ORIGINAL ARTICLE

Bergstrom, Blume, and Varian: Voluntary contributions and neutrality

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Abstract

Bergstrom, Blume, and Varian provided a neutrality result for the private provision of public goods that has inspired a considerable literature. The result has significant implications for income redistribution and broader policy interventions. This paper reviews the basic result and its applications, and discusses extensions to general private provision economies.

1 | INTRODUCTION

A surprising result in the analysis of private contributions to a public good is that the total level of provision is unaffected by any reallocation of income among consumers that leaves the set of contributors (those giving some positive amount) unchanged. This neutrality result was first established by Warr (1983) and extended by Bergstrom, Blume, and Varian (1986), henceforth BBV. That a transfer of income between consumers induces those receiving income to raise their contributions to the public good, and that this increase is offset by a reduction in contributions of those losing income, is unremarkable. What is striking is that the *exactly*

offsetting nature of the change in contributions is not special to particular utility functions but is a general consequence of the individual rationality conditions for the Nash equilibrium of the contribution game. Since the publication of BBV, the analysis has been extended to models with multiple private and public goods with the aim of identifying those redistributions of endowment, within a variety of situations, that leave the private provision equilibrium unchanged. Although the focus of this paper is upon the implications of the BBV neutrality result and its extensions, we should note, before proceeding, that the publication of BBV inspired a much broader literature on private provision. Some of the theoretical applications of BBV are noted below, whereas Muñoz-Herrera and Nikiforakis (2019) provide an extensive review of experimental investigations.

When a model is constructed to analyze an economic issue, several factors are important: the concept of equilibrium adopted, the existence and uniqueness of equilibrium, and the comparative statics of policy interventions. The insightful approach of BBV fully explored all these aspects for the scenario in which a private good can either be consumed or contributed to public good provision. To be precise, in contrast to a personalized pricing scheme, BBV analyzed a game-theoretic model in which the strategy of each player was their voluntary contribution to the provision of a public good. For this noncooperative model of public good provision, the proof of existence of a Nash equilibrium was reinforced by the description of sufficient conditions to obtain uniqueness (see also Bergstrom, Blume, & Varian, 1992; Fraser, 1992) confirming that there was no way to escape from the neutrality through appeal to multiple equilibria. In addition, to analyze the extent to which government provision of a public good impacts on private contributions, BBV also provided a comprehensive characterization of the comparative statics of redistributive policy.

An earlier literature (Chamberlin, 1974, 1976; McGuire, 1974; Young, 1982) had demonstrated the inefficiency of the private provision equilibrium and (inconclusively) discussed the consequence of changing the number of potential contributors upon the degree of efficiency. The publication of BBV sparked fresh research into private provision due to the generality of their neutrality result and its obvious implications for redistributive policies. The work of BBV also motivated research into what have since then become known as "aggregate games" because the neutrality result is a consequence of the equilibrium level of public good provision being a function of aggregate income. In fact, the neutrality result can be obtained as a special case of a more general theorem that characterizes when Nash equilibria are independent of the distribution of agents' characteristics (see Bergstrom & Varian, 1985). A detailed analysis of public good provision from the aggregate game perspective is provided in Cornes and Hartley (2005). A more recent line of research, exemplified by Allouch and King (2019), has extended the BBV model to public good provision within networks that have local interaction between contributors.

The private provision model has found useful applications in a very wide range of contexts. As examples, it has formed the basis for understanding noncooperative behavior in the household (Lundberg & Pollak, 1994) and as the basis of the threat point in bargaining models of the household (Lundberg & Pollak, 1996). It is also one of the limit points of the semi-cooperative household model of d'Aspremont and Dos Santos Ferreira (2019). The contributions of different nations to military coalitions have been modeled as contributions to a public good (Sandler & Hartley, 2001). The model has been applied in environmental economics to analyze green markets (Kotchen, 2006) and global environmental problems (Buchholz & Konrad, 1994; Murdoch & Sandler, 1997). Johnson (2004) used the model to study open source software and Lévy-Garboua, Montmarquette, Vaksmann, and Villeval (2017) to model contributions to a voluntary mutual insurance pool. This is only a small sample of the very many applications.

It is our aim in this paper to report and discuss the neutrality results that have followed the publication of BBV's pioneering article. The model of BBV not only brought strategic behavior in public good provision to the forefront of public economic theory, but it also raised a variety of challenges that motivated many further research articles. In our discussion, we specify the crucial assumptions upon which the various neutrality results rely and therefore highlight the conditions on the primitives that provide an explanation for their failure. In particular, the consideration of multiple private and public goods allows the implications of relative price changes to be investigated. Moreover, further extensions and generalizations of the work by BBV lead us to other considerations, including altruistic behavior and its effect on neutrality.

The remainder of the paper is structured as follows. In Section 2 we describe a general private provision economy with multiple private and public goods. In Section 3 we simplify the economy to a single public good to prove a version of the BBV neutrality result and to discuss some significant implications of that result. Section 4 summarizes and compares different neutrality results for economies with multiple public goods. Altruism and neutrality are considered in Section 5. In Section 6, we conclude with a few final remarks.

2 | PRIVATE PROVISION ECONOMY

We begin by describing a very general model of an economy with private provision based on the papers by Faias, Moreno-García, and Wooders (2014, 2015). Consider an economy with a finite set of private commodities that can be used either for consumption or as inputs into the production of public goods. The economy has a finite number of firms which are characterized by technologies for producing public goods, and a finite number of consumers who are endowed with private goods, have preferences over the consumption of private and public goods, and share the potential profits of firms.

To be precise, let $\mathcal{E}$ be an economy with a finite number ℓ of private goods and a finite number k of public goods. There is a finite set $C = \{1, ..., n\}$ of consumers and a finite set $\mathcal{F} = \{1, ..., k\}$ of firms. Each consumer $i \in C$ is characterized by her endowment of private goods $e_i \in \mathsf{R}_{++}^{\ell}$, and by her preference relation over the commodity space $\mathsf{R}_{+}^{\ell+k}$, represented by a continuous, concave, and monotone-increasing utility function $U_i : \mathsf{R}_{+}^{\ell+k} \to \mathsf{R}_{+}$. Define the aggregate endowment by $e \equiv \sum_{i=1}^{n} e_i$.

