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The initial value problem for the compressible Navier-Stokes equations without heat conductivity

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Abstract

In this paper, we are concerned with the global existence and convergence rates of strong solutions for the compressible Navier-Stokes equations without heat conductivity in $\mathbb{R}^3$. The global existence and uniqueness of strong solutions are established by the delicate energy method under the condition that the initial data are close to the constant equilibrium state in H^2-framework. Furthermore, if additionally the initial data belong to L^1, the optimal convergence rates of the solutions in L^2-norm and convergence rates of their spatial derivatives in L^2-norm are obtained.

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Keywords: Navier-Stokes equations; Global existence; Optimal convergence rates

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1. Introduction

In this paper, we are interested in the following well-known compressible Navier-Stokes equations, which describes the motion law of compressible viscous fluid,

$$\begin{cases} \rho_t + \mathrm{div}(\rho u) = 0, \\ (\rho u)_t + \mathrm{div}(\rho u \otimes u) + \nabla P = \mathrm{div}\mathcal{T}, \\ (\rho \mathcal{E})_t + \mathrm{div}(\rho \mathcal{E} u + Pu) = \mathrm{div}(u\mathcal{T}) + k\Delta\theta. \end{cases} \tag{1.1}$$

Here $\rho = \rho(t, x)$, $u = u(t, x)$, $P = P(\rho, \theta)$, $\theta = \theta(t, x)$ stand for the fluid density, velocity, pressure and absolute temperature functions respectively, at time $t \geq 0$ and position $x \in \mathbb{R}^3$. The specific total energy $\mathcal{E} = \frac{1}{2}|u|^2 + E$, E is the specific internal energy. $\mathcal{T}$ is the stress tensor given by

$$\mathcal{T} = \mu(\nabla u + \nabla u^T) + \lambda(\mathrm{div} u)I,$$

where I is the identity matrix, μ and λ are the coefficient of viscosity and second coefficient of viscosity satisfying

$$\mu > 0, \quad 3\lambda + 2\mu \geq 0.$$

k represents the coefficient of heat conduction. In this paper, we study the case when the coefficient of heat condition $k = 0$ and the gas is ideal and polytropic, so that the equations of state for the fluids are given by

$$P = R\rho\theta, \quad E = c_\nu\theta, \quad P = A\mathrm{e}^{\frac{\mathcal{S}}{c_\nu}}\rho^\gamma, \tag{1.2}$$

where $R > 0$ and $A > 0$ are the universal gas constant, $\gamma > 1$ is the adiabatic exponent, $\mathcal{S}$ is the entropy, and c_ν is the specific heat at constant volume satisfying $c_\nu = R/(\gamma - 1)$.

As one of the most important systems in fluid dynamics, there are many important progresses on the global existence, stability and large time behavior of solutions to the compressible Navier-Stokes equations, cf. [1–3,5–8,10–23,25,26,30,31,33–35,38–40] and the references therein. Among them, when the coefficient of heat conduction $k > 0$, we refer to Kazhikhov et al. [25,26] for the global existence of smooth solutions with strictly positive initial density in one dimension. For the multi-dimensional case, the global existence of classical solutions were first established by Matsumura-Nishida [33–35] for initial data close to a non-vacuum equilibrium in $H^\ell(\mathbb{R}^3)$ with $\ell \geq 3$. Hoff [14,15], Wen-Zhu [38] and Huang et al. [18,19] proved the global existence of weak or classical solutions for discontinuous initial data provided the initial energy is suitably small. Danchin et al. [1–3] proved the global existence and uniqueness of strong solutions in the framework of hybrid Besov spaces. For arbitrary data, Lions [30] and Feireisl et al. [12] proved the global existence of weak solutions for the isentropic flow with suitably large γ. When the coefficient of heat conduction $k = 0$, $(1.1)_3$ is a hyperbolic equation which makes the problem much more complicated and has different phenomena (see [39,41,42]). Liu and Zeng [32] reformulated the original system in the Lagrangian's coordinates and proved the elaborate pointwise estimates and large-time behavior of solutions to (1.4) by studying the Green's function and the nonlinear interaction of waves in one dimension. For the case of three dimensions,

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Kawashima [24] first announced the global existence solutions of system (1.4). Later, Duan-Ma [6] and Tan-Wang [36] proved the global existence of small strong solutions under the additional assumption that the initial data are bounded in the L^1 space. It is worth to mention that the decay-in-time estimates on (P,u) is used to establish the uniform bound of $\mathcal{S}$. Tan-Wang [37] studied the global existence of small strong solutions under the H^ℓ-framework with $\ell \geq 4$. Wu [39] investigated the global existence and asymptotic behavior to the initial boundary value problem. Compared to the Cauchy problem, the equilibrium state of pressure is a monotonically increasing function on time. Xin et al. [41,42] proved the smooth or strong solutions will blow up in finite time if the initial data have an isolated mass group, no matter how small the initial data are.

The main motivation of this paper is to establish the global existence without the low frequency assumptions of the initial data by an elaborate energy method. To begin with, we note the fact that all thermodynamics variable ρ, θ, E, P as well as the entropy $\mathcal{S}$ can be represented by functions of any two of them. To overcome the difficulties arising from the non-dissipation on θ, we will rewrite system (1.1). We select the independent two variables to be P and $\mathcal{S}$. On the basis of the state equations (1.2), we deduce that

$$\rho = A^{-\frac{c_\nu}{c_\nu+R}} P^{\frac{c_\nu}{c_\nu+R}} \mathrm{e}^{-\frac{\mathcal{S}}{c_\nu+R}}. \tag{1.3}$$

Thus under the aforementioned assumptions, the system (1.1) in terms of variables P, u and $\mathcal{S}$ reads

$$\begin{cases} P_t + \gamma P \mathrm{div} u + u \cdot \nabla P = \frac{\Psi[u]}{c_\nu}, \\ \rho u_t + \rho u \cdot \nabla u + \nabla P = \mu \Delta u + (\mu + \lambda) \nabla \mathrm{div} u, \\ \mathcal{S}_t + u \cdot \nabla \mathcal{S} = \frac{\Psi[u]}{P}, \end{cases} \tag{1.4}$$

where $\Psi[u] = \frac{\mu}{2} |\nabla u + \nabla u^T|^2 + \lambda (\mathrm{div} u)^2$ is the classical dissipation function. It is worth noting that system (1.4) is a hyperbolic-parabolic system and the dissipation property comes from viscosity. We are concerned with the initial value problem to system (1.4) with initial data satisfying

$$(P, u, \mathcal{S})(0, x) = (P_0, u_0, \mathcal{S}_0)(x) \longrightarrow (\bar{P}, 0, \bar{\mathcal{S}}), \text{ as } |x| \longrightarrow \infty. \tag{1.5}$$

For the global existence and large time behavior of strong solutions to the Cauchy problem (1.4)-(1.5), we have the following theorem.

