MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

NATIONAL TECHNICAL UNIVERSITY
«KHARKIV POLYTECHNIC INSTITUTE»

GUIDELINES
of calculation and graphical performing the task by the topic «LINEAR ELECTRIC CIRCUITS OF HARMONIC CURRENT»

for courses "Theoretical Basics of Electrical Engineering",
"Theory of Electrical and Magnetic Circuits ",
"Theory of Electric Circuits"

for students of specialties
141 "Electric Power, Electrical Engineering and Electromechanics",
151 "Automation and computer-integrated technologies",
171 "Electronics"

Approved
by editorial and publishing
university council,
record No. 2 of 25.06.2020

Kharkiv
NTU "KhPI"
2020

GUIDELINES of calculation and graphical performing the task by the topic «LINEAR ELECTRIC CIRCUITS OF HARMONIC CURRENT» for courses «Theoretical Basics of Electrical Engineering», «Theory of Electrical and Magnetic Circuits», «Theory of Electric Circuits» for students of specialties 141 «Electric Power, Electrical Engineering and Electromechanics», 151 «Automation and computer-integrated technologies», 171 «Electronics» / comp. M. M. Rezynkina, O. H. Kiessaiev, O. L. Rezynkin, S. A. Lytvynenko – Kharkiv: NTU "KhPI", 2020. – 44 p.

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CONTENT

INTRODUCTION

These guidelines contain general theoretical basis on the topic of «Linear electric circuits of harmonic current», tasks for calculation and graphing, and examples of AC circuits calculations.

Calculation and graphic work is devoted to the topic "Linear electric circuits of harmonic current" for the students of the first semester of study the disciplines: "Theoretical basics of electrical engineering", "Theory of electric and magnetic circuits", "Theory of electric circuits".

Knowledge and skills in this topic are crucial for the further successful study by students of the special disciplines of electrical engineering. In practice, the method of complex amplitudes, which uses the algebra of complex numbers and allows applying all methods of DC calculation to AC circuits, is quite widespread. The method of vector diagrams construction is a necessary condition for understating of the theory of three-phase circuits. All these methods are applied in various special disciplines of electrical engineering.

In the process of the task performing, students can use the literature at the end of these guidelines, which contains additional information on theoretical issues and examples of such problems solution.

The methodical instructions are intended for students of the specialties 141 "Electric Power, Electrical Engineering and Electromechanics", 151 "Automation and computer-integrated technologies", 171 "Electronics" of full-time and distance forms of study.

1. SCOPE

These guidelines establish the requirements for performing calculation and graphical task «Linear electric circuits of harmonic current» on the courses "Theoretical basics of electrical engineering", "Theory of electrical and magnetic circuits", "Theory of electrical circuits".

The content of the calculation and graphical task corresponds to the training program for students in electrotechnical specialties.

2. PERFORMANCE PURPOSE

As a result of this task performance, a student must:

1) know the basic laws and methods of linear AC circuits calculations;

2) be able to:

a) apply the laws and methods of electric circuits calculation to solve specific tasks;

b) build vector diagrams and timelines on the results of the calculations.

3. PERFORMANCE ORDER

The calculation and graphical task is performed in the study of the theoretical material on this topic.

4. REQUIREMENTS FOR THE TASK

4.1 Each section should contain:

1) the task;

2) the calculation part;

3) graphical part (vector diagram and timelines);

4) conclusions (evaluation of the results).

4.2 Task is performed on A4 (297x210) mm paper sheets. If necessary, A3 (297x420) mm format can be used.

Left, bottom and top margins should be at least 20 mm, at least 10 mm on the right.

4.3 The work pages are numbered in Arabic numerals in the upper right corner. Page numbering should be end-to-end for all work. There is no number on the title page, but it is included in the overall page numbering. The form of the title page is given in Attachment B.

4.4 Text of the work should be written on one side of a sheet in one of the following ways:

a) handwritten – clear, legible handwriting with letters and numbers not less than 2.5 mm high. Font density in the work should be the same;

b) printed – size 12 – 14 at 1.5 intervals.

4.5 Work must be stitched.

5. GENERAL PROVISIONS

5.1 Harmonic electrical quantities

An electromagnetic process in an electric circuit is called periodic if the values of currents and voltages are repeated at regular intervals. The time after which the instantaneous values of quantities are repeated is called period $[T]$=s.

$$f(t \pm kT) = f(t). \tag{5.1}$$

Frequency f is inversely proportional to period $[f] = \text{s}^{-1} = \text{Hz}$:

$$f = \frac{1}{T}. \tag{5.2}$$

The most common and important type of periodic currents and voltages are sinusoidal (harmonic) currents and voltages.

If applied voltage is sinusoidal

$$u = U_m \sin(\omega t + \Psi_u), \tag{5.3}$$

then the current will also be sinusoidal in the linear circuit

$$i = I_m \sin(\omega t + \Psi_i), \tag{5.4}$$

where u, i are instantaneous voltage and current values;

ω is angular frequency;

Ψ_u, Ψ_i are initial phases of voltage and current;

U_m, I_m are voltage and current amplitudes.

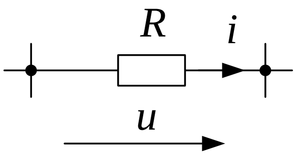
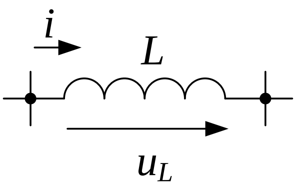
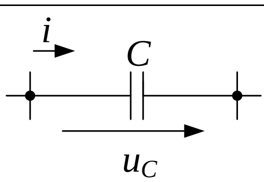
5.2 Parameters of passive circuit elements

Passive elements of AC electrical circuits are:

- resistive element with resistance *R*, in which electromagnetic energy is converted to heat;
- inductive element with inductance *L*, which characterizes the magnetic field energy $W_M = \frac{Li^2}{2}$ and the phenomenon of self-induction $e_L = -L\frac{di}{dt}$;
- capacitive element with a capacitance *C*, which characterizes the electric field energy $W_E = \frac{Cu^2}{2}$.

Table 5.1 presents the circuit passive elements and the voltage drop dependence on the elements' parameters.

Table 5.1

Element	Designation in the circuit	Connection between current and voltage
R		$u_R = iR$
L		$u_L = L\frac{di}{dt}$
C		$u_C = \frac{1}{C}\int idt$

5.3 Complex images of currents and voltages

All basic laws and methods for DC electric circuits calculation can be used to calculate linear AC circuits. To do this, the method of *complex amplitudes* (symbolic method) is used. It uses complex images of sinusoidal currents and voltages (complex amplitudes or complexes of RMS values). This allows the integration and differentiation operations (see table 5.1) to be replaced by the algebraic operations.

Complex amplitudes of voltage and current are designated by underlining of the corresponding values and written as follows:

$$\underline{U}_m = U_m e^{j\Psi_u}; \tag{5.5}$$

$$\underline{I}_m = I_m e^{j\Psi_i}, \tag{5.6}$$

where U_m is voltage amplitude;

ψ_u is initial voltage phase;

I_m is current amplitude;

ψ_i is initial current phase;

$j = \sqrt{-1}$ is imaginary unit.

From complex amplitudes, it is possible to transect to complexes of RMS values of voltage and current:

$$\underline{U} = \frac{\underline{U}_m}{\sqrt{2}} = Ue^{j\Psi_u}; \tag{5.7}$$

$$\underline{I} = \frac{\underline{I}_m}{\sqrt{2}} = Ie^{j\Psi_i}. \tag{5.8}$$

Ohm’s and Kirchhoff's laws can be used for voltage and current complexes.

Thus, for calculation of the harmonic current circuits, you can use all the methods of calculation of electric circuits used for DC circuits.

5.4 Ohm’s and Kirchhoff's laws in complex form

5.4.1 Ohm's law in complex form

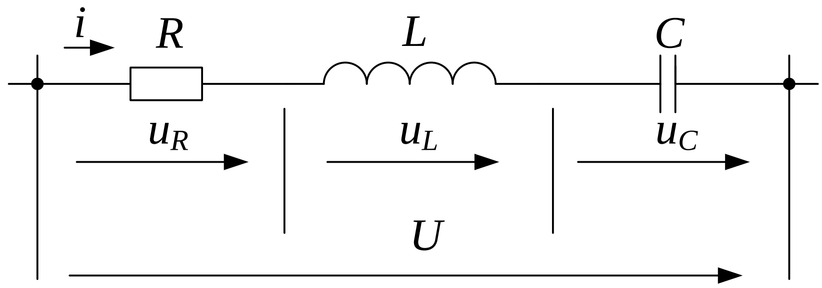


Figure 5.1 – Elementary section of the AC electric circuit

For the electric circuit passive section, the complex of current’s RMS is equal to a ratio of the complex of voltage’s RMS and its complex impedance:

$$\underline{I} = \frac{\underline{U}}{\underline{Z}}, \qquad (5.9)$$

where $\underline{Z}$ is impedance of the section.

The complex impedance is determined by an equation:

$$\underline{Z} = R + jX = Ze^{j\varphi}, \qquad (5.10)$$

where R is active resistance of the section;

$X = X_L - X_C = \omega L - \dfrac{1}{\omega C}$ is reactance of the section;

$\varphi = \Psi_u - \Psi_i$ is phase shift angle between voltage and current.

Table 5.2 presents the passive elements complexes and dependence of the complexes of RMS voltages on their parameters.

Table 5.3 presents the complexes of the *R*-*L* and *R*-*C* elements connected in series.

Table 5.2

Element	Element's complex	Complex of RMS voltage drop over an element
R	$\underline{Z} = R$	$\underline{U}_R = \underline{I}R$
L	$\underline{Z} = jX_L = j\omega L = \omega L e^{j90^\circ}$	$\underline{U}_L = \underline{I}j\omega L$
C	$\underline{Z} = jX_C = -j\dfrac{1}{\omega C} = \dfrac{1}{\omega C}e^{-j90^\circ}$	$\underline{U}_C = \underline{I}\left(-j\dfrac{1}{\omega C}\right)$

Table 5.3

Section	Impedance of a circuit section
R-L	$\underline{Z} = R + jX_L = R + j\omega L = \sqrt{R^2 + (\omega L)^2}\, e^{jarctg\frac{X_L}{R}}$
R-C	$\underline{Z} = R + jX_C = R - j\dfrac{1}{\omega C} = \sqrt{R^2 + \left(-\dfrac{1}{\omega C}\right)^2}\, e^{jarctg\frac{Xc}{R}}$

5.4.2 Kirchhoff's laws in the complex form

Kirchhoff's current law in the complex form:
an algebraic sum of the complexes of currents' RMS of the electric circuit branches that converge into a node is equal to zero:

$$\sum_{k=1}^{n} \underline{I}_k = 0. \tag{5.11}$$

Kirchhoff's voltage law in the complex form:
an algebraic sum of the complexes of RMS of voltage drops in any closed loop of an electric circuit is equal to an algebraic sum of the complexes of EMFs' RMS in the loop:

$$\sum_{k=1}^{m} \underline{Z}_k \underline{I}_k = \sum_{k=1}^{l} \underline{E}_k \, . \tag{5.12}$$

Note: complex amplitudes $\underline{E}_m, \underline{U}_m, \underline{I}_m$ can also be used in Ohm's and Kirchhoff's laws.

The number of independent equations and the sign rules at writing Kirchhoff's equations are determined in the same way as for DC circuits.

