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Optimal Groundwater Extraction under Uncertainty and a Spatial Stock Externality

Nathaniel H. Merrill and Todd Guilfoos

We introduce a model that incorporates two important elements to estimating welfare gains from groundwater management: stochasticity and a spatial stock externality. We estimate welfare gains resulting from optimal management under uncertainty as well as a gradual stock externality that produces the dynamics of a large aquifer being slowly exhausted. This groundwater model imposes an important aspect of a depletable natural resource without the extreme assumption of complete exhaustion that is necessary in a traditional single cell (bathtub) model of groundwater extraction. Using dynamic programming, we incorporate and compare stochasticity for both an independent and identically distributed as well as a Markov chain process for annual rainfall. We find that the spatial depletion of the aquifer is significant to welfare gains for a parameterization of a section of the Ogallala Aquifer in Kansas, ranging from 2.9% to 3.01%, which is larger than those found previously over the region. Surprisingly, the inclusion of stochasticity in rainfall increases welfare gains only slightly.

Key words: Groundwater management, Ogallala Aquifer, stochastic dynamic programming, welfare analysis.

JEL codes: C61, D99, Q25.

Groundwater plays an important role in mitigating the effect of weather variability on economic activity. Agriculture is highly dependent upon rainfall and is uniquely exposed to weather-related risk. Many groundwater aquifers around the world offer stable supplies of irrigation water to make up for deficits in natural rainfall. It is estimated that 50% of global cereal production depends on groundwater for irrigation (Wijnen et al. 2012). This dependence has led to aquifers being depleted at an alarming rate in many places around the world. Indeed, groundwater depletion has led to greater pumping costs, loss of access, saline intrusion in coastal systems, and land subsidence (United Nations Environment Programme and Division of Early Warning and Assessment 2003). Aquifers such as the Indus River Plains Aquifer on the India-Pakistan border, the North China Plains Aquifer, and the High Plains (Ogallala) Aquifer in the United States are experiencing declines as society's reliance on groundwater continues to grow (United Nations Environment Programme 2012).

Since 1950, total storage across the Ogallala Aquifer fell by approximately 8.3% (McGuire 2012). The initial development of irrigation technology allowed greater access to groundwater, reducing the impact of droughts (Hornbeck and Keskin 2014). Greater access to groundwater eventually led to a switch to high-value water-intensive crops such as corn, thereby exposing farmers to additional drought sensitivity and dependence on groundwater. Some areas of the aquifer experienced depletion, forcing farming to transition from irrigated crops to non-irrigated crops such as

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sorghum, wheat, and cotton (Steward et al. 2013; The Economist 2013; Hornbeck and Keskin 2014).

Groundwater depletion causes losses in the marginal value of groundwater due to higher extraction costs and the loss of groundwater as a buffer in dry years, thus increasing the variability of returns to farming (Tsur and Graham-Tomasi 1991; Knapp and Olson 1995). Despite the buffer value of groundwater being a potentially large percentage of the total value of a groundwater resource, estimated welfare gains from moving from open access to optimal extraction of groundwater are relatively small in size.[1,2] This finding is linked to Gisser and Sanchez (1980), who showed small gains from groundwater management. Koundouri (2004) reports a range of welfare gains from groundwater management across many studies, which are generally small unless there is a complete loss of access to the aquifer.

Larger estimates of the benefits of groundwater management are found when incorporating spatial heterogeneity and spatial relationships between groundwater users (Guilfoos et al. 2013; Edwards 2016). Ignoring lateral flows of groundwater leads to an under- or over-representation of the pumping cost externality when assuming a uniform single cell aquifer, known as the bathtub model (Saak and Peterson 2007; Katic 2010; Savage and Brozović 2011; Saak and Peterson 2012; Guilfoos et al. 2013; Palazzo and Brozović 2014).[3] However, the degree of the associated externalities is dissipated with increasing degrees of ownership by extractors, as pointed out by Brozović, Sunding, and Zilberman (2010).

We choose to simplify the spatial representation of the aquifer in our model to focus on the loss of access as the resource is depleted while retaining the benefits of a reduced form model. We call the gradual loss of access to groundwater "spatial depletion", and operationalize it with a specific functional form. The model we propose is useful to understand and measure the spatial stock externality of groundwater extraction. Using a depletion function is helpful for a few reasons: (*a*) it can be parameterized with aquifer specific data to simulate actual rates of depletion; (*b*) the costs associated with pumping can be separated from the costs associated with spatial depletion; and (*c*) the depletion function intuitively acts as a simple heuristic of the gradual loss of access that is observed in groundwater basins. By proposing a new reduced form model of spatial depletion of an aquifer, we illuminate the economic costs to the large-scale loss of access to groundwater.

Many papers incorporate stochasticity of rainfall and surface water supplies in their analysis of welfare gains from groundwater management (Tsur and Tomasi 1991; Provencher and Burt 1993; Knapp and Olson 1995; Koundouri and Christou 2006). However, one particular aspect that has not been addressed in these studies is the importance of uncertainty in rainfall to groundwater management when farmers gradually lose access to groundwater. Simplifying the spatial stock externality allows us to investigate the stochastic dynamics while keeping the size of the stochastic dynamic programming problem computationally manageable. The extent to which groundwater can act as a buffer against variable rainfall is a function of both extraction costs and the spatial depletion of the resource. In addition to investigating the gradual loss of access to groundwater, we explore the autocorrelated nature of rainfall, which may increase the value of groundwater as a buffer. Access to groundwater and low pumping costs allow more groundwater to be extracted to meet irrigation demand during a drought. As groundwater depletion occurs, the ability to buffer the effects of a drought decreases. We hypothesize that the loss of access to water is more acutely felt under variable rainfall and multi-year droughts.

We make two contributions to the literature. First, we introduce a model that incorporates an important spatial stock externality: the gradual loss of access to the aquifer with a backstop technology of dryland farming.[4] This contrasts with other work that models an abrupt loss of the entire aquifer (e.g., Koundouri and Christou 2006). Second, we identify the drivers of welfare gains from management under various forms of uncertainty to

[1] Buffer values were estimated at between 5% and 84% of the aquifer's value, but a comparison between myopic and optimal management was not presented (Tsur and Tomasi 1991).

[2] Knapp and Olson (1995) find gains to optimal management of 2.6% for Kern County, CA.

[3] The size of this externality depends on the size and spatial distribution of users causing welfare gains to management to be over- or underestimated when using a bathtub-type model of groundwater.

[4] We use "backstop" as the terminology to refer to the next most profitable use of the land, which could encompass many different economic activities. We assume that this is always dryland farming in this study.

understand the impact of serial correlation in rainfall and extended periods of drought. We apply our model to northwest Kansas over a section of the Ogallala Aquifer and find gains from management ranging from 2.88% to 3.01% using a 4% discount rate, with larger gains achieved under uncertainty in the rainfall process. The estimates are sensitive to the choice of discount rate, as gains from groundwater management range from 1% to 11% using corresponding discount rates of 7% to 1%. We find that including the gradual depletion of the aquifer materially impacts groundwater management gains compared to other estimates of the same region. We find that gains from management are sensitive to the shape of the spatial depletion function. Flat and wide aquifers benefit more from groundwater management than steep and deep aquifers with the same volume of storage and irrigation demand. Our results suggest that gains from groundwater management are sensitive to the relative intensity of irrigated farming but not sensitive to the autocorrelated nature of rainfall.

Conceptual Model

We model the dynamics of access to a groundwater resource by introducing a spatial depletion function where the amount of irrigable farmland above the aquifer is a function of the groundwater height (figure 1). This specification represents an upside-down cone shape, where the area above the remaining groundwater faces uniform pumping lifts, but the area no longer above the groundwater is left without any option for irrigation. Modeling in this manner creates an analytical relationship between the aggregate amount of water extracted from the aquifer and the evolution of the height of groundwater and the changing access to groundwater. This concept is different than a cone of depression at a single well or group of wells, as the depletion function represents the entire aquifer (or the section being modeled). Through assumptions about the distribution of farmers above the aquifer and the aquifer's physical properties, we argue that the function represents the spatial depletion observed in a large-scale groundwater system with heterogeneous extinction dates across the resource.

