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Profile Synthesis of Prismatic-Prismatic Variable Joints for Use In Reconfigurable Mechanisms

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PROFILE SYNTHESIS OF PRISMATIC-PRISMATIC VARIABLE JOINTS FOR USE IN RECONFIGURABLE MECHANISMS

by

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ABSTRACT

PROFILE SYNTHESIS OF PRISMATIC-PRISMATIC VARIABLE JOINTS FOR USE IN RECONFIGURABLE MECHANISMS

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Reconfigurable mechanisms provide increased flexibility in machine design and can be used in a variety of applications. Variable joints in reconfigurable mechanisms allow for machine designs that better optimize space and resources. This thesis uses a configuration space analysis to determine the motion profiles of Prismatic-Prismatic higher variable joints. Prismatic-Prismatic variable joints have never been analyzed or synthesized. In addition, no variable joints of any type have been considered with two-dimensional contact points. Variable joints enable the creation of Type II Mechanisms with Variable Topology, mechanisms whose topology changes due to changes in joint geometry. The goal of this synthesis is to synthesize Prismatic-Prismatic variable joints with one and two-dimensional contacts, outline their designs, and determine the design restrictions for each variation.

ACKNOWLEDGMENTS

Ryan D. Callahan, B.S.

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CHAPTER 1

Introduction

1.1 Motivation

Reconfigurable mechanisms are mechanisms that are able to change their topology. This can allow for a single mechanisms to serve multiple functions, creating a more efficient use of weight, space, and material.

Reconfigurable mechanisms can reduce the cost of machinery for a company in two ways: capital costs and operating costs. Actuators and the associated drives are often one of the most expensive initial costs of a machine or robot depending on a need for greater force, speed, acceleration, and/or accuracy. The use of reconfigurable mechanisms can reduce the number of actuators depending on the needed motion paths. If the actuator being eliminated would have needed to be moved itself, it can also reduce the force needed from that actuator, further reducing the cost. Not only are actuators one of the more expensive parts when purchasing a machine initially, they directly lead to the majority of the machine's operating costs in the form of electricity. For large manufacturing machinery running 24 hours per day, these costs can add up to significant sums.

Every additional actuator leads to an increase of complexity which also adds costs. Reducing the complexity of machinery reduces machine downtime. This downtime leads to lost productivity, especially for machines expected to run constantly. From a physical layout, designing the machinery around actuators can become complex depending on the kinematic arrangement of the machine, especially if the actuator needs to be attached to a moving axis itself. From a programming perspective, the complexity for each additional axis is magnified. This complexity directly leads to an increase in the difficulty to service the machine. Physical damage to an actuator can often be difficult to diagnose and many times can be mistaken for an issue in the programming. Each actuator also has a drive, which in themselves are complex components, further adding failure points for the machine.

Reconfigurable mechanisms can create more flexible machinery, providing industry with more robust tools that are more adaptable to changes. More robust machinery also allows the machinery to adapt to various product lines and different needs over time, making them more capable over the long term. Problems with physical joints are easier to diagnose and repair, especially for millwrights trained in repairing traditional mechanical machine components. These reasons combined with the lower capital and operating costs made possible by reconfigurable mechanisms allows for companies to expand their use of automation. Automation allows companies to reduce costs, allowing for increased profit margins.

Automation is increasingly becoming more and more a part of life, especially in manufacturing. The ever-growing need for automation has led to a need for more robots capable of performing complex motion. For robots to do more tasks, robot manufacturers typically add more degrees of freedom and, in turn, more actuators. Reconfigurable robots and mechanisms can simplify machines by allowing for fewer actuators to create complex motion paths which typically require the use of larger degree of freedom machines.

Reconfigurable mechanisms have the capability to revolutionize robotics and machinery; however, the field remains immature with significant opportunities for advancement to achieve its potential. A major part of this gap is in the synthesis of variable joints. Type II Mechanisms with Variable Topology are reconfigurable mechanisms whose geometry allows for a change in topology, which requires a variable joint. Prismatic-Prismatic variable joints are a type of variable joint that have yet to be systematically synthesized, a vital first step before its implementation and application. A thorough synthesis of Prismatic-Prismatic variable joints requires defining the desired motion path and outlining the design constraints and restrictions to achieve that desired motion path. The effects of non-ideal joint geometry and physical implementation need to be analyzed further to provide design constraints. Understanding the design constraints of these variable joints is crucial to being able to design them and implement them into

reconfigurable mechanisms. This work advances the design of variable joints as well as reconfigurable mechanisms as a whole, a rapidly expanding field.

This thesis will contribute a complete analysis of Prismatic-Prismatic variable joints. This thesis will analyze Prismatic-Prismatic variable joints consisting of higher pairs with both one-dimensional and two-dimensional circular contact points. This analysis will result in a synthesis of Prismatic-Prismatic variable joints. Prismatic-Prismatic variable joints will also be analyzed in conjunction with Revolute-Prismatic variable joints to determine their feasibility.

1.2   Thesis Overview

Chapter 2 overviews the pertinent previous work relevant to this thesis. This previous work includes the foundation of mechanical joints, mechanisms, and how they are synthesized. Previous work on reconfigurable mechanisms are also outlined along with how this work relates to the work of this thesis. Works on configuration space analysis are presented as these provide a tool for the synthesis performed in this thesis.

In Chapter 3, the configuration space method used for the synthesis throughout the thesis is outlined. Explaining the methodology used helps illustrate

how the synthesis of variable joints is done, as well as illuminating why the configuration space method was used.

In Chapter 4, the methodology outlined in Chapter 3 is used to validate prior research of valid prismatic higher pair configurations. These configurations are the basis of this research; validating these configurations allows for the synthesis of Prismatic-Prismatic variable joints performed in this thesis.

In Chapter 5, a configuration space method is used to investigate Prismatic-Prismatic variable joints while using ideal one-dimensional contact points. This analysis produces the design constraints of these joints.

In Chapter 6, the configuration space method is repeated for Prismatic-Prismatic variable joints while using circular contacts. The effect of circular contacts and the undesirable motions they can cause will be quantified to determine whether or not the variable joint has the potential to jam.

In Chapter 7, the Prismatic-Prismatic variable joints will be combined with the constraints of Revolute-Prismatic variable joints. Chapter 7 will determine the feasibility of these joints.

In Chapter 8, the contributions to the field of this thesis will be reviewed and outlined.

CHAPTER 2

Literature Review

The goal of this thesis is to synthesize Prismatic-Prismatic variable joints for their design and implementation into reconfigurable mechanisms. Before synthesizing variable joints, it is important to identify the need for the synthesis. A proper literature review not only identifies holes in the current state of the art but shows possible research avenues for the work. In addition to identifying need, it is vital to establish the foundation methodologies that can be used to perform the synthesis.

A proper understanding of classical joint design is the foundation of variable joint design and many of the same principles are applicable in this thesis. It is also important to paint a picture of what reconfigurable mechanisms are, how they are analyzed, and how variable joints allow for reconfigurable mechanisms to be created. Previous works, specifically with regards to variable joints and mechanisms that use them, are necessary as they are the most closely related precursors to this research and have many parallels between their synthesis. Finally, the work outlining the methodology for configuration space analyses, the primary tool used for the synthesis of Prismatic-Prismatic variable joints, is presented.

2.1 Classical Joint Design and Classification

Reuleaux [9] created the foundation to modern kinematics. With respect to this research, the most important aspects of his work were the creation of the notion of kinematic pairs, i.e., two individual machine elements that interact with each other. A machine element in this context is a broad term for the elementary components of a machine, like joints, bearings, gears, etc. These pairs are then further classified by Reuleaux into lower pairs and higher pairs. For a pair of elements to be considered a lower pair, they must have surface to surface contact between them as opposed to a point contact; the second element must be carried upon by or enveloped by the other. These works on classical joint design and classification are the foundation of this thesis.

This designation of lower pairs has served as the paradigm for the design of robots for numerous years. The 6 lower pairs (revolute, prismatic, cylindrical, helical, spherical, and planar) are fundamental to modern mechanics and are used consistently in classical mechanism analysis and synthesis. The preliminary analysis was done by Reuleaux [9] and Kennedy [1] in the 19th century and continues to be used in modern analysis of reconfigurable mechanisms by people like Tsai [14].

Higher pairs do not require the surface contact that lower pairs need; they merely require point contacts. Higher pairs are extremely important as they open

new design paradigms of reconfigurable mechanisms, especially with regards to reconfigurable mechanisms with variable joints.

2.2   Joint Classification

2.2.1   Lower Pairs

According the Reuleaux [9], closed pairs must follow three rules:

1. The two elements must have surface to surface contact.

2. The two elements must be geometrically identical.

3. One element restricts the motion of the other link to the required motion.

Reuleaux used this definition to outline the six possible types of lower pair joints: revolute, prismatic, helical, spherical, planar, cylindrical. These are denoted by R, P, H, S, E, and C, respectively.

2.2.2   Higher Pairs

Reuleaux also defined higher pair joints. For joints to be higher pairs, they must follow two rules:

1. The two elements must have a point or line contact.

2. The two elements must restrict relative sliding.

In this thesis, only higher pair joints will be analyzed due to the flexibility they have in design of variable joints; flexibility that is particularly advantageous to variable joints. A lower pair's requirement for surface contact limits the ability to design complex variable joints.

2.2.3   Prismatic Higher Pair Classification

Slaboch [2] created a notation for the types of higher pair prismatic and revolute joints that are possible without redundancies, configurations with no unneccesary contacts. A higher pair can be classified by a notation of the format X_u. X denotes the type of joint it is (e.g., R for revolute, P for prismatic, etc). The u subscript indicates what version of the joint it is, as defined by Slaboch [2]. For a subscript of 1, the contacts are external. For a subscript of 2, the contacts are internal. For a subscript of 3, the contacts are a combination of external and internal. A type 3 joint is valid for a revolute higher pair but is redundant for a prismatic higher pair. Slaboch [2] outlined the versions of revolute and prismatic higher pairs. For prismatic higher pairs, there are two versions, P_1 and P_2. These two versions restrict a translating link to translational motion without any redundant contacts.

A P_1 type joint is shown in Figure 2.1. A P_1 higher pair requires only two

contact bodies to maintain translational motion, denoted in Figure 2.1 as c_1 and c_2. These contacts are placed on the outside of a stationary link consisting of two horizontal lines. In the Figure 2.1, the stationary link is represented by blue lines. The two circles are the "point contacts" that are connected via the dotted line and translate horizontally. Whether the point contacts are considered stationary or the horizontal lines are is arbitrary as motion is relative.

A P_2 type joint can be seen in Figure 2.2. A P_2 higher pair requires three contacts to maintain translational motion. These point contacts are placed internal to the stationary link. These constraint bodies must be arranged such that the point isolated on one side (c_3 in this case) is attached between the two on the opposite side (c_1 and c_2 in this case). As with a P_1 type joint, whether the circular contacts or lines are stationary is arbitrary.

The notation used to designate variable joints were defined by Slaboch [2]. For variable joints, they are denoted as $X_u \, Y_v$, combining the notation of higher pairs. X and Y denote the type of higher pair while the u and v denote the version of the higher pair.
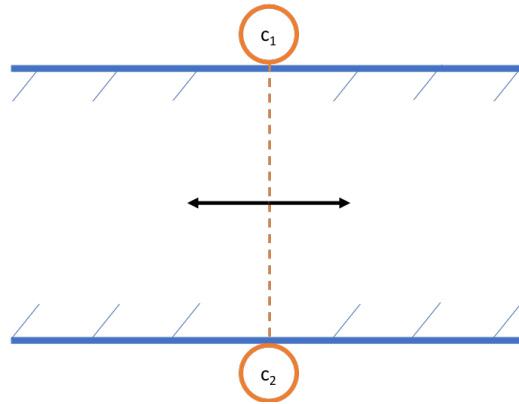
Figure 2.1: Diagram of P_1 higher pair, the dashed line designates that the contacts are fixed relative to each other.
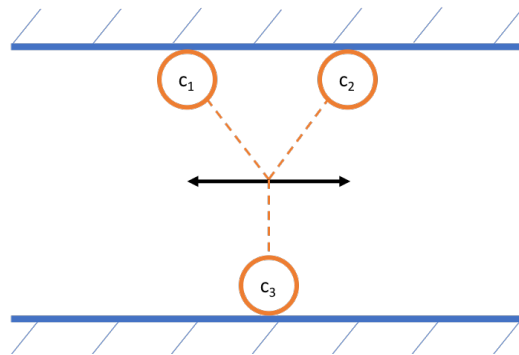
Figure 2.2: Diagram of P_2 higher pair, the dashed line designates that the contacts are fixed relative to each other.

2.3   Classification of Reconfigurable Mechanisms

Reconfigurable mechanisms can be categorized into three categories: Metamorphic

mechanisms as defined by Dai and Jones [4], Kinematotropic mechanisms as defined

by Wohlhart [18], and Mechanisms with Variable topology as defined by Kuo [5]. These are further explained below.

- Metamorphic Mechanism: A mechanism whose the total number of effective links changes as the mechanism moves from one configuration to another or a singular condition makes it behave differently [4]. As the mechanism moves through its motion path, some of the links disengage from the moving link allowing for a differing motion path.

- Kinematotropic Mechanisms: Mechanisms that, in passing a singular position in which a certain transitory infinitesimal mobility is attained, permanently change their global mobilities. A Kinematotropic Mechanism intrinsically has a singularity designed into it, and can pass through that singularity, creating a different motion profile.

- Mechanism with Variable Topology (MVT): A mechanisms with variable topology is a mechanism whose topology changes during operation. MVTs can be classified into three main types according to Shieh, Sun, and Chen [15] :

  1. MVTs that change topology due to an intrinsic constraint (Kinematotropic or Metamorphic).

  2. MVTs that change topology due to joint geometry.

3. MVTs that change topology due to an external constraint

(Metamorphic).

Type II MVTs are the most important to this research as they require a variable joint to be made possible. A variable joint is a combination of two higher pair joints. Synthesizing more types of variable joints allows for more Type II MVTs to be designed.

2.4 Mechanism Analysis

Most analyses of reconfigurable mechanisms is based on methods of analysis of classical mechanisms. Classical mechanisms have been rigorously analyzed since the 19th century and the analysis of these mechanisms has continued since. With the advent of reconfigurable mechanisms, the application of the same tools developed for classical mechanisms has been adapted and applied to reconfigurable mechanisms.

2.4.1 Classical Mechanism Analysis

Gogu [1] (and similarly by Waldron [17][16]) looked at the different methods for analyzing mobility of classical, non-reconfigurable, mechanisms. Gogu analyzes a single system by using thirty-five different mobility calculation equations. All of these equations are somewhat derived from Gruebler's equation. This work is more

centered on the analysis of mechanisms in terms of classification and determining degrees of freedom for classical mechanisms. While Gogu's work doesn't directly apply to the work of this thesis, it does illustrate a common approach for analyzing mechanisms as a whole.

Tsai [14] used matrices to enumerate classical kinematic mechanisms, using classical lower pair (non-variable) joints. He analyzed a series of classical mechanisms by representing them with graph representations (i.e., mechanism schematics) as well as matrix representation. These mechanisms span from gear trains to automotive mechanisms to robotic mechanisms. And while this work is important, and can provide significant insight into the analysis of kinematic mechanisms, this work is not usually directly applied to the synthesis of variable joints.

2.4.2   Reconfigurable Mechanism Analysis

Dai and Jones [4] adapted Tsai's work to reconfigurable mechanisms. Their matrix analysis is similar to Tsai's but is able to specifically represent reconfigurable mechanisms, in this case, metamorphic mechanisms. While this work analyzes reconfigurable mechanisms, they only investigate metamorphic mechanisms, which do not require variable joints.