5.5 Determination of complex power, power balance of a circuit, indications of a wattmeter

The complex power is determined by multiplying the complex RMS voltage by the complex conjugate value of RMS current:

$$\underline{S} = \underline{U}\,\overset{*}{\underline{I}} = Ue^{j\Psi_u} \cdot Ie^{-j\Psi_i} = Se^{j\varphi} = S\cos\varphi + jS\sin\varphi = P + jQ \tag{5.13}$$

where $\overset{*}{\underline{I}}$ is complex conjugate value of RMS current;

P is active power;

Q is reactive power.

Two conjugate complex numbers differ only by a sign before the imaginary unit j ($\underline{I}_m = I_m e^{j\Psi_i}$; $\overset{*}{\underline{I}}_m = I_m e^{-j\Psi_i}$).

Power balance for the harmonic current circuit:

$$\sum \underline{S}_{gen} = \sum \underline{S}_{cons}, \tag{5.14}$$

where $\underline{S}$ is apparent power.

The power generated is an algebraic sum of all sources complex powers.

Complex power generated by an EMF source is:

$$\underline{S}_{gen} = \underline{E}\,\overset{*}{\underline{I}} = P_{gen} + jQ_{gen}\,, \qquad (5.15)$$

where $\underline{E}$ is complex of RMS of EMF source;

P_{gen} is generated active power;

Q_{gen} is generated reactive power;

$\overset{*}{\underline{I}}$ is complex conjugate value of RMS of current of the branch to which the source is connected.

Consumed power is defined as a sum of the complex powers of each electric circuit section. The complex power consumed by the electric circuit branches is determined by the equation:

$$\underline{S}_{cons} = \underline{U}\,\overset{*}{\underline{I}} = \underline{Z}I^2 = RI^2 + jXI^2 = P_{cons} + jQ_{cons}\,, \qquad (5.16)$$

where $\underline{U}$ is complex of RMS voltage of the branch;

$\overset{*}{\underline{I}}$ is complex conjugate value of RMS current of the branch.

The active power measurement is performed by a wattmeter. The wattmeter has two windings (fig. 5.2).

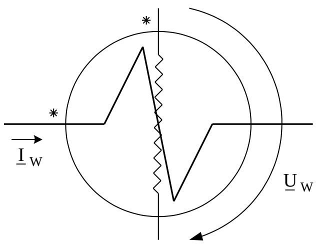


Figure 5.2

The winding through which the current flows has a small number of turns and a large cross-section of a wire with low resistance, is connected in series to the test circuit (as an ammeter is usually connected). The current must flow into the beginning of the winding indicated by an asterisk in the diagram (Fig. 5.2).

A voltage winding with a large number of turns of a wire with a small cross-section and high resistance is connected in parallel to the test section of the circuit (as a voltmeter is usually connected). The wattmeter voltage is measured between the terminal marked by an asterisk and the terminal with no mark (Fig. 5.2).

An electrical scheme of a wattmeter connection for measuring the active power consumed in an AC circuit is shown in fig. 5.3.

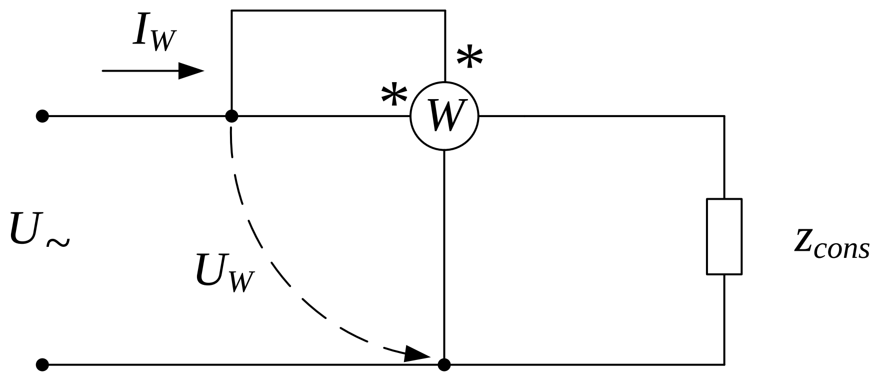


Figure 5.3

A wattmeter measures the active power of the circuit section to which it is connected. Active power is defined as the real part of the complex power of this section by the equation:

$$P_W = \mathrm{Re}[\underline{U}_W \overset{*}{\underline{I}}_W] = U_W I_W \cos\varphi \,, \qquad (5.17)$$

where $\underline{U}_w$ is complex of RMS voltage drop over the terminals of the wattmeter voltage measuring winding;

$\overset{*}{\underline{I}}_w$ is complex conjugate value of RMS current flowing across the wattmeter current measuring winding;

φ is phase shift angle between voltage and current of the wattmeter.

5.6 Complex numbers application in the electric circuits calculations

It is necessary to be able to use the complex numbers at calculation harmonic current circuits.

It is known from mathematics courses that a complex number can be presented in the algebraic and the exponential forms (table 5.4).

Transition from the exponential to the algebraic form is done by Euler's formula:

$$\underline{A} = Ae^{j\alpha} = A\cos\alpha + jA\sin\alpha = a + jb. \qquad (5.18)$$

Transition from the algebraic to the exponential form is follows:

$$\underline{A} = a + jb = \sqrt{a^2 + b^2}\, e^{jarctg\frac{b}{a}} = Ae^{j\alpha}. \qquad (5.19)$$

It should be remembered that if a real part of a complex number is negative, then to angle α additional angle $\pm\pi$ must be summed as the tangents of two angles differing by $\pm\pi$ are the same

Examples:

1. $\underline{A}_1 = -5 + j6 = \sqrt{5^2 + 6^2}\, e^{j\left(arctg\frac{6}{-5} \pm 180^0\right)} = 7{,}81e^{-j230{,}2^0} = 7{,}81e^{j129{,}8^0}$

2. $\underline{A}_2 = -4 - j3 = \sqrt{4^2 + 3^2}\, e^{j\left(arctg\frac{-3}{-4} \pm 180^0\right)} = 5e^{j216{,}9^0} = 5e^{-j143{,}1^0}$

Table 5.4

Algebraic form	$\underline{A} = a + jb$	a is the complex number real part; b is the complex number imaginary part
Exponential form	$\underline{A} = Ae^{j\alpha}$	A is the complex number module; α is the complex number argument

Operations with complex numbers

1. *Addition and subtraction* of complex numbers are performed in the algebraic form by adding (subtracting) separately the real and imaginary parts:

$$\underline{A} = \underline{A}_1 \pm \underline{A}_2 = \left(a_1 + jb_1\right) \pm \left(a_2 + jb_2\right) = \left(a_1 \pm a_2\right) + j\left(b_1 \pm b_2\right) = a + jb \quad .(5.20)$$

2. *Multiplication (division)* of complex numbers is performed in the exponential form (but it is also possible in the algebraic form) by multiplication (division) of modules and addition (subtraction) of arguments:

$$\underline{A} = A_1 e^{j\alpha_1} \cdot A_2 e^{j\alpha_2} = A_1 \cdot A_2 e^{j\left(\alpha_1 + \alpha_2\right)} = Ae^{j\alpha}; \qquad (5.21)$$

$$\underline{A} = \frac{A_1 e^{j\alpha_1}}{A_2 e^{j\alpha_2}} = \frac{A_1}{A_2} e^{j\left(\alpha_1 - \alpha_2\right)} = Ae^{j\alpha}. \qquad (5.22)$$

3. Exponentiation is performed as follows:

$$\left(Ae^{j\alpha}\right)^n = A^n e^{jn\alpha}. \qquad (5.23)$$

6. THE TASK CONTENT

Calculation of the branched electric circuit of harmonic current

1) If **there is no** magnetic coupling between coils:

a) determine currents and voltages in all branches;

b) calculate active and reactive power balance, check its fulfillment and determine the wattmeter value;

c) draw a vector diagram of currents and voltages;

d) plot the instantaneous EMF values $e(t)$ and current in the branch of source $i(t)$ in one coordinate axis.

2) If **there is** a magnetic coupling between the coils:

a) compile a system of equations according to Kirchhoff's laws in the complex form;

b) calculate the currents and check the correctness of the calculations by the method of decoupling of inductive couplings, substituting the values of the currents found in the system of equations compiled by Kirchhoff's laws;

c) determine the voltage drops over the branches;

d) find the power balance for two branches containing inductively coupled coils.

Variants of the task are given in Attachment.

7. EXAMPLES OF ELECTRIC CIRCUITS CALCULATION

7.1 Calculation of the harmonic current circuit without the magnetic coupling between the coils

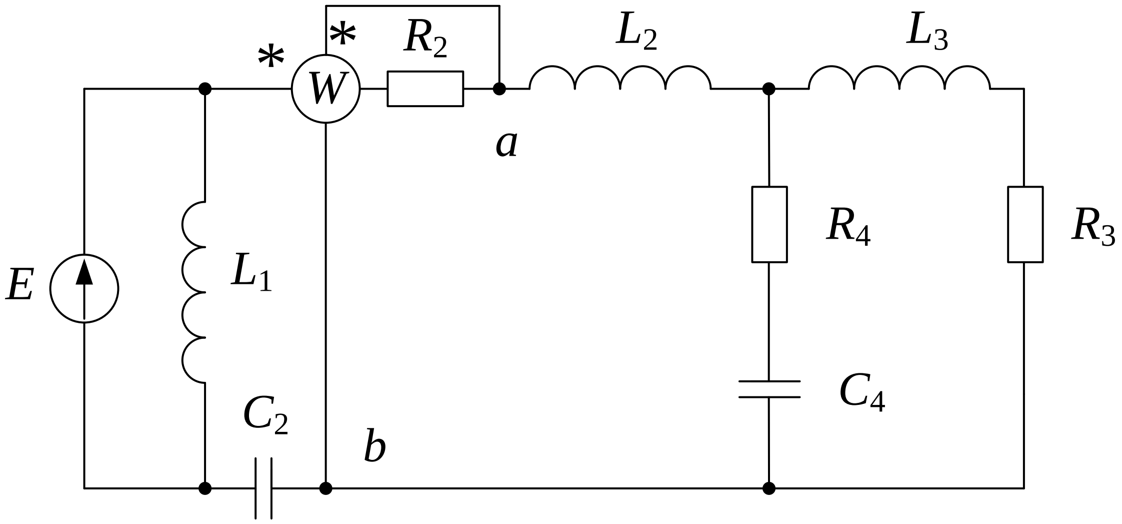


Figure 7.1

The electric circuit (fig. 7.1) parameters have the following values:

E=120 V	L_1=8 mH	C_2=66,5 μF		C_4=100 μF
Ψ_E =30^o		L_2=5.59 mH	L_3=3,194 mH	
f=200 Hz		R_2=1 Ohm	R_3=3 Ohm	R_4=6 Ohm

Important: the measuring devices (in this case, a wattmeter) should not be taken into account at the electric circuit calculation.

For the circuit shown in fig. 7.2, determine the currents and voltages of branches by the complex method, check the power balance. Draw vector diagrams and graphs of $e(t)$, $i(t)$ instant values.

