimal thinning should be anticipated, leading to lower volume levels after t_1. Thus, fire hazard induces an effect of intertemporal substitution of future clearcut revenues by thinning revenues. By opting to thin for a longer period instead of keeping a higher volume *in situ*, not only clearcut revenues decrease but also the asset (standing trees) that is being exposed to risk of destruction is lower, fostering clearcut postponement.

In what concerns the optimal decision between adopting rotation forestry or continuous cover forestry, Figure 4 depicts the conditions under which it becomes optimal to never clearcut the stand.

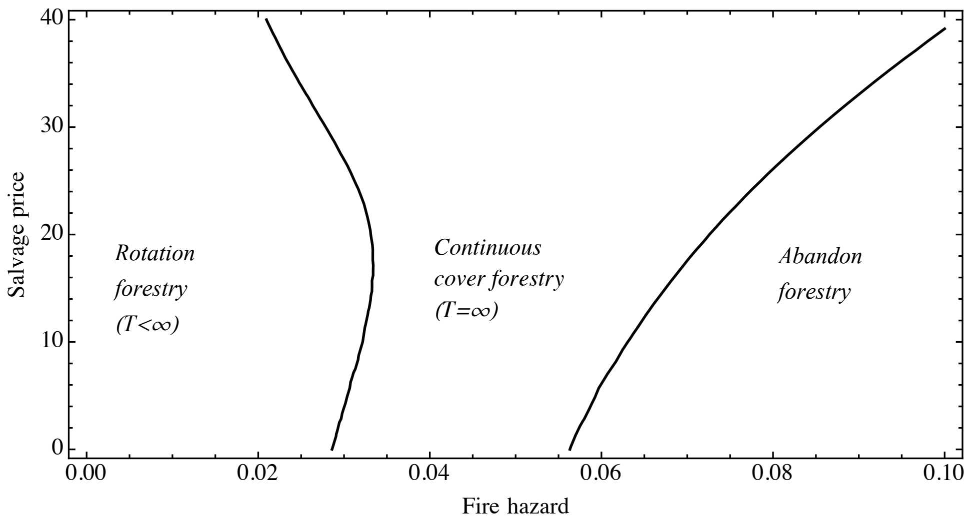

Figure 4 - The optimality of continuous cover versus rotation forestry.

$(P = 40, c = 1000, \delta = 0.03)$

For sufficiently high levels of fire hazard, the optimal harvesting policy tends to never clearcutting the stand, fully prioritizing thinnings and salvage harvests as sources of income. Moreover, forestry can become unsustainable for extremely high levels of fire occurrence. In Figure 4 the frontier after which the optimal decision should be to abandon forestry is also depicted. Evidently, when fire hazard is sufficiently high and the salvage price does not pay for the exposure to such level of risk, the land expectation value eventually becomes negative and the optimal decision should be to abandon forestry in that site.

5 Conclusion

This study investigated the optimal harvesting of a forest stand susceptible to natural destruction. The proposed model contributes to the literature of forestry economics by providing optimal solutions for the schedule of thinnings and clearcuts of an uneven-aged stand exposed to fire hazard.

By allowing for the anticipation of revenues through thinnings, this study offers new results by showing that it can be optimal to adopt longer rotations when subject to the risk of fire. For sufficiently high levels of fire hazard, it can actually become optimal to manage in a continuous cover regime, by choosing to never clearcut the stand and to depend exclusively on thinnings and salvage harvests.

Additionally, this study emphasizes the importance of thinning the stand as an essential instrument of forest management. In the adaptation to more severe fire regimes, excluding the possibility of thinning may unintentionally lead forestry in certain sites to become unsustainable which, in itself, can be extremely detrimental in terms of fire intensification. The empirical results of this work show that there can be non-negligible economic gains from thinnings that would allow sustainable forest management even at high levels of fire occurrence.

As pointed out by Tahvonen (2016), the assumption of equal timber net price between thinnings and clearcuts neglects harvest fixed costs and the possible implications these can have on the optimal harvesting. Hence, the scenario of lower net price from thinnings is part of the research agenda. Moreover, this study is limited to the analysis of homogeneous fire occurrences. Thus, fires of lower intensity that do not impose re-establishment but can still cause losses have not been addressed.