Meaningful lifespans for irrigation pumping in any specific area across an aquifer are a

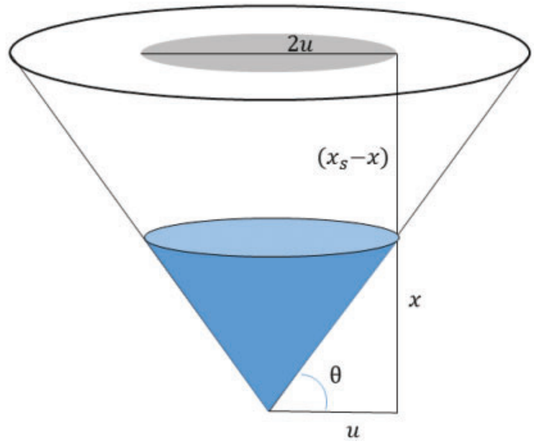

Figure 1. Spatial depletion model

Note: Variable u is the radius of the irrigable acreage, x is the groundwater height, and $(x_s - x)$ is the pumping lift, which is the difference between the surface height, x_s, and the height of the groundwater, x. The slope angle, θ, is fixed given the parameters x_s and A, the total surface area of the aquifer.

function of various location-specific confounding factors including pumping lift, pumping demand, specific yield, conductivity, and saturated thickness (Steward et al. 2013). As such, the function we propose portrays the most at-risk areas being exhausted first, which for a particular area over an aquifer could be a result of a number of factors mentioned above. Therefore, the cone-shaped function is a reduced form relationship relating the remaining groundwater to the remaining spatial extent of the aquifer in terms of irrigable acreage. The function is not intended to be spatially explicit but it represents the differences in timing of ultimate depletion across the aquifer. For example, we do not contend that edges of the aquifer will be depleted sooner. Rather, the edge of the cone represents the more marginal farmland in terms of remaining useful lifespan, which could be distributed anywhere across the resource as the result of the physical properties of the aquifer. However, saturated thickness does generally taper the further it is from the deepest sections as a result of geologic properties that create the aquifer. Figure 2 shows estimates of the remaining useful lifespans of the Ogallala in Kansas. Generally, the estimated remaining lifespan decreases moving away in space from the deepest sections.

Spatial Stock Externality

We simplified the spatial stock externality of lost irrigable acreage by making the

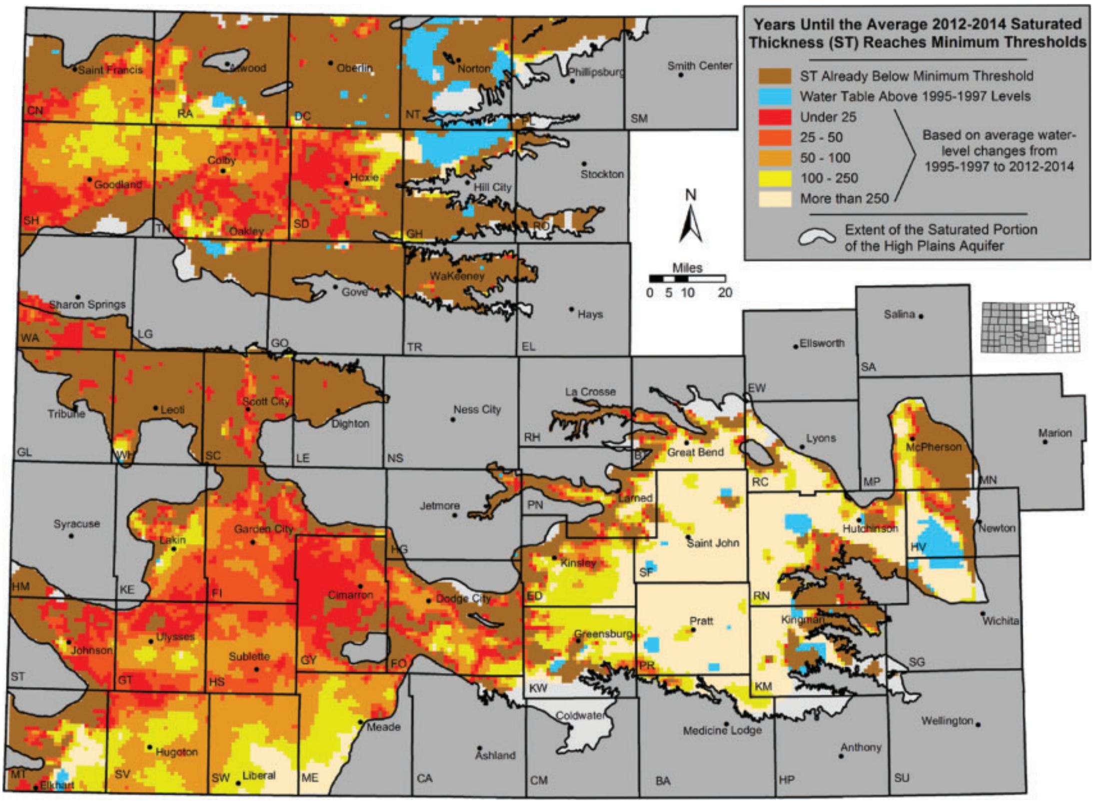

Figure 2. Estimated usable lifetime of the Ogallala in Kansas

Note: From Buchanan et al. (2015). The 1995–1997 to 2012–2014 trends of depletion were extrapolated to estimate useable lifespan.

percentage of remaining irrigable land a function of the groundwater height. Here, x_t is the height of the groundwater from the bottom of the cone (figure 1).[5] Equation (1) defines the percentage of irrigable land as a function of height based on the shape of the cone, and is bounded by 0 and 1

$$(1) \qquad \gamma_t(x_t) = \frac{\pi\left(\frac{x_t}{x_s/u_0}\right)^2}{A} \qquad \gamma_t \in [0, 1].$$

Here, x_s is the land surface height from the bottom of the cone, and u_0 is the radius of the circle that is the base of a cone representing the total surface area of the aquifer, A. The derivation of equation (1) can be found in the appendix.

The shape of this function should reflect a particular aquifer's geologic properties, as well as the distribution of economic activity across space. The presented framework is flexible to a range of assumptions and is transferable to other aquifer systems. Using the smooth function in equation (1), the model is most applicable to areas where the marginal cost of increased pumping lifts is less important than the loss of access to groundwater as an input to production. The function is also more applicable when the spatial distribution of wells and water demand is relatively uniform since the slope of the function defines the inter-temporal trade-offs at any particular groundwater stock level. Jumps or changes in the slope of the function would result in varying depletion paths and optimal pumping rules.

Rainfall Process

There are many ways to model the variability of rainfall depending on the time and spatial scale appropriate to the problem

[5] In other groundwater applications, x is often elevation from sea level. Because of the geometry, we use x to represent the height of water above the bottom of the cone, referred to throughout this paper as "groundwater height." When referring to the elevation of groundwater from sea level, we use "groundwater elevation."

(Srikanthan and McMahon 1999). We choose to model annual rainfall in two ways: the first scenario assumes independent and identically distributed (i.i.d.) realizations of rainfall. There are n discrete rainfall states, r^n, and corresponding probabilities, p^n. The probability of next year's rainfall, r_{t+1}, is independent of the current rainfall state, r_t, thus $prob(r_{t+1} = r^n) = p^n$, and

$$(2) \quad E(r_{t+1}) = \sum_{1}^{n} p^n r^n.$$

Equation (2) defines expectations when rainfall is i.i.d., where E is the expectation operator.

However, it is clear that rainfall time series are not serially independent. Many stochastic rainfall generation methods exist to model the serial dependence (Srikanthan and McMahon 1999). Therefore, the second scenario presented in this paper assumes a simple Markov chain process in order to replicate climate persistence—particularly droughts—where the probabilities of future rainfall states are a function of the current year's rainfall. This process defines probabilities of the future state conditional on the current state, $prob(r_{t+1} = r^n | r_t) = p^n(r_t)$, and

$$(3) \quad E(r_{t+1} | r_t) = \sum_{1}^{n} p^n(r_t) r^n.$$

By specifying the transition probabilities of moving from one state to another, we capture the time-dependent process and the persistence of annual rainfall.