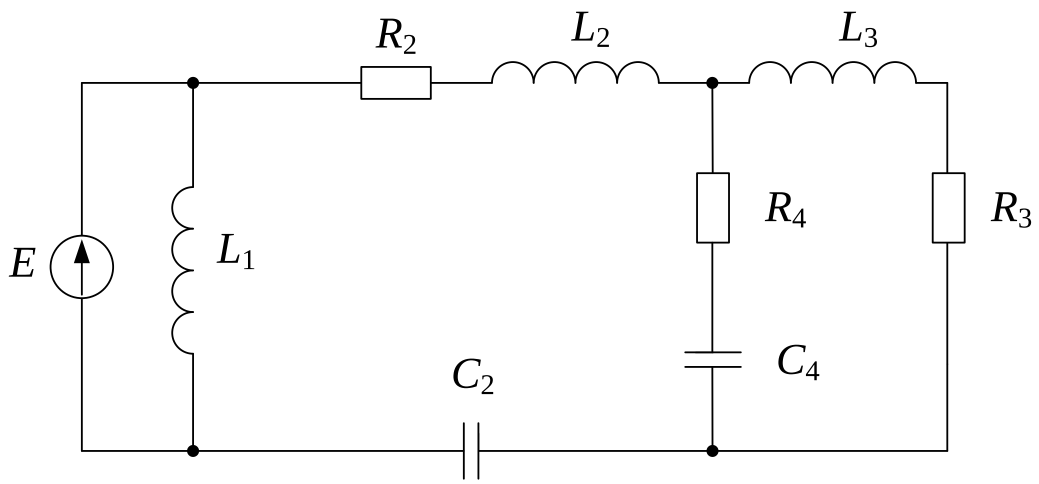


Figure 7.2

Solution:

1) Determination of angular frequency:

$$\omega = 2\pi\, f = 2\pi \cdot 200 = 1257\ \ \text{s}^{-1}.$$

2) Branches reactance calculation (inductive and capacitive resistances):

$$\boxed{X_L = \omega L;\ X_C = -\frac{1}{\omega C}}$$

$$X_{L1} = \omega L_1 = 1257 \cdot 8 \cdot 10^{-3} = 10{,}06\ \text{Ohm};$$

$$X_{L2} = \omega L_2 = 1257 \cdot 5{,}59 \cdot 10^{-3} = 7{,}03\ \text{Ohm};$$

$$X_{C2} = -\frac{1}{\omega C_2} = -\frac{1}{1257 \cdot 66{,}5 \times 10^{-6}} = -11{,}96\ \text{Ohm};$$

$$X_{L3} = \omega L_3 = 1257 \cdot 3{,}194 = 4{,}01\ \text{Ohm};$$

$$X_{C4} = -\frac{1}{\omega C_4} = -\frac{1}{1257 \cdot 100 \cdot 10^{-6}} = -7{,}96\ \text{Ohm}.$$

3) Branches complex reactance calculation:

$$\boxed{\underline{Z} = R + jX = \sqrt{R^2 + X^2}\, e^{j\,arctg\frac{X}{R}} = Ze^{j\varphi}}$$

Note: if the branch has only the reactance X (R = 0), then

$$\varphi = arctg\frac{X}{R} = arctg\frac{X}{0} = arctg \pm \infty = \pm 90^\circ$$

$\underline{Z}_1 = j\omega L_1 = j10{,}06 \ = 10{,}06 \ e^{j90^o}$ Ohm;

$$\underline{Z}_2 = R_2 + j\left(\omega L_2 - \frac{1}{\omega C_2}\right) = 1 + j(7{,}03 - 11{,}96) = 1 - j4{,}93 =$$

$= 5{,}03 \ e^{-j78{,}5^o}$ Ohm;

$\underline{Z}_3 = R_3 + j\omega L_3 = 3 + j4{,}01 = 5{,}01 \ e^{j53{,}2^o}$ Ohm;

$$\underline{Z}_4 = R_4 - j\frac{1}{\omega C_4} = 6 - j7{,}96 = 9{,}97 \ e^{j-53^o} \text{ Ohm.}$$

As can be seen from fig. 7.3, this circuit is mixed connected.

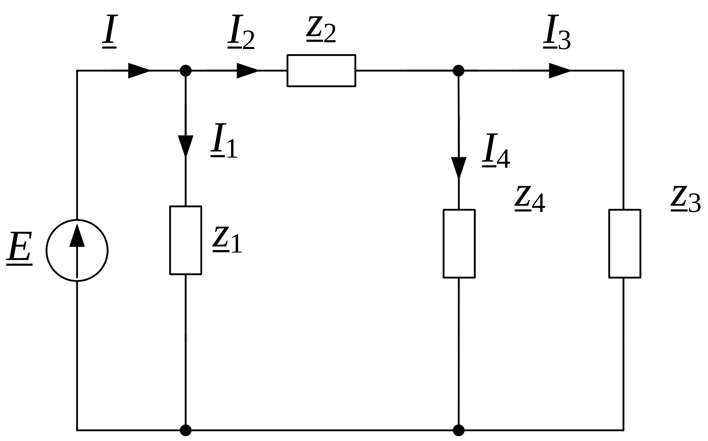


Figure 7.3

Branches $\underline{Z}_3$ and $\underline{Z}_4$ are connected in parallel. Their equivalent resistance is defined as follows:

$$\underline{Z}_{34} = \frac{\underline{Z}_3 \cdot \underline{Z}_4}{\underline{Z}_3 + \underline{Z}_4} = \frac{5{,}01 \ e^{j53{,}2^0} \cdot 9{,}97 \ e^{-j53^0}}{3 + j4{,}01 + 6 - j7{,}94} = \frac{43{,}95 \ e^{j0{,}2^0}}{9 - j3{,}95} = \frac{43{,}95 \ e^{j0{,}2^0}}{9{,}83 \ e^{-j23{,}7^0}} = 5{,}08 \ e^{j23{,}9^0} =$$

$= 4{,}64 + j2{,}06$ Ohm.

Impedances $\underline{Z}_2$ and $\underline{Z}_{34}$ are connected in series, as can be seen from fig. 7.4.

$$\underline{Z}_{234} = \underline{Z}_2 + \underline{Z}_{34} = 1 - j4{,}93 + 4{,}64 + j2{,}06 = 5{,}64 - j2{,}87 = 6{,}33 \ e^{-j27^0} \text{ Ohm.}$$

Impedances $\underline{Z}_1$ and $\underline{Z}_{234}$ are connected in parallel, but it is not necessary to determine their equivalent resistance, as the voltage drop over these impedances is known (it is equal to the source voltage of EMF). Using Ohm's law in the complex form, you can calculate the currents of these branches.

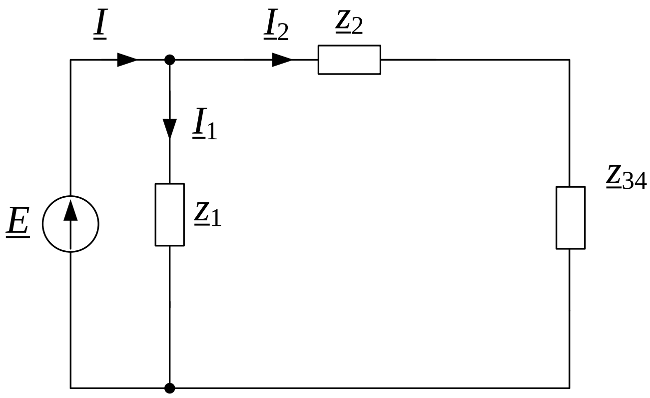


Figure 7.4

Complex form of EMF:

$$\underline{E} = Ee^{j\Psi_E} = 120e^{j30^o} \text{ V.}$$

3) Determination of branches currents:

$$\underline{I}_1 = \frac{\underline{E}}{\underline{Z}_1} = \frac{120e^{j30^o}}{10{,}06e^{j90^o}} = 11{,}93e^{-j60^0} = 5{,}96 - j10{,}33 \text{ A;}$$

$$\underline{I}_2 = \frac{\underline{E}}{\underline{Z}_{234}} = \frac{120e^{j30^o}}{6{,}33e^{-j27^0}} = 18{,}96e^{j57^0} = 10{,}32 + j15{,}9 \text{ A.}$$

Current $\underline{I}_3$ is determined by the current divider rule:

$$\underline{I}_3 = \underline{I}_2 \cdot \frac{\underline{Z}_4}{\underline{Z}_3 + \underline{Z}_4} =$$

$$= 18{,}96e^{j57^o} \cdot \frac{9{,}97e^{-j53^o}}{3 + j4{,}01 + 6 - j7{,}96} = 19{,}23e^{j27{,}7^o} = 17{,}03 + j8{,}94 \text{ A.}$$

Currents $\underline{I}_4$ and $\underline{I}$ can be determined by Kirchhoff's current law:

$\underline{I}_4 = \underline{I}_2 - \underline{I}_3 = 10{,}32 + j15{,}9 - 17{,}03 - j8{,}94 = -6{,}71 + j6{,}96 = 9{,}67e^{j134}$A;

$\underline{I} = \underline{I}_1 + \underline{I}_2 = 5{,}96 - j10{,}33 + 10{,}32 + j15{,}9 = 16{,}28 + j5{,}57 = 17{,}21e^{j18{,}9}$ A.

4) Determination of branch voltages:

$$\underline{U}_1 = \underline{E} = 120e^{j30^o} = 103{,}92 + j60 \text{ V;}$$

$$\underline{U}_2 = \underline{Z}_2 \cdot \underline{I}_2 = 5{,}03e^{-j78{,}5^0} \cdot 18{,}96e^{j57^0} = 95{,}37e^{-j21{,}5^0} = 88{,}73 - j34{,}95 \text{ V;}$$

$$\underline{U}_{34} = \underline{Z}_{34} \cdot \underline{I}_2 = 5{,}08e^{j23{,}9^0} \cdot 18{,}96e^{j57^0} = 96{,}32e^{j80{,}9^0} = 15{,}23 + j95{,}11 \text{ V.}$$

The correctness of the calculations is verified by Kirchhoff's voltage law:

$$\underline{U}_2 + \underline{U}_{34} = \underline{E};$$

$$88{,}73 - j34{,}95 + 15{,}23 + j95{,}11 = 103{,}96 + j60{,}16;$$

$$103{,}96 + j60{,}16 \approx 103{,}92 + j60.$$

Kirchhoff's voltage law is fulfilled, so the voltages of the branches are found correctly.

5) Checking the active and reactive power balance.

Complex power generated by the source:

$$\underline{S}_{gen} = \underline{E} \cdot \overset{*}{\underline{I}} = P_{gen} + jQ_{gen} = 120e^{j30} \cdot 17{,}21e^{-j18{,}9} = 2065e^{j11{,}1} = 2026 + j398 \text{VA.}$$

Calculation of the power complexes consumed by the branches:

$$\underline{S}_1 = \underline{U}_1 \cdot \overset{*}{\underline{I}} = 120e^{j30^0} \cdot 11{,}93e^{j60^0} = 1432e^{j90^0} = j1432 \text{ VA;}$$

$$\underline{S}_2 = \underline{U}_2 \cdot \overset{*}{\underline{I}}_2 = 95{,}37e^{-j21{,}5^0} \cdot 18{,}96e^{-j57^0} = 1808e^{-j78{,}5^0} = 361 - j1772 \text{ VA;}$$

$$\underline{S}_3 = \underline{U}_3 \cdot \overset{*}{\underline{I}}_3 = 96{,}32e^{j80{,}9^0} \cdot 19{,}23e^{-j27{,}7^0} = 1852e^{j53{,}2^0} = 1109 + j1483 \text{ VA;}$$

$$\underline{S}_4 = \underline{U}_{34} \cdot \overset{*}{\underline{I}}_4 = 96{,}32e^{j80{,}9^0} \cdot 9{,}67e^{-j134^0} = 931e^{-j53{,}1^0} = 559 - j745 \text{ VA.}$$

Power consumed by the whole circuit:

$$\underline{S}_{cons} = P_{cons} + jQ_{cons} = \underline{S}_1 + \underline{S}_2 + \underline{S}_3 + \underline{S}_4 =$$

$$= j1432 + 361 - j1772 + 1109 + j1483 + 559 - j745 = 2029 + j398 \text{ VA.}$$

The active power balance:

$$P_{gen}=P_{cons}\ ;\ \ 2026\ \text{W} \approx 2029\ \text{W}.$$

The reactive power balance:

$$Q_{gen}=Q_{cons}\ ;\ 398\ \text{VAr} = 398\ \text{VAr}.$$

Note: in the active and the eactive power balance equations the error should not exceed 5 %.

