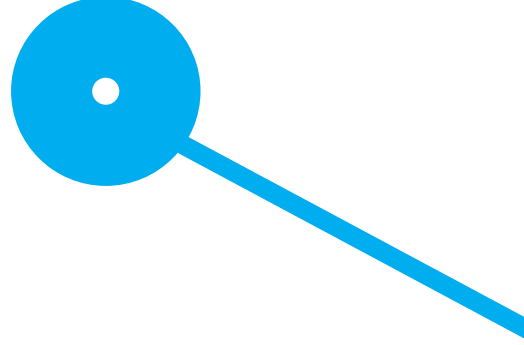


Optimization of Aluminium Profiles  
Production Planning  
Elisabete Sofia Queirós Almeida



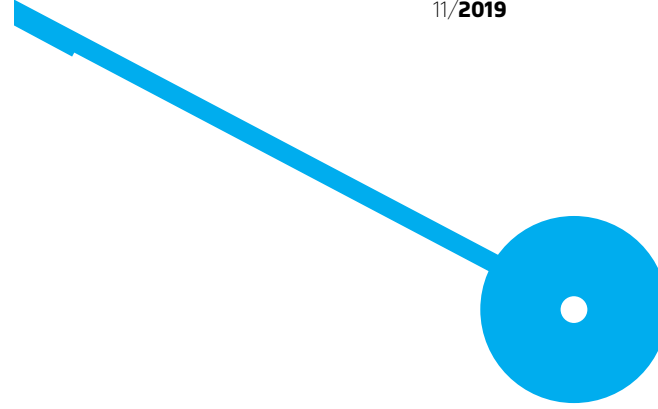
11/2019

Elisabete Sofia Queirós Almeida. Optimization of Aluminium Profiles  
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# Optimization of Aluminium Profiles Production Planning

Elisabete Sofia Queirós Almeida

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# Optimization of Aluminium Profiles Production Planning

Elisabete Sofia Queirós Almeida

Supervisors

PhD Eliana Oliveira da Costa e Silva

PhD Aldina Isabel de Azevedo Correia

PhD Maria de Fátima de Almeida Ferreira

# Jury

## President

PhD Alexandra Maria da Silva Braga

School of Management and Technology - Polytechnic of Porto

## Supervisors

PhD Eliana Oliveira da Costa e Silva

School of Management and Technology - Polytechnic of Porto

PhD Aldina Isabel de Azevedo Correia

School of Management and Technology - Polytechnic of Porto

PhD Maria de Fatima de Almeida Ferreira

School of Management and Technology - Polytechnic of Porto

## External Examiner

PhD Rui Jorge Ferreira Soares Borges Lopes

University of Aveiro

*“O sucesso é ir de fracasso em fracasso sem perder entusiasmo”*

*(Winston Churchill)*

# Acknowledgements

Um trabalho de mestrado é uma longa viagem, que inclui uma trajetória permeada por inúmeros desafios, tristezas, incertezas, alegrias e muitos percalços pelo caminho. Trilhar este caminho só foi possível com o apoio e incentivo de várias pessoas, a quem dedico especialmente este projeto e às quais estarei eternamente grata.

Especialmente às minhas orientadoras, Professora Doutora Eliana Costa e Silva, Professora Doutora Aldina Correia e Professora Doutora Fátima Almeida, pela sua orientação, total apoio, disponibilidade, ensinamentos, incentivos e dedicação incondicional. Pela partilha do saber e valioso contributo na elaboração desta dissertação, muito obrigada!

Obrigada também à instituição, coordenação do curso, bem como aos restantes professores, por me proporcionarem ferramentas e material necessário para uma boa aprendizagem.

Aos meus pais, António e Fernanda, pelas palavras de incentivo e encorajamento valorizando o meu potencial, nos momentos mais difíceis. Sem vocês não teria esta oportunidade de lutar pelos meus sonhos e objetivos.

Ao meu irmão, Miguel, e restantes familiares, pela ajuda, apoio, incentivo, colaboração e encorajamento.

Ao meu namorado, Filipe, por todo o incentivo, carinho, companheirismo durante todo este longo percurso. Obrigada por sempre acreditares nas minhas capacidades e por todo o amor e dedicação.

A minha profunda gratidão aos meus amigos, sem vocês não teria tido a mesma força de vontade, obrigada pelas palavras de conforto e apoio.

Finalmente, ao Professor Doutor Rui Borges Lopes pelas críticas construtivas que enumerou na arguência do presente trabalho e que contribuíram para esta versão final.

Agradeço por todo o apoio, palavras de incentivo constantes, carinho e amizade, pela oportunidade de crescimento académico e também pessoal.

A Todos, Muito Obrigada!



# Resumo

No atual ambiente empresarial, a crescente competitividade impulsiona as empresas a implementar estratégias de otimização para assegurar ou melhorar a sua posição no mercado.

Nesse sentido é crucial tomar as melhores decisões do ponto de vista do planejamento da produção. A produção de perfis de alumínio apresenta vários desafios ao responsável da produção. Este trabalho aborda um caso real de uma empresa do setor metalúrgico, cuja área de negócio é o desenvolvimento e produção de perfis de alumínio para aplicação em diversas áreas, como obras de engenharia, arquitetura e indústria em geral. O objetivo é minimizar o desperdício ocorrido na produção, designado de sucata, através da minimização dos tempos de preparação, que ocorrem quando a matriz é trocada, respeitando os prazos de entrega do produto e garantindo a qualidade. Este estudo de caso centra-se num problema de sequenciamento de flow shop que envolve tempos de preparação dependentes da sequência decorrentes da necessidade de alterar as ferramentas usadas no processo de extrusão de alumínio. Um modelo de programação inteira mista é desenvolvido e implementado para responder ao desafio da empresa. O problema será formulado na linguagem de modelagem AMPL e será usado o solver Gurobi para resolver instâncias reais extraídas dos dados providenciados pela empresa. Os resultados obtidos com o modelo desenvolvido são comparados com a regra de despacho FIFO, em termos da soma dos tempos de preparação (e consequentemente do número de trocas de matrizes), bem como o cumprimento do prazo de entrega. Além disso, embora o procedimento atual da empresa não seja conhecido, os resultados obtidos são comparados com a média de trocas de matriz obtidas a partir dos dados históricos de produção fornecidos pela empresa. Conclui-se que, utilizando as soluções obtidas com o modelo desenvolvido, a quantidade de sucata gerada durante o processo de produção é minimizada, uma vez que, o modelo garante a minimização da soma dos tempos de preparação e minimiza o número de vezes que ocorrem trocas de matriz.

**Palavras Chave:** Escalonamento, Tempo de Processamento, Otimização, Programação Linear Inteira Mista, Extrusão de Alumínio, Sucata.





# Abstract

In the current business world, the increasing competitiveness forces companies to adopt optimization strategies to ensure or improve their market position. It is crucial to take the best decisions from the production planning point of view.

The production of aluminium profiles poses several challenges to the production manager. This work addresses a real case of a company that operates in the aluminium market, whose core business is the development and production of aluminium profiles for application in several areas, such as, engineering, architecture and industry works in general. The aim is to minimize production waste, commonly known as scrap, through minimization of setup times, that occur when the die is changed, while respecting product delivery times and maintaining quality. This case study focuses on a scheduling flow shop problem involving sequence-dependent setup times arising from the need to change the tools used in the process of aluminium extrusion. A mixed integer programming model is developed and implemented for answering the company's challenge. The problem is formulated the AMPL modeling language and the Gurobi solver is used to solve real instances extracted from data provided by the company. The results obtained with the developed model are compared to the dispatching rule FIFO, in terms the sum of the setup times (and consequently the number of exchanges of dies), as well as the fulfillment of the deadline. Furthermore, although the company's current procedure is not known, the results obtained are compared with the average of die changes obtained from the historical production data provided by the company. It is observed that by using the solutions obtained with the developed model, the quantity of scrap that is generated during the production process is minimized, since the model guarantees the minimization of the sum of the setup times, therefore minimizes the number of times there are die changes.

**Keywords:** Scheduling, Setup Times, Optimization, Mixed Integer Linear Programming, Aluminium Extrusion, Scrap.



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# Acronyms

<b>CR</b>	Critical Ratio
<b>EDD</b>	Earliest Due Date
<b>FIFO</b>	First In First Out
<b>FSP</b>	Flowshop Scheduling Problem
<b>JSP</b>	Job Shop Scheduling Problem
<b>LP</b>	Linear Programming
<b>MP</b>	Mathematical Programming
<b>MILP</b>	Mixed Integer Linear Programming
<b>MINLP</b>	Mixed Integer Nonlinear Programming
<b>MIP</b>	Mixed Integer Programming
<b>NLP</b>	Nonlinear Programming
<b>OR</b>	Operational Research
<b>SPT</b>	Shortest Processing Time
<b>SQP</b>	Quadratic Programming



# Chapter 1

## Introduction

### 1.1 Study Contextualization

Nowadays, the increasing globalization of different types of markets leads companies to invest repeatedly in order to improve and optimize all processes that comprise their activities. Companies have to be permanently making decisions, either at a productive level or with human resources, aiming for constant cost savings, and for continuous improvement of their services. Industrial needs are becoming always more complex pushed by an ever more demanding market and an increasingly fierce competition. In fact, companies are required to meet tight deadlines that are set with the customers while maintaining high quality products. Failing to do so can result in significant financial and productivity losses, and ultimately in losing the customer [29]. The productive activities of companies are one of the areas that most studies and developments have generated, providing an increasingly efficient and effective production. It is precisely in this area that production planning and scheduling is encompassed. This aims to structure, as best as possible, the production process of various products, organizing and scheduling the different jobs that constitute it.

Thus, effective sequencing and scheduling have become a requirement for industries to maintain competitive positions into today fast-changing business environment. Production scheduling is the major component for the efficient management of production. This class of problems is among the most studied in Operational Research (OR) field and presents applications in several areas [29]. However, the reality of the production systems is more complex than the theoretical classical models and there is a noticeable gap between the theory and the application of the existing methods [29].

Scheduling refers to the allocation of resources to jobs over a set of given time periods, optimizing one or more criteria [69]. Finding the best schedule can be very easy or very difficult, depending on the shop environment, the process constraints and the performance indicators. The main goal is to find a schedule that satisfies all resource constraints while minimizing the objective.

According to Harjunkski *et al.* [40] the major decisions scheduling problem are: (i) selection and sizing of tasks; (ii) assignment of tasks to processing units; and (iii) sequencing and/or timing of tasks on each unit (see Figure 1.1).

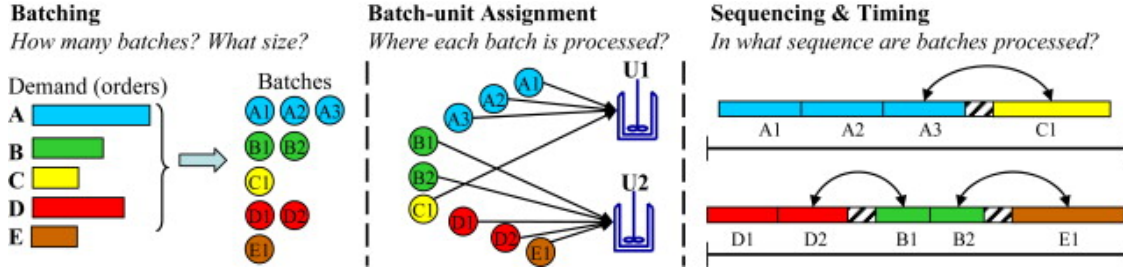


Figure 1.1: Decisions in batch continuous process scheduling. Source: [40].

Production scheduling problems have been the subject of many research works in the last decades and feature practical relevance to process industries [40]. Fuchigami and Rangel [29] in 2018, review articles published in the last 25 years and identify case studies in various productive sectors. The authors consider that most of the production scheduling literature is theoretical and does not model many of the complexities experienced in practice.

The first published work in production scheduling is the Johnson's research publication in 1954, addressing the Flow Shop Scheduling Problem (FSP) [46]. In the classical FSP, a set of jobs (indexed by  $j$ ) flow through multiple stages in the same machine (indexed by  $i$ ) order, where each stage consists of only one machine. The author presents a simple algorithm that optimally solves the FSP in polynomial time for the special case where  $i = 2$ . The Johnson's original paper assumed that setup times are included in the processing times. This is a common assumption for this type of problems.

According to Allahverdi [3], more than 90% of the literature on scheduling problems ignores setup times/costs. However the author emphasizes that setup times/costs need to be explicitly considered while scheduling decisions are made in order to increase productivity, eliminate waste, improve resource utilization, meet deadlines, increased profitability and satisfaction and increased customer satisfaction. Setup time represents the time loss incurred at each operation changeover and appear in a infinity of industrial and service applications. Allahverdi and Soroush [6] emphasize the importance of reducing setup times/setup costs across various industries, including the aluminium industry.

Scheduling problems have been receiving considerable attention since the mid-1960 and three comprehensive literature reviews have been published regarding the research on scheduling problems with setup times/costs. On the other hand, globalization and constant changes in the business environment are relevant factors that increase exportation. The national metallurgy and metal mechanics industry, is characterized by being the most exporting industry in the Portuguese economy, operating in more than 200 markets, such as in Spain, Germany, France, United Kingdom, Italy, Angola and United States of America [2].

## 1.2. The Case Study

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Specifically, in the aluminium industry, profiles are mostly produced from aluminium alloys by the extrusion process. Saha [78] points that extrusion is a process of mechanical conformation by plastic deformation of an aluminium billet, in which the material is subjected to high pressures, applied by means of a punch, and forced to pass through the hole of a die, in order to reduce and/or change the shape of its cross-section.

## 1.2 The Case Study

This work addresses a real case of a company that operates in the aluminium market, whose core business is the development and production of aluminium profiles for application in engineering, architecture and industry works in general.

In this company, after the customers' orders have been received, the production manager has to develop a production plan. Workers takes into consideration the specific characteristics of the customers' order, the production capability of the factory, the existing stock, the deadline for the delivery of the final product, among other factors. The aim is to scheduling the production of orders in a way to minimize the waste, commonly known as scrap, respecting product delivery times and maintaining quality. Always that a die change is made, scrap is generate, and a setup time is verified. So, the scrap can be reduced through minimization of setup times, which occur when the die is change. This case study focuses on a FSP involving sequence-dependent setup times arising from the need to change the tools used in the process of aluminium extrusion. For this, a mathematical model will be developed for schedule the profiles production, to minimize the setup times.

## 1.3 Contributions of this Project

With the elaboration of this project, some contributions appear. Most of the works found of the literature on scheduling problems ignores setup times [3]. In practice, all scheduling related decisions are mainly derived by managers or operators. In order to systematically improve their decisions, computer aided tools can be useful to increase productivity, taking into consideration the involvement of parameters and the dynamic demand changes. Harjunoski *et al.* [40] emphasize a need to further develop and apply optimization solutions in process industries. Since the existing commercial software does not answer to the company's specifications, the modelling of this industrial real challenge enhances the innovative contribution of the present work. In fact, a more dynamic framework, that supplements models of manufacturing with insights from the field of operational planning, is essential.

Another contribution is related to sustainability in the aluminum industry. Energy conservation and emissions reduction have been a fundamental concern for the sustainability of the aluminium industry, and an energy efficiency in the aluminium extrusion process can will play an important part in achieving this goal [39]. Reducing the generation of scrap in extrusion process is an energy efficiency measures in

the aluminium industry [39]. Moreover, improvements in operations scheduling affect resource efficiency, and thus reduce the resource consumption and waste generation of the production process [51].

Furthermore, with this work, the aim is also contributing to the academia–industry knowledge transfer. These problems belong to the class of combinatorial optimization problems which are strongly NP–hard (Non–deterministic Polynomial time hardness), whose complexity is exponential [69].

## 1.4 Outline

The present document is structured in six chapters, as presented in Figure 1.1.

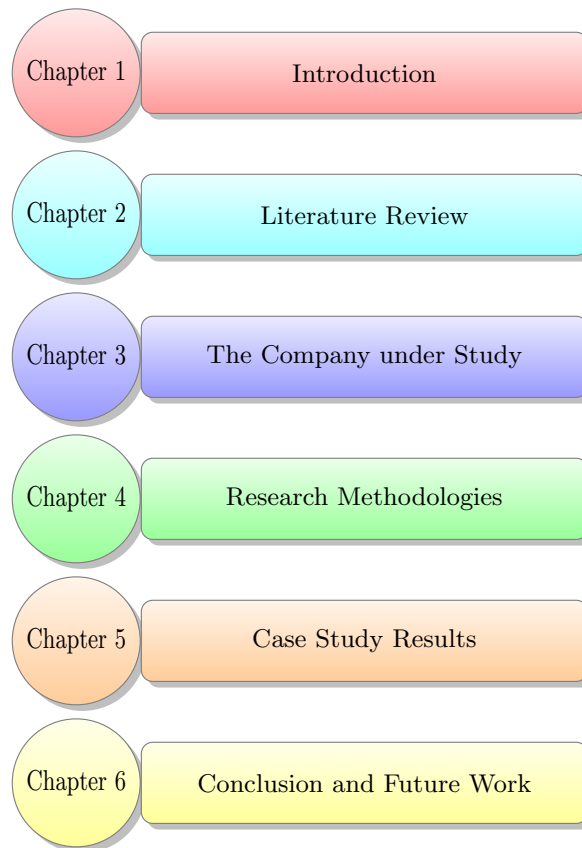


Figure 1.2: Document structure

This first chapter, includes the introduction, which provides an overview of the scheduling problem, the case study that will be addressed, the objectives that are intended to achieve, and the contributions of this project. Finally, the structure and organization of the project are exposed. In Chapter 2 a literature review of the aluminium extrusion process and production planning and scheduling is made. The company characterization, problem description and dataset are presented in Chapter 3. Chapter 4 presents the general datasets analysis and the developed mathematical model. In Chapter 5 several instances are solved to illustrate the efficiency and applicability of the suggested approach. Finally, in Chapter 6 the main conclusions and future research avenues are presented.

## Chapter 2

# Literature Review

In this section are exposed the concepts related with the aluminium extrusion process, production planning and scheduling. Companies are focused on improving their market competitiveness through increased operational performance therefore, production planning and scheduling is crucial and represents one of the main challenges that managers face currently.

### 2.1 The Aluminium Extrusion Process

In industry, extrusion is an attractive manufacturing process due to its high profitability. The first mention on the principles of extrusion is due to the famous hydraulic engineer Joseph Bramah who described a press in which the aluminium, that was kept molten in an iron pan, was forced by a pump to pass through a tube. This allowed to register the pioneer patent, with the principles of extrusion, in 1797 [81].

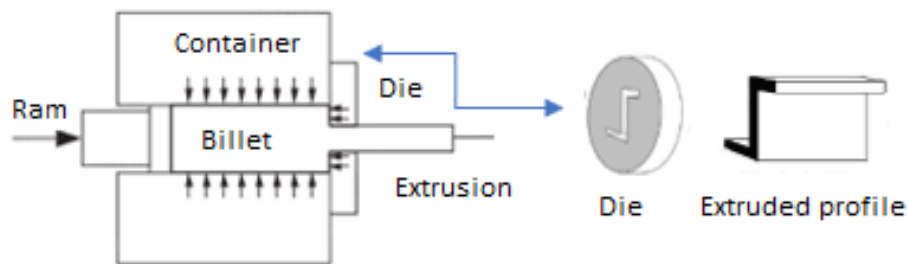


Figure 2.1: Direct extrusion process (Adapted from [78]).

Extrusion is a continuous process of production and can be defined as “a plastic deformation process in which a block of metal (billet) is forced to flow by compression through the die opening of a smaller cross-sectional area than that of the original billet” [78, p. 12] (see Figure 2.1).

This continuous process can be of two types: direct or indirect with the difference, in simple description, that the billet is the moving part in direct extrusion, whereas the die is the moving part in indirect extrusion [39], as depicted in Figure 2.2.

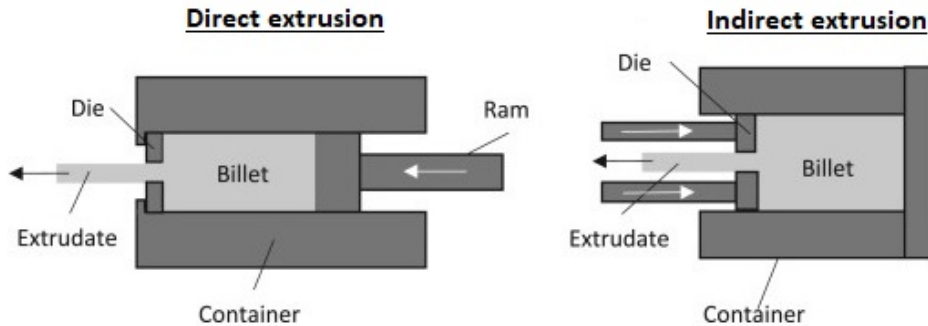


Figure 2.2: Schematic drawing of direct and indirect extrusion (Adapted from [39]).

In direct extrusion, as in the company under study, the die is located at the forward end of the container and the aluminium alloy to be extruded is pushed towards it, hence moving relative to the container. In the case of indirect extrusion, the die is placed on the end of the ram, which is bored out to allow passage of the extruded section, and moves through the container [78, 81].

Extrusion can be cold or hot [78], in a hot aluminium extrusion, aluminium billet and extrusion tools such as containers and die require preheating. During the extrusion process, the material is under high pressure and is normally driven, within the limits of the material, at high temperatures to reduce the material's resistance to deformation [8]. However, along the extrusion process temperature control is required. Temperature management plays an important role in the extrusion of aluminium, mainly the extrudate temperature at the die exit, because it largely determines the microstructure, mechanical properties and surface quality of the extruded product [101].

Extrusion is a complex process that is affected by several variables during extrusion but also in pre-extrusion and post-extrusion. All the variables have to be controlled, since they directly influence the quality of the extruded product and the scrap production. Saha [78] point that the quality of the extruded product is influenced by: the extruder type; the pressure capacity and size of container; the frictional effects at the die or both container and die; the type, layout, and design of die; the length of the billet and the type of alloy; the temperature of the billet and the container; the extrusion ratio; the die and tooling temperature, as well as speed of extrusion. Ferrás *et al.* [26] presents an empirical study concerning the extrusion process of the company in study, and analyzing the dataset provided concluded that variables concerning with extrusion temperature, time, speed, pressure and die geometry are crucial to improve and control the scrap production.

Extrusion dies, used for producing aluminium profiles, may be steel circular, rectangular or tubular, and may vary in shape from simple to quite complicated, whose inscribed configuration gives the profile the desired shape [8]. The complexity of an extruded profile, can be illustrated as a function of the ratio



## 2.1. The Aluminium Extrusion Process

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of the perimeter to the cross-sectional area of the part. The shapes generally is classified into three groups: solid, semi-hollow and hollow, and the complexity increases from solid to semi-hollow to hollow shapes [8].

In the extrusion process, the die is inserted into the container together, if necessary, with various supporting accessories, such as: backer, bolster and feed plate [78]. These accessories are also constructed of hardened steel and provide support for the die during the extrusion process, contributing to improved tolerance control and extrusion speed [78].

The performance of the extrusion is usually limited by typical failure mechanisms related to the material. Saha [78] enumerate the most common mechanisms of failure:

- Hot;
- Plastic deformation and cracking;
- The material must have high resistance to tempering;
- High hot hardness (high yield strength);
- Good wear resistance nitriding and hard and thin coatings;
- Good toughness.

Dies, for extrusion of aluminium alloys, are exposed to severe thermal, chemical and mechanical conditions. Consequently, in the direct extrusion process, die wear occurs, exposed by the loss or progressive removal of material from a die surface [8, 78]. This wear affects the process size, shape, and quality of material flowing through the die. Therefore, there is a maximum time of use of each die and tool, in order to the quality of the extruded product is always high. According to Arif *et al.* [8] various parameters have influence on the service lifetime of the extrusion die, namely the grade of the tool steel used and heat treatments; extruded profile shape complexity and die construction; die maintenance; chemical and structure composition of billet and temperature of the extruded material; technological parameters related to the hot extrusion and wear resistance characteristics of the die bearing surface layer.

In current extrusion production, continuous billet-to-billet extrusion is often used. In this process, the new billet is welded onto the back surface of the old billet in the welding chamber or feeder ring [53]. In this process transverse welds occur which introduce a discontinuity to the weld interface in the extruded product and in many structural applications can cause quality problems. At this point, it is crucial to minimize the transverse weld length of the extruded aluminium. The main purpose of this continuous extrusion production is to keep the length of the transverse welding interface to a minimum at the same time providing a high quality weld in the extruded that is strong enough to withstand subsequent processing [55]. Transverse weld length it depends strongly both on the type of alloy and on the geometries of the corners of the die or the speed with which the materials leave the corners of the die [53]. Furthermore, the relationship between ram speed and puller speed is central [55]. The puller is

close loop linked with the press cycle and it stops automatically when the press is stopped, at any time during and at the end of the extrusion process. Saha [78] states that there are many advantages of using a puller system, namely:

- Reduction of labor and increase of production;
- Reduction of the amount of scrap;
- Controlling the flow of metal through the die;
- The possibility of cutting the billet in various lengths without stopping the press.

Finally, after extrusion process, the extruded profiles are subjected to heat and/or surface treatment, which should also be considered in the production planning. This final procedure confers to the profiles greater resistance to corrosion and oxidation, in order to achieving good mechanical strength. Usually it is done by mechanical, anodizing, electrolytic, and painting (both powder and liquid) processes [78].

## **2.2 Production Planning and Scheduling**

The current business world is subject to constant change, and the ability to respond quickly to these changes has increasingly become a critical success factor. The increasing competitiveness forces companies to adopt more efficient processes to ensure their market position. The production planning system has as its primary objective to ensure that the desired products are produced at the right time, at a minimal cost (not necessarily economic), in the exact quantities, while maintaining the established quality levels [48].

Vercellis [91] and Arenales *et al.* [7] consider that the best way to proceed is to divide the production planning process into three distinct levels: strategic planning, tactical planning and operational planning. In this sense, it is crucial to take the best decisions from the production planning point of view. In the current more globally oriented market, strategic, tactical and operational issues regarding markets, products and production are fundamental to understand for any company to stay competitive [65].

Strategic planning is at the top of the pyramid and concerns long term decisions that provide the overall direction and direction of the company. The current situation of the company, the goals to be achieved and the strategies to follow to achieve those same goals should therefore be characterized. Examples of strategic level decisions include reducing production costs, choosing suppliers, locating facilities, and so on.

The tactical planning deals with the type of aggregate planning and the amount of production over a period of weeks up to six months. This type of planning acts in each functional area of the company, serving as a kind of connection between strategic and operational plans. It is in tactical planning that goals are created and the appropriate environment is built so that the actions determined in strategic

## 2.2. Production Planning and Scheduling

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planning can be realized. Examples of such plans include purchase and production decisions, inventory policy, or transportation policy.

Finally, operational planning serves as a regulator of day-to-day decisions and therefore has a short term planning horizon. Operational planning is responsible for decisions, such as scheduling, that is, the exact order in which operations are to be performed and the sizing of production lots. In fact, Fuchigami and Rangel [29] consider that for a company to remain competitive in demanding consumer markets, one of the most important activities, at the operational level, is production scheduling. The present work focus on decision making issues related to operational planning.

Scheduling is a critical issue in process operations and is crucial for improving production performance. Scheduling is a classic combinatorial problem much studied by researchers in OR [38]. Scheduling problems are very challenging from the mathematical point of view and also highly relevant and useful for industry, which explains the large research interest and the progress that has been achieved during the last years. A general overview concerning scheduling problems are review in papers, such as, for example Graves [37], Floudas and Lin [27], Méndez *et al.* [59], Maravelias [57] and Harjunkoski *et al.* [40], Fuchigami and Rangel [29].

Scheduling is not only the sequencing, but also determining the starting and completing time of each operation based on the sequence. Pinedo [69, p. 12] defines scheduling as a decision making process that deals with the allocation of resources to jobs over given time periods in order to optimize one or more objectives.

According to Pinedo [69] all schedule can be classified into one of the following three types of schedules: non-delay schedule, active schedule and semi-active schedule. Firstly it is important to know the concepts of sequence, schedule and scheduling policy. While the sequence usually corresponds to a permutation of the  $n$  jobs or the order in which jobs are to be processed on a given machine, the schedule usually corresponds to the allocation of jobs to a given set of machines at certain times. And lastly, the concept of a scheduling policy appears in stochastic settings, and aims to “prescribe” the most appropriate action for all states in which the system can be found. Deterministic models only consider the sequences and the schedules.

The first class of schedules, non-delay schedule, happens when there are free machines whenever an operation needs to be processed. This scheduling class prohibits unforced idleness. The second class, active schedule, is imposed when it is not possible to construct another schedule, through process order changes on the machines, leading to at least one operation ending earlier and none ending later than the starting schedule. The third class, semi-active schedule, identifies escalations where no operation can be completed earlier without changing the processing order on either machine.

