A Work Project presented as part of the requirements for the Award of a Master Degree in Finance from the NOVA – School of Business and Economics.

"WEATHER DERIVATIVES PRICING AND RISK MANAGEMENT APPLICATIONS"

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22/01/2020

ABSTRACT

The main objective of this paper is to discuss suitable methods for the modelling of weather variables and to bring together much of the current thinking in terms of the pricing of their respective derivative contracts (CDD, HDD) with payoffs depending on temperature. In addition to the theoretical overview provided, an empirical investigation is undertaken using historical data from the De Bilt meteorological station: we use the aforementioned data to first suggest a stochastic process that describes the evolution of the temperature. Further, such temperature modelling phase is accompanied by the numerical technique of Monte Carlo simulation for derivatives pricing. Finally, we will analyse some weather-sensitive industries and discuss possible weather hedging strategies they could apply.

Keywords: Weather Derivatives, Temperature Modelling, Derivatives Pricing, Risk Management

This work used infrastructure and resources funded by Fundação para a Ciência e a Tecnologia (UID/ECO/00124/2013, UID/ECO/00124/2019 and Social Sciences DataLab, Project 22209), POR Lisboa (LISBOA-01-0145-FEDER-007722 and Social Sciences DataLab, Project 22209) and POR Norte (Social Sciences DataLab, Project 22209).

1. INTRODUCTION

Nowadays, the entire industrialised world is somehow affected by variations in weather patterns, being apparently random or predictable. These fluctuations affect certain products and without exception have financial impacts on their producers and consumers, thus leading them to the desire of minimising as much of this risk as possible. The list of businesses subject to weather risk is long and includes, for example, energy producers and consumers, supermarket chains, the agricultural industries and many other sectors. As a matter of fact, it is primarily the energy sector that, in recent years, has driven the demand for weather derivatives, a relatively new innovation in financial engineering that has been receiving a significant attention as the world continues to realise the magnitude of the risk management applications that these contracts own. The purpose of weather derivatives is to allow an investor to hedge against undesirable weather states, i.e. it gives weather dependent industries and organizations a possibility to protect themselves against potential financial losses that could be caused by unpredictable weather changes. For example, they allow natural gas supply companies to avoid the negative impact of a mild winter when no one turns on the heating, or they allow construction companies to avoid the losses due to a period of rain when construction workers cannot work outside. Anyway, this list could be extended.

The types of impact of weather on businesses range from small reductions in revenues to total disasters, such as when a tornado destroys a factory. Tornadoes are an example of what we will call *catastrophic* weather events, causing extreme damage to property and, in the worst cases, loss of life. Companies wishing to protect themselves against the financial impact of such disasters can buy insurance that will pay them according to the losses they sustain. Weather derivatives, however, are designed to help companies insure themselves against *non-catastrophic* weather events. Non-catastrophic weather variations include warm or cold periods, rainy or dry periods, windy or calm periods, and so on. They are expected to occur reasonably frequently. Nevertheless, they can cause a significant soreness for businesses with profits that depend in a sensitive way on the weather. Hedging with weather derivatives is desirable for such businesses because it significantly reduces the year-to-year volatility of their profits. This is beneficial for a number of reasons: 1) low volatility in profits can often reduce the interest rate at which companies borrow money; 2) in a publicly traded company, low volatility in profits usually translates into low volatility in the share price, meaning that, being less volatile, shares are valued more highly; 3) low volatility in profits reduces the risk of bankruptcy.

In the following paragraphs we will analyse this argument in depth. In section 2, we will give an overview of the weather derivatives market; in section 3, we will describe the different weather contracts, the variables characterizing them and the payoff functions; section 4 is dedicated to temperature modelling; in section 5, we will discuss the weather derivatives pricing and in section 6 we will show an empirical example of it. Section 7 is dedicated to the examination of some hedging strategies using weather derivatives along with some empirical examples. Finally, in section 8, we will draft our final conclusions on the study.

2. THE WEATHER DERIVATIVES MARKET

The first transaction in the weather derivatives market took place in the US in 1997, but the demand for weather hedging products skyrocketed during the mild winter of 1997/98, also known as *El Niño*[1]. This phenomenon received huge publicity in the American press, thus many companies decided to hedge their seasonal weather risk due to the exposure to a

[1] El Niño is a periodic warming of the tropical Pacific Ocean which affects weather around the world. Typical consequences of El Niño include increased rainfall in the southern US and drought in the western Pacific. Winter temperatures in the north-central US states are typically higher than normal in El Niño years, and lower than normal in the south-east and south-west of the country.

potential significant earnings decline. After that, the market for weather derivatives expanded rapidly and contracts started to be traded over-the-counter (OTC) as individually negotiated contracts. Then, in September 1999, in order to increase the size of the market and to remove credit risk from the trading of the contracts, the Chicago Mercantile Exchange (CME) created a marketplace for weather derivatives' transactions. This was the first exchange where standard weather derivatives could be traded. In the beginning, CME only traded two weather products: Heating Degree Days (HDD) and Cooling Degree Days (CDD) for ten cities in the US. Later, in 2003, CME expanded its weather derivatives to six European cities and also launched a new weather contract, the Cumulative Average Temperature (CAT).

2.1 OTC vs Exchange-Traded Weather Derivatives: the location basis risk

As we all know, one noticeable drawback from using Exchange-traded contracts of CME is the so-called "*basis risk*". CME's weather derivatives are only written over a few cities in the US and over Amsterdam and London in Europe and, as a consequence, the underlying temperature index of an Exchange-traded contract will not correspond and be perfectly correlated with the temperature index of the targeted weather-exposed region. This situation may increase the basis risk. In general, "*basis risk is smallest when the financial loss is highly correlated with the weather, and when contracts of the optimum size and structure, based on the optimum location, are used for hedging*" (Stephen Jewson and Anders Brix, 2005). In the context of weather derivatives, we can refer to this specific type of basis risk as *"location basis risk"*.

3. THE CONTRACTS: WEATHER VARIABLES, INDICES AND PAYOFF

When we speak about weather derivatives, we refer to swaps, call and put options based on a variety of different underlying weather variables. The most commonly used is the *temperature* (we could take hourly values, daily minima or maxima, or daily averages). In most countries the daily average, the most common frequency utilized, is defined as the midpoint of the daily minimum and maximum. However, the market provides us with many derivative contracts depending on a wide variety of weather indices, like *wind-based*, *rain-based* and *snow-based* derivatives.

The exact relationship between the relevant weather variable and the impact on businesses will be different for different variables and different companies though: indeed, specific hedges are structured using indexes designed to capture as much of this dependence as possible. The most commonly used indexes for temperature-based contracts are *degree day indices (DD)*, *average temperature indices*, *cumulative average temperature indices(CAT),* and *event indices*, but in our study we will just consider the former.