Thus, the balance of active and reactive power is performed, which confirms correct calculation of the electrical circuit currents and voltages.

6) Determination of the wattmeter readings.

The active power of the circuit section is determined by (5.17):

$$P_w = \mathrm{Re}\left[\underline{U}_w \overset{*}{\underline{I}}_w\right].$$

Voltage $\underline{U}_w$ is determined by Kirchhoff's voltage law for a loop that includes this voltage (fig. 7.5):

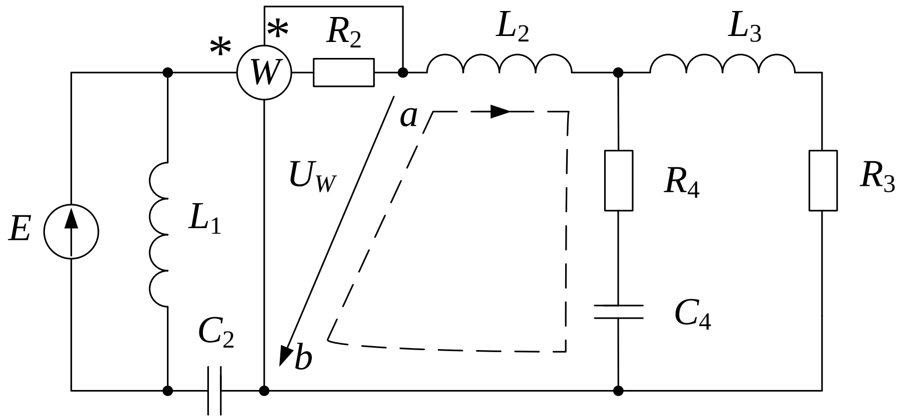


Figure 7.5

$$\underline{U}_{\mathrm{L2}} + \underline{U}_{34} - \underline{U}_w = 0.$$

Thus

$$\underline{U}_w = \underline{U}_{L2} + \underline{U}_{34} = \underline{I}_2 \cdot j\omega L + \underline{U}_{34} = (10{,}32 + j15{,}9)\cdot j7{,}03 + 15{,}23 + j95{,}11 =$$
$$= j72{,}55 - 111{,}78 + +15{,}23 + j95{,}11 = -96{,}55 + j167{,}66 =$$
$$= 193{,}36 e^{j119{,}9^\circ}\ \text{V}.$$

$\overset{*}{\underline{I}}_w$ flows in the second branch:

$$\overset{*}{\underline{I}}_w = \overset{*}{\underline{I}}_2 = 18{,}96e^{-j57^\circ} \text{ A.}$$

So, wattmeter readings is:

$$P_w = \mathrm{Re}\left[193{,}36e^{j119{,}9^\circ} \cdot 18{,}96e^{-j57^\circ}\right] = \mathrm{Re}\left[3666{,}1e^{j62{,}9^\circ}\right] = 3666{,}1 \cdot \cos 62{,}9^\circ = $$
$$= 1670 \text{ W.}$$

7) The vector diagram drawing.

The vector diagram for the branched circle is drawing in accordance with Kirchhoff's laws:

$$\underline{I} = \underline{I}_1 + \underline{I}_2; \tag{7.1}$$

$$\underline{I}_2 = \underline{I}_3 + \underline{I}_4; \tag{7.2}$$

$$\underline{U}_1 = \underline{U}_{34} + \underline{U}_2. \tag{7.3}$$

To draw the vector diagram, the RMS values of currents and voltages in the circuit sections are used, as well as the phase shift angles between them. RMS values of voltages, currents and phase shift angles are:

U_1=E=120 V	I_1=11,93 A;	φ_1=90^o
U_2=95,37 V	I_2=18,96 A	φ_2= – 78,5^o
U_{34}=96,32 V	I_3=19,23 A	φ_3=53,2^o
	I_4=9,67 A	φ_4= – 53^o
	I=17,21 A	

Important:

- the phase shift angle φ is drawn from the current vector to the voltage vector in the vector diagram.

If angle φ>0 (inductive nature of the circuit section), it is drawn counter-clockwise, and if φ<0 (capacitive nature of the circuit section), it is drawn clockwise.

- A vector sum of two vectors is done by the parallelogram rule. If it is necessary to draw a vector sum of more vectors, each subsequent vector is drawn from the end of the previous one. The sum vector connects the beginning of the first vector with the end of the last one.
- The scales of voltage m_U and of current m_I should be selected.

The vector diagram drawing for the branched circle is started from the branched section. As the third and fourth branches are connected in parallel, voltage vector U_{34} is drawn first and from it currents I_4 and I_3 are drawn at the phase shift angles φ_4 and φ_3, as shown in fig. 7.6.

In drawing, vectors of currents I_4 and I_3 are added geometrically, their sum is equal to current I_2 according to equation (7.2). According to equation (7.3), voltage vector of the second branch U_2 is added to vector U_{34}. U_2 direction is found by turning by angle φ_2 relatively current I_2 and its module is known. Obtaining voltage U_1 equal to EMF is done by adding geometrically U_{34} and U_2.

And finally, according to equation (7.1) current vector I_1 is added to vector I_2, current vector I_1 is built at angle φ_1 relatively to voltage U_1. A geometric sum of vectors I_1 and I_2 is equal to the source current I.

So, the diagram is drawn in the following order:

$$\overline{U}_{34}\begin{matrix}\rightarrow -\varphi_4 \rightarrow \overline{I}_4 \\ \rightarrow -\varphi_3 \rightarrow \overline{I}_3\end{matrix} + \rightarrow \overline{I}_2 \rightarrow \varphi_2 \rightarrow \overline{U}_{34} \rightarrow \overline{U}_{34} + \overline{U}_2 = \overline{U}_1 \rightarrow -\varphi_1 \rightarrow$$

$$\rightarrow \overline{I}_1 \rightarrow \overline{I}_1 + \overline{I}_2 = \overline{I}$$

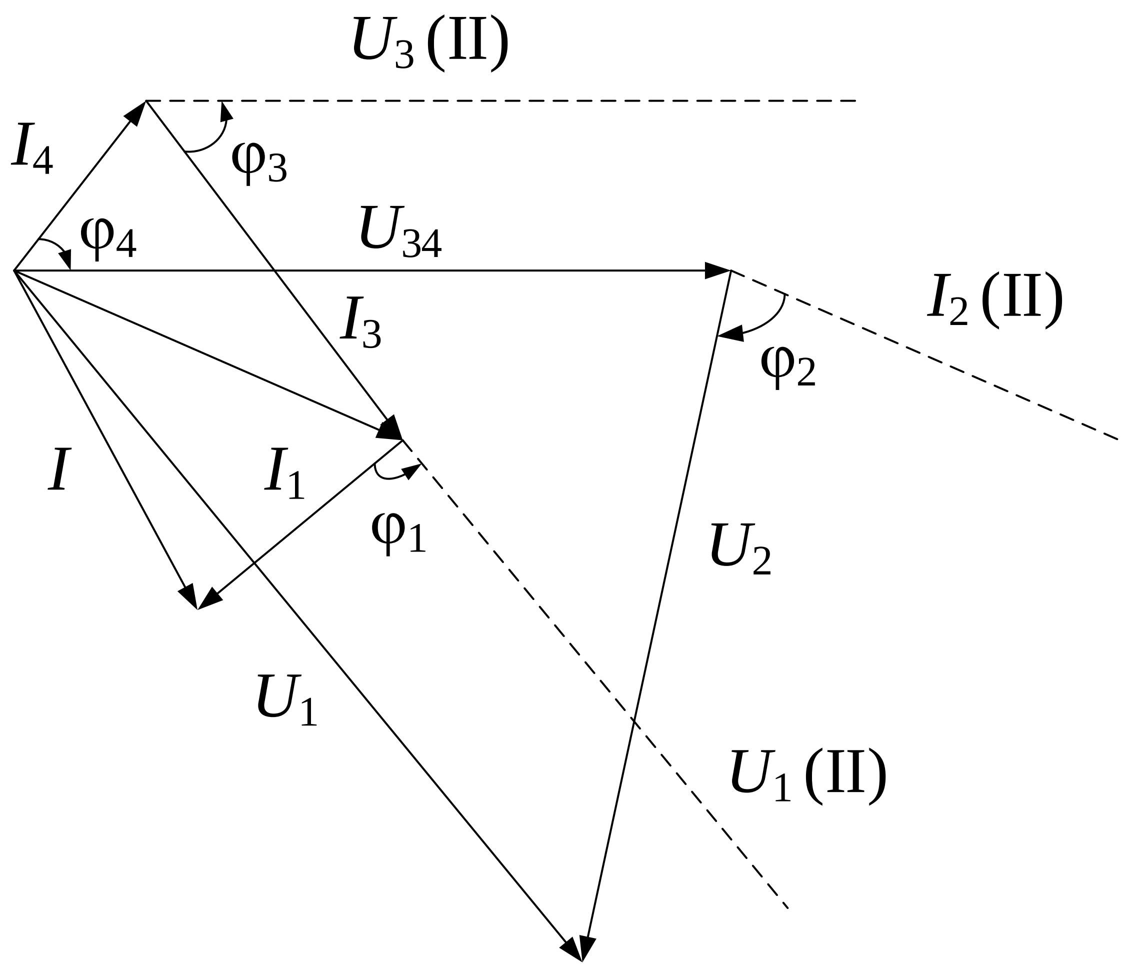


Figure 7.6

8) Timelines drawing.

Instantaneous values of EMF $e(t)$ and current $i(t)$ in the source branch can be written by complexes of RMS values of EMF $\underline{E} = 120 e^{j30^o}$ and calculated current $\underline{I} = 17{,}21 e^{j18{,}9}$:

$$e(t) = 120\sqrt{2}\sin(\omega t + 30^\circ) = 170\sin(\omega t + 30^\circ) \quad V;$$

$$i(t) = 17{,}21\sqrt{2}\sin(\omega t + 18{,}9^\circ) = 24\sin(\omega t + 18{,}9^\circ) \quad A.$$

Their graphs are shown in fig. 7.7.

The axis ωt can be marked in radians or degrees (for example, 30º corresponds to $\frac{\pi}{6}$ radians and 18.9º corresponds to 0.11π radians). A positive initial phase is drawn on the horizontal axis on the left from the origin, the negative – from the right. The positive half-wave of sine waves begins to the right of this mark.