Dai and Jones' work was continued further by Yan and Kuo [19] and Lan

and Du [6], for different types of reconfigurable mechanisms, namely MVTs. Being able to enumerate MVTs demonstrates how mechanisms may need variable joints to exist. Being able to quantitatively determine a need for variable joints is important, but these works do not look at variable joints themselves, nor the synthesis of these joints.

Slaboch and Voglewede [13] showed the limitations of the analysis approaches with their development of Mechanism State Matrices, a classification tool of reconfigurable mechanisms. However, this work does not specifically look at the synthesis of variable joints.

All of the work into the analysis of reconfigurable mechanisms are important precursors to the research of this thesis as they help demonstrate the need for variable joints as well as the tools to analyze them. Reconfigurable mechanism analyses help show limitations of traditional mechanisms as well as show how reconfigurable mechanisms can be designed using variable joints.

2.5   Variable Joint Synthesis and Implementation

2.5.1   Variable Joint Synthesis

The majority of the work on variable joints has only recently been performed. Slaboch [2] designed the motion profiles of prismatic and revolute higher pair joints

and outlined the different configurations for both prismatic and revolute joints. These different configurations varied the locations of the moving point contacts relative to the stationary link. These configurations have the minimum number of constraint bodies necessary to form the joint without redundancies, defining the fundamental designs for these joints. These configurations and motions paths were then applied to Revolute-Prismatic $(R_u P_v)$ variable joints. The combinations of the configurations of each type of joint were described along with their design constraints, an example being link interference as the joint transitioned from one type to the other. This research did not investigate prismatic-prismatic variable joints or the effects of approximating obstacles as point contacts, but did provide the fundamentals of this research and a template for how a proper synthesis of variable joints should be performed.

2.5.2   Mechanism Synthesis

Malak [8] synthesized a Revolute-Prismatic variable joint and implemented it into a mechanism. Malak designed and built a $RRRR$-$RRRP$ mechanism, or a mechanism that uses a variable joint that is $RRRR$ in one configuration, and then a $RRRP$ in another configuration. This mechanism made use of a Revolute-Prismatic variable joint, not a Prismatic-Prismatic variable joint.

Buchta [3] performed a survey of industries to determine a Type II MVT

that could be of practical use in industry, and designed a mechanism using a Revolute-Prismatic variable joint; this work took a similar design approach to Malak. Both Malak and Buchta focused on the design of Type II MVTs and the use of Revolute-Prismatic variable joints in these mechanisms, but not on the synthesis of variable joints, Prismatic-Prismatic variable joints specifically.

Currently, no work regarding the synthesis of Prismatic-Prismatic variable joints has been performed. This thesis analyzes and synthesizes the motion profile and geometry of prismatic-prismatic higher order variable joints for their use in reconfigurable mechanisms, specifically Type II MVTs.

2.6   Configuration Space Analysis

Most of the work on synthesis of variable joints has used a configuration space analysis. A configuration space is simply a two or three-dimensional area or volume that two or more rigid body objects can exist within [12]. Generally, all objects but one are moved to provide a better understanding for the mobility of the object. If multiple objects need to move relative to each other, multiple analyses can be performed and the locations at which the configuration spaces agree are the overall valid configurations for the mechanism.

Rimon and Burdick [10] [11] provided the theoretical basis behind a

configuration space analysis, specifically with regards to creating matrices characterizing two object's surfaces and their mobility with respect to each other. This work is applicable to the synthesis of variable joints as it provides a tool for this thesis.

Sacks [12] performed similar work to Rimon and Burdick, but the work was significantly more thorough in the practical implementation of configuration space approach to classical mechanisms. He used a configuration space analysis to analyze the contact of features in various mechanisms, both planar and spatial. An analysis of how tolerances relate to the configuration space method was also performed. Although he mentions higher and lower pair joints, he does not specifically use the configuration space analysis on them, nor on variable joints. Due to the more practical methodology for configuration space analysis, Sacks provides a useful tool for the synthesis of the motion profile of a prismatic-prismatic variable joint.

2.7   Summary

As illustrated through the literature review, the motion path of Prismatic-Prismatic variable joints has yet to be synthesized. Since Reuleaux outlined the basics for joint and mechanism design, a significant amount of work has been done into the analysis of classical mechanisms and reconfigurable mechanisms. Slaboch pushed the study of variable joints forward through his motion profile synthesis of

Revolute-Prismatic variable joints. Malak and Buchta took the motion profiles of Revolute-Prismatic variable joints and applied them to their synthesis of Type II MVTs. All of this work has led up to the synthesis of Prismatic-Prismatic variable joints, but none have accomplished this synthesis.

CHAPTER 3

Methodology

This work will build upon prior research by Slaboch [2] with the use of configuration spaces and geometry. A configuration space is a collection of locations at which two or more (planar) objects can physically exist based on rigid body mechanics, essentially meaning the two objects do not overlap or deform. These locations then illustrate the potential motion path for the free object relative to the stationary objects. In the case of a joint, the configuration spaces of all of the obstacles are combined, and the union is the locations at which the mechanism can physically exist. This chapter will outline the two methodologies used in this research to synthesize Prismatic-Prismatic variable joints.

3.1 Background

The configuration space analysis of Prismatic-Prismatic variable joints is performed with two different approaches: a numerical approach and a geometric approach. In the numerical approach, an algorithm written in Matlab determines if two objects are in contact with each other. The program does this by breaking the objects into line segments and then determines if any line segments overlap. If they do, it
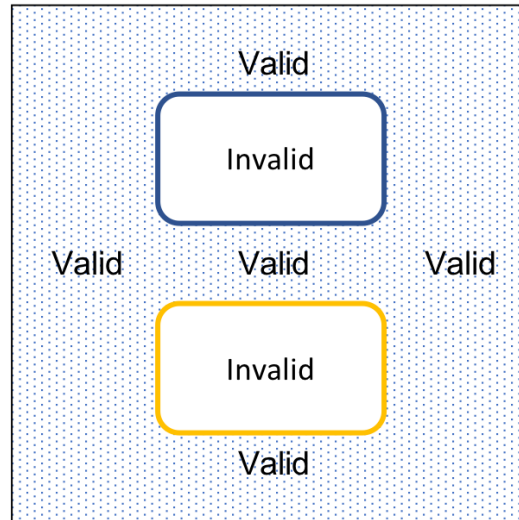
Figure 3.1: Example of how a configuration space analysis works, showing the locations of valid configurations

determines if the overlap is small enough to be able to assume surface contact. This procedure is repeated for a grid of poses to find the valid configuration space for the contact between each object. The sweep of test points is repeated for each object. The locations where the configuration spaces of each object are in union are the valid configurations as seen in Figure 3.1. The objects can then be rotated and the process repeated to get a complete picture of the valid configurations.

In the geometric approach, the geometry of the two objects is used to determine the shape of the configuration space, the locations at which one moving object can exist relative to a stationary one. The stationary object of a prismatic higher pair can be represented by a rectangle and the translating objects can be represented as circles. In configuration space, if a rectangle is interacting with a

circle of radius r, the resulting configuration space will be a rectangle with rounded

corners [7]. The configuration space's height and width will be the height and width

of the rectangular object plus $2r$, and the rounded corners of the configuration space

will be of radius r. An example of this analysis will be shown later in Figure 3.5.

The two approaches for determining the configuration space of a joint can be

chosen depending on the analysis. A numerical approach is better at determining

the configuration space of objects with more complex geometry. It is more flexible

than a geometric approach, but is more difficult to program and is an

approximation (i.e., the results are not exact). If the geometries of the two objects

are simple, a geometric approach is better. The geometric approach yields an exact

result when looking at simpler situations, but is difficult to create 3-D volumes of

configurations often caused by two-dimensional objects.

3.2 Numerical Approach

A numerical approach to determine the configuration space and the resulting

motion of a variable joint can be found by approximating both interacting objects

as polygons. The configuration space can then be determined by finding the

intersection of the polygons as they move in the plane. For the numerical approach,

a Matlab program to determine if two polygons intersected was developed as there

are no known built-in Matlab functions to determine when a circle contacts or

intersects a polygonal object. This program will be referred to as the Numerical

Program and code for it can be found in Appendix A.

The Numerical Program determines the configuration space of various

prismatic higher pairs. The program loops through a series of poses (position and

orientation) to create a two or three dimensional configuration space for the joint.

For this thesis, the x direction is the direction of the joint's first translation, the y

direction is perpendicular to the x in the plane of the translation, ψ is the

(undesired) rotations of the moving object, and θ is the angle of translation for the

second portion of the joint.

The process is summarized in the flowchart in Figure 3.4. The general

program flow is as follows:

1. The two polygons representing the stationary objects and moving object are

   broken into line segments between each of the nodes of the polygons.

2. The polygon representing the stationary object is placed at various

   configurations of x and y.

3. All of the line segments of both objects are analyzed using a built in

   mathematical formula to determine if any two lines of the polygons

   representing the object intersect, and if so, where.

4. If the objects intersect, they intersect at two points. The distance between the two intersection points is determined.

- If this distance is small based on the dimensions of the two objects and the desired accuracy, the configuration is approximated as surface contact, and the configuration is deemed to be valid.

- If the distance is large, the polygons are said to be overlapping and the configuration is deemed invalid.

- If the objects do not interact, it is considered valid and the stationary object is not in contact with the moving object.

5. Steps 1 through 4 are repeated for all of the bodies of the stationary object being analyzed.

6. The locations deemed valid in the configuration spaces of all of the constraints are the locations at which all constraints can coexist with the stationary link. The boundaries of the invalid space are the locations where the objects are in contact, the purpose of the analysis.

- If the valid points of the configuration space form a contiguous line of points, the line is a motion path for the joint.

- If there are no contiguous valid points, the joint in question is not a
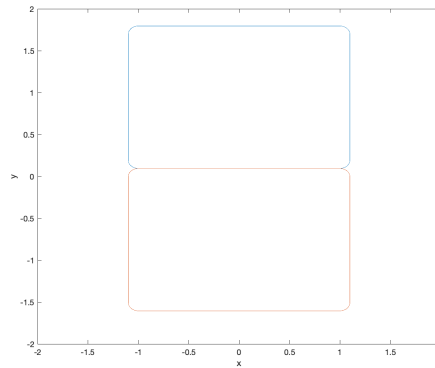
Figure 3.2: A series of valid configuration points with the inner object at 0°

functioning variable joint for a specific ψ. (An example of this can be

seen in Figures 3.2 and 3.3.)

7. Steps 1 through 6 are repeated for various rotations, ψ, of the moving object

being tested, creating a three-dimensional mesh of points showing valid

configurations based on the moving link's location in the x, y, and ψ.

3.3 Geometric Approach

The geometric approach was implemented by using the geometry of the moving and

stationary objects to determine where the configuration spaces union, showing a

motion path. The approach determines the shape and dimensions of a configuration

space by the dimensions and shapes of the objects that are interacting. This
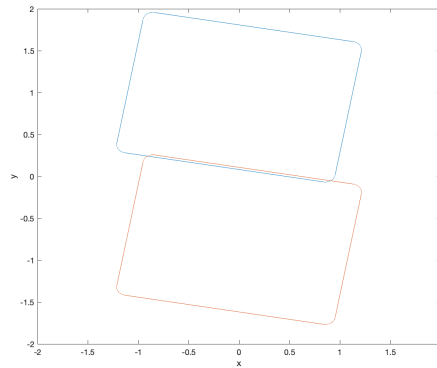
Figure 3.3: A series of valid configuration points with the inner object at -10°

approach is particularly useful for simple geometries, like those performed in

Chapter 4. An example of this type of analysis can be seen in Figure 3.5.

The geometric approach specifically relates to analyzing prismatic-prismatic

variable joints with circular objects. This approach uses the known geometry of the

stationary object, and then creates boundaries the exact same distance from the

stationary object as the radius of the circular objects. If the boundary has an angle

greater than 180°, the boundary becomes rounded to the same radius as the contact

points. This boundary creation can be seen in Figure 3.6.

While the geometric approach uses the geometries of the objects to

determine valid configurations, it is generally combined with the numerical

approach. The translating object is tested at various points, similarly to the

numerical approach, and if any of the centers of the contacts of the translating
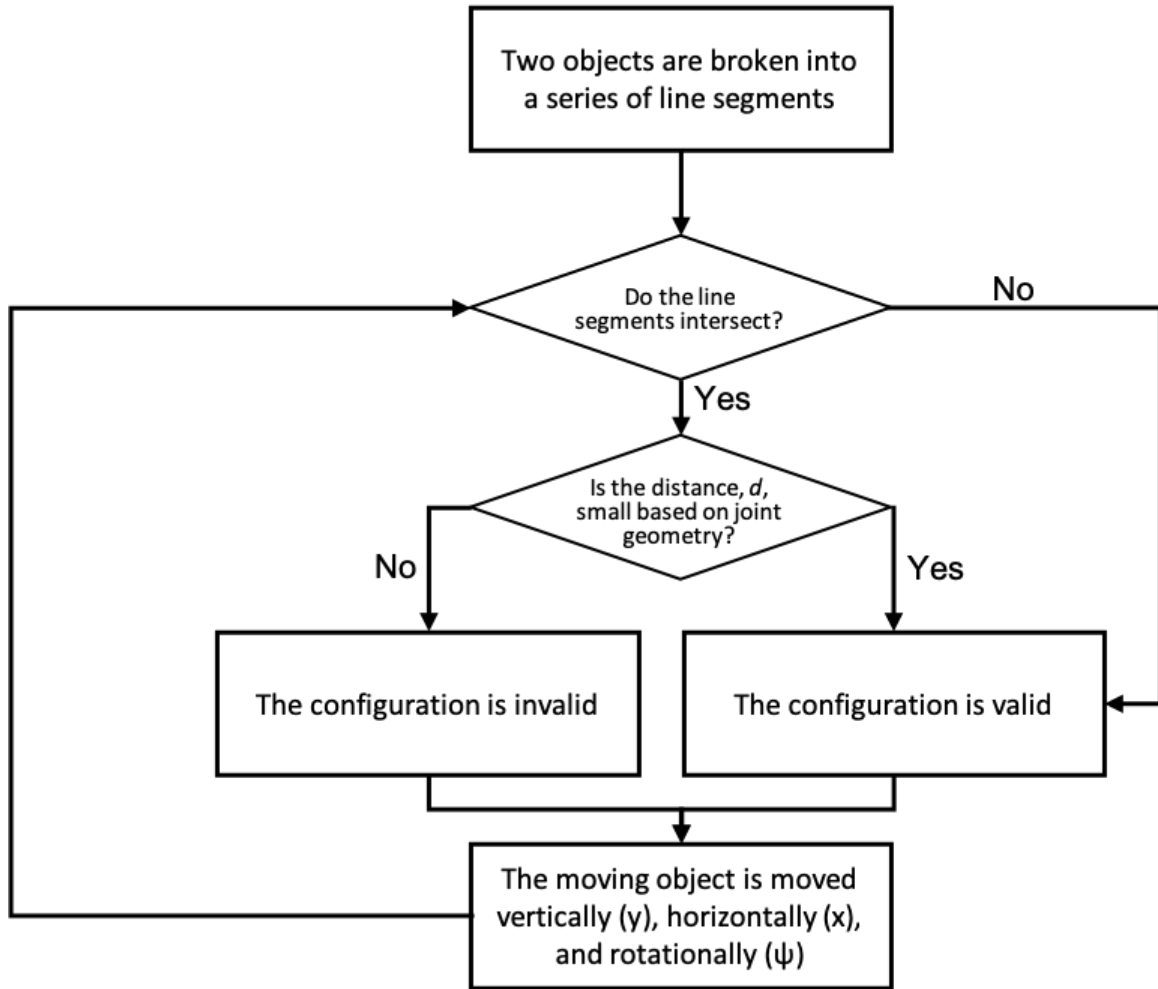
Figure 3.4: Flow chart of the numerical program

object are within the boundaries, the configuration is invalid. An example of both valid and invalid configurations is shown in Figures 3.7 and 3.8 respectively.