Scheduling problems have different characteristics. As a way of classifying, them is useful to resort to a certain nomenclature, in order to make it easier to specify, represent and solve. Graham *et al.* [34] initially introduced the  $\alpha|\beta|\gamma$  nomenclature for classification of scheduling problems.

$\alpha$

Notation	Description	Notation	Description
1	Single Machine	$FFm$	$m$ -stage flexible (hybrid) flow shop
$P$	Parallel machines (identical)	$AFm$	$m$ -stage assembly flow shop
$Q$	Parallel machines (uniform)	$J$	Job shop
$R$	Parallel machines (unrelated)	$FJ$	Flexible job shop
$Fm$	$m$ -stage flow shop	$O$	Open shop

Table 2.1: Shop type ( $\alpha$  field)

$\beta$

Notation	Description	Notation	Description
$ST_{si}$	Sequence-independent setup time	$SC_{sdf}$	Sequence-dependent family setup cost
$SC_{sd}$	Sequence-dependent setup cost	$ST_{psd}$	Past-sequence-dependent setup time
$ST_{sd}$	Sequence-dependent setup time	$PREC$	Precedent constraints
$ST_{sif}$	Sequence-independent family setup time	$r_j$	Non-zero release date
$ST_{sdf}$	Sequence-dependent family setup time		

Table 2.2: Shop characteristics ( $\beta$  field)

$\gamma$

Notation	Description	Notation	Description
$C_{max}$	Makespan	$\sum T_j$	Total tardiness
$E_{max}$	Maximum earliness	$\sum W_j$	Total waiting time
$L_{max}$	Maximum lateness	$\sum U_j$	Number of tardy (late) jobs
$T_{max}$	Maximum tardiness	$\sum w_j F_j$	Total weighted flow time
$D_{max}$	Maximum delivery time	$\sum w_j F_j$	Total weighted flow time
$TSC$	Total setup/changeover cost	$\sum w_j U_j$	Total weighted number of tardy jobs
$TST$	Total setup/changeover time	$\sum w_j E_j$	Total weighted earliness
$TNS$	Total number of setups	$\sum w_j T_j$	Total weighted tardiness
$\sum F_j$	Total flow time	$\sum w_j W_j$	Total weighted waiting time
$\sum C_j$	Total completion time	$\sum h(E_j)$	Total earliness penalties
$\sum E_j$	Total earliness	$\sum h(T_j)$	Total tardiness penalties
$TADC$	Total absolute differences in completion times		

Table 2.3: Performance ( $\gamma$  field)

This classification is also proposed in a similar way by Pinedo [69]. The  $\alpha$  describes the machine environment (Table 2.1), the second,  $\beta$ , indicates the characteristics associated with jobs (Table 2.2), and the third,  $\gamma$ , defines the performance measure (Table 2.3) [69]. For example, a single machine Flow Shop Scheduling Problem (FSP) for minimizing the maximum makespan with batch sequence dependent setup

## 2.2. Production Planning and Scheduling

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times will be noted as  $1|ST_{sd}|C_{max}$ .

The optimization criterion is defined in terms of one or several possible measures of performance. The most common performance measures in scheduling problems are, according to [7]:

- Minimize makespan ( $C_j$ ) – The makespan represents the date of the end of the last operation to be processed. It is a criterion commonly used to measure the level of utilization of the machines. It can be measured by the difference between the instant the last job finishes its processing and the instant that the first job begins. Considering  $C_{max} = \max C_j$ .

Thus, if  $C_j$  is the processing end time for job  $j$ , with  $j = 1, \dots, n$ , then:

$$C_{max} = \max_{j=1, \dots, n} \{C_j\},$$

and the objective function to use is:

$$\min C_{max}$$

- Minimize flow time ( $F_j$ ) – The flow time is equal to the sum of the end times,  $C_j$ , of all jobs. The objective function to use is:

$$\min \sum_{j=1}^n C_j$$

- Minimize tardiness ( $T_j$ ) – Tardiness, designate  $T_{max}$ , corresponds to the job with the largest difference,  $T_j$ , between the end time  $C_j$  and the due date  $d_j$ , when the job is completed after the due date. Considering  $T_{max} = \max T_j$ . In this context:

$$T_{max} \geq T_j \quad j = 1, \dots, n$$

$$T_j \geq C_j - d_j \quad j = 1, \dots, n$$

$$T_j \geq 0$$

that is,

$$T_j = \max\{0; C_j - d_j\}$$

Thus,  $T_j$  cannot be less than zero, as this would mean advance rather than delay. The objective function to use is:

$$\min T_{max}$$

- Minimize earliness ( $E_j$ ) – Earliness, in opposition to tardiness, is the amount of time that job  $j$  finished prior to its due date. The objective function to use is:

$$E_j = \max_{j=1, \dots, n} \{0; d_j - C_j\}$$

- Minimize lateness ( $L_j$ ) – Lateness,  $L_j$ , is the difference between the end date of a job  $j$  and the end time. If positive indicates a delay in delivery and if negative means an anticipation of the

delivery time. Considering  $L_{max} = \max L_j$ , the maximum lateness, the problem formulation can be presented as follows:

$$\begin{aligned} \min L_{max} \\ L_{max} &\geq L_j \quad j = 1, \dots, n \\ L_j &= C_j - d_j \quad j = 1, \dots, n \end{aligned}$$

Note that the difference between lateness  $L_j$  and tardiness  $T_j$  is that lateness can be positive or negative indicating a delay or anticipation, where  $T_j$  cannot be negative, i.e. either delayed or not delayed.

- Minimize number of tardy jobs – Consists of determining the smallest sum of the delays. The objective function to use is:

$$\min \sum_{j=1}^n T_j$$

Pinedo [69] evidences that in all scheduling problems considered, the number of jobs and the number of machines are assumed to be finite. The number of jobs are denoted by  $n$  and the number of machines by  $m$ . Subscript  $i$  and  $j$  refer to specific job and machine, respectively, where the pair  $(i, j)$  refers to the processing step of job  $j$  ( $j = 1, \dots, n$ ) on machine  $i$  ( $i = 1, \dots, m$ ). The notations introduced in Table 2.4 are considered by Pinedo [69] as extremely important when talking about production scheduling.

Notation	Description	Definition
$p_{ij}$	Processing time	Represents the processing time of job $j$ on machine $i$ . The subscript $i$ is omitted if the processing time of job $j$ does not depend on the machine or if job $j$ is only to be processed on one given machine.
$r_j$	Release date	This is the instant that job $j$ can start its processing.
$d_j$	Deadline	Represents the instant that the job $j$ should be completed. Completion of a job after its due date is allowed, but a penalty is incurred.
$w_j$	Weight	It is a priority factor denoting the importance of job $j$ relative to the other jobs in the system.
$S_{jj'}$	Sequence-dependent setup times	Represents the sequence-dependent setup time that is incurred between the processing of jobs $j$ and $j'$ . Particularly, $S_{0j'}$ denotes the setup time for job $j'$ if job $j'$ is first in the sequence and $S_{j0}$ the clean-up time after job $j$ if job $j$ is last in the sequence. If the setup time between jobs $j$ and $j'$ depends on the machine $i$ , then the subscript is included, i.e., $s_{ijj'}$ .

Table 2.4: Notations.

A widely used approach to real-world scheduling, where problems are often characterised by a highly complex and dynamic environment, are dispatching rules [68]. Dispatching rules consist of a well defined

## 2.2. Production Planning and Scheduling

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set of rules for deciding which job (or set of jobs) to process on a machine next. Advantages of dispatching rules include to be executed quickly, the ability to react to dynamic changes, their simple and intuitive nature, their ease of implementation within practical settings, and their flexibility to incorporate domain knowledge and expertise [68]. Jayamohan and Rajendran [45] classified the dispatching rules into five categories:

- Rules involving process times;
- Rules involving due-dates;
- Simple rules involving neither process times nor due-dates;
- Rules utilizing shop floor conditions;
- Rules involving two or more of the first four classes.

The amount and nature of this type of rules is quite high, so the main rules adapted in Holthaus and Rajendran [43], Slack *et al.* [82], Pinedo [69] can be defined as being:

- First In First Out (FIFO) – This is the simplest process scheduling in which, priority is given according to the sequence jobs arrive at the system, i.e. the first allocated job is executed first. This rule is often used as a bench-mark rule and it is easy to implement. [43].
- Earliest Due Date (EDD) – This is the earliest due date based rule in which, jobs are sequenced according to their due dates, i.e., priority is given to the execution of the most urgent orders in terms of due date. The goal is to reduce delays. Generally it gives good results under light load conditions, although the performance of this rule deteriorates under high loading levels.
- Shortest Processing Time (SPT) – This rule ignores the due date information of jobs. Priority is given according to the shortest total processing time, and is commonly used to minimize mean flow time and percentage of tardy jobs.
- Critical Ratio (CR) – Priority is given to the lowest critical ratio (time to due date divided by total remaining production time) among the waiting jobs. This is a dynamic rule that seeks to combine EDD with SPT, which only considers processing time.

Although these rules have been highly used with good results [20], none of these can guaranty the optimal solution [45].

### 2.2.1 Scheduling Classifications

The scheduling of production activities is accomplished through different strategies, and their application leads to processes with different structures and characteristics. Literature presents several scheduling

classifications, according to the number of machines, jobs and operations. Graves [37] covering the general characteristics of both scheduling theory and scheduling practice, classifies production scheduling problems in three dimensions:

- Requirements generation – *open shop*, when requirements are generated directly by customers' orders or *closed shop*, when requirements are generated indirectly by inventory replenishment decisions.
- Processing complexity – *one stage-one processor* (one machine problem) when all jobs require one processing step which must be done on the production facility; *one stage-parallel processors*, which is similar to the previous except that each job requires a single processing step which may be performed on any of the parallel processors; *multistage-flow shop*, when all jobs are to be processed on the same set of facilities with an identical precedence ordering of the processing steps; *multistage-job shop*, when there are no constraints on the processing steps for a job, and alternative routings for a job are allowed.
- Scheduling criteria – *scheduling costs*, when concerned with fixed costs associated with production setups or changeovers, inventory holding costs; *scheduling performance*, such as the percentage of late jobs, the average or maximum tardiness for a set of jobs.

Furthermore, scheduling problems can be classified by production environments into static and dynamic problems in which the system parameters may be either deterministic or stochastic [37]. Static problem consist in determining a solution to a scheduling problem instance, i.e., characterizing the starting times of a known set of activities subject to deterministic resource and precedence constraints. While dynamic problem consist on reacting to some changes of the initial problem characteristics that generally occur in real time, either by performing a global rescheduling each time such a change occurs, or by adapting an initial static schedule to each disruption.

The two most important scheduling problem types are the job shop and the flow shop [69]. Figure 2.3 illustrates a comparison between the flow shop strategy with the job shop strategy. In the flow shop strategy all products run the same type of route (in the same direction), however there is no obligation to go through all machines. On the other hand, in job shop there is a route that has the opposite direction, i.e., routes are customized to products.

The Job Shop Scheduling Problem (JSP) may be described as given a set of  $n$  jobs, each composed of several operations that must be processed on  $m$  machines, to find the best schedule of the operations on the machines, taking account the precedence constraints. The objective of JSP is to minimize the makespan value  $C_{max}$  [32]. Commonly, it is assumed that: jobs are independent; machines and jobs are all continuously available; the setup times can be ignored or included into processing times; each machine can process at most one operation at a time and once an operation initiates processing on a given machine it must complete processing on that machine without interruption; the operations of a given job have to be processed in a given order.



## 2.2. Production Planning and Scheduling

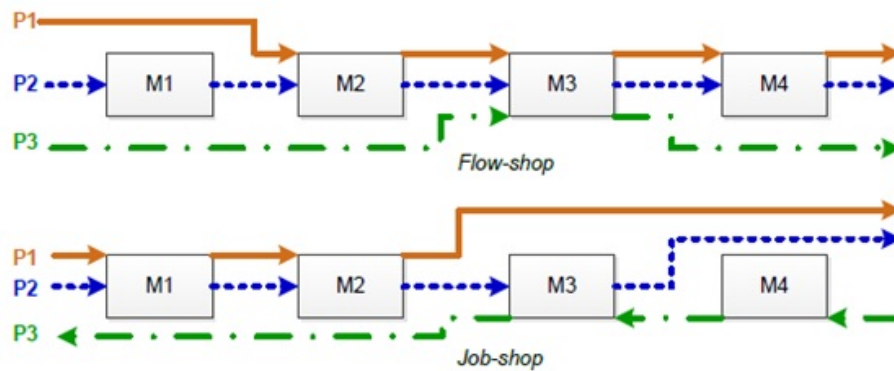


Figure 2.3: Flow Shop Scheduling *vs* Job Shop Scheduling

The Flow Shop Scheduling Problem (FSP) is a particular case of JSP which consists on scheduling  $n$  jobs with the same order and given processing times on  $m$  machines [15]. When the sequence of tasks is the same on all machines, it is known as a permutational flow shop. The classical FSP has as criterion the makespan minimization. Considering  $p_{ij}$  as the processing time of job  $j$  ( $j = 1, \dots, n$ ) on machine  $i$  ( $i = 1, \dots, m$ ), the goal is to find the desired schedule in which these  $n$  jobs should be processed on each of the  $m$  machines in order to minimize a well defined measure. This problem is known to be NP-Complete in the strong sense when  $m \geq 3$  [69]. Bachman and Janiak [9] deal with the single machine maximum lateness minimization problem where the job processing times are given as a non-decreasing linear function. The authors consider that the processing time of job is a function dependent on its position in a sequence.

Gupta and Stafford Jr. [38] enumerates 21 assumptions for the traditional FSP originally conceived in Johnson paper in 1954 [46], concerning with characteristics of jobs, machines and operating policies of the shop. The assumptions concerning operation jobs are the following:

- (J1) Each job is released to the shop at the beginning of the scheduling period.
- (J2) Each job may have its own due date which is fixed and is not subject to change.
- (J3) Each job is independent of each other.
- (J4) Each job consists of specified operations, each of which is performed by only one machine.
- (J5) Each job has a prescribed technological order which is the same for all jobs and is fixed.
- (J6) Each job requires a known and finite processing time to be processed by various machines. This processing time includes transportation and setup times, if any, and is independent of preceding and succeeding jobs.
- (J7) Each job is processed no more than once on any machine.
- (J8) Each job may have to wait between machines and thus in process inventory is allowed.

The assumptions concerning machines are the following:

- (M1) Each machine center consists of only one machine, i.e., the shop has only one machine of each type.
- (M2) Each machine is initially idle at the beginning of the scheduling period.
- (M3) Each machine in the shop operates independently of other machines and thus is capable of operating at its own maximum output rate.
- (M4) Each machine can process at most one job at a time. This eliminates those machines that are designed to process several jobs simultaneously, like multi-spindle drill.
- (M5) Each machine is continuously available for processing jobs throughout the scheduling period and there are no interruptions due to breakdowns, maintenance or other such causes.

The assumptions concerning operating policies are the following:

- (P1) Each job is processed as early as possible. Thus, there is no intentional job waiting or machine idle time.
- (P2) Each job is considered an indivisible entity even though it may be composed of a number of individual units.
- (P3) Each job, once accepted, is processed to completion, i.e., no cancellation of jobs is permitted.
- (P4) Each job, once started on a machine, is completed to its completion before another job can start on that machine, i.e., no preemptive priorities are assigned.
- (P5) Each job is processed on no more than one machine at a time. (This is a result of assumptions J5 and P2).
- (P6) Each machine is provided with adequate waiting space for allowing jobs to wait before starting their processing.
- (P7) Each machine is fully allocated to the jobs under consideration for the entire scheduling period, i.e., machines are not used for any other purpose throughout the scheduling period.
- (P8) Each machine processes jobs in the same sequence, i.e., no passing or overtaking of jobs is allowed.

The literature on FSP is extensive. Recent reviews are those by Hejazi and Saghafian [74], Minella *et al.* [60], Xu and Zhou [95], Sun *et al.* [85].

Gupta and Stafford [38] gave a summarized version of the evolution of this research over the period from 1955 to 2005. The current FSP that are being solved and the approaches taken to solve (optimally or approximately) are presented. The authors consider that although there are significant progress in FSP theory, it is not appreciable to solve practical FSP optimally and efficiently.

## 2.2. Production Planning and Scheduling

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### 2.2.2 Scheduling with Setup Times/Costs

According to Allahverdi *et al.* [5], the first studies assume setup times as negligible or as part of job processing time. For the same authors, these assumptions simplify the problem analysis, but ultimately affect the quality of the solutions of many of the scheduling problems that need to explicitly address setup times. In fact, scheduling with setup times/costs plays a crucial role in industry. Allahverdi [3] remarks that the interest in scheduling problems with setup times/costs is growing, however, research on this problems is still less than 10 percent of the available research on scheduling problems.

A formal definition of setup times/costs is provided by Allahverdi and Soroush [6]. Namely, setup time can be defined as the time required to prepare the necessary resource (people, machines) to perform a task (operation, job). Setup cost is the cost to set up any resource used prior to the execution of a job. These authors present a comprehensive classification on scheduling with setup time/cost, and divide the scheduling problems with setup times/costs into two broad groups: Batch and Non-Batch. The big difference is that, in the first production is done in batches. So, the problem to analyze and solved is influenced by factors such as the existent number of batches, the size of each batch, or the number of batches jobs per batch produced. The second group (Non-Batch) can be regarded as a particular case of the first group in which batches have dimension one.

Batch and Non-Batch scheduling are also divided into two problem sub-types, depending on whether or not setup times are influenced by the sequence of jobs. Ulungu *et al.* [89] also divide setup time into two categories: sequence-independent and sequence-dependent. While in sequence-independent setup time is only dependent on the job that is being processed, in sequence-dependent the setup time depends on the job to be processed and on the one already processed. From these two categories the most difficult occurs when setup times are dependent on sequence, specifically when setup times are unsymmetrical. This means that the setup time from job  $j$  to  $j'$  is different when compared to the setup time from job  $j'$  to  $j$ .

The importance of explicitly treating setups times/costs in production scheduling has been addressed several times in literature. Allahverdi *et al.* [4] in 1999 make a comprehensive review of the literature on static scheduling problems involving setup times/costs, from mid-1960s to mid-1988s, covering about 200 papers. The authors also classify scheduling problems into batch and non-batch, sequence-independent and sequence-dependent setup. In addition, they categorize literature according to the shop environments on single machine, parallel machines, flow shops, and job shops. Gaps on existing research, including in the underlying theoretical underpinnings and in the treatment of multiple objectives are identified. Further, the author emphasize that few papers have treated the many industrial applications having sequence-dependent setups.

Allahverdi *et al.* [5] in 2008 also address scheduling problems with setup times/costs, covered about 300 papers published from mid-1998s to mid-2006s. The authors, further categorize scheduling literature on models with setup times/costs according to shop environments, including single machine, parallel

machines, flow shop, no–wait flow shop, flexible flow shop, job shop, open shop, and others. The common solution methods that are used are branch and bound algorithms, dynamic programming algorithms, mathematical programming formulations, heuristics and meta–heuristics.

In 2015, Allahverdi [3], once again conducts a survey on these problems, involving static, dynamic, deterministic and stochastic problems for different shop environments with setup times/costs, covering about 500 papers published from mid–2006 to the end of 2014. It further classifies the problems as family and non–family, where each family is a set of jobs that has similar characteristics in terms of setups, tooling and operation sequence. There is a negligible setup time/cost to change from one job to another with the same family but a major setup time/cost is needed between job families.

### 2.2.3 Schematic Representation of Scheduling Problems

The Gantt chart has been one of the most important tools for visual schematic representation of schedules in industrial settings [67]. The horizontal axis of the Gantt chart represents the time and the vertical axis represents the processing units. A rectangle on the chart stands for a task. Its location and length indicate when the task is started and how long the task is executed.



Figure 2.4: Illustrative example of a Gantt chart.

A Gantt chart is pictured in Figure 2.4, for one machine, where each order is represented by a rectangle that is labeled with the job number. For example, Job 1 is processed on machine followed by jobs Job 3, Job 5, Job 2 and Job 4.

Panwalkar and Koulamas [67] point others schematics to depict a schedule, namely: cartesian coordinates, disjunctive graph model and the string analog schematic using strings, weights and pulleys for scheduling and sequencing problems involving general precedence relations. However, Gantt chart are by far the most used schematic representation of scheduling solutions.

### 2.2.4 Modeling and Resolution of Scheduling Problem

Optimization refers to the process of finding optimal values for the decision variables of a given system from all the feasible values which maximize or minimize the output, and mathematical optimization is the process of finding the best solution to a problem from a set of available alternatives [64]. In literature

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there are several classifications on optimization problems [48]. Nguyen *et al.* [64] present several aspects that need to be considered during the optimization:

- Number of decision variables – Optimization problems can be classified as *one-dimensional* or *multi-dimensional* optimization, depending on the number of decision variables.
- Nature of the decision variables – Decision variables can be independent or mutually dependent. Optimization problems can be stated as *static* or *dynamic* if decision variables are independent, and *deterministic* if decision variables are subject to small uncertainty or have no uncertainty. Opposite, optimization problems can be seen as probabilistic optimization if decision variables are subject to uncertainty.
- Types of decision variables – Variables can be *continuous* if it can assume any real value in a range, and *discrete* if it can assume only integer values or discrete values, or both. The latter is referred to as MILP.
- Number of objective functions – Problems can be classified as *single-objective* if only one objective is considered, or *multi-objective* if more than one objective is considered.
- Natures of the objective function – Different optimization techniques can be established depending on whether the objective function is linear or nonlinear, convex or non-convex, uni-modal or multi-modal, differentiable or non-differentiable, continuous or discontinuous, and computationally expensive or in-expensive. These result in *linear* and *nonlinear programming*, *convex* and *non-convex optimization*, *derivative-based* and *derivative-free optimization* methods, *heuristic* and *meta-heuristic optimization* methods, *simulation-based* and *surrogate-based optimization*.
- Presence of constraints and constraint nature – Optimization can be classified as *constrained* or *unconstrained problems* based on the presence of constraints which define the set of feasible solutions within a larger search-space. Two major types of constraints are equality or inequality. A constraints function may have similar attributes to those of objective functions, and can be separable or inseparable.
- Problem domains – *Multi-disciplinary optimization* or *single-domain optimization*.

Scheduling problems are very challenging from the mathematical point of view, and are known to be very difficult to model and solve in an efficient way, being that Mathematical programming (MP) methods are the most widely reported methods for scheduling industrial problems [83]. MP involves the development of mathematical models that represent real world situations and can be used to determine the optimal [94].

Optimization problems are generally classified according to the complexity of the functions involved in the problem. The most relevant subareas are as follows:

- Nonlinear Programming (NLP) – In this kinds of problems one or more of the constraints and/or the objective function are nonlinear functions. When the production processes under consideration possess significant nonlinear characteristics as often in real time production scheduling or in control problems, it is sometimes necessary to make linear approximations to the nonlinear functions [83].
- Linear Programming (LP) – This is the simplest case, with the easiest resolution. The objective function and all constraints are linear functions. The exact form of these constraints may differ from one problem to another. A problem LP can be transformed into the following standard form [54]:

$$\min \quad c_1 x_1 + c_2 x_2 + \dots + c_n x_n \quad (2.1)$$

$$\text{s.t.} \quad a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \quad (2.2)$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \quad (2.3)$$

$$\vdots \qquad \qquad \qquad \vdots \quad (2.4)$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \quad (2.5)$$

$$x_1, x_2, \dots, x_n \geq 0. \quad (2.6)$$

where the  $b_i$ 's,  $c_i$ 's and  $a_{ij}$ 's are fixed real constants with  $b_i \geq 0$ , and the  $x_i$ 's are real numbers to be determined.

- Mixed Integer Linear Programming (MILP) – Involves real variables usually related to operation and discrete variables (integers or binary), such as decision variables, allocation and decision, or any other quantity that requires modeling with discrete variables. Scheduling problems usually involve discrete decisions such as equipment assignment and task allocation over time and hence MILP formulation is used [83].
- Mixed Integer Nonlinear Programming (MINLP) – This is the area of optimization that addresses nonlinear problems with continuous and integer variables, which have characteristics of the MIP and NLP problems already described. MINLP has proven to be a powerful tool for modeling. At the same time, it combines algorithmic design challenges from combinatorial and nonlinear optimization [79].

Three different very common ways of formulating the scheduling problems are next presented, distinguished by the type of binary variables used. The three types of variables are the proposed by Wagner [92], Bowman [13] or Manne [56]. These variables are called precedence variables [10].

Wagner [92], whose work focused on minimizing makespan in three machine FSP, presents the following key binary variables:

- $x_{jki} \in \{0, 1\}$ , that is 1 if job  $j$  is scheduled at position  $k$  for being processing on machine  $i$ , and 0 otherwise.

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Bowman [13], whose work focused on minimizing makespan in the JSP, the key binary variables are:

- $x_{jit} \in \{0, 1\}$ , being 1 if job  $j$  is processed by machine  $i$  at discrete time interval  $t$ , and 0 otherwise.

Manne [56], who also focused on the makespan problem for the JSP, presented the binary variables:

- $x_{jj'i} \in \{0, 1\}$ , 1 if job  $j$  precedes order  $j'$  on machine  $i$  (not necessarily immediately before), and 0 otherwise.

Many algorithms for solving optimization problems involve a large number of decision choices and algorithms' specific parameters that need to be carefully set to reach their best performance. Maximizing the performance of these algorithms may involve the proper setting of tens to hundreds of parameters. A good introduction to OR models for scheduling is given by Pinedo [69] where a review on different models and solution approaches is presented.

Next several approaches for solving and modelling scheduling problems are described. According to the nature of the solution approach, these can be classified as: exact algorithms, heuristics, hybrid approaches and simulation/decision support system (DSS) [75] (see Figure 2.5).

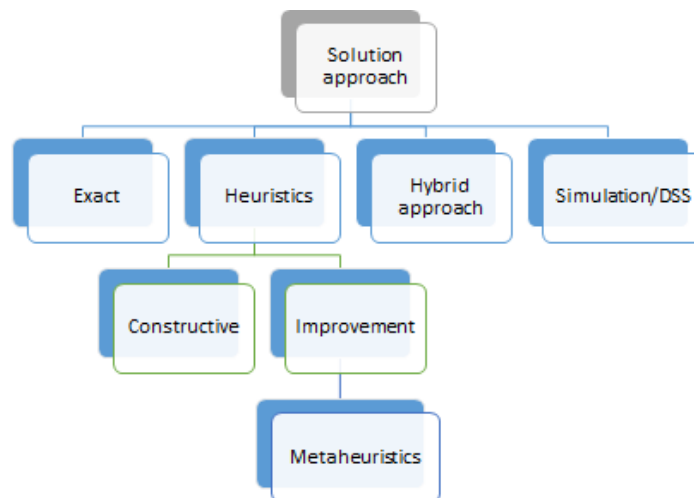


Figure 2.5: Classification proposed according to the solution approach (Adapted from [75]).

Exact algorithms propose a method to obtain the optimal solution. The heuristics can be classified either as constructive or improvement heuristics, in which the first builds a feasible schedule from scratch, step by step, and the latter try to improve a previously generated schedule by normally applying some form of specific problem knowledge [76]. In the improvement procedures category there are the metaheuristic methodologies. Hybrid approaches propose a procedure that combines two or more approaches to obtain a solution. The simulation/DSS approach analyse the performance of systems restricted by certain rules or constraints.

### Exact Approaches

Exact algorithms have been developed with the objective of optimizing certain well defined criterion find for problems involving a limited number of jobs and machines [84]. Branch and Bound (B&B) is the most common technique to solve FSP and differ in terms of the lower bounds used and the branching strategies. McMahon and Burton [58] applied the B&B technique to three machines FSP with the objective to minimize makespan.

Lee and Kim [52] proposed a B&B algorithm for the problem with two stages and any number of identical parallel machines at the second stage, with the objective the minimization of total tardiness. Choi and Lee [16] have studied a two-stage problem with multiple identical parallel machines at each stage for the minimization of tardy jobs. The authors propose a B&B method as well as some ad-hoc heuristics.

Rabadi *et al.* [70] addressed a single machine early/tardy (E/T) scheduling problem with setup times and sequence-dependent, whose objective minimize the total amount of earliness and tardiness. B&B algorithm is developed to obtain optimal solutions for the problem and will be shown that this algorithm solved problems three times larger than what has been reported in the literature.

Exact approaches have demonstrating limited results for medium or large problems and are specially sensible to the number of jobs and/or machines as can be observed in the work of Ladhari and Haouari [50] and Ruiz and Maroto [76]. However, with the development of computational resources and algorithms, competitive exact approaches are already begin to verified. Recently Nesello *et al.* [63] proposed an exact algorithm based on the iterative solution of three alternative arc-time-indexed models to solve the single machine scheduling problem with periodic maintenance and sequence-dependent setup times with aim of minimizing the makespan.

Wang and Ye [93] presented an order acceptance and scheduling problem on unrelated parallel machines, where a B&B based formulation has been developed. The branching determines how many jobs should be accepted followed by addressing of the unrelated parallel machine scheduling sub problem with the minimization of the total weighted tardiness. Two Mixed Integer Programming (MIP) models were formulated and the results demonstrate the efficiency of the enhancement techniques for the formulations, as well as the effectiveness and efficiency of the B&B based formulation algorithm.