It should be mentioned that, when modelling uneven-aged stands, Faustmann models have limited practical application since, by definition, the forest size structure is ignored. Nonetheless, in contrast to class-structured approaches, this model is useful to provide clear theoretical principles in which to ground forestry practice. Further research can focus on extensions of this model to the integration of non-timber benefits such as carbon sequestration and the implications brought by fires in terms of carbon release. Finally, future empirical estimations of salvage price functions of uneven-aged stands can bring significant insight into the ongoing research on the optimal rotation under risk of fire.

References

[1] FAO, 2018. *Forests Pathways to Sustainable Development*. State of the World's Forests 2018. Rome: FAO.

[2] Martell, D. L. 1980. "The Optimal Rotation of a Flammable Forest Stand". *Canadian Journal of Forest Research* 10 (1): 30–34.

[3] Routledge, R. D. 1980. "The Effect of Potential Catastrophic Mortality and Other Unpredictable Events on Optimal Forest Rotation Policy". *Forest Science* 26 (3): 389–99.

[4] Reed, William J. 1984. "The Effects of the Risk of Fire on the Optimal Rotation of a Forest". *Journal of Environmental Economics and Management* 11 (2): 180–90.

[5] Englin, Jeffrey, Peter Boxall, and Grant Hauer. 2000. "An Empirical Examination of Optimal Rotations in a Multiple-Use Forest in the Presence of Fire Risk". *Journal of Agricultural and Resource Economics* 25 (1): 14–27.

[6] Reed, William J. 1987. "Protecting a Forest against Fire: Optimal Protection Patterns and Harvest Policies". *Natural Resource Modeling* 2 (1): 23–53.

[7] Reed, William J., and Joseph Apaloo. 1991. "Evaluating the Effects of Risk on the Economics of Juvenile Spacing and Commercial Thinning". *Canadian Journal of Forest Research* 21 (9): 1390–1400.

[8] Thorsen, Bo Jellesmark, and Finn Helles. 1998. "Optimal Stand Management with Endogenous Risk of Sudden Destruction". *Forest Ecology and Management* 108 (3): 287–99.

[9] Kilkki, Pekka, and Unto Väisänen. 1969. "Determination of the Optimum Cutting Policy for the Forest Stand by Means of Dynamic Programming". *Acta Forestalia Fennica* 102: 100–112.

[10] Clark, Colin W. 1976. *Mathematical Bioeconomics: The Optimal Management of Renewable Resources*. Pure and Applied Mathematics. New York: Wiley.

[11] Cawrse, David C., David R. Betters, and Brian M. Kent. 1984. "A Variational Solution Technique for Determining Optimal Thinning and Rotational Schedules". *Forest Science* 30 (3): 793–802.

[12] Betters, David R., Eric A. Steinkamp, and Matthew T. Turner. 1991. "Singular Path Solutions and Optimal Rates for Thinning Even-Aged Forest Stands". *Forest Science* 37 (6): 1632–40.

[13] Tahvonen, Olli. 2016. "Economics of Rotation and Thinning Revisited: The Optimality of Clearcuts versus Continuous Cover Forestry". *Forest Policy and Economics* 62: 88–94.

[14] Assmuth, Aino, and Olli Tahvonen. 2018. "Optimal Carbon Storage in Even- and Uneven-Aged Forestry". *Forest Policy and Economics* 87: 93–100.

[15] Sydsæter, Knut, ed. 2005. *Further Mathematics for Economic Analysis*. Harlow: Financial Times/Prentice Hall.

A Expected NPV over one rotation (π)

The expected revenues net of costs earned over one cycle can be expressed as:

$$\begin{aligned}\pi = \quad & \int_0^T \rho(X)(\int_0^X Ph(t)e^{-\delta t}dt + \{P_{fire}[x(X)]x(X) - c\}\, e^{-\delta X})dX \\ & +S(T)\{\int_0^T Ph(t)e^{-\delta t}dt + [Px(T) - c]\; e^{-\delta T}\}.\end{aligned}$$

After performing integration by parts in the first term, π can be written as:

$$\begin{aligned}\pi = \quad & [-S\,(X)\,(\int_0^X Ph(t)e^{-\delta t}dt + \{P_{fire}[x(X)]x(X) - c\}\, e^{-\delta X})]_0^T \\ & -\int_0^T - S(t)(Ph(t) + P'_{fire}[x(t)]\dot{x}(t)x(t) + P_{fire}[x(t)]\dot{x}(t) \\ & -\delta\{P_{fire}[x(t)]x(t) - c\})e^{-\delta t}dt + S(T)\int_0^T Ph(t)e^{-\delta t}dt \\ & +\, S(T)[Px(T) - c]\; e^{-\delta T}.\end{aligned}$$