Economic Benefits

In order to estimate the per-acre return to farming, we model a section over the aquifer that is irrigated acreage, and a section over the aquifer that is dryland acreage. The returns to irrigated land are a function of per-acre yield, crop prices, irrigation pumping cost, and the quantity of irrigation water extracted and applied. Equation (4) gives the per-acre returns to irrigated farmland

$$(4) \quad F_I(w_t, x_t | r_t) = y_I(w_t,\ r_t) p_c - C_1(x_s - x_t) w_t$$

where y_I is the per-acre yield, w_t is the irrigation water applied per acre, p_c is the price of the irrigated crop, and C_1 is the pumping lift-dependent marginal cost of one acre-foot of groundwater extraction. The cost function gives the cost of pumping one acre-foot of water to the surface for a given pumping lift, $(x_s - x_t)$.[6] Extraction behavior is based on profit-maximizing decisions of irrigation rates. Farmers that irrigate do so to maximize profits in each time period by choosing w_t to maximize returns—see equation (4). Farmland that does not have access to the aquifer only has rainfall as a water input for the production of crops and its returns are represented by

$$(5) \quad F_D(r_t) = y_d(r_t) p_d$$

where $y_d(r_t)$ is the per-acre yield of dryland crops as a function of rainfall, and p_d represents the prices of those crops.

As mentioned before, irrigable farmland is itself a function of the groundwater height. So a single period's aquifer-wide return would be the area-weighted sum of irrigated and dryland profits,

$$(6) \quad \Pi_t = A\phi[\gamma_t(x_t) F_I(w_t, x_t | r_t) + (1 - \gamma_t(x_t))\ F_D(r_t)]$$

where ϕ is the percentage of irrigable land above an aquifer that is farmed. We assume homogeneous per-acre irrigation water demand across the irrigated portion of the aquifer, based on profit-maximizing behavior, so the total water extracted is

$$(7) \quad W_t = [A\phi\gamma_t(x_t)]\ w_t.$$

Based on the cumulative extraction, W_t, the groundwater height changes through time following the equation of motion,

$$(8) \quad g(x_t, w_t) = x_{t+1} = x_t + \frac{R - (1 - \alpha) W_t}{\gamma_t(x_t) AS}$$

where R is the natural recharge, α is the percentage of irrigation water returning to the aquifer, A is the total surface area of the aquifer, and S is storativity or the volume of

[6] The pumping cost, while linear, is increasing as pumping lifts increase. Therefore, even without a function for the extinction of parts of the aquifer, the model would reach a steady state based on increasing marginal pumping costs. This is similar to the bathtub model in Gisser and Sanchez (1980).

water released per change in groundwater height.[7] Further, $\gamma_t(x_t)$ is the percentage of irrigable land remaining over the initial aquifer from equation (1).[8]

The social planner's problem, representing optimal management of the resource, is to find the extraction path to maximize the discounted sum of future profits, subject to the equation of motion (equation [8]), and physical constraints, as given here:

$$\max_{w_t} \mathrm{E}_r\left[\sum_{t=0}^{\infty}\beta^t\Pi_t(w_t,x_t,\gamma_t|r_t)\right] \tag{9}$$

$$s.t.$$

$$x_{t+1}=x_t+\frac{R-(1-\alpha)W_t}{\gamma_t AS}$$

$$\gamma_t=\frac{\pi\left(\frac{x_t}{x_s/u_0}\right)^2}{A}$$

$$x_t\in[\ \bar{x},\ \underline{x}\ )$$

$$x_0=z.$$

Here, E_r is the expected value operator over the stochastic rainfall variable, r, β is the discount factor, t is the year index, $\bar{x}$ and $\underline{x}$ are the maximum and minimum groundwater heights, and x_0 is the initial groundwater height at some level z. Rainfall, r_t, either follows the deterministic i.i.d. scenario, or the Markov chain assumption described above. The choice of extraction, w_t, is made after r_t is observed, or *ex post*.[9]

In contrast to looking at the problem from a social-planner's perspective, each individual farmer does not have an incentive to restrict their own pumping due to the open access properties of the groundwater resource. The benefits of water savings from one year to the next are not guaranteed to accrue to those curtailing their water use. This leads to myopic extraction behavior since each individual farmer over the aquifer will extract groundwater up until their net marginal benefit of irrigation equals their marginal cost of extraction for each year instead of saving additional water for the future (Gisser and Sanchez 1980).

Myopic extraction of the resource leads to the over-extraction of water over time, causing economic inefficiencies. In our model, we can attribute the inefficiencies to two externalities; the pumping cost externality, or the additional costs imposed due to greater pumping lifts for all extractors through time, and a stock externality driven by the loss of access to the aquifer for farmers as the resource shrinks spatially, referred as the spatial stock externality.

The difference between the optimal and myopic extraction of the resource generates the welfare gains from moving from open access to optimal use of the resource. We expect that optimal groundwater management will decrease extraction, which reduces pumping costs on farmers in future periods, similar to other models of groundwater extraction. Since our model includes the gradual loss of access to the aquifer for farmers over time, we expect optimal management to maintain more irrigated acreage. We expect that the gains from optimal management will not be distributed evenly across time with early sacrifices in profit made to maintain higher groundwater levels and larger aquifer-wide profits in the future. Finally, we expect that maintaining a greater amount of irrigated acreage will lead to larger welfare gains since groundwater mitigates the impact of stochastic rainfall. However, the size of the open access externalities and the resulting gains from optimal management is an empirical question.

Dynamic Programming Problem

To describe the solution to the dynamic social planner's problem under stochastic rainfall and the time path of depletion, we use a dynamic programming approach (Bellman 1957). A discussion of dynamic programming as applied to groundwater extraction can be found in Provencher and Burt (1994). By applying the principle of dynamic programming, the first-order conditions for this problem are given by the Hamilton-Jacobi-Bellman equation for this discrete time stochastic model (Brito 2008):

[7] Though rainfall is treated as stochastic, we use a long-run average for recharge to the aquifer. This presumes that water from recharge areas percolate through the ground to deep production wells over time, and that when the wells take up groundwater there are multiple ages of recharge water brought up, which smooths the variability in recharge. Therefore, a long-run average is more suitable.

[8] This formulation of the equation of motion has consequences for the rate of depletion. Given a similar per-acre pumping demand, w_t, height changes will be smaller under this spatial depletion model than under the assumption of a traditional bathtub model. A more formal comparison of these rates of depletion to the bathtub model is discussed in section 3 of the supplementary online appendix.

[9] See Tsur and Tomasi (1991) for a discussion of *ex post* and *ex ante* information treatments. This problem does not turn deterministic because irrigation decisions are made after rainfall is observed, *ex post*.

$$(10)\quad V_t(x_t) = \max_{w_t}\{\Pi_t(w_t, x_t, \gamma_t|r_t) + \beta E_r[V_{t+1}(g(w_t, x_t)|r_{t+1})]\}.$$

The value function, V, represents the expected present value of future benefits of the system assuming optimal management in all future periods, and is, therefore, only a function of a stock variable, in our case groundwater heights. Further, β is the discount factor equal to $\frac{1}{1+\delta}$, and δ is the discount rate; E_r is the expectation operator over the random variable r and $V_{t+1}(g(w_t, x_t))$ is the value of next period's stock assuming optimal behavior in all subsequent periods. Further, $g(w_t, x_t)$ represents the groundwater stock transition equation as a function of current extraction decisions and groundwater height, which in our case is the same as equation (8) above.

Intuitively, the optimal extraction of groundwater balances the marginal benefit of extraction today against the discounted marginal costs imposed on all subsequent time periods given the expectations of the random variable. The first-order condition with respect to withdrawals for the Bellman equation above implies the following along the optimal extraction path:

$$(11)\quad \frac{\partial \Pi(w_t, x_t, \gamma_t|r_t)}{\partial w_t} + \beta E_r\left[\frac{\partial V}{\partial x_{t+1}}\frac{\partial g(w_t, x_t)}{\partial w_t}\right] = 0.$$

Inputting equations (6) and (8) into equation (11), we have

$$(12)\quad \gamma_t(x_t)A\phi\left[\frac{\partial F_I(w_t, x_t|r_t)}{\partial w_t}\right] + \beta E_r\left[\frac{\partial V}{\partial x_{t+1}}\left(-\frac{(1-\alpha)\phi}{S}\right)\right] = 0.$$

The marginal value of extraction today (the first term) should equal the discounted marginal cost (the second term) imposed by period t's extraction, the opportunity cost.

