

Bryant University

HONORS THESIS



A Stochastic Approach to Portfolio Optimization Using Competing Risk Metrics

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Submitted in partial fulfillment of the requirements for graduation
with honors in the Bryant University Honors Program

MAY 2020

A Stochastic Approach to Portfolio Optimization Using Competing Risk Metrics

Bryant University Honors Program

Honors Thesis

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May, 2020

Acknowledgements

I would like to thank my advisor Professor Rick Gorravett whose valuable feedback and support proved to be vital to the development of this Thesis. Similarly, I would like to thank Professor David Louton for his advice and valuable comments. Finally, I would like to thank all of the Honors Program Faculty and Staff who helped along the way, this would not have been feasible without your help.

A special note of gratitude to my parents for providing me with all the support and resources needed to successfully finalize my studies at Bryant University.

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Abstract

This thesis presentation presents a stochastic approach to portfolio construction using various risk metrics as underlying models for portfolio optimization. The risk models utilized in this thesis include Mean-Variance, Minimum-Variance, Value-at-Risk (VaR), Conditional Value-at-Risk (CVaR). To evaluate the efficiency and overall performance of these models, historical data for 30 specific stocks was selected. The stock selection process focused on the selecting stocks that are highly volatile and correlated with one another. Empirical results reveal that portfolio optimization strategies outperform the benchmark. Additionally, results showed that the Minimum-Variance model constructed the best portfolio for the predetermined backtesting time period.

Keywords: Portfolio construction models, empirical analysis, stochastic models, portfolio optimization.

1 Notation

R_i = Return of an asset(i) at time t_{t+1}

R_p = Return of a portfolio

r_i = Adjusted return profile of an asset (i) at time t_{t+1}

r_μ = Average returns of asset (i) for a given time range

n = Number observations in data set

σ_i = Standard deviation of an asset (i)

σ_p = Standard deviation of a portfolio (p)

$\rho_{i,j}$ = Covariance between asset (i) and asset (j)

N = Number of assets

x_i = Investment amount of asset/stock (i)

$dt = \max\{0, \varphi - \sum_{i=1}^N r_{it}x_i\}$

σ = Standard deviation of returns for an investment (i)

D_1 = Dividend at time t_{t+1}

R_{rf} = Return of a risk-free investment (usually 10-year T-Bill)

2 Introduction

The goal of portfolio optimization is to allocate funds to an asset following some objective and relevant parameters. An asset, or investment vehicle, is anything from stock and bonds, to real estate and foreign currencies. Typically, optimizers focus on maximizing factors of expected returns, while minimizing costs or financial risk. Harry Markowitz is considered the founding father of modern portfolio management theory. His work on portfolio selection in the 1950s set up the analytical framework of portfolio selection and optimization. To understand the mathematical models proposed by Markowitz, the following underlying concepts must be considered.

2.1 Risk and Returns

Consider a financial asset with an initial price of p_0 dollars at the time of purchase. That same asset at time t_{t+1} will now have a price of p_1 dollars. Therefore, the return profile for that specific financial asset can be calculated as:

$$R_i = \frac{p_1 - x_0}{p_0}$$

Additionally, since this paper is concerned with publicly traded companies i.e. equities, one must consider the effect of dividends (D_1) on the price of the stock. The adjusted price of a stock, over given time period, is therefore given by:

$$r_i = \frac{p_1 - x_0}{p_0} + \frac{D_1}{p_0}$$

Furthermore, consideration must be given to the risk incurred as a result of investing in an asset (i). Financial literature states that the risk of an investment can be measured by its variance (σ^2), and consequently the standard deviation of returns:

$$\sigma = \sqrt{\frac{\sum (r_i - r_\mu)^2}{n - 1}}$$

2.2 Portfolio Construction

The considerations for risk and return become important when constructing a portfolio of financial assets. As a result, consider a set of random financial

assets $N = 1, 2, 3, \dots, n$. For a given time frame, we can make the argument that these assets generate the following returns:

$$\xi = (\xi_1, \xi_2, \xi_3, \dots, \xi_n)$$

However, because of the budget constraint, an individual is only able to invest a specified amount on a given portfolio. Therefore, it is up to his/her discretion on how they would allocate or distribute their funds among a set of financial assets. Fund allocation is therefore described as the weight (as a percentage of the total funds) allocated to each asset in a portfolio. This is represented by the following equation:

$$w = (w_1, w_2, \dots, w_n)$$

In the 1950s, Harry Markowitz pioneered and developed the analytical framework for portfolio selection. He stated that the returns of a portfolio can be categorized as the total returns of the individual financial assets, relative to the funds allocated to each asset [1].

$$R_p = \sum_{i=1}^N w_i \xi_i \tag{1}$$

Additionally, Markowitz classified the risk of a portfolio as the covariance between the returns of an asset (i) and the returns of an asset (j) multiplied by their relative standard deviations. Therefore, the risk of a portfolio can be categorized as:

$$\sigma_p = \sqrt{\sum_{i=1}^N \sum_{j=1}^N \sigma_i \sigma_j \rho_{ij}} \tag{2}$$

2.2.1 Mean-variance optimization

Based on the Markowitz equations for risk and return, we are able to set up an optimization problem that allows us to calculate the optimal portfolios with the best risk-adjusted returns. The mean variance optimization model is described as:

$$\min \sum_{i=1}^N \sum_{j=1}^N \sigma_{i,j} x_i x_j$$

$$\begin{aligned} s.t. \quad & \sum_{i=1}^N w_i \xi_i = r_p \\ & \sum_{i=1}^N w_i = 1 \\ & 0 \leq w_i \leq 1, \forall_i \end{aligned}$$

The mean-variance optimization model, will enable us to calculate what the optimum portfolios are, at a given level of risk. Plotting each individual portfolio on a risk vs. return graph, will enable us to show the efficient frontier, or the Markowitz Efficient frontier. Figure 1 shows an example of an efficient frontier.