Theorem 1.1. *Given two constants $\bar{P} > 0$ and $\bar{\mathcal{S}}$. Assume that $(P_0 - \bar{P}, u_0, \mathcal{S}_0 - \bar{\mathcal{S}}) \in H^2(\mathbb{R}^3)$ with $\delta_0 := \|(P_0 - \bar{P}, u_0, \mathcal{S}_0 - \bar{\mathcal{S}})\|_2$ small, then the Cauchy problem (1.4)-(1.5) admits a unique globally strong solution $(P, u, \mathcal{S})(t)$ satisfying*

$$\begin{aligned} P - \bar{P}, \mathcal{S} - \bar{\mathcal{S}} &\in C^0([0,\infty); H^2(\mathbb{R}^3)) \cap C^1([0,\infty); H^1(\mathbb{R}^3)), \\ u &\in C^0([0,\infty); H^2(\mathbb{R}^3)) \cap C^1([0,\infty); L^2(\mathbb{R}^3)), \end{aligned}$$

$$\|(P - \bar{P}, u)(t)\|_2^2 + \int_0^t \|\nabla P(\tau)\|_1^2 + \|\nabla u(\tau)\|_2^2 d\tau \lesssim \|(P_0 - \bar{P}, u_0)\|_2^2, \tag{1.6}$$

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$$\|\mathcal{S}(t)-\bar{\mathcal{S}}\|_2 \lesssim \|\mathcal{S}_0-\bar{\mathcal{S}}\|_2 \exp\left\{C\|(P_0-\bar{P},u_0)\|_2\right\}, \text{ for any } t\geq 0. \tag{1.7}$$

Moreover, if $(P_0-\bar{P},u_0)\in L^1(\mathbb{R}^3)$*, then for any* $t\geq 0$*, it holds*

$$\|(P-\bar{P},u)(t)\|_{L^q} \lesssim (1+t)^{-\frac{3}{2}(1-\frac{1}{q})}, \quad 2\leq q\leq 6, \tag{1.8}$$

$$\|\nabla(P,u)(t)\|_1 \lesssim (1+t)^{-\frac{5}{4}}, \tag{1.9}$$

$$\|\partial_t(P,u,\mathcal{S})(t)\| \lesssim (1+t)^{-\frac{5}{4}}. \tag{1.10}$$

Finally, denote

$$(\mathcal{P}_0,m_0)=(\frac{c_\nu P_0}{R}-\frac{c_\nu\bar{P}}{R}+\frac{1}{2}\rho_0|u_0|^2,\rho_0u_0)$$

and assume that the Fourier transform $(\hat{\mathcal{P}}_0,\hat{m}_0)$ *satisfies*

$$\hat{\mathcal{P}}_0(\xi)\geq c_0, \hat{m}_0(\xi)=0, \text{ for } 0\leq|\xi|\ll 1, \tag{1.11}$$

where c_0 *is a positive constant, then we also have the following lower bound time decay rates*

$$\min\{\|(P-\bar{P})(t)\|,\ \|u(t)\|\}\geq c_1(1+t)^{-\frac{3}{4}}, \tag{1.12}$$

where c_1 *is a positive constant independent of time.*

Remark 1.1. The boundedness of $\|(P_0-\bar{P},u_0)\|_{L^p}$ is not used in proving the global existence, this is different from the previous work [6,36] where the condition on L^1-norm boundedness of the initial perturbation plays a key role in the proof of the global existence.

Now let us outline the main points of the study and explain some of the major difficulties and techniques presented in this article. By a continued argument, the global existence of strong solutions can be proven by combining the local existence and the a priori estimates. The local well-posedness can be proven by the standard argument of the contracting map theorem as [34]. The key point is to obtain the a priori estimates of the strong solution. More specifically, since the dissipative variables P and u satisfy $(1.4)_1$ and $(1.4)_2$ whose linear parts possess the same structure as that of the compressible isentropic Navier-Stokes equations, the uniform bound of (P,u) can be established by a direct energy method as in [34] under a priori assumption that $\|(P-\bar{P},u,\mathcal{S}-\bar{\mathcal{S}})\|_2$ is sufficiently small. Due to the appearance of the non-dissipative variable $\mathcal{S}$, the main difficulty lies in the closure of the energy estimates of the variable $\mathcal{S}$. To overcome this difficulty, we introduce the useful tools of Besov spaces and exploit some delicate energy estimates to get a L^1-decay on the velocity u, which enables us to close the energy estimates of the non-dissipative variable $\mathcal{S}$.

Notation. In the following part of the paper, ∇^ℓ with an integer $\ell\geq 0$ stands for the usual any spatial derivatives of order ℓ. For the sake of conciseness, we do not precise in functional space names when they are concerned with scalar-valued or vector-valued functions, $\|(f,g)\|_X$ denotes $\|f\|_X+\|g\|_X$. We will employ the notation $a\lesssim b$ to mean that $a\leq Cb$ for some positive constant C which may change from line to line but whose meaning is clear from the paper. We use

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$H^m(\mathbb{R}^3)$ and $W^{m,p}(\mathbb{R}^3)$ to denote the usual Sobolev spaces with norm $||\cdot||_m$ and $||\cdot||_{m,p}$ for $m \geq 0$ and $1 \leq p \leq +\infty$. If $m = 0$, we just use $||\cdot||$ and $||\cdot||_{L^q}$ for convenience. The integration domain $\mathbb{R}^3$ will be always omitted without any ambiguity. $\langle \cdot, \cdot \rangle$ represents the inner-product in $L^2(\mathbb{R}^3)$. Finally, $\mathfrak{F}(f)$ or $\hat{f}$ denotes the Fourier transform of the function f.

Following that, let us introduce the Littlewood-Paley decomposition. Let $\varphi(\xi)$ be a smooth function such that φ is supported in the shell $\{\xi \in \mathbb{R}^3, \frac{3}{4} \leq |\xi| \leq \frac{8}{3}\}$ and

$$\sum_{q \in \mathbb{Z}} \varphi(2^{-q}\xi) = 1, \quad \text{for all} \quad \xi \neq 0.$$

We denote the space $\mathcal{D}'(\mathbb{R}^3)$ is the dual space of $\mathcal{D}(\mathbb{R}^3) = \{f \in S(\mathbb{R}^3);\ D^\alpha \hat{f}(0) = 0, \text{ for all}$ multi-index $\alpha \in \mathbb{N}^3\}$, it also can be identified by the quotient space of $S'(\mathbb{R}^3)/\mathcal{P}$ with polynomials space $\mathcal{P}$.

Defining the following the dyadic blocks

$$\Delta_q f = \mathfrak{F}^{-1}\varphi(2^{-q}\xi)\mathfrak{F}f \ \text{ for } \ q \in \mathbb{Z}.$$

Then the formal equality

$$f = \sum_{q \in \mathbb{Z}} \Delta_q f$$

holds true for all $f \in \mathcal{D}'(\mathbb{R}^3)$ which is the homogeneous Littlewood-Paley decomposition.