A firm $h \in \mathcal{F}$ produces public good h by means of a technology that is described by a production function $F_h : \mathsf{R}_{+}^{\ell} \to \mathsf{R}_{+}$, converting private goods into that firm's public good. We assume that each F_h is continuous and concave. Each consumer $i \in C$ receives a share $\delta_i^h \geq 0$ of firm h's profit and $\sum_{i=1}^{n} \delta_i^h = 1$ for each h. A *price system* is a vector $(p, q) \in \mathsf{R}_{+}^{\ell+k}$, where $p = (p^j, j = 1, ..., \ell)$ denotes the vector of prices for the ℓ private commodities and $q = (q^h, h = 1, ..., k)$ denotes the vector of prices for the k public goods.

Given a price system $(p, q) \in \mathsf{R}_{+}^{\ell+k}$, each firm $h \in \mathcal{F}$ chooses the vector of inputs in R_{+}^{ℓ} that maximizes its profits $\Pi_h(y) \equiv q_h \cdot F_h(y) - p \cdot y$. Given a price system $(p, q) \in \mathsf{R}_{+}^{\ell+k}$ and profits Π_h for each firm h, each consumer $i \in C$ chooses private goods consumption and voluntary contributions to public good provision. Each consumer takes as given the contributions of the other consumers to public goods. That is, given a vector $(g_j, j \in C, j \neq i)$ of voluntary contributions, each consumer i solves the problem:

$$\max_{(x_i, \varrho_i) \in \mathsf{R}_+^\ell \times \mathsf{R}_+^k} U_i(x_i, g_{-i} + \varrho_i)$$

$$\text{such that} \quad p{\cdot}x_i + q{\cdot}\varrho_i \leq p{\cdot}e_i + \sum_{h=1}^k \delta_i^h \Pi_h,$$

where $g_{-i} = \sum_{j=1, j \neq i}^n \varrho_j.$

Definition   A private provision equilibrium for the economy $\mathcal{E}$ is a price system (p, q), a vector of inputs $y = (y_h \in \mathsf{R}_+^\ell; \ h = 1, ..., k)$ for firms, an allocation of private commodities $x = (x_i \in \mathsf{R}_+^\ell; \ i = 1, ..., n)$, and an assignment of voluntary contributions $\sum_{i=1}^n \varrho_i = (g^h \in \mathsf{R}_+; \ h = 1, ..., k) = g$ such that

(i) (x_i, ϱ_i) solves the optimization problem of consumer i for every $i \in C$;
(ii) y_h maximizes firm h's profit, for every $h \in \mathcal{F}$;
(iii) $\sum_{i=1}^n x_i + \sum_{h=1}^k y_h \leq \sum_{i=1}^n e_i$;
(iv) $g^h \leq F_h(y_h)$ for every public good h.

An existence result for this equilibrium can be found in Faias et al. (2014). The proof of existence is based on an adaptation of the Shapley–Shubik market game with a continuum of players (consumers and firms) which has the property that the strategic Nash equilibria of the market game induce a private provision equilibrium of the original finite economy. For the purpose of that proof, the technologies of the firms are assumed to exhibit constant returns to scale.[1]

3 | NEUTRALITY AND INCOME DISTRIBUTION

By reducing the economy of Section 2 to a single public good and a single private good, the model includes as a particular case the one formulated by Bergstrom et al. (1986). BBV provided a theorem that generalized Warr's (1983) neutrality result and relied only on properties resulting from optimization by individual agents without recourse to first-order conditions. BBV identified the property required for a redistribution of endowments to define a perturbed economy with an equilibrium in which all consumers had the same private goods allocation as before the redistribution and the total public good contribution was unchanged. To be precise, the income redistributions that guarantee neutrality are those made among contributors and leave contributors able to afford their original equilibrium private good allocation. This condition is analogous to that required for the demonstration of Ricardian equivalence. This is because of the shared underlying logic that individuals find it optimal to make behavioral changes that undo government intervention.

We now prove a variant of the BBV neutrality results for a private provision economy with a single public good but multiple private goods. The single firm producing the public good is labeled $h = 1$. Define the wealth of consumer i by $w_i \equiv p{\cdot}e_i + \delta_i^1 \Pi_1$, where the public good is

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[1]We remark that Florenzano (2009) considers a model with externalities that are more general than all of those cited in this manuscript. Among further additional results, she shows the existence of a variety of equilibria (see also Florenzano, 2003).

chosen as numeraire so that its unit price $q = 1$. The optimization for consumer i can therefore be written as

$$\max_{(x_i, g^i) \in \mathbb{R}_+^\ell \times \mathbb{R}_+} U_i(x_i, g^i)$$

subject to

$$p \cdot x_i + g^i \leq w_i + g_{-i},$$

$$g^i \geq g_{-i}.$$

The second constraint reflects the fact that the contribution of i to the public good, $\varrho_i = g^i - g_{-i}$, must be nonnegative.

From the optimization we obtain the demand function for the public good

$$g^i = \max\{\zeta_i(w_i + g_{-i}), g_{-i}\}.$$

The demand for the public good is assumed to be normal. This assumption is adopted to ensure that the inverse of the demand function can be used in the proof of neutrality below. Neutrality can be established without this assumption but uniqueness cannot.

Assumption SI. The public good is normal: $\zeta_i(w_i + g_{-i})$ is strictly increasing in $w_i + g_{-i}$.

The important distinction for the analysis of neutrality is between consumers who contribute to the public good and those who do not. This is because the subsequent focus is upon redistribution of income among contributors.

Definition Given an income distribution $\{w_i, i \in C\}$, consumer i is *a contributor* at the private provision equilibrium if $g^i > g_{-i}$, otherwise consumer i is *a noncontributor*.