Note that $-S(X)$ is an anti-derivative of $\rho(X)$. Developing the first term leads to:

$$\begin{aligned}\pi = \quad & -S\,(T)\int_0^T Ph(t)e^{-\delta t}dt - S\,(T)\,\{P_{fire}[x(T)]x(T) - c\}\, e^{-\delta T} \\ & +S\,(0)\,\{P_{fire}[x(0)]x(0) - c\}\, e^0 - \int_0^T - S(t)(Ph(t) + P'_{fire}[x(t)]\dot{x}(t)x(t) \\ & +P_{fire}[x(t)]\dot{x}(t) - \delta\{P_{fire}[x(t)]x(t) - c\})e^{-\delta t}dt + S(T)\int_0^T Ph(t)e^{-\delta t}dt \\ & +S(T)[Px(T) - c]e^{-\delta T}.\end{aligned}$$

Since S(0) = 1 and $x(0) = 0$, it can be simplified to:

$$\begin{aligned}\pi = \quad & -c + S\,(T)\,\{P - P_{fire}[x(T)]\}x(T)e^{-\delta T} + \int_0^T S(t)(Ph(t) + P'_{fire}[x(t)]\dot{x}(t)x(t) \\ & +P_{fire}[x(t)]\dot{x}(t) - \delta\{P_{fire}[x(t)]x(t) - c\})e^{-\delta t}dt.\end{aligned}$$

After substituting $\dot{x}(t) = g(t)f[x(t)] - h(t)$, it simplifies to:

$$\begin{aligned}\pi = \quad & -c + S\,(T)\,\{P - P_{fire}[x(T)]\}x(T)e^{-\delta T} + \int_0^T S(t)(P - P_{fire}[x(t)] \\ & -P'_{fire}[x(t)]x(t)\}h(t) + \{P_{fire}[x(t)] + P'_{fire}[x(t)]x(t)\}g(t)f[x(t)] \\ & -\delta\{P_{fire}[x(t)]x(t) - c\})e^{-\delta t}dt.\end{aligned}$$

B Optimality conditions

The current-value Hamiltonian is given by $H = S[(P - P_{fire} - P'_{fire}x)h + (P_{fire} + P'_{fire}x)gf - \delta(P_{fire}x - c)] + \varphi(gf - h)$. Conditions (8) are necessary for H to be maximized with respect to the control variable h, i.e.:

$$\begin{aligned}
\tfrac{\partial H}{\partial h} < 0 \quad &\Rightarrow \quad h = 0, \\
\tfrac{\partial H}{\partial h} = 0 \quad &\Rightarrow \quad h \in [0, h_{MAX}], \\
\tfrac{\partial H}{\partial h} > 0 \quad &\Rightarrow \quad h = h_{MAX}.
\end{aligned}$$

Differentiating H with respect to h leads to $S(P - P_{fire} - P'_{fire}x) - \varphi$.

Condition (9) sets $\varphi' = -\frac{\partial H}{\partial x} + \delta\varphi$. Solving the first derivative of H with respect to the state variable x, the equation becomes:

$$\begin{aligned}
\varphi' = \; & SP'_{fire}h + SP''_{fire}xh + SP'_{fire}h - SP'_{fire}gf - SP_{fire}gf' - SP''_{fire}xgf \\
& - SP'_{fire}gf - SP'_{fire}xgf' + S\delta P'_{fire}x + S\delta P_{fire} - \varphi gf' + \delta\varphi.
\end{aligned}$$

Rearranging and simplifying, it can be expressed as

$$\varphi' = \varphi(\delta - gf') - S[P''_{fire}\dot{x}x + P'_{fire}x(gf' - \delta) + 2P'_{fire}\dot{x} + P_{fire}(gf' - \delta)].$$

