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Manzoor A. Khanday
Vikram University Ujjain, manzoorstat@gmail.com

J. R. Singh
Vikram University Ujjain

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Economic Design of $\bar{X}$ Control Chart under Double EWMA

Manzoor A. Khanday
Vikram University Ujjain
Ujjain, India

J. R. Singh
Vikram University Ujjain
Ujjain, India

Designing of parameters plays an important role in economic design of control charts for lowering the cost and time. Manipulating sample size (n) and sampling interval (h), the effect of double exponentially weighted moving average (DEWMA) model was studied for the Economic Design (ED) of $\bar{X}$ control chart. Optimum sizes and level were obtained when the characteristics of an item possesses DEWMA model. When shifts are uncertain the optimal design for DEWMA chart should be more conservative and should be implemented for benefiting the consumers as well as producers.

Keywords: Control chart, economic design of control chart, DEWMA

Introduction

Control charts are the essential statistical tools for evaluating process presentation, dealing process capability, observing processes and refining processes. During the observation, the statistical control chart is employed to detect the occurrence of assignable cause. The Shewhart $\bar{X}$ chart assumed to be the best statistical tool in Statistical process control. However, it has limitations int the detection of small and moderate process mean shifts. Therefore, a widespread control chart used for small alterations of process mean moved to the EWMA by Roberts (1959).

The exertion to rise the sensitivity of EWMA control chart for identifying small alterations and logic in a process, a double EWMA (DEWMA) control chart was developed. Zhang (2002) considered the DEWMA control charts for the mean. The traditional EWMA and DEWMA control charts for watching process means were developed under the likelihood of normality. Other extensions of control charts to improve process monitoring were Saghaei et al. (2014), Amiri et al. (2015), and Lee et al. (2014). Akhavan Niaki et al (2013a, b) presented the ESD of the VSI

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Correspondence: Manzoor A. Khanday, manzoorstat@gmail.com

$\bar{X}$ control charts for the correlated and non-normal data. Akhavan Niaki et al. (2013c) studied the ESD of the VSI $\bar{X}$ control charts in the existence of the multiple assignable causes with normal and correlated data.

The design of a control chart depends on sample size (n), the control limits (k), and the time interval between samples (h). Various costs are linked with control chart usage, which includes sampling cost and testing, the cost related with examining out-of-control signals and modifying the assignable causes, and the costs of allowing non-conforming units reaching the customer. Khanday and Singh (2015) studied the effect of Markoff's model on Economic design of $\bar{X}$ control charts under independent observations.

Duncan's Model for the Cost Function

Duncan (1956) obtained an approximate function for the average net income per hour of using the control chart for mean of normal variables as

$$\mathrm{I} = V_0 - \frac{\eta M\,\mathrm{B} + \left(\alpha T/h\right) + \eta W}{1 + \eta\,\mathrm{B}} - \frac{b + cn}{h}. \tag{1}$$

Duncan's cost model indicates

(i) The rate of an out-of-control conditions;
(ii) The rate of false alarms;
(iii) The rate of finding an assignable cause; and
(iv) The rate of sampling inspection, evolution, and plotting.

The average cost per hour involved for maintaining the control chart is $(b + cn) / h$. The average gross income per hour of the process under the investigation of the control chart for mean can be redrafted as $\mathrm{I} = V_0 - \mathrm{L}$, where

$$\mathrm{L} = \frac{\eta M\,\mathrm{B} + \left(\alpha T/h\right) + \eta W}{1 + \eta\,\mathrm{B}} - \frac{b + cn}{h}. \tag{2}$$

L can be treated as the per hour cost due to the surveillance of the process under the control chart. The probability density function is denoted by P′ and α′, which are determined from the sampling distribution of mean and are written as

$$\mathrm{P}' = 1 - \Phi(\xi), \tag{3}$$

$$\alpha' = \alpha_N = 2\Phi(-k), \tag{4}$$

where $\xi = \left(k - \delta\sqrt{n}\right)$.

Derivation for Ideal Value of Sample Size *n* and Sampling Interval *h*

The optimum value of sample size (n) and sampling interval (h) may be obtained either by maximizing the gain function I or by minimizing the cost function L. Differentiating L with respect to n and h and equating to zero,

$$\frac{\partial \mathrm{L}}{\partial n} = \frac{(1+\eta\,\mathrm{B})\left(\eta M\,\frac{\partial \mathrm{B}}{\partial n} + \frac{T}{h}\frac{\partial \alpha'}{\partial n}\right) - \left(\eta M\,\mathrm{B} + \frac{\alpha' T}{h} + \eta W\right)\eta\,\frac{\partial \mathrm{B}}{\partial n}}{(1+\eta\,\mathrm{B})^2} + \frac{c}{h} = 0\,, \tag{5}$$