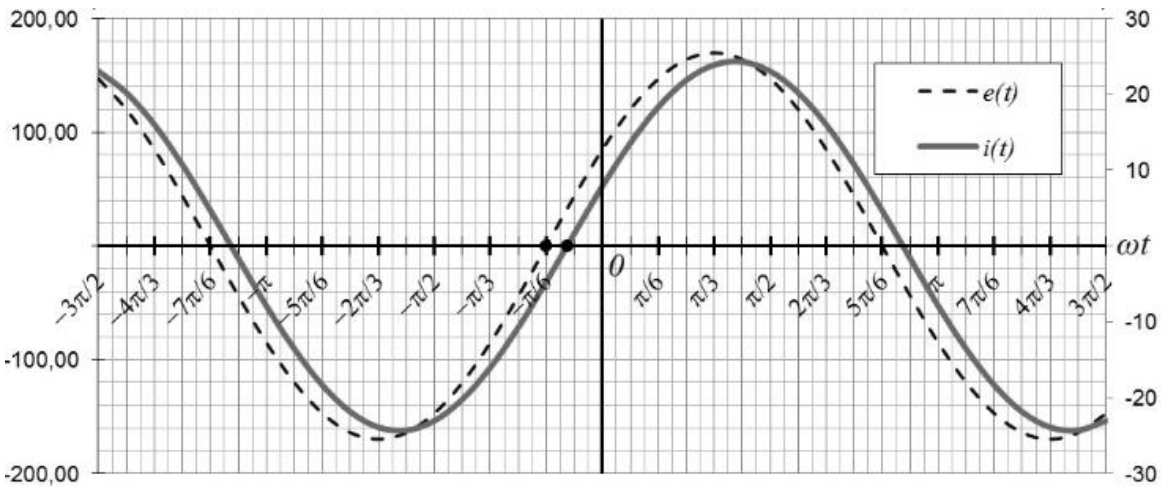


Figure 7.7

7.2 Calculation of the electric circuit of harmonic current with the inductive coupling between the coils

If an alternating current in one of the coils causes the EMF in the other one, then these two coils have a magnetic coupling (they are inductively coupled) and the resulting EMF is called a mutual induction EMF. Presence of the mutual induction EMF must be taken into account for the electrical circuit calculation.

The voltage drop over coil with inductance L_k depends on its own current i_k, which creates a self-inducing EMF $e_s = -L_k \frac{di_k}{dt}$ and on current i_m of inductively coupled coil with inductance L_m that creates EMF of mutual induction $e_m = \mp M \frac{di_m}{dt}$ in coil with inductance L_k:

$$u_k = L_k \frac{di_k}{dt} \pm M \frac{di_m}{dt}. \tag{7.4}$$

where M is the mutual inductance of coils k and m.

If the coils' currents have the same direction relative to the same coil terminals, the coils have *aiding* connection. In this case, the self- and the mutual magnetic fluxes in each of them coincide in the direction, amplifying the resulting flux and voltage, so in the expression for voltage (7.4) the second term has a positive sign.

If the coils' currents are opposite, the coils are connected in the *opposite* direction. In this case, the self- and the mutual magnetic fluxes in each of them have opposite directions, and they weaken the resulting flux and voltage, so in the expression for voltage (7.4) the second term has a negative sign.

The expression for this voltage in the complex form has the form:

$$\underline{U}_k = j\omega L_k \underline{I}_k \pm j\omega M \underline{I}_m. \tag{7.5}$$

Kirchhoff's laws, the loop current method, or a special technique called the decoupling of inductive couplings method are used for branching circuit calculations.

For example, the scheme of the previous task was used (see fig. 7.1). It includes the inductive coupling between the coils of the second and third branches, as shown in fig. 7.6.

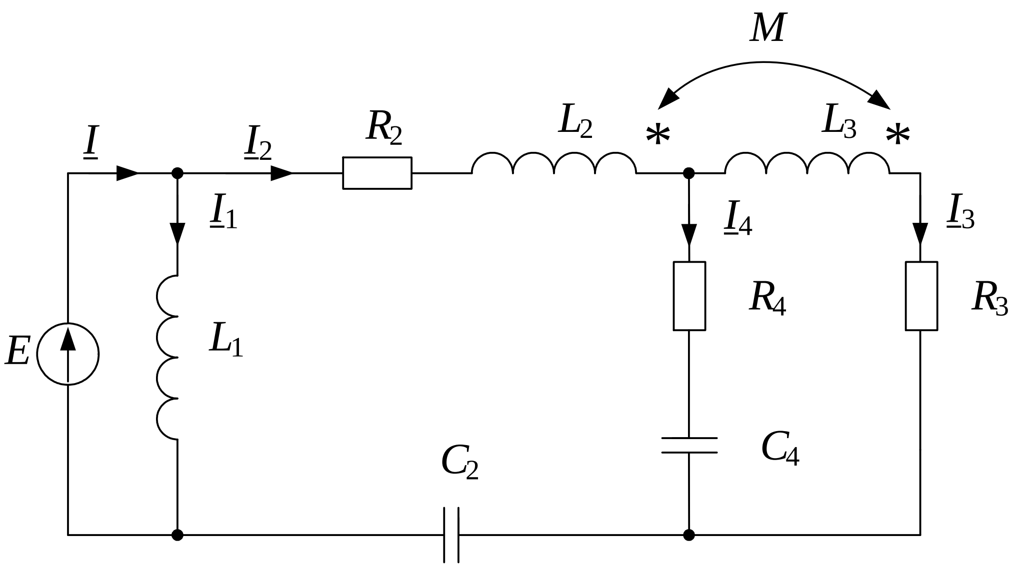


Figure 7.6

The values of the circuit parameters are:

$E = 120$ V; $\Psi_E = 30^\circ$; $\omega L_1 = 10{,}05$ Ohm; $R_2 = 1$ Ohm; $\omega L_2 = 7{,}02$ Ohm; $\dfrac{1}{\omega C_2} = 11{,}97$ Ohm; $\omega L_3 = 4{,}01$ Ohm; $R_3 = 3$ Ohm; $R_4 = 6$ Ohm; $\dfrac{1}{\omega C_4} = 7{,}96$ Ohm; $\omega M = 2$ Ohm.

1) *The system of equations in the complex form according to Kirchhoff's laws.*

The directions of currents in the branches should be selected. The loops' directions are selected clockwise.

$$\begin{cases} \underline{I} = \underline{I}_1 + \underline{I}_2; \\ \underline{I}_2 = \underline{I}_3 + \underline{I}_4; \\ j\omega L_1 \cdot \underline{I}_1 = \underline{E}; \\ (R_2 + j\omega L_2 - j\dfrac{1}{\omega C_2})\underline{I}_2 + j\omega M \underline{I}_3 + (R_4 - j\dfrac{1}{\omega C_2})\underline{I}_4 = \underline{E}; \\ (R_3 + j\omega L_3)\underline{I}_3 + j\omega M \underline{I}_2 - (R_4 - j\dfrac{1}{\omega C_4})\underline{I}_4 = 0. \end{cases} \tag{7.6}$$

Currents I_2 and I_3 have the same direction relative to the same terminals of coils with inductance L_2 and L_3, i.e. we have aiding connection. (In the case of different directions of currents I_2 and I_3 relative to the same coils' terminals, it will be opposite connection). So, terms $j\omega M \underline{I}_2$ and $j\omega M \underline{I}_3$ in equations (7.6) have a “plus” sign because directions of currents I_2 and I_3 coincide with the loops' directions and there is the aiding connection.

2) *Decoupling of inductive couplings*.

The meaning of this approach is replacing a circuit containing inductive couplings by an equivalent circuit without inductive couplings.

The general rule of decoupling of inductive couplings is follows:

if two inductively coupled branches are connected to a node of a branched electrical circuit by the **same** clamps, then inductance "–*M*" is additionally connected into these branches, and in the third branch free from magnetic coupling (also connected to this node), inductance "+*M*" is connected. If inductively coupled branches are connected to such a node by the different clamps, then inductance "+*M*" is connected into these branches, and inductance "–*M*" is connected in the third branch.

Note: *Only currents* can be calculated using the decoupling of inductive couplings method. Voltages are calculated by Kirchhoff's voltage law according to the initial scheme (fig. 7.6).

Figure 7.7 shows examples of several different circuits with two inductively coupled coils and their decoupling.

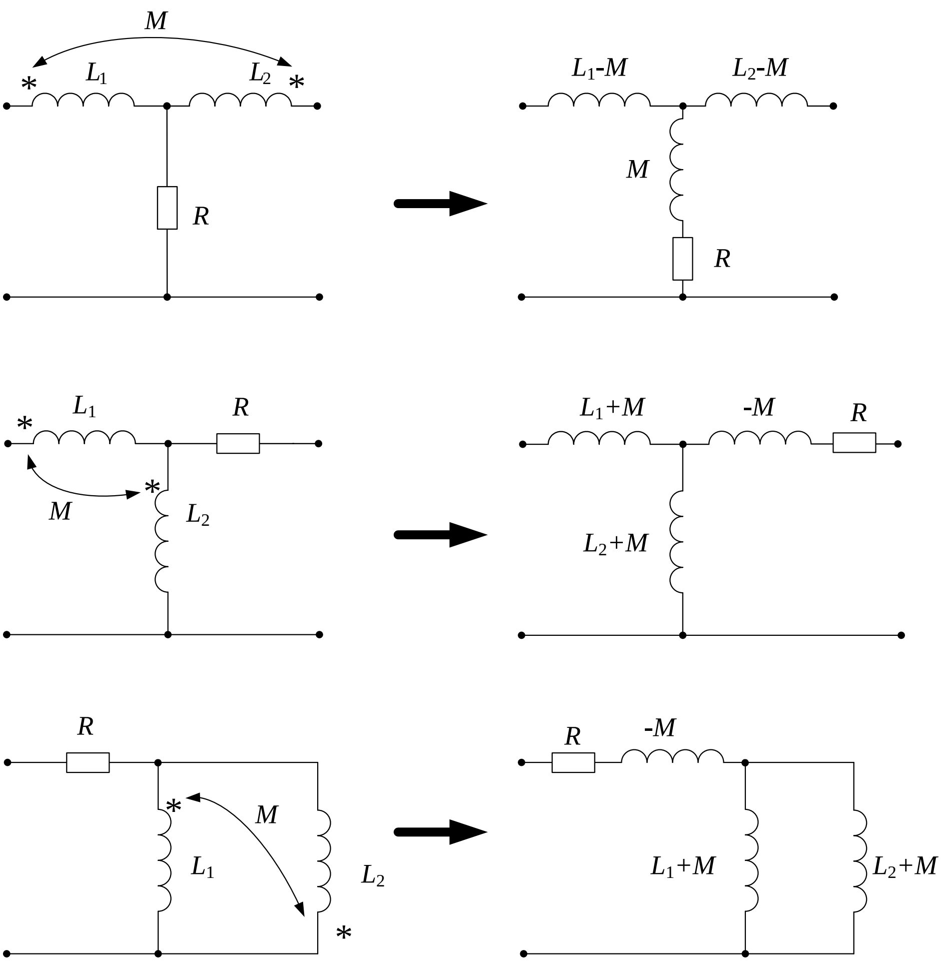


Figure 7.7

3) For calculations of circuit currents (see fig. 7.6), the decoupling of inductive couplings method is used. By this method, we obtain the circuit shown in fig. 7.8.