### Heuristic Approaches

A major drawback of the above techniques, that has been pointed in literature, is that there is a limit on the size of the problems that can be solved using such techniques in a reasonable time. On the other hand, it is argued that heuristic algorithms can solve large FSP instances and are able to find high quality solutions in short computation time. Several such algorithms have been proposed and can be classified into the following categories: (i) construction methods; (ii) improvement heuristics; (iii) metaheuristics [44].



## 2.2. Production Planning and Scheduling

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*Construction methods* build a feasible schedule from scratch [61]. The final sequence is obtained by integrating jobs (usually in an iterative manner) on an incomplete sequence [24]. An example is Johnson's algorithm [46]. Among the existing heuristics, the insertion method of Nawaz *et al.* in 1983 [62], referred to as NEH (Nawaz–Enscore–Ham) is considered one of the best heuristics for the FSP (see work Ruiz and Maroto [76]). In according Ribas *et al.* [75] the NEH procedure can be divided into two steps: the generation of an initial order for the jobs applying the largest processing time rule and the iterative insertion of jobs in a partial sequence in accordance with the initial order obtained in the first step. More recent methods, such as those of Dong *et al.* [18], Kalczynski and Kamburowski [47], Rad *et al.* [71] have demonstrated NEH outperforming algorithms.

*Improvement heuristics* are usually composed of two phases. The first is a construction phase where a complete sequence is computed [24]. In the second phase is improved the previously generated schedule, typically by specific knowledge of the problem [41, 84].

*Metaheuristics* are defined as search methodologies or algorithms of a solution space for a near optimal solution [86]. The initial solution for the metaheuristics is provided by constructive and/or improvement heuristics, because it requires a fast initial solution to start the procedure [76]. The quality of this initial solution can influence the performance of the metaheuristic [24]. This type of algorithm basically combines heuristic methods in higher level frameworks. These methods were first introduced by Glover [31]. An advantage of metaheuristics over simple heuristics is solution robustness. However metaheuristics are usually more difficult to implement and adapt since in order to obtain good results special information about the problem must be provided [102]. Nearly all metaheuristic algorithms share the same following characteristics: are nature-inspired; make use of random variables; do not require substantial gradient information; and have several parameters that need to be fitted to the problem. Factors that affect the quality of the search result include the type of problem model, applied metaheuristic algorithm, size of the search space, time allocated for the search, and computer capacity. According Boussaïd *et. al* [12] metaheuristics can be divided in single point search working only on one solution at a time and population based working on multiple solutions at the same time.

Exact algorithms did not prove efficient in solving real life instances of this problem until recently, in way more papers on heuristic and metaheuristic approaches can be found in the literature [22]. Examples of matheuristics are the algorithms of Simulated Annealing (SA), Tabu Search (TS), Genetic Algorithms (GA), Ant Colony Algorithms (ACO), Particle Swarm Optimization (PSO) and Greedy Randomized Adaptive Search Procedure (GRASP).

Simulated Annealing (SA) is an evolutionary metaheuristic approach which is inspired from the physical process of annealing, in which a solid is heated to a liquid state and, when cooled sufficiently slowly, takes up the configuration with minimal inner energy. The principles of SA is first to change a current solution randomly to obtain other various solutions (diversification phase). An example of SA algorithms is the are presented by Osman and Potts [66]. This algorithm is fairly simple in the sense that it uses a constant temperature and an insertion neighborhood along with a FIFO dispatching rule initialization

procedure. Ying *et al.* [98] point out that the basic SA algorithm has a major weakness, the number of neighborhood solutions generated grows enormously as the number of jobs increases, meaning that finding an efficient solution requires a large computational burden. In this view the authors propose a restricted simulated annealing, which demonstrate superior results with respect to the existing literature at the moment. A paralleled version of a SA is presented in [25].

The Tabu Search (TS) was originally developed by Glover in 1989 [31]. This technique aims to guide the search by exploring the solution space of a problem beyond local optimality. Ben-Daya and Al-Fawzan in 1998 [11] implemented a TS algorithm with some extra features such as intensification and diversification schemes that provides better moves in the TS process. Grabowski and Wodecki [33] deals with a classic FSP with makespan criterion, and to propose a new very fast local search procedure based on a tabu search approach. The results indicate that the algorithm proposed solves the flow shop instances with high accuracy in a very short time.

Genetic Algorithms (GA) are adaptive methods based on the genetic process of biological organisms, since the original idea proposed by Holland in 1975 [42]. Unlike the conventional optimization methods, a GA maintains for each generation a set of potential solutions a population and encodes the factors of a problem by chromosomes, where each gene represents a feature. The combinations of genes evolved through genetic operators so that the chromosomes would approach the optimal solution from generation to generation. There are three genetic operators: reproduction, crossover, and mutation. Reproduction is a process which forms a new population of strings by selecting strings in the old population based on their fitness values. Crossover is used as the main genetic operator on which the performance of a GA is extremely dependent. The mutation operator plays a very substantial role in GA and helps to maintain the diversity in the population to prevent premature convergence. Chen *et al.* [14] generated a GA based heuristic for FSP with makespan as criterion. Ruiz and Maroto [77] employed a GA to minimize makespan on a  $m$ -stage problem with unrelated parallel machines, sequence-dependent setup times and machine eligibility. Zhang *et al.* [100] proposed an hybrid GA for permutation FSP, with total flow time minimization. GA for the unrelated parallel machine scheduling problem in which machine and job sequence-dependent setup times, presented by Vallada and Ruiz [90] was able to give even smaller deviations from best known solutions. Another GA for this problem was proposed by Yilmaz Eroglu *et al.* [97].

The Ant Colony Algorithms (ACO) algorithm described is competitive and presents advantage for larger problems, when compared with a GA, a SA, a local search method and a branch and bound algorithm. Ying and Lin [99] proposed an ACO metaheuristic for the multiprocessor task problem with precedence relationships. While, Rajendran and Ziegler [72] investigate scheduling in permutation flow shops by using two ACO algorithms, with the objective of minimizing the makespan.

Particle Swarm Optimization (PSO) was first proposed by Kennedy and Eberhart in 1995 [49]. This is one of the most recent and hopeful evolutionary metaheuristics, which is inspired by adaptation of a natural system based on the metaphor of social communication and interaction [44]. The challenge has

## 2.2. Production Planning and Scheduling

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been to employ the algorithm to combinatorial problems, thus new PSO variants have been developed. A PSO metaheuristic was proposed by Tseng and Liao [88] for the scheduling of multiprocessor tasks in a multistage hybrid flow shop with makespan minimization. The proposed PSO algorithm has a new encoding scheme, an implementation of the best velocity equation and neighbourhood topology among several different variants, and an effective incorporation of local search.

Feo and Resende [23] proposed Greedy Randomized Adaptive Search Procedure (GRASP). This is a memory-less multi-start metaheuristic for combinatorial optimization problems. There are two phases within each GRASP iteration: (i) construction and (ii) local search [12]. The construction step builds a feasible solution using a randomized greedy heuristic. In the second phase, this solution is used as the initial solution for a local search procedure. After a given number of iterations, the GRASP algorithm terminates and the best solution found is returned. Resende *et al.* [73] devise several versions of an hybrid algorithm based on GRASP and path relinking methodologies for the max-min diversity problem.

### 2.2.5 Scheduling in the Aluminium Industry

This last subsection aims at referencing scheduling papers that have been developed for the aluminium industry.

Duman *et al.* [19] presents a study of a problem scheduling casting lines of an aluminium casting and processing plant in a parallel machine environment with sequence-dependent setups. A mathematical formulation for scheduling jobs to minimize the total setup time while achieving workload balance between the production lines is presented. A four-step algorithm for finding good solutions in a reasonable amount of time is proposed. In this process, a set of asymmetric travelling salesman problems is followed by a pairwise exchange heuristic. The solutions presented are better compared to those verified by the company in study.

Gravel *et al.* [35] addresses a GA for the solution of an industrial scheduling problem of  $n$  orders with sequence-dependent setup times on  $m$  parallel machines in aluminium foundry. Multiple criteria are considered, namely: meeting due dates, number and duration of required setups and metal flow. The algorithm presents provides a better solution than the manual process.

Schwindt and Trautmann [80] address a scheduling problem arising in the context of a rolling ingots production, that consists of computing a feasible production schedule with minimal makespan. A B&B solution procedure is presented in a two-phase approach. In the first phase, the resource requirements of the changeover operations are disregarded and, once a feasible schedule for the latter relaxation has been found, the job sequences on the resources which are changed over between consecutive jobs are fixed. In the second phase, a feasible schedule are determined by resuming the branching process of the first phase and resolving the conflicts arising from the resource requirements of the changeover operations. The algorithm has been implemented as a beam search heuristic enumerating alternative sets of precedence

relationships.

Gagne *et al.* [30] proposed an ACO algorithm for the problem non–batch sequence–dependent setup times in a casting centre and their objective is to minimize total tardiness. The authors showed that it performs competitively with the best results of Tan *et al.* [87], in terms of solution quality while taking much less computational time, presenting a certain advantage for larger problems.

Xu *et al.* [96] focused on minimizing the energy consumption and have developed a hybrid PSO for a LP of aluminium industry continuous casting and rolling production. Simulation results demonstrate the effectiveness of both the models and the optimization method, and that the optimal scheduling scheme with minimum energy consumption can be obtained efficiently.

Esteban and Penya [21] applying a set of GA, SA and backtracking algorithms to the foundry scheduling problem. The objective is to decrease the energy spend of foundries. The scheduling of the orders and the optimisation of the foundry process it self is carried out based on expert’s knowledge acquired over years of experience, neglecting issues such as, energy costs and hourly electricity prices.

Gravel *et al.* [36] present an ACO metaheuristic fin order to treat the multiobjective industrial problem of  $n$  orders, with sequence-dependent setup times on one machine, taking into account the technological and logical constraints on equipment and the management of the supply of liquid metal. The objectives are the minimization of unused capacity, minimization the total tardiness of the set of orders, minimization of the total number of drainings required when changing alloys, and minimization the total unused vehicle capacity. The results could not be compared with an *ex ante* situation because was a new continuous casting facility with no previous work history, however the solutions presented of a better overall quality, than those found by single objective optimization.

## Chapter 3

# The Company under Study

This section describes the company where the present work was developed. A brief presentation is made and the production process is described. The company provided us with two datasets and they are presented. The first dataset with the extrusion process to first half of 2018 and the second concerned the information the order of its customers for the month of May 2018.

### 3.1 Company Characterization and Problem Description

The present work is motivated by an industrial challenge proposed by ADLA – Aluminium Extrusion, S.A., a company that operates in the aluminium market, located in the region of Celorico de Basto. The company’s core business is the extrusion and treatment of aluminium profiles (Figure 3.1). Innovation, quality, technology and environment are at its core values. The company considers internationalization a vital aspect for its survival. Investments in the production of new goods and services or significant improvements in production, contemplating concerns with principles of ecodesign, efficient use of resources, and durability of materials and products.



Figure 3.1: Type of aluminium profiles production in company. Source: [1]

At the company, as mentioned before, aluminium profiles are produced through direct extrusion (Figure 2.1). The raw material used is supplied in the form of aluminium alloy cylinders, called a billet. In a phase prior to extrusion, the billet, in order to have the necessary plasticity, is pre-heated gradually in

an oven (Figure 3.3, left) and then placed in the container.

The extrusion press receives the billet on one side and the die opposite side, which is also preheated to avoid thermal shocks. Next, in the process of extrusion by the action of the hydraulic force of the press, the billet is pressed against the die, causing the change of the aluminium form and the formation of the profile. The dies (Figure 3.2) are steel blocks with one or more holes, solid and resistant, whose inscribed configuration gives the profile the desired shape. If the profile has to be changed or the die has already reached its maximum use time, the process is interrupted and the die is changed.



Figure 3.2: Type of dies. Source: [1]

Because extrusion is a continuous process, a large automated circular saw cuts the billet to the required length as it is produced. After the pressing operation, profile is quickly cooled (with air or water) along a lane, the tip of the profile being held by a puller, which pulls it to the end of the lane. It is then subjected to a cold stretching work to relieve any tension and obtain the desired straightness and cut (Figure 3.3, right).

Finally, the profile under goes a heat treatment of tempering to give it the specified mechanical properties. In the company under study only thermal finishing is carried out, and the surface treatment is outsourcing, with an average time of five days to be ready. The surface treatment can be anodizing or lacquering, according to the customers' requirements.

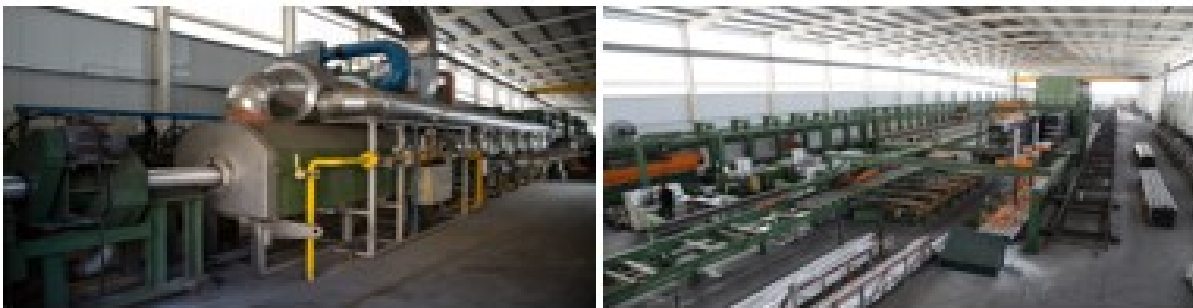


Figure 3.3: Production of aluminium profiles in the company. Source: [1].

Summarising, aluminium extrusion is a continuous process and is essentially divided into four moments: (i) pressing operation – extrusion, (ii) stretching, (iii) cutting and (iv) heat treatment. Figure 3.4 presents

## 3.2. Datasets

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the entire process.



Figure 3.4: Production process in company.

The customers order are quite diversified. According characteristics, these orders are scheduled based on the alloy type, the die, the order quantity, the delivery date, and others. The production is planning every week. The focus of the current study is to reduce scrap at the moment of extrusion, through minimization of setup times, occurred by die change. Thus, it is intended to optimize production scheduling in order to increase productivity, maintaining quality and reducing delivery times.

According to Saha [78], productivity is defined by the quantity of good extrusion produced per unit time. There are a number of ways to maximize productivity, namely: (i) minimize the amount of avoidable scrap per each billet run for the same press speed, billet size, and runout length; (ii) maximize the billet length to best utilize press capacity and runout table length; (iii) optimize the speed, predicting/controlling the variation of the exit temperature, that rise during extrusion.

Process scrap, can originate from high purity and specification requirements, quality problems, defects, over-ordering, mismatches between batch and order volumes, subtractive processing, scalping and trimming during processing and start-up losses [39]. Reducing the generation of process scrap can provide both energy and economic benefits.

Both the company under study and the literature point out that die changes generates scrap [39]. In Figure 3.4 the production process in company can be seen. The die changes occurs in the extrusion stage. These changes occur whenever the type of profile to be produced changes, or when the die exceeds its use time. Thus, a good production scheduling that minimizes die changes will allow significant scrap reduction. This is where our study is centered.

## 3.2 Datasets

The company provided two datasets. The first dataset, named Production process dataset, concerns the extrusion process and contains information collected between January 1st and June 30th, 2018. The second dataset, named Customers' orders dataset, concerns information of the customers' orders for the month of May 2018, with a total of 498 orders. Other information concerning production was also provided by the company.

The dataset contains 42 827 observations, each corresponding to an extruded billet, and is also used in work of Ferrás *et al.* [26]. This dataset contains 65 variables, 62 of which are quantitative (discrete

and continuous) variables and three are nominal qualitative. These variables can be divided into three categories: pre-extrusion ( $PR$ ), extrusion ( $E$ ) and post-extrusion ( $PO$ ), depending on the phase of the process in which they are included, except the variable order and the production date. In the present study, and according to the literature review in Section 2.1, only some of the extrusion and post-extrusion variables are considered for the developed scheduling model. However, all interfere with the quality of the extruded profile. The variables considered for the development of model presented in Section 4.2 are depicted in Table 3.1.

	Acronym	Description	Classification
Extrusion	$E\_D$	Die	Qualitative nominal
	$E\_To$	Tool	Qualitative nominal
	$E\_NH$	Number of holes	Quantitative discrete
	$E\_t$	Extrusion time (s)	Quantitative continuous
Post_Extrusion	$PO\_LB$	Length bars (mm)	Quantitative continuous
	$PO\_NB$	Number of bars	Quantitative discrete
	$PO\_WB$	Weight bars (kg)	Quantitative continuous

Table 3.1: Extrusion process dataset variables used.

The  $E\_D$  variable corresponds to the die used, and  $E\_To$  represents the tool that was used. The  $E\_NH$  variable indicates the number of holes in the die, and it is through these holes that the amount of kg of extruded product is determined. Finally, the  $E\_t$  variable indicates the time each billet took to be extruded. For the post-extrusion variables, the  $PO\_LB$  indicates the length of each bar,  $PO\_NB$  the number of bars, and  $PO\_WB$  the total weight of the bar of each extruded billet.

The variables  $E\_t$  and  $PO\_WB$ , will be used for estimate the processing time for each order (see Section 4.3.2).

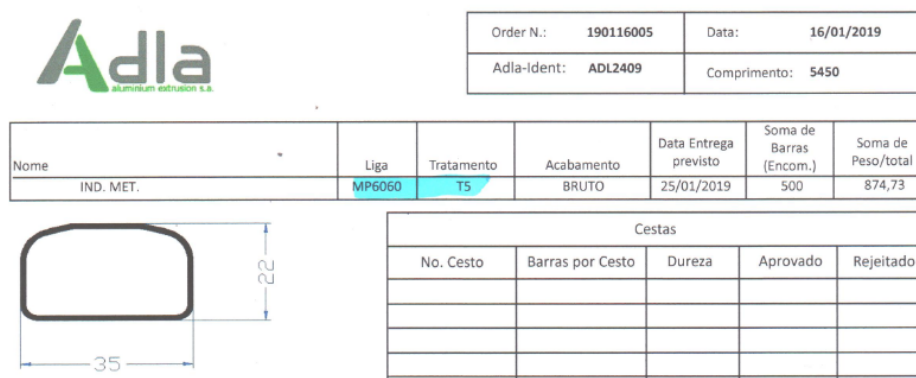


Figure 3.5: Example of a customer's order [1].

The Customers' orders dataset, contains information on each customer's order (see Figure 3.5) and includes the following specifications:



### 3.2. Datasets

- *Order N* – code that the company uses for identifying each order.
- *Data* – day that the company has received the order.
- *Adla-Ident* – the identification of the used die – *E\_D*.
- *Comprimento* – length of the bars to be produced, in mm – *LB*.
- *Nome* – Name of customer which is omitted for confidentiality reasons.
- *Liga* – alloy that is used.
- *Tratamento* – finishing treatment that is used.
- *Acabamento* – description of the type of finishing that is used and can be surface or thermal. As mentioned in Section 3.1, the company only carries out thermal treatment and therefore it has to recur to external companies for surface finishing.
- *Data Entrega previsto* – date of delivery of the order, i.e. deadline of the order.
- *Soma de barras (Encom.)* – total number of bars to be produced – *NB*.
- *Soma de Peso/total* – total weight of the bars, in kg –  $c_j$ .
- Drawing of the profile geometry, including the measurements.

From the data provided by the company, further information was computed, namely the release date and the deadline, whose calculation is presented in Section 4.3.1. In Figure 3.6 it is shown an example of an order received on 02/05/2018. The order has a deadline of 44 days to be delivered to the customer.

The order characteristics are given, namely, the desired profile type the die to be used has the code ADL4581, the order consists of 150 bars with a length of 4 meters, whose total weight is 208.8 kg.

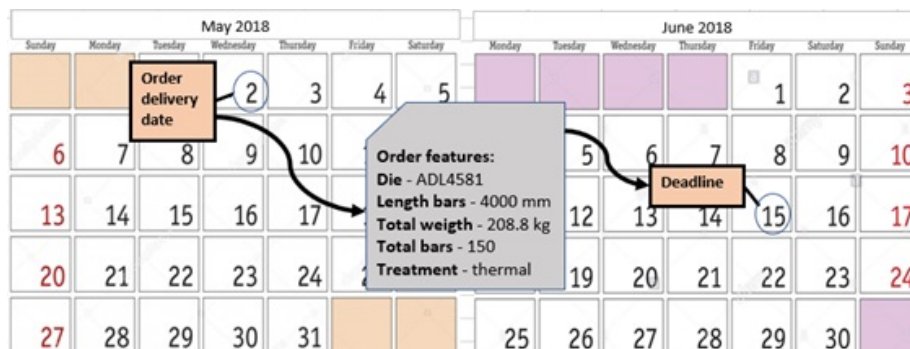


Figure 3.6: Example of an order received on 02/05/2018.

Furthermore, for performing the production scheduling more information was provided by the production manager, namely:

- Raw material dimensions – maximum 7 m.
- When to lubricate the material – every 30 billets, on average.
- Number of dies and tools available – more than 1 500.
- Maximum and minimum allowed usage time for each die – less than 25 hours but depends on the nutrition (steel hardening process).
- Maximum processing weight (kg) with a given die – 30 000 kg.
- Minimum processing weight (kg) with die – 250 kg.
- Time to exchange a die – 1.5 minutes, on average.
- Maximum length of the bars after extrusion and before cutting – 56 m.
- Time for surface treatment – 5 days.
- Time specified by the company for the order to reach production – 60 minutes.
- Time specified by the company to perform other tasks such as, cutting the bars, packing, cleaning, surface treatment and thermal treatment – 1 day, with to 5 extra days if surface treatment.
- Expected stop periods – Sundays, holidays and scheduled maintenance.

To know the exact number of copies ( $E.T_o$ ) of the dies used for producing the orders of May, information from the Production process dataset was used.

These datasets will be study in detail in the next section.

# Chapter 4

## Research Methodologies

In this chapter the research methodologies are presented. More specifically, a general data analysis of the two data sets that were provided by the company (presented in Section 3.2) is made. The developed optimization model is presented, namely the parameters, decision variables, objective function and their constraints, in Section 4.2. Based on historical data of the extrusion process, some unknown parameters of the model are computed and others are estimated, in Section 4.3.

### 4.1 General Datasets Analysis

#### 4.1.1 Production Process Dataset

Regarding the Production process dataset it can be seen from the variable ( $E_D$ ), which represents the die used for the extrusion of each billet, that a total of 3 101 orders were produced, where 764 different dies were used. Analyzing the tools of each die ( $E_{To}$ ), in the dataset can be seen that the tools can range from 1 to 27. According to company and confirmed in the dataset the maximum of number of holes ( $E_{NH}$ ) is 12. The extrusion time ( $E_t$ ) of each billet ranges from 56 to 1 200 seconds, and it can be seen that 75% of cases do not have a time greater than 195 seconds (can see in Table 4.1).

Variable	Min	Q1	Median	Mean	Q3	Max
$E_t$	56.00	143.00	167.00	173.93	195.00	1 200.00
$PO_{LB}$	2 050.00	6 000.00	6 500.00	6 542.00	6 500.00	12 050.00
$PO_{NB}$	0.00	5.00	14.00	17.24	20.00	228.00
$PO_{WB}$	0.00	59.00	66.00	63.47	72.00	88.00

Table 4.1: Statistics of the extrusion variables present in Table 3.1.

The length bars obtained in the extrusion of each billet ( $PO_{LB}$ ) has bellows between 2 050 and 12 250 with mean value 6 542, the corresponding number of bars produced is between 0 and 228, and

bars weight  $PO\_WB$  is between 0 and 88 kg.

### 4.1.2 Customers' Orders Dataset

In this section a characterization of the Customers' orders dataset is presented.

A large diversity in the order of customers is observed (Table 4.2). In fact, the lengths of each bar to be produced ( $LB$ ) ranges from 1 875 mm to 10 050 mm, and half of the ordered bars have lengths bellow 6 500 mm. Also in terms of the number of bars to be produced ( $NB$ ) there is a large variability, ranging from 14 to 20 165 bars, while half of the orders correspond to 135 bars or less. The total length of bars ( $TLB$ ) of each order varies from 60 to 120 969.80 meters and the total weight ( $c_j$ ) of the order varies between 30.75 kg and 30 000.52 kg. Half of orders have 750 meters or less, and a total weight bellow 488.87 kg.

Variable	Min	1st Q	Median	Mean	3rd Q	Max
$LB$	1 875.00	5 594.00	6 500.00	5 903.00	6 500.00	10 050.00
$NB$	14.00	53.88	135.00	278.17	299.50	20 165.00
$TLB$	60.00	343.33	750.00	1 626.80	1 749.00	120 969.80
$c_j$	30.75	254.59	488.87	908.95	999.50	30 000.52

Table 4.2: Descriptive statistics of the customers' orders.

The variable  $TLB$  stands for the total length of bars, in meters, to each order, by the equation (4.1):

$$TLB = NB \times LB \quad (4.1)$$

The month of May was characterized by a total of 498 orders from 67 distinct customers. A total of 305 different profiles are to be produced. Therefore, there is a wide variety of profiles to produce, which translates into a great complexity in the production planning. Figure 4.1 shows that the day with less orders is Friday (67 orders, or 13%). On the other hand, Thursday is the day with more orders, in a total of 134 (27%).

Upon the reception of an order, a delivery period is established which is expressed by the week of the year (between week 1 and 52). For the purposes of the present analysis the day of delivery is Friday, and consequently the deadline (last day of delivery time) can be computed.

The number of days to produce the orders, i.e., the delivery time, varies between 5 and 98 days, 50% of which have between 38 and 44 days for delivery. Only 25% of orders have delivery deadlines exceeding 44 days. Given that, on average the company has approximately 40 days to produce the orders, lead us to conclude that the deadlines are relatively small. This information can be observed in Table 4.3.

Analysing the medians of delivery time per day of the week (Table 4.3), can be concluded that the day of week that presents smaller delivery time is Thursday while Friday is the one with the highest value. For average values the minimum occurs on Tuesday and the maximum on Friday.

#### 4.1. General Datasets Analysis

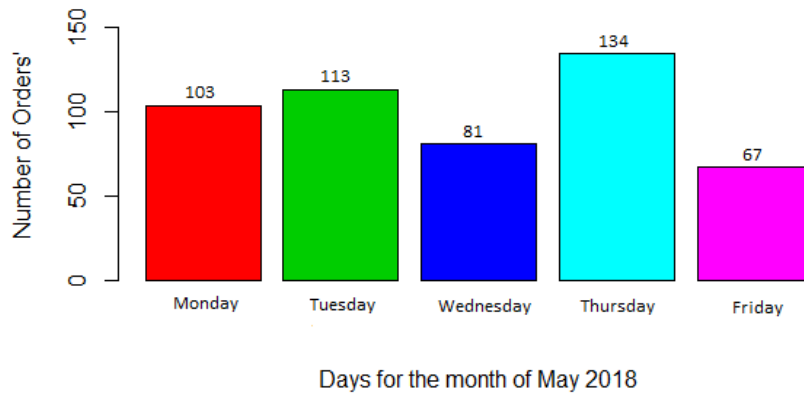


Figure 4.1: Total number of orders' per day of week.

Delivery time	Min	1st Q	Median	Mean	3rd Q	Max
All days of week	5.00	38.00	40.00	39.77	44.00	98.00
Monday	5.00	41.00	41.00	38.96	41.00	53.00
Tuesday	16.00	34.00	40.00	37.66	46.00	76.00
Wednesday	33.00	39.00	39.00	41.74	45.00	51.00
Thursday	8.00	38.00	38.00	39.61	38.00	98.00
Friday	19.00	43.00	43.00	42.46	49.00	49.00

Table 4.3: Delivery time of the orders statistics of May 2018.

The violin plot in Figure 4.2 suggests that most of the orders around 40 days. The weekday with the highest variability is the Thursday. The day with the smallest variability is the Wednesday.

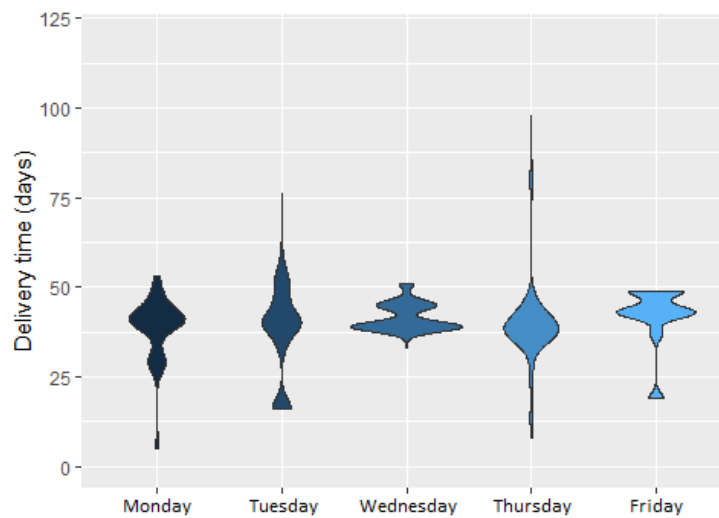


Figure 4.2: Delivery time per order day of week.