3.1 Degree day indices

Degree day (DD) indices are usually employed in the energy sector for planning energy systems and predicting seasonal domestic demand for heating and cooling. They can be divided into two main categories: 1) *Heating Degree Days* (HDD); 2) *Cooling Degree Days* (CDD).

3.1.1 Heating Degree Days

Commonly used in the U.S. and Europe, but seldom in Japan, *Heating Degree Days* (HDDs) are utilized during the winter to measure the demand for heating, and are thus a measure of how cold it is (the colder it is, the more HDDs there will be). Given that patterns of energy usage vary from location to location, we could find different definitions describing HDDs, but the mostly used in the weather market is the following:

$$z_i = \max(T_0 - T_i, 0) \\ = (T_0 - T_i)^+ \tag{3.1}$$

This formula will provide us with the number of HDDs (z_i) on a particular day i, where T_i is the average temperature on day i, while, T_0 is the baseline temperature.

In all the countries, where temperature is measured in Celsius, the baseline is usually taken to be 18°C (64.4°F), while, in the United States, where temperature is measured in Fahrenheit, the baseline is usually set at 65°F (≈ 18°C).

An HDD index x over an N_d day period is usually defined as the sum of the HDDs over all days during that period:

$$x = \sum_{i=1}^{N_d} z_i \tag{3.2}$$

As would be expected, what we can usually observe is a large number of HDDs in winter, and fewer, or none, in summer.

3.1.2 Cooling Degree Days

Mainly used in the U.S. and rarely in Europe and Japan, *Cooling Degree Days* (CDDs) are used in summer to measure the demand for energy used for cooling, and are thus a measure of how hot it is (the hotter it is, the more CDDs there will be). Cooling is almost regularly driven by electricity, and so CDDs are most relevant to the electricity market (even if more and more electricity is being generated from natural gas, and so CDDs are also becoming relevant for the gas industry). The number of CDDs z_i on a particular day i is defined as:

$$z_i = \max(T_i - T_0, 0) \\ = (T_i - T_0, 0)^+ \tag{3.3}$$

where T_0 is again the baseline temperature (18°C). A CDD index x over a certain period is defined as the sum of the CDDs over all days during that period, as in equation (*3.2*).

3.1.3 Derivatives Payoff: Swaps, Call Options and Put Options

Once the index value is measured, it is used as input to a payoff function in order to financially settle the derivative contract. The function defines who should pay what to whom at the end of the contract's period. In the following paragraphs, we are going to define the payoff functions of weather swaps, calls and puts from the point of view of the buyer of a contract, as reported by Stephen Jewson and Anders Brix (2005).

A *long swap contract* has the aim of insuring against high future values of the index. In fact, for low values (more precisely, when the index is lower than the strike), the buyer has to pay the seller. To understand the logic behind that, let's analyse the payoff function (*p(x)*) of a CME's *weather swap* contract. We can write it as:

$$p(x) = D(x - K) \tag{3.4}$$

where, x arises from equation (*3.2*), K is the strike price and D is the "*tick size*", the monetary value associated to one index point. We can call this "*linear swap*". The majority of swaps are costless, meaning that there is no premium, and the profit or loss for a swap are equal to the payoff. Swap contracts traded on exchanges (like the CME) involve a daily settlement and they are known as *futures* contracts, while those exchanged OTC usually involve a settlement at the end of the contract. The latter are also named *forward* contracts, whose payoff includes limits. An important feature to be considered by a hedger using a linear swap to shield his business risk is the size of the hedge. The *optimum size* (the one that minimises the variance of the basis risk) "*is given by the regression coefficient obtained by regressing the business profits onto the weather index*" (Stephen Jewson and Anders Brix, 2005).

To protect themselves against future high values of the index, companies could also make use of *weather call options*. The difference is that long calls involve the payment of a single, fixed upfront payment. The pricing of call options consists of determining that premium.

The payoff (*p(x)*) determining the amount the seller of a long weather call option has to pay (or receive) depending on the value of the index can be expressed as:

$$p(x) = D * \max(x - K, 0) \tag{3.5}$$

On the other hand, if the hedger needs an insurance against low future values of the index He could take a long position on a *weather put option*. At the start of the contract, the buyer has to pay an upfront payment to the seller, who in turn, at the end, will pay (or receive) a certain amount on the basis of the payoff dependent on the value of the index.

The payoff function (*p(x)*) in that case is:

$$p(x) = D * \max(K - x, 0) \tag{3.6}$$

The graphs representing the payoffs of these three weather contracts and the financial contracts of the same name are identical.

4. TEMPERATURE MODELLING

Temperature-based derivatives are the most frequently traded and, since they are the cornerstone of our study and we would like to price them, it is important to focus on another fundamental concept, useful for reaching that purpose: the daily temperature modelling, a stochastic process describing the temperature's behaviour. Even if a single and precise model does not exist, the most commonly used is the one proposed by Peter Alaton (2002). In the following paragraphs, we will show the different steps characterizing this stochastic process, as described by Alaton. After this first theoretical section, we will apply the model on a database with temperatures from the last 10 years from the De Bilt meteorological station.

4.1 Mean Temperature

If we plot a time series of temperatures, we could observe that it is strongly affected by seasonality. It should be possible to model such seasonal dependence with a sine-function, for example, that could be expressed as follows:

$$\sin(\omega t + \varphi) \tag{4.1}$$

where t denotes the time (expressed in days) and $\omega = 2\pi/365$, given that the oscillation is one years and we are neglecting leap years. In addition to that, "*because the yearly minimum and maximum mean temperatures do not usually occur at January 1 and July 1 respectively, we have to introduce a phase angle* φ" (Alaton, 2002).

Furthermore, if we analyse the data more in depth we could notice a positive trend. The reasons for that are multiple:

1) *Random and predictable internal climate variability*. The simplest explanation for an apparent trend is that it is part of the random internal variability of the climate system.

2) *Urbanisation*. This phenomenon generally has a warming effect (not by chance, it is also called "*urbanisation heating effect*") which is not only local: temperatures, indeed, tend to rise in areas nearby big cities, meaning that they warm the surroundings.

3) *Anthropogenic climate change*. Man's activities, mainly the release of carbon dioxide (CO_2) into the atmosphere from burning fossil fuels, have had an effect on the climate system: for example, warming in some regions and cooling in others.

In our study, in order to catch this trend from data, we will assume, as a first approximation, that the warming trend is linear.

Thus, summing up, we could express a deterministic model to describe the mean temperature at time *t* (T_t^m) as follows:

$$T_t^m = A + Bt + Csin(\omega t + \varphi) \qquad (4.2)$$

where A, B, C and φ have to be determined and chosen so that the curve will fit well the data.