The set of contributors is denoted by Γ, where $\Gamma \subseteq C$, and $\#\Gamma = \gamma$. Similarly, the set of noncontributors is denoted by Ξ, where $\Xi \equiv C/\Gamma$.

Neutrality refers to the fact that the equilibrium does not change when income is redistributed. To make this statement precise it is necessary to have a formal statement of redistribution. The definition below allows both endowments and profit shares to be redistributed between consumers. The key features of the redistribution are that no consumer is driven to negative wealth and that no resources are lost during the process of redistribution.

Definition A *redistribution* among the set $R \subseteq C$ is a vector

$$r = (\hat{e}_i, \hat{\delta}_i^1), \quad r \in \mathbb{R}^\ell \times \mathbb{R}, \quad i \in R,$$

such that $0 \leq e_i + \hat{e}_i$, $\sum_{i \in R} \hat{e}_i = 0$, $0 \leq \delta_i^1 + \hat{\delta}_i^1 \leq 1$, and $\sum_{i \in R} \hat{\delta}_i^1 = 0$.

Denote the vector of wealth before a redistribution r by $(\tilde{w}_i, i \in R)$, and the vector after the redistribution by $(\bar{w}_i, i \in R)$, where

$$\tilde{w}_i = p{\cdot}e_i + \delta_i^1 \Pi_1,$$

and

$$\bar{w}_i = p{\cdot}[e_i + \hat{e}_i] + [\delta_i^1 + \hat{\delta}_i^1]\Pi_1.$$

Observe that no resources are lost by redistribution since

$$\sum_{i \in R}\bar{w}_i = \sum_{i \in R}(p{\cdot}e_i + \delta_i^1 \Pi_1) + \sum_{i \in R}(p{\cdot}\hat{e}_i + \hat{\delta}_i^1 \Pi_1) = \sum_{i \in R}\tilde{w}_i.$$

Similarly, denote the private provision equilibrium before a redistribution r by $(\tilde{x}_i, i \in C, \tilde{g})$ and the equilibrium after the redistribution by $(\bar{x}_i, i \in C, \bar{g})$. Finally, it is necessary to define what is meant by a neutral redistribution.

Definition A redistribution is *neutral* if $\bar{x}_i = \tilde{x}_i \ \forall \ i \in C$ and $\bar{g} = \tilde{g}$.

Our version of the neutrality result of BBV can now be stated.

Theorem 1. *A redistribution r among the set of contributors, Γ, that satisfy $p{\cdot}\hat{e}_i + \hat{\delta}_i^1 \Pi_1 < g^i - g_{-i} \ \forall \ i \in \Gamma$ is neutral.*

Proof. At the private provision equilibrium $g^i = g \ \forall \ i \in \Gamma$ and $g = \zeta_i(w_i + g_{-i})$. Under assumption SI, $\zeta_i^{-1}(g)$ exists, so we can write $w_i + g_{-i} = \zeta_i^{-1}(g)$. From the definition of g_{-i}, $\sum_{i \in \Gamma}(w_i + g_{-i}) = \sum_{i \in \Gamma}w_i + (\gamma - 1)g$. The equilibrium value of g is therefore the solution to $\sum_{i \in \Gamma}w_i + (\gamma - 1)g = \sum_{i \in \Gamma}\zeta_i^{-1}(g)$. Since $\sum_{i \in \Gamma}\bar{w}_i = \sum_{i \in \Gamma}\tilde{w}_i$ we have $\bar{g} = \tilde{g}$. The identity $\bar{g} = \tilde{g}$ implies that $\zeta_i(\bar{w}_i + \bar{g}_{-i}) = \zeta_i(\tilde{w}_i + \tilde{g}_{-i})$ and hence from the individual budget constraints that $\bar{x}_i = \tilde{x}_i$. □

Clearly, it is necessary that the redistribution is conducted within the set of contributors. Any noncontributor involved in a redistribution will change their demand for the private goods and neutrality will fail. Similarly, if the redistribution causes one or more contributors to become noncontributors then their level of private consumption will not be sustainable and neutrality will fail. Hence, it is critical that the redistribution involves only contributors and does not change the set of contributors.

It was noted by Warr (1983) that the levels of private consumption are also unaffected by redistribution of income. Itaya, de Meza, and Myles (1997) used this result to show that if the consumers have the same preferences, the level of private consumption must be identical for all contributors to the public good. Consequently, since the level of public good is by definition the same for all contributors, there must be equalization of utilities. Hence, income distribution (within a range) is irrelevant for the distribution of utility. The equalization of utility and the independence of total public good provision from the distribution of income derive from the private provision economy being an aggregate game where the key statistic is total income in

society.[2] From this follows the additional implication that an extra dollar of income for contributor i raises i's utility by the same amount as an extra dollar to contributor j raises i's utility. That is, contributors are indifferent as to who should receive any extra income that society may generate.

To prove these statements we now assume:

Assumption IU.   For all $i \in C$, $U_i(x_i, g) = U(x_i, g)$.

Under Assumption IU the demand function for the public good is given by $g^i = \max\{\zeta(w_i + g_{-i}), g_{-i}\}$ for all $i \in C$, and for the set of contributors $w_i + g_{-i} = \zeta^{-1}(g)$. Since $g_{-i} = g - \varrho_i$ we have $\varrho_i = w_i - \zeta^{-1}(g) + g$. Let $w^* = \zeta^{-1}(g) + g$. Clearly, $\varrho_i > 0$ if $w_i > w^*$ and $\varrho_i = 0$ if $w_i \leq w^*$, so w^* is the minimum income required to choose to be a contributor.