Myopic behavior is used to simulate perfect competition, where farmers behave as if they expect the open access properties of the aquifer to restrict future benefits of water savings in the current period. Therefore, this behavior is represented by solving one period's profit maximization problem, maximizing equation (6) with respect to withdrawals with no regard to future states, and equating marginal benefit to marginal cost for each period sequentially. This leads to the following first-order condition:

$$(13)\quad \gamma_t(x_t)A\phi\left[\frac{\partial F_I(w_t, x_t|r_t)}{\partial w_t}\right] = 0.$$

Comparing equations (12) and (13) shows the difference between optimal per-acre withdrawals and myopic withdrawals. The difference depends on the size of the opportunity cost. Myopic per-acre withdrawals are greater than optimal per-acre withdrawals along the optimal path when $F_I(w_t, x_t|r_t)$ is concave with respect to w_t, and when $\frac{\partial V}{\partial x_{t+1}} > 0$. The function F_I is concave when there are decreasing marginal returns to irrigation and linear or increasing marginal costs to irrigation (see equation [4]). The second condition is more complicated but is met when there is always a larger value to having higher rather than lower groundwater heights. This depends on aspects of our spatial depletion function and the equation of motion, including recharge, all conditional on optimal extraction.

We can use the principles of dynamic programming to arrive at a more specific condition for optimality, the discrete time Euler equation for our model, which is not a function of $\frac{\partial V}{\partial x_{t+1}}$ itself. To do this, we utilize the above first-order condition, equation (12), combined with the envelope theorem (please see section 1 of the supplementary online appendix for a detailed derivation and steps for this process):

$$(14)\quad \gamma(x_t)\left[\frac{\partial F_I(w_t, x_t|r_t)}{\partial w_t}\right] = \beta E_r\Bigg[\bigg(\frac{\partial \gamma(x_{t+1})}{\partial x_{t+1}}(F_I(w_{t+1}, x_{t+1}|r_{t+1}) - F_D(r_{t+1})) + \frac{\partial F_I(w_{t+1}, x_{t+1}|r_{t+1})}{\partial x_{t+1}} \times \gamma(x_{t+1})\bigg)\left(\frac{(1-\alpha)\phi}{S}\right) + \gamma(x_{t+1}) \times \left[\frac{\partial F_I(w_{t+1}, x_{t+1}|r_{t+1})}{\partial w_{t+1}}\right] \times \left(1 - \frac{\frac{\partial \gamma(x_{t+1})}{\partial x_{t+1}} R}{\gamma(x_{t+1})^2 AS}\right)\Bigg].$$

The opportunity cost of groundwater extraction in our model, that is, the right-hand side of equation (14), includes a number of components. The myopic extractors ignore these opportunity costs and generate the pumping cost and spatial stock externality. The effect of a marginal reduction in groundwater height is $\frac{\partial \gamma(x_{t+1})}{\partial x_{t+1}}(F_I(w_t, x_t) - F_D)$. This positive term is our spatial stock externality. This externality is larger under small realizations of rainfall because irrigated farming has the ability to increase yields, while dryland farming is a victim of circumstance. The value of the backstop plays an important role in management, since as $(F_I - F_D)$ grows, or as dryland farming returns become negligible compared to irrigated farming returns, the magnitude of the spatial stock externality increases.

The next term, $\frac{\partial F_I(w_{t+1}, x_{t+1}|r_{t+1})}{\partial x_{t+1}} \gamma(x_{t+1})$, captures the increased pumping cost on returns from irrigated agriculture, the pumping cost externality, scaled by the amount of irrigated land. The figure is positive, since pumping costs are decreasing with increasing groundwater heights. The term $\gamma(x_{t+1})\left[\frac{\partial F_I(w_{t+1}, x_{t+1}|r_{t+1})}{\partial w_{t+1}}\right]$ captures the change in future marginal benefits of irrigation, given x_{t+1}, which is always positive given our yield functions and rainfall range. This term is multiplied by $\left(1 - \frac{\frac{\partial \gamma(x_{t+1})}{\partial x_{t+1}} R}{\gamma_t(x_{t+1})^2 AS}\right)$, which captures the additional marginal change in groundwater height due to recharge at the new groundwater height, x_{t+1}. The expression is a result of the non-linearities of the cone model of the spatial depletion function. It is unclear whether these set of terms together always provide an additional opportunity cost to extraction and is a matter of parameterization.

The magnitude of the difference between myopic and optimal extraction is dependent on the size of the opportunity cost. The greater the opportunity cost of extraction along the optimal path, the larger the difference between extraction rates. When the depletion function becomes less significant, that is, when $\gamma'(x)$ is relatively small, a change in groundwater height has a smaller effect on spatial depletion; this occurs in aquifers that do not have exhaustion concerns or have a relatively thick saturated thickness. When there are small losses of irrigated land, the gap between the dryland and irrigation returns does not matter as much (a comparison to the traditional bathtub model, an extreme case of no spatial depletion, can be found in section 2 of the supplementary online appendix). We explore the sensitivity of the welfare estimates to the shape of the spatial depletion function numerically in the results section.

Solving equation (10) to find the policy functions representing optimal management of the resource means finding the function $V(x)$, either explicitly or numerically. The addition of a stochastic element and a nonlinear spatial depletion function makes an analytic solution intractable in this case (Brito 2008). We use the numerical method of value function iteration.

Parameterized Model

We quantify welfare gains in this spatial depletion model under uncertainty with an application to the northwest Kansas section of the Ogallala Aquifer. The Kansas Water Authority provided the physical parameters of the aquifer, including saturated thickness, storativity, and natural recharge (Guilfoos, Khanna, and Peterson 2016). The pumping cost estimates from this region are taken from Hendricks and Peterson (2012) for Groundwater Management District 4 in Kansas.

The physical parameters of the model are presented in table 1. The spatial stock externality is modeled using the function in equation (1). Given the initial groundwater height, the initial aquifer surface area remaining above the groundwater is set at 1.68 million acres, of which 373,200 acres, or 22% of the total land, are irrigated. We assume that this total acreage of farmland (irrigated plus dryland) remains constant as the aquifer area is depleted.[10]

The bottom of the aquifer is set at 2,892 feet above sea level based on the minimum water table found over management district 4. The initial water height was set at 3,069 feet based on the irrigated acreage and initial pumping lift to the surface of 26 feet, which is the average across management district 4. Therefore, the surface elevation is 3,094 feet. Since x is the height of water from the bottom of the aquifer, $x_0 = 177$ feet and $x_s = 202$ feet. The depth of the aquifer and initial total surface area make up the two physical

[10] A sensitivity analysis allowing the amount of farmland to vary is found in supplementary online appendix figure 1.

Table 1. Parameter Values for Northwest Kansas Groundwater Management District 4

Parameter	Description	Value
C_1	Cost of pumping	\$.1044 /a-ft/ft
R	Natural recharge	131,400 a-ft
A	Aquifer area	2.19 million acres*
$\bar{x}$	Land surface	202 ft, or 3094 ft above sea level
$\underline{x}$	Lower aquifer bound	0 ft, or 2892 ft above sea level
x_0	Initial water height	177 ft, or 3069 ft above sea level
$\gamma(x_0)A$	Initial irrigable area	1.68 million acres
ϕ	% of aquifer farmed	22% or 373,200 irrigated acres initially**
S	Storativity	.17
α	Irrigation water return	20%
β	Discount factor	.96%
r	Rainfall states	
	High	2 ft
	Medium	1.58 ft
	Low	1.25 ft
p	Crop Prices	
	Corn	\$4.45 /bushel
	Sorghum	\$4.25 /bushel
	Winter Wheat	\$6.53 /bushel

Note: The physical parameters of the aquifer were provided by the Kansas Water Authority. The pumping cost estimates from this region were taken from Hendricks and Peterson (2012) for Groundwater Management District 4 in Kansas. Prices were obtained from USDA ERS Yearbook Tables and average U.S. prices in 2013/2014 (USDA ERS 2014). Asterisk * denotes that since x is measured in feet, A should also be in square feet for numerical analysis. We refer to areas in acres in the text for simplicity; ** denotes initial dryland acres were set to zero at initial groundwater height to make the results reflect the gains of groundwater management moving forward.

parameters needed to define the cone-shaped function used to capture the spatial depletion (figure 1).