4.2 Is temperature deterministic?

In the last paragraph we described temperature as a deterministic variable, but is it really deterministic? The answer is: "No". Hence, to create a more realistic model able to describe better the temperature's behaviour, we need to add a stochastic component, some sort of noise. What Alaton discovered in his analysis was that the quadratic variation ($\sigma_t^2 \in R^+$) of the temperature varies across the different months of the year, but it remains nearly constant within each month. Especially, during winter the quadratic variation is much higher than during the rest of the year. Therefore, we assume that σ_t is a piecewise constant function with a constant value during each month and it can be written as:

$$\sigma_t = \sigma_i \, , i = 1, 2, \dots, 12$$

$$\sigma_t = \begin{cases} \sigma_1 & , during\ January \\ & \vdots \\ \sigma_{12} & , during\ December \end{cases}$$

Thus, such additional driving noise process of temperature would be defined as $\sigma_t W_t \, , t \geq 0$, where W_t is a standard Brownian motion.

4.3 Mean reversion

Another important aspect to take into account is that temperature cannot increase day by day for a long period, because we would get unrealistic values. Therefore, one solution to this problem could be to add a mean reverting component to our model in order to "*not allow the temperature to deviate from its mean value for more than short periods of time*" (Alaton, 2002). Now, by mixing all the above assumptions, we get the following stochastic differential equation (SDE):

$$dT_t = a(T_t^m - T_t)dt + \sigma_t dW_t \qquad (4.3)$$

where a denotes the speed of mean reversion, the rate at which the process mean reverts (the larger will be the value of a, the faster will be the mean-reverting process). The solution to this equation can be defined as "*Ornstein-Uhlenbeck* process", a stationary, Gaussian, and Markovian process that satisfies the following stochastic differential equation:

$$dX_t = \alpha(\mu - X_t)dt + \sigma dW_t \qquad (4.4)$$

where W_t is a Brownian motion, α is a positive number representing the rate of mean reversion, μ is the long-term mean of the process, and σ is the volatility, per square root time, of the random fluctuations that are modelled as Brownian motions. By looking at formula *(4.4)*, "*if we ignore the random fluctuations in the process due to dW_t, then we see that X_t has an overall drift towards a mean value μ. The process X_t reverts to this mean exponentially, at rate α, with a magnitude in direct proportion to the distance between the current value of X_t and μ*" (Planetmath.org). This can be seen by looking at the solution to the equation *(4.4)*, without considering random fluctuations:

$$X_t = \mu + (X_0 - \mu)e^{-\alpha(t-t_0)}. \qquad (4.5)$$

For this reason, the Ornstein-Uhlenbeck process is also called a mean- reverting process.

Though, there is a problem with this stochastic differential equation: it does not reverse to T_t^m in the long run. To solve this issue, we need to add another term to the drift:

$$\frac{dT_t^m}{dt} = B + \omega C cos(\omega t + \varphi) \; . \tag{4.6}$$

Now, if we assume the starting point to be T_s, we get the following model for temperature:

$$dT_t = \left\{\frac{dT_t^m}{dt} + a(T_t^m - T_t)\right\} dt + \sigma^t dW_t \; , \; t > s. \tag{4.7}$$

Through the help of the Ito's Lemma (used to determine the derivative of a time-dependent function of a stochastic process), we can find the solution to this differential equation, which is:

$$T_t = (T_s - T_s^m)e^{-a(t-s)} + T_t^m + \int_s^t e^{-a(t-\tau)}\sigma_\tau dW_\tau \; , \tag{4.8}$$

where

$$T_t^m = A + \; Bt + Csin(\omega t + \varphi).$$

4.4 Parameters Estimation

In this section we will follow the Alaton's method to estimate the unknown parameters in equation *(4.2)*. In order to do that we will fit the function:

$$Y_t = a_1 + a_2 t + a_3 \, sin(\omega t) + a_4 cos\,(\omega t) \tag{4.9}$$

to the temperature data using the method of least squares, meaning that we need to find a parameter vector $\xi = (a_1, a_2, a_3, a_4)$ that solves $\min_{\xi} ||\boldsymbol{Y} - \boldsymbol{X}||^2$, where **Y** is the vector with the elements contained in Y_t, while **X** is the temperature data vector. Hence, by applying this method, we will obtain the first four constants we were looking for:

$$A = a_1 \tag{4.10}$$

$$B = a_2 \tag{4.11}$$

$$C = \sqrt{a_3^2 + a_4^2} \quad (4.12)$$

$$\varphi = arctan\left(\frac{a_4}{a_3}\right) - \pi \quad (4.13)$$

At this point, only the variation of temperatures and the speed of mean reversion are missing. Alaton (2005) proposes the following formulas to estimate them:

$$\hat{\sigma}_\mu^2 = \frac{1}{N_\mu - 2}\sum_{j=1}^{N_\mu}\left(\tilde{T}_j - \hat{a}T_{j-1}^m - (1-\hat{a})T_{j-1}\right)^2 , \quad (4.14)$$

where, given a specific month $\mu = 1,2,...,12,\ \tilde{T}_j \equiv T_j - (T_j^m - T_{j-1}^m)$ and a can be defined as $\hat{a}_n = -\,log\left(\frac{\sum_{i=1}^n Y_{i-1}\{T_i - T_i^m\}}{\sum_{i=1}^n Y_{i-1}\{T_{i-1} - T_{i-1}^m\}}\right)$, inside which $Y_{i-1} = \frac{T_{i-1}^m - T_{i-1}}{\sigma_{i-1}^2}$ (with $i = 1, 2, ..., n$), where $\sigma_{i-1}^2 = \sigma_\mu^2 = \frac{1}{N_\mu}\sum_{j=0}^{N_\mu - 1}\left(T_{j+1} - T_j\right)^2$. Now, we have everything to simulate trajectories of the Ornstein-Uhlenbeck process.

5. WEATHER DERIVATIVES PRICING: THE ACTUARIAL PRICING METHOD

The pricing of weather derivatives is a harsh topic of discussion in the academic literature and an adequate pricing model has not still been found. This makes it difficult to market this kind of products. Andreas Müller and Marcel Grandi (2000) assert that "*the familiar option price model of Black - Scholes cannot be applied in the case of weather derivatives, simply because this model presupposes the existence of a negotiable underlying, or, in other words, derives the price of the derivative from the price of the actually existing underlying. This prerequisite is obviously not fulfilled in the case of weather derivatives - after all, what does weather cost?*". Also, the researchers Sean D. Campbell and Francis X. Diebold (2005) state that "*standard approaches to arbitrage-free pricing are inapplicable in weather derivative contexts*", so again the Black-Scholes model cannot be applied. As a consequence, the best way to price weather derivatives is through stochastic processes modelling the underlying variable.