In order to verify if there is a significant difference between the delivery time per day of the week some

statistical tests were conducted.

In order to compare the mean delivery times per day of the week, the parametric statistical test Analysis of Variance (ANOVA) can be used. Before using ANOVA it is necessary to verified normality, homogeneity of variances and independence of groups. The normality is reject since the p-values of the test Shapiro–Wilk presents a value less than 5%. However, since the sample dimension is higher than 30, by the Central Limit Theorem, the normality can be assumed.

The Levene test is used to test the homogeneity of the variances. Their p-value is approximately zero ( $p = 1.291e^{-05}$ ), so,  $H_0$  (group variances are equal) at significance level  $\alpha = 0.05$  is rejected. Thus, there is heterogeneity of variances, therefore ANOVA cannot be used and the Kruskal Wallis (KW) for equality for medians is used. Its p-value ( $3.395e^{-12}$ ) is less than 0.05, thus for a significance level of 5%, the null hypothesis is rejected, i.e., there are significant differences between medians delivery times on different days of the week.

Next comparisons between the medians delivery times on two different days of the week is made using the Wilcoxon test. Table 4.4 shows the p-values. It can verified that there are significant differences between the delivery times for all pairs of weekdays except for Wednesday – Monday, and Wednesday – Tuesday.

	Monday	Tuesday	Wednesday	Thursday
Tuesday	0.02566	-	-	-
Wednesday	0.69536	0.69536	-	-
Thursday	0.00104	0.00497	$2.0 e^{-16}$	-
Friday	$8.2 e^{-08}$	0.00097	0.03645	$6.4 e^{-07}$

Table 4.4: P-values for Wilcoxon test.

## 4.2 The Developed Optimization Model

This work addresses a deterministic scheduling problem with sequence dependent job setup times, where the set of jobs to process is known before their scheduling. Note that in this problem the data is deterministic and known in advance. The objective is to minimize the setups and therefore contribute to the reduction of scrap produced.

In this section the formulation of the production scheduling for aluminium extrusion as an optimization problem is presented.

The scheduling problem studied in this work is a flowshop. In this kind of problems each job  $j$  is processed in several machines. However, in accordance with goal of this work, that is to minimize the amount of scrap produced, the present mathematical formulation considers only one machine, the extruder, with several dies. This situation can be interpreted as the existence of several machines, that cannot work simultaneously.

## 4.2. The Developed Optimization Model

---

There is evidence that if the setup times at the extrusion are minimized, there will be a decrease in scrap production. These setup times occur whenever die changes are performed. Die changes when there is a profile change in the production or when the time/weight limit of the utilization of the die has been achieved.

The proposed model will be intended to allow for short term production scheduling. Moreover sufficient raw materials are assumed and there is no limitation on storage area. In this deterministic problem the demands are known *a priori* and maximum inventory level is assumed.

### 4.2.1 Parameters

The parameters of the model concern: orders, dies and setups. The following is a description of the parameters used in the model.

- $j \in Orders$  are the orders, with  $\#Orders = n$ .
- $M_j$  is the set of dies<sup>1</sup> that can be used for processing order  $j$ .
- $d \in Dies$  are all the available dies, with  $\#Dies = m$ .
- $p_j$  is the extrusion processing time of order  $j$ .  
This parameter is not available, therefore it is estimated using linear regression models as explained in Section 4.3.2. This time depends on the quantity, profile geometry, number of holes, among other factors.
- $r_j$  is the release date of order  $j$ , in minutes.  
The computation of this parameter is explained in Section 4.3.1.
- $d_j$  is the deadline of order  $j$ , in minutes.  
The computation of this parameter is explained in Section 4.3.1.
- $c_j$  is the weight, in kg, to extrude of order  $j$ .
- $DMax_d$  is the maximum allowed processing extrusion weight in kg of die  $d$ .
- $DMin_d$  is the minimum allowed processing extrusion weight in kg of die  $d$ .
- $\alpha$  is the time for changing the die, and is given in Section 3.2.
- $S_{jj'}$  is the setup time that is such that:

$$\begin{cases} \alpha & \text{if from order } j \text{ to } j' \text{ the die must be changed} \\ 0 & \text{otherwise} \end{cases}$$

---

<sup>1</sup>In fact, this is the combination of die  $E\_D$  and tool  $E\_To$ .

- $t_{prod}$  is the time specified by the company, for the order to reach production, in minutes. This is the average time that the company estimates it takes to get the order to the production manager, including the time of evaluating if the die is good for production (see Section 3.2).
- $t_{after}$  is the time specified by the company that takes to perform other tasks such as, cutting the bars, packing, cleaning, surface treatment and thermal treatment (see Section 3.2).
- $M$  represents a sufficiently large positive number.  
This scalar is introduced in such a way that the constraints in which it is used can be made redundant when necessary.

### 4.2.2 Decision Variables

Three sets of decision variables are used, namely:

- $s_j \geq 0$  is the starting time of extrusion process of order  $j \in \{1, \dots, n\}$ .
- $x_{jj'} \in \{0, 1\}$ , and is 1 if order  $j'$  immediately follows  $j$ ,  $\forall j \neq j'$ , and is 0 otherwise.
- $m_j^d \in \{0, 1\}$ , is 1 if die  $d$  is used for processing order  $j$ , and is 0 otherwise.

The decision variables,  $s_j$ , are continuous. The other decision variables are binary and are related to the ordering of the orders and die assignment.

### 4.2.3 Mathematical Model

In this section a Mixed Integer Linear Programming (MILP) model is developed. In the formulation of the model the notation of Areanales [7] is used. Taking into account the characteristics of the problem under study, and the definitions of the previous section, the objective function and constraints are presented next.

#### Objective Function

Literature presents several objective functions. However none of these will be used, since the goal is to minimize the amount of scrap produced. As peak production amount of scrap is observed in the production process dataset at instances when die change occurs, the minimization of setup times will be used, since this minimization will imply the minimization of die changes.

The objective is to find a schedule that minimizes the sum of setup times:

$$\min \sum_{j \neq j' \in Orders} S_{jj'} x_{jj'} \quad (4.2)$$



## 4.2. The Developed Optimization Model

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### Constraints

$$s_j \geq r_j + t_{prod}, \quad \forall j \in Orders \quad (4.3)$$

$$s_{j'} \geq s_j - M + p_j + Mx_{jj'} + S_{jj'}x_{jj'}, \quad \forall j \neq j' \in Orders \quad (4.4)$$

$$\sum_{d \in M_j} m_j^d = 1, \quad \forall j \in Orders \quad (4.5)$$

$$\sum_{d \notin M_j} m_j^d = 0, \quad \forall j \in Orders \quad (4.6)$$

$$s_j + p_j + t_{after} \leq d_j, \quad \forall j \in Orders \quad (4.7)$$

$$\sum_{j \in Orders} m_j^d c_j \leq DMax_d, \quad \forall d \in Dies \quad (4.8)$$

$$\sum_{j \in Orders} m_j^d c_j \geq DMin_d, \quad \forall d \in Dies \quad (4.9)$$

$$x_{jj'} + x_{j'j} \leq 1, \quad \forall j \neq j' \in Orders \quad (4.10)$$

$$\sum_{j' \in Orders \setminus \{j\}} x_{jj'} = 1, \quad \forall j \in \{0\} \cup Orders \quad (4.11)$$

$$\sum_{j \in \{0\} \cup Orders \setminus \{j'\}} x_{jj'} = 1, \quad \forall j' \in \{0\} \cup Orders \quad (4.12)$$

$$s_j \geq 0, \quad \forall j \in Orders \quad (4.13)$$

$$x_{jj'} \in \{0, 1\}, \quad \forall j \neq j' \in Orders \quad (4.14)$$

$$m_j^d \in \{0, 1\}, \quad \forall j \in Orders, \quad d \in Dies \quad (4.15)$$

Constraints (4.3) guarantee that each order  $j$  can only be processed after all procedures prior to extrusion production are executed. Constraints (4.4) ensures that if a order  $j$  precedes order  $j'$  (i.e.,  $x_{jj'} = 1$ ) then the starting time of order  $j$ ,  $s_j$ , plus its the processing time,  $p_j$ , and setup time of  $j$ ,  $S_{jj'}$ , is less than the starting time of order  $j'$ ,  $s_{j'}$ . Otherwise, that is, if  $x_{jj'} = 0$ , where  $M$  is a sufficiently high constant, the respective constraint becomes redundant. Constraints (4.5) require that each order  $j$  is only assigned to exactly one die  $d$ , among those that can be used. Conversely, constraints (4.6) guarantee that the order  $j$  cannot be processed by an die  $d$  not in the  $M_j$  set. Constraints (4.7) force each order  $j$  to be ready for delivery to the customers before the deadline, including the time to be extruded and the time it takes to perform other tasks such as, cutting the bars, packing, cleaning, treatment and finishing or thermic, whose time is given by  $t_{after}$ . Constraints (4.8) and (4.9) define the maximum and minimum of extruded weight that can be extruded by each die  $d$ . Constraints (4.10) guarantees that either order  $j$  precedes (not immediately) order  $j'$  or order  $j'$  precedes order  $j$ . Constraints (4.11) defines that if every order  $j'$  has a predecessor order  $j$  that may be the initial dummy product. Constraints (4.12) are similar to the previous. Finally, constraints (4.13), (4.14) and (4.15) define the domain of the decision variables.

This model has a total of  $n(n+m)$  decision variables and  $2n^2 + 4n + 2m + 2$  constraints. The  $n(n+m)$  decision variables are distributed between (4.13), (4.14) and (4.15), with  $n$ ,  $n(n-1)$  and  $nm$ , respectively. While (4.3), (4.5), (4.6) and (4.7) correspond to  $n$  constraints each, (4.4) and (4.10) correspond to  $n(n-1)$

constraints each, (4.11) and (4.12) correspond to  $n + 1$  constraints each, (4.8) and (4.9) correspond to  $m$  constraints each.

The model here proposed assumes that the weights of the individual orders satisfy the limits set by the production manager. If this does not occur, i.e., if the order's weight is larger than allowed the order is subdivided in several smaller orders in a preliminary step prior to scheduling.

### 4.3 Parameters Estimation and Computation

In Section 3.2 a brief description of the two datasets used for the development of the optimization model presented in Section 4.2 was made. However, information that is not available is necessary for being included in the model. In this section the calculations to determine the releasing date ( $r_j$ ) and the deadline ( $d_j$ ) are presented. Furthermore, the methods used for estimating the processing time ( $p_j$ ) are presented.

#### 4.3.1 Release Date and Deadline Computation

In the Customers' orders dataset there is information of the date in which each order is received. However, for the development of the proposed model it was necessary to make some adjustments. The dataset starts on 02/05/2018, which was considered the reference date, i.e.,  $t = 0$ .

The release date of order  $j$ ,  $r_j$ , was calculated as the minutes from the reference date until the date in which the order is received, excluding Sundays. For example,  $r_j = 0$  corresponds to the release date of the order  $j$  that arrived at the company on 02/05/2018. Since production works 24 hours per day (in continuous) the release date of an order that arrives at the company on 03/05/2018 is  $24 \text{ hours} \times 60 \text{ minutes} \times 1 \text{ day} = 1440 \text{ minutes}$ . An order that arrives at the company on 07/05/2018 is  $24 \text{ hours} \times 60 \text{ minutes} \times 4 \text{ days} = 5760 \text{ minutes}$ , since there are four working days from 02/05/2018 to 07/05/2018<sup>2</sup>, as illustrated in Figure 4.3.

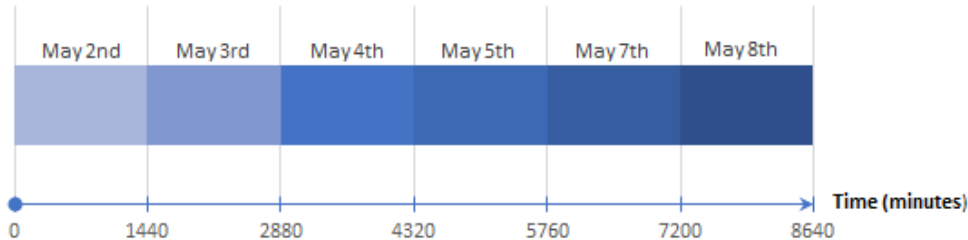


Figure 4.3: Release date illustration.

The deadline of order  $j$ ,  $d_j$ , corresponds to the limit moment for delivery of the order  $j$ , and was calculated as the minutes from the reference date until the date the delivery of the order  $j$ , excluding

<sup>2</sup>Note that 06/05/2018 was a Sunday, i.e., a non working day.

### 4.3. Parameters Estimation and Computation

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Sundays. That is,  $d_j$  is equal to  $r_j$  plus the delivery time of the order  $j$ . For example the order  $j = 36$ , that arrives on 03/05/2018, thus has a release date of  $r_{36} = 1\,440$  minutes, and must be delivery until 15/06/2018 with a delivery time of 38 working days, corresponds to a deadline of  $d_{36} = 54\,720$  minutes.

#### 4.3.2 Processing Time Estimation

In classical deterministic scheduling, the job processing times are generally assumed as constants. However, there are situations in which the real processing times may vary due to the learning, aging or deteriorating effect.

The processing time of each order,  $p_j$ , is not available, therefore it is necessary to estimate it. For this purpose, for each die (in a total of 764), a linear regression models was estimated from historical data from the extrusion process.

From the Production process dataset a new dataset was created aggregating all billets of each produced order  $j$  in a single row. The variables in this dataset correspond to the total processing time ( $\sum E.t$ ) and total weight ( $\sum PO.WB$ ) for each order  $j$ .

Variable	Min	1st Q	Median	Mean	3rd Q	Max
Total processing time	113.0	693.0	1 218.0	2 405.0	2 450.0	49 508.0
Total weight	19.0	244.0	439.0	878.8	922.0	20 865.0

Table 4.5: Statistics for total processing time and weight of each order.

The main statistics for these new variables are presented in Table 4.5. A large variability in terms of total processing times and total weights can be observed. The total time of each order is between 113 and 49 508 seconds, and the total weight variable is between 19 and 20 865 kg. Note that 75% of orders present total processing time less than 2 450 seconds and a total weight bellow 922 kg. Thus the majority of the orders are small. Therefore, assuming that the weight limits are respected in a realistic assumption. Figure 4.4 suggests a linear relation between total processing time and total weight.

For exemplification, Figure 4.5 shows the behavior of the two most used dies for which we have historical data on extrusion, that is dies in the Production process dataset. For these two dies the same linear relation between the processing time and weight, can be observed.

The dispersion diagrams in Figures 4.4 and 4.5 show, as expected, a positive correlation between the variables, i.e., on average, an increase in weight leads to an increased processing time. From the graph on the left, for Die A, the processing time is expected two increase 2.7752 seconds per each additional kg, starting with a default time of 211.30939 seconds. For Die B, the ratio of increase per kg is different from the one of Die A. In fact, the graph on the right, shows expected default processing time of 109.68341 seconds, that increases 2.97825 seconds per each additional kg. In both cases, the coefficient of determination,  $R^2$ , which is used as a measure of fit quality, is approximately 0.97. Therefore, in both the linear regression models, 97% of the total processing time is explained by the total weight.

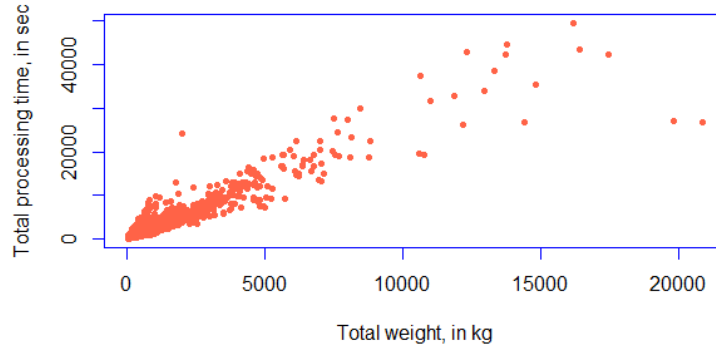


Figure 4.4: Total processing time *vs.* total weight, for all orders.

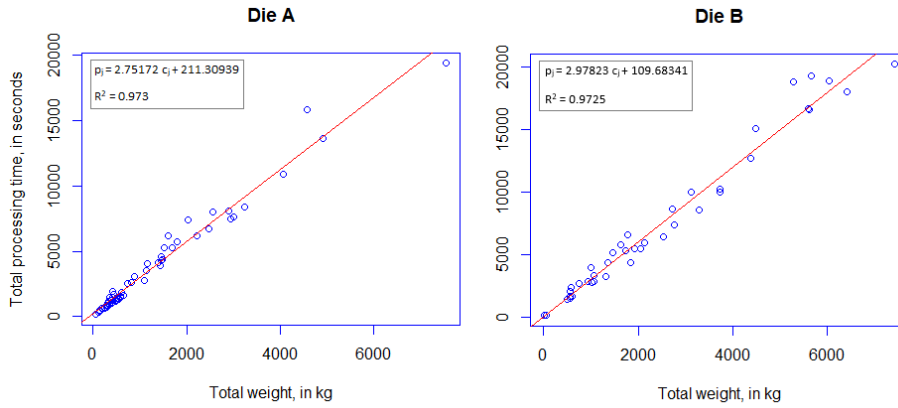


Figure 4.5: Linear regression models.

To estimate the linear regression models the software R, version 3.6.1, was used. For an order  $j$  processed using die  $d$ , the processing time will be estimated based on following equation:

$$p_j = m^d c_j + b^d \quad (4.16)$$

where,

- $p_j$  is the extrusion processing time of job  $j$ , in seconds.
- $c_j$  is the quantity produced, in kg.
- $b^d$  is the intercept.
- $m^d$  is the slope.

Three situations where this process cannot be applied are verified:

### 4.3. Parameters Estimation and Computation

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- The historical data just have one order  $j$  for the die  $d$  and is not possible estimate the linear model;
- The die  $d$  is new and not available in the historical data, because the dies were not used in production from January to June 2018;
- The linear regression estimation resulted in a negative slope, which is not realistic. This occurs in 9 orders.

In the first case the slope was computed using the following:

$$m^d = \frac{\text{Total processing time}^d}{\text{Total weight}^d} \quad (4.17)$$

In the second and third cases the processing time was calculated similarly to the previous one, but using average values of the entire Production Process Dataset (presented in Table 4.1), i.e.:

$$m^d = \frac{173.93}{63.47} \quad (4.18)$$

For all previous cases the intercept,  $b^d$ , is set to zero.

There was one case for which the estimated processing time was negative. For this cases the procedure used was the same used in the second and third cases.



## Chapter 5

# Case Study Results

In the previous chapter, a model that aims at answering the company's specifications was developed. In this chapter, instances extracted from the Customers' orders dataset (see Section 3.2), provided by the company, as well as the computational results obtained for these instances are presented. Since there is no information of the company's scheduling procedure for the dies being schedule, it is not possible to make a direct comparison of the results of the model and the company's reality.

To overcome this situation, the average die changes per week was computed, from the historical production process data. Furthermore, the FIFO dispatching rule (c.f. Section 2.2) very frequently used in practice, was also used with the instanced tested. In this rule, it is selected the first job that arrived in the queue.

Finally, in order to perceive the improvements obtained with the developed model, in Section 5.6 the results obtained with the MILP model are compared to the average die changes per week of company and to the results of the dispatching rule FIFO.

The MILP model (4.2)–(4.10) was implemented using AMPL<sup>1</sup> (A Modeling Programming Language) modeling system and the real instances were solved using the Gurobi solver, available at the NEOS<sup>2</sup> (Network Enabled Optimization System) platform.

AMPL is one of the most used modeling languages for mathematical programming [28]. Due to the generality of its syntax and the similarity of its expressions with algebraic notations, which enables to easily specify and understand the objective function, the constraints, as well as the logical relations between variables. NEOS is an environment for remotely solving optimization problems [17]. It is supported by the Optimization Technology Center of Northwestern University & Argonne National Laboratory and is a free service, requiring no registration, downloads or installations. It supports a wide variety of problem formats (e.g., AMPL, GAMS, etc) and features an easy to use interface.

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<sup>1</sup><http://www.ampl.com>

<sup>2</sup><https://neos-server.org/neos/solvers/>

For obtaining the numerical results of the instances, a single mod file is used. This file presents the specification of the model without any values attributed to the parameters. Also, a single run file containing solver and display specification is used. While, several data files (one per instance), were created (see Figure 5.1).

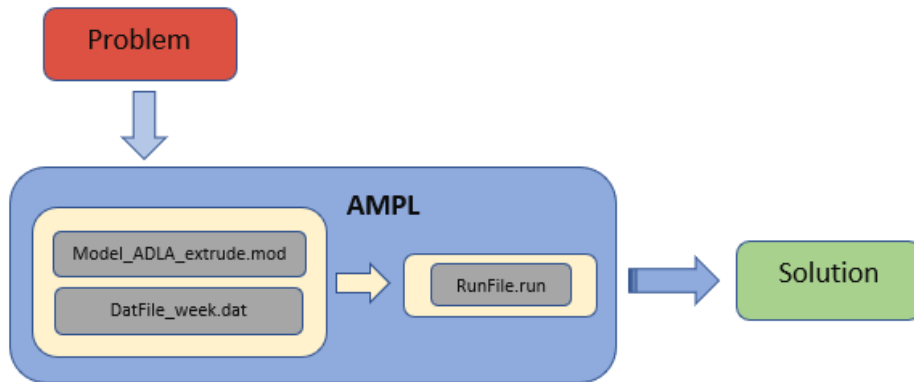


Figure 5.1: Implementation of the model in AMPL

The model is given in the `Model_ADLA_extrude.mod` file and includes the codification of all the parameters, decision variables, objective, constraints. This model is used to test all the instances. For writing the dat files for each instance, the file `WriteDatFiles.R` was created. This R file reads the information from the Customers' orders dataset and writes a dat file for each instance with the syntax used by AMPL. Finally, `RunFile.run` contains the code for the selection of the solver and the formatting information of the output that is produced, i.e., the optimal solution.

## 5.1 The Procedure of the Company

There is no information on the company's scheduling procedure for the dies being schedule, therefore it is not possible to make a direct comparison of the results of the model and the company's reality.

However, a characterization of the orders from the historical production process data was made and an average die changes, per week, was computed. Analyzing the historical production data provided in the Production process dataset it could be observed that between between January 1st and June 30th a total of 3 101 orders were produced and 2 046 die changes were made. That gives an average of approximately 119 orders and 79 die exchanges, per week.

Figure 5.2 indicates the number of the orders produced and the number of die changes made in each month. This figure shows differences for different months. Specially may is the month with the highest number of produced orders, in a total of 664, but a number of die changes of 368, which is similar to the ones observed in the other months.

A more detailed analysis is performed to verify the ratio of die changes per produced orders, in each month. In Table 5.1 the percentage of produced orders that have die changes is presented. March is the



## 5.2. Instances Selection

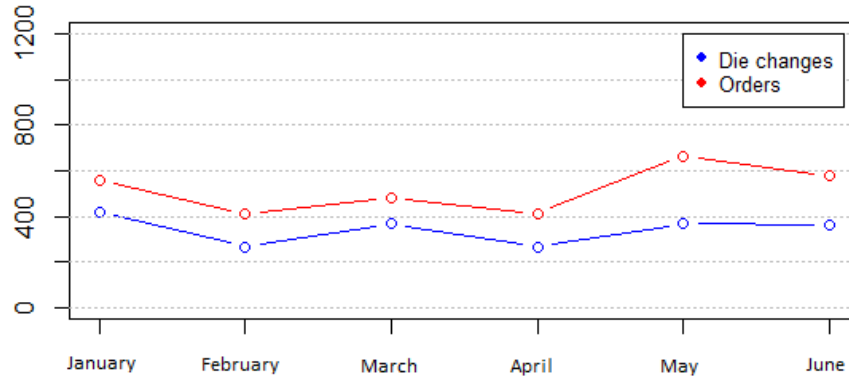


Figure 5.2: Illustration of the number of orders produced and number of die changes, per month, between January 1st and June 30th, 2018.

	Orders	Die changes	Ratio
January	556	416	74.82%
February	409	266	65.04%
March	478	368	76.99%
April	414	265	64.01%
May	664	368	55.42%
June	580	363	62.59%

Table 5.1: Percentage ratio of die changes for the orders produced in each month.

month which presented a highest percentage, approximately 77% , of die changes. On the other hand, May presented the lowest percentage, approximately 55%. Overall, there was an average of approximately 67% of produced orders that have die changes. Furthermore, there was, on average, 79 die changes per week. Note that these values will serve as a reference to compare the numerical results obtained using the model developed.

In the next subsection, the selection of instances, as well as the computational results are presented and discussed.

## 5.2 Instances Selection

The real data provided by the company concerning the customers' orders for the month of May 2018, presented in Section 4.1.2, was used to construct the instances for which the numerical results were obtained. Taking into account the expected stop periods, specified in Section 3.2, a total of 26 working days and 498 orders were considered.

Instances were selected to test the scheduling of incoming customers' orders. The objective was to

consider instances that present the business environment reality, depict the recurring volatility of the markets. These results will allow to validate the model proposed in Section 4.2 which aims to minimize die changes by minimizing the setups times, that consequently will contribute to the reduction of scrap production in extrusion process.

For each instance, the orders relative to a week with six working days are considered. The selection of considering instances with six working days also had into account the statistics results obtained in Table 4.3 (see Section 4.1.2), i.e., the existence of orders that have five days to delivery time.

Information from the company concerning if there are specific days to schedule the production of orders was not provided. Therefore, our selection began with the first working day. In fact, since Wednesday, 01/05/2018, is an holiday, therefore, the weeks are considered from Wednesday to Tuesday, namely:

- Week 1: corresponds the orders from May 2nd to 8th.
- Week 2: corresponds the orders from May 9th to 15th.
- Week 3: corresponds the orders from May 16th to 22nd.
- Week 4: corresponds the orders from May 23rd to 29th.

Note that following this procedure the last days of May, i.e. 30th and 31st, were not considered. Figure 5.3 presents the characterization of the four weeks, relatively to the number of orders received, the number of customers and the different dies used.

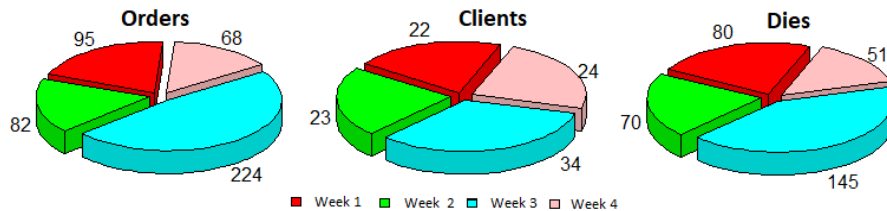


Figure 5.3: Characterization of the weeks considered. From left to right: number of orders, number of customers and number of dies.

In Figure 5.3 it can be seen that in the third week, from May 16th to 22nd, the total number of orders received was 224. This presents approximately 45% of the total number of orders of entire month of May. For the sake of brevity, only 50% of the orders received in the month of May 2018 will be used to test the model. In order to analyze three different scenarios, the selection of instances to test the scheduling of order production relapsed on the period from May 2nd to 8th (Week 1), 16th to 22nd (Week 2) and 23rd to 29th (Week 4).

Table 5.2 describes the instances to be tested, as well the total number of orders in each. In the following sections the results and discussion for the three instances are presented in detail.

### 5.3. May 2nd to 8th

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Days	Total Orders
May 2nd to 8th	95
May 9th to 15th	82
May 23rd to 29th	68

Table 5.2: Characterization of the real instances used to test the developed model.

### 5.3 May 2nd to 8th

The first instance corresponds to orders from May 2nd to 8th, and includes the scheduling of 95 orders from 22 different customers (see Figure 5.3). The release date this instance starts at  $t = 0$  minutes and finishes at  $t = 7\,200$ . Table 5.3 presents information concerning the total weight, the processing time and the deadline of these orders.

	Min	Q1	Median	Mean	Q3	Max
Total weight ( $c_j$ )	200.50	252.00	440.70	1 075.10	1 001.90	14 051.40
Processing time ( $p_j$ )	7.45	11.83	18.37	52.68	48.13	570.19
Deadline ( $d_j$ )	12 960.00	56 160.00	56 160.00	62 435.00	73 440.00	73 440.00

Table 5.3: Total weight, the processing time and the deadline of the orders received from May 2nd to 8th.

In Table 5.3 it can be observed a large variability in the total weight of each order, with orders with weights from 200.50 kg to 14 051.40 kg. However, 75% of orders have a total weight less or equal than to 1 001.90 kg. In the orders produced, 50% have processing times less or equal than to 18.37 minutes and the maximum was of 570.19 minutes. Deadline average is 62 435.00 minutes, ranging from 12 960.00 to 73 440.00 minutes.