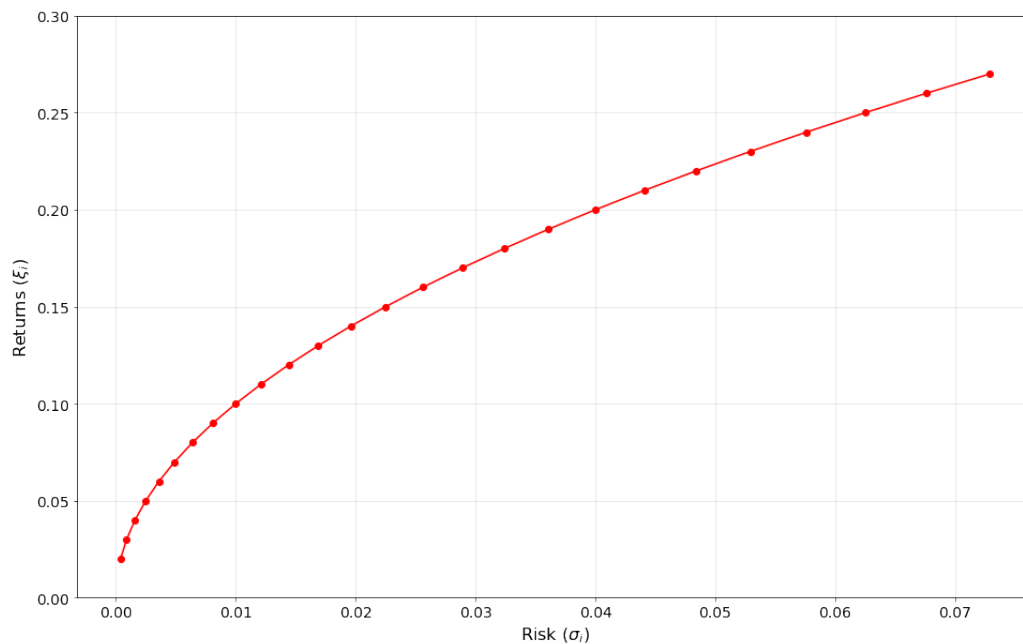


Figure 1: Efficient Frontier

2.2.2 Minimum-Variance Optimization

Another optimization model that will be utilized in this paper, is the minimum variance model. This model seeks to optimize for portfolios that

are the least volatile i.e. have the lowest risk, and therefore, lowest standard deviation (σ), without regard for optimizing based on portfolio returns. The minimum variance model can be described as:

$$\begin{aligned} \min \quad & \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{i,j} \\ \text{s.t.} \quad & \sum_{i=1}^N w_i \xi_i = r_p \end{aligned}$$

2.2.3 The Optimal Portfolio: Sharpe Ratio

Alternatively, William Sharpe's work on the Sharpe Ratio allowed an investor to identify the best risk-adjusted returns, relative to a risk-free asset. The Sharpe Ratio is a measure that allows an investor to calculate the excess returns that an asset (or portfolio) has earned, relative to a risk-free investment, in terms of per unit of risk incurred [2]. Thus, in mathematics, the Sharpe Ratio acts as the utility function for Markowitz' efficient frontier. The Sharpe Ratio is given by:

$$\text{SharpeRatio} = \frac{R_p - R_{rf}}{\sigma_p} \quad (3)$$

Using the Markowitz model we are able to calculate the optimal portfolios at each risk level. However, optimizing for the Sharpe Ratio, allowed an investor to identify the portfolio that had the best risk-adjusted returns, relative to a risk-free asset. This optimization can be described as:

$$\begin{aligned} \max \quad & \frac{R_p - R_{rf}}{\sigma_p} \\ \text{s.t.} \quad & \sum_{i=1}^N w_i = 1 \\ & 0 \leq w_i \leq 1, \forall_i \end{aligned}$$

2.2.4 Application

To understand how these optimization models work, I have retrieved three years worth of adjusted returns for five companies: Apple (AAPL), J.P. Morgan (JPM), Exxon Mobile (XOM), Honeywell (HON), and Procter & Gamble (PG). These companies will have different return and risk profiles. Applying the optimization models for the return and risk profiles of these companies allows us to create the Markowitz Efficient frontier. This can be seen in Figure 2.

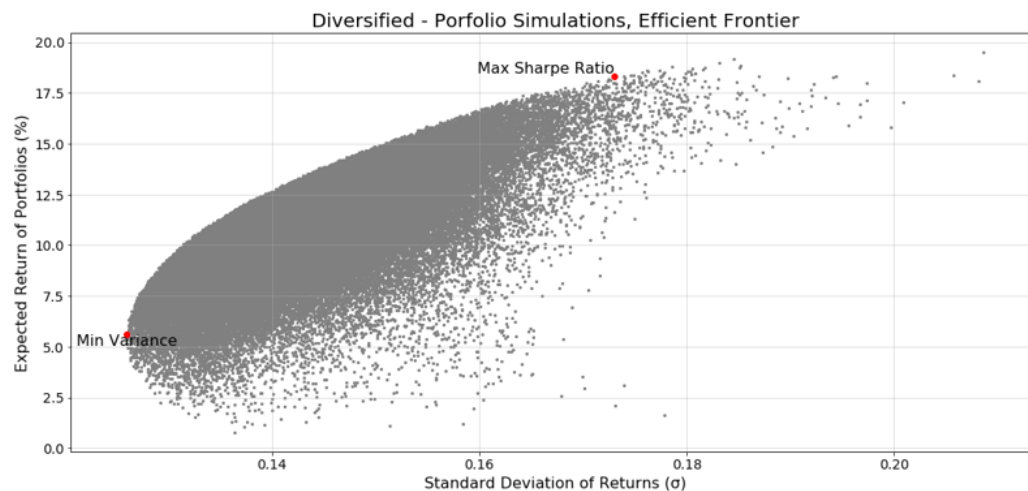


Figure 2: Efficient Frontier – Application with 5 financial assets

Additionally, Figure 2 highlights the minimum variance and max Sharpe portfolio. It must be noted that all the portfolios under the efficient frontier are considered inefficient because they are not able to provide suitable risk-adjusted returns.