The researcher Hélène Hamisultane (2008) focuses attention on the actuarial pricing method (the most commonly used in the weather market) and asserts that it "*evaluates the weather derivatives as being the conditional expectation of the future payment of these products, defined under the real probability of the underlying asset and to which is added a discounted compensation for the risk supported by the seller of the contract*". Thus, for example, the actuarial prices of weather call and put options and futures[2] on a HDD index at time *t* can be expressed as follows:

$$Call(t, T_t, I_t^H) = \delta e^{-r(T-t)} \left(E[\max{(I_T^H - K, 0)}|F_t] + \kappa\sigma_{max(I_T^H - K, 0)} \right) \quad (5.1)$$

$$Put(t, T_t, I_t^H) = \delta e^{-r(T-t)} \left(E[\max{(K - I_T^H, 0)}|F_t] + \kappa\sigma_{max(K - I_T^H, 0)} \right) \quad (5.2)$$

$$Futures(t, T_t, I_t^H) = \delta \left(E[I_T^H|F_t] + \kappa\sigma_{I_T^H} \right) \quad (5.3)$$

where δ is the tick size, K is the strike price (for the options), r is the risk-free rate, F_t indicates the available information about temperature until time *t*, and time *T* represents the maturity date of the contracts. Moreover, $\kappa\sigma_{max(I_T^H - K, 0)}$, $\kappa\sigma_{max(K - I_T^H, 0)}$ and $\kappa\sigma_{I_T^H}$, the so-called "*risk loading*" (Stephen Jewson and Anders Brix), represent the risk premiums, where $\sigma_{max(I_T^H - K, 0)}$, $\sigma_{max(K - I_T^H, 0)}$ and $\sigma_{I_T^H}$ measure the volatility of payoffs (in the case of options) and the volatility of the HDD index (in the case of futures). In our analysis, for sake of simplicity and given that we have not been able to find the market values of these instruments in order to derive κ , we will assume $\kappa = 0$ as Jewson and Brix did. In addition, as the researcher states in her paper, the actuarial method "*is based on the law of large numbers which clarifies that by repeating a large number of times an experience, in an independent way, we obtain a more and more reliable estimate of the true value of the expectation of the observed phenomenon*". The expectation under the real probability can be computed in either

[2] As defined by Hélène Hamisultane (2008, equations 8 and 9)

of the two following ways: 1) by using historical data ("*Burn Analysis*"); 2) by using the technique of Monte Carlo simulation. With the first approach, we accumulate the degree days of a specific year, we estimate the payoff of the derivative for this year, and then we repeat the process for other years. Finally, the expected price of the derivative will be defined by the average of annual payoffs. Instead, with Monte Carlo simulation technique, we use a model for daily average temperatures to generate a set of paths; for each of these paths we construct the HDD index which is used to calculate the payoff. Finally, the average of the payoffs from all the generated paths will be equivalent to the expectation of the derivative's price.

6. WEATHER DERIVATIVES PRICING: AN EMPIRICAL EXAMPLE WITH MONTE CARLO SIMULATION

In this section, we will show how to price a HDD call and a HHD put for the month of November (2019): first, we will simulate the temperature trajectories through the Ornstein-Uhlenbeck process, then we will price these financial products by means of Monte Carlo simulation. As already mentioned, this model is applied on a database with temperatures from the last 10 years from the De Bilt meteorological station, the main one in the Netherlands.

6.1 Temperature trajectories simulation

In order to create temperature simulations paths, first, we need to discretize equation[3] *(4.7)*. If we discretize dT_t to a time interval $\delta = T_j - T_{j-1}$, we should obtain the following result:

$$\delta T = T_j - T_{j-1} = \delta T^m + a\left(T_{j-1}^m - T_{J-1}\right)\delta t + \sigma_j \epsilon \sqrt{\delta t} \qquad (6.1)$$

where $\epsilon_{j=1}^{N-1}$ represent independent standard normally distributed variables.

[3] As represented by Konstantina Kordi (2012, equation 4.1)

Now, if we assume that the time interval δt is equivalent to 1 day, equation *(6.1)* can be written as:

$$T_j = T_{j-1} + \delta T^m + a\left(T_{j-1}^m - T_{J-1}\right) + \sigma_j \epsilon \qquad (6.2)$$
$$= (1-a)\left(T_{j-1}^m - T_{J-1}\right) + T_j^m + \sigma_j \epsilon$$

where $T_{j-1}^m = A + \ B(j-1) + Csin(\omega(j-1) + \varphi)$ and $T_j^m = A + \ B(j) + Csin(\omega(j) + \varphi)$. Finally, by applying the Alaton's suggested method in Excel, we can obtain the following parameters: $A = 9.89304$, $B = 0.00046$, $C = 7.49985$, $\varphi = -1.94611$, $\hat{a}_n = 0.22197$, $\sigma_{November} = 3.991$. Now, we just need to place them into equation (*6.2*) to obtain temperature trajectories.

6.2 Monte Carlo simulation

As we have already mentioned, in this section, we are going to calculate the prices of HDD call and HDD put options by means of Monte Carlo simulation.

The first step is to simulate the number of temperature trajectories for a certain period of time (starting from today's temperature). Then we accumulate each path in order to build a HDD index for each of them and we calculate their payoffs at maturity ($[max\ (I_T^H - K, 0)]$ for the HDD call and $[max(K - I_T^H, 0)]$ for the HDD put). Thereafter, we approximate the expectations of the actuarial prices of a HDD call and a HDD put via Monte Carlo simulation: once we have determined all the possible payoffs, we calculate the related call and put prices using equations *(5.1)* and *(5.2)* and, finally, we compute their arithmetic means in order to obtain a single price for each of them.

An important aspect to underlying is the strike price (*K*). In our study, since we have not been able to find a value of *K* from the CME, we create an "artificial strike price" through two different methods: 1) we calculate the average of all past Novembers' cumulative HDD indexes; 2) we determine the cumulative HDD index for each Ornstein-Uhlenbeck trajectory and then we calculate the average in order to find a single value. Thereafter, in order to analyse different scenarios, we also estimate, for each method, other two values of *K*: one by adding one standard deviation and the other one by subtracting it to the strike previously found.

6.3 Empirical Results

In order to replicate the aforementioned model, we use, first, Excel in order to find the parameters and to check the validity of such model, and then, we make use of Python to generate a Monte Carlo simulation that allow us to price a HDD call and a HDD put.

First, let's take a look at *Figure 1*. Here, we can observe how the simulated mean temperature, described in equation *(4.2)*, fits the historical daily mean temperature's path.