Definition 1.1. Given $s \in \mathbb{R}$, $1 \leq p, r \leq +\infty$, then the homogeneous Besov space $\dot{B}^s_{p,r}(\mathbb{R}^3)$ is defined by

$$\dot{B}^s_{p,r}(\mathbb{R}^3) = \{f \in D'(\mathbb{R}^3) : \|f\|_{\dot{B}^s_{p,r}} < +\infty\},$$

here

$$\|f\|_{\dot{B}^s_{p,r}} \triangleq \|2^{qs}\|\Delta_q f\|_{L^p}\|_{l^r}.$$

For $s_1, s_2 \in \mathbb{R}$, $1 \leq p, r \leq +\infty$, we define the hybrid Besov spaces $\tilde{B}^{s_1,s_2}_{p,r}$ with norm $\tilde{B}^{s_1,s_2}_{p,r}$ given by

$$\|f\|_{\tilde{B}^{s_1,s_2}_{p,r}} \triangleq (\sum_{q \leq 0} \|2^{qs_1}\Delta_q f\|^r_{L^p})^{\frac{1}{r}} + (\sum_{q > 0} \|2^{qs_2}\Delta_q f\|^r_{L^p})^{\frac{1}{r}}.$$

The rest of the paper is structured as follows. In Section 2, we will reformulate the problem and state the equivalent results, Proposition 2.2. Section 3 is devoted to prove the global existence part of Proposition 2.2. In Section 4 and Section 5, we will give the decay part of Proposition 2.2.

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2. Reformulated system

In this section, we first reformulate system (1.4). Set

$$\alpha_1=\sqrt{\frac{c_v}{(R+c_v)\bar{\rho}\bar{P}}},\ \alpha_2=\sqrt{\frac{(R+c_v)\bar{P}}{c_v\bar{\rho}}},\ \kappa_1=\frac{\mu}{\bar{\rho}},\ \kappa_2=\frac{\mu+\lambda}{\bar{\rho}},$$

where $\bar{\rho}=\rho(\bar{P},\bar{s})$. After the change of variables

$$n=P-\bar{P},\ v=\frac{1}{\alpha_1}u,\ \sigma=\mathcal{S}-\bar{\mathcal{S}},$$

we rewrite the initial value problem (1.4)-(1.5) in the perturbation form as

$$\begin{cases} n_t+\alpha_2\mathrm{div}v=F,\\ v_t+\alpha_2\nabla n-\kappa_1\Delta v-\kappa_2\nabla\mathrm{div}v=G,\\ \sigma_t+\alpha_1 v\cdot\nabla\sigma=H,\\ (n,v,\sigma)|_{t=0}=(n_0,v_0,\sigma_0)=(P_0-\bar{P},\frac{1}{\alpha_1}u_0,\mathcal{S}_0-\bar{\mathcal{S}}), \end{cases} \tag{2.1}$$

where the nonlinear terms are given by

$$F\triangleq-\frac{(R+c_v)\alpha_1}{c_v}n\nabla\cdot v-\alpha_1 v\cdot\nabla n+\frac{\Psi[\alpha_1 v]}{c_v},$$

$$G\triangleq-\alpha_1 v\cdot\nabla v-\frac{1}{\alpha_1}(\frac{1}{\rho}-\frac{1}{\bar{\rho}})\nabla n+\mu(\frac{1}{\rho}-\frac{1}{\bar{\rho}})\Delta v+(\mu+\lambda)(\frac{1}{\rho}-\frac{1}{\bar{\rho}})\nabla\mathrm{div}v,$$

and

$$H\triangleq\frac{\Psi[\alpha_1 v]}{n+\bar{P}}.$$

For any $T>0$, we define the solution space by

$$\begin{aligned} X(0,T)=\Big\{\ &(n,v,\sigma): n,\sigma\in C^0([0,T];H^2(\mathbb{R}^3))\cap C^1([0,T];H^1(\mathbb{R}^3)),\\ &v\in C^0([0,T];H^2(\mathbb{R}^3))\cap C^1([0,T];L^2(\mathbb{R}^3)),\\ &\nabla n\in L^2((0,T);H^1(\mathbb{R}^3)),\nabla v\in L^2((0,T);H^2(\mathbb{R}^3))\ \Big\}, \end{aligned}$$

and the solution norm by

$$\mathcal{X}(T)=\sup_{0\le t\le T}\|(n,v,\sigma)(t)\|_2^2+\int_0^T\|\nabla n(\tau)\|_1^2+\|\nabla v(\tau)\|_2^2d\tau.$$

Taking a standard contraction mapping argument as in [34], the local existence result for the Cauchy problem (2.1) can be stated as the following proposition.

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Proposition 2.1. *Suppose that the initial data satisfy* $(n_0, v_0, \sigma_0) \in H^2(\mathbb{R}^3)$ *and*

$$\inf_{x\in\mathbb{R}^3}\{n_0(x) + \bar{P}\} > 0.$$

Then there exists a positive constant T_0 *depending on* $\mathcal{X}(0)$ *such that the Cauchy problem* (2.1) *has a unique solution* $(n, v, \sigma) \in X(0, T_0)$ *satisfying*

$$\inf_{t\in[0,T_0],x\in\mathbb{R}^3}\{n(t, x) + \bar{P}\} > 0, \text{ and } \mathcal{X}(T_0) \le 2\mathcal{X}(0).$$

It is easy to check that Theorem 1.1 is equivalent to the following proposition.

Proposition 2.2. *Assume that* $\|(n_0, v_0, \sigma_0)\|_2$ *is sufficiently small, then the Cauchy problem* (2.1) *has a unique global solution* $(n, v, \sigma) \in X(0, \infty)$ *such that for any* $t \in [0, \infty)$,

$$\|(n, v)(t)\|_2^2 + \int_0^t \|\nabla n(\tau)\|_1^2 + \|\nabla v(\tau)\|_2^2 d\tau \lesssim \|(n_0, v_0)\|_2^2, \tag{2.2}$$

$$\|\sigma(t)\|_2 \lesssim \|\sigma_0\|_2 \exp\{C\|(n_0, v_0)\|_2\}, \text{ for all } t \ge 0. \tag{2.3}$$

Moreover, if $(n_0, v_0) \in L^1(\mathbb{R}^3)$, *then for any* $t \ge 0$, *it holds*

$$\|(n, v)(t)\|_{L^q} \lesssim (1+t)^{-\frac{3}{2}(1-\frac{1}{q})}, \quad 2 \le q \le 6, \tag{2.4}$$

$$\|\nabla(n, v)(t)\|_1 \lesssim (1+t)^{-\frac{5}{4}}, \tag{2.5}$$

$$\|\partial_t(n, v, \sigma)(t)\| \lesssim (1+t)^{-\frac{5}{4}}. \tag{2.6}$$

Finally, if the Fourier transform $(\hat{\mathcal{P}}_0, \hat{m}_0)$ *satisfies*

$$\hat{\mathcal{P}}_0(\xi) \ge c_0, \hat{m}_0(\xi) = 0, \text{ for } 0 \le |\xi| \ll 1,$$

where c_0 *is a positive constant, then we also have the following lower bound time decay rates*

$$\min\{\|n(t)\|, \ \|v(t)\|\} \ge c_2(1+t)^{-\frac{3}{4}}, \tag{2.7}$$

where c_2 *is a positive constant independent of time.*

For later use, we show some useful analytic tools. First, we recall some Sobolev inequalities as follows, cf. [9].