The benefits of optimizing portfolios for better fund allocation can be highlighted when the individual risk-return profiles for each company are plotted on the efficient frontier curve. This allows us to understand how diversifying a portfolio's holdings can lead to better returns at a lower risk. Figure 3 shows the Markowitz Efficient Frontier, and the individual risk and return profiles for the companies listed at the beginning of this section.

For example, Figure 3 shows that an investor that has 100% fund allocation

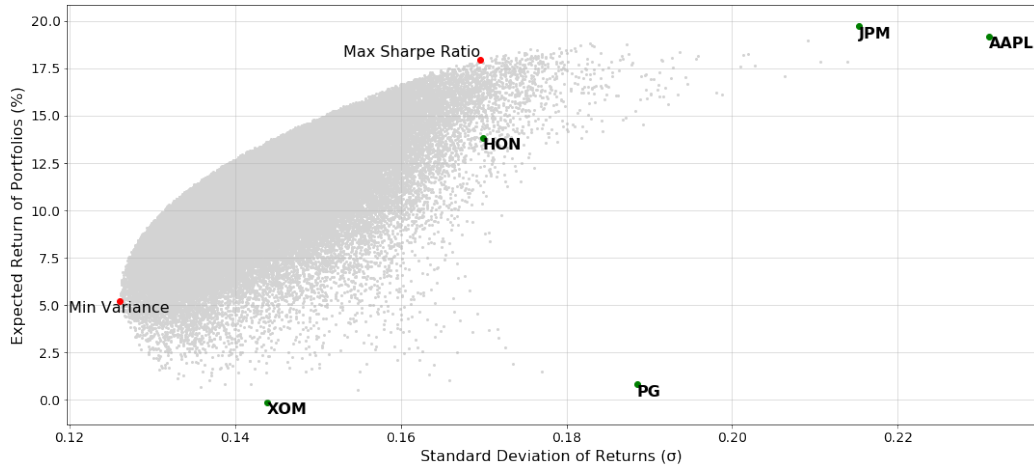


Figure 3: Efficient Frontier and Individual Risk & Return Profiles

in J.P Morgan can earn better returns than the optimal portfolio. However, the investor achieves these returns at a significantly higher risk. In this example, the expected return of the Sharpe portfolio is 17.97% and a standard deviation of 0.17. Whereas J.P Morgan has returns of 19.70% but a standard deviation of 0.21. Therefore, the optimal portfolio can reduce the risk, while maximizing the returns of the portfolio. The optimization yields the following weight recommendations Apple: 32%, J.P Morgan: 43%, Exxon: 0.91%, Procter & Gamble: 0.07%, and Honeywell: 23%.

2.3 Additional Risk Metrics

2.3.1 Value-at-Risk (VaR)

Value-at-Risk (VaR), just like variance and standard deviation, is another method for measuring/estimating the potential loss of investment. VaR estimates the value that might be lost on an investment, given a confidence interval (α), for a specific time period. Artzner et. al. provided the general definition for VaR [3]. For a given $\alpha \in (0, 1)$, VaR is defined as:

$$\begin{aligned}
 VaR_{\alpha}(X) &= -\inf\{x \in \mathbb{R} : F_x(x) > \alpha\} \\
 VaR_{\alpha}(X) &= \min\{c : P(X \leq c) \geq \alpha\}^{[10]} \\
 F_Y^{-1} &= (1 - \alpha)^{[4]}
 \end{aligned}
 \tag{4}$$

It is important to note that this paper assumes that returns of the companies follow a normal distribution ($r \sim N(\mu, \sigma^2)$). Thus, the VaR of the returns of a portfolio can be visually represented in Figure 4. The curve in Figure

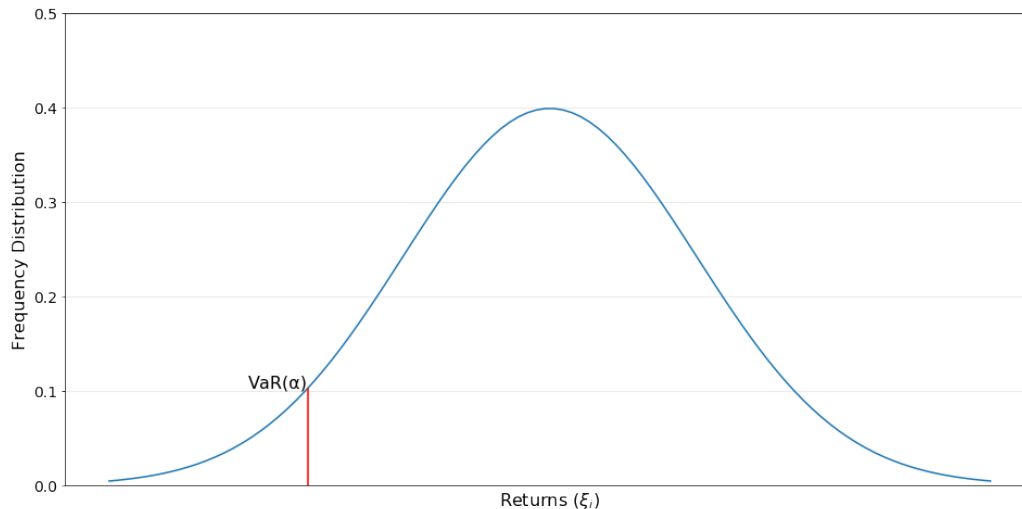


Figure 4: Distribution of Returns & Value-at-Risk

4, represents the distribution of returns for a given portfolio. Assuming that it has a mean (μ) of 1 and a standard deviation (σ) 1. Therefore, the area to the left of the VaR represents the potential loss of investment given a confidence interval of α . Typically, confidence intervals are set at 1%, 5% and 10%. This approach to modelling risk was first introduced following a report from the Basle Committee on Banking Supervision, in 1996 [5]. J.P Morgan created one of the more precise definitions for VaR in 1996, saying that “VaR answers the question: how much can I lose with x% probability over a given time horizon” [6][13]. At this point, it must be noted that the confidence interval that will be used in this paper is 5% ($\alpha = 0.05$).