To estimate crop yields as a function of rainfall and applied irrigation water, we used Kansas State's Crop Yield Predictor tool, which is parameterized for the Colby, Kansas area (Klocke et al. 2010). We fit functions for corn, sorghum, and winter wheat. By running the tool for the full range of water applied to crops and rainfall, we estimated per-acre yield as a function of rainfall, or rainfall plus irrigation water applied. We assume that an inch of rainfall is equivalent to an inch of irrigation water applied. We fit cubic functions to the yield data and present the yield functions in figure 3 (see supplementary online appendix, section 4 for yield function details). Corn is assumed to be grown on irrigated acreage and a rotation of wheat, fallow, and sorghum on dryland acreage (Hansen et al. 2012). The crop yields from dryland are assumed to be one-third of the per-acre sorghum yield at a particular rainfall amount, plus one-third of wheat. This assumes that, in any year, various dryland farms, which are represented as homogeneous, are at various stages of the rotation. Prices were obtained from USDA ERS Yearbook Tables, and average U.S. prices in 2013/2014 were used in the simulation (USDA ERS 2014).

Expectations of rainfall play an important role in defining optimal management. The presented model is flexible to various definitions of rainfall expectations and stochastic processes. For clarity of interpretation and limiting the computational burden, we chose to use three, roughly equally likely, levels of rainfall representing low, medium, and high amounts of yearly rainfall.[11] We fit an empirical, time-homogeneous Markov chain process to binned rainfall amounts observed rainfall at the Colby, Kansas gauge.[12] The transition probability matrix and details are found in table 2.

Numerical Solutions

Optimal policy functions are found numerically by means of stochastic dynamic

[11] Yearly rainfall is the input to the Crop Yield Predictor. From there, a weather generator is used to create growing season weather to estimate crop yield.

[12] This implies a stationary distribution and constant transition matrices. By simulating this process for 250,000 years, we estimated the non-conditional probabilities of each state, which are used for our stochastic case, as well as the average of the process for the deterministic treatment to match the conditions for each scenario for comparison.

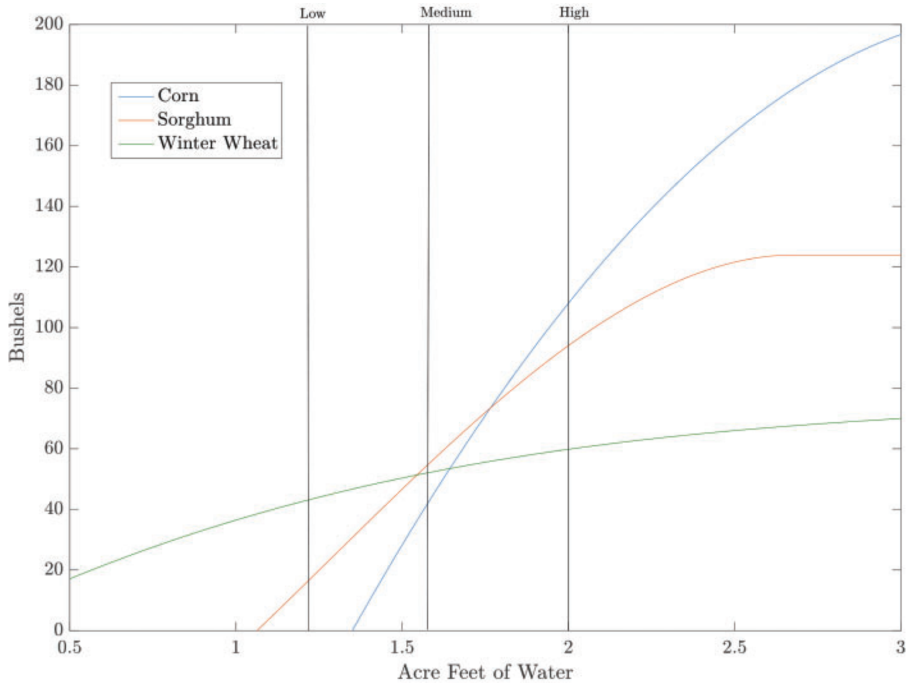

Figure 3. Crop yield functions

Note: Derived from Kansas State University's Crop Yield Predictor. Low, medium, and high rainfall amounts are shown as vertical lines. The Y-axis represents yield in terms of bushels per acre as a function of rainfall for sorghum and winter wheat, or rainfall plus irrigation water for corn.

Table 2. Markov Chain Transition Probability Matrix

Future State	Current State			Non-conditional
	Low	Med	High	
Low – 1.25 ft	40.48%	21.62%	42.50%	35.29 %
Med – 1.58 ft	30.95%	37.84%	25.00%	31.10 %
High - 2 ft	28.57%	40.54%	32.50%	33.61 %

Note: "Low", "Med", and "High" refer to annual precipitation. Probabilities are empirically found from a precipitation time series from Colby, Kansas. The last column represents the non-conditional (on current state) probabilities of future rainfall used in the i.i.d. stochastic scenario.

programming using MATLAB 2013 (see the supplementary online appendix for code). The optimal extraction decisions as a function of water levels are approximated by first estimating the value function, $V(x)$, by means of value function iteration, solving equation (10) for discrete levels of x, replacing the optimized values for $V(x)$, and repeating until the functions converge within a specified tolerance (Putterman 1994). Creating the transition probability matrix for each realization of rainfall incorporates the Markov chain yearly rainfall process. Once the optimal value functions are found, the corresponding policy functions can be recovered for any realization of rainfall and groundwater height. The optimal policy functions are displayed in figure 4.[13]

The derived optimal decision rules are iterated through time starting at the initial groundwater height. Realizations of rainfall in each year are generated from the i.i.d or the Markov process fitted above to yearly rainfall to match expectations used for transition probabilities. To evaluate gains from optimal management, a myopic decision is made in each year by using the policy of maximizing equation (6) with respect to

[13] The corresponding value functions can be found in supplementary online appendix figure 2.

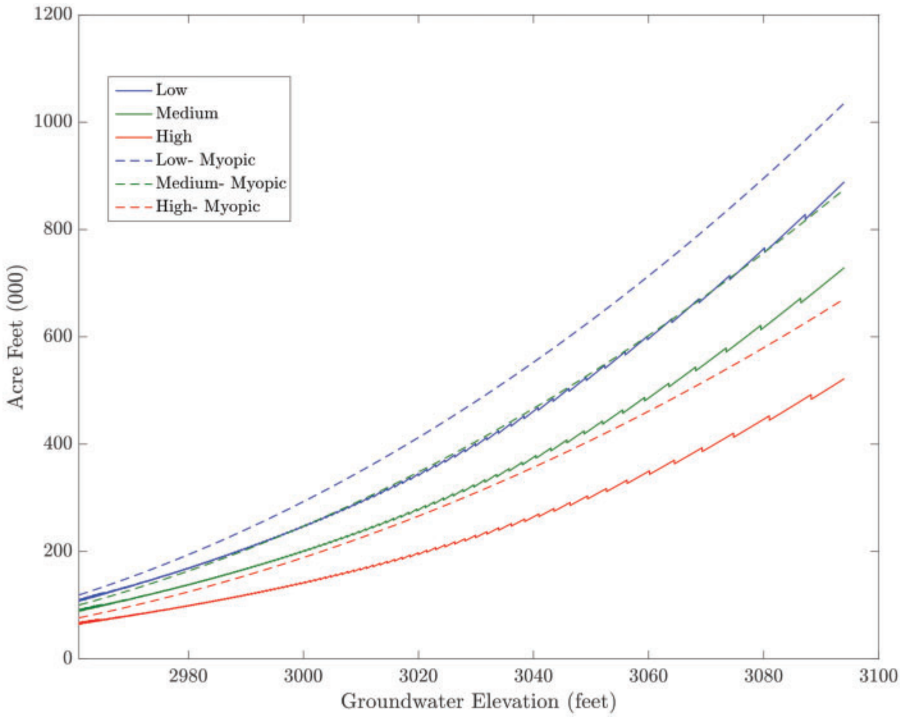

Figure 4. Optimal and myopic policy functions at groundwater elevations, i.i.d. rainfall scenario

Note: The three states of rainfall (Low, Medium, High); each has a corresponding policy function represented by the three colors. At low levels of rainfall, demand for irrigation water is higher. The dotted lines represent the decisions made by a myopic extractor, while the solid lines are results of the optimal dynamic programming policy functions. The jaggedness is the result of the discretization of the stock variable (groundwater height) in the numerical methods.

groundwater withdrawals. These paths through time for the i.i.d. case can be found in figure 5.