The geometric approach with the numerical addition is a non-exact approach similar to the numerical approach. The results are only at the poses tested. As the number of poses tested approaches infinity, the amount of error approaches zero.
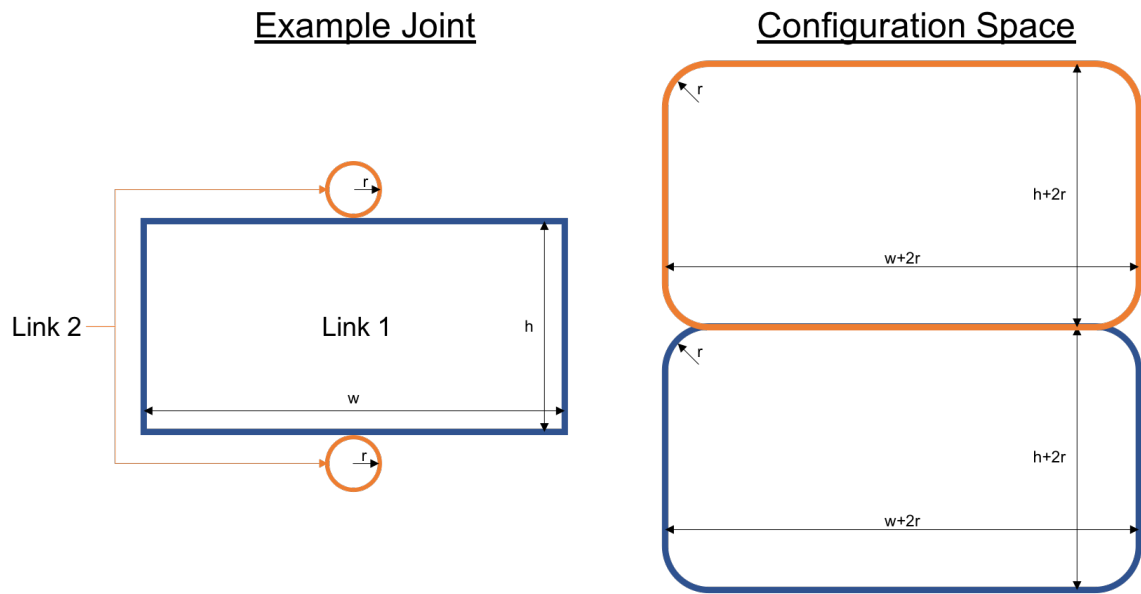
Figure 3.5: An example P_1 higher pair with its corresponding configuration space boundaries; all space outside these boundaries is valid

3.4   Summary

This chapter overviewed the use of configuration space to analyze prismatic-prismatic variable joints in two approaches: the numerical and geometric approach. For the numerical approach, the determination of valid configurations is approximate, as the number of test points is not infinite. This approach is more exact as opposed to the numerical approach, but is less capable of extracting data and is difficult to iterate for multiple configurations like the numerical approach.

Two configuration space approaches (i.e., numerical and geometric) are combined to best analyze the motion profiles of the joints. The configuration space
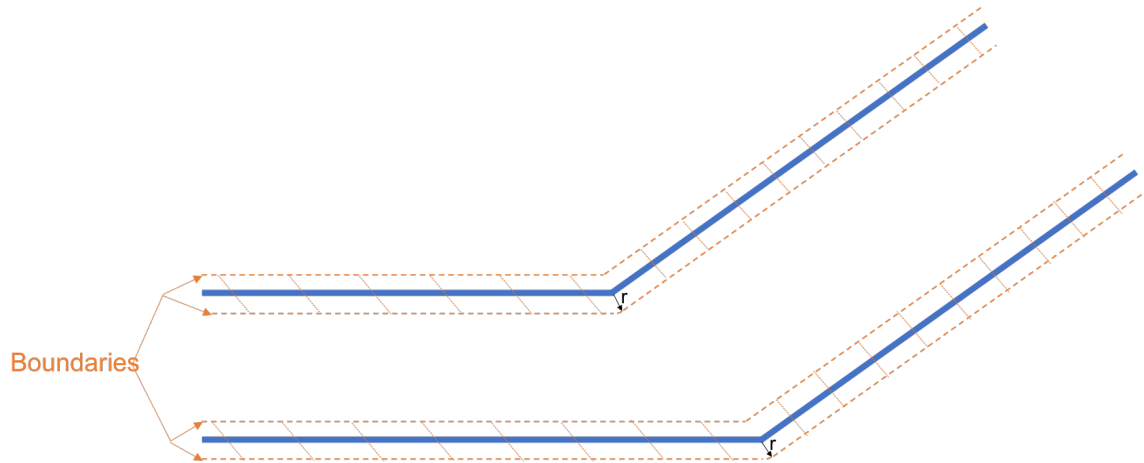
Figure 3.6: A stationary object for a P_2P_2 variable joint with the boundaries for a geometric configuration space analysis
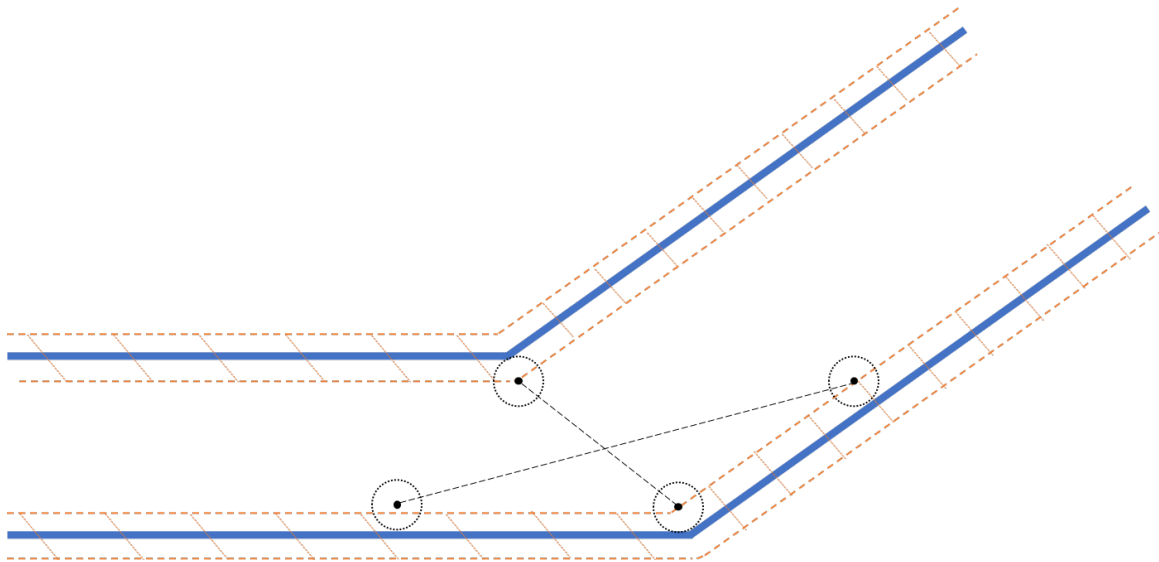
Figure 3.7: An example of a valid configuration in the geometric approach combined with the numerical approach

analysis allows for the determination of the motion path of the translating object in the case of a Prismatic-Prismatic variable joint relative to the stationary object and
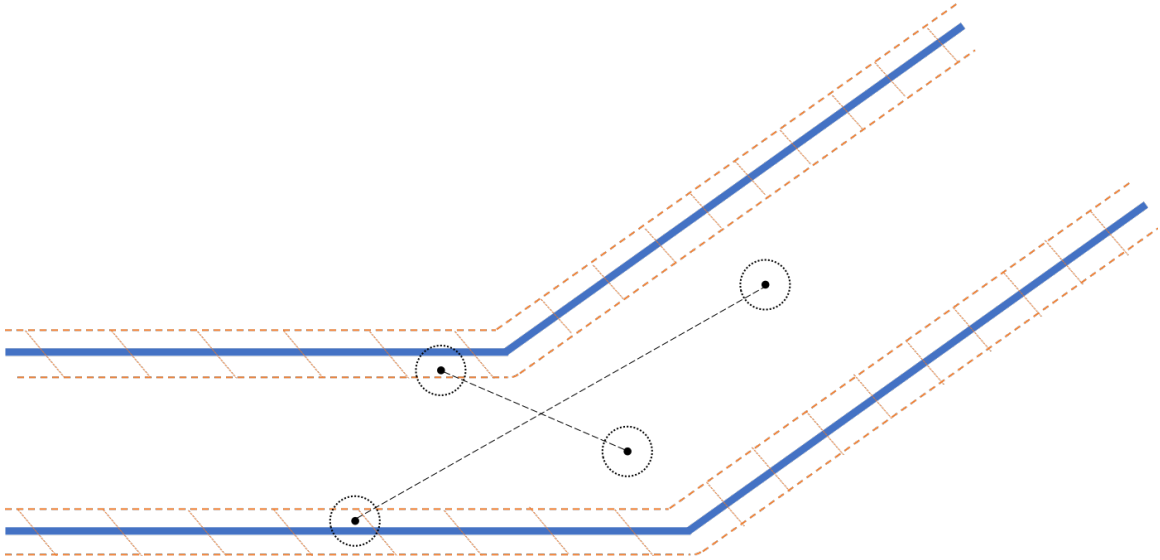
Figure 3.8: An example of an invalid configuration in the geometric approach combined with the numerical approach

thus synthesize a joint. This motion path can then be analyzed, and its limitations and restrictions can be outlined. The two approaches are important to be able to extract any potential undesired motions when the variable joints are considered with circular contacts. The two approaches will allow for Prismatic-Prismatic variable joints to be analyzed, understood, and implemented.

CHAPTER 4

Verification of the Synthesis of Prismatic Higher Pairs

A prismatic higher pair is a slider joint created by two objects called links, one stationary, and the other restricted to translational motion by the stationary link. This stationary link only touches each contact of the moving link at a single point, i.e., no surface contact. To be able to synthesize the motion profile of prismatic-prismatic variable joints, it is important to first validate the configuration space analysis of prismatic higher pairs before application to variable joints. Previous work analyzed various configurations of prismatic higher pairs with various numbers of contact points to determine the minimum number of contacts for each configuration.

4.1   General Assumptions

Both P_1 and P_2 prismatic higher pairs as outlined by Slaboch [2] will be re-analyzed to validate this work as well as the methodology used. A diagram of a P_1 higher pair can be seen in Figure 2.1. The rectangular link shown is assumed to be the moving link. While in actuality it can be two infinite parallel lines, it is represented as a polygon and used as the moving link to provide more intuitive results. It does
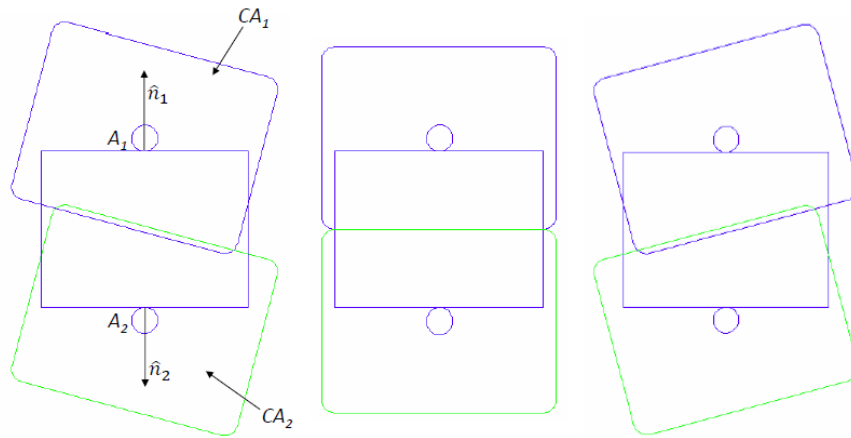
Figure 4.1: Results of Slaboch's configuration space analysis of P_1 joints [2]

not matter which link is assumed to be moving because the links only need to be moving relative to each other.

The dimensions of the links are arbitrary and chosen for convenience. In these figures, the width of the rectangular moving link is 1.92 units, its height is 1.45 units and the radii of the stationary link's circle contacts are .12 units. For the P_1 analysis, the contacts are centered at $(0,.845)$ and $(0,-.845)$. For the P_2 analysis, the contacts are centered at $(-.4,.6)$, $(.4,.6)$, and $(0,-.6)$. These dimensions were chosen to best recreate Slaboch's work [2].
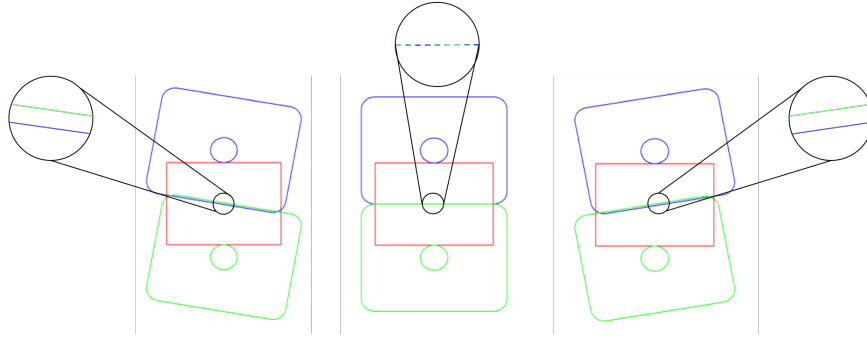
Figure 4.2: Validation of Slaboch's configuration space analysis of P_1 joints

4.2 Configuration Space Analysis of P_1 Joint

A geometric approach was first taken, then confirmed with a numerical approach. The results of Slaboch [2] are shown in Figure 4.1 which shows a prismatic joint with a rectangular moving link and two fixed stationary circular contacts to represent the fixed link. The rectangles with rounded corners are the configuration spaces of the rectangular moving link and its interaction with each stationary link's contacts.

With the rectangular link not rotated (center in Figure 4.2), there is a clear line where the configuration spaces line up without overlapping. This line is the line of motion for the moving link, showing that the joint restricts the motion to pure translation with no rotation, (i.e., a properly functioning prismatic joint).

With the rectangular moving link rotated to either -10° or 10° (left and right in Figure 4.2), the analysis indicates an invalid configuration due to the two

configuration spaces overlapping. It indicates that the moving link would have to obstruct the stationary link's contact.

The geometric and numerical method performed in this thesis, shown in Figure 4.2, found identical results to previous research, validating both methods. When the moving link is not rotated (center), there is a clear line where the configuration spaces of each constraint coincide but do not overlap, indicating that the moving link is allowed to translate along this line. With the moving link rotated, the configuration spaces overlap, showing that it is not a valid configuration. Not only does Figure 4.2 corroborate Slaboch's analysis of P_1 higher pair joints, it also serves to validate the methodologies used and outlined in Chapter 3.

4.3   Configuration Space Analysis of P_2 Joint

The procedure performed in Section 4.2, was repeated for a P_2 type higher pair with the same dimensions of stationary link contacts and rectangular moving link as used for the P_1. In this pair, the translational motion is restricted by three internal contacts on the stationary link as opposed to the two external contacts of the P_1.