We can prove the following:

Theorem 2.   *Under Assumption IU, at the private provision equilibrium: (a) $x_i = x$ $\forall i$ with $w_i \geq w^*$ and (b) $U(x_i, g) = U_\Gamma$ $\forall i$ with $w_i \geq w^*$, and $U(x_i, g) < U_\Gamma$ $\forall i$ with $w_i < w^*$.*

Proof. Take any $k, l \in \Gamma$. Then $\varrho_k - \varrho_l = [w_k - \zeta^{-1}(g) + g] - [w_l - \zeta^{-1}(g) + g]$ $= w_k - w_l$. Evaluating the budget constraints at the equilibrium, $p \cdot x_k + g = w_k + g_{-k}$ and $p \cdot x_l + g = w_l + g_{-l}$ so that $p \cdot x_k - p \cdot x_l = [w_k + g_{-k}] - [w_l + g_{-l}] = [w_k - w_l] - [\varrho_k - \varrho_l] = 0$. Given Assumption IU, it follows $x_k = x_l$. Since k and l were arbitrarily chosen this is true for all i with $w_i > w^*$. It holds for $w_i = w^*$ by continuity. Given (a), (b) follows immediately. □

Observe that if all private goods are normal, then it follows that $x_i < x$ $\forall i$ with $w_i < w^*$. With a single private good, this is true directly from the budget constraint. The content of this theorem is that the consumption levels, and the utilities, of all contributors are equalized at the private provision equilibrium. This occurs despite potential inequality in the distribution of income. Noncontributors by definition have a lower utility level.

The result that utilities are equalized seems to suggest that as long as the social welfare function is symmetric and concave in utilities, income redistribution is redundant as a tool of economic policy. This is not the entirely the case for two reasons. First, the neutrality result only applies to redistribution between contributors. Different issues arise when noncontributors are affected by redistribution. Second, neutrality only applies to redistributions that do not change the set of contributors. For more general redistributions, Itaya et al. (1997) proved that social welfare is not a concave function of income, with the nonconcavity caused by changes in the set of contributors. In particular, a welfare increase necessarily arises if a *small* transfer is made such that the consumer from whom income is taken just ceases contributing to the public good. The policy implication of this result is that a uniform distribution of income does not maximize social welfare. To maximize welfare it is always necessary to have sufficient inequality to ensure that some consumers lack the income to contribute to the public good.

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[2]Recall that in the proof of Theorem 1 the equilibrium of provision is the solution to $\sum_{i \in \Gamma} w_i + (\gamma - 1)g = \sum_{i \in \Gamma} \zeta_i^{-1}(g)$, so g necessarily depends on $\sum_{i \in \Gamma} w_i$.

An even more surprising result emerges when noncontributors are involved in the redistribution. Cornes and Sandler (2000) showed that when there are three or more consumers a redistribution of income from noncontributors to contributors can be a Pareto improvement. To see how this works, assume there are three consumers and that only one of them contributes to the public good. Now transfer $1 from each of the noncontributors to the contributor. The income of the contributor has risen by $2 so, if the public good is normal, the level of contribution will increase. If the increased contribution has value greater than $1 to each of the noncontributors, a Pareto improvement has been realized. Notice that the transfers require collective action to be effective, and that it becomes easier to generate a Pareto improvement as the number of noncontributors rises. This result should be considered alongside the demonstration by Andreoni (1988) that when the number of potential contributors is large, only the richest consumer contributes and, in a slightly different model, by Fries, Golding, and Romano (1991) that in a large but finite economy only one type will contribute. This creates precisely the conditions under which an inequality-increasing redistribution will be a Pareto improvement. A related result is demonstrated by Varian (1994) who shows (with two contributors) that a Pareto improvement can be obtained if each contributor subsidizes the cost of provision for the other. This is because the cost of the induced extra provision of public good is equal to the subsidy. If the size of the subsidies is chosen optimally, then a Lindahl equilibrium is obtained. In a similar vein, Liu (2019) demonstrates that small matching grants will always secure a Pareto improvement if all consumers are contributors and may secure an improvement when there are some noncontributors.

Building on these contributions, the impact of private provision on measured inequality has been explored by Dasgupta and Kanbur (2011). They show that private provision increases the absolute difference in real income between any two noncontributors relative to the difference in nominal incomes. If the noncontributors are sufficiently poorer than the contributors, or if the contributors are sufficiently numerous, the gap between contributors and noncontributors increases as well. Private provision therefore increases inequality if an absolute measure is used, and may increase a relative measure.

These observations about the potential benefits of inequality carry implications for the design of tax instruments and institutions. Itaya, de Meza, and Myles (2002) explore optimal income and commodity taxes when there is private provision. The nonconcavity in the social welfare function implies that standard results do not extend to this setting since some inequality is always desirable even when lump-sum taxes are available. The structure of the optimal tax system depends on the extent of "see-through": The degree to which the consumers comprehend the link between government revenue and government public good provision. With see-through there is neutrality with respect to any taxes that fall only upon contributors— a conclusion based again on the observation that consumers find it rational to react in a manner that neutralizes the government intervention. Olszewski and Rosenthal (2004) analyze alternative political institutions for choosing the marginal rate of a proportional income tax. They show that many institutions (including majority voting and Leviathan government) deliver a Pareto improvement over nonintervention unless the society has a small number of members and there is considerable inequality in the initial distribution. Uler (2008) demonstrates that the level of private provision increases with the tax rate when contributions are tax deductible and that in a society that uses the tax system to ensure perfect equality the public good is provided efficiently.

4 | EXTENSIONS

There is no unique extension of BBV's model to more general settings since alternative choices can be made about the nature of production technology and the form of redistribution. As a consequence, the study of neutrality for economies with multiple private and public goods is not an easy task. In this section we aim to summarize and compare some generalizations of the neutrality result and explore further implications of income redistribution.

We observe that BBV's model ruled out relative price changes since it had a single private good and a linear production technology for the public good. This is in contrast to the general model of Section 2 in which redistribution involving noncontributors can affect the demand for private goods and impact on relative prices. We also observe that when the technologies of the firms exhibit constant returns to scale the distribution of firms' profits among shareholders has no impact on the equilibrium outcome since profit is zero in equilibrium. This is not the case when nonlinear technologies are considered for the production of public goods. To illustrate the consequence of relative price changes, we state a (partial) neutrality result for redistribution among the noncontributors in the BBV model and then contrast this with the potential outcomes in more general models.