The welfare implications of each extraction path are estimated by discounting and summing each year's profit as defined by equation (6) from the initial year to the end of the time horizon—in our case 500 years—which is well after reaching a steady state and where future benefits are essentially discounted to zero. The differences between the optimal and myopic welfare values are the estimated gains of moving from open access to optimal management of the aquifer.

Results

The welfare results from groundwater management are found in table 3 under various rainfall assumptions. Gains from optimal management are in the range of 2.88% to 3.01% using a 4% discount rate. The stochastic scenarios generate slightly larger relative gains (.09% to .13% points larger) and reflect the magnitude of including stochasticity in assessing welfare impacts to management. This relatively small increase in welfare reflects the additional buffer value of groundwater gained from optimal management.

There is a lack of consensus as to the appropriate discount rate in long-term, inter-generational resource problem (Freeman, Herriges, and Kling 2014). The choice of a discount rate affects the welfare calculations, the shape of the optimal extraction path, and the distribution of benefits over time. The gains from management found in the deterministic scenario range from 10.87% using a 1% discount rate, to 1.08% with a 7% discount rate, highlighting the importance of this choice for the welfare analysis of long-term resource problems. Results of welfare gains using a range of rates can be found in figure A1 in the appendix. We continue by presenting and discussing results using a 4% discount rate, which is within the suggested range by the U.S. Environmental Protection Agency and the Office of Management and Budget, and similar to previous groundwater studies (EPA 2010).

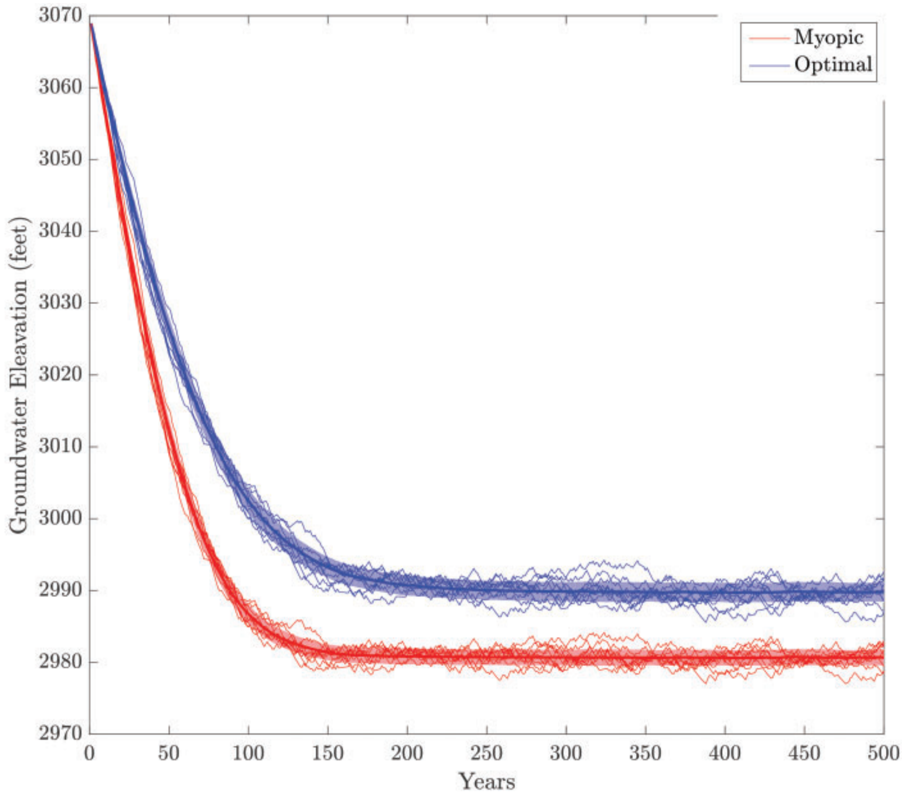

Figure 5. Groundwater elevation over time, i.i.d rainfall scenario

Note: The blue lines represent groundwater levels under optimal management. The average (thicker line), standard deviation (blue shaded region), and a number of individual runs are plotted. The red line is the myopic extractor's groundwater elevation through time.

Table 3. Welfare Gains From Groundwater Management, Total Discounted Profit (Billions of dollars)

	Myopic	Optimal	Difference	% Gain
Deterministic	$ 6.683	$ 6.876	$ .193	2.88
Stochastic	$ 6.589	$ 6.785	$ .196	2.97
	(.0937)	(.0938)	(.006)	(.04)
Stochastic-MC	$ 6.515	$6.711	$.196	3.01
	(.0408)	(.0350)	(.006)	(.03)

Note: Results were found using a 4% discount rate; see appendix figure A1 for a sensitivity analysis using a range of rates. Standard errors of the stochastic figures from 1,000 iterations through rainfall realizations are in parentheses. The deterministic scenario assumes average annual rainfall each year. "Stochastic" assumes i.i.d. random draws from high, medium, and low rainfall states based on empirical probabilities. "Stochastic-MC" assumes draws from a Markov chain process where the transition probabilities are found in table 2.

The results show that including the loss of the spatial extent of the aquifer leads to larger estimated welfare losses due to open access when compared to other modeling assumptions for welfare impact studies. To put our work into perspective, our estimates are larger than Lee, Short, and Heady (1981) for the Ogallala Aquifer in Texas (0.3%), Nieswiadomy (1985) for the High Plains Aquifer in Texas (0.28%), and similar to Kim et al. (1989), also for the High Plains in Texas (1% to 3.7%). Kim's paper includes endogenous technological change as a function of groundwater heights and therefore similarly provides backstop technologies in a sense, albeit under certainty. However, our analysis looks at an area over the Ogallala that has greater depletion concerns than other areas with thicker saturated thickness. Our study area has a much greater loss in saturated thickness since pre-development, that is, between 15% and 60% (Buchanan et al. 2015), than the 8.3% loss of total storage over the entire aquifer (McGuire 2012). Therefore, the gains from management in percentage terms will be larger in our study area than in other areas without depletion concerns due to the scarcity of the initial stock of groundwater.

The welfare gains arise from the difference between the optimal decision rules and the myopic pumpers' choices (figure 4). The optimal planner internalizes the intertemporal externalities, while the myopic extractor does not. Optimal total withdrawals in the deterministic scenario policy function are 18% less, on average. For the stochastic rainfall scenarios, the realized rainfall determines the corresponding optimal policy function, one for each state of rainfall. Compared to the myopic decision maker, total withdrawals across the aquifer from the optimal policy functions average about 16% less in low rainfall, 19% less in medium rainfall, and 25% less in high rainfall, and vary through groundwater heights (figure 4). The optimal policy rules result in relatively larger water savings in better rainfall years.

The sources of welfare losses to myopic extraction are higher pumping costs and the loss of irrigated acres. The magnitude of these losses is less obvious as they depend on the time path of depletion, the discount rate, the recharge rate, and the relative benefits of irrigated farmland compared to dryland farming. When optimal policy functions are iterated through time, meaning the decisions are made optimally based on the current rainfall and groundwater state, the groundwater stock is depleted more slowly under optimal decisions (figure 5). The lower extractions lead to a different time path of benefits (table 4). For about the first ten years, the aquifer-wide returns to optimal management are lower than under myopic extraction due to forgone profits from irrigation (figure 6). However, these near-term profits are foregone in order to maintain irrigated acreage and shorter pumping lifts for a longer period, resulting in delayed benefits. This shift accounts for the welfare gains to management.

The lower optimal extractions, in equivalent groundwater heights and rainfall amounts compared to the myopic pumper, lead groundwater heights to settle at a steady state elevation of approximately 2,990 feet, on average, as opposed to 2,981 feet under myopic extraction for a difference of 9 feet in height at the steady state and between 0 and 14 feet through the time path. This may seem to be a small difference in height, but given our spatial cone model, where the irrigated acreage is a function of the groundwater height, these steady state groundwater levels translate to an optimal steady state with 114,697 irrigated acres, or 31% of the initial irrigated acreage, as opposed to 94,162 irrigated acres under myopic extraction, or 25% of the initial irrigated acreage. While at the steady state the undiscounted aquifer-wide welfare gain is nearly 9%, the early sacrifices in irrigation water, combined with the long planning horizon and the discount rate result in a more modest discounted welfare gain of approximately 3%.