The results of the P_2 higher pair analysis done by Slaboch is shown in Figure 4.3. Just as with the P_1 joint analysis in Section 4.2, the moving link translates horizontally as designed. With the case of the P_2, when the moving link is rotated,
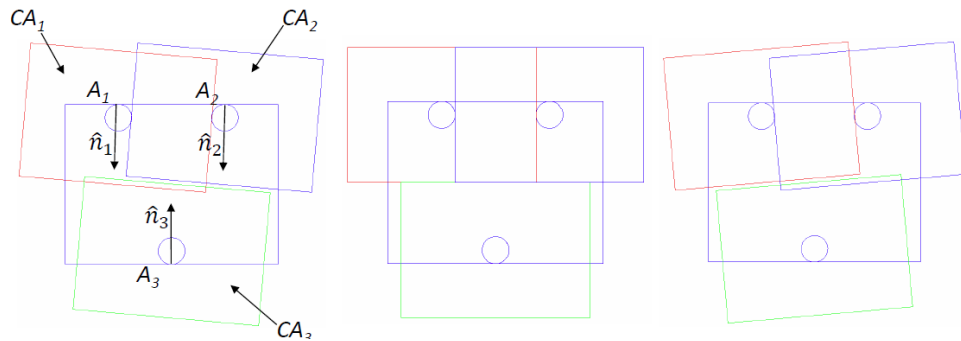
Figure 4.3: Results of Slaboch's configuration space analysis of P_2 joints [2]

it loses contact with one of the stationary link. When it is rotated to $10°$(left), the

A_1 and A_3 contacts' configuration spaces (CA_1 and CA_3) overlap, while the A_2 and

A_3 lose contact with each other. The moving link cannot exist at this configuration

due to the overlap of the A_1 and A_3 configuration spaces. When the moving link is

rotated to $-10°$ (right), the A_2 and A_3 contacts' configuration space overlap,

indicating an invalid configuration.

The results using the methodology in Chapter 3 seen in Figure 4.4 show the

configuration space analysis of Slaboch applied to a P_2 higher pair. The results for

the P_2 higher pair are the same as the P_1 higher pair; horizontal translation is

possible at $0°$, but two of the configuration spaces overlap at both a positive and

negative rotation. Similarly to the P_1 analysis of Section 4.2, the work corroborates

Slaboch's work as well as validates the methodology used in this thesis.
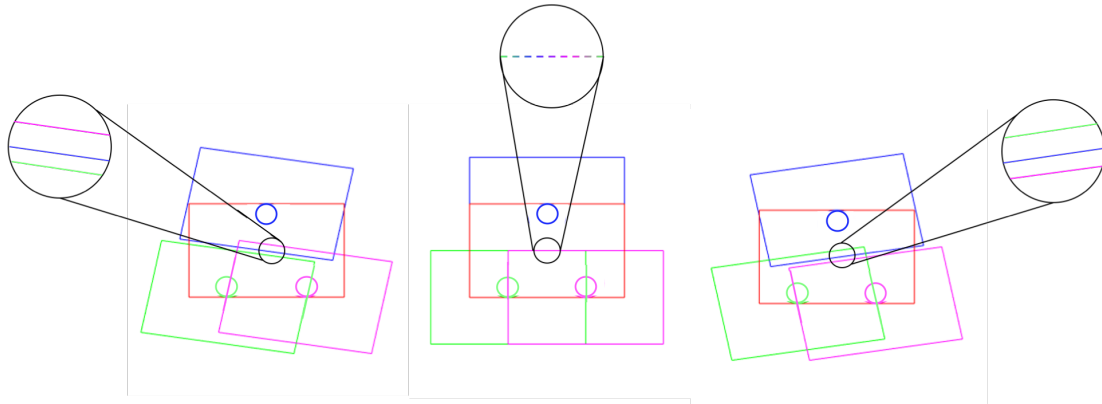
Figure 4.4: Validation of Slaboch's configuration space analysis of P_2 joints

4.4 Summary

Slaboch [2] used a configuration space analysis to determine the minimum number of contacts required for the stationary link to restrict the moving link to translational motion. Slaboch also analyzed these pairs with fewer contacts then the ones shown in this chapter, all of those failed, showing that Slaboch had determined the minimum number of contacts for each type of prismatic higher pair. This chapter validates this work. This information will be used in the design and analysis of Prismatic-Prismatic variable joints in Chapter 5 and 6. Also seen in this chapter, the configuration space of a circular contact and rectangle is the shape of the rectangle with rounded edges of the same radius as the circles, if the circles have no radius, the configuration space is simply the dimensions of the rectangular link.

This chapter shows the tools used for this thesis applied to previous works, ensuring

the validity of the results of the methodologies discussed in Chapter 3.

CHAPTER 5

Prismatic-Prismatic Joint Synthesis With Point Contacts

Slaboch [2] outlined two main types of prismatic higher pairs, type P_1 and type P_2. In Chapter 4, the two types of prismatic higher pairs were verified using a configuration space method. This verification defines the two types of prismatic higher pairs and their configurations. These configurations use the minimum number of contacts of a stationary link required to restrain a moving link to translational motion based on where the contact points are placed. As seen in Figure 3.5 in Chapter 3, if the stationary link's contacts are one-dimensional (i.e., $r = 0$), then the configuration space boundaries are equal to the dimensions of the moving link.

There are only three potential types of Prismatic-Prismatic variable joints: $P_1 P_1$, $P_1 P_2$, and $P_2 P_2$ [2]. This chapter will demonstrate the design and limitations of each of these Prismatic-Prismatic variable joints when point contacts are utilized.

5.1   Desired Motion Path

The desired motion path for a Prismatic-Prismatic Variable Joint is intuitive and easily defined. A Prismatic-Prismatic Variable joint should translate in one direction, and then another without any rotation. In Figure 5.1, a sample desired
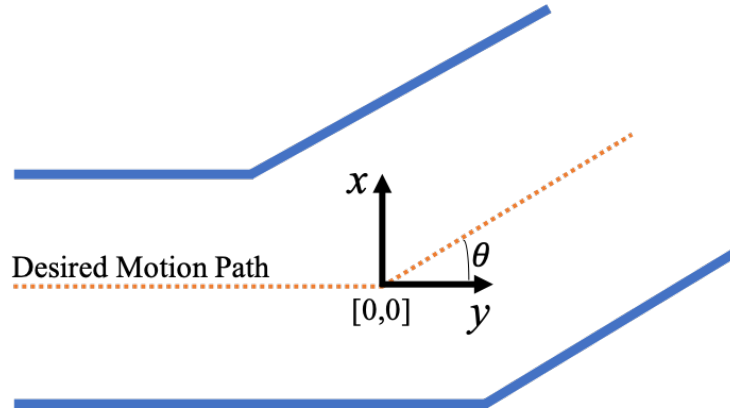
Figure 5.1: Diagram of the motion path of a Prismatic-Prismatic variable joint

motion path is shown relative to the stationary link. The desired motion path is the two lines of translation with the angle between these two lines being the desired translation angle, θ. The center-point of transition from the one portion of the joint to the other is defined to be [0,0].

The motion path of the moving link is:

$$q_y = \begin{cases} 0 & q_x \leq 0 \\ \\ q_x \tan \theta & q_x > 0 \end{cases}$$

where $q = (q_x, q_y)$ is the location of the centroid of the moving link. q is located at [0,0] when the joint transitions from one prismatic pair to another, as seen in Figure 5.1.

5.2   Valid and Invalid Prismatic-Prismatic Variable Joint Types

This chapter analyzes the three potential types of Prismatic-Prismatic variable

joints. The analysis will be done using the geometric approach discussed in Chapter

3. This analysis is performed by defining the geometry of each of the two links, and

the design dimensions that can be changed. Using the geometries of the links, it is

possible to determine if the joints in question are valid by looking at the desired

motion path, and determining if there are any locations at which the configuration

space analysis would show it is not possible. Unlike in Chapter 4, the link with

contacts will now be the moving link, and the other link consisting of lines will now

be the stationary link. This notation was done to match Slaboch's work, but now

with the analysis of Prismatic-Prismatic variable joints, it is important logically to

view the links as they will be used, with the moving link moving and the stationary

link stationary.

This chapter will use the terminology of first pairs and second pairs. The

first pair is the initial prismatic higher pair as defined previously, and the second

pair is the second. This can be seen more clearly in Figure 5.2. A summary of the

analysis of this chapter is shown in Table 5.5. The joints feasible will have their

design constraints outlined and will be further analyzed in Chapter 6 when

non-ideal contacts are considered. This table shows a synthesis of

Table 5.1: Summary of analysis of various Prismatic-Prismatic variable joints with point contacts

Joint Type	Variation	Validity	Number of Contacts	Redundant	Constraints
P_1P_1	1	Invalid	2	No	NA
P_1P_1	2	Invalid	2	No	NA
P_1P_1	3	Valid	3	Yes	Contact coordinates outlined in Table 5.2 $\theta < 90°$
P_1P_2	1	Invalid	5	Yes	NA
P_1P_2	2	Valid	5	Yes	Contact coordinates outlined in Table 5.3 $\theta < 90°$
P_2P_2	1	Valid	4	Yes	Contact coordinates outlined in Table 5.4 Equation 5.1 $\theta < 90°$

Prismatic-Prismatic variable joints, showing the different variations of

Prismatic-Prismatic variable joints and their constraints.
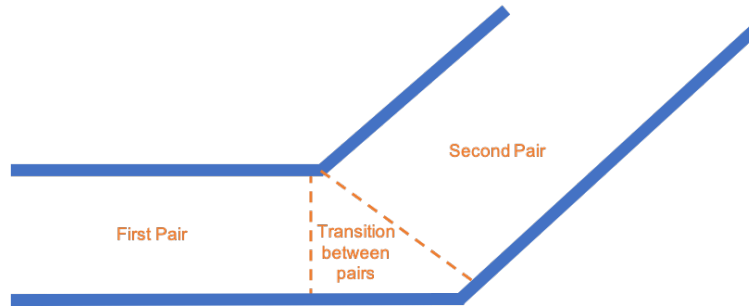
Figure 5.2: Definition of the first and second pair of a Prismatic-Prismatic variable joint

5.2.1 P_1P_1 Variable Joints

First Variation

Figure 5.3 shows a diagram of a potential P_1P_1 joint using three moving contact points; c_2 and c_3 are each used for the first pair and second pair of the joint respectively, while c_1 is used for both. This design ensures there are two perpendicular contacts for each prismatic higher pair, as discussed in Chapter 4. The issue with this design is that the c_2 contact will keep the joint from transitioning into the second prismatic pair due to interference with the stationary link. The interference of the c_2 contact and the direction of restricted motion is denoted by the arrow in Figure 5.3.

One possible change in the stationary link would be to move the transition point of the stationary link's bottom line, from the first pair to the second, which
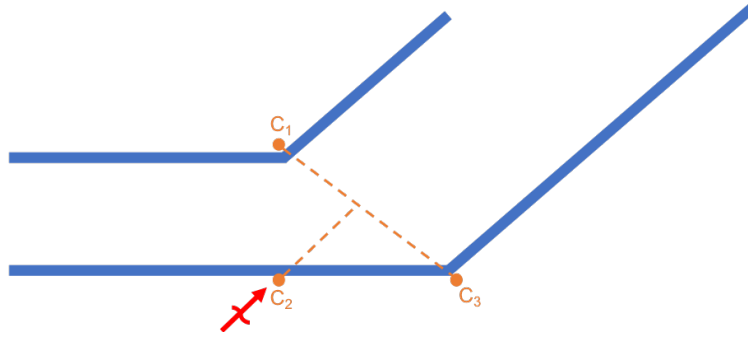
Figure 5.3: Diagram of P_1P_1 first variation with point contacts and a large offset of the bottom of the stationary link
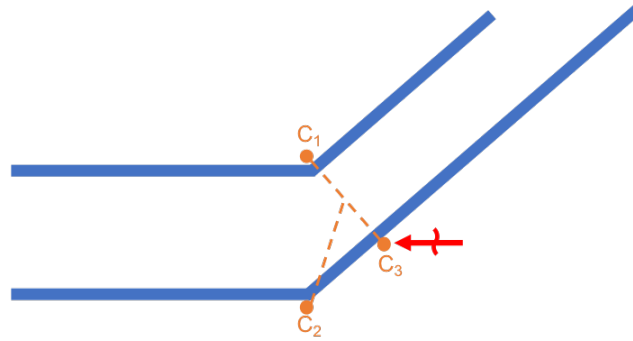
Figure 5.4: Diagram of P_1P_1 first variation with point contacts and a smaller offset of the bottom of the stationary link

can be seen in Figure 5.4. While this change would enable the c_2 contact to

translate along the the first portion of the joint, there would then be interference

with the c_3 contact. The interference of the bottom right contact and the direction

it is unable to move in is denoted by the red arrow in Figure 5.4.

Thus, the first variation of the P_1P_1 variable joints in Figures 5.3 and 5.4 are

not valid Prismatic-Prismatic variable joints.

Second Variation

Figure 5.5 shows a diagram of a potential P_1P_1 joint using two moving link contact points: c_1 and c_2. The two contact points are oriented in the same configuration as in the P_1 higher pair analyzed in Chapter 4. While this variation of the joint would allow for the desired motion path, it is also not a fully defined joint. The second prismatic pair of the joint allows for motions other than the desired motion path, both undesired translations and rotations. For a P_1 higher pair to work, the two moving link contacts must be perpendicular to the lines of the stationary link, otherwise the moving link will be able to rotate and then translate perpendicular to the desired motion path.

To show why this design is not feasible more analytically, a configuration space analysis using the geometric method of the second prismatic pair of the joint is shown in Figure 5.6. In the second portion of the joint, the moving link is able to rotate as seen in Figure 5.6 (left) because when the link is rotated, the configuration space boundaries do not overlap. If the configuration space boundaries do not overlap, it is a valid configuration. If the configuration space when the moving link is rotated is valid, it means the link is allowed to rotate in the second pair of the joint, which is undesired. In the second variation of the P_1P_1 variable joint, the two contact points do not form a line of contact perpendicular to the stationary link, resulting in an invalid joint.
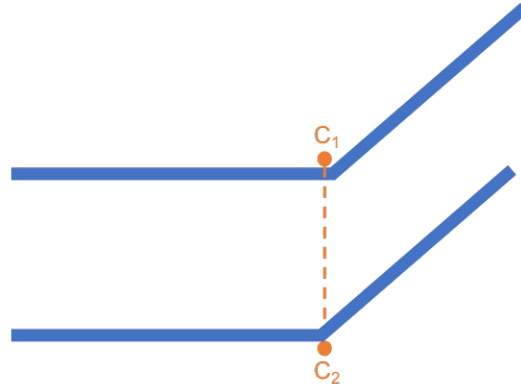
Figure 5.5: Diagram of P_1P_1 second variation with point contacts
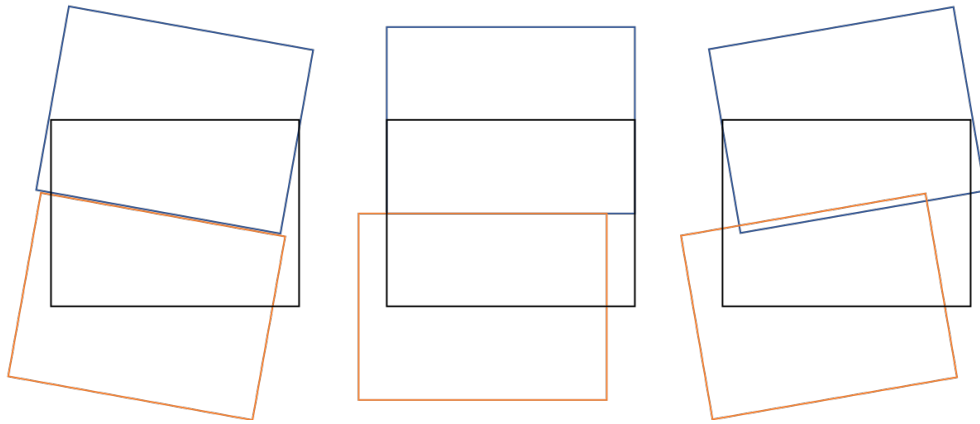
Figure 5.6: Geometric configuration space analysis of the second portion of the second variation of a P_1P_1 at -10 (CCW) ° (left), 0° (center), and 10° (CW) (right)

Thus, the second variation of the P_1P_1 variable joints in Figure 5.5 is not a valid Prismatic-Prismatic variable joint.