$$\frac{\partial \mathrm{L}}{\partial h} = \frac{(1+\eta B)\left(\eta M\,\frac{\partial \mathrm{B}}{\partial h} - \frac{\alpha' T}{h^2}\right) - \left(\eta M\,\mathrm{B} + \frac{\alpha' T}{h} + \eta W\right)\eta\,\frac{\partial \mathrm{B}}{\partial h}}{(1+\eta\,\mathrm{B})^2} - \left(\frac{b+cn}{h^2}\right) = 0, \tag{6}$$

where

$$\frac{\partial \mathrm{B}}{\partial n} = \frac{-h}{\mathrm{P}'^2}\frac{\partial \mathrm{P}'}{\partial n} + c, \quad \frac{\partial \mathrm{B}}{\partial h} = \frac{1}{\mathrm{P}'} - \frac{1}{2} + \frac{\eta h}{6}, \quad \frac{\partial \alpha'}{\partial n} = 0\,, \tag{7}$$

$$\frac{\partial \mathrm{P}'}{\partial n} = \frac{\delta}{2\sqrt{n}}\phi(\xi). \tag{8}$$

The solutions of the equations (5) and (6) for n and h yield the required optimum values, and may be rewritten

$$0=\eta h\left(M-\eta M\,\mathrm{B}-\frac{\alpha' T}{h}-\eta W\right)\frac{\partial \mathrm{B}}{\partial n}+\frac{T}{h}\frac{\partial \alpha'}{\partial n} \\ +\eta\,\mathrm{B}\left(\eta M\,\frac{\partial \mathrm{B}}{\partial n}+\frac{T}{h}\frac{\partial \alpha'}{\partial n}\right)+c\left(1+\eta\,\mathrm{B}\right)^2 \tag{9}$$

$$0=\eta h^2\left(M-\eta M\,\mathrm{B}-\frac{\alpha' T}{h}-\eta W\right)\frac{\partial \mathrm{B}}{\partial n}-\alpha' T\left(1+\eta\,\mathrm{B}\right) \\ +\eta^2 h^2 M\,\mathrm{B}\frac{\partial \mathrm{B}}{\partial n}-\left(b+cn\right)\left(1+\eta\,\mathrm{B}\right)^2 \tag{10}$$

If η is small, and the optimum h is crudely of order of $1/\sqrt{\eta}$, ignore terms with ηB containing ηWc, $(\alpha' T)\,/\,h$, and the terms equating higher powers of η. The equations (9) and (10) are simplified:

$$\frac{-\eta h^2 M}{\mathrm{p}'^2}\frac{\partial\,\mathrm{p}'}{\partial n}-\eta\alpha' T+c=0\,, \tag{11}$$

$$\eta M h^2\left(\frac{1}{\mathrm{P}'}-\frac{1}{2}\right)-\left(\alpha' T+b+cn\right)=0\,. \tag{12}$$

From equation (12),

$$h=\left\{\frac{\alpha' T+b+cn}{\eta M\left(\frac{1}{\mathrm{P}'}-\frac{1}{2}\right)}\right\}^{\frac{1}{2}}\,. \tag{13}$$

By eliminating h from equation (11),

$$\frac{-\alpha' T+b+cn}{\mathrm{P}'^2\left(\frac{1}{\mathrm{P}'}-\frac{1}{2}\right)}\frac{\partial\,\mathrm{p}'}{\partial n}-\eta\alpha' T+c=0\,. \tag{14}$$

The values of n for which equation (14) holds true gives the optimum value of sample size n. Using this value of n in equation (13), obtain the optimum value of the sampling interval (h).

Derivation of the Optimum Values of Sample Size (*n*) and Sampling Interval (*h*) under DEWMA

Suppose a process is on target μ initially and successive measurements $\bar{X}_t$, (t = 1, 2, 3,…) are taken it may be average of several measurements taken at time 't' to check whether there is a shift from the target. To use a control chart based on the statistic

$$Y_t = \lambda X_t + (1-\lambda) Y_{t-1} \quad \text{and} \quad Z_t = \lambda Y_t + (1-\lambda) Z_{t-1}, \tag{15}$$

such that $0 < \lambda < 1$ and $Y_0 = Z_0 = \mu_0$,

$$Z_t = \lambda^2 \sum_{j=1}^{t} (t-j+1)(1-\lambda)^{t-j} X_j + t\lambda(1-\lambda)^t Y_0 + (1-\lambda)^t Z_0 \tag{16}$$

where $E(Z_t) = \mu_0$ and

$$\sigma_{Z_t}^2 = \lambda^4 \left[\frac{1+(1-\lambda)^2 (1-\lambda)^{2t} + (2t^2+2t-1)(1-\lambda)^{2t+2} - t^2 (1-\lambda)^{2t+4}}{\left(1-(1-\lambda)^2\right)^3} \right] \sigma_0 . \tag{17}$$