Consider the BBV model with a single private good ($\ell = 1$), a single public good ($k = 1$), and a linear production technology allowing units to be chosen so that $p = q = 1$. Since the production technology is linear, we have $\Pi_1 = 0$. Define $w_i^* = \zeta_i^{-1}(g) + g$ as the minimum income required for i to become a contributor. Wealth before the redistribution r is $(\tilde{w}_i, i \in R)$, and after the redistribution is $(\bar{w}_i, i \in R)$. These wealth levels generate corresponding equilibria $(\tilde{x}_i, i \in C, \tilde{g})$ and $(\bar{x}_i, i \in C, \bar{g})$. The definition of partial neutrality can now be given.

Definition　A redistribution is *partially neutral* if $\bar{g} = \tilde{g}$.

It is now possible to state a partial neutrality result for redistribution among noncontributors.

Theorem 3.　*In the BBV model, a redistribution r among the set of noncontributors, Ξ, that satisfies $\hat{e}_i < w_i^* - w_i \; \forall \; i \in \Xi$ is partially neutral.*

Proof.　The redistribution does not change the set of contributors, nor does it affect the existing contributors. Hence, $\sum_{i \in \Gamma} \bar{w}_i = \sum_{i \in \Gamma} \tilde{w}_i$ which implies $\bar{g} = \tilde{g}$. ☐

Redistribution among the noncontributors cannot be neutral because it will affect the consumption of private good by noncontributors. Each noncontributor will increase (or decrease) consumption of the private good exactly by the size of the change in income caused by the redistribution. Total demand for the private good is unchanged and there is no impact on the contributors. Noting this, the consequence of having many private goods can now be explored. Redistribution among the noncontributors will raise the income of some, and reduce the income of others. Except in very special cases, these income effects will cause changes in the aggregate demand for private goods and consequent changes in equilibrium relative prices. The relative price changes will also affect the contributors to the public good. Although their endowments have not changed, the relative price changes can affect the real incomes of contributors. This will impact on private consumption and contribution to the public good.

These observations about relative price changes are formalized in a series of papers by Villanacci and Zenginobuz (2005, 2006, 2007). They consider an economy with several private goods and a finite set of firms producing one public good, which is privately financed, with nonlinear production technologies described by strictly concave transformation functions.

Villanacci and Zenginobuz (2006) show that an increase in the quantity of public good provided through a redistribution involving contributors and noncontributors does not imply a Pareto improvement. Reciprocally, a Pareto improvement need not require an increase in the public good level. They also show that there exists a generic set of economies for which a government intervention—the tax-financed purchase of private goods to use as inputs to public good production—Pareto-improves upon the equilibrium outcome in which there is only private provision of the public good. In the BBV model with a single private good and a single public good, this intervention would not affect the equilibrium level of public good. With many private goods, the change in private good use caused by government purchases for public good production can affect relative prices and, hence, generate nonneutrality. In this way, a nonneutrality result in terms of utility levels that involves an intervention policy beyond redistribution of endowments or incomes is obtained.

On the other hand, Villanacci and Zenginobuz (2007) demonstrate an extension of the neutrality result of BBV to the case of many private goods by considering a redistribution of the numeraire commodity among the contributors. Consequently, this confirms that for a redistribution of the numeraire commodity to be nonneutral, the participation of noncontributors in the redistribution is required. For those cases of nonneutrality, Villanacci and Zenginobuz (2007) show that no clear conclusion concerning the impact of redistribution on the level of public good is possible with multiple private commodities: redistribution between contributors and noncontributors can increase or decrease the equilibrium level of public good. This result is also driven by relative price changes since, in the BBV model with one private good, redistribution from a noncontributor to a contributor will increase the level of public good.

A different approach has been explored by Villanacci and Zenginobuz (2012) who introduce a nonprofit firm that has the aim of maximizing the amount of public good produced, and whose costs are financed by the total voluntary contributions that consumers provide. Analogous neutrality and nonneutrality results to Villanacci and Zenginobuz (2007) are obtained by Villanacci and Zenginobuz (2012) for the case in which the public good is provided by a nonprofit firm instead of competitive firms. In all of this sequence of papers, relative price effects and the nonconstant returns to scale play a crucial role. The implications that each of these features (i.e., impact of relative prices and strictly concavity of transformation function) has separately are not studied.

5 | MULTIPLE PUBLIC GOODS

The neutrality results for economies with a single public good apply to redistribution among contributors such that no individual gives up an amount of income in excess of her initial individual contribution level. This argument is very general, and applies equally to the cases of identical preferences and heterogeneous preferences. The extension of the result to an economy with multiple public goods is not immediate when preferences are heterogeneous.

When consumers have identical preferences the argument extends naturally if there are no relative prices changes: the multiple public goods can be viewed as a single Hick's aggregate and

the model essentially collapses back to the single public good case. Kemp (1984) provided an informal extension of Warr's argument to the case of multiple public goods with heterogeneous preferences by assuming away relative price changes. In contrast, Villanacci and Zenginobuz (2007) show that relative price changes can eliminate neutrality. There is an additional distinction between the single and multiple public good cases. With a single public good it is possible to establish uniqueness of equilibrium. In contrast, with multiple public goods there is a continuum of equilibria. The continuum arises because the private provision equilibrium identifies the total amount of each public good that is provided but not the individual contributions that make up the total.

The case of preference heterogeneity raises an additional issue since different consumers potentially contribute to different subsets of the total set of public goods. If redistribution is conducted within the group of contributors to a particular public good, then the same neutrality argument applies as for a single public good. For redistribution among contributors to different public goods, Cornes and Itaya (2010) show that neutrality holds if there are consumers that provide links across the sets of contributors. With a single private commodity and two public goods, A and B, the neutrality theorem of BBV holds if redistribution leaves unchanged the aggregate income of the three following sets of agents: those who only contribute to A, those who only contribute to B, and those who contribute to both A and B. To extend these results to more than two public goods Cornes and Itaya (2010) define the following concepts:

> Two individuals share an active interest in a public good A at an allocation if, at that allocation, both individual's marginal rates of substitution between that public good and the private good equal the relative cost ratio q_A/p, where q_A is the unit cost of producing A and p is the price of the single private good. Moreover, individuals h and $h + k$ are linked if there exists a set of agents, indexed by $1, ..., k$, and a set of public goods indexed by A_j, such that individuals $h + j - 1$ and $h + j$ share an active interest in the public good A_j, for each $j \in \{1, ..., k\}$.