Table 5.2: Coordinates of point contacts for moving link of a P_1P_1 variable joint

	c_1	c_2	c_3
x	0	0	$w_{ch1}\tan(\theta)$
y	$.5w_{ch1}$	$-.5w_{ch1}$	$-.5w_{ch1}$

Third Variation

The third variation of P_1P_1 joints is valid, but contains redundancy. An example of

this configuration of P_1P_1 joints can be seen in Figure 5.7 with the design

parameters of the moving link given in Table 5.2 where w_{ch1} is the width of the first

channel and θ is the angle of translation. It is identical to the stationary link in

Figure 5.3, but with an additional stationary link line for the second prismatic pair

of the joint. The moving link is then redesigned so that c_3 point meets this new line

of the stationary link. c_3 must also form a line perpendicular to the stationary link

with the c_1 contact to avoid the same situation that was discussed in Figure 5.6; if

the contacts are not perpendicular to the stationary link line, undesired rotations

are possible. This design change allows for two contacts to be in use at a time, with

both forming a normal contact line, preventing any undesired rotations.

In Figure 5.7, the third variation's contacts c_1 and c_2 form a prismatic higher

pair for the first pair of the joint and contacts c_1 and c_3 form a prismatic higher pair

for the second pair of the joint. Both these pairs are functional P_1 higher pairs as
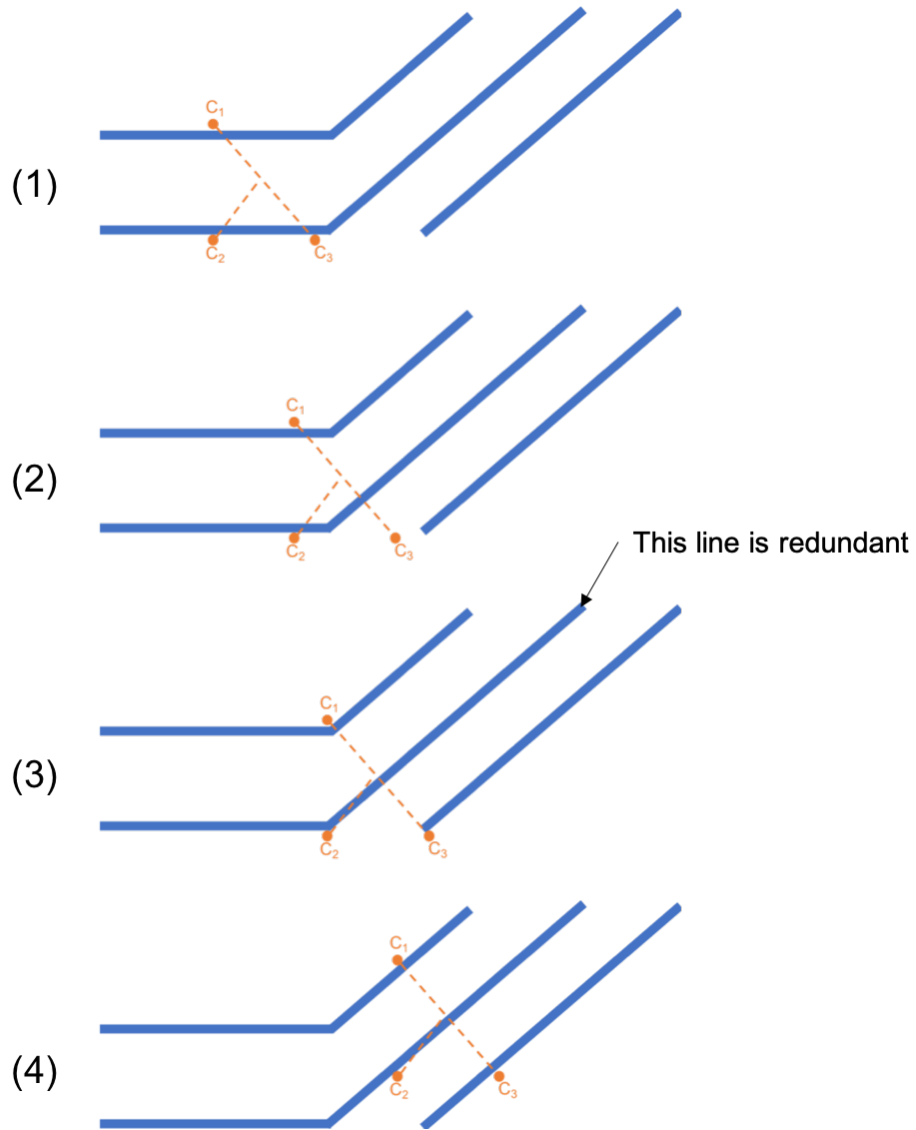
Figure 5.7: Diagram of P_1P_1 third variation with point contacts shown as the moving link moves from (1) to (4)

both contacts are on the outside of the stationary joint and perpendicular to the line of motion which is required to avoid undesired rotation. But for both pairs of the joint there is a third contact that is unnecessary making this joint redundant.

Thus, Variation 3 is the only variation of P_1P_1 variable joints that is valid

with the minimum number of redundancies possible. The other two variations are incapable of restricting the moving link to translational motion along the desired motion path without rotation.

5.2.2 P_1P_2 and P_2P_1 Variable Joints

Variable joints are designed to be able to translate in both directions transitioning to and from each higher pair of the joint. Thus, any analysis performed of a P_1P_2 variable joint applies for a P_2P_1 as they are simply the reverse of each other. Thus the analysis will only be done for a P_1P_2.

First Variation

Figure 5.3 shows a diagram of a potential P_1P_1 joint using five moving contact points: c_1, c_2, c_3, c_4, and c_5. The P_1P_2 joint is shown using two external point contacts (c_1, c_2) for the first prismatic pair (P_1), and three internal contacts (c_3, c_4, c_5) for the second prismatic pair (P_2). The design of the moving link comes from combining the P_1 and P_2 higher pair contacts into one.

The first variation of the P_1P_2 variable joint runs into the same restriction as the first variation of the P_1P_1 variable joint. As the joint transitions into the second prismatic pair of the joint, the c_2 contact restricts the translation of the link due to
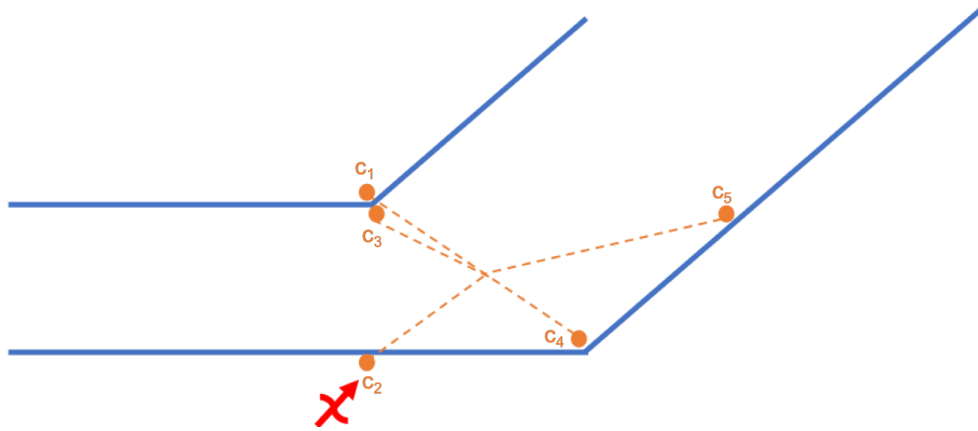
Figure 5.8: Diagram of $P_1 P_2$ with point contacts

interference with the stationary link. In Figure 5.8, the direction that the contact

cannot travel is denoted with an arrow.

Thus, the first variation of the $P_1 P_2$ variable joint in Figure 5.8 is not a valid

Prismatic-Prismatic variable joint.

Second Variation

To counter the effect of the interference exhibited in Figure 5.8, the stationary link

needs to be redesigned to accommodate a contiguous line. In the second variation,

the transition from one prismatic pair to the other at the bottom of the stationary

link is lined up with the same transition point at the top. This design allows for the

moving link contact that was formerly immobilized to translate for either prismatic

pair.
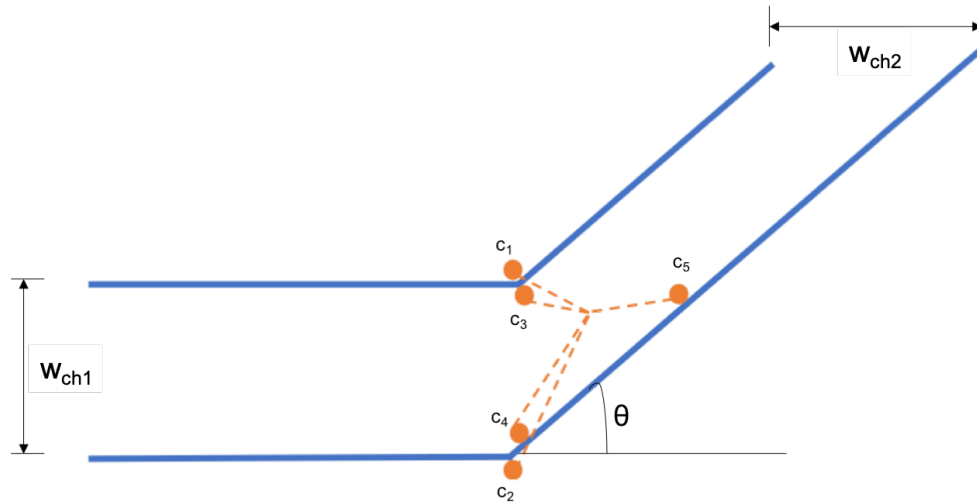
Figure 5.9: Diagram of P_1P_2 with point contacts and no offset in the stationary link

The configuration shown in Figure 5.9 is a valid Prismatic-Prismatic variable

joint. c_1 and c_2 form a P_1 pair and do not interfere with the transition to the second

prismatic pair. c_3, c_4, c_5 form a P_2 pair and do not interfere with the functioning of

the first prismatic pair. However, redesigning the stationary link necessitates design

constraints. The width of the second channel is directly related to the width of the

first channel and the translation angle. Also translation at an angle beyond $90°$ is

not possible. The design coordinates for the moving link's contacts are outlined in

Table 5.3, where w_{ch1} is the width of channel 1 and w_{ch2} is the width of channel 2.

Thus, the second variation is the only variation of P_1P_2 variable joints that is

valid with the minimum number of redundancies possible. The other variation is

Table 5.3: Coordinates of point contacts for moving link of a P_1P_2 variable joint

	c_1	c_2	c_3	c_4	c_5
x	0	0	0	0	w_{ch2}
y	$.5w_{ch1}$	$-.5w_{ch1}$	$.5w_{ch1}$	$-.5w_{ch1}$	$.5w_{ch1}$

incapable of restricting the moving link to translation motion along the desired

motion path without rotation.

5.2.3 P_2P_2 Variable Joints

P_2 pairs require three internal constraints to retain a link to translational motion.

In Figure 5.10, a P_2P_2 variable joint is designed using three internal contacts for the

moving link for each portion of the joint: c_1, c_2, and c_3 for the first prismatic pair

and c_2, c_3, and c_4 for the second prismatic pair. A fourth contact is required as the

same three contacts cannot create a valid P_2 higher pair for both prismatic pairs

that make up the joint. A fourth contact point means that the joint is redundant.

There is an additional design requirement due to the required geometry of

the joint; a P_2 pair must have two contacts with another opposite and between

them. If the desired translation angle does not meet the criteria,

$$\theta \geq \tan^{-1} \frac{w_{ch1}}{w_{ch2}}, \tag{5.1}$$

the joint will not function along the first portion as the points will not be aligned. The design requirement of Equation 5.1 is due to the single contact on one side being outside of the two contacts on the opposite side. This design requirement ensures the moving link is not able to rotate, similar to the second variation of a P_1P_1 in Figure 5.5. As seen in the configuration space analysis performed in Figure 5.6, the joint will be able to rotate, which is not a desired motion. The design coordinates for the moving link's contacts are outlined in Table 5.4, where w_{ch1} is the width of channel 1 and w_{ch2} is the width of channel 2.

Thus, Variation 1 is the only variation of P_2P_2 variable joints that is valid with the minimum number of redundancies possible.

Table 5.4: Coordinates of point contacts for moving link of a P_2P_2 variable joint

	c_1	c_2	c_3	c_4
x	$-w_{ch1}\tan(90-\theta) - .5w_{ch2}$	$w_{ch1}\tan(90-\theta) - .5w_{ch2}$	$w_{ch1}\tan(90-\theta) + .5w_{ch2}$	$-w_{ch1}\tan(90-\theta) + .5w_{ch2}$
y	$-.5w_{ch1}$	$.5w_{ch1}$	$.5w_{ch1}$	$-.5w_{ch1}$

5.3   Summary

While all three types of prismatic-prismatic variable joints, P_1P_1, P_1P_2 and P_2P_2, with point contacts for the moving link, are able to be designed, they each have their own strengths and weaknesses as outlined in the Constraints column of Table 5.5. In addition, it was also found that Prismatic-Prismatic variable joints cannot
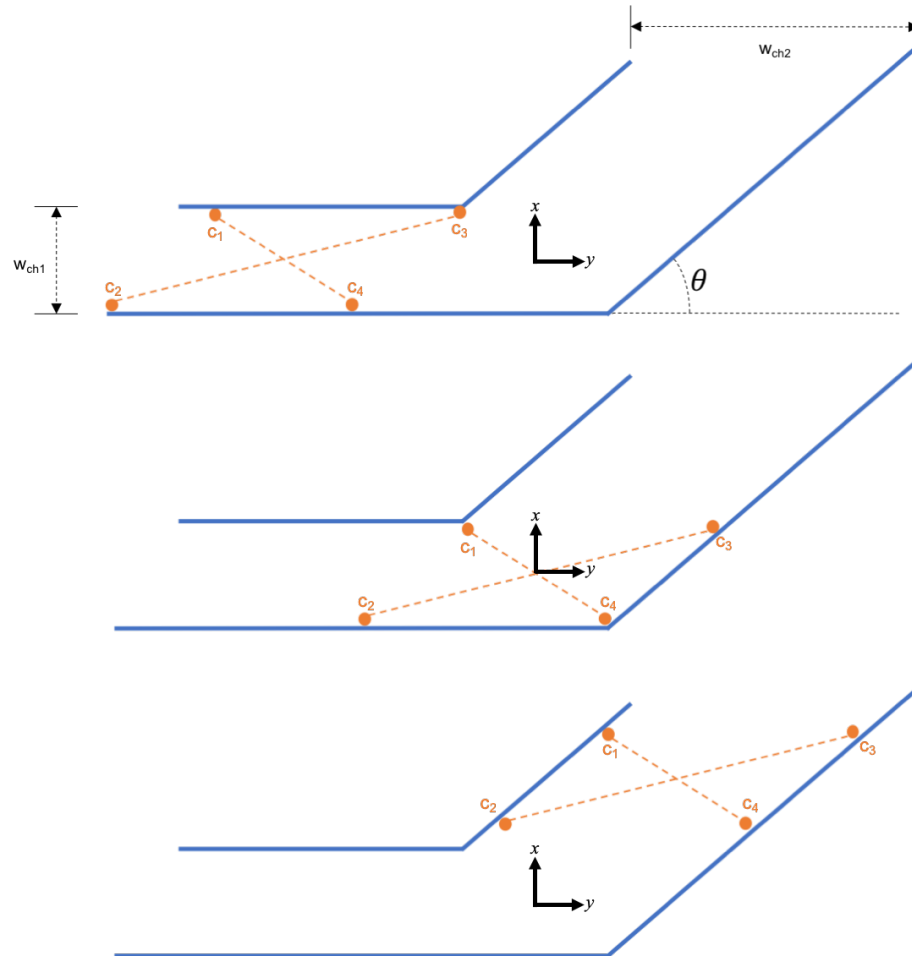
Figure 5.10: Diagram of P_2P_2 with point contacts

be designed without redundant contacts. This means that these redundancies have

the possibility to either make tolerances worse for joints or potentially restrict

undesired motions even more depending on the joint.