2.3.2 Conditional Value-at-Risk (CVaR)

The final risk model that will be explored in this paper is Conditional Value-at-Risk (CVaR). CVaR is similar to VaR, in the sense that they measure expected losses of a financial asset or portfolio of assets. However, CVaR has superior mathematical capabilities, and it is considered to be a coherent risk measure. CVaR can account for losses that exceed VaR. This

second-order quantile measure was explored by Artzner et. al. [3], Rockafellar and Uryasev [7], and Ogryczak [8].

CVaR, also known as Expected Shortfall (ES), can be defined as the maximum expected loss of a portfolio for a given period of time under a confidence interval (α). Rockafellar and Urasev [7] showed that CVaR can be mathematically defined as

$$CVaR_{\alpha}(X) = -\frac{1}{\alpha}[E(X * 1_{\{x \leq -VaR_{\alpha}(X)\}}) - VaR_{\alpha}(X)(\alpha - P(X \leq -VaR_{\alpha}(X)))]$$

$$CVaR_{\alpha}(X) = -\frac{1}{\alpha} \int_0^{\alpha} VaR_p(X) dp \quad (5)$$

Hu and Zhang [9] set up the optimization of CVaR as the following: “Suppose that the rate of return is a discrete random variable for each stock, then the CVaR optimization model can be described as follows:”

$$\begin{aligned} \min \quad & \left\{ \frac{1}{\alpha} \sum_{t=1}^T d_t p_t - \varphi \right\}^{[9]} \\ \text{s.t.} \quad & d_t \geq \varphi - \sum_{i=1}^n r_{i,j} x_i, t = 1, \dots, T \\ & d_t \geq 0, t = 1, \dots, T \\ & \sum_{i=1}^n r_i x_i \geq \rho M_0 \\ & \sum_{i=1}^n x_i = M_0 \end{aligned}$$

Here p_t represents the probability of this event occurring given specific scenarios and φ represents an ‘unbounded auxiliary variable’, which equals $VaR_{(\alpha)}$.

Therefore, the visual representation of the efficient frontier for a CVaR and VaR portfolio based on optimization can be seen in Figure 5. This figure was

retrieved from a paper written by Vinay Kaura from the Imperial College London, where portfolio optimization for value-at-risk is investigated. It must be noted that they used an confidence interval of 99% ($\alpha = 0.01$).

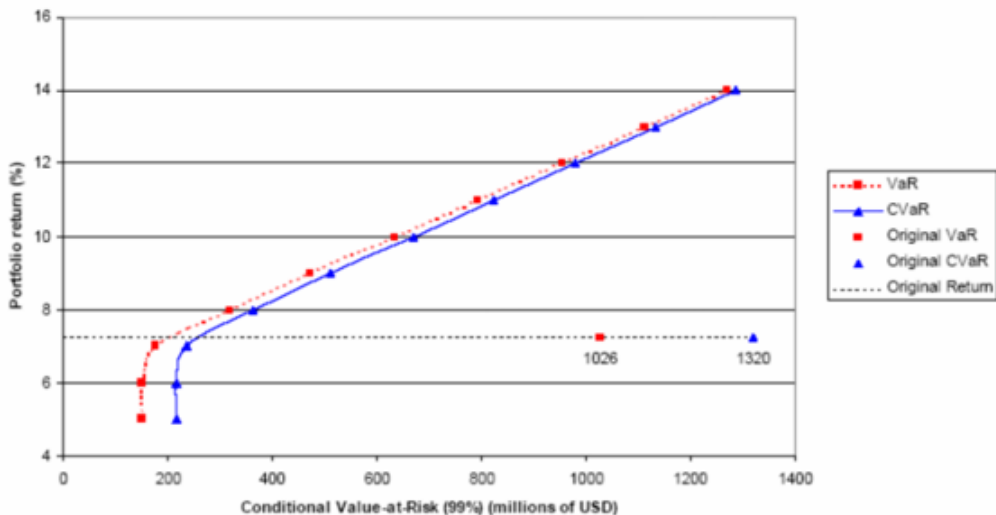


Figure 5: Efficient Frontier for VaR and CVaR Portfolio, as per optimizations done by Vinay Kaura [10]

2.3.3 Application of VaR & CVaR Optimization

If we refer back to the companies in section (2.2.4), and apply the optimization models for CVaR and VaR, we are able to generate an efficient frontier similar to that shown in Figure 5. Figures 6 and 7, highlight the minimum VaR and minimum CVaR Portfolios, respectively. Figure 6 shows the efficient frontier for the $VaR_{(\alpha)}$ portfolios, and Figure 7 shows the efficient frontier for the $CVaR_{(\alpha)}$ portfolios.