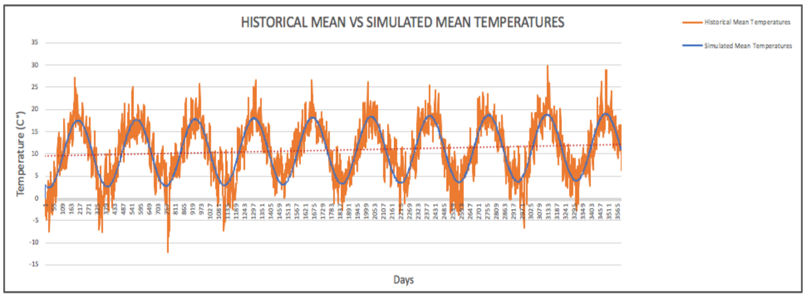

Figure 1: *Historical daily mean temperatures vs simulated mean temperatures (01/01/2010-28/10/2019)*

Once we have found all parameters (A, B, C, φ, a, σ_i) we can create the Ornstein-Uhlenbeck trajectories. In *Figure 2*, it is possible to see one trajectory we created in Excel: in this case too, the simulated temperatures (blue) seem to follow quite well the historical ones (orange) even with some spikes due to the presence of independent standard normally distributed variables. Later, in order to validate the model, first, we draw a graph to compare the historical temperatures and the simulated Ornstein-Uhlenbeckin trajectory for the month of

November 2018 (*Figure 3* below), then, we calculate the Relative Standard Errors (RSE) for each observation, and we plot them. The RSE tells us how much an estimate deviates from the actual population and it is calculated as follows:

$$\frac{Standard\ Error\ (= Obs.Temperature - Sim.Temperature)}{Obs.Temperature} \qquad (6.3)$$

Estimates with a RSE of 25% or greater are subject to high sampling error and should be used carefully. Now, if we take a look at *Figure 4*, we can observe that even if half of the values are greater than 25%, the simulated temperatures seem to follow more or less the same path.

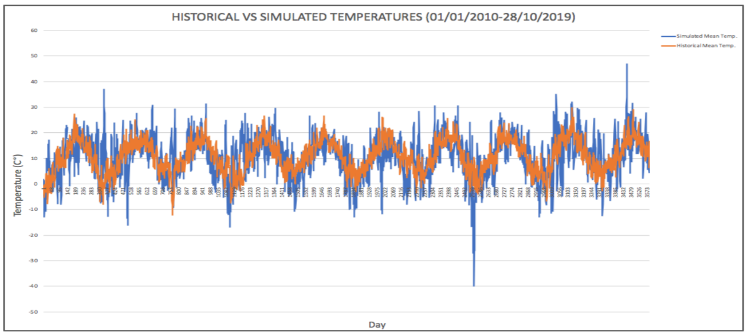

Figure 2: *Historical temperatures vs Ornstein-Uhlenbeck trajectory (01/01/2010-28/10/2019)*

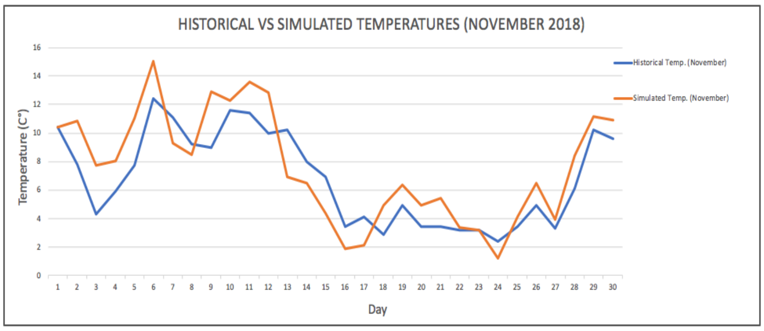

Figure 3: *Historical temperatures vs Ornstein-Uhlenbeck trajectory (November 2018)*

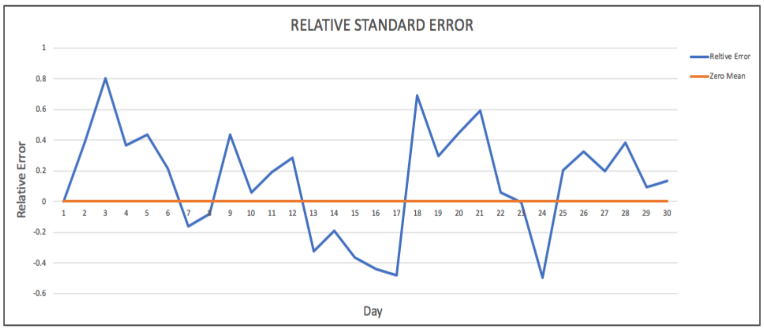

Figure 4: *Relative Standard Error (November 2018)*

Now that our temperature model has been created, we can price both the HDD call and the HDD put for the month of November 2019. As we can observe in the Python codes (*Appendix* - "*Empirical Analysis*"), we need other parameters for that purpose: 1) the one-month risk free rate of -0.443% (retrieved from the ECB website); 2) the tick size, equal to $20 per index point (as stated in the weather contracts section of the CME website); 3) the number of steps (*N_Steps*), which is the number of days in the month; 4) the number of simulations (*N_Reps*), to which we decided to assign a value of 10000; 5) the initial temperature, which is the one on the 31/10/2019 ($T_0 = 3.4\ C°$); 6) the exchange rate on the 01/11/2019 (€/1.1168$) in order to convert the tick size from $ to €, since we are dealing with European weather derivatives.

First, we create 10000 Ornstein-Uhlenbeck trajectories for the temperature of November 2019 and then, for each path, we calculate the payoff at maturity (30/11/2019). Before that step, we need to determine the strike price by making use of the two different aforementioned approaches. Thereafter, by applying the formulas (*5.1*) for the call and (*5.2*) for the put we find 10000 prices for each financial instrument. Finally, by simply averaging all the prices we will determine a single price for the HDD call and the HDD put. In the table below, we can observe the strike price results arising from these two methods[4].

[4] They can be observed respectively in *Screen 1* and *Screen 3* in the "*Empirical Analysis*" section of the *Appendix*.

METHOD 1			METHOD 2		
K_{PAST_1}	$K_{PAST\_1} + \sigma$	$K_{PAST\_1} - \sigma$	K_{EXP_1}	$K_{EXP\_1} + \sigma$	$K_{EXP\_1} - \sigma$
326.31	330.30	322.32	349.96	353.95	345.97

Table 1: *Strike Prices*

In the following table, the different option prices are presented.

	METHOD 1			METHOD 2		
	K_{PAST_1}	$K_{PAST\_1} + \sigma$	$K_{PAST\_1} - \sigma$	K_{EXP_1}	$K_{EXP\_1} + \sigma$	$K_{EXP\_1} - \sigma$
HDD Call	€ 2823.90 = $ 3153.73	€ 2747.63 = $ 3068.55	€ 2900.62 = $ 3239.42	€ 2379.34 = $ 2657.24	€ 2306.36 = $ 2575.76	€ 2453.02 = $ 2739.53
HDD Put	€ 48.01 = $ 53.62	€ 53.37= $ 59.61	€ 43.09 = $ 48.13	€ 87.08 = $ 97.25	€ 95.75 = $ 106.93	€ 79.13 = $ 88.38

Table 2: *HDD Call and Put prices*

Unfortunately, we did not find the market values to compare the results with, but we can say that they are reasonable. In fact, by looking at other papers, we noticed high values like ours (in case of call options). Furthermore, the huge difference between calls and puts is given by the fact that with the latter you are “gambling” on temperatures higher than 18°C, but it is very rare in the Netherlands during the period of November. The consequence is a payoff equal to zero in the majority of simulations, implying a low price. The opposite happens with call options.