Eq. (10) comes from the fact that this control problem includes a scrap value function $\Psi[T, x(T)] = -c + S\,(T)\,\{P - P_{fire}[x(T)]\}x(T)$. Thus, following Sydsaeter et al. (2005, p. 341) the transversality condition reads as:

$$(\varphi(T) - \frac{\partial \Psi}{\partial x})x(T) = 0.$$

Since $\frac{\partial \Psi}{\partial x} = -S(T)P'_{fire}[x(t)]x(T) + S(T)\{P - P_{fire}[x(T)]\}$, the following condition can be derived:

$$(\varphi(T) - S(T)\{P - P_{fire}[x(T)] - P'_{fire}[x(T)]x(T)\})x(T) = 0.$$

The condition for the optimal rotation age (11) comes from the maximization of J with respect to T. Knowing that $J = \frac{\pi}{\delta\int_0^T S(t)e^{-\delta t}dt}$ and $\delta\int_0^T S(t)e^{-\delta t}dt > 0$, we can set the first derivative of J equal to zero and simplify:

$$\begin{aligned}
\frac{\partial J}{\partial T} = 0 \quad &\Leftrightarrow \frac{\frac{\partial \pi}{\partial T}\delta\int_0^T S(t)e^{-\delta t}dt - \pi\delta S(T)e^{-\delta T}}{(\delta\int_0^T S(t)e^{-\delta t}dt)^2} = 0 \Leftrightarrow \\
&\Leftrightarrow \frac{\frac{\partial \pi}{\partial T} - J\delta S(T)e^{-\delta T}}{\delta\int_0^T S(t)e^{-\delta t}dt} = 0 \Leftrightarrow \\
&\Leftrightarrow \frac{\partial \pi}{\partial T} - J\delta S(T)e^{-\delta T} = 0.
\end{aligned}$$

Knowing that:

$$\begin{aligned}
\frac{\partial \pi}{\partial T} = \;& S'(T)\{P - P_{fire}[x(T)]\}x(T)e^{-\delta T} - S(T)P'_{fire}[x(T)]\dot{x}(T)x(T)e^{-\delta T} \\
& -\delta S(T)\{P - P_{fire}[x(T)]\}x(T)e^{-\delta T} + S(T)\{P - P_{fire}[x(T)]\}\dot{x}(T)e^{-\delta T} \\
& +S(T)(\{P - P_{fire}[x(T)] + P'_{fire}[x(T)]x(T)\}h(T) + \{P_{fire}[x(T)] \\
& +P'_{fire}[x(T)]x(T)\}g(T)f[x(T)] - \delta\{P_{fire}[x(T)]x(T) - c\})e^{-\delta T}.
\end{aligned}$$

And since $e^{-\delta T} > 0$ and $-\frac{S'(T)}{S(T)} = \frac{\rho(T)}{S(T)} = \lambda$, condition $\frac{\partial J}{\partial T} = 0$ can be simplified to:

$$y(T) \equiv -\lambda\{P - P_{fire}[x(T)]\}x(T) + Pg(T)f[x(T)] - \delta[Px(T) - c + J] = 0.$$

Taking the first derivative of y with respect to T:

$$\begin{aligned}
y'(T) = \;& \lambda P'_{fire}[x(T)]\dot{x}(T)x(T) - \lambda\{P - P_{fire}[x(T)]\}\dot{x}(T) + Pg(T)f'[x(T)]\dot{x}(T) \\
& -\delta P\dot{x}(T) - \delta\frac{\partial J}{\partial T}.
\end{aligned}$$

Rearranging:

$$\begin{aligned}
y'(T) = \;& \dot{x}(T)(Pg(T)f'[x(T)] - \lambda\{P - P_{fire}[x(T)]\} + \lambda P'_{fire}[x(T)]x(T) - \delta P) \\
& +Pg'(T)f[x(T)] - \delta\frac{\partial J}{\partial T}.
\end{aligned}$$

Assuming that $T > t_1$, since the stumpage net price is the same whether it came from a clearcut or from thinnings, one should expect condition (14) to be satisfied at the moment of clearcut, T. Applying this condition and the fact that $y(T) = 0 \Rightarrow \frac{\partial J}{\partial T} = 0$, it follows that $y'(T)|_{y(T)=0} = Pg'(T)f[x(T)] < 0$. Therefore, if a finite T that maximizes J exists, it must be the global optimum.