The larger welfare gains when including uncertainty reflects the additional benefit maintained from groundwater when taking into account its value as a steady resource to buffer variation in rainfall. When comparing welfare across rainfall scenarios, modeling uncertainty increases estimated gains from management, resulting in an increase in welfare gains of 0.09% to 0.13% points over the deterministic scenario welfare gains. The addition of a Markov chain process added little additional welfare gains (0.04% points) when optimal policy was matched to a stochastic process.

The concavity of the benefit function implies risk aversion in the sense that irrigation water limits the range of available water in future years, which is preferred over a wider range of possible states, as described in Provencher and Burt (1993) and under risk aversion in Knapp and Olson (1996). However, the yield of the backstop of dryland farming is less variable (flatter in figure 3) than irrigated crop yields. In a sense, the transition to dryland farming attenuates the loss of the buffer value of the groundwater, as the crops are more resilient to a range of rainfall than non-irrigated corn would be, shown by Hornbeck and Keskin (2014). These authors find that water scarce counties are no more sensitive to drought than irrigated ones. The small additional value of groundwater as a buffer, found in our numerical application, is consistent with this finding. Still, our goal is to evaluate the buffer value from a dynamically optimal groundwater extraction perspective. We capture the lower water-sensitivity of dryland practices as compared to irrigated through the shape of the crop yield functions.

The magnitude of the water demand relative to the volume of groundwater storage available is likely to be important to these results since this defines the scarcity of the resource. We investigate this by varying the initial irrigated acreage in the model and find that with 50% initial irrigated acreage, as

Table 4. Time Paths of Key Variables, Deterministic Rainfall Scenario

Year	1	20	40	60	80	100	120	140	160	180	200
Groundwater Elevation (feet)											
Optimal	3069	3049	3033	3019	3009	3002	2997	2994	2992	2991	2990
Myopic	3069	3044	3023	3006	2995	2988	2984	2982	2982	2981	2981
Difference	0	5	10	13	14	14	13	11	10	10	9
Irrigated acreage											
Optimal	372300	293852	234620	191835	162257	142833	130538	123048	118602	116052	114697
Myopic	372300	274254	202703	154566	125241	109124	100993	97122	95336	94527	94162
Difference	0	19598	31918	37269	37016	33709	29545	25926	23265	21525	20534
Dryland acreage											
Optimal	0	78448	137680	180465	210043	229467	241762	249252	253698	256248	257603
Myopic	0	98046	169597	217734	247059	263176	271307	275178	276964	277773	278138
Difference	0	−19598	−31918	−37269	−37016	−33709	−29545	−25926	−23265	−21525	−20534
Profit irrigated ($/acre)											
Optimal	888	879	875	872	870	871	869	868	868	871	868
Myopic	904	900	896	893	891	889	889	888	888	888	888
Difference	−16	−20	−21	−21	−21	−19	−20	−20	−20	−18	−20
Profit dryland $/acre	196	196	196	196	196	196	196	196	196	196	196
Total profit (Million $)											
Optimal	330.5	273.7	232.2	202.7	182.3	169.4	160.9	155.7	152.7	151.3	150.1
Myopic	336.6	266.0	214.8	180.7	160.0	148.7	143.0	140.3	139.0	138.5	138.2
Difference	−6.1	7.8	17.4	22.0	22.3	20.7	17.9	15.5	13.7	12.9	11.9

Note: The path of variables through time show that optimal management maintains higher groundwater heights and irrigated acreage. This is at the expense of short-term profits, which are made up for later in the time horizon. Key variables in the time horizon show stocks and flows at points in time and do not represent the interval between time periods. Results are presented undiscounted.

opposed to 22%, the estimate of welfare gain is approximately 5% (supplementary online appendix, figure 1). Unsurprisingly, the gains are sensitive to water demand parameters.

The shape of the cone impacts the returns from groundwater management. To explore this aspect of model, we hold constant all the parameters and vary the shape of the cone. Importantly, we keep the volume of groundwater constant and the initial groundwater demand constant, but change the angle of the cone, the depth of the aquifer, and the area of the base of the cone. We find that as the cone becomes shallower and wider, gains from management increase, as shown in table 5 and appendix figure A2. Gains from management appear to be relatively sensitive to the shape of the cone, varying between 2.71% and 4.71%. Flat and wide aquifers, *ceteris paribus*, can be expected to benefit more from groundwater management than steep and deep aquifers. Therefore, the rate of depletion, which is determined by the aquifer-specific properties, is relatively important to overall management gains. These results lead us to believe that the drivers of welfare gain are primarily the gap between returns of irrigated and dryland farming, the volume of storage available relative to groundwater demand, and the shape of the aquifer.

Discussion

Unlike other approaches, our spatial depletion model captures the gradual loss of access to a stabilizing resource as a result of the overall extraction of water from the aquifer. The embedded assumption of infinite conductivity results in uniform pumping lifts across the irrigated acreage above the remaining aquifer, representing the average lift faced by the remaining pumpers. The intertemporal pumping cost externality is relatively small when the marginal pumping cost, as a function of lifts, is small compared to the marginal benefit of groundwater extraction (Koundouri 2004). In our specification, the important externality affecting welfare is the loss of access over time. Certain areas have thinner saturated thickness than others, and even with a relatively short pumping lift, can lose access as a result of the aquifer-wide extractions. To compare the size of this externality to the traditional

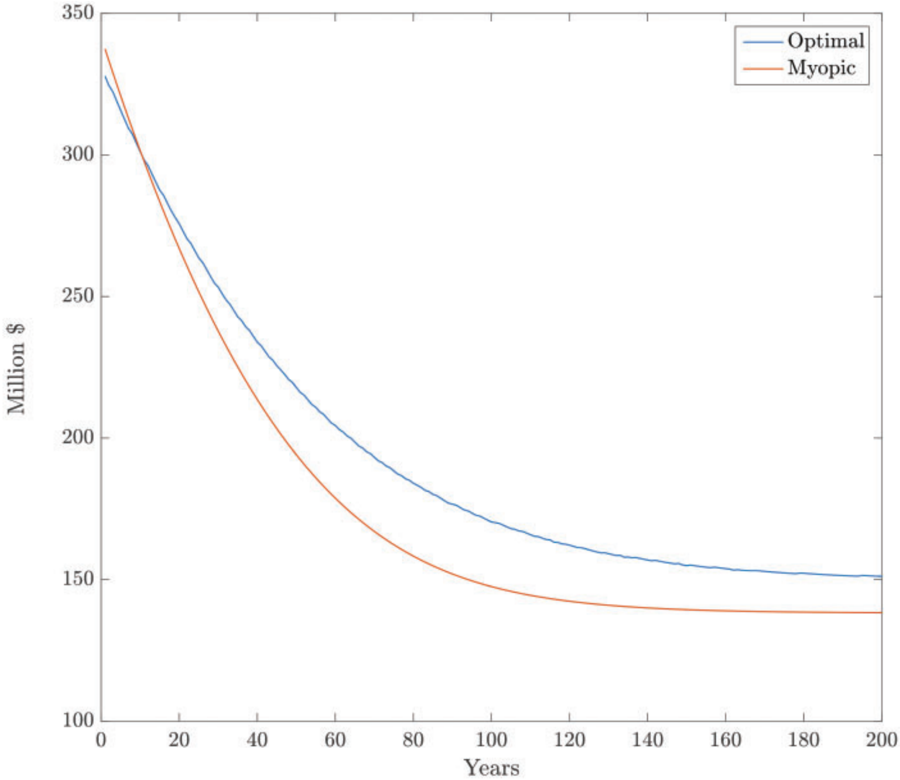

Figure 6. Total undiscounted profit, deterministic rainfall scenario

Note: The optimal path gives up some near-term benefits to allow for a longer sustained period of larger returns. Benefits are presented undiscounted for the deterministic scenario. A discounted version of this figure is included in the supplementary online appendix figure 3.

