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Chiral magnetization configurations in magnetic nanostructures in the presence of Dzyaloshinskii-Moriya interactions

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Many low-dimensional systems, such as nanoscale islands, thin films, and multilayers, as well as bulk systems, such as multiferroics, are characterized by the lack of inversion symmetry, a fact that may give rise to a Dzyaloshinskii-Moriya (DM) interaction. For sufficient strength, the DM interaction will favor spiral spin configurations of definite chirality. In order to harness such systems for applications, it is important to understand the conditions under which these spiral spin configurations form and how they can be controlled via an external field. Here, we present exact solutions of the 1D magnetization profiles in such systems for arbitrary material parameters in closed form. Determining the energy per unit length exactly, we are able to present the critical strength of the DM interaction, at which spiral solutions are energetically favorable. These magnetization profiles, in general, take the form of a domain wall or soliton lattice, with all solitons having the same chirality, whose sign is dictated by DM interaction. Conversely, given an energetically favorable spiral solution, we determine quantitatively how the magnetization profile changes as a function of the applied field. © *2012 American Institute of Physics*. [doi:10.1063/1.3671784]

I. INTRODUCTION

Low-dimensional magnetic systems are a fundamental component of many prospective magnetic memory devices, such as Parkin's proposed "racetrack" memory,[1] where a sequence of domain walls in a magnetic nanowire encodes the binary states. The reduced dimensionality seen in such a system and also present in other magnetic systems, such as nanoscale islands and thin films, can give rise to a lack of inversion symmetry, which may give rise to parity breaking Dzyaloshinskii-Moriya (DM) interactions that favor spiral spin configurations of definite spin chirality, such as those that have been seen in recent spin-polarized scanning tunneling microscopy and photoemission electron microscopy experiments on quasi-1D spin structures.[2,3] To harness such systems for novel applications, it is important that we understand the mechanisms and conditions under which such spiraling configurations can form.

Utilizing a "micromagnetic" continuum approach,[4] we are able to describe, in closed form, the resulting magnetic configurations for quasi-1D spin structures subject to the DM interaction with arbitrary material parameters and subject to an external field. These configurations take the form of a domain wall or soliton lattice, with chirality determined by the DM interaction. By calculating the energy densities per unit length for these configurations, this method allows us to determine the critical strength of the DM interaction at which spiraling configurations are energetically favorable. We conclude by briefly relating our model to recent experimental results, which have seen size-dependent spiraling spin structures in iron nanoparticles.[3]

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II. THEORETICAL MODEL

We consider the following 1D energy density per unit area, which is valid for sufficiently narrow wires,[4] and extend it to include the DM interaction,

$$E = \int_{-L/2}^{L/2} \mathrm{d}x \Big\{ A(\partial_x \mathbf{m})^2 - K_e m_z^2 + K_h m_x^2 - HM_0 m_z - D(\mathbf{m}\times\partial_x\mathbf{m})_x \Big\}, \quad (1)$$

where L is the sample length (or the period of soliton-lattice) and $\mathbf{m}$ is the unit vector of the magnetization, $\mathbf{m} \equiv \mathbf{M}/M_0$, with M_0 the saturation magnetization. Eq. (1) describes the exchange interaction, easy and hard-axis anisotropies, and coupling to an external field aligned along the easy axis, with the final term describing our additional parity-breaking DM term. The sign of the DM term, D, can be either positive or negative and determines the "handedness" of our configuration. It is more convenient to work with dimensionless quantities and with polar coordinates $\mathbf{m} = (\cos\theta, -\sin\theta\sin\phi, \sin\theta\cos\phi)$, so we write

$$\mathcal{E} = \int_{-L/2\delta_0}^{L/2\delta_0} \mathrm{d}\xi \left\{ \frac{1}{2}(\partial_\xi\phi)^2 + \frac{1}{2}\sin^2\phi - h\cos\phi - \mathcal{D}\partial_\xi\phi \right\}, \quad (2)$$

where dimensionless quantities ξ and $\mathcal{E}$ have been introduced by re-scaling the length along the x-axis and energy per unit area as $\xi = x/\delta_0$ and $\mathcal{E} = E/2\sqrt{AK_e}$, where $\delta_0 = \sqrt{A/K_e}$ is the characteristic width and $2\sqrt{AK_e}$ is half the energy per unit area of the static π-Bloch wall. In Eq. (2), to confine our solutions to the easy plane, further simplifications were introduced by setting $\theta = \pi/2$ and neglecting polar fluctuations, and we have also defined the dimensionless terms,