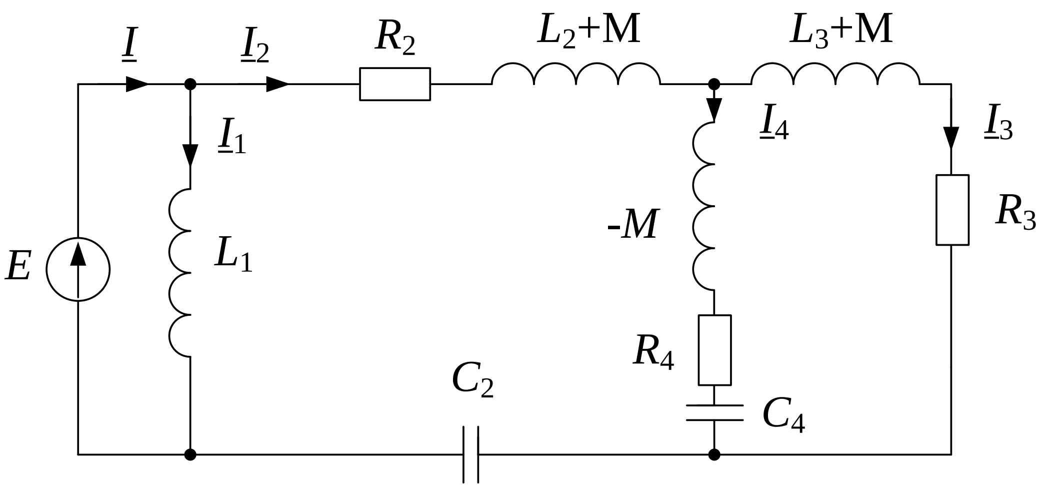


Figure 7.8

The resulting scheme has the mixed connection. In this scheme, the currents of the branches will be the same as in the original scheme (see fig. 7.6), and the voltages will be different.

4) Calculation of the branches’ impedances:

$$\underline{Z}_1 = j\omega L_1 = j10{,}06 = 10{,}06e^{j90^0} \quad \text{Ohm;}$$

$$\underline{Z}_2 = R_2 + j(\omega L_2 + \omega M - \frac{1}{\omega C_2}) = 1 + j(7{,}02 + 2 - 11{,}97) = 1 - j2{,}95 =$$

$$= 3{,}11e^{-j71{,}5^0} \quad \text{Ohm;}$$

$$\underline{Z}_3 = R_3 + j(\omega L_3 + \omega M) = 3 + j(4{,}01 + 2) = 3 + j6{,}01 = 6{,}72e^{j63{,}5^0} \quad \text{Ohm;}$$

$$\underline{Z}_4 = R_4 + j(-\omega M - \frac{1}{\omega C_4}) = 6 + j(-2 - 7{,}96) = 6 - j9{,}96 = 11{,}63e^{-j58{,}9^0} \quad \text{Ohm;}$$

$$\underline{Z}_{34} = \frac{\underline{Z}_3 \cdot \underline{Z}_4}{\underline{Z}_3 + \underline{Z}_4} = \frac{6{,}72e^{j63{,}5^0} \cdot 11{,}63e^{-j58{,}9^0}}{3 + j6{,}01 + 6 - j9{,}96} = 7{,}95e^{j28{,}3^0} = 7 + j3{,}77 \quad \text{Ohm;}$$

$$\underline{Z}_{234} = \underline{Z}_2 + \underline{Z}_{34} = 1 - j2{,}95 + 7 + j3{,}77 = 8 + j0{,}82 = 8{,}04e^{j5{,}9^0} \quad \text{Ohm.}$$

5) Calculation of the currents:

$$\underline{I}_1 = \frac{\underline{E}}{\underline{Z}_1} = 11{,}94e^{-j60^0} = 5{,}97 - j10{,}34 \ \text{A;}$$

$$\underline{I}_2 = \frac{\underline{E}}{\underline{Z}_{234}} = \frac{120e^{j30^0}}{8{,}04e^{j5{,}9^0}} = 14{,}93e^{j24{,}1^0} = 13{,}63 + j6{,}1 \ \text{A;}$$

$$\underline{I}_3 = \underline{I}_2 \frac{\underline{Z}_4}{\underline{Z}_3 + \underline{Z}_4} = 14{,}93e^{j24{,}1^0} \frac{11{,}63e^{-j58{,}9^0}}{3 + j6{,}01 + 6 - j9{,}96} = 17{,}66e^{-11{,}1^0} =$$

$$= 17{,}33 - j3{,}4 \quad \text{A;}$$

$$\underline{I}_4 = \underline{I}_2 - \underline{I}_3 = 13{,}63 + j6{,}1 - 17{,}33 + j3{,}4 = -3{,}7 + j9{,}5 = 10{,}2e^{111{,}3^0} \quad \text{A;}$$

$$\underline{I} = \underline{I}_1 + \underline{I}_2 = 5{,}97 - j10{,}34 + 13{,}63 + j6{,}1 = 19{,}6 - j4{,}24 = 20{,}05e^{-j12{,}2^0} \quad \text{A.}$$

The correctness of current calculations is checked by substituting the currents found in the system in the equations written by Kirchhoff's laws (7.4).

6) Calculation of voltages of individual circuit sections.

The decoupling by the inductive couplings method (see fig. 7.8) **<u>is used only to calculate currents</u>**. To determine the voltages of branches the initial scheme is used (see fig. 7.6).

$$\underline{U}_1 = \underline{E} = 120 e^{j30^0} = 103{,}92 + j60 \text{ V};$$

$$\underline{U}_2 = (R_2 + j\omega L_2 - j\frac{1}{\omega C_2})\underline{I}_2 + j\omega M\,\underline{I}_3 = (1 + j7{,}02 - j11{,}97)(13{,}63 + j6{,}1) +$$

$$+ j2(17{,}33 - j3{,}4) = 50{,}63 - j26{,}71 = 57{,}24 e^{-j27{,}8^0} \text{ V};$$

$$\underline{U}_3 = (R_3 + j\omega L_3)\underline{I}_3 + j\omega M\,\underline{I}_2 = (3 + j4{,}01)(17{,}33 - j3{,}4) + j2(13{,}63 + j6{,}1) =$$

$$= 53{,}42 + j86{,}55 = 101{,}71 e^{58{,}3^0} \text{ V};$$

$$\underline{U}_4 = (R_4 - j\frac{1}{\omega C_4})\underline{I}_4 = \underline{U}_3.$$

7) Power balance for the second and third branches.

Calculation of powers incoming into the second and third branches from the source:

$$\underline{S}_{2incom} = \underline{U}_2 \overset{*}{\underline{I}}_2 = 57{,}24 e^{-j27.8} \cdot 14{,}93 e^{-j24.1} = 854{,}6 e^{-j51..9} = 527 - j672 \text{ VA};$$

$$\underline{S}_{3incom} = \underline{U}_2 \overset{*}{\underline{I}}_3 = 101{,}71 e^{j58.3} \cdot 17{,}66 e^{j11.1} = 1796{,}2 e^{j69.4} = 632 + j1681 \text{ VA}.$$

Calculation of the powers consumed by the elements of the branches:

$$\underline{S}_{2cons} = R_2 I_2^2 + j(\omega L_2 - \frac{1}{\omega C_2})I_2^2 = 1 \cdot 14{,}93^2 + j(7{,}02 - 11{,}97)14{,}93^2 =$$

$$= 223 - j1103 \text{ VA};$$

$$\underline{S}_{3cons} = R_3 I_3^2 + j\omega L_3 I_3^2 = 3 \cdot 17{,}66^2 + j4{,}01 \cdot 17{,}66^2 = 936 + j1251 \text{ VA}.$$

Calculations of the powers transferred by magnetic fluxes because of the phenomenon of mutual induction:

$$\underline{S}_{2M} = j\omega M \underline{I}_3 \overset{*}{\underline{I}}_2 = 2e^{j90^0} \cdot 17{,}66 e^{-j11.1^0} \cdot 14{,}93 e^{-j24.1^0} = 527{,}3 e^{j54.8^0} =$$

$$= 304 + j431 \quad \text{VA};$$

$$\underline{S}_{3M} = j\omega M \underline{I}_2 \overset{*}{\underline{I}}_3 = 2e^{j90^0} \cdot 14{,}93 e^{j24.1^0} \cdot 17{,}66 e^{j11.1^0} = 527{,}3 e^{j125.2^0} =$$

$$= -304 + j431 \quad \text{VA}.$$

The power incoming in the branches from the source is equal to a sum of the power consumed by the elements of that branch and the power transferred by the magnetic flux:

$$\underline{S}_{2\text{cons}} + \underline{S}_{2M} = 223 - j1103 + 304 + j431 = 527 - j672 = \underline{S}_{2\text{const}};$$

$$\underline{S}_{3\text{cons}} + \underline{S}_{3M} = 936 + j1251 - 304 + j431 = 632 + j1682 = \underline{S}_{2\text{const}}.$$

Powers $\underline{S}_{2M}$ and $\underline{S}_{3M}$ can also be presented as generated by sources in the second and third branches, the EMFs of which is equal to the EMF of the mutual induction ($\underline{E}_{2M} = -j\omega M \underline{I}_3; \underline{E}_{3M} = -j\omega M \underline{I}_2$):

$$\underline{S}_{2M} = -j\omega M \underline{I}_3 \overset{*}{\underline{I}}_2 = -2e^{j90^0} \cdot 17{,}66 e^{-j11.1^0} \cdot 14{,}93 e^{-j24.1^0} = 527{,}3 e^{-j125.2^0} =$$

$$= -304 - j431 \quad \text{VA};$$

$$\underline{S}_{3M} = -j\omega M \underline{I}_2 \overset{*}{\underline{I}}_3 = -2e^{j90^0} \cdot 14{,}93 e^{j24.1^0} \cdot 17{,}66 e^{j11.1^0} = 527{,}3 e^{-j54.8^0} =$$

$$= 304 - j431 \quad \text{VA}.$$

Then the power balance is:

$$\underline{S}_{2\text{cons}} = \underline{S}_{2\text{const}} + \underline{S}_{2M} = 527 - j672 - 304 - j431 = 223 - j1103 \quad \text{VA};$$

$$\underline{S}_{3\text{cons}} = \underline{S}_{3\text{const}} + \underline{S}_{3M} = 632 + j1681 + 304 - j431 = 936 + j1251 \quad \text{VA}.$$

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ATTACHMENT

Table 1 – Variants of the task according to the calculations of the harmonic current circuit.