7. WEATHER RISK MANAGEMENT: SOME EXAMPLES OF WEATHER RISK HEDGING

As we have seen until this point, the weather variable cannot be controlled, implying that “*as long as an enterprise's fortune is subject to the mercy of mother nature, weather risk will be a crucial part of the overall risk to manage.*” (Cao, Li and Wei, 2003)

7.1 Weather risk management strategies using options

Francisco Perez-Gonzalez and Hayong Yun (2010), using data from U.S. energy firms, found out that weather derivatives lead to higher market valuations, investments and leverage, demonstrating how risk management meaningfully affects valuation, investments, and financing decisions. As can be observed in *Table 3* (*Appendix* – "*Tables*"), there are four main weather hedging strategies using weather options traded at the CME.

7.2 Weather exposed industries

The industries dealing with a weather exposure may have an incentive in using weather derivatives: for soft drink producers, ski resorts, utility companies, construction companies and agriculture companies these instruments may prove beneficial for hedging weather exposure.

In the following lines, we will develop a broad overview of how the volumetric risk, caused by weather variables, could affect the energy sector and we will show some risk management strategies used to hedge against it.

7.2.1 Energy Sector

Weather has always been recognized as a source of risk in energy sector since it affects both energy consumption (in the short run) and energy production (in the long run). When we talk about energy consumption, temperature, for example, seems to have the highest effect on consumption of natural gas in winter and consumption of electricity mainly during summer. Regarding energy production, hydroelectric plants are strictly dependent on rainfall and wind power plants on wind speed.

If we take a look at *Figure 5* (*Appendix*– "*Figures*"), describing the relationship between the outdoor temperature and the residential energy consumption, it can be observed that it is not linear and has two branches. Temperature of 18°C plays the role of a threshold level since at that temperature the energy consumption is minimal: at lower temperatures the relationship is negative and there is a larger demand for heating, while at higher temperatures there is a positive relationship and the consumption of electricity is greater.

Now, let's consider the energy production[5]. In wind power plants, for example, weather risk occurs in the form of too low or too high wind speeds. To have a clear image of that, *Figure 6* (*Appendix*– "*Figures*") shows the relationship between wind speed (m/s) and power production (kW) for Vestas V90 wind turbine: as we may observe, the primary risk for wind power plants are unexpected wind speed variations generating lower than planned production outputs and consequently a lower than planned income.

In the following paragraphs, we will illustrate a weather hedging strategy, applied in the energy sector: a long put HDD option that a gas supplier could use to protect itself against a warm winter.

Let us assume that the company had analysed his historical sales and determined that November is the most volatile month, meaning that is the riskiest among the winter months. Thus, it is decided to buy an HDD option for November. Then, suppose that the strike value, calculated as historical average, is 120. The tick size of 1 HDD point is worth 1.000 monetary units of natural gas sales. Given that HDD index measures deviation of winter temperatures underneath 18°C, the higher the value of accumulated index the higher will be gas consumption. In other words, the company is worried about a drop in the accumulated HDD index under 120 points, so it takes a long position in put HDD option with strike of 120 HDDs and tick size of 1.000 monetary units. For this protection, he has to pay an upfront premium to the trader who, thinking the accumulated HDD index will rise above 120, is selling it: let's say, for example, 5.000 monetary units. Hence, the payoff formula for this weather option can be expressed by formula (*6.3*), implying that the profit function can be written as:

[5] As described by Ivana Stulec, Tomislav Bakovic and Domagoj Hruska (2012)

$$P_p = tick\ size * max(\mathrm{K} - \mathrm{HDD}, 0) - premium \qquad (7.1)$$

$$P_p = 1.000 * max(120 - HDD, 0) - 5.000$$

where HDD represents the observed accumulated value of weather index during the covered time period. Now, let's suppose that during November the HDD's value accounted for 100, meaning 20 HDDs under historical average (20.000 monetary units of sales less than predicted by average). In such circumstances, the gas supplier will choose to exercise the option, thus receiving a net payment of 15.000 monetary units. This payment would, to some extent, cover the loss of reduced sales of natural gas caused by the mild winter. On the other hand, in case of cold winter, the gas company would reach higher sales of natural gas and would use these extra profits to cover the upfront premium paid to enter the option contract.

9. CONCLUSION

Weather derivatives are complex financial products and their market is not well developed yet (in Europe even less than in the U.S.). However, it has great potentials to increase, since climate change is a very important factor to take into account, an element that affects businesses more and more due to its unpredictability.

Finally, let's consider some aspects of the presented pricing model that could be improved. Maybe, the main issue when pricing weather derivatives is to find a good model able to describe the weather. Our model is a simplification of the reality even if it seems to work quite well. Hence, one thing that it could be nice to develop it would be a more sophisticated model that takes into account a changing volatility. In that way, a model including stochastic volatility should give us more realistic results. Another good improvement to consider would be to include a term describing the jumps affecting temperature paths.