Table 5. Cone Shape Sensitivity Analysis, Deterministic Rainfall Scenario

	Shape	Cone height (feet)	Total surface area (million acres)	Welfare gains (%)	V' (million $)
1	Deep and narrow	394	1.12	2.71	$34
2		344	1.29	2.74	$38
3		294	1.5	2.77	$44
4		244	1.81	2.84	$52
5		194	2.28	2.92	$63
6		144	3.07	3.06	$80
7		94	4.71	3.49	$107
8	Shallow and wide	44	10.05	4.71	$116

Note: Holding the total storage volume constant by changing the surface area and holding constant the number of initial irrigated acreage, the shallower the cone, the larger the welfare gains to management. For a given groundwater height change, more irrigated acreage is transition to dryland; V' is the derivative of the value function at the initial conditions representing the initial value of a foot of water height, or the cost of extracting enough water to change the height by one foot (see discussion of the Euler equation in the appendix). The cost of one foot of water height for a shallower cone is larger given the larger irrigated acreage that one foot of height supports. Appendix figure A2 shows the functions relating groundwater height to irrigated acreage for each shape (1–8).

pumping cost externality, we simulated the same section of the aquifer under the assumption of the standard bathtub model, which resulted in small estimated welfare gains of 0.06% (see supplementary online appendix table 1).

We include a backstop, dryland farming, which reduces the gains from management since in its absence we would have assumed infinitely expensive alternatives (Koundouri and Christou 2006). Without a backstop, the welfare gains from management are larger

(5.2% in the stochastic rainfall scenario) and stochasticity in rainfall becomes more important (see supplementary online appendix table 1).

We simplified droughts with a Markov chain rainfall process, where the lengths of the droughts were defined by the transition probabilities. Drought refers to extended periods of lower than normal soil moisture due to climatic variables including rainfall (used in this paper), temperature, and even wind speed. Drought measures such as the Palmer Drought Severity Index include a duration aspect, where the progressions of the climatic variables matter, and not just the current conditions. We do not carry over soil conditions or other stock variables (except the groundwater height) that could be affected by the series of yearly rainfall realizations. This could be explored by modeling additional processes in the benefit function and is left to future work. As modeled, the addition of an autocorrelation process mattered little to the welfare gains from management.

The discussion of rainfall processes leads to an important point about the assumptions used to generate the optimal paths. We matched the rainfall processes to the optimal policy rules, in each case assuming perfect information about the process that generates the annual rainfall. The method we use to estimate optimal management assumes that the decision makers not only act optimally in each period given the realization of rainfall, but also have accurate expectations of the process that generates rainfall. In reality, the processes that generate weather and longer-term climate trends are complex and the distribution of rainfall is not stationary. As such, the extent to which the policy maker's expectation of rainfall coincides with the actual process may have implications to welfare gains, with mistakes or misspecifications causing inefficiencies.

Conclusion

We introduce a dynamic spatial depletion model of groundwater extraction that incorporates stochastic rainfall and a gradual spatial stock externality, leaving more farmland at the mercy of variable rainfall as groundwater levels fall, and also less available for irrigation across a smaller area. Incorporating the spatial depletion of the aquifer substantially added to welfare estimates of moving from open access to optimal management of the resource. This highlights the welfare implications of the loss of access to groundwater as opposed to increased pumping lifts explored previously. By building a novel model flexible to rainfall expectations and various stochastic processes, we showed the importance to total welfare over an aquifer of stochastic processes that leave farm producers more exposed to the elements of nature. We find that the addition of randomness and persistence of rainfall does not materially affect welfare gains, largely due to the relatively more consistent yields from dryland farming as a backstop.

The shape of the aquifer has important implications for the rate of depletion and the size of welfare gains. Greater irrigated acreage is lost in flat and wide aquifers, which leads to greater gains from groundwater management. Not only does the relationship between the volume of storage to the total groundwater demand matter to the economic value of conservation, so does the shape of the aquifer.

Supplementary Material

Supplementary material are available at *American Journal of Agricultural Economics* online.

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Appendix

Derivation of Equation 1

Starting with area of a circle representing the irrigable agricultural acreage above the aquifer,

$$A = \pi u^2$$

we can relate the geometry of the aquifer to the radius of the irrigable acreage

$$u = \frac{x}{\tan(\theta)}.$$

Using these two, we obtain irrigable area as a function of the groundwater height and geometry of the aquifer.

$$I = \pi \left(\frac{x}{\tan(\theta)} \right)^2.$$

With the physical parameters of the aquifer being known, x_s and A, we can solve for total aquifer radius, u_0, and $\tan(\theta)$:

$$u_0 = \frac{x_s}{\tan(\theta)}$$

$$\tan(\theta) = \frac{x_s}{u_0}.$$

With these, we can come to equation (1) in the paper by making γ a percentage of the total aquifer area remaining at a groundwater height x_t:

$$\gamma_t(x_t) = \frac{\pi \left(\frac{x_t}{x_s/u_0} \right)^2}{A}.$$

Since x and u are measured in feet, A should also be in square feet for numerical analysis. We refer to areas in terms of acres in the text for simplicity.

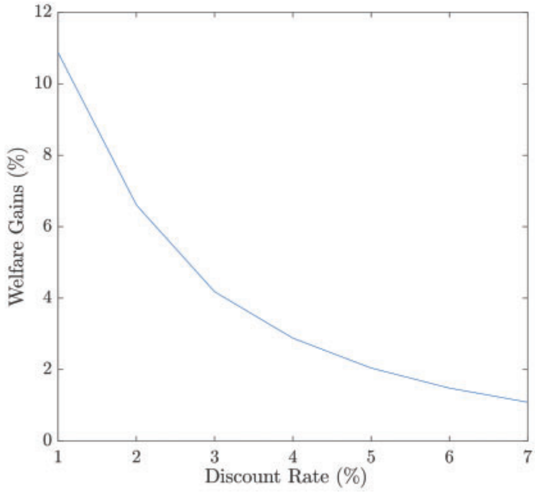

Figure A1. Welfare gains through a range of discount rates, deterministic rainfall scenario

Note: The choice of a discount rate affects the estimates of welfare gains from moving from a myopic to optimal extraction of the resource. For long-term, inter-generational problems, the choice of discount rates is controversial. See Freeman, Herriges, and Kling (2014) and U.S. EPA (2010) for an overview of the issue.

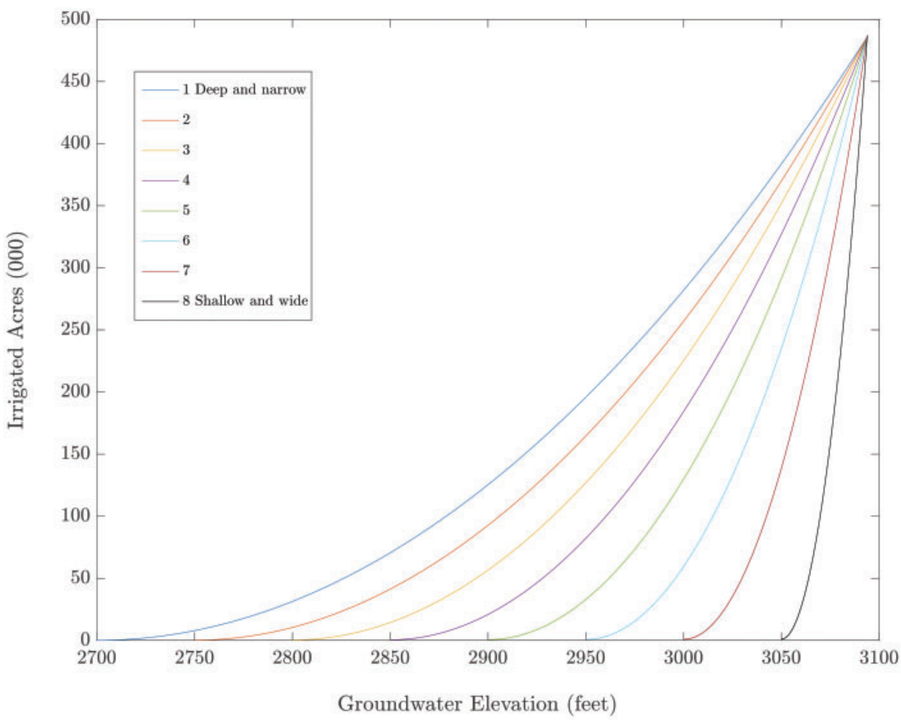

Figure A2. Groundwater elevation to irrigated acres

Note: The lines show the irrigated acreage as a function of groundwater elevation. A shallower cone has a steeper curve in this figure, showing that irrigated land depleted faster for a given height change. Table 5 shows the corresponding welfare implications. The total cone volume is held constant by changing the surface area. The initial irrigated acreage is held constant by changing the farming intensity.