Table 5.5: Summary of analysis of various Prismatic-Prismatic variable joints with point contacts

Joint Type	Variation	Validity	Number of Contacts	Redundant	Constraints
P_1P_1	1	Invalid	2	No	N/A
P_1P_1	2	Invalid	2	No	N/A
P_1P_1	3	Valid	3	Yes	Contact coordinates outlined in Table 5.2 $\theta < 90°$
P_1P_2	1	Invalid	5	Yes	N/A
P_1P_2	2	Valid	5	Yes	Contact coordinates outlined in Table 5.3 $\theta < 90°$
P_2P_2	1	Valid	4	Yes	Contact coordinates outlined in Table 5.4 Equation 5.1 $\theta < 90°$

CHAPTER 6

Prismatic-Prismatic Joint Synthesis With Circular Contacts

Chapter 4 and 5 outlined and validated the two types of prismatic higher pairs, P_1 and P_2, and three potential variations of Prismatic-Prismatic variable joints, P_1P_1, P_1P_2, and P_2P_2. All potential prismatic-prismatic variable joints are possible and come with various design constraints. However, these constraints were designed for joints with one-dimensional point contacts for the moving link. In Chapter 6, these same joints will be analyzed with the contacts being more realistic circular contacts. Prismatic-Prismatic Variable Joints with circular contacts have different design and performance issues than those with one-dimensional contacts.

6.1 Preliminaries

The desired motion path for a Prismatic-Prismatic variable joint is the same as the motion path defined in Chapter 4. In this chapter, the contacts will be analyzed as two-dimensional circular contacts which can cause a variation from this ideal motion path, both in translation and rotation. For this reason, the numerical approach outlined in Chapter 3 will be used to analyze this undesired motion.

6.2   Valid and Invalid Prismatic-Prismatic Variable Joint Types

Table 6.1 shows the conclusions from the various types of Prismatic-Prismatic variable joints including the constraints caused by the moving links consisting of circular contacts. This table allows the reader a map to the chapter and a complete summary of the contributions made in this chapter.

6.2.1 P_1P_1 Variable Joints

First Variation

It was determined in Chapter 5 that this variation is not possible with point contacts. Thus, it is also not possible with circular contacts.

Second Variation

It was determined in Chapter 5 that this variation is not possible with point contacts. Thus, it is also not possible with circular contacts.

Table 6.1: Summary of analysis of various Prismatic-Prismatic variable joints with circular contacts

Joint Type	Variation	Validity	Number of Contacts	Redundant	Constraints
P_1P_1	1	Invalid	2	No	N/A
P_1P_1	2	Invalid	2	No	N/A
P_1P_1	3	Valid	3	Yes	Contact coordinates outlined in Table 5.2 Figure 6.5 $\theta < 90°$
P_1P_2	1	Invalid	5	Yes	N/A
P_1P_2	2	Valid	5	Yes	Contact coordinates outlined in Table 5.3 $\theta < 90°$
P_2P_2	1	Valid	4	Yes	Contact coordinates outlined in Table 5.4 Equation 5.1 Figure 6.5 $\theta < 90°$

Third Variation

To allow for a P_1P_1 joint to function using circular contacts, both links must be redesigned. The centers of the point contacts must be offset from the stationary link by the radius of the circular contacts. The stationary link must also be redesigned
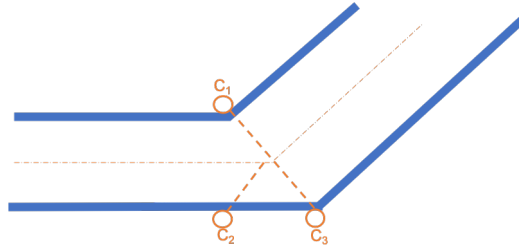
Figure 6.1: Diagram of P_1P_1 with circular contacts and no offset

to accommodate circular contacts. The bottom line of the stationary link must be offset by

$$Offset_{StationaryLink} = r_{contact} \cos(\theta) \tag{6.1}$$

where $r_{contact}$ is the radius of the contacts and θ is the desired translation angle. This offset is to ensure the bottom circular contact is able to translate when the link transitions over to the second prismatic pair of the joint. A P_1P_1 joint with no offset can be seen in Figure 6.1 and an example of the same joint with an offset can be seen in Figure 6.2. An offset designed into the stationary link is necessary, but then enables for the moving link to have a small amount of undesired rotation.
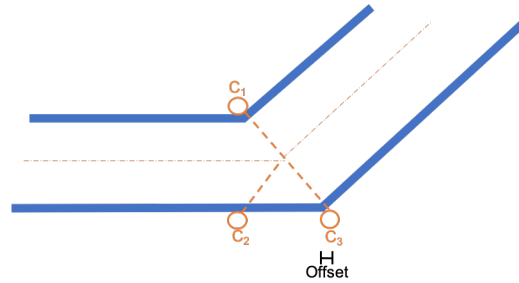
Figure 6.2: Diagram of P_1P_1 with circular contacts and an offset

6.2.2 P_1P_2 and P_2P_1 Variable Joints

First Variation

It was determined in Chapter 5 that this variation is not possible with point

contacts. Thus, it is also not possible with circular contacts.

Second Variation

The point contact design for a P_1P_2 variable joint was synthesized in Chapter 5, but

there are additional design considerations when the moving link has circular

contacts. The design considerations for a stationary link for a P_1P_2 variable joint

are similar to that of a P_1P_1. An offset needs to be designed into the stationary link

to allow the circular constraint to translate.
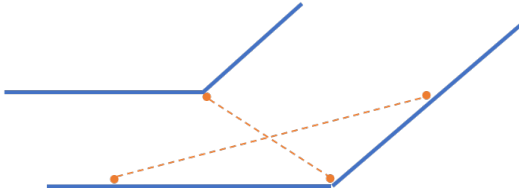
Figure 6.3: Sample P_2P_2 with point contacts
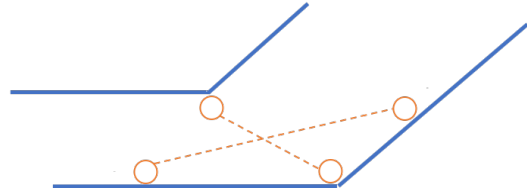
Figure 6.4: Sample P_2P_2 with circular contacts

6.2.3 P_2P_2 Variable Joints

Similarly to P_1P_2 and P_1P_1 joints, the P_2P_2 joint needs to be redesigned to accommodate the circular contacts to be valid. Unlike the P_1P_2 joint, the P_2P_2 joint only requires a redesign of the moving link. In this case, the moving link contact's centers need to be spaced one radius from the stationary link vertically. The redesign of the link can be seen in Figures 6.3 and 6.4.

6.3 Numerical Analysis of Prismatic-Prismatic Variable Joints

Due to the use of circular contacts in this chapter, the configuration space analysis must be performed using the numerical approach outlined in Chapter 3. This approach analyzes the various poses at which the moving link can exist in relation to the stationary link that vary from the ideal motion path in the form of undesired rotations. In particular, all three types of Prismatic-Prismatic variable joints will be analyzed to determine if the moving link is able to rotate and the effects of various

design variables on this rotation. In particular, the analysis in this section is done at

the transition point between the two prismatic pairs as this is the part of the joint

allows for the circular contacts to cause undesired rotation.

6.3.1 P_1P_1 Variable Joints

The numerical method described in Chapter 3 determined that the moving link of

P_1P_1 variable joints with circular contacts is able to have undesired rotation. Figure

6.6 shows the configuration space of a P_2P_2 joint in the x and ψ plane where x is

the horizontal direction and ψ is the undesired rotation of the moving link. Figure

6.7 shows the maximum rotation of the moving link with respect to x. The

maximum rotation of the moving link is the standard by which a joint's

performance is determined.

For a P_1P_1 variable joint, there are two design parameters that need to be

considered when determining how susceptible the joint is to undesired rotations:

circular contact radius and translation angle. The ratio of the sizes of the channels

do not need to be taken into consideration as the ratio is dependent on the

translation angle. Because changes in the contact radius and translation angle both

effect the undesired rotation of the moving link, it is important to analyze how they

interact. For this reason, the maximum undesired rotation extracted from the

numerical analysis is presented in a designed experiment shown in Figure 6.5. The
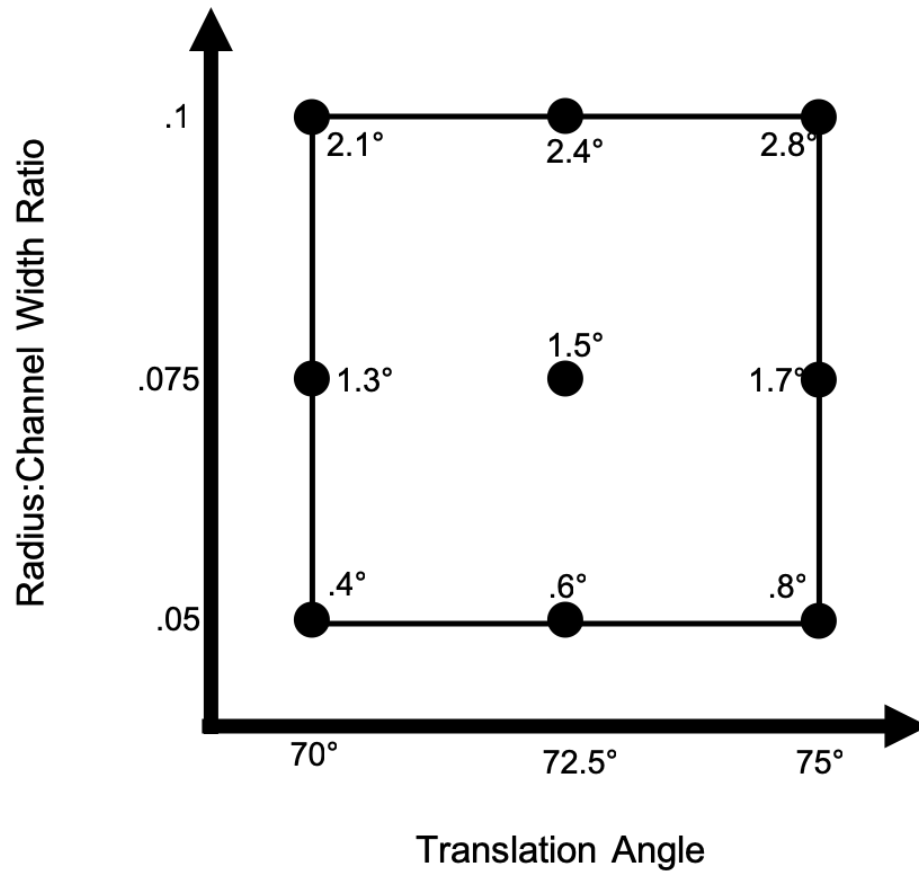
Figure 6.5: Maximum undesired rotations of P_1P_1 joints with respect to changes in contact radius and translation angle

ranges represented in Figure 6.5 were chosen as they show the range at which the

greatest undesired rotation happen.

Figure 6.5 shows that while translation angle has an important impact on

the potential for the moving link to rotate, it does not have as big of an impact as

the radius of the circular contacts for the range analyzed. The radius of the contacts

have a greater impact than the translation angle because their size is the direct

cause of the undesired rotations of the moving link.

6.3.2 P_1P_2 Variable Joints

After using the numerical method described in Chapter 3, it was determined that

the moving link of P_1P_2 variable joints with circular contacts do not have undesired

rotation. Although the design of the joint needs to be different from the point

contact case, the resulting joint geometry results in a joint with zero undesired

rotation. The lack of undesired rotation of the moving link is an advantage of this

type of Prismatic-Prismatic variable joint and should be taken into consideration

when implementing these joints into reconfigurable mechanisms.

6.3.3 P_2P_2 Variable Joints

Unlike the P_1P_2 joint, the P_2P_2 joint is highly susceptible to undesired rotation of

the moving link when circular contacts are used. This undesired rotation is due to

the way the points need to be aligned to restrict both portions to translational

motion. For a P_2P_2 variable joint, all of the contacts are internal. Thus, unlike with

the P_1P_2 variable joint, there is no additional external contact to restrict undesired

rotation. There is also less restriction on undesired translational deviations from the

desired motion path, which can be seen with the joint geometry, and through a

configuration space analysis.

Figures 6.6 through 6.7 show an example the results of a configuration space

analysis of a P_2P_2 variable joint with circular contacts. The joint in this analysis

has a translation angle of 75°, channel width of 1 unit for each portion of the joint,

and a contact radius of .1 units.

Figure 6.7 shows the maximum potential rotations of the moving link of a

P_2P_2 joint as a function of horizontal movement. This analysis is repeated and

analyzed while varying aspects of the joint design to determine the impact of these

aspects on the performance of the joint.

Similar to Figure 6.5 for a P_1P_1 variable joint, a parametric study for a P_2P_2

variable joint, is presented in a design of experiment format in Figure 6.8. The

design of experiment format allows for all of the factors that contribute to undesired

rotation be examines as a whole.