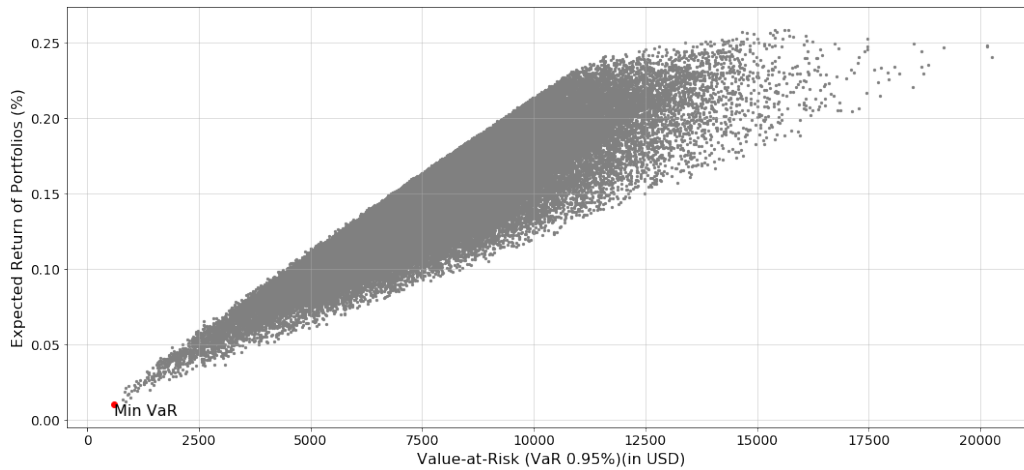


Figure 6: VaR Efficient Frontier – Application

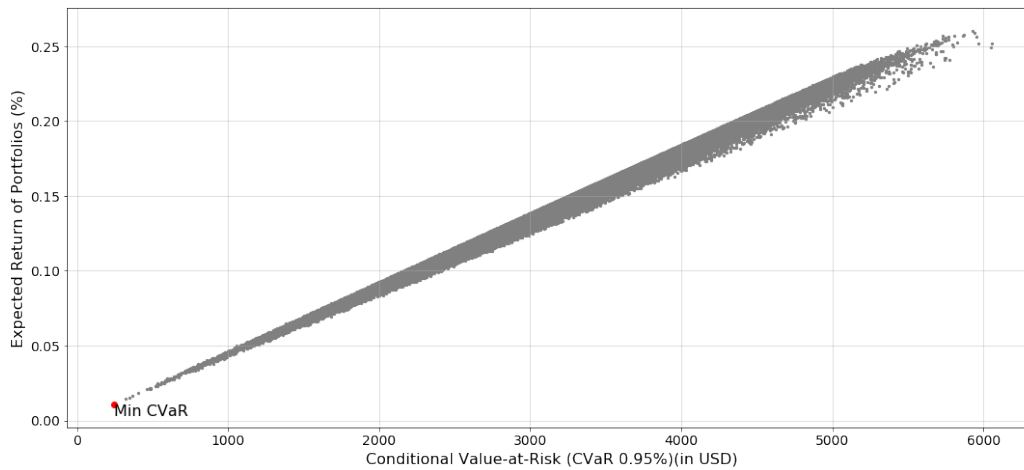


Figure 7: CVaR Efficient Frontier – Application

To summarize, VaR provides an investor with a range of potential losses for a specified confidence interval, while CVaR measures the average expected loss. Looking at Figures 6 and 7 we can see that predicted losses for VaR are larger than those predicted by the CVaR model. Since VaR is not a coherent risk measure, these differences could be attributed to CVaR being a coherent risk measure.

2.3.4 Coherent Risk Measures

Artzner et. al. [3] pioneered the development of coherent risk measures, in their paper “Coherent measures of risk”. In this paper, it is argued that for a risk model or measure to be coherent, they have to satisfy a set of axioms. For a risk measure to be coherent, it has to show normalization, monotonicity, sub-additivity, positive homogeneity, and translation invariance. Consider a risk measure (ρ):

Axiom 1. Normalization The normalization axiom states that the risk of nothing is zero.

$$\rho[0] = 0 \tag{6}$$

Axiom 2. Monotonicity ([3, p. 210] Monotonicity) This axiom states that a portfolio with greater future returns will have lower expected losses. Consider two random variables X and Y that represent losses, then:

$$X \leq Y \longrightarrow \rho(X) \leq \rho(Y) \tag{7}$$

Axiom 3. Sub-additivity ([3, p. 209] Sub-additivity) This axiom states that diversification reduces the risk of a portfolio. The risk of two portfolios added together, cannot be greater than the individual risk of each standalone portfolio.

$$\rho(X + Y) \geq \rho(X) + \rho(Y) \tag{8}$$

Axiom 4. Translation Equivariance ([3, p. 209] Translation Equivariance) This axiom states that an increase in losses (c) will increase risk by the same amount.

$$\rho(X + c) = \rho(X) + c \tag{9}$$

Axiom 5. Positive Homogeneity ([3, p. 209] Positive Homogeneity) This axiom states that growing the size of a portfolio by a factor of λ , will increase risk by the same amount.

$$\rho(\lambda X) = \lambda \rho(X) \tag{10}$$

VaR is not considered to be a coherent risk measure since it does not meet the sub-additivity axiom, axiom 3. [11]

3 Empirical Analysis

3.1 Data Selection

The portfolio selection pool is constructed of all the stocks currently found in the PSI - Invesco Dynamic Semiconductors ETF [12]. One of the goals of this paper is to analyze how these optimization models perform with highly volatile and correlated underlying assets. Thus, limiting stock selection to one sector ensured that the return profiles of the selected stocks would be highly correlated with one another.

Furthermore, the stocks found in the PSI ETF are overweight low volatility, which enabled me to conduct analysis on the effects of volatility on the optimizing models. There was a total of 30 stocks selected. The data was collected from the 3rd of March 2017 until the 3rd of March 2020 (3 years). The data showed daily prices for each one of the companies. The PSI ETF will be used as the benchmark for backtesting.

Additionally, the 10-year US treasury rate was used as the risk-free asset. This allows the optimizing model to identify the optimal portfolio. The data was collected for the same time period and using the same frequency i.e. daily.

3.2 Expected Returns

Expected Returns were calculated based on a series of research reports. Using FactSet Market Research, I retrieved implied target prices for all the companies in the PSI [14]. The same implied target prices were selected from Morningstar Research [15] and ValueLine [16]. An average of implied target prices was used as the expected returns.