C Effect of fire hazard on the optimal rotation

From condition $y(T) = 0$ we get the optimal rotation age. Differentiating $y(T)$ with respect to fire hazard λ:

$$\begin{aligned}\frac{\partial y(T)}{\partial \lambda} &= \frac{\partial \{Pg(T)f[x(T)]\}}{\partial x(T)}\frac{\partial x(T)}{\partial \lambda} - \{P - P_{fire}[x(T)]\}x(T) + \lambda \frac{\partial P_{fire}[x(T)]}{\partial x(T)}\frac{\partial x(T)}{\partial \lambda}x(T) \\ &-\lambda\{P - P_{fire}[x(T)]\}\frac{\partial x(T)}{\partial \lambda} - \delta P\frac{\partial x(T)}{\partial \lambda} - \delta\frac{\partial J}{\partial \lambda} \Leftrightarrow\end{aligned}$$

$$\begin{aligned}\Leftrightarrow \frac{\partial y(T)}{\partial \lambda} &= \frac{\partial x(T)}{\partial \lambda}(Pg(T)f'[x(T)] - \lambda\{P - P_{fire}[x(T)]\} + \lambda P'_{fire}[x(T)]x(T) - \delta P) \\ &-\{P - P_{fire}[x(T)]\}x(T) - \delta\frac{\partial J}{\partial \lambda}.\end{aligned}$$

Using condition (14) this expression can be simplified to:

$$\frac{\partial y(T)}{\partial \lambda} = -\{P - P_{fire}[x(T)]\}x(T) - \delta\frac{\partial J}{\partial \lambda}.$$

D Numerical results with $\delta = 1\%$ and $\delta = 5\%$

$(P = 40, c = 1000, \delta = 0.01)$

λ	*Thinning*	P_{fire}	t_1	*T*	*J*	*E(X)*	
0%	No	-	-	41.62	€18,171.6	41.62	(1)
	Yes	-	29.71	127.21	€21,581.1	127.21	(2)
1%	No	€0	-	38.17	€13,600.1	31.73	(3)
	Yes	€0	27.60	118.69	€16,113.8	69.48	
	Yes	€10	28.12	134.86	€17,068.3	74.04	
	Yes	€20	28.64	157.11	€18,063.9	79.22	
	Yes	€30	29.17	190.00	€19,100.2	85.04	
	Yes	€40	29.71	244.56	€20.176.3	91.33	
2%	No	€0	-	35.15	€9,827.58	25.25	(3)
	Yes	€0	25.54	109.47	€11,699.3	44.40	
	Yes	€10	26.56	141.68	€13,196.5	47.04	
	Yes	€20	27.60	212.04	€14,806.2	49.28	
	Yes	€30	28.64	524.06	€16,519.7	50.00	
	Yes	€40	29.71	∞	€18,329.8	50.00	
3%	No	€0	-	32.47	€6,670.27	20.75	(3)
	Yes	€0	23.51	106.34	€8,131.73	31.96	
	Yes	€10	25.03	152.21	€9,900.83	32.99	
	Yes	€20	26.56	341.75	€11,855.0	33.33	
	Yes	€30	28.12	∞	€13,980.7	33.33	
	Yes	€40	29.71	∞	€16,268.2	33.33	

(1) Faustmann rule.
(2) Tahvonen (2016).
(3) Reed (1984).

$(P = 40, c = 1000, \delta = 0.05)$

λ	*Thinning*	P_{fire}	t_1	T	J	$E(X)$	
0%	No	-	-	30.09	€1,596.66	30.09	(1)
	Yes	-	21.48	124.71	€1,843.07	124.71	(2)
1%	No	€0	-	27.98	€1,130.93	24.41	(3)
	Yes	€0	19.45	∞	€1,358.34	100.0	
	Yes	€10	19.96	∞	€1,426.71	100.0	
	Yes	€20	20.47	549.36	€1,498.15	99.59	
	Yes	€30	20.98	391.01	€1,572.57	98.00	
	Yes	€40	21.48	369.47	€1,649.90	97.51	
2%	No	€0	-	26.12	€719.76	20.35	(3)
	Yes	€0	17.41	∞	€948.02	50.00	
	Yes	€10	18.44	∞	€1,055.76	50.00	
	Yes	€20	19.45	∞	€1,174.39	50.00	
	Yes	€30	20.47	∞	€1,303.24	50.00	
	Yes	€40	21.48	∞	€1,441.71	50.00	
3%	No	€0	-	24.50	€350.21	19.37	(3)
	Yes	€0	15.36	∞	€595.37	33.33	
	Yes	€10	16.90	∞	€721.20	33.33	
	Yes	€20	18.44	∞	€869.78	33.33	
	Yes	€30	19.96	∞	€1,037.89	33.33	
	Yes	€40	21.48	∞	€1,224.32	33.33	

(1) Faustmann rule.
(2) Tahvonen (2016).
(3) Reed (1984).