The radius of the contacts for a P_2P_2 variable joint relates to the deviation of

the moving link from the ideal motion path. As seen in Figure 6.8, contact radius

has the largest effect on undesired rotation for a P_2P_2, for the range studied. While

both the channel width ratio and translation angle have an impact on the ability for

the moving link to rotate, they are less impactful compared to contact radius for the
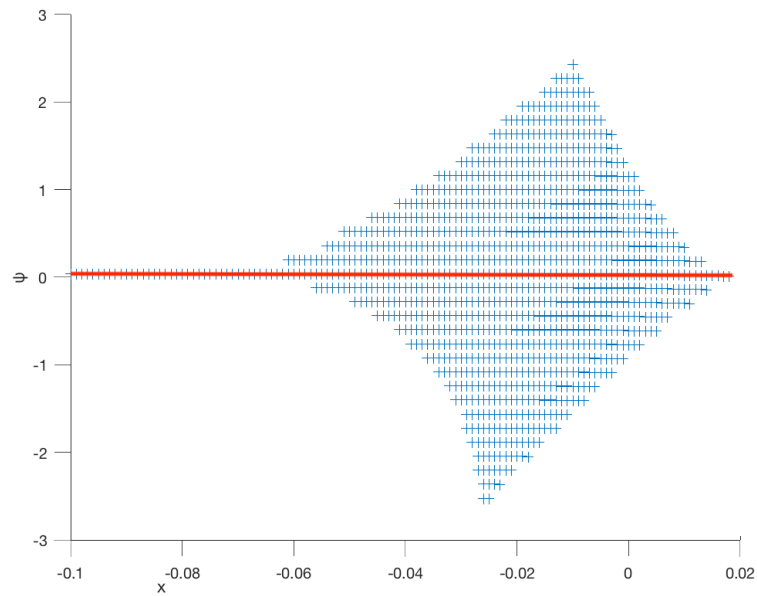
Figure 6.6: Configuration space of a P_2P_2 in the x and ψ directions. The blue is the potential configurations, the red is the desired motion path
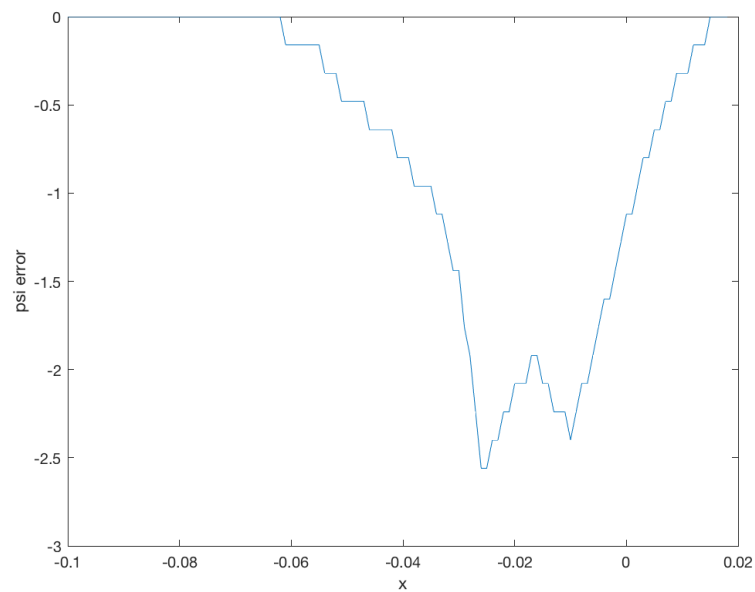
Figure 6.7: The maximum ψ error from the desired motion path for each corresponding x
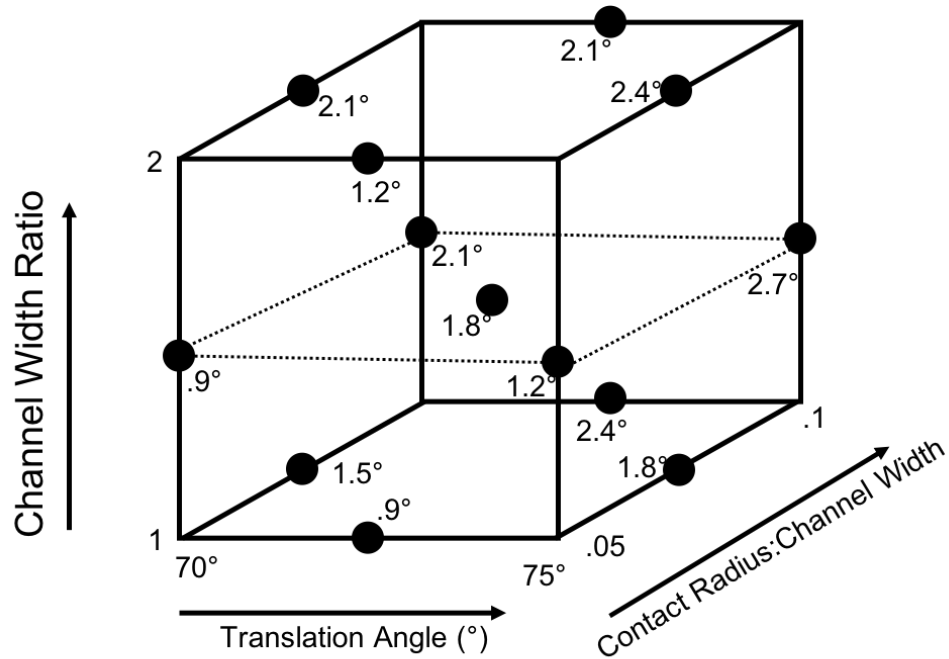
Figure 6.8: Maximum undesired rotations of a P_2P_2 variable joint with respect to various factors.

values shown. Contact radius being the biggest factor on undesired rotations of the

joint makes sense, as it is the direct cause of the rotation. As seen in Chapter 5,

when the contact radius is 0 (contacts are point contacts) there are no undesired

rotations. Translation angle is dependent on the application and is not a design

choice, unlike the channel width ratio.

6.4   Comparison of Prismatic-Prismatic Variable Joints

The analysis of P_1P_1, P_1P_2, and P_2P_2 variable joints illustrates that there is no

perfect design. The design is impacted by the ability to minimize the contact radius

of the moving link, and the desired translation angle and the ratio of the widths of

the stationary link's channels. It is important to clearly compare the three joints

and the constraints that come with them to properly implement the joints based on

a mechanism's requirements.

P_2P_2 variable joints with or without circular contacts are subject to the

limitation on translation as defined by Equation 5.1, which is a ratio of the widths

of the channels to ensure that the contacts form a triangle for each prismatic pair.

P_1P_2 and P_1P_1 are not subject to this same limitation. Technically, P_1P_2 variable

joints and P_1P_1 variable joints are limited only by

$$\theta_{P_1P_1} < 90° \tag{6.2}$$

and

$$\theta_{P_1P_2} < 90°. \tag{6.3}$$

The ratio of the channel widths of a P_2P_2 joint are defined in Equation 5.1 and the

ratio of the channel widths of a P_1P_2 and P_1P_1 are defined by Equation 6.4. These

constraints limit the ability of the engineer to design the joint to the specification

required by the mechanism. These limitations are especially true once the dynamics of the joint are considered. If the ratio of channel widths needs to be small as defined by

$$\frac{w_{ch1}}{w_{ch2}} = \sin^{-1}(90 - \theta), \tag{6.4}$$

the one channel becomes over-sized as needed to handle the forces just so the second channel is capable of handling the loads required. Such a design would lead to an over-sized joint for the application and an important consideration when selecting a joint.

Another design requirement of the joint comes with the offset between the two lines of the stationary link as seen on a P_1P_2 joint in Figure 6.2. For a P_1P_1 or P_1P_2 joint, this offset is defined by Equation 6.5. While this consideration is important when designing the stationary link, it has little impact on joint selection for a mechanism. The offset of the stationary link,

$$Offset_{Link2} = w_ch2\cos(90 - \theta) \tag{6.5}$$

is not a design advantage or disadvantage for either type of Prismatic-Prismatic variable joint but must be considered.

6.5   Summary

Each type of Prismatic-Prismatic variable joints comes with its own design considerations and constraints. P_2P_2 joints are highly susceptible to undesired rotational and translational motions when the moving link's contacts are assumed to be circular, where P_1P_2 joints are not. Also, P_1P_2 and P_1P_1 joints are able to translate at shallow angles (i.e., $\theta < 20°$) much more easily than P_2P_2 joints.

Understanding how the various joint design aspects impact the moving link's ability to rotate will inform engineers on the design disadvantages of these joints, especially when considering the dynamic forces within the joint, and material choice when considering the frictional forces that may result in the joint jamming. The constraints outlined in this chapter are purely kinematic; a dynamic analysis would include these limitations in addition to others. An example being that while a P_2P_2 joint is valid below 90°, the slightest deflection would break the joint if it is near 90°.

CHAPTER 7

Potential of a Revolute-Prismatic-Prismatic Variable Joint

With the joints developed in this thesis, there is a potential for

Revolute-Prismatic and Prismatic-Prismatic variable joints to be combined into a

Revolute-Prismatic-Prismatic variable joint. Slaboch [2] outlined six types of

Revolute-Prismatic variable joints, with both P_1 and P_2 higher pairs for the

Prismatic portion of the joint. All three Prismatic-Prismatic variable joint will be

analyzed in combination with a Revolute-Prismatic joint to determine if they are

possible to be used in a Revolute-Prismatic-Prismatic variable joint.

7.1 $R_n P_1 P_1$ Joints

With the design of the translating link of a $P_1 P_1$, it is not possible for the link to be

rotated into position by a revolute pair. As shown in Figure 7.1, contact c_3

interferes with the stationary link and is unable to get into position. As the link

transitions from an Revolute-Prismatic to a Prismatic-Prismatic joint, c_3 interferes

with the stationary link and is unable to allow the translating link to get into the

proper position. Thus, $R_n P_1 P_1$ variable joints are not possible.
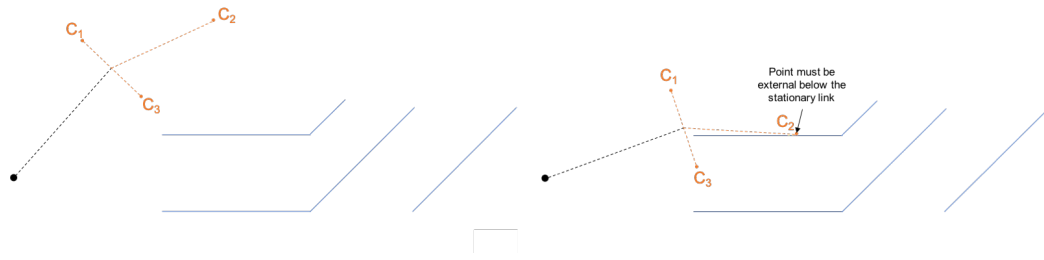
Figure 7.1: Analysis of a P_1P_1 variable joint with a Revolute pair

7.2 $R_nP_1P_2$ Joints

Any possible $R_nP_1P_2$ joints are subject to the design considerations and constraints of a P_1P_2 variable joints. Due to the design of the translating link of P_1P_2 joints outlined in Chapter 5, it is not possible for a $R_nP_1P_2$ joint to exist. As the joint rotates in the revolute portion and approaches the transition to the first prismatic portion, the contacts for the P_2 portion of the joint will interfere with the stationary link of the joint, as shown in Figure 7.2. The contact c_5 of the P_2 higher pair is unable to become internal to the stationary link, keeping the joint from properly transitioning to the P_1 higher pair of the joint. Thus, $R_nP_1P_2$ joints are not possible.

7.3 $R_nP_2P_2$ Joints

A $R_nP_2P_2$ is technically possible to exist, but there is a significant portion of the joint where the moving link is not constrained to the desired motion. Because the
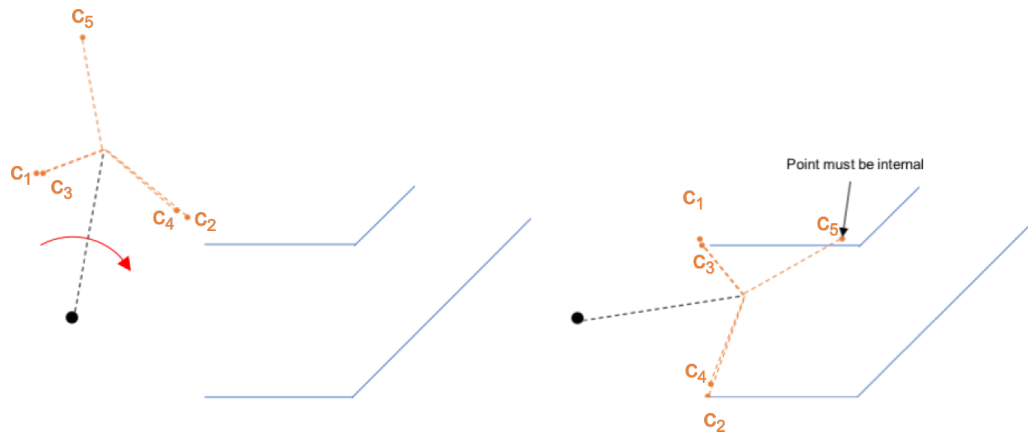
Figure 7.2: Diagram showing the interference caused by a $R_n P_1 P_2$ joint

moving link is not properly constrained, the link is effectively invalid. The stationary link needs to be designed to accommodate the rotation of the moving link. Specifically, the length of the upper line of the stationary link, l_1 needs to be shorter than the length of the lower line, l_2, by the relationship

$$l_2 - l_1 = w_{ch1} \sin \theta \tag{7.1}$$

where l_1 and l_2 are the lengths of the stationary link for the first prismatic pair and w_{ch1} is the width of the channel. As the joint rotates to allow for the first portion of translational motion, only two moving link contact points are in contact with the stationary link, which can be seen in Figure 7.3. In Chapter 4, it was noted that a P_2 higher pair requires three contact points to restrict a link to translational
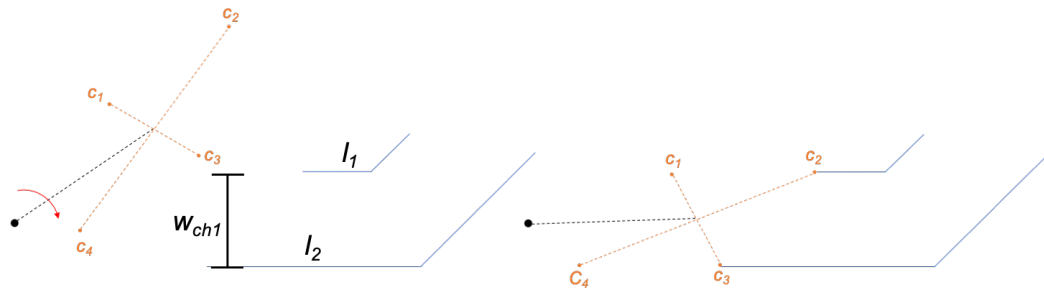
Figure 7.3: Diagram of a $R_n P_2 P_2$ joint, showing the two contact points available for the initial prismatic motion

motion. For this reason, until the moving link translates to allow c_1 to make contact with the stationary link, the moving link is able to rotate.

A configuration space analysis shows that after the link has rotated, it is also able to translate in the y direction due to only two of the three contacts necessary for a P_2 higher pair. Because of this undesired rotation and translation, $R_n P_2 P_2$ is not a feasible design for a Revolute-Prismatic-Prismatic variable joint.

7.4 Summary

$R_n P_1 P_2$ variable joints are impossible to be designed due to interference of the moving link with the stationary link. While the desired motion path of a $R_n P_2 P_2$ variable joint is possible, numerous other motion paths are also available to the moving link, meaning the joint is not functional. Using the research performed by Slaboch [2] in combination with the synthesis performed in Chapter 5,

Revolute-Prismatic-Prismatic variable joints are not possible under the assumptions

of this thesis.

CHAPTER 8

Summary and Future Work

This thesis has provided the design requirements of the synthesis of

Prismatic-Prismatic variable joints. The understanding of these design requirements

allow for engineers to design Prismatic-Prismatic variable joints which can be used

in Type II Mechanisms with Variable Topology. Three types of Prismatic-Prismatic

variable joints are possible and outlined: P_1P_1, P_1P_2 and P_2P_2. These joints have

their own design considerations and constraints, especially when designing them

with circular contacts. The advantages and disadvantages of each joint are outlined

completing a complete and thorough synthesis of Prismatic-Prismatic variable joints.

8.1 Contributions

Chapter 5 uses a geometric approach to determine the validity of various

Prismatic-Prismatic variable joints. Because there are two forms of prismatic higher

pairs, there are three combinations of potential Prismatic-Prismatic variable joints.

The design constraints of P_1P_2 and P_2P_2 are defined via a thorough synthesis of

these joints. This synthesis is a contribution that has never been performed and

enables the design and implementation of Prismatic-Prismatic variable joints. The

ultimate goal of this research is to allow for Prismatic-Prismatic variable joints to be selected and implemented into reconfigurable mechanisms. Implementing these joints would not be possible without the design constraints defined in this thesis.