For example, if consumers 1 and 2 contribute to public good A, whereas 2 and 3 contribute to public good B, then 2 links the contributor sets. Neutrality applies: If income is redistributed from 1 to 3, then 1 will reduce contribution to A, 2 will increase contribution to A and reduce that to B, whereas 3 will increase contribution to B. Cornes and Itaya (2010) extend Theorem 7 in BBV and show that an income redistribution that is restricted to a set of linked individuals, and that maintains the links between them, has no effect on the original equilibrium allocation. This result is an example of the general proposition of Bernheim and Bagwell (1988) that neutrality applies whenever there are sufficient connections for the private sector to react in a way that undoes government actions.

More recently, in a context with multiple public and private goods, Faias et al. (2015) consider redistribution of endowments that may involve all commodities and not only the numeraire good. These redistributions are restricted to the set of consumers that contribute a positive amount to each and every public good and who can be referred to as *full contributors*. Assuming that public goods are produced by means of concave production functions, they prove that neutrality still holds for redistributions among full contributors that allow each consumer to get her initial equilibrium bundle of private goods. The proof of this result is constructive, and uses an algorithm that is able to transfer the impact of endowment reallocation into the individual private provision of public goods. To be precise, the first step focuses on the agents whose value of endowment has decreased after the redistribution, and defines for them new lower contributions given by the difference between the value of the new

endowment and the value of equilibrium private consumption bundle. Analogously, in a second step, those contributors for whom the value of the endowments has increased, increase their contributions to public goods. In this way, one specifies new contributions such that the new private provision sequentially fills in the gap between equilibrium provision and the lower provision that may arise after the first step. By construction, under this mechanism, the variation of the value of individual equilibrium contributions equals the value of the endowment variation and neutrality is established.

To summarize, the aforementioned works lead us to argue that, to obtain neutrality results addressing multiple public goods, it becomes crucial both to discern the properties of the reallocation of resources and to identify the set of consumers among which the corresponding redistribution takes place. The table recapitulates the results we have described in abbreviated form.

Production function	Private goods	Public goods	Neutrality redistribution requirements	Nonneutrality redistribution requirements
Linear	One	One	BBV (1986, Th. 1)	BBV (1986, Th. 4)
			Among contributors	Between contributors and noncontributors
Linear	One	Two	BBV (1986, Th. 7)	
			Among contributors of the same public good, and among contributors to both	
Linear	One	Multiple	CI (2010, Prop. 4)	
			Among linked agent	
Strictly concave	Multiple	One	VZ (2007, Th. 5) and VZ (2012)	VZ (2007, Th. 7 and Th. 8) and VZ (2012, Th. 1)
			Private numeraire good	Private numeraire good
			Among contributors	Among noncontributors, and between contributors and noncontributors
Concave	Multiple	Multiple	FMW (2015, Th. 4.1)	
			Private multiple goods	
			Among contributors to all public goods	

Abbreviations: BBV, Bergstrom, Blume, and Varian; CI, Cornes and Itaya; FMW, Faias, Moreno-García, and Wooders; VZ, Villanacci and Zenginobuz.

6 | ALTRUISM, CONJECTURES, AND CUSTOMS

The private provision model has widespread application but that has not prevented discontent about the model on the basis of the results that it generates. Boadway and Keen (2000) describe the neutrality and inequality results as "apparently implausible" and use this motivation to

justify exploration of modified versions of the model. This section reviews some of the modifications and their implications for neutrality.

Individual private contributions to the provision of a public good can be thought of as an example of altruistic behavior. Actually, in the related literature, the fact that someone contributes without caring about their own contribution per se, has been referred to as pure altruism[3] and gives rise to research connecting public good provision and the degree of altruism.

In this regard, there is an interesting reinterpretation of neutrality. As shown by Warr (1982) and Roberts (1984), in the standard model with one private good and one public good, government contributions to the public good financed by lump-sum taxes will crowd out private contributions dollar-for-dollar. This complete crowding-out effect is related to the neutrality of income redistribution by recasting it as a family of neutral tax increases and tax decreases. Andreoni (1988) argues that this invariance proposition leads us to the paradox that an exogenous change in gifts has no significant impact on the total provision of the public good.

To explain empirical observations on charitable giving, Andreoni (1989) develops a model in which individuals contribute to public goods not only because they benefit directly from the public good but also because they get some benefit from their gift per se (the *warm glow* effect). The addition of this second, seemingly selfish reason for contributing, gives rise to impure altruism and allows us to shed light on how the relative degrees of altruism matter.

Andreoni (1989) shows that a redistribution of income will increase the total supply of the public good if and only if the person receiving the transfer is more altruistic than the person losing income. If all individuals have purely altruistic preferences, then the redistribution will be neutral. Thus, pure altruism is a sufficient condition for redistributions to have a neutral effect on the amount of public good that is provided. However, pure altruism becomes a necessary condition for the redistribution to have a neutral effect on all consumption, including private goods. Further, Andreoni (1990) formalizes the application of the impure altruism setting to charitable giving, and calibrates the model to measure the effects of possible policies.

The original BBV model of private provision has been generalized not only to multiple private and public goods and to the consideration of impure altruism but also to further scenarios that allow the deepening of neutrality results. For instance, Gradstein, Nitzan, and Slutsky (1994) introduce incomplete information about income, which implies several difficulties for demonstrating neutrality, and identify sufficient conditions for neutrality to hold. Recently, Faias and Moreno-García (2019) argue that, when analyzing the provision of public goods, the distribution of the level of utilization may be significant. They consider utilization as an additional variable, and extend the analyses of altruistic behavior, neutrality, congestion, and other externalities that can be captured using this approach.