Lemma 2.1. *If* $f \in H^2(\mathbb{R}^3)$, *then it holds*

(1) $\|f\|_{L^\infty} \le C\|\nabla f\|^{\frac{1}{2}}\|\nabla^2 f\|^{\frac{1}{2}} \le C\|\nabla f\|_1$;
(2) $\|f\|_{L^6} \le C\|\nabla f\|$;
(3) $\|f\|_{L^q} \le C\|f\|_1, \ 2 \le q \le 6$.

We also need the following tame estimates for Besov spaces, cf. [2,4].

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Lemma 2.2. *Let* $s_1, s_2 \le \frac{3}{2}$ *such that* $s_1 + s_2 > 0$, $u \in \dot{B}^{s_1}_{2,1}$ *and* $v \in \dot{B}^{s_2}_{2,1}$, *then* $uv \in \dot{B}^{s_1+s_2-\frac{3}{2}}_{2,1}$ *and*

$$\|uv\|_{\dot{B}^{s_1+s_2-\frac{3}{2}}_{2,1}} \le C\|u\|_{\dot{B}^{s_1}_{2,1}}\|v\|_{\dot{B}^{s_2}_{2,1}}.$$

The next two lemmas are about some embedding estimates for the hybrid Besov spaces, cf. [2,4].

Lemma 2.3. *The following embeddings for hybrid Besov spaces hold:*
(1) We have $\tilde{B}^{s,s}_{2,1} = \dot{B}^s_{2,1}$;
(2) If $s \le t$, *then* $\tilde{B}^{s,t}_{2,1} = \dot{B}^s_{2,1} \bigcap \dot{B}^t_{2,1}$; *Otherwise,* $\tilde{B}^{s,t}_{2,1} = \dot{B}^s_{2,1} + \dot{B}^t_{2,1}$.

Lemma 2.4. *There exists a positive constant* C *such that for all* $s \in \mathbb{R}$, *we have*

$$\frac{1}{C^{|s|+1}}\|u\|_{H^s} \le \|u\|_{\tilde{B}^{0,s}_{2,2}} \le C^{|s|+1}\|u\|_{H^s}.$$

For $\alpha, \beta \in \mathbb{R}$, let us define the following characteristic function on $\mathbb{Z}$

$$\phi^{\alpha,\beta}(r) = \begin{cases} \alpha, & \text{if} \quad r \le 0, \\ \beta, & \text{if} \quad r \ge 1. \end{cases}$$

In order to get a L^1-decay on the velocity v, we also need the following lemma which is devoted to estimates for the convection terms, cf. [2,3].

Lemma 2.5. *Let* F *be a homogeneous smooth function of degree* m. *Suppose that* $-\frac{3}{2} < s_1, t_1, s_2, t_2 \le \frac{5}{2}$. *The following two estimates hold:*

$$\left|\langle F(D)\Delta_q(u\cdot\nabla a), F(D)\Delta_q a\rangle\right| \lesssim c_q 2^{-q(\phi^{s_1,s_2}(q-m))}\|u\|_{\dot{B}^{\frac{5}{2}}_{2,1}}\|a\|_{\tilde{B}^{s_1,s_2}_{2,1}}\|F(D)\Delta_q a\|,$$

$$\begin{aligned} &\left|\langle F(D)\Delta_q(u\cdot\nabla a), \Delta_q b\rangle + \langle\Delta_q(u\cdot\nabla b), F(D)\Delta_q a\rangle\right| \\ &\lesssim c_q\|u\|_{\dot{B}^{\frac{5}{2}}_{2,1}} \times \Big(2^{-q\phi^{t_1,t_2}(q-m)}\|b\|_{\tilde{B}^{t_1,t_2}_{2,1}}\|F(D)\Delta_q a\| + 2^{-q\phi^{s_1,s_2}(q-m)}\|a\|_{\tilde{B}^{s_1,s_2}_{2,1}}\|\Delta_q b\|\Big), \end{aligned}$$

where the operator $F(D)$ *is defined by* $F(D)f := \mathfrak{F}^{-1}F(\xi)\mathfrak{F}f$ *and* $\sum_{q\in Z} c_q \le 1$.

3. The proof of global existence

By the standard continuity argument, the global existence of strong solution for the Cauchy problem (2.1) follows from Proposition 2.1 and a priori estimate. Suppose that the Cauchy problem (2.1) has a unique solution $(n, v, \sigma) \in X(0, T)$, with some $T \in (0, +\infty]$, we make a priori assumption

$$\sup_{0\le t\le T}\|(n, v, \sigma)(t)\|_2 \le \delta \ll 1. \tag{3.1}$$

Under the assumption (3.1), we derive the following energy estimates on (n, v, σ) which have been proven in [36], thus we omit the proofs of them for simplicity.

Lemma 3.1. *Under the a priori assumption (3.1), there exists a positive constant $D_1 > 0$ suitably large which is independent of δ such that for any $t \in [0, T]$,*

$$\frac{d}{dt}\big\{D_1\|(n, v)(t)\|^2 + \langle\nabla n, v\rangle(t)\big\} + \|\nabla(n, v)(t)\|^2 \lesssim \delta\|\nabla^2 v(t)\|^2, \tag{3.2}$$

$$\frac{d}{dt}H_1(n(t), v(t)) + \big(\|\nabla^2 n(t)\|^2 + \|\nabla^2 v(t)\|_1^2\big) \lesssim \delta\|\nabla(n, v)(t)\|^2, \tag{3.3}$$

and

$$\frac{d}{dt}\|\sigma(t)\|_2^2 \lesssim (\|\nabla v(t)\|_{L^\infty} + \|\nabla^2 v(t)\|_{L^3})\|\sigma(t)\|_2^2 + \delta\|\nabla v(t)\|_2^2, \tag{3.4}$$

here $H_1(n, v) = D_1\|\nabla(n, v)\|_1^2 + \sum_{|\alpha|=1}\langle\partial_x^\alpha\nabla n, \partial_x^\alpha v\rangle$ and is equivalent to $\|\nabla(n, v)\|_1^2$ since $D_1 > 0$ is large.

In order to close a prior estimate on σ, we need to derive a L^1-decay on the velocity v.