While the work of Chapter 5 will allow for Prismatic-Prismatic variable joints to be implemented into reconfigurable mechanisms, Chapter 6 serves to provide the same tools while assuming the joints use circular two-dimensional contacts. For practical purposes, one-dimensional objects do not exist. When manufacturing variable joints in real world applications, it is important to understand the effect of imperfect manufacturing processes that may lead to non-ideal contacts. A direct comparison of the two types of Prismatic-Prismatic variable joints and how they are able to handle non-ideal contacts was presented. This information will allow for engineers to properly select and implement these joints.

8.2   Future Work

There are 21 possible variable joints while only 2 (R_nP_m) have been synthesized up until this point. As more variable joints are synthesized, the opportunities for reconfigurable mechanisms greatly expands as well. With more variable joints, engineers will be able to be more creative in the design of reconfigurable mechanisms, not only pushing the field of reconfigurable mechanisms forward, but provides all of the benefits that comes with reconfigurable mechanisms.

While this thesis has helped in the design of reconfigurable mechanisms, there is still a considerable amount of work to be done. There are two main opportunities that should be tackled next with this work. First would be a quasi-static analysis of Prismatic-Prismatic variable joints to consider links with dynamic forces as there would be in real life. As an example, when P_2P_2 variable joints approach 90 degrees, while they would be valid in a rigid body situation, the slightest flex of either of the links would allow the moving link to slip out of place and break the joint. Understanding how these joints can be driven and react under dynamic load would be the logical next step in this research.

The second opportunity would be to look at Prismatic-Prismatic variable joints with non-circular two-dimensional contacts. This would allow for unique designs that would mitigate the undesired motions seen in Chapter 6 and better optimize Prismatic-Prismatic variable joints. Examples of shapes that could reduce the undesired rotations would be triangles or crescents. Analyzing Prismatic-Prismatic variable joints or any variable joints with various two-dimensional shapes could be key to unlocking their potential. Different shapes may be able to bridge the gap of realistic two-dimentional design while minimizing the design constraints outlined in Chapters 5 and 6. This thesis, specifically Chapter 6, outlines the procedure for analyzing these various different shapes and

their impact on undesired rotations, allowing for research to happen rapidly, advancing the field.

This thesis lays the foundation for future work to further advance the field of variable joints. Chapter 5 is a synthesis of Prismatic-Prismatic variable joints and defines their design and limitations. Chapter 6 goes a step further and analyzes Prismatic-Prismatic variable joints with circular contacts. The methodology outlined in Chapter 6 will allow for future synthesis of variable joints with two-dimensional contact points to be done. Chapter 7 also provided a framework for analyzing variable joints with three or more higher pairs. The work in this thesis provides tools and insights into Prismatic-Prismatic variable joints but in addition it outlined methodologies for numerous future advancements in the design, synthesis, and implementation of variable joints.

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APPENDIX A

Matlab Code

A.1 Geometric Analysis of Higher Pairs and Variable Joints

This Matlab program allows for the configuration space analysis of higher pairs and variable joints via a geometric methodology.

```matlab
clear all
clc
%% Inputs
n_circle = 100;
ch1 = 10;
ch2 = 20;
theta = 40;


initial_center = [0;0];
initial_radius = 0;

circle_1_center = [ch1;ch1+initial_radius];
circle_2_center = [-.2*ch1;-initial_radius];
circle_3_center = [ch1+ch2+2.81*initial_radius;ch1+initial_radius];
circle_4_center = [1.52*ch1+ch2*initial_radius;0];

thetaR = degtorad(theta);
corner_bl_center = [initial_center(1);initial_center(2)];
corner_ul_center = ...
    [initial_center(1)+ch1/tan(thetaR);initial_center(2)+ch1];
corner_ur_center = ...
    [initial_center(1)+ch1/tan(thetaR)+ch2;initial_center(2)+ch1];
corner_lr_center = [initial_center(1)+ch2;initial_center(2)];

%% Creating Initial C.S.
t = 1.5*pi;
i = 1;
n=1;
while i ≤ n_circle
    x_1 = initial_radius * cos(t);
    y_1 = initial_radius * sin(t);
    s_prime(1,n) = x_1 + corner_bl_center(1);
    s_prime(2,n) = y_1 + corner_bl_center(2);
    t = t-((.5*pi+thetaR)/n_circle);
    i = i+1;
    if n == 1
        start = s_prime;
    end
    n=n+1;
end
i=0;
while i ≤ n_circle
    x_1 = initial_radius * cos(t);
    y_1 = initial_radius * sin(t);
    s_prime(1,n) = x_1 + corner_ul_center(1);
    s_prime(2,n) = y_1 + corner_ul_center(2);
    t = t-((.5*pi-thetaR)/n_circle);
    i = i+1;
    n=n+1;
end
i = 0;
while i ≤ n_circle
    x_1 = initial_radius * cos(t);
    y_1 = initial_radius * sin(t);
    s_prime(1,n) = x_1 + corner_ur_center(1);
    s_prime(2,n) = y_1 + corner_ur_center(2);
```

```matlab
56      t = t-((.5*pi+thetaR)/n_circle);
57      i = i+1;
58      n=n+1;
59  end
60  i = 0;
61  while i ≤ n_circle
62      x_1 = initial_radius * cos(t);
63      y_1 = initial_radius * sin(t);
64      s_prime(1,n) = x_1 + corner_lr_center(1);
65      s_prime(2,n) = y_1 + corner_lr_center(2);
66      t = t-((.5*pi-thetaR)/n_circle);
67      i = i+1;
68      n=n+1;
69  end
70  s_prime(:,n) = start;
71  b=1;
72  cs = transpose(s_prime);
73  plot(cs(:,1),cs(:,2))
74  %% C.S.
75  size = size(cs);
76  a=1;
77  while a≤size(1)
78      config_1(a,1) = circle_1_center(1) + cs(a,1);
79      config_1(a,2) = circle_1_center(2) + cs(a,2);
80      a = a+1;
81  end
82  a=1;
83  while a≤size(1)
84      config_2(a,1) = circle_2_center(1) + cs(a,1);
85      config_2(a,2) = circle_2_center(2) + cs(a,2);
86      a = a+1;
87  end
88  a=1;
89  while a≤size(1)
90      config_3(a,1) = circle_3_center(1) + cs(a,1);
91      config_3(a,2) = circle_3_center(2) + cs(a,2);
92      a = a+1;
93  end
94  a=1;
95  while a≤size(1)
96      config_4(a,1) = circle_4_center(1) + cs(a,1);
97      config_4(a,2) = circle_4_center(2) + cs(a,2);
98      a = a+1;
99  end
100 plot(config_1(:,1),config_1(:,2),config_2(:,1),config_2(:,2),...
101     config_3(:,1),config_3(:,2),config_4(:,1),config_4(:,2))
102 xlabel('x')
103 ylabel('y')
104 %axis([-1 50 -1 50])
```

A.2 Configuration Space Analysis of P_2P_2 With 2-D Contacts

```
 1  clear all
 2  clc
 3  clf
 4
 5  %% Input
 6
 7  ch1 = 1; %units
 8  ch2 = 1; %units
 9  angle = 76; %degrees
10  initial_rot = -3; %degrees
11  final_rot = 3; %degrees
12  contact_radius = .1; %units
13  initial_q = [0,0,0];
14  q_start = [-.1,0,initial_rot];
15  n_circle = 1000;
16  n_matrix_width =  300;
17  n_matrix_height = 300;
18  n_matrix_angle = 200;
19  matrix_height = .2;
20  matrix_width = .3;
21  tic
22
23  %% Initialization
24
25  rot = initial_rot;
26  q = initial_q;
27
28  %% Link 1 Creation
29  p_bl = [q(1)-.5*ch2-.5*ch1*tand(90-angle),q(2)-.5*ch1];
30  p_ul = [q(1)-.5*ch2+.5*ch1*tand(90-angle),q(2)+.5*ch1];
31  p_ur = [q(1)+.5*ch2+.5*ch1*tand(90-angle),q(2)+.5*ch1];
32  p_br = [q(1)+.5*ch2-.5*ch1*tand(90-angle),q(2)-.5*ch1];
33  P1 = [p_bl;p_ul;p_ur;p_br];
34  initialP1 = P1;
35  %% Link 2 Creation
36  anglerad = degtorad(angle);
37
38  slope3 = (p_ul(2)-p_bl(2))/(p_ul(1)-p_bl(1)); %slope of link
39
40  %creating limiting line
41  coefficients3limit = polyfit([p_ul(1), p_bl(1)], [p_ul(2), ...
        p_bl(2)], 1);
42  alimit = coefficients3limit (1);
43  blimit = coefficients3limit (2);
44
45  %creating actual link line
46  coefficients3link = ...
        polyfit([p_ul(1)-contact_radius*cosd(90-angle), ...
        p_bl(1)-contact_radius*cosd(90-angle)], ...
        [p_ul(2)+contact_radius*sind(90-angle), ...
        p_bl(2)+contact_radius*sind(90-angle)], 1);
47  alink = coefficients3link (1);
48  blink = coefficients3link (2);
49
50  xc_start = (.5*ch1 + contact_radius - blink) / alink;
51  circle_center = [xc_start,alink*xc_start+blink];
52  xc_end = (circle_center(1) + contact_radius*sind(angle));
```

```matlab
53
54
55  % Accounting for lack of y-intercept of 90 degrees
56  if slope3 == inf
57      x3 = linspace(-.5*ch2,-.5*ch2,2);
58      y3 = [-2,2];
59  else
60      x3 = linspace(xc_end,2,2);
61      y3 = alimit*x3+blimit;
62  end
63
64  t = .5*pi;
65  i = 1;
66  circle = zeros(2,n_circle);
67  circle_center = [xc_start,alink*xc_start+blink];
68  while i <= n_circle
69      circle(1,i) = contact_radius * cos(t) + circle_center(1);
70      circle(2,i) = -contact_radius * sin(t) + circle_center(2);
71      t = t-(anglerad/n_circle);
72      i = i+1;
73  end
74  i = 0;
75
76  slope4 = (p_ur(2)-p_br(2))/(p_ur(1)-p_br(1));
77  coefficients4 = polyfit([p_ur(1), p_br(1)], [p_ur(2), p_br(2)], 1);
78  c = coefficients4 (1);
79  d = coefficients4 (2);
80  if slope4 == inf
81      x4 = linspace(.5*ch2,.5*ch2,2);
82      y4 = [-1000,1000];
83  else
84      x4 = linspace(.5*ch2 - .5*ch1/tand(angle),2,2);
85      y4 = c*x4+d;
86  end
87
88  x1 = linspace(-2,xc_start,2);
89  y1 = linspace(.5*ch1,.5*ch1,2);
90
91  x2 = linspace(-2,.5*ch2 - .5*ch1/tand(angle),2);
92  y2 = linspace(-.5*ch1,-.5*ch1,2);
93
94  circle_rev = fliplr(circle);
95  link2 = [x2 x4 x3(2) x3(1) circle_rev(1,:) x1(2) x1(1);y2 y4 ...
96          y3(2) y3(1) circle_rev(2,:) y1(2) y1(1)];
96
97  %% Test Points
98  q = q_start;
99  xd = 1;yd = 1;
100 td = 1;
101 invalid_n_3 = 1;
102 invalid_config_3 = [;];
103 valid_n_3 = 1;
104 valid_config_3 = [;];
105 distance = 0;
106 distance_test = [];
107 n_distance_test_3 = 0;
108 test=0;
```

```
109  n_valid=1;
110  n_invalid=1;
111  br_x_limit = (-.5*ch1-d)/slope4;
112
113  valid_config = ...
         zeros(n_matrix_width*n_matrix_height*n_matrix_angle,3);
114  invalid_config = ...
         zeros(n_matrix_width*n_matrix_height*n_matrix_angle,3);
115  for td = 1:n_matrix_angle
116      for xd = 1:n_matrix_height
117          for yd = 1:n_matrix_width
118
119              %Rotating link to current rotation
120              R = [cosd(q(3)) -sind(q(3));sind(q(3)) cosd(q(3))];
121              P1 = initialP1;
122              P1=R*P1.';
123              P1 = P1.';
124              P1(:,1) = P1(:,1)+[q(1)];
125              P1(:,2) = P1(:,2)+[q(2)];
126
127
128              %Creating the limits for points based on x and ...
                     testing them
129              i=1;
130              for i = 1:4
131                  x = P1(i,1);
132                  y = P1(i,2);
133
134                  if x < xc_start
135                      y_limit_upper(i) = .5*ch1;
136                      y_limit_lower(i) = -.5*ch1;
137                  elseif (x >= xc_start) && (x < xc_end)
138                      y_limit_upper(i) = circle_center(2) - ...
                             sqrt(-circle_center(1)^2 + ...
                             contact_radius^2 + 2*circle_center(1)*x ...
                             - x^2);
139                      y_limit_lower(i) = -.5*ch1;
140                  elseif (x >= xc_end) && (x < br_x_limit)
141                      y_limit_upper(i) = slope3 * x + blimit;
142                      y_limit_lower(i) = -.5*ch1;
143                  else
144                      y_limit_upper(i) = slope3 * x + blimit;
145                      y_limit_lower(i) = c * x + d;
146                  end
147
148                  if (y <= y_limit_upper(i)) && (y >= y_limit_lower(i))
149                      isvalid(i) = 1;
150                  else
151                      isvalid(i) = 0;
152                  end
153  %                i = i+1;
154              end
155
156              %Determining if configuration is valid and archiving
157              test_valid = nnz(isvalid);
158              if test_valid == 4
159                  valid_config(n_valid,:) = q;
```

```matlab
160                    n_valid = n_valid + 1;
161                else
162                    invalid_config(n_invalid,:) = q;
163                    n_invalid = n_invalid + 1;
164                end
165                q(1) = q(1) + matrix_width/n_matrix_width;
166                q(1) = round(q(1),12);
167
168            end
169
170            q(2) = q(2) + matrix_height/n_matrix_height;
171             q(2) = round(q(2),12);
172            q(1) = q_start(1);
173            q(1) = round(q(1),12);
174        end
175        q(1) = q_start(1);
176        q(1) = round(q(1),12);
177        q(2) = q_start(2);
178        q(2) = round(q(2),12);
179        q(3) = q(3) + (final_rot-initial_rot)/n_matrix_angle;
180        q(3) = round(q(3),12);
181
182  end
183  %% Eliminating redundant values and comparing configurations to ...
         ideal
184  valid_config = unique(valid_config, 'rows');
185  A = arrayfun(@(x) valid_config(valid_config(:,1) == x, :), ...
         unique(valid_config(:,1)), 'uniformoutput', false);
186  j = 1;
187  sizeA = size(A);
188  while j <= sizeA(1)
189      B = A{j};
190      xcond(j) = B(1,1);
191      t_ideal(j) = 0;
192      if xcond(j) <= 0
193          y_ideal(j) = 0;
194      else
195          y_ideal(j) = slope3*xcond(j);
196      end
197
198      y_max(j) = max(abs(B(:,2)));
199      t_max(j) = max(abs(B(:,3)));
200
201      y_error(j) = y_ideal(j)-y_max(j);
202      t_error(j) = t_ideal(j)-t_max(j);
203
204      j = j+1;
205  end
206  t_error_max = max(abs(t_error))
207
208
209  %Plotting Data
210  hold on
211  plot3(valid_config(:,1),valid_config(:,2),valid_config(:,3),'+');
212  plot(xcond,y_error)
213  plot(xcond,t_error)
214  %axis([-2 2 -2 2])
```