An alternative modification of the BBV model is to replace the Nash assumption with some form of conjectural variation. Cornes and Sandler (1985) show that if the conjectural variation is positive (meaning that every consumer expects an increase in the contributions of others in response to an increase in own contribution) the equilibrium will have greater total public good supply than the Nash equilibrium. Even though the level of provision changes, Dasgupta and Itaya (1992) demonstrate that the neutrality result still remains valid for any constant conjecture. Turning to nonconstant conjectures, Sugden (1985) argues that the only consistent conjectures are negative, so that the equilibrium provision of the public good will be zero under

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[3]A terminology that can be questioned since there is a private benefit arising from the increase in the total quantity of public good.

reasonable assumptions. Moving to non-Nash conjectures can therefore alter the equilibrium level of the public good but does not necessarily eliminate the neutrality. Overall, the conjectural approach is arbitrary given the good game-theoretic motives for focusing upon the Nash equilibrium. If the Nash equilibrium of the private provision economy does not agree with observations, it would seem that the objectives of the households and the social rules they observe should be reconsidered, not the conjectures they hold when maximizing.

The individualism embodied in the BBV model can be removed by modifying the rules of social behavior. Sugden (1984) considers the principle of reciprocity by which each consumer considers the contributions of others and contrasts them to what they feel they should make. If the contributions of others match, or exceed, what is expected then the consumer obliged by the social rule to make a similar contribution. Formalizing this notion, Sugden (1984) proves that the outcome is Pareto efficient if all consumers agree upon the expected contributions. In all other cases a Pareto improvement can be attained by increasing the supply of public good. Bordignon (1990) has provided a formalization of Kantian behavior that leads to similar conclusions. It has been shown experimentally (Chakravarty & Fonseca, 2017) that a sense of social group identity can raise contributions to group-specific public goods. A similar impact on provision can be obtained if contributors can conduct coordinated punishment of noncontributors, and this mechanism can function more effectively as group size increases (Hwang, 2017).

7 | FINAL REMARKS

The work of Bergstrom et al. (1986) on the private provision of public goods has had a profound impact on the public economics literature. This highly influential paper has proved to be an inspiration for a large number of researchers. It has been influential in a wide range of topics in economic theory, policy, and experiments. Moreover, the noncooperative approach to the private provision of public goods pioneered by BBV has become a standard component of graduate courses all over the world.

This paper has focused on neutrality theorems for private provision economies. We have proved a version of the original BBV theorem and have discussed the extension of the result to more general settings. The focus upon the neutrality result is justified because of the important implications it has for the impact of redistributive policies. As we have noted, such policies will be ineffective if they involve only the set of contributors, and policies that reduce income inequality may actually cause social welfare to fall if they do not take into account the impact on private provision. In addition, the logic of the neutrality result carries over into the ineffectiveness of government provision of the public good due to complete crowding-out. These observations are significant for policy design and have merited empirical investigation of the extent to which they apply in practice. The results in BBV have been an inspiration for many subsequent papers, and many more papers inspired by BBV can be expected to follow.

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REFERENCES

Allouch, N., & King, M. (2019). Constrained public goods in networks. *Journal of Public Economics Theory*, *21*, 895–902.

Andreoni, J. (1988). Privately provided public goods in a large economy: The limits of altruism. *Journal of Public Economics*, *35*, 57–73.

Andreoni, J. (1989). Giving with impure altruism: Applications to charity and Ricardian equivalence. *Journal of Political Economy*, *97*, 1447–1458.

Andreoni, J. (1990). Impure altruism and donations to public goods: A theory of warm glow giving. *Economic Journal*, *100*, 464–477.

Bergstrom, T., Blume, L., & Varian, H. (1986). On the private provision of public goods. *Journal of Public Economics*, *29*, 25–49.

Bergstrom, T., Blume, L., & Varian, H. (1992). Uniqueness of Nash equilibrium in private provision of public goods, an improved proof. *Journal of Public Economics*, *49*, 391–392.

Bergstrom, T., & Varian, H. (1985). When are Nash equilibria independent of the distribution of agents' characteristics? *Review of Economic Studies, LII*, *52*, 715–718.

Bernheim, B. D., & Bagwell, K. (1988). Is everything neutral? *Journal of Political Economy*, *96*, 308–338.

Boadway, R., & Keen, M. (2000). Redistribution. In A. B. Atkinson, & F. Bourguignon (Eds.), *Handbook of income distribution*. Amsterdam: Elsevier.

Bochet, O., Nikiforakis, N., Reuben, E., Wooders, J., & Wooders, M. (2019). Introduction to the JPET special issues commemorating works of James Andreoni, Theodore Bergstrom, Larry Blume, and Hal Varian. *Journal of Public Economic Theory*, *21*, 799–803.

Bordignon, M. (1990). Was Kant right? Voluntary provision of public goods under the principle of unconditional commitment. *Economic Notes*, *3*, 342–372.

Buchholz, W., & Konrad, K. A. (1994). Global environmental problems and the strategic choice of technology. *Journal of Economics*, *60*, 299–321.

Chakravarty, S., & Fonseca, M. A. (2017). Discrimination via exclusion: An experiment on group identity and club goods. *Journal of Public Economic Theory*, *19*, 244–263.

Chamberlin, J. (1974). Provision of collective goods as a function of group size. *American Political Science Review*, *68*, 707–716.

Chamberlin, J. (1976). A diagrammatic exposition of the logic of collective action. *Public Choice*, *26*, 59–74.

Cornes, R. C., & Hartley, R. (2005). Aggregative public good games. *Journal of Public Economic Theory*, *9*, 201–219.

Cornes, R. C., & Itaya, J. (2010). On the private provision or two or more public goods. *Journal of Public Economic Theory*, *12*(2), 363–385.

Cornes, R. C., & Sandler, T. (1985). The simple analytics of pure public good provision. *Economica*, *52*, 103–116.

Cornes, R. C., & Sandler, T. (2000). Pareto-improving redistribution and pure public goods. *German Economic Review*, *1*, 159–186.