#### 5.3.1 Results of FIFO Dispatching Rule

In this subsection the results from May 2nd to 8th obtained using the dispatching rule FIFO are presented. This rule gives priority to the order of arrival, i.e., the first order that arrives is the first to be produced. For this purpose, the data from Customers' orders dataset was sorted by order of arrival and whenever the die that must be used in a given order is different from the previous one a setup time of 1.5 minutes<sup>3</sup> is added. Note that before each order is processed there is a prior time to consider since its arrival, i.e., the average time that the company estimates it takes to get the order to the production manager, including the time of evaluating if the die is good for production. Considering the information provided by the company this was set to 60 minutes.

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<sup>3</sup>This value was provided by the company

Applying the dispatching rule FIFO from May 2nd to 8th, a total of 85 die changes is obtained, with a total setup time equal 127.50 minutes. The first order started to be produced in the minute 60.00 and the last order finished in the minute 8 257.50. The time to process all orders is approximately 5 005.00 minutes. Table 5.3 indicates that the minimum order deadline verified for these instance is 12 960.00 minutes, with is quite below the processing time. Therefore, it is guaranteed that all orders were produced on time, i.e., before the deadline.

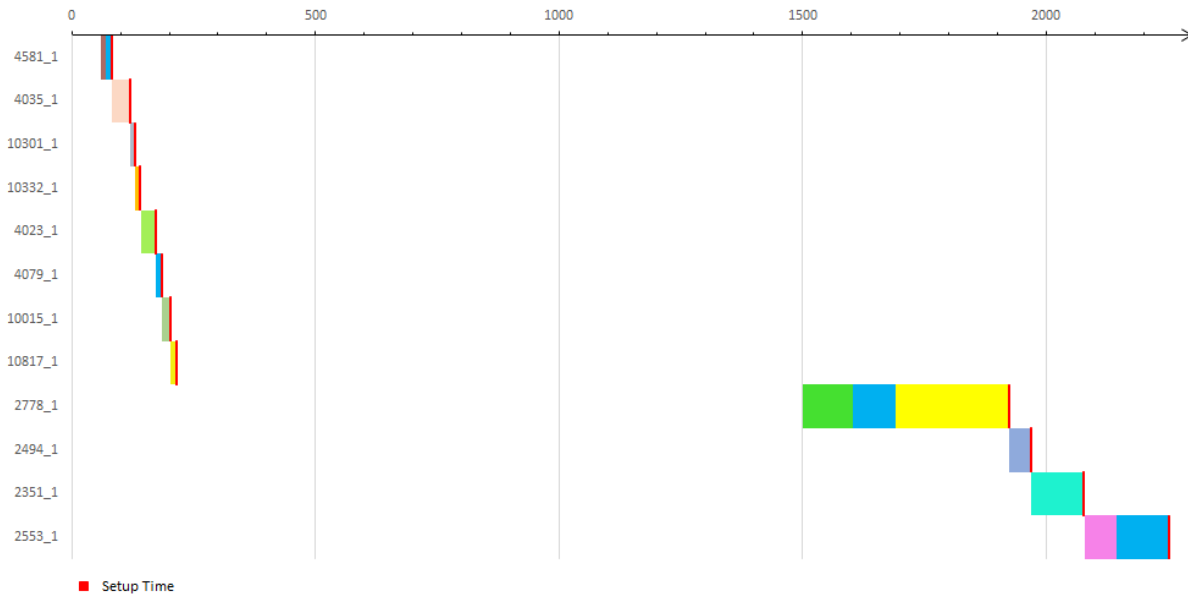


Figure 5.4: Description, per die, of the orders production scheduling obtained by the dispatching rule FIFO on the 16 first orders for the May 2nd to 8th instance.

Figure 5.4 shows the sequence of the 16 first orders and corresponding setup times for each die. The solution for the complete instance is presented in Figure A.1, in Appendix. The 16 first orders for the week 1 instance, can be described as:

- The first order starts to be produced at minute  $t = 60$  and die 4581\_1 is used, with a processing time of 10.66 minutes. The second order also uses the same die and the processing time was also the same. As the third order does not use the same die, a setup time of 1.5 minutes is considered.
- The next orders to be produced use different dies, namely dies 4035\_1, 10301\_1, 10332\_1, 4023\_1, 4079\_1, 10015\_1 and 10817\_1. They have different processing times, and at the end of each production order a setup time of 1.5 minutes is verified.
- The next order only arrived at the minute 1440, so it can only be produced from there, taking into account the 60 minutes needed to be able to be produced. Die 2778\_1 is used to produced the next three orders. The first one starts at 1 500.00 minutes and the third one ends at minute 1 921.76. Next, the die is changed, generating a setup time of 1.5 minutes.

### 5.3. May 2nd to 8th

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- Order 13 and 14 use different dies. The first start at minute 1 923.26 and the second starting at minute 1 968.36. Between the two orders, there is a 1.5 minute setup time resulting from the die exchange.
- Lastly, the 2553\_1 die is used for the production of the last two orders, where the first one starts at minute 2 077.97 and has a processing time of 65.57 minutes. The second has a processing time of 108.06 minutes and ends at minute 2 251.60.

#### 5.3.2 Results of the Developed Model

The optimal solution found for the instance of May 2nd to 8th, presents and objective function value of 118.50. Therefore, in total the optimal solution has a minimum total setup times of 118.50 minutes, which corresponds to 79 die changes. The optimization problem presented a total of 9 251 variables, the large majority of which are binary and only 95 are linear, 17 860 inequality constraints and 207 equality constraints (see Table 5.9).

Figure 5.6 presents the result of the scheduling of this instance using the model developed in Section 4.2. Analogously to the FIFO dispatching rule, the first order could only start to be produced 60.00 minutes after arriving to the company, and when die is changed a setup time of 1.50 minute is added.

The first order started to be produced at minute 1 500.00 and the last finished your production at minute 54 735.78. Figure 5.6, shows that that none of the orders produced by the same die overlap. The existence of three dead times in the scheduling of orders was verified (see Figure 5.6), i.e., time intervals in which no production is being made. In fact, the developed model is only minimizing setup times and there is no guarantee that dead times will not be verified.

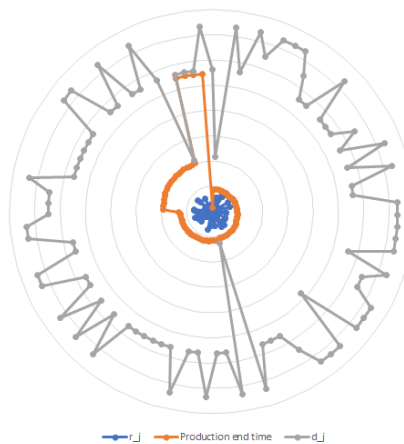


Figure 5.5: Representation of release date, production end time and deadline for each order from May 2nd to 8th.

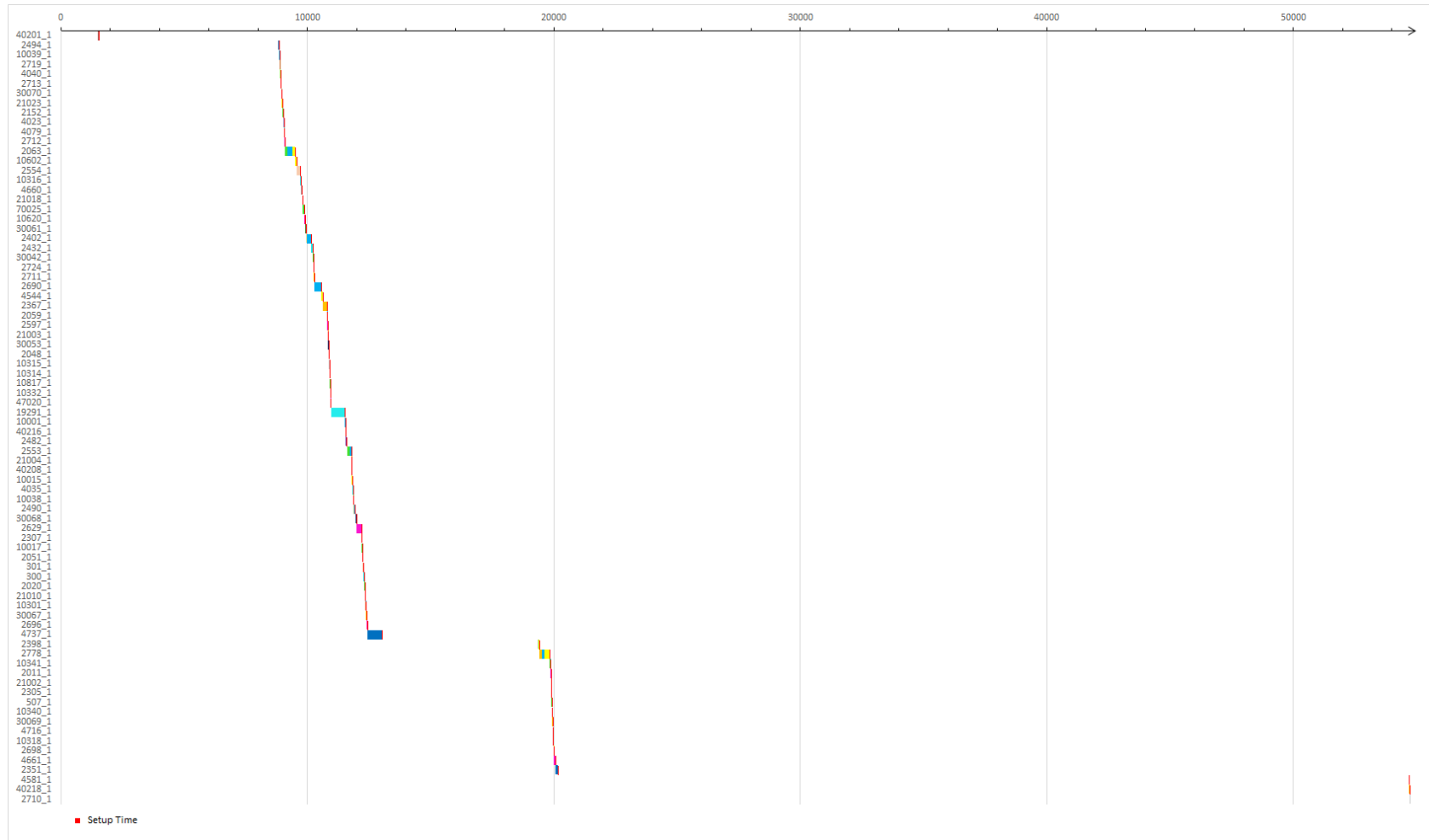


Figure 5.6: Solution, per die, for the orders from May 2nd to 8th.

### 5.3. May 2nd to 8th

On the other hand, from Figure 5.5 it is possible to verify that the time constraints are respected, i.e., orders are only processed after release date and finish before the deadline, including processing time and the time to perform other tasks such as, cutting the bars, packing, cleaning, surface treatment and thermal treatment. Therefore, all constraints imposed in the model are being guaranteed.

In order to clarify the results obtained with the developed model, the scheduling solution for a six orders of this instance is presented in Figure 5.7.

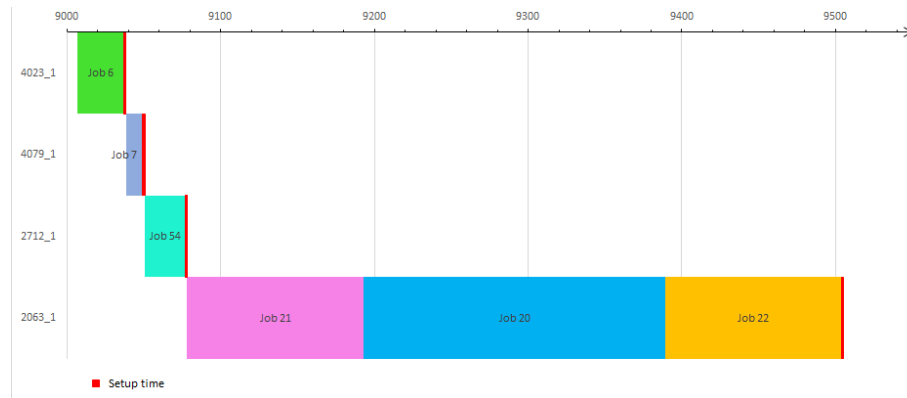


Figure 5.7: Solution, per die, from 11st to 16th produced order, from May 2nd to 8th.

The first order illustrated, Job 6, corresponds to the order was produced in 11st place, and the last order, Job 22, was produced in 16th place. This period is selected since it depicts two distinct cases that may occur, i.e., the die produced only one order, or a die produced multiple orders. The scheduling solution was:

- Job 6 was produced with die 4023\_1 and had a processing time of 30.26 minutes. The release date is at minute 0 and started production at minute 9 006.62. This order has a deadline of 56 160.00 minutes to be delivered to the customer. As the next order will use a new die, a setup time of 1.50 minutes is verified.
- The next order to be produced is number 7, Job 7, which also arrived at time 0 and started production at minute 9038.38. The die used is 4079\_1. With a processing time of 10.99 minutes per minute, a setup time of 1.50 minutes occurs at minute 9 049.37. The deadline is 56 160.00 minutes.
- Job 54 arrived at the minute 2 880.00 and began to be produced at minute 9 050.87, on the die 2712\_1. It has a processing time of 25.95 minutes and in the end die exchange occurs. The deadline is 73 440.00 minutes.
- Jobs 21, 20 and 22 are produced with the die 2063\_1. These orders arrived at minute 1 440 and began to be produced at minute 9 078.32, minute 9 193.06 and minute 9 389.13, respectively. At the end a setup time of 1.50 minutes is verified, which denote that the next order will use a different die. The deadline for each order is 56 160.00 minutes.

The optimization model was developed based on the information provided by the company. However, a computational test considering the parameter  $t_{prod}$  equal to minute 1 440 was made, i.e., the orders can only start being produced the next day your arrival. This parameter,  $t_{prod}$ , is the time specified by the company, for the order to reach production, and in the developed model is considered to minute 60. The results obtained yield an objective function value 118.50 minutes, i.e., there were 79 exchanges. This was also the number of die exchanges found previously. The first order started production at minute 8 640 and the last order finished at minute 13 763.20. Deadline fulfillment was verified and all orders were completed before the delivery date to the customer and all constraints imposed were verified. The results of this model can be seen in Figure A.2, in Appendix.

### 5.3.3 Performance Comparison

Table 5.4 presents the results obtained by dispatching rule FIFO and the developed model, to a  $t_{prod}$  equal to 60 or 1 440 minutes, for the instance of May 2nd to 8th. The results are relative to the total number of die changes, the total of setup times, and the beginning and end of the first and last order produced. Total processing time is also showed.

	Setup times	Die changes	First order		Last order		Processing time
			Start time	End time	Start time	End time	
FIFO	127.50	85	60.00	70.66	8 235.48	8 257.50	5 004.69
OPT	118.50	79	1 500.00	1 544.85	54 721.50	54 735.78	5 004.69
OPT <sup>1</sup>	118.50	79	8 640.00	8 662.63	13 749.00	13 763.20	5 004.69

Table 5.4: Comparison of results for the instance of May 2nd to 8th.

Regarding the results obtained, it can be seen that if the company applies the FIFO rule, 85 die changes will be made, corresponding to 127.50 minutes of setup times. On the other hand, the developed model presents lower values, i.e., for the same period there are 79 die changes with a total 118.50 minutes of setup times. The results refer to a total orders processing time of 5 004.69 minutes.

Considering the purpose of this study, the minimization of the scrap produced, through the minimizing of die changes, the developed model presents the most advantageous solution for the company when compared to the solution found with the FIFO rule. In relation to the beginning and end of the production of orders, it is verified by the FIFO method the orders start and finish earlier. The results of the developed model obtained for this instance, show that the first order begins production the day after its release date, and the last order finishes at minute 54 735.78. However, few orders are being produced so late and meet the deadline (see Figure 5.6). Comparing the results on the developed model for a  $t_{prod} = 60$  (company reference) and a  $t_{prod} = 1 440$ , the setup times and exchanges are the same, however in the

<sup>1</sup>Developed model for a  $t_{prod} = 1 440$ .



#### 5.4. May 9th to 15th

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second case a more favorable starting time schedule is verified. Orders start production later but finish earlier, with no production stoppages (see Figure A.2).

### 5.4 May 9th to 15th

The second instance corresponds to orders from May 9th to 15th, whose release date corresponds to minute  $t = 8\,640$  and to minute  $t = 15\,840$ , respectively. This instance includes the production scheduling of 82 orders from 23 different customers (see Figure 5.3). Table 5.5 presents information on the total weight, the processing time and the deadline of these orders.

	Min	Q1	Median	Mean	Q3	Max
Total weight ( $c_j$ )	99.45	413.57	701.13	1 270.54	1 589.48	8 421.20
Processing time ( $p_j$ )	7.33	18.71	30.27	56.14	58.67	440.99
Deadline ( $d_j$ )	23 040.00	43 200.00	73 440.00	66 679.00	83 520.00	144 000.00

Table 5.5: Total weight, processing time and deadline of the orders received from May 9th to 15th.

From Table 5.5 it can be observed that the total weight per order ranged from 99.45 kg to 8 421.20 kg, and the processing time ranged from 7.33 minutes to 440.99 minutes. The deadline varies between 23 040.00 and 144 000.00 minutes.

#### 5.4.1 Results of FIFO Dispatching Rule

Applying the dispatching rule FIFO for the instance from May 9th to 15th, a total of 71 die changes is obtained, with a total setup time equal 106.50 minutes. The first order was produced at minute 8 700.00 and the last order to minute 17 572.74 with a processing time of 11.65 minutes. The minimum deadline observed in Table 5.5 is 23 040.00 minutes, so deadlines are met. The solution for this instance is presented in Figure A.3, in Appendix. The orders received per day are not enough for the company to produce all day, and down times can verified.

#### 5.4.2 Results of the Developed Model

The optimization problem solved for the instance from May 9th to 15th has a total of 6 939 variables, the large majority of which are binary and only 82 are linear. The problem has 13 284 inequality constraints and 189 equality constraints, and the objective function is 103.50.

Figure A.4, in Appendix, present the result of the scheduling of this instance using the model developed in Section 4.2. During this period 69 die changes were made, the first order to be produced started at minute 11 580.00 and the last order at minute 71 999.99. In Table 5.5 can see that 50% of orders have deadlines of less than 73 440 minutes, so, is necessary ensure deadlines are being met. Note that, in the

developed model, after the production the orders take 1 440 minutes (one day) to be delivered. The time of surface treatment, in case to need, it is a previous procedure done by the manager, and it is already included in the deadline.

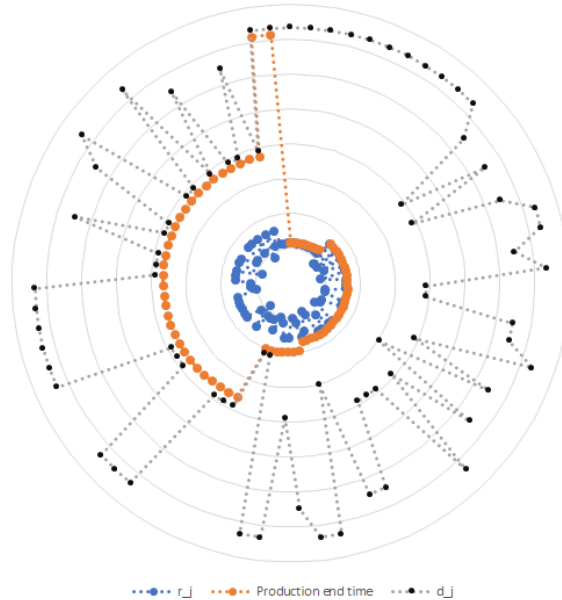


Figure 5.8: Graphic representation of release date, production end time and deadline for each order from May 9th to 15th.

Figure 5.8 illustrates the releasing date, production end time and deadline of each order, and one can verify that none order were starting produced before the release date and ended after the deadline. Only orders with a deadline equal or less to 73 440 minutes are presented, in the others the deadline is guaranteed.

### 5.4.3 Performance Comparison

Table 5.6 presents information relative to the total number of die changes, the total of setup times and time to produced first and last order, for the instance 9th to 15th, for dispatching rule FIFO and for the developed model.

	Setup times	Die changes	First order		Last order		Processing time
			Start time	End time	Start time	End time	
FIFO	106.50	71	8 700.00	8 735.79	17 572.74	17 584.39	4 538.07
OPT	103.50	69	11 580.00	11 634.06	71 559.00	71 999.99	4 538.07

Table 5.6: Comparison of results for the instance from May 9th to 15th.

In Table 5.6 less die changes with the developed model are verified. Analogous to the results for the previous instance, by the FIFO rule orders start and finish earlier. However in the developed model,

## 5.5. May 23rd to 29th

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although the time discrepancy, the deadlines are fulfilled. The model developed for this instance is more favorable for company, because the exchanges are minimized, which consequently reduces the scrap, are concluded.

## 5.5 May 23rd to 29th

The last instance corresponds the orders from May 23rd to 29th. Orders arrived between minute  $t = 99\,360.00$  and the minute  $t = 90\,720.00$ . This instance includes the production scheduling of 68 orders from 24 different customers (see Figure 5.3).

	Min	Q1	Median	Mean	Q3	Max
Total weight ( $c_j$ )	30.75	256.56	383.66	630.81	772.68	2 566.20
Processing time ( $p_j$ )	1.81	12.03	22.01	36.82	43.94	225.21
Deadline ( $d_j$ )	47 520.00	90 720.00	95 040.00	94 405.00	108 000.00	116 640.00

Table 5.7: Total weight, the processing time and the deadline of the orders received from May 23rd to 29th.

In Table 5.7 a large variability in the total weight and processing times is observed. However, with the verified values one can assume that the orders are relatively small. This variability also can be seen in deadline, 47 520.00 to 116 640.00 minutes, being that 75% of orders have deadlines between 90 720.00 to 116 640.00 minutes, considerably high.

### 5.5.1 Results of FIFO Dispatching Rule

Applying the dispatching rule FIFO for the instance from May 23rd to 29th, yield a total of 53 die changes, with a total setup time equal 79.50 minutes. The processing time of all the orders is 2 503.77 minutes. The first order was produced at minute 25 980.00 and the last order was minute 34 960.72, with a processing time of 11.86 minutes. Since deadlines are higher then 47 520.00 minutes (see Table 5.7), delivery time are guaranteed.

The solution for the complete instance is presented in Appendix, in Figure A.5. Similar to what happened in the previous instance, the orders do not have enough quantities to produce all day, so down times are verified.

### 5.5.2 Results of the Developed Model

The optimization problem solved for the instance from May 23rd to 29th has a total of 4 805 variables, the large majority of which are binary and only 68 are linear. The problem has 9 112 inequality constraints and 157 equality constraints, and the objective function is 75.00.

Figure A.6, in Appendix, presents the result of the scheduling of this instance using the model developed in Section 4.2. A total of 50 die changes are verified to a total processing time of 2 503.77 minutes.

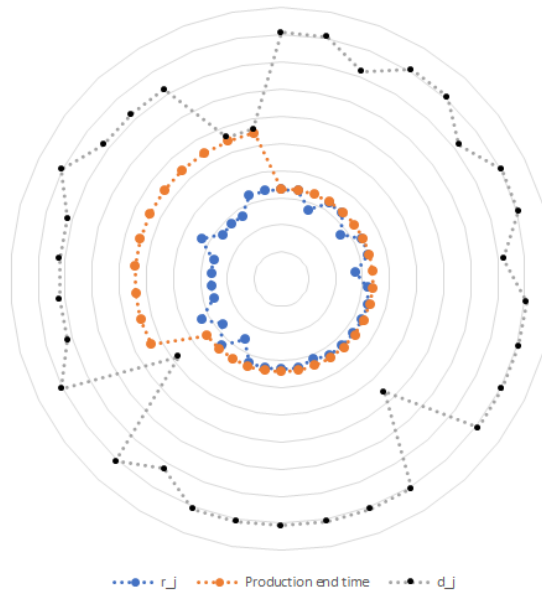


Figure 5.9: Representation of release date, production end time and deadline for each order from May 23rd to 29th.

Deadlines are fulfilled, as you can see in Figure 5.9. Since the last order to be produced ended at minute 97 919.97, we only need to verify orders with a deadline shorter than this one. In Figure 5.9 it is possible to see that we have a large number of orders where the production end time is very close to the release date.

To better understand the schedule made in this instance, a brief explanation of 11 orders produced will be made (see Figure 5.10):

- Job 411, 410, 409 and 408 are produced by die 2391\_1, started at 53 961.30 and ended at minute 54287.24. In total 2 198.20 kg were produced over 325.94 minutes. Both orders arrived the company at the minute 25 920.00.
- Job 455 arrived at the company in minute 33 120.00 and started production in minute 54 288.80. The weight total of order is 308.74 kg and it takes 9.35 minutes to produce, in die 4109\_1. This order has a deadline of 90 720.00 minutes. Next orders use different die, so a setup time at 1.50 minute are existed.
- Job 414 and 413 whose release date is minute 27 360.00, began production on die 2649\_1 in minute 54 299.60 and ended on minute 54 333.78, and a setup time at 1.5 minute are verified. During 34.16 minutes 998.95 kg were produced. Orders have a deadline of 82 080.00 minutes.
- Job 446 arrived at the company in minute 33 120.00 and started production in minute 54 335.30. The weight total of order is 1 164.00 kg and it takes 53.16 minutes to produce, in die 2723\_1. This

## 5.5. May 23rd to 29th

order has a deadline of 108 000.00 minutes. Next orders use different die, so a setup time at 1.50 minute is existed.

- Job 441, 440 and 418, produced by die 2001\_1, started at minute 54 390.00 and ended at minute 54 563.11. These orders have different release dates, i.e., Job 418 arrived at the company at minute 27 360.00, and the others at minute 33 120.00. The deadlines are also different, 82 080.00 and 108 000.00, respectively. A total 3 544.65 kg were produced for 173.13 minutes. In the end there is a setup time, which indicates that the next orders to be produced use different dies.

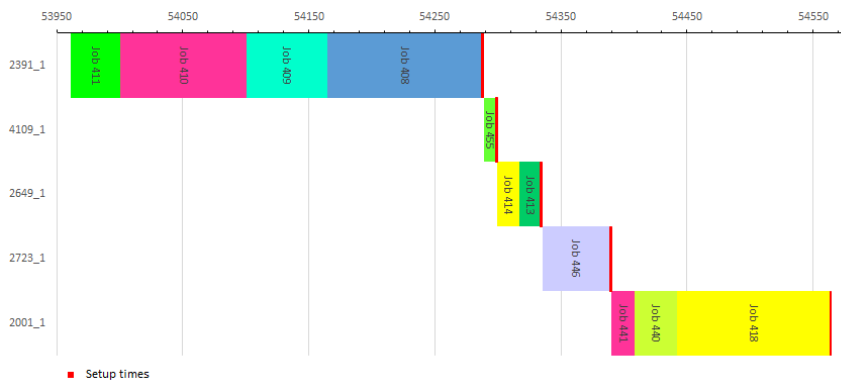


Figure 5.10: Solution, per die, from 11 produced order, from May 23rd to 29th.

### 5.5.3 Performance Comparison

Table 5.8 presents information relative to the total number of die changes, the total of setup times and time to produced first and last order, for the instance 23rd to 29th, for dispatching rule FIFO and developed model.

	Setup times	Die changes	First order		Last order		Processing time
			Start time	End time	Start time	End time	
FIFO	79.50	53	25 980.00	26 018.40	34 960.72	34 972.58	2 503.77
OPT	75.00	50	33 180.00	33 185.72	97 876.30	97 919.97	2 503.77

Table 5.8: Comparison of results from instance May 23rd to 29th.

Applying the FIFO rule for the last instance there are 53 die changes with a total time of 79.50 minutes of setup times. Applying the developed model, there are 50 die changes with a total time of 75.00 minutes of setup times. In this instance, the developed model presents better results than the dispatching rule FIFO. Regarding production times, the same as in previous instances occurs.

## 5.6 Final Considerations

A total of three instances were solved. The results are summarized in Table 5.9. For all instances, the optimal solution was found in a small computational time. The setup times varied from 75.00 to 118.50. The problems presented between 4 805 and 9 251 variables binary and linear decision variables, mostly binary. The constraints range from 9 269 to 18 067, mostly inequalities.

	May 2nd to 8th	May 9nd to 29th	May 23rd to 29th
Objective function	118.50	103.50	75.00
Number of variables	9 251	6 939	4 805
binary	9156	6 857	4 737
linear	95	82	68
Number of constraints	18 067	13 473	9 269
equality	207	189	157
inequality	17860	13284	9 112

Table 5.9: Results and dimension of the developed model obtained for each instance.

Table 5.10 presents the results obtained for the instances, applying the FIFO method and the developed model.