3.3 Computational Methods

Using the equations described in Section 2, a stochastic simulator was created using Jupyter Lab. The Python code was set up so that random weights were generated, which would allow for calculation expected portfolio returns and portfolio standard deviation. The simulation was run 250,000 times i.e. 250,000 different portfolios were generated. This allowed

for a more accurate calculation of optimal and minimum variance portfolio. The python optimizer then produced the weights for both the optimal, minimum variance, minimum VaR, and minimum CVaR portfolio.

Using the Quantopian platform, these weights were incorporated in a buy-and-hold - long-only - strategy, and the model was backtested for 2 years. The Quantopian platform provides performance metrics that were used to determine which optimization model yielded performed better, relative to the benchmark (Invesco PSI).

4 Results

4.1 Mean-Variance & Minimum-Variance Portfolios

Using the Markowitz optimization model, the python optimizer was able to compute the returns and standard deviations for 250,000 portfolios. Additionally, using the Sharpe Ratio as the utility function, I was able to identify the optimal portfolio given the data inputted into the model. Also, the optimizer found the minimum variance portfolio. The weight allocations for these portfolios were stored for use in Quantopian backtesting platform. This information can be seen in Figure 8.

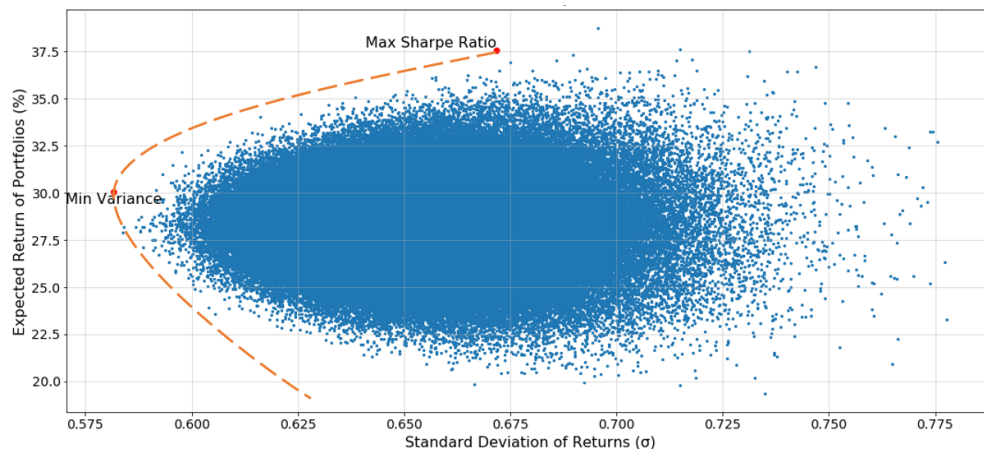


Figure 8: Markowitz Efficient Frontier using the 30 Stocks in PSI

Thus, looking at figures 9 and 10 we can see how the mean-variance and

minimum variance models, respectively, distributed the allocation of funds amongst the companies in the PSI.

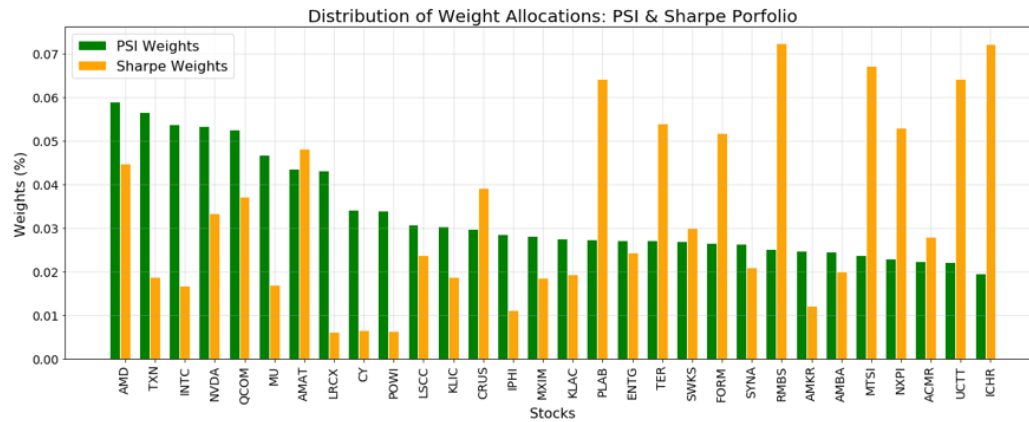


Figure 9: Weight Allocation Mean-Variance Optimal Portfolio Vs. PSI Index

Figure 9 shows that the Sharpe ‘Optimal’ portfolio allocates more weights to the companies on the right-hand side of the figure. In contrast, the index ETF portfolio allocates weights more heavily towards big names such as AMD, TXN, and INTC.

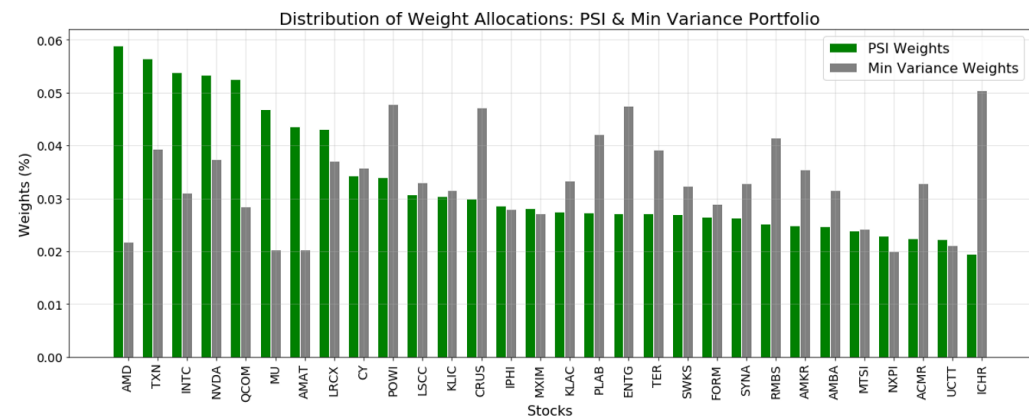


Figure 10: Weight Allocation Minimum-Variance Portfolio Vs. PSI Index

Conversely, as Figure 10 show, the minimum variance portfolio spread the fund allocation more evenly across the 30 companies. There is greater weight

allocation towards companies in the ‘middle of the pack’. The minimum-variance portfolio only allocates more than 5% to one company: ICHR.