Variant	Group index					
	I	II	III	IV	V	VI
1	25 $\boxed{19}$	19 $\boxed{7}$	13 $\boxed{25}$	7 $\boxed{13}$	1 $\boxed{1}$	25 $\boxed{7}$
2	2 $\boxed{26}$	8 $\boxed{8}$	14 $\boxed{8}$	20 $\boxed{14}$	26 $\boxed{26}$	2 $\boxed{14}$
3	27 $\boxed{21}$	21 $\boxed{9}$	15 $\boxed{27}$	9 $\boxed{15}$	3 $\boxed{3}$	21 $\boxed{15}$
4	10 $\boxed{10}$	16 $\boxed{22}$	22 $\boxed{10}$	4 $\boxed{4}$	4 $\boxed{22}$	16 $\boxed{16}$
5	23 $\boxed{5}$	17 $\boxed{11}$	5 $\boxed{23}$	11 $\boxed{17}$	5 $\boxed{5}$	17 $\boxed{23}$
6	24 $\boxed{18}$	12 $\boxed{6}$	18 $\boxed{12}$	6 $\boxed{24}$	12 $\boxed{12}$	18 $\boxed{24}$
7	7 $\boxed{7}$	13 $\boxed{19}$	19 $\boxed{1}$	25 $\boxed{13}$	1 $\boxed{7}$	13 $\boxed{7}$
8	26 $\boxed{14}$	20 $\boxed{8}$	14 $\boxed{2}$	8 $\boxed{26}$	2 $\boxed{20}$	14 $\boxed{20}$
9	15 $\boxed{21}$	9 $\boxed{9}$	21 $\boxed{3}$	27 $\boxed{15}$	3 $\boxed{27}$	21 $\boxed{9}$
10	22 $\boxed{16}$	16 $\boxed{4}$	10 $\boxed{22}$	4 $\boxed{4}$	10 $\boxed{16}$	22 $\boxed{22}$
11	17 $\boxed{17}$	5 $\boxed{11}$	23 $\boxed{23}$	17 $\boxed{5}$	11 $\boxed{11}$	5 $\boxed{17}$
12	6 $\boxed{18}$	12 $\boxed{12}$	18 $\boxed{24}$	6 $\boxed{12}$	24 $\boxed{18}$	12 $\boxed{6}$
13	19 $\boxed{25}$	13 $\boxed{13}$	19 $\boxed{19}$	25 $\boxed{1}$	19 $\boxed{13}$	1 $\boxed{25}$
14	2 $\boxed{14}$	8 $\boxed{20}$	14 $\boxed{26}$	20 $\boxed{2}$	26 $\boxed{8}$	14 $\boxed{14}$
15	27 $\boxed{9}$	21 $\boxed{27}$	15 $\boxed{15}$	21 $\boxed{21}$	27 $\boxed{3}$	9 $\boxed{15}$
16	16 $\boxed{22}$	16 $\boxed{4}$	10 $\boxed{10}$	22 $\boxed{4}$	22 $\boxed{22}$	4 $\boxed{16}$
17	11 $\boxed{5}$	23 $\boxed{17}$	5 $\boxed{5}$	17 $\boxed{11}$	23 $\boxed{23}$	5 $\boxed{11}$

Continuation of Table 1.

Variant	Group index											
	I		II		III		IV		V		VI	
18	24	24	12	18	24	12	18	6	24	6	18	24
19	13	1	7	19	1	7	25	25	1	25	19	13
20	26	2	20	20	14	20	8	14	2	8	26	8
21	21	15	15	3	9	21	3	9	27	27	15	21
22	4	22	16	10	10	4	22	22	4	16	10	22
23	11	17	5	23	17	17	11	23	5	11	23	11
24	6	6	12	24	24	6	18	18	12	6	24	12
25	1	13	7	25	13	7	7	1	1	19	25	25
26	2	2	20	26	8	2	14	14	26	20	8	26
27	3	15	9	27	15	9	3	21	9	3	27	3
28	30	29	28	28	29	28	30	30	29	30	28	30
29	28	30	30	28	28	29	29	29	30	29	29	28
30	29	30	28	30	28	28	29	30	30	30	30	29
31	7	1	1	19	1	25	13	7	19	13	25	7
32	8	32	2	2	14	14	26	26	20	32	20	8
33	9	33	21	15	3	9	21	21	21	33	15	3
34	16	22	22	4	16	4	4	16	16	10	22	16

Variants of harmonic current circuits are shown in fig. 1.

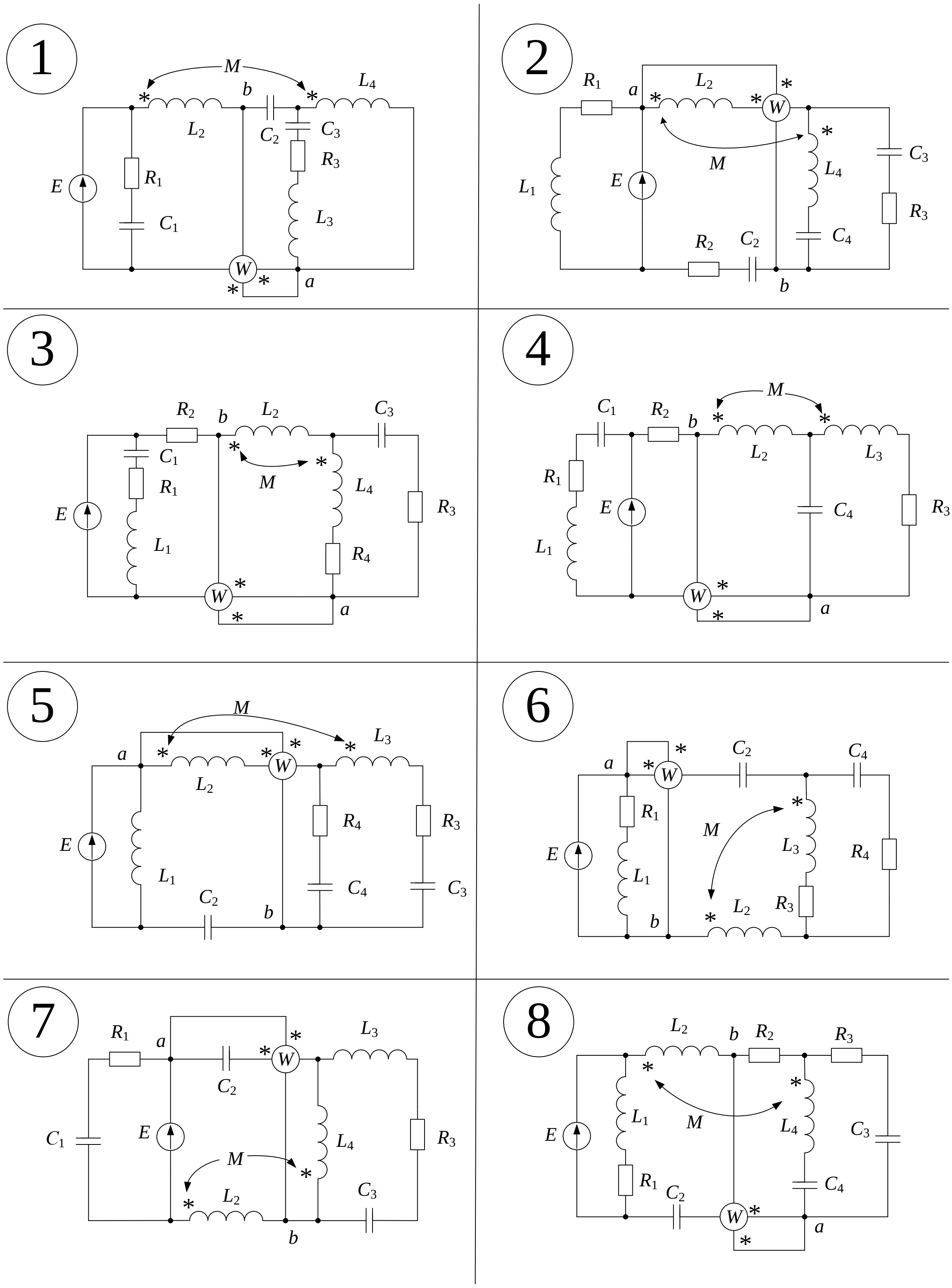

Figure 1 (sheet 1)

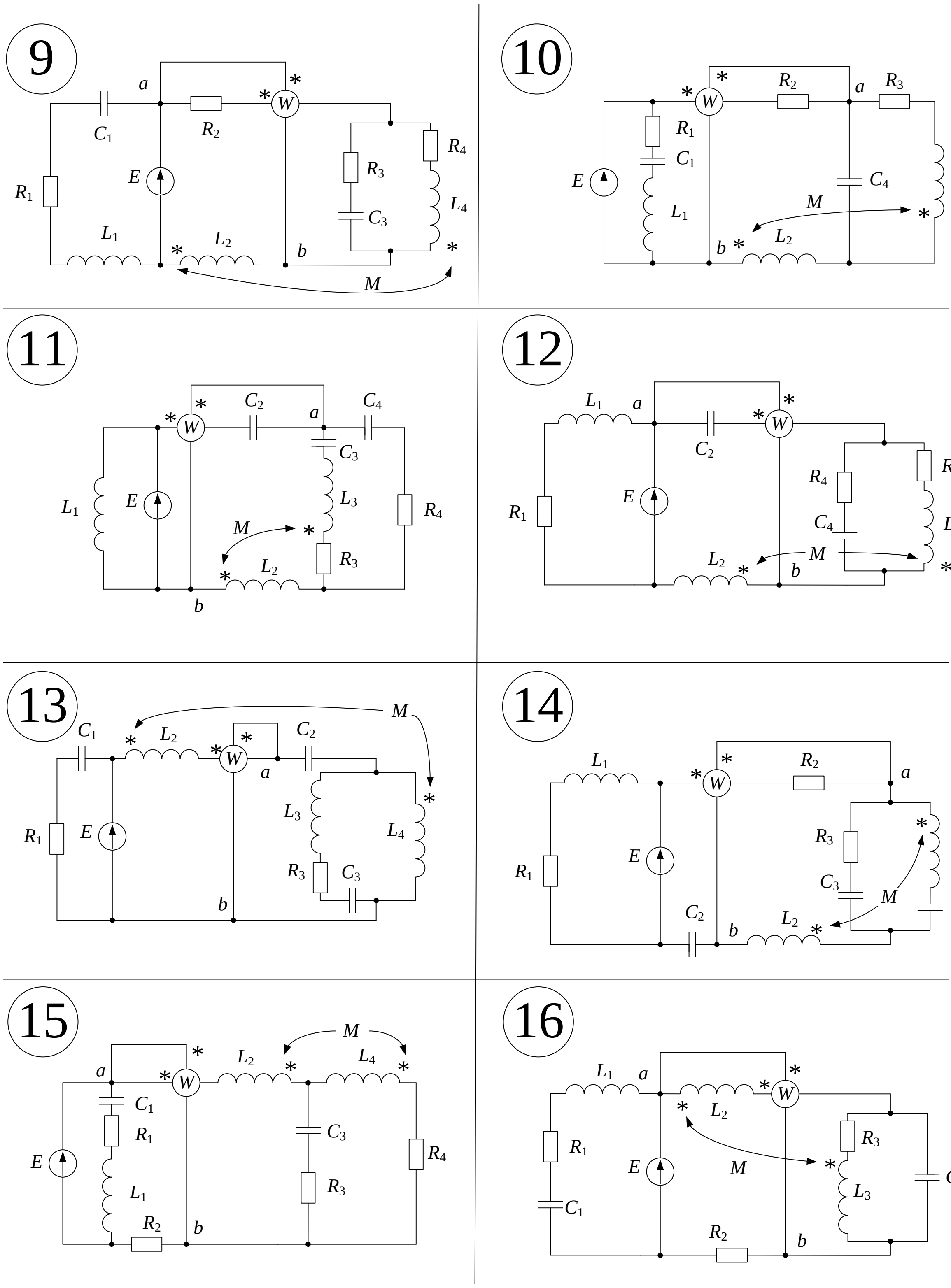

Figure 1 (sheet 2)