Lemma 3.2. *Under the a priori assumption (3.1), it holds*

$$\begin{aligned}&\|n(t)\|_{\tilde{B}_{2,1}^{\frac{1}{4},\frac{3}{2}}} + \|v(t)\|_{\tilde{B}_{2,1}^{\frac{1}{4},\frac{1}{2}}} + \int_0^t \|n(\tau)\|_{\tilde{B}_{2,1}^{\frac{9}{4},\frac{3}{2}}}d\tau + \int_0^t \|v(\tau)\|_{\tilde{B}_{2,1}^{\frac{9}{4},\frac{5}{2}}}d\tau \\ &\lesssim \|n(0)\|_{\tilde{B}_{2,1}^{\frac{1}{4},\frac{3}{2}}} + \|v(0)\|_{\tilde{B}_{2,1}^{\frac{1}{4},\frac{1}{2}}} + \int_0^t \|\nabla v(\tau)\|_1^2 d\tau,\end{aligned} \tag{3.5}$$

for any $t \geq 0$.

Proof. First step: Low frequencies
Let $q_0 \triangleq 5 + [\log_2 \frac{\alpha_2}{\kappa_1+\kappa_2}]$, then $1 < 2^{q_0-4}\frac{\kappa_1+\kappa_2}{\alpha_2} \leq 2$. For any $q \leq q_0$, applying the operator Δ_q to the first two equations of (2.1), and taking the L^2 product of the result equations with $\Delta_q n$ and $\Delta_q v$ respectively, we obtain the following two identities:

$$\frac{1}{2}\frac{d}{dt}\|\Delta_q n\|^2 + \alpha_2\langle\Delta_q \mathrm{div} v, \Delta_q n\rangle = \langle\Delta_q F, \Delta_q n\rangle, \tag{3.6}$$

and

$$\frac{1}{2}\frac{d}{dt}\|\Delta_q v\|^2 - \alpha_2\langle\Delta_q \mathrm{div} v, \Delta_q n\rangle + \kappa_1\|\wedge\Delta_q v\|^2 + \kappa_2\|\Delta_q \mathrm{div} v\|^2 = \langle\Delta_q G, \Delta_q v\rangle. \tag{3.7}$$

We also can obtain an identity involving $\langle\Delta_q\wedge^{-1}\mathrm{div} v, \wedge\Delta_q n\rangle$. To achieve it, we apply $\Delta_q\nabla$ to $(2.1)_1$ and Δ_q to $(2.1)_2$ and take the L^2 product of the result equations with $\Delta_q v$ and $\Delta_q\nabla n$ respectively, we arrive at

$$\frac{d}{dt}\langle \Delta_q \wedge^{-1} \mathrm{div} v, \wedge\Delta_q n\rangle - \alpha_2\| \wedge \Delta_q n\|^2 + \alpha_2\|\Delta_q \mathrm{div} v\|^2 + (\kappa_1+\kappa_2)\langle \wedge\Delta_q \mathrm{div} v, \wedge\Delta_q n\rangle$$
$$= \langle \Delta_q F, \Delta_q \mathrm{div} v\rangle + \langle \Delta_q \mathrm{div} G, \Delta_q n\rangle. \tag{3.8}$$

A linear combination of (3.6), (3.7) and (3.8) we arrive at

$$\begin{aligned}&\frac{d}{dt}\Big\{\frac{1}{2}D_2\|(\Delta_q n, \Delta_q v)\|^2 - \langle \Delta_q \wedge^{-1} \mathrm{div} v, \wedge\Delta_q n\rangle\Big\} + D_2\kappa_1\| \wedge \Delta_q v\|^2 \\ &+ D_2\kappa_2\|\Delta_q \mathrm{div} v\|^2 + \alpha_2\| \wedge \Delta_q n\|^2 - \alpha_2\|\Delta_q \mathrm{div} v\|^2 - (\kappa_1+\kappa_2)\langle \wedge\Delta_q \mathrm{div} v, \wedge\Delta_q n\rangle \\ =\ & D_2\langle \Delta_q F, \Delta_q n\rangle + D_2\langle \Delta_q G, \Delta_q v\rangle - \langle \Delta_q F, \Delta_q \mathrm{div} v\rangle - \langle \Delta_q \mathrm{div} G, \Delta_q n\rangle.\end{aligned} \tag{3.9}$$

Noticing that

$$\frac{3}{4}2^q\|\Delta_q f\| \le \| \wedge \Delta_q f\| \le \frac{8}{3}2^q\|\Delta_q f\| \le \frac{8}{3}2^{q_0}\|\Delta_q f\|$$

for $q \le q_0$, we have

$$|\langle \Delta_q \wedge^{-1} \mathrm{div} v, \wedge\Delta_q n\rangle| \le \|\Delta_q v\|\| \wedge \Delta_q n\| \le \frac{8}{3}2^{q_0}\|\Delta_q v\|\|\Delta_q n\|, \tag{3.10}$$

$$|\langle \kappa_1\Delta\Delta_q v + \kappa_2\nabla(\mathrm{div}\Delta_q v), \nabla\Delta_q n\rangle| \le (\kappa_1+\kappa_2)\| \wedge^2 \Delta_q v\|\| \wedge \Delta_q n\|$$
$$\le (\kappa_1+\kappa_2)2^{q_0}\| \wedge \Delta_q v\|\| \wedge \Delta_q n\|. \tag{3.11}$$

Thus if we choose $D_2 > 0$ suitably large, denote for $q \le q_0$

$$\mathcal{H}_q^2 \triangleq \frac{1}{2}D_2\|(\Delta_q n, \Delta_q v)\|^2 - \langle \Delta_q \wedge^{-1} \mathrm{div} v, \wedge\Delta_q n\rangle,$$

then $\mathcal{H}_q$ is equivalent to $\|(\Delta_q n, \Delta_q v)\|$. It follows from Cauchy-Schwarz's inequality, Hölder's inequality, (3.9), (3.10) and (3.11) that

$$\frac{d}{dt}\mathcal{H}_q^2 + 2^{2q}\mathcal{H}_q^2 \lesssim \|(\Delta_q F, \Delta_q G)\|\|(\Delta_q n, \Delta_q v)\|. \tag{3.12}$$

Let $\epsilon_1 > 0$ be a small parameter and denote $\mathcal{G}_q = \sqrt{\mathcal{H}_q^2 + \epsilon_1^2}$. From (3.12) we get

$$\frac{d}{dt}\mathcal{G}_q + 2^{2q-1}\mathcal{G}_q \lesssim (\|(\Delta_q F, \Delta_q G)\| + \epsilon_1 2^{2q}).$$

Integrating over $[0, t]$ and making ϵ_1 tend to 0, we arrive at

$$2^{\frac{1}{4}q}\mathcal{H}_q(t) + \int_0^t 2^{\frac{9}{4}q}\mathcal{H}_q(\tau)d\tau \lesssim 2^{\frac{1}{4}q}\mathcal{H}_q(0) + \int_0^t 2^{\frac{1}{4}q}\|(\Delta_q F, \Delta_q G)(\tau)\|d\tau, \text{ for any } t \ge 0. \tag{3.13}$$

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Second step: High frequencies

