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5 Theoretical luminosity precision for the FCC-ee: overview of the path to 0.01%

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We present an overview of the pathways to the required theoretical precision for the luminosity targeted by the FCC-ee precision studies. We put the discussion in context with a brief review of the situation at the time of the LEP. We then present the current status and an overview of routes to the desired 0.01% targeted by the FCC-ee (as well as by the ILC).

We use the situation that existed at the end of the LEP as our starting point. At the end of the LEP, the error budget for the `BHLUMI`4.04 MC used by all LEP collaborations to simulate the luminosity process was that calculated in Ref. [1]. For reference, we reproduce this result here in Table B.5.1. In this table, we show the published works on which the various error estimates are based, as discussed in Ref. [1].

Table B.5.1: Summary of the total (physical + technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, this estimate is valid for a generic angular range of 1°–3° (18–52 mrad), and for LEP2 it is valid for energies up to 176 GeV and an angular range of 3°–6°. Total uncertainty is taken in quadrature. Technical precision is included in (a).

Type of correction or error	LEP1 1996 (%)	 1999 (%)	LEP2 1996 (%)	 1999 (%)
(a) Missing photonic $\mathcal{O}(\alpha^2)$ [2,3]	0.10	0.027	0.20	0.04
(b) Missing photonic $\mathcal{O}(\alpha^3 L_e^3)$ [4]	0.015	0.015	0.03	0.03
(c) Vacuum polarisation [5,6]	0.04	0.04	0.10	0.10
(d) Light pairs [7,8]	0.03	0.03	0.05	0.05
(e) Z and s-channel γ [9,10]	0.015	0.015	0.0	0.0
Total	0.11 [10]	0.061 [1]	0.25 [10]	0.12 [1]

One way to address the 0.01% precision tag needed for the luminosity theory error for the FCC-ee is to develop the corresponding improved version of the `BHLUMI`. This problem is addressed in Ref. [11], wherein the path to 0.01% theory precision for the FCC-ee luminosity is presented in some detail. The results of this latter reference are shown in Table B.5.2, wherein we also present the current state of the art for completeness, as discussed in more detail in Ref. [11].

The key steps in arriving at Table B.5.2 are as follows. The errors associated with the photonic corrections in lines (a) and (b) in the LEP results in Table B.5.1 are due to effects that are known from Refs. [2–4] but were not implemented into `BHLUMI`. In Table B.5.2, we show what these errors will become after these known results are included in `BHLUMI`, as discussed in

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Table B.5.2: Anticipated total (physical + technical) theoretical uncertainty for a FCC-ee luminosity calorimetric detector with angular range 64–86 mrad (narrow), near the Z peak. Description of photonic corrections in square brackets is related to the second column. The total error is summed in quadrature.

Type of correction or error	Update 2018 (%)	FCC-ee forecast
(a) Photonic $[\mathcal{O}(L_e\alpha^2)]$ $\mathcal{O}(L_e^2\alpha^3)$	0.027	0.1×10^{-4}
(b) Photonic $[\mathcal{O}(L_e^3\alpha^3)]$ $\mathcal{O}(L_e^4\alpha^4)$	0.015	0.6×10^{-5}
(c) Vacuum polarisation	0.014 [12]	0.6×10^{-4}
(d) Light pairs	0.010 [13, 14]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090 [9]	0.1×10^{-4}
(f) Up–down interference	0.009 [15]	0.1×10^{-4}
(f) Technical precision	(0.027)	0.1×10^{-4}
Total	0.097	1.0×10^{-4}

Ref. [11]. Similarly, in line (c) of Table B.5.1, the error is due to the uncertainty at the time of LEP on the hadronic contribution to the vacuum polarisation for the photon at the respective momentum transfers for the luminosity process; in Table B.5.2, we show the improvement of this error that is expected for the FCC-ee, as discussed in Refs. [12, 16].

Continuing in this way, in line (d) in Table B.5.2, we show the expected improvement [11], with reference to the LEP time for Table B.5.1, in the light pairs error for the FCC-ee. As explained in Ref. [11], the complete matrix element for the additional real e^+e^- pair radiation should be used, because non-photonic graphs can contribute as much as 0.01% for the cut-off, $z_{\rm cut} \sim 0.7$. This can be done with the MC generators developed for the $e^+e^- \to 4f$ processes for LEP2 physics—see Ref. [11] for further discussion. With known methods [11], the contributions of light quark pairs, muon pairs, and non-leading, non-soft additional $e^+e^- + n\gamma$ corrections can be controlled such that the error on the pairs contribution is as given in line (d) for the FCC-ee. As noted, we also show the current state of the art [11] for this error in line (d) of Table B.5.2.

Turning to line (e) in Table B.5.2, we show the improvement of the error for the Z and s-channel γ exchange for the FCC-ee as well as its current state of the art. In Ref. [11], a detailed discussion is presented of all of the six interference and three additional squared modulus terms that result from the s-channel γ, s-channel Z, and t-channel Z exchange contributions to the amplitude for the luminosity process. It is shown that, if the predictions of `BHLUMI` for the luminosity measurement at FCC-ee are combined with those from `Bhwide` [17] for this Z and s-channel γ exchange contribution, then the error in the second column of line (e) of Table B.5.2 could be reduced to 0.01%. To reduce the uncertainty of this contribution practically to zero we would include these Z and γ_s exchanges within the CEEX-type matrix element at $\mathcal{O}(\alpha^1)$ in `BHLUMI` [18]. Here, CEEX stands for coherent exclusive exponentiation, which acts at the level of the amplitudes, as compared with the original Yennie–Frautschi–Suura [19] (YFS) exclusive exponentiation (EEX), which is used in `BHLUMI`4.04 and which acts at the level of the squared amplitudes. It is expected to be enough to add the EW corrections to the large angle Bhabha (LABH) process in the form of effective couplings in the Born amplitudes. This leads to the error estimate shown in Table B.5.2 in line (e) for the FCC-ee.