$h = HM_0/2K_e$, as the reduced external field and the re-scaled DM constant, $\mathcal{D} = D/2\sqrt{AK_e}$. So defined, the configurations which satisfy Eq. (2) are consistently described by the azimuthal angle $\phi(x)$. This energy density, Eq. (2), leads to the following Euler-Lagrange equation $\partial_\xi^2\phi - U'(\phi) = 0$, which has the following first integral:

$$\frac{1}{2}(\partial_\xi\phi)^2 - U(\phi) = c, \tag{3}$$

where we have defined the potential $U(\phi) = \frac{1}{2}\sin^2\phi - h\cos\phi$ and c is an integration constant larger than the minimum of $-U(\phi)$. We note that, as a total derivative, the chiral DM term contributes to the energy, but does not enter the Euler-Lagrange equation. As such, the set of all possible solutions is not affected by the value of $\mathcal{D}$ and the DM term only determines the most energetically favorable configuration.

As the first integral Eq. (3) is separable, it can be formally integrated to determine the possible configurations. Here, we are mainly interested in chiral, spiraling solutions, where $\partial_\xi\phi$ never vanishes. This is the case for $c > h$, where we obtain explicitly

$$w(\xi) = -|w_2|\frac{\mathrm{cn}\left(\frac{\xi}{\delta}, k\right)}{\mathrm{sn}\left(\frac{\xi}{\delta}, k\right)}, \tag{4}$$

where sn(x, k) and cn(x, k) are Jacobian elliptic functions of modulus k. Here, the intrinsic width δ and modulus k are given by

$$\delta^{-1} = |w_2|\sqrt{\frac{1}{2}(c+h)}, \quad k = \sqrt{1 - |w_1^2|/|w_2^2|}.$$

Here, $h > 0$ and the constants $w_{1,2}^2$ are defined as

$$w_{1,2}^2 = \left[-(1+c) \pm \sqrt{1+h^2+2c}\right]/(h+c).$$

Note that our system is aligned such that $\phi(x=0) = \pi$ and the "unit cell" of our soliton lattice extends over length $L = 2K(k)\delta$, where K is the complete elliptic integral of the first kind.

Rather than discussing this solution in full generality, we shall focus on a few very simple cases:

A. Absence of anisotropies and external fields

In this case, the Euler Lagrange equations are simply $\partial_\xi^2\phi = 0$ and may be immediately integrated to $\phi = kx + \phi_0$, with k the wavenumber. The energy density is then $Ak^2 - Dk$, which is minimal for the spiraling wavevector $k_0 = D/2A$. Note that the uniform ferromagnetic state is unstable with respect to a spiral structure with $k_0 \neq 0$ for arbitrarily small values of D. The chirality of the spiral is determined by the sign of D.

B. Nonvanishing anisotropy

Consider now a sample or nanowire of length $L \gg \delta_0$ with an easy-axis anisotropy, which stabilizes the ferromagnetic state. We may now give a simple criterion for the instability toward spiral structures for $h = 0$ by, considering simple domain wall configurations. With $h = 0$, we have $U(\phi) = \frac{1}{2}\sin^2\phi$, and so we can directly integrate Eq. (3) for $c = 0$ to find the following domain wall solution for $L \to \infty$:

$$\phi(\xi) = 2\tan^{-1} e^{\xi}, \tag{5}$$

which describes a rotation of the azimuthal angle ϕ by π within the easy plane and which is contained to a localized region, i.e., a π-soliton or a Bloch wall. Since the magnetization winds by a total angle π, we have $\int d\xi\partial_\xi\phi = \pi$ and, hence, we find for the Bloch wall energy,[2] $\mathcal{E} = 2 - \pi\mathcal{D}$, where the last term is due to the DM interaction. In physical units, we have $E = 4\sqrt{AK_e} - \pi D$. Since the uniform ferromagnetic state has zero energy,[2] the anisotropic ferromagnet becomes unstable with respect to the formation of a single π Bloch wall if the DM constant obeys $D > D_c$, where the critical strength is given by

$$D_c = \frac{4}{\pi}\sqrt{AK_e}. \tag{6}$$

In particular, in contrast to the previous case of no anisotropy, there is now a finite value of D required to induce a spiral structure. Also, it is evident that the resulting equilibrium spiral structure will no longer have constant pitch, but be of the type in Eq. (4), i.e., have the character of a lattice of domain walls or a so-called "soliton lattice". We now include an external field in our considerations.

C. Nonvanishing anisotropy and nonvanishing external field

Also, here it is convenient to consider the case of $L \to \infty$, where we have the benefit of a simple analytical solution[4] of Eq. (3), which may be expressed as

$$\tan(\phi/2) = \cosh R_{ss}/\sinh(\xi/\delta_{ss}). \tag{7}$$

This corresponds to a soliton-soliton pair stabilized in an external field. Here, we introduced the dimensionless relative distance R_{ss} of the two constituents of the soliton-soliton pair and the intrinsic width δ_{ss}, which are both related to the external field as

$$\mathrm{csch}^2 R_{ss} = h, \quad \delta_{ss} = \tanh R_{ss}. \tag{8}$$

Thus, a strong applied field leads to decrease of the intersoliton distance R_{ss} with a concomitant decrease of the intrinsic width δ_{ss}. The (dimensionless) energy per area $\mathcal{E}$ for this soliton-soliton pair is given by

$$\mathcal{E} = 4\{\coth R_{ss} + R_{ss}\,\mathrm{csch}^2 R_{ss}\} - 2\pi\mathcal{D}, \tag{9}$$

since, in this case, the soliton-soliton pair consists of two π domain walls with the same twist and, therefore, $\int d\xi\partial_\xi\phi = 2\pi$. We may now convince ourselves that the limit $h \to 0$ is correct, since then, $R_{ss} \to \infty$ and the first term equals 4 and one recovers the condition in Eq. (6). For finite fields, however, we have $D > D_c$, as the applied field has a

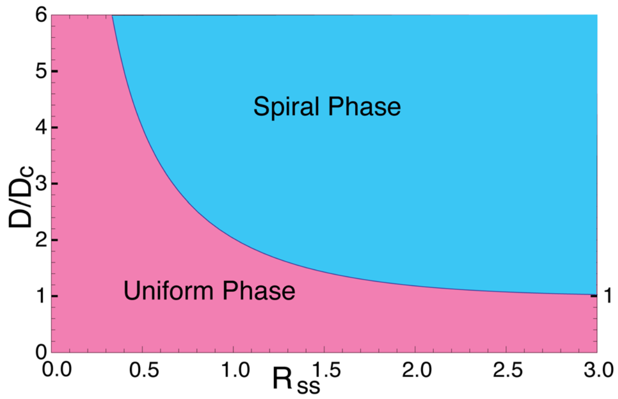

FIG. 1. (Color online) Phase boundary between spiral and uniform states in the $D-R_{ss}$ plane, where R_{ss} is related to the applied field as $\mathrm{csch}^2 R_{ss} = h$.

stabilizing influence on the uniform state. Conversely, we can stabilize the uniform state with a field and, after switching it off, the magnetization will relax into a spiral state. This spiral state has a lower energy than the uniform state for

$$D > D_c\{\coth R_{ss} + R_{ss}\,\mathrm{csch}^2 R_{ss}\}, \quad (10)$$

where D_c is given by Eq. (6). The spiral and the uniform phase in the $D-R_{ss}$ plane are shown in Fig. 1. Note that the value of D considerably increases for large fields. Vice versa, by varying h and, hence, R_{ss}, one can pass between the two phases.