Instance	Method	Setup times	Die changes	First order		Last order	
				Start time	End time	Start time	End time
May 2nd to 8th	FIFO	127.50	85	60.00	70.66	8 235.48	8 257.50
	OPT	118.50	79	1 500.00	1 544.85	54 721.50	54 735.78
	OPT <sup>1</sup>	118.50	79	8 640.00	8 662.63	13 749.00	13 763.20
May 9th to 15th	FIFO	106.50	71	8 700.00	8 735.79	17 572.74	17 584.39
	OPT	103.50	69	11 580.00	11 634.06	71 559.00	71 999.99
May 23rd to 29th	FIFO	79.50	53	25 980.00	26 018.40	34 960.72	34 972.58
	OPT	75.00	50	33 180.00	33 185.72	97 876.30	97 919.97

Table 5.10: Comparison of results from all instances.

The developed model, compared to the rule FIFO for instances considered, presents a smaller number of die changes and consequently a minor total setup time. The developed model only takes into consideration the minimization of setup times and does not require orders to be produced the closest possible to the release date. This is an improvement that could be implemented in the model by adding minimizing starting times to the objective function. However, this does not invalidate the results obtained, because all constraints are being met.

<sup>1</sup>Developed model for a  $t_{prod} = 1 440$

## 5.6. Final Considerations

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In the first instance an extra comparison was made, i.e., to the developed model we changed a parameter provided by the company.

It was considered that each order could only be produced after one day of arriving at the company. The results in terms of number of die exchanges were the same as if it was considered that they can be produced 60 minutes after their arrival. However there was a better schedule on production times, and orders were produced continuously without interruption. This situation needed further study, because it could be an improvement in production planning for both the model and the company.

As already mentioned, it was not possible to make a direct comparison of the results obtained in this work and those practiced in the company. In Section 5.1 an average of orders and exchanges of die per week, considering the historical production data provided in the Production process dataset are computed. An average of 119 orders were produced and 79 die changes per week were made.

With the developed model, in any of the weeks it was exceeded 79 exchanges of die, per week. In the first instance, where the highest number of orders was produced, close to the estimated average, 79 die exchanges were made. In other instances lower values were verified.

In conclusion, the developed model presents a viable solution for production planning, in the order to decrease the level of scrap produced. It has the advantages of optimizing the use of dies and being an automatic tool. With this solver the production manager can more quickly set up production sequencing for incoming orders, and the responsiveness to the customer is better. Assuming that the production manager in a pre-processing phase, already includes in the deadline all conditions needed, such as treatment type, available dies, alloy type, etc.

Also in a pre-processing phase it is necessary to verify whether there are orders with weights above the set limit, and subdivided into weights below. The model presented is an advantageous solution for the company to sequence orders automatically, ensuring deadlines, minimizing die changes, and consequently reducing scrap.





## Chapter 6

# Conclusion and Future Work

Production scheduling is one of the major component for efficient management of production. This class of problems is among the most studied in Operational Research (OR) and presents applications in several areas [29]. Scheduling refers to the allocation of resources to jobs over a set of given time periods, optimizing one or more criteria [69], and are strongly NP-hard.

Although studied by many researchers, the majority of the literature ignores setup times/costs. However, in many applications it is crucial to consider setups, since they are essential due to scheduling decisions are made in order to increase productivity, eliminate waste, improve resource utilization, meet deadlines, increase customer satisfaction [3].

In this work, the challenge proposed by a Portuguese aluminium extrusion and treatment company was presented. The main objective of the company was to optimize the production sequencing in order to reduce scrap, while maintaining the quality standards of the extruded aluminium and respecting the deadline of delivery to the customers. Since, whenever a die exchange is made, a setup time occurs and scrap is produced. Therefore, an direct approach for minimizing the amount of scrap produced in the extrusion process is to minimize the die exchanges. For this reason and since there is no estimation on the amount of scrap production, this work focuses on minimizing the setup times.

This work, starts with a literature review on the industrial extrusion process and on production planning and scheduling. The problem proposed by the company and the information necessary for developing the mathematical model are also discussed. The characteristics of the production scenario implies that the dies must be changed for producing the aluminium profiles order by the costumers. These profiles have different geometries and sizes, and it is essential to consider setup times.

The company provided two datasets. The first dataset concerns the extrusion production process for a period of six months, while the second dataset concerns information of the customers' orders for one month.

A mathematical model, more precisely a MILP model, was developed. The objective was to minimize

the sum of setup times, and therefore minimize the exchanges die, which consequently decrease the scrap produced. To this end, three real instances, taken from the Customers' orders dataset provided by the company regarding orders received in May 2018, were used to test the model.

Since the scheduling of orders practiced by the company for these instances is not known, a direct comparison is not possible. This represents a limitation of the present study. For having a comparison measure, the results obtained with the developed model were compared the results obtained using the dispatching rule FIFO.

It may be concluded that, for the real instances considered in this work, the developed model presents smaller setup times, and consequently a smaller number of die changes. Furthermore, the results obtained guarantee that the orders' are proceeded only with a single die on a set of allowed ones, all order in are produced, the minimum and maximum weight usage of each die is not exceeded.

As already mentioned it was not possible to make a direct comparison of the results found in the present work and the company's practice. However, by using the six month of historical production data in the production process dataset, provided by the company it was possible to observe an average of the number of orders and die exchanges, per week, that are made by the company. On average, on the company there are, per week, 119 orders and 79 die changes. In the results obtained with the developed model it was possible to verified that in any of the weeks the 79 exchanges of die were observed.

The approach presented in this paper has several advantages over the current procedure used in the company. Currently, scheduling is performed by the production manager by hand and the time it takes to find an admissible sequence for the orders' production is time consuming. In fact, it sequences the customers' orders automatically, ensuring deadlines, minimizing die changes, and consequently reducing scrap. For each instance, the necessary parameters, related to the order characteristics (i.e. identification of the order, the set of dies that can be used to produce the order, its weight and processing time, the release and deadline dates) are automatically extracted from an excel an written in a format that can be interpret by the solver. The optimal results of each instance are obtained automatically in a computational time that is by far smaller than the time that the production manager takes to find a solution by hand. Therefore, the approach here presented presents itself as an useful tool for decision support in the aluminum industry.

However, the model is far from being perfect. In fact, a closer analysis of the results obtained with the developed model showed the existence of dead times. This occurs because the model does not include in this formulation the minimization of dead times. There are several possibilities to overcome this limitation, such as, modifying the objective function such that it included a term for guaranteeing minimization of starting times. In this case it would be necessary to balance the contribution of the setup times and the starting times in the objective function, since they present quite different orders of magnitude. Another possibility is to consider a bi-objective model for minimizing the setups and starting times. Yet another different approach would be to consider additional constrains to the problem that obliges that the difference between the starting time of the last order,  $S_{max}$ , and the starting time of

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the first order,  $S_{min}$ , do not exceed a given percentage of the total processing time, i.e.  $S_{max} - S_{min} \leq (1 + \lambda) \sum_{j=1}^n p_j$ , for which the  $\lambda$  threshold must be tuned. Due to time limitations it was not possible to test the above approaches, but the work has already been started.

Another interesting extension of this work would be to study the problem presented in this work in the context of robust stochastic programming and robust optimization.



## Appendix A

# Gantt Charts



Figure A.1: Description, per die, of the orders production scheduling obtained by the dispatching rule FIFO from instance May 2nd to 8th.

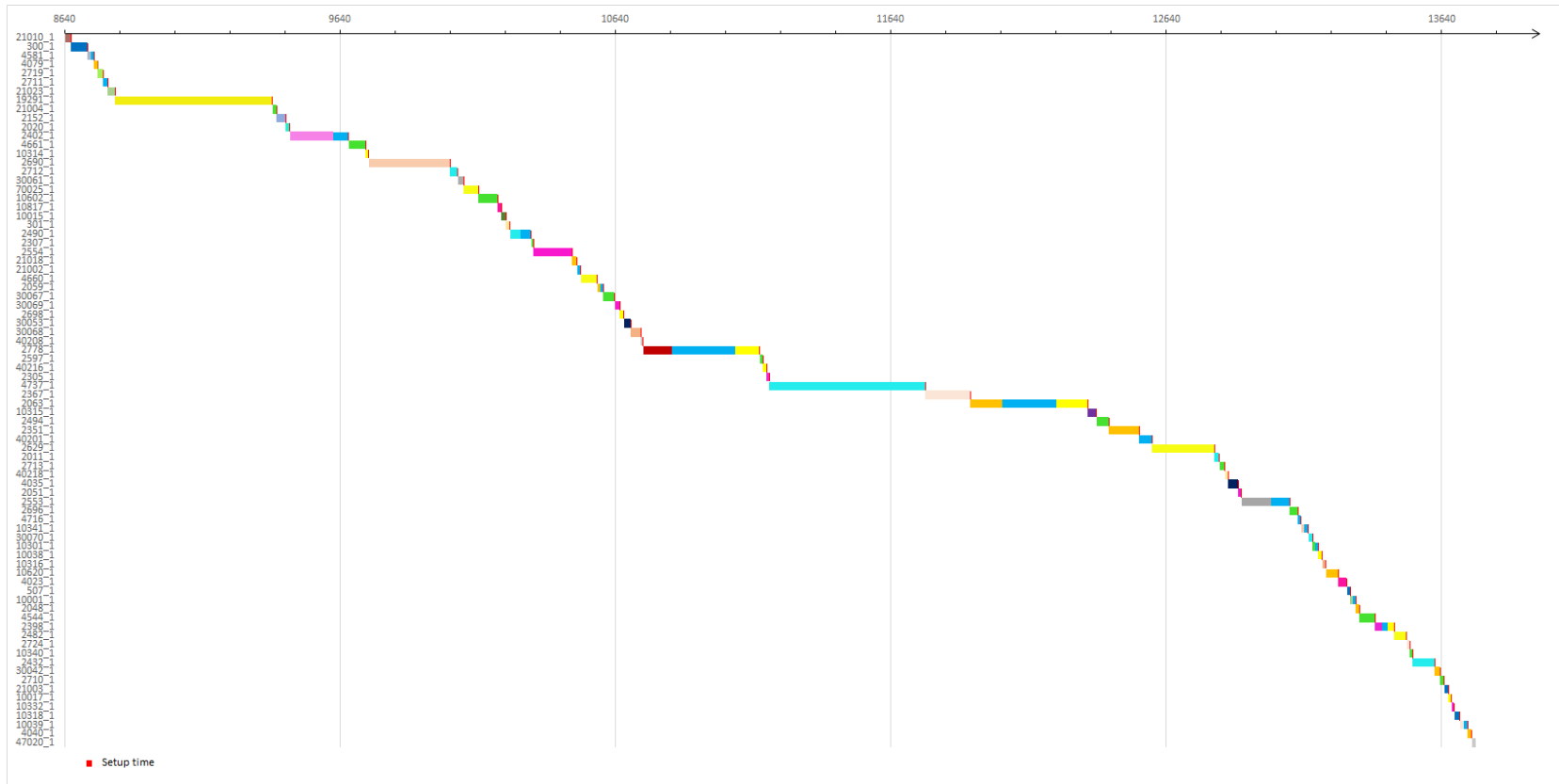


Figure A.2: Solution, per die, for the orders of the May 2nd to 8th, with  $\beta$  equal 1 440 minutes.

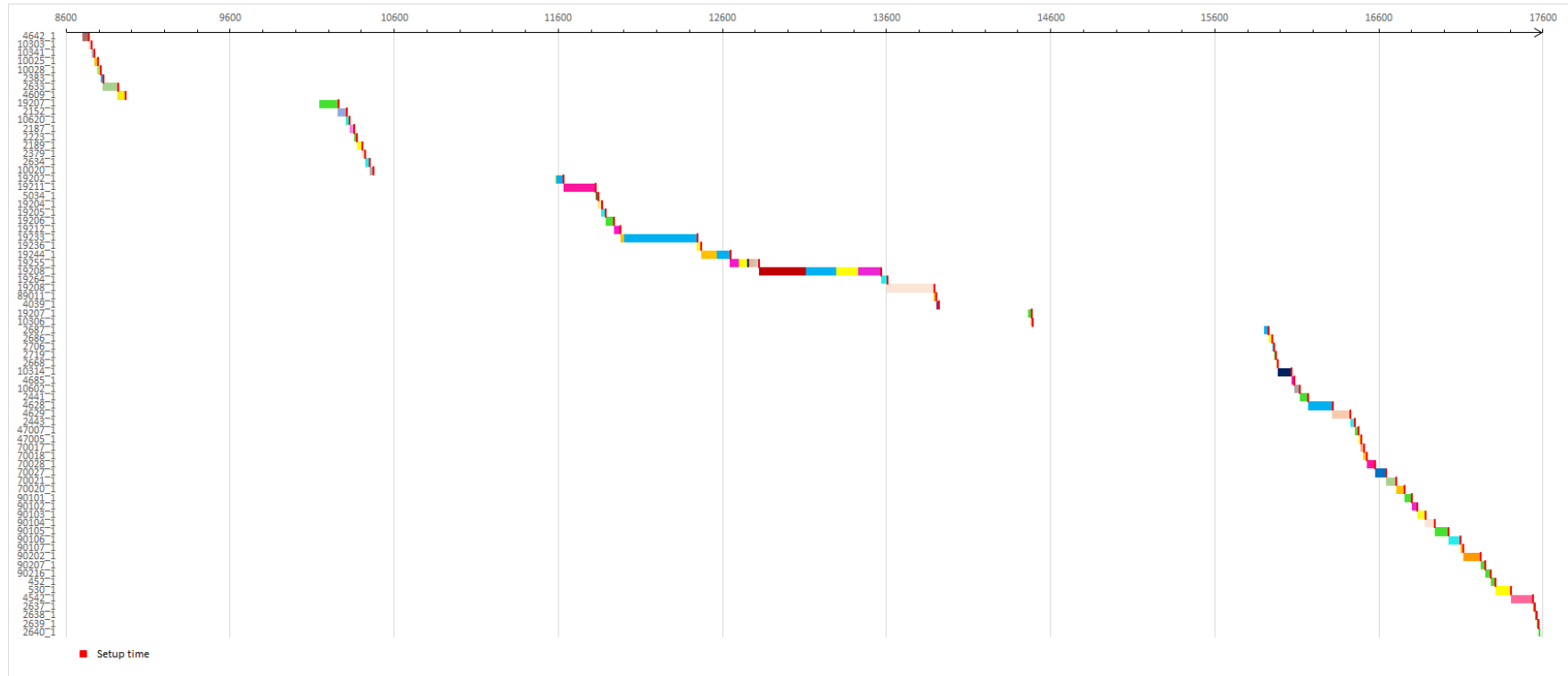


Figure A.3: Description, per die, of the orders production scheduling obtained by the dispatching rule FIFO from instance May 9th to 15th.



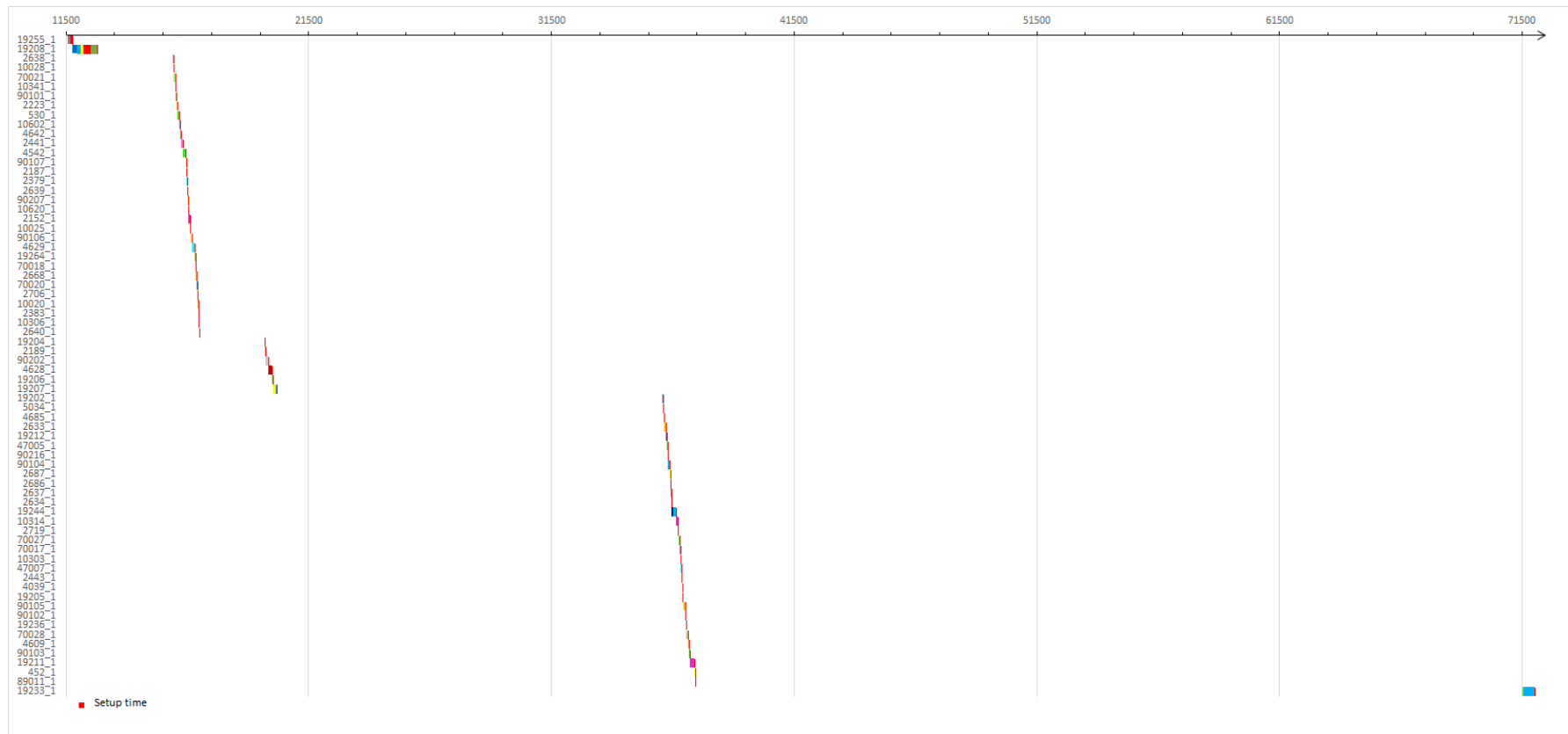


Figure A.4: Solution, per die, for the orders of the May 9th to 15th.

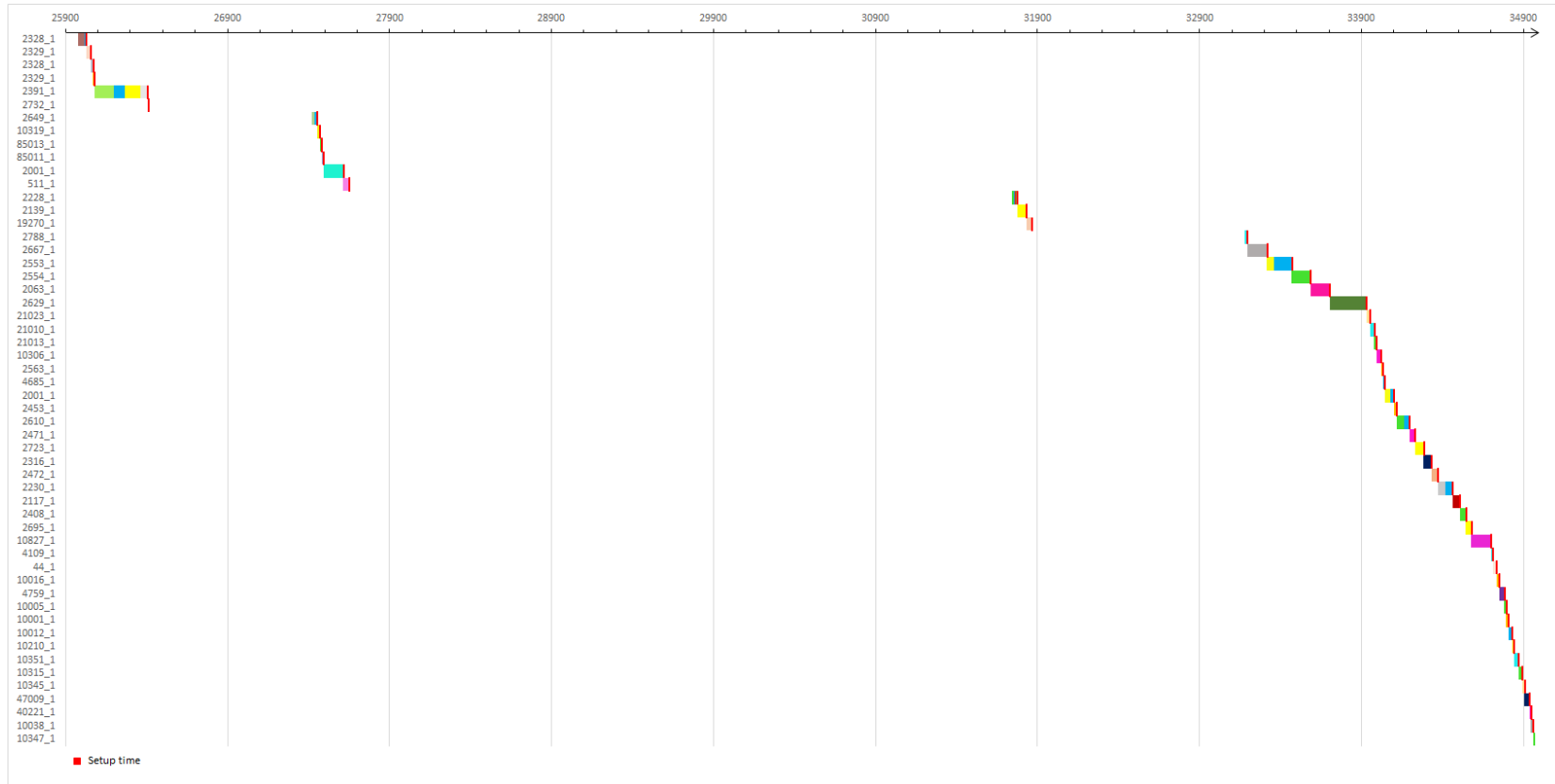


Figure A.5: Description, per die, of the orders production scheduling obtained by the dispatching rule FIFO from instance May 23rd to 29th.



Figure A.6: Solution, per die, for the orders of the May 23rd to 29th.

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#
# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
#
# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
#
# Set of orders
set Orders;

# Set of Dies that can be used to extrude order j
set M_j {Orders};

# Set of Die available
set Die:= union {j in Orders} M_j[j];

# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
#
# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
#
# PARAMETERS

# Processing time of order j
param p_j {j in Orders};

# release date of order j, in minutes since 02/05/2018
param r_j {j in Orders};

# deadline of order j, in minutes since 02/05/2018
param d_j {j in Orders};

# quantity, in kg, to extrude in order j
param c_j {j in Orders};

# maximum processing kg or time with die d
param DMax_d := 30000; #{d in Die};

# minimum processing kg or time with die d
param DMin_d := 0; #{d in Die};

# time specified by the company for the order to each production, in min
param beta := 60;
param otherprocess := 24*60;
param MGrande := 10^10;

# tempo do setup
param S_jj{Orders, Orders};

# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
#
# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
#
# DECISION VARIABLES
# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -

```

---

```

# is the starting time of order $j$
var s_j {j in Orders} >= 0;

# 1 j precede j1 # <> -> significa diferente
var x_jj {j in (Orders union {0}), j1 in (Orders union {0}): j<>j1 } binary;

# 1 j precede j1 # <> -> significa diferente
var x_jjd {j in Orders, j1 in Orders, d in Die: j<>j1 } binary;

# 1 if die $d$ is used for order $j$
var m_jd {j in Orders, d in Die} binary;

# - * - * - * - * - * - * - * - * - * -
#           OBJECTIVE FUNCTION
# - * - * - * - * - * - * - * - * - * -

# Minimizing Setup Times

minimize f_obj:
    sum {j in Orders, j1 in Orders: j<>j1} S_jj[j,j1]*x_jj[j,j1];

# - * - * - * - * - * - * - * - * - * -
#           CONSTRAINTS
# - * - * - * - * - * - * - * - * - * -

#1 Each Order can only be processed after "arriving"
s.t. Arrival {j in Orders}:
    s_j[j] >= r_j[j]+beta;

#2
s.t. startingtime {j in Orders, j1 in Orders: j<>j1 }:
    s_j[j1] >= s_j[j]-MGrande+ p_j[j]+MGrande*x_jj[j,j1]+
    S_jj[j,j1]*x_jj[j,j1];

#3 each order is processed by a single die from among those that can be used
s.t. rest1 {j in Orders}:
    sum{d in M_j[j]} m_jd[j,d] = 1;

# the order cannot be processed by a matrix that is not in M_j
s.t. rest_nova {j in Orders}:
    sum{d in (Die diff M_j[j])} m_jd[j,d] = 0;

#6
s.t. startingtime2 {j in Orders}:
    s_j[j]+p_j[j]+otherprocess <= d_j[j];

#
s.t. MaxMinWeight {d in Die}:
    DMin_d <= (sum {j in Orders:d in M_j[j]}
    m_jd[j,d]*c_j[j]) <= DMax_d;

#5

```

```

s.t. precedencia {j in Orders, j1 in Orders: j<>j1}:
    x_jj[j,j1] + x_jj[j1,j]<=1;

s.t. resty {j in (Orders union {0})}:
    (sum{j1 in (Orders union {0}): j<>j1} x_jj[j,j1]) = 1;

s.t. resty2 {j1 in (Orders union {0})}:
    (sum{j in (Orders union {0}): j<>j1} x_jj[j,j1]) = 1;

s.t. resty2 {j1 in Orders}:
    (sum{j in Orders: j<>j1} x_jj[j,j1]) = 1;

s.t. resty {d in Die, j in Orders}:
    (sum{ j1 in Orders: j<>j1 and card(M_j[j] inter
M_j[j1])<>0} x_jj[j,j1]) = m_jd[j,d];

s.t. resty2 {d in Die, j1 in Orders}:
    (sum{ j in Orders: j<>j1} x_jj[j,j1]) =
m_jd[j1,d];

#4
#s.t. rest2 {d in Die, j in Orders}:
#
#    (sum{ j1 in Orders: j<>j1} x_jjd[j,j1,d]) <=
m_jd[j,d];

#s.t. precedencia {j in Orders}:
#
#    sum {j1 in Orders, d in Die: j<>j1} x_jjd[j,j1,d] =1;
#

# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
#
#    DATA Instance 1 (May 2nd to 8th)
# - * - * - * - * - * - * - * - * - * - * - * - * -

set Orders := 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52
53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79
80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 ;

set M_j[1] := 4581_1 ;
set M_j[2] := 4581_1 ;
set M_j[3] := 4035_1 ;
set M_j[4] := 10301_1 ;
set M_j[5] := 10332_1 ;
set M_j[6] := 4023_1 ;
set M_j[7] := 4079_1 ;
set M_j[8] := 10015_1 ;
set M_j[9] := 10817_1 ;
set M_j[10] := 2778_1 ;
set M_j[11] := 2778_1 ;
set M_j[12] := 2778_1 ;
set M_j[13] := 2494_1 ;
set M_j[14] := 2351_1 2351_2 ;

```

---

```
set M_j[15] := 2553_1 2553_2 ;
set M_j[16] := 2553_1 2553_2 ;
set M_j[17] := 2554_1 ;
set M_j[18] := 2490_1 ;
set M_j[19] := 2490_1 ;
set M_j[20] := 2063_1 ;
set M_j[21] := 2063_1 ;
set M_j[22] := 2063_1 ;
set M_j[23] := 2629_1 2629_2 ;
set M_j[24] := 70025_1 ;
set M_j[25] := 40201_1 40201_2 ;
set M_j[26] := 40208_1 ;
set M_j[27] := 40218_1 ;
set M_j[28] := 40216_1 ;
set M_j[29] := 10314_1 ;
set M_j[30] := 10340_1 ;
set M_j[31] := 2724_1 ;
set M_j[32] := 10017_1 ;
set M_j[33] := 10301_1 ;
set M_j[34] := 10341_1 ;
set M_j[35] := 10039_1 ;
set M_j[36] := 10038_1 ;
set M_j[37] := 10001_1 ;
set M_j[38] := 10318_1 ;
set M_j[39] := 4660_1 ;
set M_j[40] := 4661_1 ;
set M_j[41] := 4716_1 ;
set M_j[42] := 2011_1 ;
set M_j[43] := 10620_1 ;
set M_j[44] := 2710_1 ;
set M_j[45] := 30068_1 ;
set M_j[46] := 30042_1 ;
set M_j[47] := 30053_1 ;
set M_j[48] := 30069_1 30069_2 ;
set M_j[49] := 30061_1 ;
set M_j[50] := 30067_1 ;
set M_j[51] := 30070_1 ;
set M_j[52] := 47020_1 ;
set M_j[53] := 2711_1 2711_2 ;
set M_j[54] := 2712_1 ;
set M_j[55] := 2713_1 ;
set M_j[56] := 10001_1 ;
set M_j[57] := 10039_1 ;
set M_j[58] := 2020_1 ;
set M_j[59] := 10316_1 ;
set M_j[60] := 2398_1 2398_2 2398_3 2398_4 ;
set M_j[61] := 2432_1 ;
set M_j[62] := 507_1 ;
set M_j[63] := 10341_1 ;
set M_j[64] := 19291_1 ;
set M_j[65] := 300_1 ;
set M_j[66] := 301_1 ;
set M_j[67] := 4544_1 ;
set M_j[68] := 2402_1 2402_2 ;
```