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APPENDIX

1. FIGURES

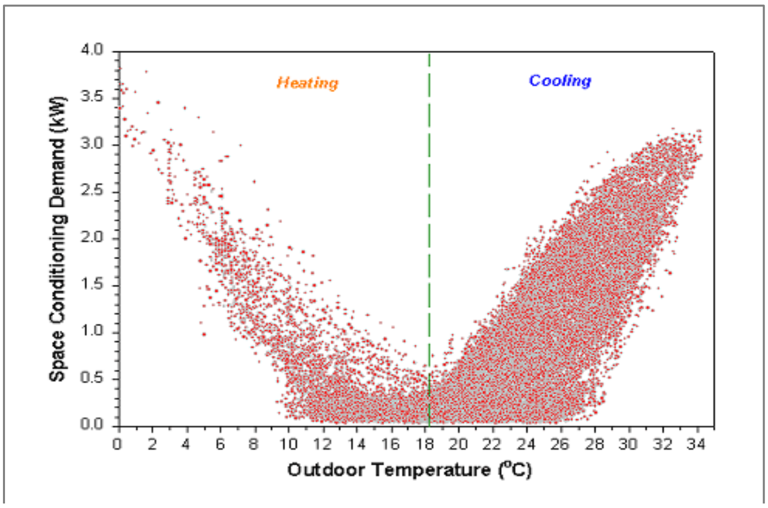

Figure 5: *Correlation between outdoor temperature and energy consumption*

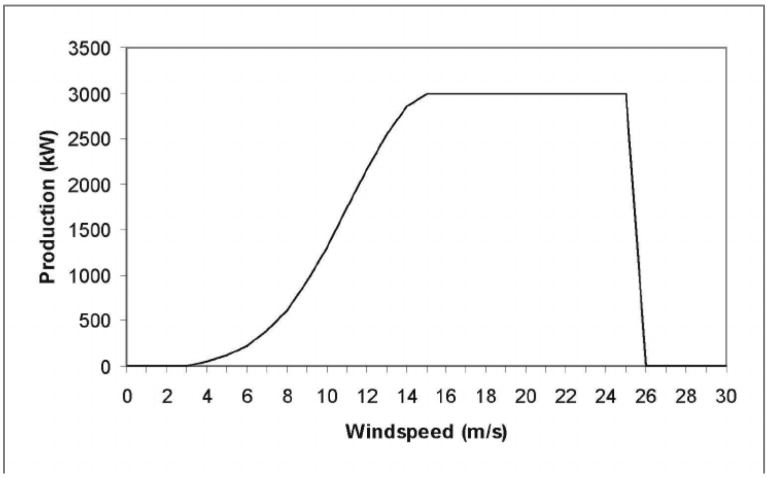

Figure 6: *Relationship between wind speed (m/s) and power production (kW)*

2. TABLES

Protection against	Contract's availability period	Hedging Strategy	Example of industry using it
Cold winter months	November to March	Call HDD	Construction company
Warm winter months	November to March	Put HDD	Utility company
Cold summer months	May to September	Put CDD or Put CAT	Beverage (or ice-cream) industry
Warm summer months	May to September	Call CDD or Call CAT	Agriculture

Table 3: *Weather risk management strategies using options*

```
import numpy as np
import pandas as pd
import os
import datetime

base_path = os.path.dirname(os.path.realpath("Weather Derivatives Pricing"))

data_folder_path = os.path.join(base_path, 'Temp_Input')

Sigma_path = os.path.join(data_folder_path, 'Variables.csv')
Sigma_dataset = pd.read_csv(Sigma_path, sep=',', header=0,
                            names=['Sigma'])

#PARAMETERS

days_per_year = 365
w = (2 * np.pi) / days_per_year

a_1 = 9.89303600254283
a_2 = 0.000463647111447223
a_3 = -2.7491474735184
a_4 = -6.97781690014465

A = a_1
B = a_2
C = 7.49984936669377
phi = -1.94610567092677
alpha = 0.221969716024534

N_Steps = 30 # NUMBER OF DAYS IN NOVEMBER
N_Steps_range = np.arange(N_Steps)
N_Reps = 10000 # NUMBER OF SIMULATIONS
N_Reps_range = np.arange(N_Reps)
T_0 = 3.4 # Temperature in De Bilt on the 31/10/2019

r = -0.00443 # 1-month risk-free rate (ECB) on the 01/11/2019

K_PAST_1 = 326.311111111111 # Average of cumulative HDD indexes in all
                              November months of the past years
K_PAST_2 = 330.302354513258 # Obtained by summing one sigma to the Strike
K_PAST_3 = 322.319867708965 # Obtained by detracting one sigma to the Strike

exchange_rate_Euro_Dollar = 1.1168 # Exchange rate on the 01/11/2019
tick_size = 20/exchange_rate_Euro_Dollar

#GENERATION OF O-U TRAJECTORIES

def Temperature_Paths(T_0, A, B, C, alpha, w, phi, Sigma_dataset, N_Steps, N_Reps):
    T_Paths = np.zeros((1+N_Reps, N_Steps+1))
    T_Paths[:,0] = T_0
    for t_i in N_Reps_range:
        for t_j in N_Steps_range:
            epsilon = np.random.normal(0,1)
            T_Paths[t_i, t_j+1] = A+B*(t_j+1)+C*np.sin(w*(t_j+1)+phi)+(1-alpha)*(T_Paths[t_i,t_j]
                                  -(A+B*t_j+C*np.sin(w*t_j+phi)))+Sigma_dataset['Sigma'][t_j]*epsilon
    return T_Paths

OU_Trajectories = Temperature_Paths(T_0, A, B, C, alpha, w, phi, Sigma_dataset, N_Steps, N_Reps)
```

Screen 1: *Parameters and Temperature O-U paths simulation*

```
#METHOD 1: USE AN ARTIFICIAL STRIKE PRICE BASED ON THE AVERAGE OF ALL PAST NOVEMBERS CUMULATIVE HDD
          (K_PAST_1) AND CALCULATE HDD PUT AND CALL PRICES

def HDD_call_put_price_1(OU_Trajectories, N_Steps, N_Reps, K_PAST_1, r, tick_size):
    Payoffs_HDD_call = np.zeros((N_Reps,1))
    Payoffs_HDD_put = np.zeros((N_Reps, 1))
    OU_Trajectories
    for t_i in N_Reps_range:
        HDD_daily = np.zeros((N_Steps, 1))
        for t_j in N_Steps_range:
            HDD_daily[t_j] = max(18-OU_Trajectories[t_i][1:N_Steps+1][t_j],0)
        HDD_cumulative = sum(HDD_daily)
        Payoffs_HDD_call[t_i] = np.exp(-r*N_Steps)*max((HDD_cumulative - K_PAST_1),0)
        Payoffs_HDD_put[t_i] = np.exp(-r * N_Steps) * max((K_PAST_1 - HDD_cumulative), 0)
    HDD_Call_Price = tick_size*np.average(Payoffs_HDD_call)
    HDD_Put_Price = tick_size*np.average(Payoffs_HDD_put)
    return [HDD_Call_Price,HDD_Put_Price]

HDD_CALL_PUT_1 = HDD_call_put_price_1(OU_Trajectories, N_Steps, N_Reps, K_PAST_1 , r, tick_size)

def HDD_call_put_price_2(OU_Trajectories, N_Steps, N_Reps, K_PAST_2, r, tick_size):
    Payoffs_HDD_call = np.zeros((N_Reps,1))
    Payoffs_HDD_put = np.zeros((N_Reps, 1))
    OU_Trajectories
    for t_i in N_Reps_range:
        HDD_daily = np.zeros((N_Steps, 1))
        for t_j in N_Steps_range:
            HDD_daily[t_j] = max(18 - OU_Trajectories[t_i][1:N_Steps + 1][t_j], 0)
            HDD_cumulative = sum(HDD_daily)
        Payoffs_HDD_call[t_i] = np.exp(-r*N_Steps)*max((HDD_cumulative - K_PAST_2),0)
        Payoffs_HDD_put[t_i] = np.exp(-r * N_Steps) * max((K_PAST_2 - HDD_cumulative), 0)
    HDD_Call_Price = tick_size*np.average(Payoffs_HDD_call)
    HDD_Put_Price = tick_size*np.average(Payoffs_HDD_put)
    return [HDD_Call_Price,HDD_Put_Price]

HDD_CALL_PUT_2 = HDD_call_put_price_2(OU_Trajectories, N_Steps, N_Reps, K_PAST_2, r, tick_size)

def HDD_call_put_price_3(OU_Trajectories, N_Steps, N_Reps, K_PAST_3, r, tick_size):
    Payoffs_HDD_call = np.zeros((N_Reps,1))
    Payoffs_HDD_put = np.zeros((N_Reps, 1))
    OU_Trajectories
    for t_i in N_Reps_range:
        HDD_daily = np.zeros((N_Steps, 1))
        for t_j in N_Steps_range:
            HDD_daily[t_j] = max(18 - OU_Trajectories[t_i][1:N_Steps + 1][t_j], 0)
            HDD_cumulative = sum(HDD_daily)
        Payoffs_HDD_call[t_i] = np.exp(-r*N_Steps)*max((HDD_cumulative - K_PAST_3),0)
        Payoffs_HDD_put[t_i] = np.exp(-r * N_Steps) * max((K_PAST_3 - HDD_cumulative), 0)
    HDD_Call_Price = tick_size*np.average(Payoffs_HDD_call)
    HDD_Put_Price = tick_size*np.average(Payoffs_HDD_put)
    return [HDD_Call_Price,HDD_Put_Price]

HDD_CALL_PUT_3 = HDD_call_put_price_3(OU_Trajectories, N_Steps, N_Reps, K_PAST_3, r, tick_size)
```