The upper and lower control limits for DEWMA control chart are

$$\begin{aligned} UCL &= \mu_0 \\ UCL &= \mu_0 + \mathrm{L}\,\sigma \sqrt{\frac{\lambda\left(2-2\lambda+\lambda^2\right)}{(2-\lambda)^3}} \\ UCL &= \mu_0 - \mathrm{L}\,\sigma \sqrt{\frac{\lambda\left(2-2\lambda+\lambda^2\right)}{(2-\lambda)^3}} \end{aligned} \tag{18}$$

Assume X_t is drawn independently from a normal distribution with variance σ^2 and t is sufficiently large. Unfortunately, λ is arbitrary and lies between 0 and 1. Suppose a machine whose performance can be effectively represented by a single unknown quality μ is inspected regularly to see whether the quality of performance is deteriorated. The successive performance level $\mu_1, \mu_2, \mu_3, \ldots, \mu_t$ are tracked by the observations $x_1, x_2, x_3, \ldots, x_t$. The operation continues until a decision is made to overhaul it in which case the level is set to zero instantaneously and the whole sequence begins again. This resetting after overhaul may be subject to error. Assuming μ_0 is $\mathrm{N}(0, \sigma^2 / n)$ and each subsequent state of repair is drawn independently from this distribution:

$$\mathrm{E}(Z_t) = \mu_0 \quad \text{and} \quad \mathrm{Var}(Z_t) = \frac{\sigma^2}{n} \frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3} = \frac{\sigma^2}{n} g^2,$$

where

$$g^2 = \frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}. \tag{19}$$

For the DEWMA model, the probability density function is represented as

$$\mathrm{P}'_e = 1 - \Phi(\xi_e) \quad \text{for} \quad \delta > 0, \tag{20}$$

$$\alpha'_e = \alpha_{Ne}, \tag{21}$$

where

$$\xi_e = \frac{(k - \delta\sqrt{n})}{g}, \alpha_{Ne} = 2\Phi\left(\frac{-k}{g}\right),$$

and equations (5) and (6) are reduced to

$$\frac{\partial \mathrm{L}}{\partial n}=\frac{(1+\eta\,\mathrm{B})\left(\eta M\,\frac{\partial \mathrm{B}}{\partial n}+\frac{T}{h}\frac{\partial \alpha_e'}{\partial n}\right)-\left(\eta M\,\mathrm{B}+\frac{\alpha_e' T}{h}+\eta W\right)\eta\frac{\partial \mathrm{B}}{\partial n}}{(1+\eta\,\mathrm{B})^2}+\frac{c}{h}=0 \quad (22)$$

$$\frac{\partial \mathrm{L}}{\partial h}=\frac{(1+\eta\,\mathrm{B})\left(\eta M\,\frac{\partial \mathrm{B}}{\partial n}-\frac{\alpha_e' T}{h}\right)-\left(\eta M\,\mathrm{B}+\frac{\alpha_e' T}{h}+\eta W\right)\eta\frac{\partial \mathrm{B}}{\partial n}}{(1+\eta\,\mathrm{B})^2}-\left(\frac{b+cn}{h^2}\right) \quad (23)$$
$$=0$$

where

$$\frac{\partial B}{\partial n}=\frac{-h}{\mathrm{P}_e'^2}\frac{\partial \mathrm{P}_e'}{\partial n}+c,\quad \frac{\partial B}{\partial h}=\frac{1}{\mathrm{P}_e'}-\frac{1}{2}+\frac{\eta h}{6},$$
$$\frac{\partial \alpha_e'}{\partial n}=0,\quad \frac{\partial \mathrm{P}_e'}{\partial n}=\frac{\delta}{2\sqrt{ng}}\phi(\xi_e) \quad (24)$$

By solving equations (22) and (23),

$$h=\left\{\frac{\alpha_e' T+b+cn}{\eta M\left(\frac{1}{\mathrm{P}_e'}-\frac{1}{2}\right)}\right\}^{\frac{1}{2}} \quad (25)$$

and

$$-\frac{\alpha_e' T+b+cn}{\mathrm{P}_e'^2\left(\frac{1}{\mathrm{P}_e'}-\frac{1}{2}\right)}\frac{\partial \mathrm{P}_e'}{\partial n}-\eta\alpha_e' T+c=0\,. \quad (26)$$

The values of n for which the equation (26) holds true yield us the required optimum value of sample size n. Substituting this value n in equation (25) yields the ideal value of the sampling interval (h) under non-normality for DEWMA model.