```
set M_j[69] := 2696_1 ;
set M_j[70] := 2698_1 ;
set M_j[71] := 2048_1 ;
set M_j[72] := 2402_1 2402_2 ;
set M_j[73] := 2307_1 ;
set M_j[74] := 2051_1 ;
set M_j[75] := 4737_1 ;
set M_j[76] := 2690_1 ;
set M_j[77] := 2719_1 ;
set M_j[78] := 2482_1 ;
set M_j[79] := 2367_1 2367_2 ;
set M_j[80] := 2597_1 ;
set M_j[81] := 4040_1 ;
set M_j[82] := 2059_1 ;
set M_j[83] := 2059_1 ;
set M_j[84] := 2305_1 ;
set M_j[85] := 2152_1 2152_2 ;
set M_j[86] := 10602_1 ;
set M_j[87] := 10315_1 10315_2 ;
set M_j[88] := 21010_1 ;
set M_j[89] := 21003_1 ;
set M_j[90] := 21004_1 ;
set M_j[91] := 21002_1 ;
set M_j[92] := 21023_1 ;
set M_j[93] := 21018_1 ;
set M_j[94] := 2398_1 2398_2 2398_3 2398_4 ;
set M_j[95] := 2398_1 2398_2 2398_3 2398_4 ;
```

```
param p_j :=
1  10.66294
2  10.66294
3  34.3367
4  10.53665
5  8.789786
6  30.25577
7  10.99138
8  14.61625
9  11.46727
10 104.1959
11  87.40049
12 230.1611
13  43.60334
14 108.1048
15  65.56749
16 108.0649
17 137.6118
18  35.50596
19  39.88648
20 196.0681
21 114.7443
22 114.7625
23 225.2053
24  51.40086
25  44.85341
```



---

26	7.947351
27	10.97595
28	13.04914
29	10.14951
30	8.907717
31	9.631804
32	10.02448
33	9.174276
34	12.09708
35	11.94924
36	13.31032
37	9.13314
38	18.2222
39	58.69931
40	59.14022
41	10.05259
42	16.54125
43	43.42484
44	14.2814
45	34.03441
46	18.58305
47	23.67932
48	17.17474
49	21.45786
50	39.3711
51	12.45729
52	14.19982
53	14.7993
54	25.9515
55	18.36969
56	8.319319
57	14.21624
58	13.09651
59	11.70876
60	25.08295
61	77.90976
62	10.43946
63	13.66471
64	570.191
65	58.06731
66	13.46324
67	55.0539
68	53.485
69	27.90257
70	13.83541
71	12.21496
72	158.724
73	7.446766
74	11.35283
75	565.1711
76	294.5054
77	19.88905
78	42.46876
79	161.264

---

```
80  8.727331
81  13.78486
82  9.911346
83  9.904068
84  8.590687
85  32.92109
86  68.27741
87  29.50234
88  22.63333
89  13.56596
90  13.22459
91  12.96265
92  24.15
93  16.76479
94  22.02403
95  22.02403
;
param r_j :=
1    0
2    0
3    0
4    0
5    0
6    0
7    0
8    0
9    0
10   1440
11   1440
12   1440
13   1440
14   1440
15   1440
16   1440
17   1440
18   1440
19   1440
20   1440
21   1440
22   1440
23   1440
24   1440
25   1440
26   1440
27   1440
28   1440
29   1440
30   1440
31   1440
32   1440
33   1440
34   1440
35   1440
36   1440
```

---

37	1440
38	1440
39	1440
40	1440
41	1440
42	2880
43	2880
44	2880
45	2880
46	2880
47	2880
48	2880
49	2880
50	2880
51	2880
52	2880
53	2880
54	2880
55	2880
56	2880
57	2880
58	2880
59	2880
60	2880
61	2880
62	2880
63	2880
64	5760
65	5760
66	5760
67	5760
68	5760
69	5760
70	5760
71	5760
72	5760
73	5760
74	5760
75	5760
76	5760
77	5760
78	5760
79	5760
80	5760
81	5760
82	5760
83	5760
84	5760
85	7200
86	7200
87	7200
88	7200
89	7200
90	7200

---

```
91  7200
92  7200
93  7200
94  7200
95  7200
;
param d_j :=
1    56160
2    56160
3    56160
4    56160
5    56160
6    56160
7    56160
8    56160
9    56160
10   56160
11   56160
12   56160
13   21600
14   21600
15   56160
16   56160
17   56160
18   56160
19   56160
20   56160
21   56160
22   56160
23   56160
24   56160
25   56160
26   56160
27   56160
28   56160
29   56160
30   56160
31   56160
32   56160
33   56160
34   56160
35   56160
36   56160
37   56160
38   56160
39   56160
40   56160
41   56160
42   56160
43   56160
44   73440
45   73440
46   73440
47   73440
```

---

```
48 73440
49 73440
50 73440
51 73440
52 73440
53 73440
54 73440
55 73440
56 73440
57 73440
58 73440
59 73440
60 73440
61 73440
62 56160
63 56160
64 12960
65 56160
66 56160
67 64800
68 73440
69 73440
70 73440
71 73440
72 73440
73 73440
74 73440
75 64800
76 64800
77 73440
78 73440
79 73440
80 47520
81 64800
82 73440
83 73440
84 73440
85 64800
86 64800
87 64800
88 73440
89 73440
90 73440
91 73440
92 73440
93 73440
94 64800
95 64800
;
param c_j :=
1 208.8
2 208.8
3 993.86
4 300.15
```

5	301.35
6	494.165
7	304.017
8	300.636
9	251.076
10	2278.2
11	1898.5
12	5125.95
13	1002.005
14	2498.514
15	1500.734
16	2501.018
17	2998.8
18	699.4933
19	799.771
20	3561.67
21	1780.264
22	1780.663
23	2500.588
24	300.196
25	1001.702
26	248.729
27	252.252
28	250.2825
29	249.3855
30	250.8155
31	248.4755
32	251.68
33	250.4125
34	301.3465
35	248.56
36	298.584
37	250.107
38	396.175
39	1156.272
40	683.1552
41	260.304
42	250.0225
43	1600.95
44	266.5
45	979.875
46	440.7
47	489.45
48	448.5
49	479.05
50	1064.7
51	292.5
52	408.85
53	488.15
54	722.215
55	616.2
56	227.37
57	310.7
58	265.85

---

```

59 294.775
60 566.8
61 1976
62 249.444
63 350.2135
64 9413.674
65 1830
66 939.6576
67 1205.406
68 500.1772
69 499.2988
70 249.429
71 254.2806
72 1499.839
73 249.555
74 249.4947
75 14051.4
76 6448.2
77 867.4575
78 1510.044
79 2817.72
80 250.0085
81 481.4145
82 250.2324
83 250
84 250
85 200.46
86 478.4
87 589.68
88 247
89 314.6
90 295.75
91 319.8
92 330
93 311.4
94 499.9176
95 499.9176
;
param S_jj : 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52
53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79
80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 :=
1 0 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
2 0 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5

```

























Appendix A. Gantt Charts

```

93  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  0  1.5  1.5
94  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  0  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  0  0
95  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  0  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5
1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  1.5  0  0
;

```

```

# - * - * - * - * - * - * - * - * - * -
#           SOLUTION AND VISUALIZATION
# - * - * - * - * - * - * - * - * - * -

```

```
#option solver gurobi;
```

```

solve;
option omit_zero_rows 1;
display f_obj;
display Orders;
display c_j, r_j, s_j, p_j, d_j;
display x_jj;
display m_jd;

```

```

# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
#                                     Instance 1 (May 2nd to 8th)
# - * - * - * - * - * - * - * - * - * -

```

Presolve eliminates 458 constraints and 8799 variables.

Adjusted problem:

9251 variables:

    9156 binary variables

    95 linear variables

18067 constraints, all linear; 62926 nonzeros

    207 equality constraints

    17860 inequality constraints

1 linear objective; 8894 nonzeros.

Gurobi 8.1.0: threads=4

Gurobi 8.1.0: optimal solution; objective 118.5

254963 simplex iterations

1067 branch-and-cut nodes

plus 91 simplex iterations for intbasis

f\_obj = 118.5

---

set Orders :=

1	7	13	19	25	31	37	43	49	55	61	67	73	79	85	91
2	8	14	20	26	32	38	44	50	56	62	68	74	80	86	92
3	9	15	21	27	33	39	45	51	57	63	69	75	81	87	93
4	10	16	22	28	34	40	46	52	58	64	70	76	82	88	94
5	11	17	23	29	35	41	47	53	59	65	71	77	83	89	95
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90;	

:	c_j	r_j	s_j	p_j	d_j	:=
1	208.8	0	54696.9	10.6629	56160	
2	208.8	0	54686.2	10.6629	56160	
3	993.86	0	11814.4	34.3367	56160	
4	300.15	0	12359.9	10.5366	56160	
5	301.35	0	10923.8	8.78979	56160	
6	494.165	0	9006.62	30.2558	56160	
7	304.017	0	9038.38	10.9914	56160	
8	300.636	0	11798.3	14.6163	56160	
9	251.076	0	10910.9	11.4673	56160	
10	2278.2	1440	19410.5	104.196	56160	
11	1898.5	1440	19514.7	87.4005	56160	
12	5125.95	1440	19602.1	230.161	56160	
13	1002	1440	8803.28	43.6033	21600	
14	2498.51	1440	20051.9	108.105	21600	
15	1500.73	1440	11707	65.5675	56160	
16	2501.02	1440	11599	108.065	56160	
17	2998.8	1440	9575.17	137.612	56160	
18	699.493	1440	11904.9	35.506	56160	
19	799.771	1440	11865	39.8865	56160	
20	3561.67	1440	9193.06	196.068	56160	
21	1780.26	1440	9078.32	114.744	56160	
22	1780.66	1440	9389.13	114.763	56160	
23	2500.59	1440	11977.5	225.205	56160	
24	300.196	1440	9805.96	51.4009	56160	
25	1001.7	1440	1500	44.8534	56160	
26	248.729	1440	11788.8	7.94735	56160	
27	252.252	1440	54709	10.9759	56160	
28	250.282	1440	11540.5	13.0491	56160	
29	249.386	1440	10899.2	10.1495	56160	
30	250.815	1440	19915.6	8.90772	56160	
31	248.476	1440	10239.9	9.6318	56160	
32	251.68	1440	12213.1	10.0245	56160	
33	250.412	1440	12350.8	9.17428	56160	
34	301.346	1440	19847.4	12.0971	56160	
35	248.56	1440	8862.6	11.9492	56160	
36	298.584	1440	11850.2	13.3103	56160	
37	250.107	1440	11529.8	9.13314	56160	
38	396.175	1440	19956.2	18.2222	56160	
39	1156.27	1440	9727.49	58.6993	56160	
40	683.155	1440	19991.3	59.1402	56160	
41	260.304	1440	19944.6	10.0526	56160	
42	250.023	2880	19861	16.5413	56160	
43	1600.95	2880	9858.86	43.4248	56160	
44	266.5	2880	54721.5	14.2814	73440	

45	979.875	2880	11941.9	34.0344	73440
46	440.7	2880	10219.9	18.5831	73440
47	489.45	2880	10829.3	23.6793	73440
48	448.5	2880	19926	17.1747	73440
49	479.05	2880	9903.78	21.4579	73440
50	1064.7	2880	12372	39.3711	73440
51	292.5	2880	8932.59	12.4573	73440
52	408.85	2880	10934.1	14.1998	73440
53	488.15	2880	10251.1	14.7993	73440
54	722.215	2880	9050.87	25.9515	73440
55	616.2	2880	8912.72	18.3697	73440
56	227.37	2880	11521.5	8.31932	73440
57	310.7	2880	8848.38	14.2162	73440
58	265.85	2880	12312	13.0965	73440
59	294.775	2880	9714.28	11.7088	73440
60	566.8	2880	19383.9	25.083	73440
61	1976	2880	10140.5	77.9098	73440
62	249.444	2880	19903.6	10.4395	56160
63	350.214	2880	19833.8	13.6647	56160
64	9413.67	5760	10949.8	570.191	12960
65	1830	5760	12252.5	58.0673	56160
66	939.658	5760	12237.5	13.4632	56160
67	1205.41	5760	10563.4	55.0539	64800
68	500.177	5760	9926.74	53.485	73440
69	499.299	5760	12412.8	27.9026	73440
70	249.429	5760	19975.9	13.8354	73440
71	254.281	5760	10854.5	12.215	73440
72	1499.84	5760	9980.23	158.724	73440
73	249.555	5760	12204.2	7.44677	73440
74	249.495	5760	12224.6	11.3528	73440
75	14051.4	5760	12442.2	565.171	64800
76	6448.2	5760	10267.4	294.505	64800
77	867.457	5760	8876.05	19.8891	73440
78	1510.04	5760	11555	42.4688	73440
79	2817.72	5760	10619.9	161.264	73440
80	250.008	5760	10804	8.72733	47520
81	481.414	5760	8897.44	13.7849	64800
82	250.232	5760	10792.6	9.91135	73440
83	250	5760	10782.7	9.90407	73440
84	250	5760	19893.5	8.59069	73440
85	200.46	7200	8972.2	32.9211	64800
86	478.4	7200	9505.4	68.2774	64800
87	589.68	7200	10868.2	29.5023	64800
88	247	7200	12326.6	22.6333	73440
89	314.6	7200	10814.2	13.566	73440
90	295.75	7200	11774.1	13.2246	73440
91	319.8	7200	19879.1	12.9627	73440
92	330	7200	8946.55	24.15	73440
93	311.4	7200	9787.69	16.7648	73440
94	499.918	7200	19361.9	22.024	64800
95	499.918	7200	19339.9	22.024	64800

;

x\_jj [\*,\*]

:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
:=																			
2	0	1	.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	.	1	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	1	0	0	0	0	.	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	1	0	0	0	.	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	.	1	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	.	1	0	0	0	0	0	0
14	0	0	1	0	0	0	0	0	0	0	0	0	0	0	.	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	.	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
25	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
26	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
33	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
44	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
78	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
85	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
86	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
:	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
:=																			
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
20	0	.	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	1	.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.	0
37	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	.
45	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
54	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
62	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
73	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
87	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
88	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
90	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
93	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
:	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
:=																			
4	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
18	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

Appendix A. Gantt Charts

34	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	1	0	0	.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	.	0	0	0	0	0	1	0	0	0	0	0	0	0
48	0	0	0	1	0	0	0	0	0	0	.	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	.	0
59	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
70	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
81	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
89	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0

:	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
:=																			
12	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
13	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
52	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
65	0	1	0	0	0	0	0	0	.	0	0	0	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	1	.	0	0	0	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	0	0	.	0	0	0	1	0	0	0	0
69	0	0	0	0	0	0	0	0	0	0	0	0	.	0	0	0	0	0	1
72	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	.	0	0	0
74	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	.	0
76	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
84	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
94	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

:	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
:=																			
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
28	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
53	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
67	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
71	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
77	0	.	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
79	0	0	0	.	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
80	0	0	0	0	.	0	0	0	0	0	0	0	0	1	0	0	0	0	0
82	0	0	0	0	1	0	.	0	0	0	0	0	0	0	0	0	0	0	0
83	0	0	0	0	0	0	1	.	0	0	0	0	0	0	0	0	0	0	0
91	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	.	0	0	0
92	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	.	0	0

---

```
95  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1
```

```
: 95 :=  
75 1  
;
```

```
m_jd [*,*]
```

```
# $1 = 10001_1  
# $2 = 10015_1  
# $3 = 10017_1  
# $4 = 10038_1  
# $5 = 10039_1  
# $6 = 10301_1  
# $7 = 10314_1  
# $8 = 10315_1  
# $9 = 10315_2  
# $10 = 10316_1  
# $11 = 10318_1  
# $12 = 10332_1  
# $13 = 10340_1  
# $14 = 10341_1  
# $15 = 10602_1  
# $16 = 10620_1  
# $17 = 10817_1  
# $18 = 19291_1  
# $19 = 2011_1
```

```
:  $1  $2  $3  $4  $5  $6  $7  $8  $9  $10  $11  $12  $13  $14  $15  $16  $17  $18  $19  
:=
```

```
4  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  
5  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  
8  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  
9  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  
29 0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  
30 0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  
32 0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  
33 0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  
34 0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  
35 0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  
36 0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  
37 1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  
38 0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  
42 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  
43 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  
56 1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  
57 0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  
59 0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  
63 0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  
64 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  
86 0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  
87 0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0
```

```
# $1 = 2020_1  
# $2 = 2048_1  
# $3 = 2051_1
```

Appendix A. Gantt Charts

```

# $4 = 2059_1
# $5 = 2063_1
# $6 = 21002_1
# $7 = 21003_1
# $8 = 21004_1
# $9 = 21010_1
# $10 = 21018_1
# $11 = 21023_1
# $12 = 2152_1
# $13 = 2152_2
# $14 = 2305_1
# $15 = 2307_1
# $16 = 2351_1
# $17 = 2351_2
# $18 = 2367_1
# $19 = 2367_2
:  $1  $2  $3  $4  $5  $6  $7  $8  $9  $10  $11  $12  $13  $14  $15  $16  $17  $18  $19
:=
14  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0
20  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
21  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
22  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
58  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
71  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
73  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0
74  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
79  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0
82  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
83  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
84  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0
85  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0
88  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0
89  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0
90  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0
91  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0
92  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0
93  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0

# $1 = 2398_1
# $2 = 2398_2
# $3 = 2398_3
# $4 = 2398_4
# $5 = 2402_1
# $6 = 2402_2
# $7 = 2432_1
# $8 = 2482_1
# $9 = 2490_1
# $10 = 2494_1
# $11 = 2553_1
# $12 = 2553_2
# $13 = 2554_1
# $14 = 2597_1
# $15 = 2629_1
# $16 = 2629_2

```



```

# $17 = 2690_1
# $18 = 2696_1
# $19 = 2698_1
:   $1  $2  $3  $4  $5  $6  $7  $8  $9  $10 $11 $12 $13 $14 $15 $16 $17 $18 $19
:=
13  0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0
15  0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0
16  0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0
17  0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0
18  0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0
19  0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0
23  0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0
60  1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
61  0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0
68  0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0
69  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0
70  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
72  0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0
76  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
78  0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0
80  0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0
94  1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
95  1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0

```

```

# $1 = 2710_1
# $2 = 2711_1
# $3 = 2711_2
# $4 = 2712_1
# $5 = 2713_1
# $6 = 2719_1
# $7 = 2724_1
# $8 = 2778_1
# $9 = 30042_1
# $10 = 30053_1
# $11 = 30061_1
# $12 = 30067_1
# $13 = 30068_1
# $14 = 30069_1
# $15 = 30069_2
# $16 = 30070_1
# $17 = 300_1
# $18 = 301_1
# $19 = 40201_1
:   $1  $2  $3  $4  $5  $6  $7  $8  $9  $10 $11 $12 $13 $14 $15 $16 $17 $18 $19
:=
10  0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0
11  0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0
12  0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0
25  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
31  0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0
44  1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
45  0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0
46  0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0
47  0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0

```

Appendix A. Gantt Charts

48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
53	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
77	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0

```
# $1 = 40201_2
# $2 = 40208_1
# $3 = 40216_1
# $4 = 40218_1
# $7 = 4040_1
# $8 = 4079_1
# $9 = 4544_1
# $10 = 4581_1
# $11 = 4660_1
# $12 = 4661_1
# $13 = 47020_1
# $14 = 4716_1
# $15 = 4737_1
# $17 = 70025_1
```

```
: $1 $2 $3 $4 4023_1 4035_1 $7 $8 $9 $10 $11 $12 $13 $14 $15 507_1 $17
:=
1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
2 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
3 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
6 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
7 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
24 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
26 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
27 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
28 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
39 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
40 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
41 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
52 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
62 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
67 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
75 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
81 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
;
```

```
# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
#
# DATA Instance 2 (May 9th to 15th)
# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
```

```
set Orders := 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112
113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 132
133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152
```

---

153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172  
173 174 175 176 177 ;

set M\_j[96] := 4642\_1 ;  
set M\_j[97] := 10303\_1 10303\_2 ;  
set M\_j[98] := 10341\_1 ;  
set M\_j[99] := 10025\_1 ;  
set M\_j[100] := 10028\_1 10028\_2 ;  
set M\_j[101] := 2383\_1 ;  
set M\_j[102] := 2633\_1 ;  
set M\_j[103] := 4609\_1 ;  
set M\_j[104] := 19207\_1 ;  
set M\_j[105] := 2152\_1 2152\_2 ;  
set M\_j[106] := 10620\_1 ;  
set M\_j[107] := 2187\_1 ;  
set M\_j[108] := 2223\_1 ;  
set M\_j[109] := 2189\_1 ;  
set M\_j[110] := 2379\_1 ;  
set M\_j[111] := 2634\_1 ;  
set M\_j[112] := 10020\_1 10020\_2 ;  
set M\_j[113] := 19202\_1 19202\_2 ;  
set M\_j[114] := 19202\_1 19202\_2 ;  
set M\_j[115] := 19211\_1 ;  
set M\_j[116] := 5034\_1 ;  
set M\_j[117] := 19204\_1 ;  
set M\_j[118] := 19205\_1 ;  
set M\_j[119] := 19206\_1 ;  
set M\_j[120] := 19212\_1 ;  
set M\_j[121] := 19233\_1 19233\_2 ;  
set M\_j[122] := 19233\_1 19233\_2 ;  
set M\_j[123] := 19236\_1 ;  
set M\_j[124] := 19244\_1 ;  
set M\_j[125] := 19244\_1 ;  
set M\_j[126] := 19255\_1 19255\_2 ;  
set M\_j[127] := 19255\_1 19255\_2 ;  
set M\_j[128] := 19255\_1 19255\_2 ;  
set M\_j[129] := 19255\_1 19255\_2 ;  
set M\_j[130] := 19255\_1 19255\_2 ;  
set M\_j[131] := 19208\_1 19208\_2 19208\_3 ;  
set M\_j[132] := 19208\_1 19208\_2 19208\_3 ;  
set M\_j[133] := 19208\_1 19208\_2 19208\_3 ;  
set M\_j[134] := 19208\_1 19208\_2 19208\_3 ;  
set M\_j[135] := 19264\_1 ;  
set M\_j[136] := 19208\_1 19208\_2 19208\_3 ;  
set M\_j[137] := 89011\_1 ;  
set M\_j[138] := 4039\_1 ;  
set M\_j[139] := 19207\_1 ;  
set M\_j[140] := 10306\_1 ;  
set M\_j[141] := 2687\_1 ;  
set M\_j[142] := 2686\_1 ;  
set M\_j[143] := 2706\_1 ;  
set M\_j[144] := 2719\_1 ;  
set M\_j[145] := 2668\_1 ;  
set M\_j[146] := 10314\_1 ;

```
set M_j[147] := 4685_1 ;
set M_j[148] := 10602_1 ;
set M_j[149] := 2441_1 ;
set M_j[150] := 4628_1 ;
set M_j[151] := 4629_1 ;
set M_j[152] := 2443_1 ;
set M_j[153] := 47007_1 ;
set M_j[154] := 47005_1 ;
set M_j[155] := 70017_1 ;
set M_j[156] := 70018_1 70018_2 ;
set M_j[157] := 70028_1 ;
set M_j[158] := 70027_1 70027_2 ;
set M_j[159] := 70021_1 70021_2 ;
set M_j[160] := 70020_1 70020_2 ;
set M_j[161] := 90101_1 ;
set M_j[162] := 90102_1 ;
set M_j[163] := 90103_1 ;
set M_j[164] := 90104_1 ;
set M_j[165] := 90105_1 ;
set M_j[166] := 90106_1 ;
set M_j[167] := 90107_1 ;
set M_j[168] := 90202_1 90202_2 ;
set M_j[169] := 90207_1 ;
set M_j[170] := 90216_1 ;
set M_j[171] := 452_1 ;
set M_j[172] := 530_1 ;
set M_j[173] := 4542_1 ;
set M_j[174] := 2637_1 ;
set M_j[175] := 2638_1 ;
set M_j[176] := 2639_1 ;
set M_j[177] := 2640_1 ;
```

```
param p_j :=
96 35.7947
97 13.86693
98 18.58867
99 16.65896
100 16.89655
101 13.25166
102 88.68128
103 47.24854
104 114.8753
105 49.77128
106 18.92034
107 26.1545
108 15.81302
109 29.86206
110 16.28231
111 25.65468
112 19.26041
113 7.331263
114 41.40627
115 192.7507
116 14.20616
```

---

117	20.87502
118	22.24842
119	52.15229
120	35.72907
121	24.87886
122	440.9867
123	23.53082
124	95.53954
125	78.74459
126	51.64094
127	54.05556
128	9.521329
129	29.36398
130	31.59707
131	283.7264
132	183.9778
133	137.3561
134	137.3561
135	34.0647
136	283.7264
137	13.00493
138	12.44395
139	18.66251
140	9.220029
141	26.39253
142	18.85519
143	9.535127
144	12.76163
145	7.99733
146	83.11176
147	13.53641
148	35.1599
149	45.65583
150	148.1162
151	106.1596
152	25.75153
153	22.5908
154	12.17489
155	16.28157
156	16.12046
157	50.52824
158	65.37407
159	59.37108
160	51.27824
161	43.5852
162	29.44251
163	47.0337
164	56.56225
165	81.15084
166	70.7052
167	16.56541
168	104.1056
169	27.29003
170	30.68519

---

```
171  27.18896
172  94.44909
173  131.2079
174  11.00786
175  10.04344
176  10.96737
177  11.65062
;
param r_j :=
96   8640
97   8640
98   8640
99   8640
100  8640
101  8640
102  8640
103  8640
104  10080
105  10080
106  10080
107  10080
108  10080
109  10080
110  10080
111  10080
112  10080
113  11520
114  11520
115  11520
116  11520
117  11520
118  11520
119  11520
120  11520
121  11520
122  11520
123  11520
124  11520
125  11520
126  11520
127  11520
128  11520
129  11520
130  11520
131  11520
132  11520
133  11520
134  11520
135  11520
136  11520
137  11520
138  11520
139  14400
140  14400
```

---

```
141 15840
142 15840
143 15840
144 15840
145 15840
146 15840
147 15840
148 15840
149 15840
150 15840
151 15840
152 15840
153 15840
154 15840
155 15840
156 15840
157 15840
158 15840
159 15840
160 15840
161 15840
162 15840
163 15840
164 15840
165 15840
166 15840
167 15840
168 15840
169 15840
170 15840
171 15840
172 15840
173 15840
174 15840
175 15840
176 15840
177 15840
;
param d_j :=
96 64800
97 64800
98 64800
99 64800
100 64800
101 64800
102 73440
103 64800
104 21600
105 30240
106 64800
107 64800
108 64800
109 64800
110 64800
```

111	73440
112	64800
113	38880
114	38880
115	38880
116	38880
117	73440
118	73440
119	73440
120	73440
121	73440
122	73440
123	73440
124	73440
125	73440
126	73440
127	73440
128	73440
129	73440
130	73440
131	73440
132	73440
133	73440
134	73440
135	38880
136	73440
137	38880
138	64800
139	21600
140	30240
141	73440
142	73440
143	125280
144	82080
145	82080
146	38880
147	73440
148	73440
149	73440
150	73440
151	73440
152	38880
153	38880
154	38880
155	38880
156	38880
157	38880
158	38880
159	38880
160	38880
161	38880
162	38880
163	38880
164	38880



---

```
165 38880
166 38880
167 38880
168 38880
169 38880
170 38880
171 64800
172 73440
173 38880
174 73440
175 73440
176 73440
177 73440
;
param c_j :=
96 999.096
97 398.208
98 503.706
99 498.96
100 502.26
101 300
102 2020.95
103 1376.55
104 2550.528
105 300.69
106 822.642
107 700.28
108 295.85
109 454.45
110 334.768
111 1001.25
112 421.889
113 215.1028
114 1348.81
115 4166.01
116 445.284
117 531.2006
118 576.1822
119 871.2704
120 1001.123
121 455.4396
122 8421.197
123 638.88
124 2226.217
125 1752.3
126 1378.416
127 1451.736
128 99.45
129 701.974
130 769.782
131 5679.18
132 3669.624
133 2730.375
134 2730.375
```

135 919.524  
 136 5679.18  
 137 267.54  
 138 504.0693  
 139 431.3088  
 140 196.746  
 141 608.4  
 142 488.28  
 143 280  
 144 315.7164  
 145 264.384  
 146 2001.348  
 147 398.736  
 148 249.665  
 149 999.635  
 150 4870  
 151 2865.432  
 152 622.05  
 153 425.49  
 154 486.85  
 155 435.5  
 156 410.8  
 157 1148.55  
 158 1263.6  
 159 1248  
 160 1084.2  
 161 1210.625  
 162 856.7  
 163 1635.4  
 164 1664  
 165 1838.2  
 166 1826.5  
 167 362.7  
 168 2496  
 169 716.3  
 170 422.5  
 171 253.536  
 172 2998.198  
 173 2872.8  
 174 308.22  
 175 300.162  
 176 306.84  
 177 308.256

;  
 param S\_jj : 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113  
 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 132 133  
 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152 153  
 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172 173  
 174 175 176 177 :=  
 96 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5  
 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5  
 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5  
 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5  
 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5





















```

1.5 1.5 0
;

# - * - * - * - * - * - * - * - * - * -
#      SOLUTION AND VISUALIZATION
# - * - * - * - * - * - * - * - * - * -

#option solver gurobi;

solve;
option omit_zero_rows 1;
display f_obj;
display Orders;
display c_j, r_j, s_j, p_j, d_j;
display x_jj;
display m_jd;

# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
#
#                               Instance 2 (May 9th to 15th)
# - * - * - * - * - * - * - * - * - * -

Presolve eliminates 389 constraints and 6837 variables.
Adjusted problem:
6939 variables:
    6857 binary variables
    82 linear variables
13473 constraints, all linear; 46873 nonzeros
    189 equality constraints
    13284 inequality constraints
1 linear objective; 6594 nonzeros.