4.2 Value-at-Risk & Conditional Value-at-Risk Portfolios

The optimization equations defined in earlier equations were also used to generate the efficient frontier for the returns of portfolios, relative to the VaR and CVaR of each portfolio. Again, the simulation was run 250,000 times, and thus, generated 250,000 portfolios. Figure 11 and 12 show the efficient frontier for VaR and CVaR optimization models.

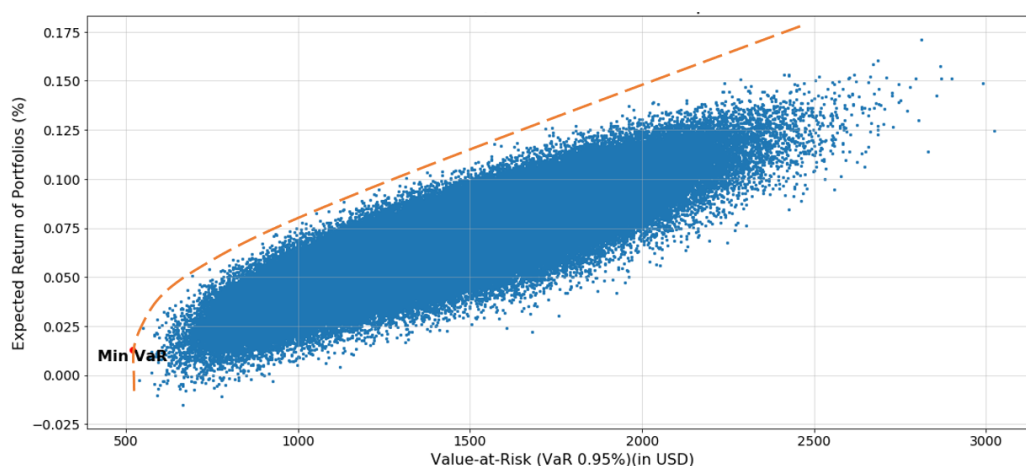


Figure 11: VaR Efficient Frontier using the 30 Stocks in PSI

One of the interesting findings between these two optimization models was the dispersion of inefficient portfolios. Figure 11 shows that there is a broader range of inefficient portfolios, while Figure 12 shows that these portfolios are more tightly packed along the frontier. The parameters for both optimization models were the same, yet they yielded significantly different distribution of portfolios. While I am still investigating the possible cause of this, I believe that because VaR provides a range and not an average, then there is a broader possibility of outcomes. However, this not clear at this moment.

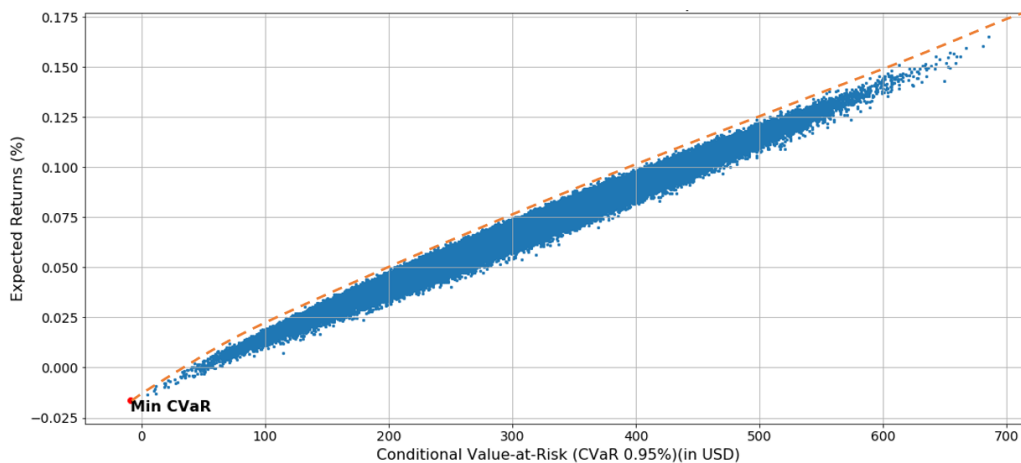


Figure 12: CVaR Efficient Frontier using the 30 Stocks in PSI

Furthermore, the weight allocations of each model are significantly different from one another. The distribution of weights among the 30 semiconductor companies can be seen in Figure 13 and 14.

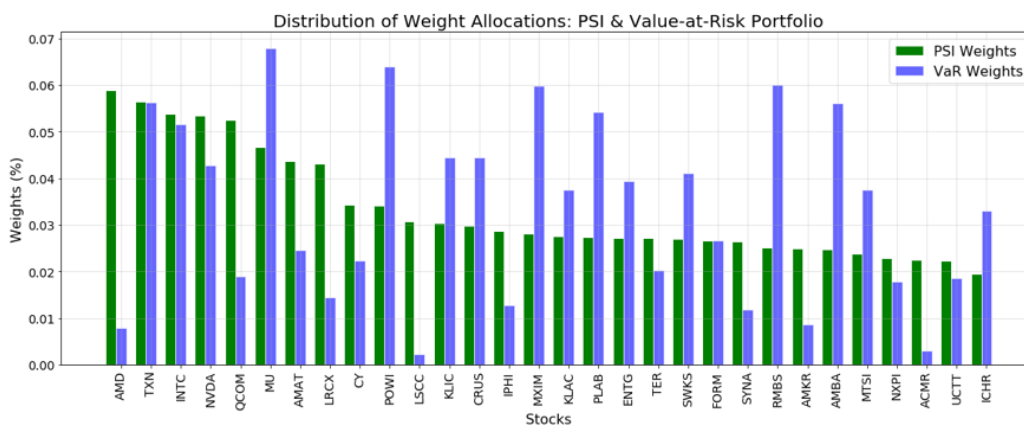


Figure 13: Weight Allocation VaR Portfolio Vs. PSI Index

Figure 13 shows that a total of eight companies received weight allocations that were greater than 5%. This is significant since the PSI only allocates 5% or more to only five companies. Furthermore, there were a total of four companies that were allocated less than 1%. Similar to the minimum-

variance portfolio, weights are heavily allocated toward the middle of the pack.