D. Soliton-Antisoliton

We conclude with remarks on soliton-antisoliton configurations, which may be obtained for $c = -h$ in the first integral. One may then obtain the following magnetization profile:[5]

$$\tan(\phi/2) = \cosh(\xi/\delta_{s\bar{s}})/\sinh R_{s\bar{s}}, \quad (11)$$

with radius and width defined by

$$\mathrm{sech}^2 R_{s\bar{s}} = h, \quad \delta_{s\bar{s}} = \coth R_{s\bar{s}},$$

with energy density in the $L \to \infty$ limit of

$$\mathcal{E} = 4\{\tanh R_{s\bar{s}} - R_{s\bar{s}}\,\mathrm{sech}^2 R_{s\bar{s}}\}. \quad (12)$$

We note that the DM term does not contribute to the domain profile, as it does not appear in Eq. (3), nor the resultant energy density, since $\int d\xi \partial_\xi \phi = 0$ in this case. Thus, nucleation is not affected by the existence of a Dzyaloshinskii-Moriya term.

E. Applications

We describe a potential application of this model to recent photoemission electron microscopy results,[3] where spiraling spin structures were observed in iron nanoparticles. These particles were coupled to a ferromagnetic cobalt support for particle sizes $L > 6$ nm, with collinear alignment for smaller sizes. Numerical analysis attributed this transition to an exchange to anisotropy-dominated energy density on increasing particle height.

We can adapt our model to suit these types of experiment by taking an approach similar to the *two-grain model* of Ref. 6. By treating the substrate and nanoparticle as a coupled quasi-1D system, if we consider "penetration" of the spin configuration into the substrate, we may determine the nanoparticle's spin structure by matching boundary conditions and accounting for differing anisotropy constants, the DM interaction, or slippage at the interface. The connection of our model with the two-grain model can be made more explicit by considering the limit of grain sizes $L \to \infty$.

III. CONCLUSIONS

In conclusion, here, we have extended the "micromagnetic" continuum approach to account for the chiral DM interaction in quasi-1D systems with arbitrary material parameters. Our method allows the calculation of exact solutions of the 1D magnetization profiles and their corresponding energy densities. We demonstrated that these solutions can take the form of analytic hyperbolic functions or, in more general cases, the configurations can be described by Jacobian elliptic functions. We find it remarkable that these configurations and their associated energy densities are determined solely by the values of two parameters: c, an integration constant, and h, the reduced field. Our method allows us, in principle, to determine the critical strength of the DM interaction, for which spiraling solutions will be energetically favorable, even in the presence of applied fields for any given parameters, and we demonstrated its critical value $D_c = (4/\pi)\sqrt{AK_e}$ when $h = 0$. We suggest that our micromagnetic continuum approach is suitable for application to many spin systems, including nanowires and nanoparticles, and can provide exact solutions that can assist in understanding the mechanisms under which spin spirals form for use in novel spintronic devices.

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[1] S. S. P. Parkin, M. Hayashi, and L. Thomas, Science **320**, 190 (2008).
[2] S. Meckler, N. Mikuszeit, A. Preßler, E. Y. Vedmedenko, O. Pietzsch, and R. Wiesendanger, Phys. Rev. Lett. **103**, 157201 (2009).
[3] A. Fraile Rodríguez, A. Kleibert, J. Bansmann, A. Voitkans, L. J. Heyderman, and F. Nolting, Phys. Rev. Lett. **104**, 127201 (2010).
[4] H.-B. Braun, Phys. Rev. B **50**, 16485 (1994).
[5] H. B. Braun, Phys. Rev. Lett. **71**, 3557 (1993).
[6] J. F. Löffler, H.-B. Braun, and W. Wagner, Phys. Rev. Lett. **85**, 1990 (2000).