Numerical Illustration

In order to illustrate, let k = 1.0, 1.5, 2.0, 2.5, and 3.0; δ = 1.0, 1.5, and 2.0; η = 0.01, M = 100, W = 25, T = 50, C = 0.05, D = 2, b = 0.5, c = 0.1, and λ = 1, 0.8, 0.6, and 0.2 to determine the optimum values of sample size and sampling interval. The values of n and h are presented in Table 1, it is clearly seen that for given k and δ the sample size n and sampling interval h decrease with the decrease in the values of λ. However, the sample size n has a dramatic effect while decreasing of k, while the sampling interval h has dramatic effect decreasing k. When the rate of occurrences of assignable cause is fixed, the value of sample size and sampling interval are different for different values of λ (shifts). The effect is more serious for DEWMA model for different parameters. In general, the in-control ARL acts of DEWMA control charts were more vigorous. The gradation of strength of DEWMA control chart increases for smaller values of flattening parameter. Thus, the performance of DEWMA is conservative and is recommended as it performs better results than that of EWMA model for smaller shifts. The DEWMA chart employed together with economic design of the control chart scheme when minor shifts to restrained shifts in the mean of the controlled parameter are expected. From an economic point of view DEWMA chart performs better when there are small shifts and spoiled data thus, we recommend DEWMA model in such cases.

Table 1. Optimum sample size (n) and sampling interval (h) under DEWMA model for different values of λ

		k = 3.0		k = 2.5		k = 2.0		k = 1.5		k = 1.0	
λ	δ	n	h	n	h	n	h	n	h	n	h
1.0	1.0	23	2.3371	20	2.4134	19	3.0258	35	4.6216	29	6.2076
	1.5	11	1.8026	9	1.9963	9	2.7064	16	4.1978	14	5.9551
	2.0	6	1.5648	6	1.8157	6	2.5740	9	4.0314	8	5.8593
0.8	1.0	19	2.1217	15	1.9265	11	1.8554	10	2.4322	17	4.3902
	1.5	9	1.6199	7	1.5147	5	1.5448	5	2.2319	8	4.1850
	2.0	5	1.3972	4	1.3342	3	1.4116	3	2.1495	5	4.1063
0.6	1.0	16	2.0240	12	1.8264	9	1.6409	6	1.5734	6	2.6078
	1.5	7	1.5565	6	1.4445	4	1.3437	3	1.3669	3	2.4991
	2.0	4	1.3514	3	1.2794	3	1.2170	2	1.2802	2	2.4559
0.2	1.0	13	1.8646	9	1.6780	7	1.5029	4	1.3440	2	1.2088
	1.5	6	1.4562	4	1.3517	3	1.2567	2	1.1736	1	1.1058
	2.0	3	1.2810	2	1.2147	2	1.1557	1	1.1049	1	1.0646

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Appendix

V_0 = the rate per hour at which income accrues from operation of the process is in control and process average is μ,

V_1 = the rate per hour at which income accrues from operation of the process when process is not in control and process average is $\mu' = \mu + \delta\sigma$,

$M = V_0 - V_1$,

η = the average number of times the assignable cause occurs within an interval of time, starting in a state of control at time $t = 0$, the probability that the process will still be in control at time t_1 is $e^{-\eta t_1}$,

B = $ah + Cn + D$,

$$a = \frac{1}{P} - \frac{1}{2} + \frac{\eta h}{12},$$

h = the interval between samples measured in hours,

e = the rate at which the time taken between the sample and plotting of a point on the $\bar{X}$ chart increases with the sample size n,

C_n = the time required to take and inspect a sample of size n,

D = average time taken to find the assignable cause after a point plotted on the chart falls outside the control limits,

P = probability of detecting an assignable cause when it exists,

$$= \int_{-\infty}^{\mu - \frac{k\sigma}{\sqrt{n}}} g\left(\bar{x}/\mu'\right) d\bar{x} + \int_{\mu + \frac{k\sigma}{\sqrt{n}}}^{\infty} g\left(\bar{x}/\mu'\right) d\bar{x}$$

$$\cong 1 - \Phi\left(k - \delta\sqrt{n}\right) \text{ for } \delta > 0$$

where $g(\bar{x} / \mu')$ is the density function of $\bar{x}$ when the true mean μ and $\Phi(x)$ is the normal probability,

α = probability of incorrectly indicating the occurrence of assignable cause

$$= \int_{\mu - \frac{k\sigma}{\sqrt{n}}}^{\mu + \frac{k\sigma}{\sqrt{n}}} g\left(\bar{x}/\mu'\right) d\bar{x} = 2\Phi\left(-k\right),$$

T = the cost per cases of looking for an assignable cause,

W = the average cost per event of finding the assignable cause when it exists,

b = per-sample cost of sampling and plotting, that is independent of sample size, and

c = the cost per unit of calculating an item in a sample.