Gurobi 8.1.0: threads=4
Gurobi 8.1.0: optimal solution; objective 103.5
5022516 simplex iterations
235147 branch-and-cut nodes
plus 80 simplex iterations for intbasis
f_obj = 103.5

set Orders :=
96  103  110  117  124  131  138  145  152  159  166  173
97  104  111  118  125  132  139  146  153  160  167  174
98  105  112  119  126  133  140  147  154  161  168  175
99  106  113  120  127  134  141  148  155  162  169  176
100 107  114  121  128  135  142  149  156  163  170  177
101 108  115  122  129  136  143  150  157  164  171
102 109  116  123  130  137  144  151  158  165  172;

:      c_j      r_j      s_j      p_j      d_j      :=
96    999.096    8640    16205.9    35.7947    64800
97    398.208    8640    36811.7    13.8669    64800
98    503.706    8640    15990.8    18.5887    64800
99    498.96     8640    16599.5    16.659     64800
100   502.26     8640    15911.5    16.8966    64800
101   300        8640    16944.8    13.2517    64800

```

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102	2020.95	8640	36133.2	88.6813	73440
103	1376.55	8640	37106.8	47.2485	64800
104	2550.53	10080	20026.5	114.875	21600
105	300.69	10080	16548.2	49.7713	30240
106	822.642	10080	16527.8	18.9203	64800
107	700.28	10080	16441.1	26.1545	64800
108	295.85	10080	16056	15.813	64800
109	454.45	10080	19686.2	29.8621	64800
110	334.768	10080	16468.8	16.2823	64800
111	1001.25	10080	36425.3	25.6547	73440
112	421.889	10080	16924	19.2604	64800
113	215.103	11520	36052.2	7.33126	38880
114	1348.81	11520	36059.5	41.4063	38880
115	4166.01	11520	37204.1	192.751	38880
116	445.284	11520	36102.4	14.2062	38880
117	531.201	11520	19663.9	20.875	73440
118	576.182	11520	36892.4	22.2484	73440
119	871.27	11520	19972.8	52.1523	73440
120	1001.12	11520	36223.3	35.7291	73440
121	455.44	11520	71534.1	24.8789	73440
122	8421.2	11520	71559	440.987	73440
123	638.88	11520	37029.7	23.5308	73440
124	2226.22	11520	36531.2	95.5395	73440
125	1752.3	11520	36452.4	78.7446	73440
126	1378.42	11520	11695	51.6409	73440
127	1451.74	11520	11580	54.0556	73440
128	99.45	11520	11746.7	9.52133	73440
129	701.974	11520	11665.7	29.364	73440
130	769.782	11520	11634.1	31.5971	73440
131	5679.18	11520	12216.4	283.726	73440
132	3669.62	11520	11757.7	183.978	73440
133	2730.38	11520	11941.7	137.356	73440
134	2730.38	11520	12079	137.356	73440
135	919.524	11520	16797.5	34.0647	38880
136	5679.18	11520	12500.1	283.726	73440
137	267.54	11520	37427	13.0049	38880
138	504.069	11520	36878.4	12.4439	64800
139	431.309	14400	20141.3	18.6625	21600
140	196.746	14400	16959.5	9.22003	30240
141	608.4	15840	36364.5	26.3925	73440
142	488.28	15840	36392.4	18.8552	73440
143	280	15840	16913	9.53513	125280
144	315.716	15840	36712.8	12.7616	82080
145	264.384	15840	16850.7	7.99733	82080
146	2001.35	15840	36628.2	83.1118	38880
147	398.736	15840	36118.1	13.5364	73440
148	249.665	15840	16169.2	35.1599	73440
149	999.635	15840	16243.2	45.6558	73440
150	4870	15840	19823.2	148.116	73440
151	2865.43	15840	16689.9	106.16	73440
152	622.05	15840	36851.2	25.7515	38880
153	425.49	15840	36827.1	22.5908	38880
154	486.85	15840	36260.6	12.1749	38880
155	435.5	15840	36793.9	16.2816	38880

Appendix A. Gantt Charts

156	410.8	15840	16833.1	16.1205	38880
157	1148.55	15840	37054.7	50.5282	38880
158	1263.6	15840	36727.1	65.3741	38880
159	1248	15840	15929.9	59.3711	38880
160	1084.2	15840	16860.2	51.2782	38880
161	1210.62	15840	16010.9	43.5852	38880
162	856.7	15840	36998.8	29.4425	38880
163	1635.4	15840	37155.5	47.0337	38880
164	1664	15840	36306.4	56.5622	38880
165	1838.2	15840	36916.1	81.1508	38880
166	1826.5	15840	16617.7	70.7052	38880
167	362.7	15840	16423.1	16.5654	38880
168	2496	15840	19717.6	104.106	38880
169	716.3	15840	16499	27.29	38880
170	422.5	15840	36274.2	30.6852	38880
171	253.536	15840	37398.3	27.189	64800
172	2998.2	15840	16073.3	94.4491	73440
173	2872.8	15840	16290.4	131.208	38880
174	308.22	15840	36412.7	11.0079	73440
175	300.162	15840	15900	10.0434	73440
176	306.84	15840	16486.6	10.9674	73440
177	308.256	15840	16970.3	11.6506	73440

;

x\_jj [\*,\*]

:	0	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113
:=																			
105	0	0	0	0	1	0	0	0	0	0	.	0	0	0	0	0	0	0	0
106	0	0	0	0	0	0	0	0	0	0	1	.	0	0	0	0	0	0	0
107	0	0	0	0	0	0	0	0	0	0	0	0	.	0	0	1	0	0	0
112	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	.	0
117	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
119	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
122	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
139	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
143	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
147	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
148	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
155	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
157	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
159	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
161	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
167	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
169	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
174	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
175	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
:	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132
:=																			
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
102	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
111	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
113	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
114	.	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

121	0	0	0	0	0	0	0	.	1	0	0	0	0	0	0	0	0	0	0
125	0	0	0	0	0	0	0	0	0	0	1	.	0	0	0	0	0	0	0
126	0	0	0	0	0	0	0	0	0	0	0	0	.	0	1	0	0	0	0
127	0	0	0	0	0	0	0	0	0	0	0	0	0	.	0	0	1	0	0
128	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.	0	0	0	1
129	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	.	0	0	0
130	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	.	0	0
134	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
137	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
138	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
150	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
162	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
163	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
177	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

: 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151  
:=

96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
101	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
104	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
116	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
124	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
131	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
132	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
133	.	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
141	0	0	0	0	0	0	0	0	.	1	0	0	0	0	0	0	0	0	0
146	0	0	0	0	0	0	0	0	0	0	0	1	0	.	0	0	0	0	0
151	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.
152	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
156	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
160	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
164	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
166	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
168	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
171	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
172	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

: 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170  
:=

97	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
98	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
99	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
100	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
103	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
109	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
118	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
120	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
123	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
135	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
144	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
145	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
153	1	.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
154	0	0	.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
158	0	0	0	1	0	0	.	0	0	0	0	0	0	0	0	0	0	0	0

Appendix A. Gantt Charts

```

165  0  0  0  0  0  0  0  0  0  0  1  0  0  .  0  0  0  0  0
170  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  .
173  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0
176  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0

```

```

:   171 172 173 174 175 176 177   :=
108  0  1  0  0  0  0  0
110  0  0  0  0  0  1  0
115  1  0  0  0  0  0  0
136  0  0  0  0  1  0  0
140  0  0  0  0  0  0  1
142  0  0  0  1  0  0  0
149  0  0  1  0  0  0  0
;

```

m\_jd [\*,\*] (tr)

```

:   96  97  98  99 100 101 102 103 104 105 106 107 108 109 110 111 112 113
:=
10020_1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0
10025_1  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
10028_1  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0
10303_1  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
10341_1  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
10620_1  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0
19202_1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1
19207_1  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0
2152_1  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0
2187_1  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0
2189_1  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0
2223_1  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0
2379_1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0
2383_1  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0
2633_1  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0
2634_1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0
4609_1  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0
4642_1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0

```

```

:   114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131
:=
19202_1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
19204_1  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
19205_1  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0
19206_1  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0
19208_1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1
19211_1  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
19212_1  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0
19233_1  0  0  0  0  0  0  0  1  1  0  0  0  0  0  0  0  0  0
19236_1  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0
19244_1  0  0  0  0  0  0  0  0  0  0  1  1  0  0  0  0  0  0
19255_1  0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  0
5034_1  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0

```

```

:   132 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149
:=

```



10306_1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
10314_1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
10602_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
19207_1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
19208_1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
19264_1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2441_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
2668_1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
2686_1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
2687_1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
2706_1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
2719_1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
4039_1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
4685_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
89011_1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0

: 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167  
:=

2443_1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4628_1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4629_1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47005_1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
47007_1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
70017_1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
70018_1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
70020_1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
70021_1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
70027_1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
70028_1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
90101_1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
90102_1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
90103_1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
90104_1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
90105_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
90106_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
90107_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

: 168 169 170 171 172 173 174 175 176 177 :=

2637_1	0	0	0	0	0	0	1	0	0	0
2638_1	0	0	0	0	0	0	0	1	0	0
2639_1	0	0	0	0	0	0	0	0	1	0
2640_1	0	0	0	0	0	0	0	0	0	1
452_1	0	0	0	1	0	0	0	0	0	0
4542_1	0	0	0	0	0	1	0	0	0	0
530_1	0	0	0	0	1	0	0	0	0	0
90202_1	1	0	0	0	0	0	0	0	0	0
90207_1	0	1	0	0	0	0	0	0	0	0
90216_1	0	0	1	0	0	0	0	0	0	0

```

;
# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
#
# DATA Instance 3 (May 23rd to 29th)
# - * - * - * - * - * - * - * - * - * - * - * -

```

## Appendix A. Gantt Charts

```
set Orders := 402 403 404 405 406 407 408 409 410 411 412 413 414 415 416 417
418 419 420 421 422 423 424 425 426 427 428 429 430 431 432 433 434 435 436 437
438 439 440 441 442 443 444 445 446 447 448 449 450 451 452 453 454 455 456 457
458 459 460 461 462 463 464 465 466 467 468 469 ;
```

```
set M_j[402] := 2328_1 ;
set M_j[403] := 2328_1 ;
set M_j[404] := 2329_1 ;
set M_j[405] := 2329_1 ;
set M_j[406] := 2328_1 ;
set M_j[407] := 2329_1 ;
set M_j[408] := 2391_1 2391_2 ;
set M_j[409] := 2391_1 2391_2 ;
set M_j[410] := 2391_1 2391_2 ;
set M_j[411] := 2391_1 2391_2 ;
set M_j[412] := 2732_1 ;
set M_j[413] := 2649_1 ;
set M_j[414] := 2649_1 ;
set M_j[415] := 10319_1 ;
set M_j[416] := 85013_1 ;
set M_j[417] := 85011_1 ;
set M_j[418] := 2001_1 2001_2 2001_3 ;
set M_j[419] := 511_1 ;
set M_j[420] := 2228_1 ;
set M_j[421] := 2228_1 ;
set M_j[422] := 2228_1 ;
set M_j[423] := 2228_1 ;
set M_j[424] := 2228_1 ;
set M_j[425] := 2139_1 ;
set M_j[426] := 19270_1 ;
set M_j[427] := 2788_1 ;
set M_j[428] := 2667_1 2667_2 2667_3 ;
set M_j[429] := 2553_1 2553_2 ;
set M_j[430] := 2553_1 2553_2 ;
set M_j[431] := 2554_1 ;
set M_j[432] := 2063_1 ;
set M_j[433] := 2629_1 2629_2 ;
set M_j[434] := 21023_1 ;
set M_j[435] := 21010_1 ;
set M_j[436] := 21013_1 ;
set M_j[437] := 10306_1 ;
set M_j[438] := 2563_1 ;
set M_j[439] := 4685_1 ;
set M_j[440] := 2001_1 2001_2 2001_3 ;
set M_j[441] := 2001_1 2001_2 2001_3 ;
set M_j[442] := 2453_1 ;
set M_j[443] := 2610_1 ;
set M_j[444] := 2610_1 ;
set M_j[445] := 2471_1 ;
set M_j[446] := 2723_1 ;
set M_j[447] := 2316_1 ;
set M_j[448] := 2472_1 ;
set M_j[449] := 2230_1 2230_2 2230_3 ;
set M_j[450] := 2230_1 2230_2 2230_3 ;
```

---

```
set M_j[451] := 2117_1 ;
set M_j[452] := 2408_1 2408_2 ;
set M_j[453] := 2695_1 2695_2 ;
set M_j[454] := 10827_1 10827_2 ;
set M_j[455] := 4109_1 ;
set M_j[456] := 44_1 ;
set M_j[457] := 10016_1 10016_2 ;
set M_j[458] := 4759_1 ;
set M_j[459] := 10005_1 ;
set M_j[460] := 10001_1 ;
set M_j[461] := 10012_1 ;
set M_j[462] := 10210_1 ;
set M_j[463] := 10351_1 ;
set M_j[464] := 10315_1 10315_2 ;
set M_j[465] := 10345_1 ;
set M_j[466] := 47009_1 47009_2 47009_3 ;
set M_j[467] := 40221_1 ;
set M_j[468] := 10038_1 ;
set M_j[469] := 10347_1 ;
```

```
param p_j :=
402 38.40363
403 10.18409
404 19.78328
405 5.062095
406 12.43733
407 6.236595
408 122.52
409 64.24642
410 99.81185
411 39.36341
412 2.985742
413 17.075
414 17.08729
415 13.81941
416 8.632669
417 9.299062
418 121.2127
419 37.05272
420 12.09297
421 6.046487
422 4.837189
423 1.813946
424 10.27903
425 53.5181
426 34.38481
427 13.96482
428 119.3161
429 43.67136
430 108.0649
431 115.0998
432 114.7703
433 225.2053
434 24.15
```

---

```
435 22.63333
436 12.76546
437 27.90204
438 9.385087
439 9.967096
440 33.30668
441 18.6104
442 18.94657
443 45.90086
444 29.37655
445 32.51882
446 53.16279
447 45.51781
448 38.67104
449 44.73489
450 44.73489
451 45.70585
452 32.9048
453 32.26078
454 121.6434
455 9.354437
456 21.38328
457 13.58439
458 30.26453
459 10.38603
460 14.45428
461 20.30771
462 9.349624
463 27.44171
464 19.45464
465 12.58457
466 31.23212
467 5.724701
468 13.31032
469 11.85876
;
param r_j :=
402 25920
403 25920
404 25920
405 25920
406 25920
407 25920
408 25920
409 25920
410 25920
411 25920
412 25920
413 27360
414 27360
415 27360
416 27360
417 27360
418 27360
```

---

419 27360  
420 31680  
421 31680  
422 31680  
423 31680  
424 31680  
425 31680  
426 31680  
427 33120  
428 33120  
429 33120  
430 33120  
431 33120  
432 33120  
433 33120  
434 33120  
435 33120  
436 33120  
437 33120  
438 33120  
439 33120  
440 33120  
441 33120  
442 33120  
443 33120  
444 33120  
445 33120  
446 33120  
447 33120  
448 33120  
449 33120  
450 33120  
451 33120  
452 33120  
453 33120  
454 33120  
455 33120  
456 33120  
457 33120  
458 33120  
459 33120  
460 33120  
461 33120  
462 33120  
463 33120  
464 33120  
465 33120  
466 33120  
467 33120  
468 33120  
469 33120  
;  
param d\_j :=  
402 99360

403	99360
404	99360
405	99360
406	99360
407	99360
408	82080
409	82080
410	82080
411	82080
412	82080
413	82080
414	82080
415	47520
416	82080
417	82080
418	82080
419	82080
420	108000
421	108000
422	108000
423	108000
424	108000
425	90720
426	90720
427	108000
428	108000
429	99360
430	99360
431	99360
432	99360
433	99360
434	108000
435	108000
436	108000
437	90720
438	90720
439	90720
440	108000
441	108000
442	108000
443	108000
444	108000
445	108000
446	108000
447	108000
448	108000
449	108000
450	108000
451	116640
452	56160
453	56160
454	56160
455	90720
456	99360

---

```
457 90720
458 90720
459 90720
460 90720
461 90720
462 90720
463 90720
464 90720
465 90720
466 90720
467 90720
468 90720
469 90720
;
param c_j :=
402 847.0884
403 216.505
404 146.5428
405 37.497
406 266.855
407 46.197
408 816.928
409 437.2232
410 668.964
411 275.088
412 126
413 499.2328
414 499.7135
415 255.507
416 247.455
417 247.975
418 2566.2
419 250.6002
420 205.03
421 102.515
422 82.012
423 30.7545
424 174.2755
425 1496
426 752.856
427 305.76
428 2156.88
429 985.3536
430 2501.018
431 2501.856
432 1780.835
433 2500.588
434 330
435 247
436 279.5
437 500
438 256.9125
439 295.36
440 649.446
```

```

441 329
442 450
443 1005
444 643.2
445 712
446 1164
447 1029
448 705
449 360
450 360
451 1000.73
452 668.9182
453 772.2365
454 774
455 308.736
456 468.1872
457 400.2375
458 998.634
459 301.086
460 398.772
461 502.398
462 300.672
463 883.025
464 368.55
465 252.72
466 299.4875
467 252.7785
468 298.584
469 302.8935
;
param S_jj : 402 403 404 405 406 407 408 409 410 411 412 413 414 415 416 417 418
419 420 421 422 423 424 425 426 427 428 429 430 431 432 433 434 435 436 437 438
439 440 441 442 443 444 445 446 447 448 449 450 451 452 453 454 455 456 457 458
459 460 461 462 463 464 465 466 467 468 469 :=
402 0 0 1.5 1.5 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5
403 0 0 1.5 1.5 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5
404 1.5 1.5 0 0 1.5 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5
405 1.5 1.5 0 0 1.5 0 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5
1.5 1.5 1.5 1.5

```















```
# - * - * - * - * - * - * - * - * - * -
```

```
#option solver gurobi;
```

```
solve;
option omit_zero_rows 1;
display f_obj;
display Orders;
display c_j, r_j, s_j, p_j, d_j;
display x_jj;
display m_jd;
```

```
# - * - * - * - * - * - * - * - * - * - * - * - * - * - * - * -
```

```
# Instance 3 (May 23rd to 29th)
```

```
# - * - * - * - * - * - * - * - * - * -
```

Presolve eliminates 389 constraints and 6837 variables.

Adjusted problem:

6939 variables:  
    6857 binary variables  
    82 linear variables  
13473 constraints, all linear; 46873 nonzeros  
    189 equality constraints  
    13284 inequality constraints  
1 linear objective; 6594 nonzeros.

Gurobi 8.1.0: threads=4  
Gurobi 8.1.0: optimal solution; objective 103.5  
5022516 simplex iterations  
235147 branch-and-cut nodes  
plus 80 simplex iterations for intbasis  
f\_obj = 103.5

set Orders :=

96	103	110	117	124	131	138	145	152	159	166	173
97	104	111	118	125	132	139	146	153	160	167	174
98	105	112	119	126	133	140	147	154	161	168	175
99	106	113	120	127	134	141	148	155	162	169	176
100	107	114	121	128	135	142	149	156	163	170	177
101	108	115	122	129	136	143	150	157	164	171	
102	109	116	123	130	137	144	151	158	165	172;	

:	c_j	r_j	s_j	p_j	d_j	:=
96	999.096	8640	16205.9	35.7947	64800	
97	398.208	8640	36811.7	13.8669	64800	
98	503.706	8640	15990.8	18.5887	64800	
99	498.96	8640	16599.5	16.659	64800	
100	502.26	8640	15911.5	16.8966	64800	
101	300	8640	16944.8	13.2517	64800	
102	2020.95	8640	36133.2	88.6813	73440	
103	1376.55	8640	37106.8	47.2485	64800	
104	2550.53	10080	20026.5	114.875	21600	
105	300.69	10080	16548.2	49.7713	30240	
106	822.642	10080	16527.8	18.9203	64800	

Appendix A. Gantt Charts

107	700.28	10080	16441.1	26.1545	64800
108	295.85	10080	16056	15.813	64800
109	454.45	10080	19686.2	29.8621	64800
110	334.768	10080	16468.8	16.2823	64800
111	1001.25	10080	36425.3	25.6547	73440
112	421.889	10080	16924	19.2604	64800
113	215.103	11520	36052.2	7.33126	38880
114	1348.81	11520	36059.5	41.4063	38880
115	4166.01	11520	37204.1	192.751	38880
116	445.284	11520	36102.4	14.2062	38880
117	531.201	11520	19663.9	20.875	73440
118	576.182	11520	36892.4	22.2484	73440
119	871.27	11520	19972.8	52.1523	73440
120	1001.12	11520	36223.3	35.7291	73440
121	455.44	11520	71534.1	24.8789	73440
122	8421.2	11520	71559	440.987	73440
123	638.88	11520	37029.7	23.5308	73440
124	2226.22	11520	36531.2	95.5395	73440
125	1752.3	11520	36452.4	78.7446	73440
126	1378.42	11520	11695	51.6409	73440
127	1451.74	11520	11580	54.0556	73440
128	99.45	11520	11746.7	9.52133	73440
129	701.974	11520	11665.7	29.364	73440
130	769.782	11520	11634.1	31.5971	73440
131	5679.18	11520	12216.4	283.726	73440
132	3669.62	11520	11757.7	183.978	73440
133	2730.38	11520	11941.7	137.356	73440
134	2730.38	11520	12079	137.356	73440
135	919.524	11520	16797.5	34.0647	38880
136	5679.18	11520	12500.1	283.726	73440
137	267.54	11520	37427	13.0049	38880
138	504.069	11520	36878.4	12.4439	64800
139	431.309	14400	20141.3	18.6625	21600
140	196.746	14400	16959.5	9.22003	30240
141	608.4	15840	36364.5	26.3925	73440
142	488.28	15840	36392.4	18.8552	73440
143	280	15840	16913	9.53513	125280
144	315.716	15840	36712.8	12.7616	82080
145	264.384	15840	16850.7	7.99733	82080
146	2001.35	15840	36628.2	83.1118	38880
147	398.736	15840	36118.1	13.5364	73440
148	249.665	15840	16169.2	35.1599	73440
149	999.635	15840	16243.2	45.6558	73440
150	4870	15840	19823.2	148.116	73440
151	2865.43	15840	16689.9	106.16	73440
152	622.05	15840	36851.2	25.7515	38880
153	425.49	15840	36827.1	22.5908	38880
154	486.85	15840	36260.6	12.1749	38880
155	435.5	15840	36793.9	16.2816	38880
156	410.8	15840	16833.1	16.1205	38880
157	1148.55	15840	37054.7	50.5282	38880
158	1263.6	15840	36727.1	65.3741	38880
159	1248	15840	15929.9	59.3711	38880
160	1084.2	15840	16860.2	51.2782	38880



161	1210.62	15840	16010.9	43.5852	38880
162	856.7	15840	36998.8	29.4425	38880
163	1635.4	15840	37155.5	47.0337	38880
164	1664	15840	36306.4	56.5622	38880
165	1838.2	15840	36916.1	81.1508	38880
166	1826.5	15840	16617.7	70.7052	38880
167	362.7	15840	16423.1	16.5654	38880
168	2496	15840	19717.6	104.106	38880
169	716.3	15840	16499	27.29	38880
170	422.5	15840	36274.2	30.6852	38880
171	253.536	15840	37398.3	27.189	64800
172	2998.2	15840	16073.3	94.4491	73440
173	2872.8	15840	16290.4	131.208	38880
174	308.22	15840	36412.7	11.0079	73440
175	300.162	15840	15900	10.0434	73440
176	306.84	15840	16486.6	10.9674	73440
177	308.256	15840	16970.3	11.6506	73440

;

x\_jj [\*,\*]

:	0	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113
:=																			
105	0	0	0	0	1	0	0	0	0	0	.	0	0	0	0	0	0	0	0
106	0	0	0	0	0	0	0	0	0	0	1	.	0	0	0	0	0	0	0
107	0	0	0	0	0	0	0	0	0	0	0	0	.	0	0	1	0	0	0
112	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	.	0
117	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
119	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
122	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
139	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
143	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
147	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
148	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
155	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
157	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
159	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
161	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
167	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
169	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
174	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
175	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
:	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132
:=																			
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
102	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
111	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
113	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
114	.	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
121	0	0	0	0	0	0	0	.	1	0	0	0	0	0	0	0	0	0	0
125	0	0	0	0	0	0	0	0	0	1	.	0	0	0	0	0	0	0	0
126	0	0	0	0	0	0	0	0	0	0	0	.	0	1	0	0	0	0	0
127	0	0	0	0	0	0	0	0	0	0	0	0	0	.	0	0	1	0	0
128	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.	0	0	0	1

Appendix A. Gantt Charts

129	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	.	0	0
134	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
137	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
138	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
150	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
162	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
163	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
177	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

: 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151  
:=

96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
101	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
104	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
116	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
124	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
131	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
132	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
133	.	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
141	0	0	0	0	0	0	0	0	.	1	0	0	0	0	0	0	0	0	0
146	0	0	0	0	0	0	0	0	0	0	0	1	0	.	0	0	0	0	0
151	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.
152	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
156	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
160	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
164	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
166	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
168	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
171	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
172	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

: 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170  
:=

97	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
98	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
99	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
100	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
103	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
109	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
118	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
120	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
123	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
135	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
144	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
145	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
153	1	.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
154	0	0	.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
158	0	0	0	1	0	0	.	0	0	0	0	0	0	0	0	0	0	0	0
165	0	0	0	0	0	0	0	0	0	0	1	0	0	.	0	0	0	0	0
170	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	.
173	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
176	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

```

: 171 172 173 174 175 176 177 :=
108 0 1 0 0 0 0 0
110 0 0 0 0 0 1 0
115 1 0 0 0 0 0 0
136 0 0 0 0 1 0 0
140 0 0 0 0 0 0 1
142 0 0 0 1 0 0 0
149 0 0 1 0 0 0 0
;

```

```

m_jd [*,*] (tr)
: 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113
:=
10020_1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
10025_1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
10028_1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
10303_1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
10341_1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
10620_1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
19202_1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
19207_1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
2152_1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
2187_1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
2189_1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
2223_1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
2379_1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
2383_1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
2633_1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
2634_1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
4609_1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
4642_1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

```

```

: 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131
:=
19202_1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
19204_1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
19205_1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
19206_1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
19208_1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
19211_1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
19212_1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
19233_1 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0
19236_1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
19244_1 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0
19255_1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1
5034_1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0

```

```

: 132 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149
:=
10306_1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
10314_1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
10602_1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
19207_1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
19208_1 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0

```

Appendix A. Gantt Charts

19264_1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2441_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
2668_1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
2686_1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
2687_1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
2706_1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
2719_1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
4039_1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
4685_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
89011_1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0

:           150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167  
:=

2443_1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4628_1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4629_1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47005_1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
47007_1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70017_1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
70018_1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
70020_1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
70021_1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
70027_1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
70028_1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
90101_1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
90102_1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
90103_1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
90104_1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
90105_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
90106_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
90107_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

:           168 169 170 171 172 173 174 175 176 177           :=

2637_1	0	0	0	0	0	0	1	0	0	0								
2638_1	0	0	0	0	0	0	0	1	0	0								
2639_1	0	0	0	0	0	0	0	0	1	0								
2640_1	0	0	0	0	0	0	0	0	0	1								
452_1	0	0	0	1	0	0	0	0	0	0								
4542_1	0	0	0	0	0	1	0	0	0	0								
530_1	0	0	0	0	1	0	0	0	0	0								
90202_1	1	0	0	0	0	0	0	0	0	0								
90207_1	0	1	0	0	0	0	0	0	0	0								
90216_1	0	0	1	0	0	0	0	0	0	0								

;  
# - \* -

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