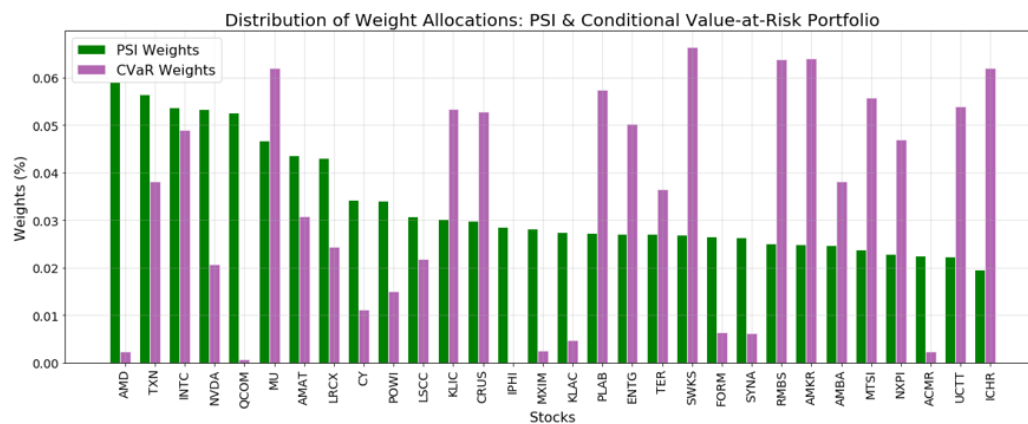


Figure 14: Weight Allocation CVaR Portfolio Vs. PSI Index

Figure 14 shows a great deviation from the PSI ETF weight allocations. IPHI was the one company that the model did not allocate any weight to. Furthermore, 11 stocks had weight allocations greater than 5%, and eight stocks had weight allocations of less than 1%.

4.3 Backtest & Results

As mentioned in section 3, I created a backtesting model that followed a buy-and-hold (long-only) strategy that invested based on the weights allocated to them by the optimization model. As a result, I backtested the data for a total of 2 years, from 23/03/2018 until 23/03/2020. The Quantopian algorithm took the weight allocation profiles from each one of the optimization models and backtested the model.

The performance metrics used to assess the performance of the optimization model, relative to the benchmark, by measuring Total Returns, Alpha, Portfolio Beta, Sharpe Ratio, and max drawdown. Table 1 shows the results of running the backtest.

Performance metrics show that the minimum variance model performs better than the other models. This model generated significant

Table 1 - Performance Measurement for Sample Period

Models	Returns (%)	Alpha (%)	Beta	Sharpe (%)	Drawdown (%)
Sharpe (optimal)	28.71	0.07	1.04	0.49	-40.04
Minimum Variance	40.96	0.11	1.02	0.61	-36.38
VaR	18.71	0.03	1.02	0.39	-35.56
CVaR	11.16	0.00	1.05	0.31	-40.19

outperformed in measures of alpha, returns, Sharpe, and drawdown. The alpha measures the returns of the portfolio relative to the benchmark. Positive alpha shows that the model outperforms the benchmark, while negative alpha shows that the model underperformed the benchmark. Furthermore, drawdown shows the maximum observed loss, from peak to a trough. Therefore, models that can generate lower maximum drawdown figures are said to be more risk-averse.

A note about timing:

At the time of making the backtesting algorithm, I did not take notice of potential issues that may arise as a result of investment timing. Therefore, I backtested the model using information that was included in my original dataset. Thus, the overall results may not accurately reflect the performance of the optimization model.

However, I have run the models setting the starting date as the date in which the expected returns were calculated. Therefore, there are no timing errors in the results of the optimization model. However, this means that the models can only be run for just over a month. For the results to be significant, the model would have to be tested against at least 6 months of future data, before being optimized again. This is a consideration that I will keep, and will potentially add as an adjustment at a later date. This can be seen in Table 2. It seems to suggest that the Sharpe portfolio is the best portfolio, but not enough data has been collected to produce a coherent answer.

Table 2 - Performance Measurement for Adjusted Sample Period

Models	Returns (%)	Alpha (%)	Beta	Sharpe (%)	Drawdown (%)
Sharpe (optimal)	13.34	0.71	1.03	2.15	9.60
Minimum Variance	11.00	0.51	0.99	1.91	-9.64
VaR	9.66	0.40	0.95	1.77	-9.53
CVaR	12.14	0.60	1.03	2.00	-9.74

5 Conclusion

In conclusion, this paper has shown that it is possible to use different risk models to generate better returns than those generated by Exchange Traded Funds. However, in this paper, there were no considerations of the possible implications of transaction costs, and the effect they may have on the alpha generated by the optimization method. In future studies, one could explore the effects of transaction costs, and other real market features, on the optimization models. This would enable an investor (most likely a Portfolio Manager) to determine if they should just buy the ETF or try their own investment method.

Furthermore, this paper does not address the implication of different market phases. For example, one could make the argument that risk-minimizing models are more likely to perform better in volatile and bearish market, while the Sharpe mean-variance model is more likely to outperform during a bullish market. This could be an interesting consideration for future studies. Finally, the incorporation of timing into the Backtesting algorithm is of crucial importance to the validity of results. This paper will be updated in 5 months, to ensure that there are no timing issues.

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