In this subsection, we suppose that $q > q_0$, thus $2^{q\frac{\kappa_1+\kappa_2}{\alpha_2}} \geq 2$. Applying the operator $\wedge\Delta_q$ to the first equation of (2.1) and multiplying the result equation by $\wedge\Delta_q n$ and integrate over $\mathbb{R}^3$, we get

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\|\wedge\Delta_q n\|^2 + \alpha_2\langle\wedge\Delta_q \mathrm{div} v, \wedge\Delta_q n\rangle + \alpha_1\langle\wedge\Delta_q(v\cdot\nabla n), \wedge\Delta_q n\rangle\\ =\ &\langle\wedge\Delta_q(F+\alpha_1 v\cdot\nabla n), \wedge\Delta_q n\rangle.\end{aligned}\tag{3.14}$$

Summing up $\frac{2\alpha_2^2}{\kappa_1+\kappa_2}\times$ (3.7) $+ (\kappa_1+\kappa_2)\times$ (3.14) $-\alpha_2\times$ (3.8), we get

$$\begin{aligned}&\frac{d}{dt}\Big[\frac{\alpha_2^2}{\kappa_1+\kappa_2}\|\Delta_q v\|^2 + \frac{1}{2}(\kappa_1+\kappa_2)\|\wedge\Delta_q n\|^2 - \alpha_2\langle\Delta_q\wedge^{-1}\mathrm{div} v, \wedge\Delta_q n\rangle\Big]\\
&+\frac{2\kappa_1\alpha_2^2}{\kappa_1+\kappa_2}\|\wedge\Delta_q v\|^2 + \frac{2\kappa_2\alpha_2^2}{\kappa_1+\kappa_2}\|\Delta_q \mathrm{div} v\|^2 - \alpha_2^2\|\Delta_q\mathrm{div} v\|^2 + \alpha_2^2\|\wedge\Delta_q n\|^2\\
&-\frac{2\alpha_2^3}{\kappa_1+\kappa_2}\langle\Delta_q\mathrm{div} v, \Delta_q n\rangle + \alpha_1(\kappa_1+\kappa_2)\langle\wedge\Delta_q(v\cdot\nabla n), \wedge\Delta_q n\rangle\\
=&\langle\frac{2\alpha_2^2}{\kappa_1+\kappa_2}\Delta_q G, \Delta_q v\rangle + \langle(\kappa_1+\kappa_2)\wedge\Delta_q(F+\alpha_1 v\cdot\nabla n), \wedge\Delta_q n\rangle\\
&-\alpha_2\langle\wedge\Delta_q(F+\alpha_1 v\cdot\nabla n), \Delta_q\wedge^{-1}\mathrm{div} v\rangle\\
&-\alpha_2\langle\Delta_q(\wedge^{-1}\mathrm{div} G + \alpha_1 v\cdot\nabla(\wedge^{-1}\mathrm{div} v)), \wedge\Delta_q n\rangle\\
&+\alpha_2\alpha_1\big\{\langle\wedge\Delta_q(v\cdot\nabla n), \Delta_q\wedge^{-1}\mathrm{div} v\rangle + \langle\Delta_q[v\cdot\nabla(\wedge^{-1}\mathrm{div} v)], \wedge\Delta_q n\rangle\big\}.\end{aligned}$$

Since $q > q_0$, it follows from Cauchy-Schwarz's inequality and Hölder's inequality that

$$|\langle\Delta_q\wedge^{-1}\mathrm{div} v, \wedge\Delta_q n\rangle| \leq \|\Delta_q v\|\|\wedge\Delta_q n\|,$$

$$\begin{aligned}\Big|\frac{2\alpha_2^3}{\kappa_1+\kappa_2}\langle\Delta_q\mathrm{div} v, \Delta_q n\rangle\Big| &\leq \frac{\alpha^2}{2}\|\wedge\Delta_q n\|^2 + \frac{2\alpha_2^4}{(\kappa_1+\kappa_2)^2}\|\wedge^{-1}\Delta_q\mathrm{div} v\|^2\\
&\leq \frac{\alpha^2}{2}\|\wedge\Delta_q n\|^2 + \frac{128\alpha_2^4 2^{-2q}}{9(\kappa_1+\kappa_2)^2}\|\Delta_q\mathrm{div} v\|^2\\
&\leq \frac{\alpha^2}{2}\|\wedge\Delta_q n\|^2 + \frac{128\alpha_2^4 2^{-2q_0}}{9(\kappa_1+\kappa_2)^2}\|\Delta_q\mathrm{div} v\|^2\\
&\leq \frac{\alpha^2}{2}\|\wedge\Delta_q n\|^2 + \frac{\alpha_2^2}{2}\|\Delta_q\mathrm{div} v\|^2.\end{aligned}$$

Thus for $q > q_0$, we denote

$$\mathcal{H}_q^2 \triangleq \frac{\alpha_2^2}{\kappa_1+\kappa_2}\|\Delta_q v\|^2 + \frac{1}{2}(\kappa_1+\kappa_2)\|\wedge\Delta_q n\|^2 - \alpha_2\langle\wedge^{-1}\Delta_q\mathrm{div} v, \wedge\Delta_q n\rangle,$$

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then $\mathcal{H}_q$ is equivalent to $\|(\wedge\Delta_q n, \Delta_q v)\|$. So that straightforward computations and Lemma 2.5 lead to

$$\begin{aligned}\frac{d}{dt}\mathcal{H}_q^2+\mathcal{H}_q^2 &\lesssim \|\big(\Delta_q G, \wedge\Delta_q(F+\alpha_1 v\cdot\nabla n), \Delta_q(v\cdot\nabla(\wedge^{-1}\mathrm{div}v))\big)\|\mathcal{H}_q\\ &\quad + c_q\|v\|_{\dot B^{\frac52}_{2,1}}\big(2^{-2q}\|v\|_{\dot B^2_{2,1}}\|\wedge\Delta_q n\|+2^{-\frac32 q}\|n\|_{\dot B^{\frac32}_{2,1}}\|\Delta_q v\|\big)\\ &\lesssim \Big[\|\big(\Delta_q G, \wedge\Delta_q(F+\alpha_1 v\cdot\nabla n), \Delta_q(v\cdot\nabla(\wedge^{-1}\mathrm{div}v))\big)\|\\ &\quad + c_q\|v\|_{\dot B^{\frac52}_{2,1}}(2^{-2q}\|v\|_{\dot B^2_{2,1}}+2^{-\frac32 q}\|n\|_{\dot B^{\frac32}_{2,1}})\Big]\mathcal{H}_q.\end{aligned} \tag{3.15}$$

Let $\epsilon_2>0$ be a small parameter and denote $\mathcal{G}_q=\sqrt{\mathcal{H}_q^2+\epsilon_2^2}$. By (3.15), we obtain

$$\begin{aligned}\frac{d}{dt}\mathcal{G}_q+C\mathcal{G}_q &\lesssim \|\big(\Delta_q G, \wedge\Delta_q(F+\alpha_1 v\cdot\nabla n), \Delta_q(v\cdot\nabla(\wedge^{-1}\mathrm{div}v))\big)\|\\ &\quad + c_q\|v\|_{\dot B^{\frac52}_{2,1}}(2^{-2q}\|v\|_{\dot B^2_{2,1}}+2^{-\frac32 q}\|n\|_{\dot B^{\frac32}_{2,1}})+\epsilon_2.\end{aligned}$$