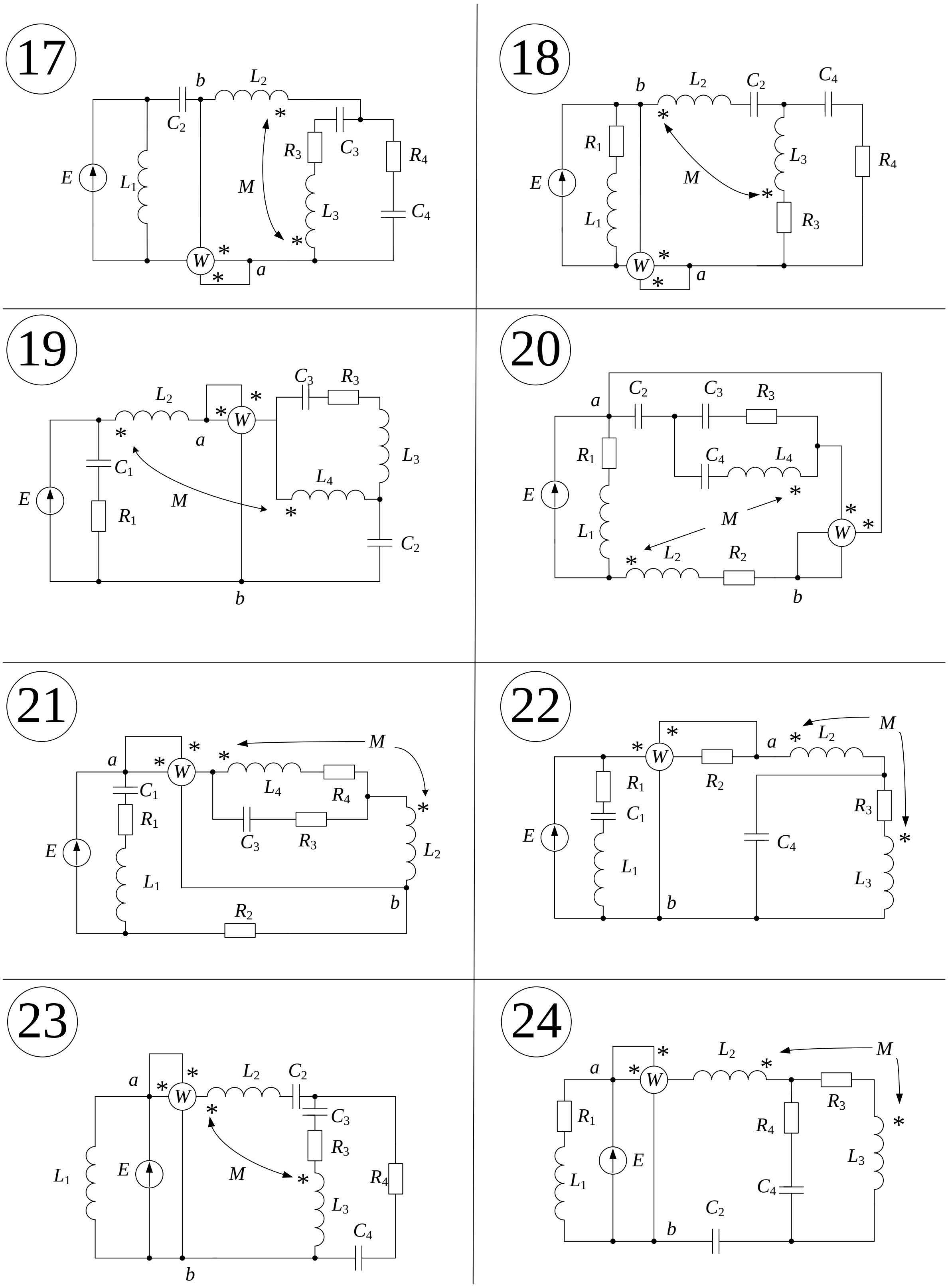

Figure 1 (sheet 3)

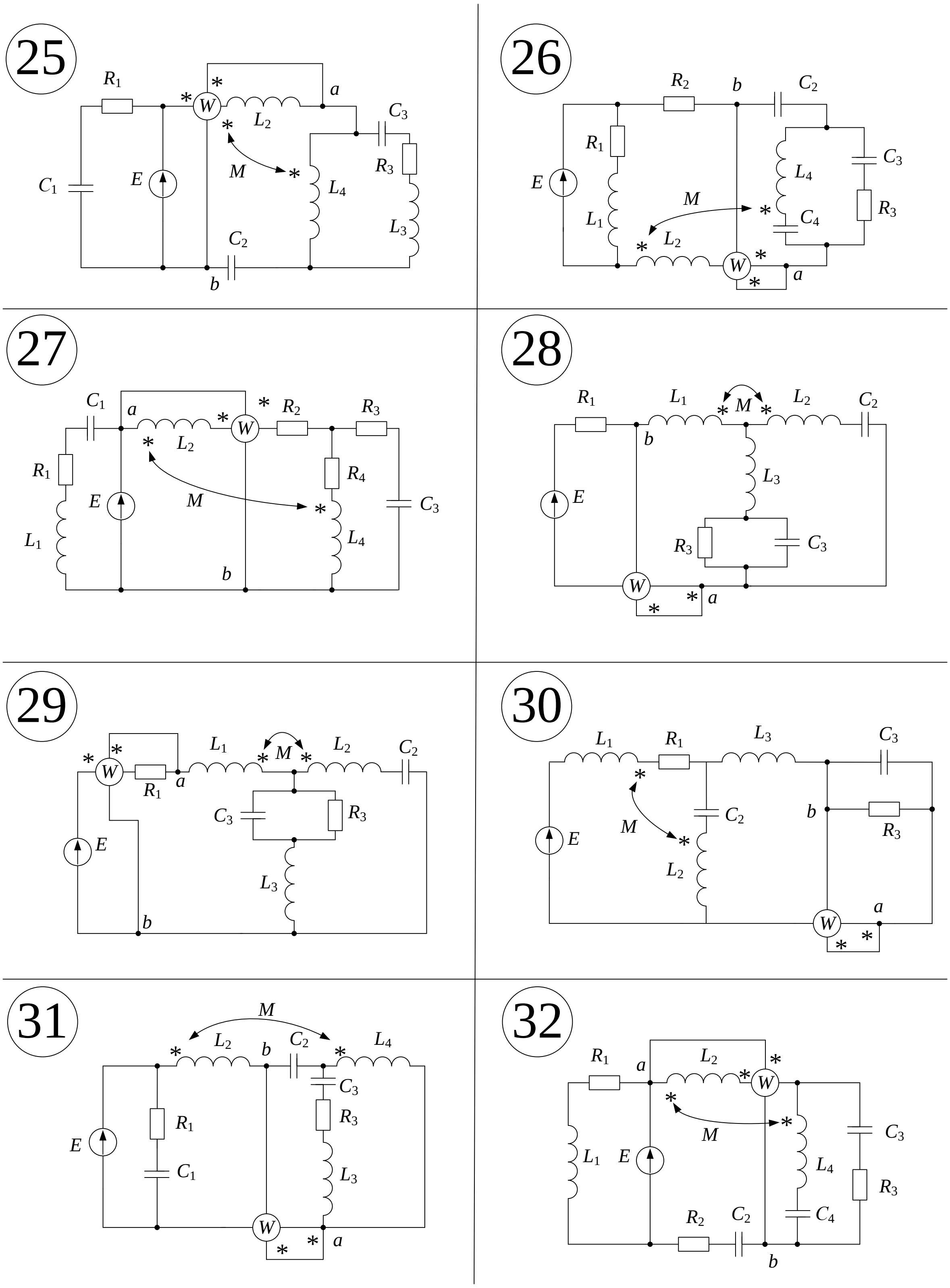

Figure 1 (sheet 4)

Variants of the numerical data of the circuits' parameters of the harmonic current are given in table 2.

Table 2 – Circuit diagram options

Data number	E V	ψ_e rad	f Hz	R_1 Ohm	L_1 mH	C_1 uF	R_2 Ohm	L_2 mH	C_2 uF	R_3 Ohm	L_3 mH	C_3 uF	R_4 Ohm	L_4 mH	C_4 uF	M mH
1	120	$\pi/2$	500	4	–	20	–	1	30	5	10,13	10	–	2	–	0,5
2	60	$\pi/6$	100	4	40	-	2	25,5	50	10	–	125	–	50	102	10
3	140	$\pi/3$	200	15	25,32	25	5	10	–	6	–	100	3	5	–	2
4	220	$\pi/4$	250	35	32	40	20	4	–	40	40	–	–	–	20	10
5	400	$\pi/6$	100	–	50	–	–	50	200	30	63,7	39,8	10	–	75	25
6	50	$-\pi/4$	400	6	10	–	–	12,75	6,25	40	12,5	–	60	–	5	5
7	240	$\pi/2$	50	4	–	200	–	10	300	5	101,3	100	–	20	–	5
8	300	$\pi/6$	500	4	8	–	2	5,1	10	10	–	25	–	10	20,4	2
9	280	$\pi/3$	250	15	20,26	20	5	8	–	6	–	80	3	4	–	1,6
10	110	$\pi/4$	200	35	40	50	20	5	–	40	50	–	–	–	25	12,5
11	40	$\pi/6$	200	–	25	–	–	25	100	30	31,85	19,9	10	–	37,5	12,5
12	100	$-\pi/4$	250	6	16	–	–	20,4	10	40	20	–	60	–	8	8
13	360	$\pi/2$	250	4	–	40	–	2	60	5	20,26	20	–	4	–	1
14	360	$\pi/6$	50	4	80	–	2	51	100	10	–	250	–	100	204	20
15	70	$\pi/3$	500	15	10,13	10	5	4	–	6	–	40	3	2	–	0,8
16	220	$\pi/4$	50	35	160	200	20	20	–	40	200	–	–	–	100	50
17	200	$\pi/6$	250	–	20	–	–	20	80	30	25,48	15,92	10	–	30	10
18	500	$-\pi/4$	100	6	40	–	–	51	25	40	50	–	60	–	20	20
19	240	$\pi/2$	100	4	–	100	–	5	150	5	50,65	50	–	10	–	2,5
20	120	$\pi/6$	200	4	20	–	2	12,75	25	10	–	62,5	–	25	51	5
21	140	$\pi/3$	50	15	101,3	100	5	40	–	6	–	400	3	20	–	8
22	110	$\pi/4$	100	35	80	100	20	10	–	40	100	–	–	–	50	25
23	400	$\pi/6$	50	–	100	–	–	100	400	30	127,4	79,6	10	–	150	50

Continuation of Table 2.

Data number	E V	ψ_e rad	f Hz	R_1 Ohm	L_1 mH	C_1 uF	R_2 Ohm	L_2 mH	C_2 uF	R_3 Ohm	L_3 mH	C_3 uF	R_4 Ohm	L_4 mH	C_4 uF	M mH
26	60	$\pi/6$	250	4	16	–	2	10,2	20	10	–	50	–	20	40,8	4
24	100	$-\pi/4$	200	6	20	–	–	25,5	12,5	40	25	–	60	–	10	10
25	120	$\pi/2$	200	4	–	50	–	2,5	75	5	25,32	25	–	5	–	1,25
27	70	$\pi/3$	100	15	50,65	50	5	20	–	6	–	200	3	10	–	4
28	1000	$\pi/2$	50	20	200	–	–	30	160	5	2	160	–	–	–	50
29	1200	$\pi/2$	400	20	25	–	–	3,75	20	5	2,5	20	–	–	–	6,25
30	600	$\pi/2$	200	20	50	–	–	7,5	40	5	5	40	–	–	–	12,5
31	240	$\pi/2$	50	4	–	200	–	10	300	5	101,3	100	–	20	–	5
32	300	$\pi/6$	500	4	8	–	2	5,1	10	10	–	25	–	10	20,4	2
33	140	$\pi/3$	200	15	25,32	25	5	10	–	6	–	100	3	5	–	2
34	110	$\pi/4$	200	35	40	50	20	5	–	40	50	–	–	–	25	12,5