Dasgupta, I., & Kanbur, R. (2011). Does philanthropy reduce inequality?*Journal of Economic Inequality*, *9*, 1–21.

Dasgupta, D., & Itaya, J.-I. (1992). Comparative statics for the private provision of public goods in a conjectural variations model with heterogeneous agents. *Public Finance*, *47*, 17–31.

d'Aspremont, C., & Dos Santos Ferreira, R. (2019). Enlarging the collective model of household behavior: A revealed preference analysis. *Economic Theory*, *68*, 1–19.

Faias, M., & Moreno-García, E. (2019). *On the private provision and use of public goods*. MPRA Paper No. 95244.

Faias, M., Moreno-García, E., & Wooders, M. (2014). A strategic market game approach for the private provision of public goods. *Journal of Dynamics and Games*, *1*(2), 283–298.

Faias, M., Moreno-García, E., & Wooders, M. (2015). On neutrality with multiple private and public goods. *Mathematical Social Sciences*, *76*, 103–106.

Florenzano, M. (2003). *General equilibrium analysis*. Boston, Dordrecht, London: Kluwer Academic Press.

Florenzano, M. (2009). *Walras–Lindahl–Wicksell: What equilibrium concept for public goods provision? I—The convex case*. CES Working Papers 2009.09. Documents de Travail du Centre d'Economie de la Sorbonne.

Fraser, C. D. (1992). The uniqueness of Nash equilibrium in the private provision of public goods. *Journal of Public Economics*, *49*, 389–390.

Fries, T. L., Golding, E., & Romano, R. (1991). Private provision of public goods and the failure of the neutrality property in large finite economies. *International Economic Review*, *32*, 147–157.

Gradstein, M., Nitzan, S., & Slutsky, S. (1994). Neutrality and the private provision of public goods with incomplete information. *Economics Letters*, *46*, 69–75.

Hwang, S.-H. (2017). Conflict technology in cooperation: The group size paradox revisited. *Journal of Public Economics Theory*, *19*, 875–898.

Itaya, J.-I., de Meza, D., & Myles, G. D. (1997). In praise of inequality: Public good provision and income distribution. *Economics Letters*, *57*, 289–296.

Itaya, J.-I., de Meza, D., & Myles, G. D. (2002). Income distribution, taxation, and the private provision of public goods. *Journal of Public Economic Theory*, *4*, 273–297.

Johnson, J. P. (2004). Open source software: Private provision of a public good. *Journal of Economics & Management Strategy*, *11*, 637–662.

Kemp, M. C. (1984). A note of the theory of international transfers. *Economics Letters*, *14*, 259–262.

Kotchen, M. J. (2006). Green markets and private provision of public goods. *Journal of Political Economy*, *114*, 816–834.

Liu, W. (2019). Participation constraints of matching mechanisms. *Journal of Public Economic Theory*, *21*, 488–511.

Lévy-Garboua, L., Montmarquette, C., Vaksmann, J., & Villeval, M. C. (2017). Voluntary contributions to a mutual insurance pool. *Journal of Public Economic Theory*, *19*, 198–218.

Lundberg, S., & Pollak, R. A. (1994). Noncooperative bargaining in marriage. *American Economic Review*, *84*, 132–137.

Lundberg, S., & Pollak, R. A. (1996). Bargaining and distribution in marriage. *Journal of Economic Perspectives*, *10*, 139–158.

McGuire, M. (1974). Group homogeneity and aggregate provision of a pure public good under Cournot behavior. *Public Choice*, *18*, 107–126.

Muñoz-Herrera, M., & Nikiforakis, N. (2019). James Andreoni and the quest for others in our utility function. *Journal of Public Economic Theory*, *21*, 804–811.

Murdoch, J. C., & Sandler, T. (1997). The voluntary provision of a pure public good: The case of reduced CFC emissions and the Montreal Protocol. *Journal of Public Economics*, *63*, 331–349.

Olszewski, W., & Rosenthal, H. (2004). Politically determine income inequality and the provision of public goods. *Journal of Public Economic Theory*, *6*, 707–735.

Roberts, R. D. (1984). A positive model of private charity and public transfers. *Journal of Political Economy*, *92*, 136–148.

Sandler, T., & Hartley, K. (2001). Economics of alliances: The lessons for collective action. *Journal of Economic Literature*, *39*, 869–896.

Sugden, R. (1984). Reciprocity: The supply of public goods through voluntary contributions. *Economic Journal*, *94*, 772–787.

Sugden, R. (1985). Consistent conjectures and voluntary contributions to public goods: Why the conventional theory does not work. *Journal of Public Economics*, *27*, 117–124.

Uler, N. (2008). Public goods provision and redistributive taxation. *Journal of Public Economics*, *93*, 440–453.

Varian, H. R. (1994). Sequential contributions to public goods. *Journal of Public Economics*, *53*, 165–186.

Villanacci, A., & Zenginobuz, Ü (2005). Existence and regularity of equilibria in a general equilibrium model with private provision of a public good. *Journal of Mathematical Economics*, *41*, 617–636.

Villanacci, A., & Zenginobuz, Ü. (2006). Pareto improving interventions in a general equilibrium model with private provision of public goods. *Review of Economic Design*, *10*, 249–271.

Villanacci, A., & Zenginobuz, Ü. (2007). On the neutrality of redistribution in a general equilibrium model with public goods. *Journal of Public Economic Theory*, *9*(2), 183–200.

Villanacci, A., & Zenginobuz, Ü. (2012). Subscription equilibrium with production: Nonneutrality and constrained suboptimality. *Journal of Economic Theory*, *147*, 407–425.

Warr, P. G. (1982). Pareto optimal redistribution and private charity. *Journal of Public Economics*, *19*, 131–138.

Warr, P. G. (1983). The private provision of a public good is independent of the distribution of income. *Economics Letters*, *13*, 207–211.

Young, D. (1982). Voluntary purchase of public goods. *Public Choice*, *38*, 73–86.