Screen 2: *HDD call and HDD put pricing method using a strike price based on the average of all past Novembers' cumulative HDD indexes*

```
#METHOD 2: CREATE AN ARTIFICIAL STRIKE PRICE BASED ON O-U TRAJECTORIES AND CALCULATE HDD PUT
          AND CALL PRICES

def Strike_Price(OU_Trajectories, N_Steps):
    OU_Trajectories
    for t_i in N_Reps_range:
        HDD_daily = np.zeros((N_Steps, 1))
        for t_j in N_Steps_range:
            HDD_daily[t_j] = max(18 - OU_Trajectories[t_i][1:N_Steps + 1][t_j], 0)
            HDD_cumulative = sum(HDD_daily)
    K = np.average(HDD_cumulative)
    return K

K_EXPECTED_1 = Strike_Price(OU_Trajectories, N_Steps)
K_EXPECTED_2 = K_EXPECTED_1 + Sigma_dataset['Sigma'][0]
K_EXPECTED_3 = K_EXPECTED_1 - Sigma_dataset['Sigma'][0]

def HDD_call_put_price_TRY_1(OU_Trajectories, N_Steps, N_Reps, K_EXPECTED_1, r, tick_size):
    Payoffs_HDD_call = np.zeros((N_Reps,1))
    Payoffs_HDD_put = np.zeros((N_Reps, 1))
    OU_Trajectories
    for t_i in N_Reps_range:
        HDD_daily = np.zeros((N_Steps, 1))
        for t_j in N_Steps_range:
            HDD_daily[t_j] = max(18-OU_Trajectories[t_i][1:N_Steps+1][t_j],0)
        HDD_cumulative = sum(HDD_daily)
        Payoffs_HDD_call[t_i] = np.exp(-r*N_Steps)*max((HDD_cumulative - K_EXPECTED_1),0)
        Payoffs_HDD_put[t_i] = np.exp(-r * N_Steps) * max((K_EXPECTED_1 - HDD_cumulative), 0)
    HDD_Call_Price = tick_size*np.average(Payoffs_HDD_call)
    HDD_Put_Price = tick_size*np.average(Payoffs_HDD_put)
    return [HDD_Call_Price,HDD_Put_Price]

HDD_CALL_PUT_TRY_1 = HDD_call_put_price_TRY_1(OU_Trajectories, N_Steps, N_Reps, K_EXPECTED_1 , r, tick_size)

def HDD_call_put_price_TRY_2(OU_Trajectories, N_Steps, N_Reps, K_EXPECTED_2, r, tick_size):
    Payoffs_HDD_call = np.zeros((N_Reps,1))
    Payoffs_HDD_put = np.zeros((N_Reps, 1))
    OU_Trajectories
    for t_i in N_Reps_range:
        HDD_daily = np.zeros((N_Steps, 1))
        for t_j in N_Steps_range:
            HDD_daily[t_j] = max(18 - OU_Trajectories[t_i][1:N_Steps + 1][t_j], 0)
            HDD_cumulative = sum(HDD_daily)
        Payoffs_HDD_call[t_i] = np.exp(-r*N_Steps)*max((HDD_cumulative - K_EXPECTED_2),0)
        Payoffs_HDD_put[t_i] = np.exp(-r * N_Steps) * max((K_EXPECTED_2 - HDD_cumulative), 0)
    HDD_Call_Price = tick_size*np.average(Payoffs_HDD_call)
    HDD_Put_Price = tick_size*np.average(Payoffs_HDD_put)
    return [HDD_Call_Price,HDD_Put_Price]

HDD_CALL_PUT_TRY_2 = HDD_call_put_price_TRY_2(OU_Trajectories, N_Steps, N_Reps, K_EXPECTED_2, r, tick_size)

def HDD_call_put_price_TRY_3(OU_Trajectories, N_Steps, N_Reps, K_EXPECTED_3, r, tick_size):
    Payoffs_HDD_call = np.zeros((N_Reps,1))
    Payoffs_HDD_put = np.zeros((N_Reps, 1))
    OU_Trajectories
    for t_i in N_Reps_range:
        HDD_daily = np.zeros((N_Steps, 1))
        for t_j in N_Steps_range:
            HDD_daily[t_j] = max(18 - OU_Trajectories[t_i][1:N_Steps + 1][t_j], 0)
            HDD_cumulative = sum(HDD_daily)
        Payoffs_HDD_call[t_i] = np.exp(-r*N_Steps)*max((HDD_cumulative - K_EXPECTED_3),0)
        Payoffs_HDD_put[t_i] = np.exp(-r * N_Steps) * max((K_EXPECTED_3 - HDD_cumulative), 0)
    HDD_Call_Price = tick_size*np.average(Payoffs_HDD_call)
    HDD_Put_Price = tick_size*np.average(Payoffs_HDD_put)
    return [HDD_Call_Price,HDD_Put_Price]

HDD_CALL_PUT_TRY_3 = HDD_call_put_price_TRY_3(OU_Trajectories, N_Steps, N_Reps, K_EXPECTED_3, r, tick_size)
```

Screen 3 : *HDD call and HDD put pricing method using an artificial strike price based on O-U trajectories*