Integrating over $[0,t]$ and making ϵ_2 tend to 0, we get

$$\begin{aligned}&2^{\frac12 q}\mathcal{H}_q(t)+\int_0^t 2^{\frac12 q}\mathcal{H}_q(\tau)d\tau\\ &\lesssim 2^{\frac12 q}\mathcal{H}_q(0)+\int_0^t 2^{\frac12 q}\|\big(\Delta_q G, \wedge\Delta_q(F+\alpha_1 v\cdot\nabla n), \Delta_q(v\cdot\nabla(\wedge^{-1}\mathrm{div}v))\big)(\tau)\|d\tau\\ &\quad + c_q\int_0^t \|v(\tau)\|_{\dot B^{\frac52}_{2,1}}(2^{-\frac32 q}\|v(\tau)\|_{\dot B^2_{2,1}}+2^{-q}\|n(\tau)\|_{\dot B^{\frac32}_{2,1}})d\tau, \quad \text{for any } t\ge 0.\end{aligned} \tag{3.16}$$

Combining (3.13) and (3.16), we conclude after summation over q in $\mathbb{Z}$ that

$$\begin{aligned}&\|n(t)\|_{\tilde B^{\frac14,\frac32}_{2,1}}+\|v(t)\|_{\tilde B^{\frac14,\frac12}_{2,1}}+\int_0^t\|n(\tau)\|_{\tilde B^{\frac94,\frac32}_{2,1}}d\tau+\int_0^t\|v(\tau)\|_{\tilde B^{\frac94,\frac12}_{2,1}}d\tau\\ &\lesssim \|n(0)\|_{\tilde B^{\frac14,\frac32}_{2,1}}+\|v(0)\|_{\tilde B^{\frac14,\frac12}_{2,1}}+\int_0^t\|\big(G, \wedge(F+\alpha_1 v\cdot\nabla n), v\cdot\nabla(\wedge^{-1}\mathrm{div}v)\big)(\tau)\|_{\tilde B^{\frac14,\frac12}_{2,1}}d\tau\\ &\quad +\int_0^t\|v(\tau)\|_{\dot B^{\frac52}_{2,1}}(\|v(\tau)\|_{\dot B^2_{2,1}}+\|n(\tau)\|_{\dot B^{\frac32}_{2,1}})d\tau.\end{aligned} \tag{3.17}$$

By Hölder's inequality, Cauchy's inequality, Lemma 2.1, Lemma 2.2 and Lemma 2.4, the terms on the right hand side of the above equation can be estimated as follows.

$$
\begin{aligned}
&\|\big(G, v\cdot\nabla(\wedge^{-1}\mathrm{div}v)\big)\|_{\tilde{B}_{2,1}^{\frac14,\frac12}}\\
\lesssim{}& \|v\cdot\nabla v\|_{\tilde{B}_{2,1}^{\frac14,\frac12}}+\|v\cdot\nabla\wedge^{-1}\mathrm{div}v\|_{\tilde{B}_{2,1}^{\frac14,\frac12}}+\|(\frac{1}{\rho}-\frac{1}{\bar\rho})\nabla n\|_{\tilde{B}_{2,1}^{\frac14,\frac12}}+\|(\frac{1}{\rho}-\frac{1}{\bar\rho})\nabla^2 v\|_{\tilde{B}_{2,1}^{\frac14,\frac12}}\\
\lesssim{}& \|v\|_{\dot{B}_{2,1}^{\frac32}}\|\nabla v\|_{\tilde{B}_{2,1}^{\frac14,\frac12}}+\|\frac{1}{\rho}-\frac{1}{\bar\rho}\|_{\tilde{B}_{2,1}^{\frac12,\frac32}}\|\nabla n\|_{\tilde{B}_{2,1}^{\frac54,\frac12}}+\|\frac{1}{\rho}-\frac{1}{\bar\rho}\|_{\dot{B}_{2,1}^{\frac32}}\|\nabla^2 v\|_{\tilde{B}_{2,1}^{\frac14,\frac12}}\\
\lesssim{}& \|\nabla v\|_1^2+\|\frac{1}{\rho}-\frac{1}{\bar\rho}\|_2\|n\|_{\tilde{B}_{2,1}^{\frac94,\frac32}}+\|\frac{1}{\rho}-\frac{1}{\bar\rho}\|_2\|v\|_{\tilde{B}_{2,1}^{\frac94,\frac52}}\\
\lesssim{}& \|\nabla v\|_1^2+\delta\|n\|_{\tilde{B}_{2,1}^{\frac94,\frac32}}+\delta\|v\|_{\tilde{B}_{2,1}^{\frac94,\frac52}},
\end{aligned}
\tag{3.18}
$$

and

$$
\begin{aligned}
\|\big(\wedge(F+\alpha_1 v\cdot\nabla n)\big)\|_{\tilde{B}_{2,1}^{\frac14,\frac12}} &\lesssim \|F+\alpha_1 v\cdot\nabla n\|_{\tilde{B}_{2,1}^{\frac54,\frac32}}\\
&\lesssim \|n\nabla\cdot v\|_{\tilde{B}_{2,1}^{\frac54,\frac32}}+\||\nabla v|^2\|_{\tilde{B}_{2,1}^{\frac54,\frac32}}\\
&\lesssim \|n\|_{\dot{B}_{2,1}^{\frac32}}\|\nabla v\|_{\tilde{B}_{2,1}^{\frac54,\frac32}}+\|\nabla v\|_{\dot{B}_{2,1}^{\frac32}}\|\nabla v\|_{\tilde{B}_{2,1}^{\frac14,\frac32}}\\
&\lesssim \|\nabla v\|_1^2+\delta\|v\|_{\tilde{B}_{2,1}^{\frac94,\frac52}}.
\end{aligned}
\tag{3.19}
$$

Substituting (3.18) and (3.19) into (3.17) gives

$$
\begin{aligned}
&\|n(t)\|_{\tilde{B}_{2,1}^{\frac14,\frac32}}+\|v(t)\|_{\tilde{B}_{2,1}^{\frac14,\frac12}}+\int_0^t\|n(\tau)\|_{\tilde{B}_{2,1}^{\frac94,\frac32}}d\tau+\int_0^t\|v(\tau)\|_{\tilde{B}_{2,1}^{\frac94,\frac12}}d\tau\\
\lesssim{}&\|n(0)\|_{\tilde{B}_{2,1}^{\frac14,\frac32}}+\|v(0)\|_{\tilde{B}_{2,1}^{\frac14,\frac12}}+\int_0^t\|\nabla v(\tau)\|_1^2d\tau.
\end{aligned}
\tag{3.20}
$$