For completeness, we note that for our discussion of the Z- and s-channel γ exchanges we made [11] a numerical study using `Bhwide` for the the calorimetric LCAL-type detector,

Table B.5.3: Results from `Bhwide` for the Z and γ_s exchange contributions to the FCC-ee luminosity with respect to the $\gamma_t \otimes \gamma_t$ process for the calorimetric LCAL-type detector [20] with the symmetric angular range 64–86 mrad; no acoplanarity cut was applied. MC errors are marked in brackets.

E_{CM} (GeV)	Δ_{tot} (%)	$\delta^{\mathrm{QED}}_{\mathcal{O}(\alpha)}$ (%)	$\delta^{\mathrm{QED}}_{\mathrm{h.o.}}$ (%)	$\delta^{\mathrm{weak}}_{\mathrm{tot}}$ (%)
90.1876	+0.642 (12)	−0.152 (59)	+0.034 (38)	−0.005 (12)
91.1876	+0.041 (11)	+0.148 (59)	−0.035 (38)	+0.009 (12)
92.1876	−0.719 (13)	+0.348 (59)	−0.081 (38)	+0.039 (13)

as described in Ref. [20], for the symmetric angular range 64–86 mrad without any cut in acoplanarity. The pure weak corrections were calculated with the `ALIBABA` EW library [21,22]. The results, shown in Table B.5.3, were obtained for three values of the centre-of-mass (CM) energy: $E_{\mathrm{CM}} = M_Z$, $M_Z \pm 1$ GeV, where the latter two values have Z contributions that are close to maximal in size. The results in the second column for the total size of the Z and γ_s exchanges are consistent with our expectations, as explained in Ref. [11]: the contribution is positive below the Z peak, where it reaches a size ~0.64%, is close to zero near the peak, and changes sign above the peak, where it reaches a size ~−0.72%. The third column features the fixed-order (non-exponentiated) $\mathcal{O}(\alpha)$ QED correction and shows that it is sizeable and up to a half of the size of the Born level effect, with a sign that is opposite to that of the latter effect. The fourth column shows the size of the higher-order QED effects from YFS exponentiation, which also change their sign near the Z peak, in opposition to the corresponding change of the $\mathcal{O}(\alpha)$ corrections. We see that the size of the former effects is about a quarter of that of the latter. The effects in the fourth column allow us to make a conservative estimate of the size of the missing higher-order QED effects in `Bhwide` using the big log factor $\gamma = \alpha \ln(|\bar{t}|/m_e^2)/\pi = 0.042$ from Section 4 of Ref. [11] and the safety factor of 2 from Ref. [9], together with the largest higher-order effect in Table B.5.3, 0.081%, as $0.081\% \times \gamma \times 2 \simeq 0.007\%$. The last column shows that the size of the pure weak corrections, as implemented within the $\mathcal{O}(\alpha)$ YFS exponentiation scheme, is at a level of 0.01% below and at M_Z and increases up to ~0.04% above M_Z. We may use the same factor as we did for the higher-order corrections to estimate the size of the missing higher-order pure weak corrections in `Bhwide` as ~0.003%. Altogether, by adding the two estimates of its massing effects, we obtain a conservative estimate of 0.01% for the physical precision of `Bhwide` to justify our remarks concerning the error in line (e) of Table B.5.2 that would result from the combination of the prediction of `BHLUMI` and that of `Bhwide` for this contribution.

In line (f) in Table B.5.2, we show the estimate of the error in the up–down interference between radiation from the e^- and e^+ lines. Unlike in LEP1, where it was negligible, for the FCC-ee, this effect, calculated in Ref. [15] at $\mathcal{O}(\alpha^1)$, is ten times larger and must be included in the upgraded `BHLUMI`. Once this is done, the error estimate shown in line (f) for the FCC-ee is obtained [11].

This brings us to the issue of the technical precision. In an ideal situation, to get the upgraded `BHLUMI`'s technical precision at a level of 10^{-5} for the total cross-section and 10^{-4} for single differential distributions, one would need to compare it with another MC program developed independently, which properly implements the soft-photon resummation, LO corrections up to $\mathcal{O}(\alpha^3 L_e^3)$, and the second-order corrections with the complete $\mathcal{O}(\alpha^2 L_e)$. In principle, an extension of a program like `BabaYaga` [23–25], which is currently exact at NLO with a matched QED shower, to the level of NNLO for the hard process, while keeping the correct soft-photon resummation, would provide the best comparison with the upgraded `BHLUMI` to establish the technical precision of both programs at the 10^{-5} precision level.[†] During the intervening time period, a very good test of the technical precision of the upgraded `BHLUMI` would follow from the comparison of its results with EEX and CEEX matrix elements; the basic multiphoton phase space integration module of `BHLUMI` was already well tested in Ref. [27] and such a test can be repeated at an even higher precision level.

In summary, we conclude that, with the appropriate resources, the path to 0.01% precision for the FCC-ee luminosity (and the ILC luminosity) at the Z peak is open via an upgraded version of `BHLUMI`.

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