Third step: Further regularity properties for v

By (3.7) and taking the similar argument to (3.16), we arrive at

$$
\begin{aligned}
&\|v(t)\|_{\tilde{B}_{2,1}^{\frac54,\frac12}}+\int_0^t\|v(\tau)\|_{\tilde{B}_{2,1}^{\frac{13}{4},\frac52}}d\tau\\
\lesssim{}&\|v(0)\|_{\tilde{B}_{2,1}^{\frac54,\frac12}}+\int_0^t\|n(\tau)\|_{\tilde{B}_{2,1}^{\frac94,\frac32}}d\tau+\int_0^t\|G(\tau)\|_{\tilde{B}_{2,1}^{\frac54,\frac12}}d\tau
\end{aligned}
\tag{3.21}
$$

$$\lesssim \|n(0)\|_{\tilde{B}^{\frac{1}{4},\frac{3}{2}}_{2,1}} + \|v(0)\|_{\tilde{B}^{\frac{1}{4},\frac{1}{2}}_{2,1}} + \int_0^t \|\nabla v(\tau)\|_1^2 d\tau.$$

Combining (3.20) and (3.21), we obtain (3.5). The proof of lemma is completed. □

The proof of global existence. Now we are in a position to verify (2.2) and (2.3). From (3.2) and (3.3), there is a function $H_2(n, v)$ which is equivalent to $\|(n, v)\|_2^2$ and such that

$$\frac{d}{dt} H_2\big(n(t), v(t)\big) + C\big(\|\nabla n(t)\|_1^2 + \|\nabla v(t)\|_2^2\big) \le 0, \text{ for any } t \ge 0, \tag{3.22}$$

which implies (2.2) since δ is sufficiently small. Adding the above inequality to (3.4), then by Gronwall's inequality, we obtain

$$\|\sigma(t)\|_2^2 \lesssim \|(n(0), v(0), \sigma(0)\|^2 \exp\left\{C \int_0^t \|\nabla v(\tau)\|_{L^\infty} + \|\nabla^2 v(\tau)\|_{L^3} d\tau\right\}. \tag{3.23}$$

Taking (2.2) into (3.5), we obtain

$$\begin{aligned} &\|n(t)\|_{\tilde{B}^{\frac{1}{4},\frac{3}{2}}_{2,1}} + \|v(t)\|_{\tilde{B}^{\frac{1}{4},\frac{1}{2}}_{2,1}} + \int_0^t \|n(\tau)\|_{\tilde{B}^{\frac{9}{4},\frac{3}{2}}_{2,1}} d\tau + \int_0^t \|v(\tau)\|_{\tilde{B}^{\frac{9}{4},\frac{5}{2}}_{2,1}} d\tau \\ &\lesssim \|n(0), v(0)\|_2. \end{aligned} \tag{3.24}$$

Noting that $\tilde{B}^{\frac{5}{4},\frac{3}{2}}_{2,1} \hookrightarrow L^\infty$ and $\tilde{B}^{\frac{1}{4},\frac{1}{2}}_{2,1} \hookrightarrow L^3$, substituting (3.24) into (3.23) yields (2.3). Thus the proof of the global existence result of Proposition 2.2 is completed. □

4. Decay rates

In this section, we are devoted to prove the decay rates (2.4)-(2.6) in Proposition 2.2. We shall first obtain the energy inequality for the derivatives which can be controlled by the first order derivatives. Then a decay-in-time estimate for the first order derivatives is derived by the error related to the derivatives of the higher order. Combining these estimates we obtain the optimal decay rates. More specifically, adding $\|\nabla(n, v)\|^2$ to both sides of (3.3), we arrive at

$$\frac{d}{dt} H_1(t) + H_1(t) + \|\nabla^3 v(t)\|^2 \le C\|\nabla(n, v)(t)\|^2. \tag{4.1}$$

In order to close the estimate (4.1), we shall establish the decay rates estimate for $\|\nabla(n, v)(t)\|$. By Duhamel's principle, we can represent the solution of the first two equations of (2.1) in term of the semigroup

$$U(t) = e^{-t\mathbb{A}} U(0) + \int_0^t e^{-(t-\tau)\mathbb{A}} (F, G)(\tau) d\tau, \tag{4.2}$$

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where $U(t) = (n(t), v(t))$ and $\mathbb{A}$ is a matrix-valued differential operator given by

$$\mathbb{A} = \begin{pmatrix} 0 & \alpha_2 \mathrm{div} \\ \alpha_2 \nabla & -\kappa_1 \Delta - \kappa_2 \nabla \mathrm{div} \end{pmatrix}.$$

In terms of the Fourier expression $\mathbb{A}$, the solution semigroup $\mathrm{e}^{-t\mathbb{A}}$ has the following property on the decay in time, which has been proven in [27–29,31].

Lemma 4.1. *Let $\ell \geq 0$ be an integer, $1 \leq p < \frac{6}{5}$ and $q \geq 2$. Then for any $t \geq 0$, it holds that*

$$\|\nabla^\ell e^{-t\mathbb{A}} U(0)\|_{L^q} \lesssim (1+t)^{-\sigma(p,q;\ell)} \|U(0)\|_{L^p} + e^{-c(1+t)} \|U(0)\|_\ell, \tag{4.3}$$

where $0 < c < 1$ is a constant and the decay rate is measured by

$$\sigma(p, q; \ell) = \frac{3}{2}(\frac{1}{p} - \frac{1}{q}) + \frac{\ell}{2}. \tag{4.4}$$

Moreover, if $\hat{n_0} \leq c_0$, $\hat{v_0} = 0$ for $|\xi| \ll 1$, then there exists a positive constant c_2 such that

$$\min\{\|n(t)\|, \|v(t)\|\} \geq c_2 (1+t)^{-\frac{3}{4}}. \tag{4.5}$$

The following lemma is concerned with the elementary but useful inequality, see [8].

Lemma 4.2. *Let $r_1 > 1$ and $r_2 \in [0, r_1]$, then for any $t \geq 0$, it holds that*

$$\int_0^t (1+t-\tau)^{-r_1} (1+\tau)^{-r_2} d\tau \leq C(r_1, r_2)(1+t)^{-r_2}. \tag{4.6}$$

Now we use the $L^p - L^q$ estimates of the linear problem for the nonlinear problem $(2.1)_1$-$(2.1)_2$. Define the function

$$\mathcal{N}(t) = \sup_{0 \leq \tau \leq t} (1+\tau)^{\frac{5}{2}} H_1(\tau),$$

then

$$\|\nabla(n, v)(t)\|_1 \lesssim \sqrt{H_1(t)} \lesssim (1+t)^{-\frac{5}{2}} \sqrt{\mathcal{N}(t)}, \tag{4.7}$$

and the time decay rate for the first order derivatives of the solutions is obtained in the following lemma.

Lemma 4.3. *Let (n, v, σ) be the solution of the initial value problem (2.1), then it holds*

$$\|\nabla(n, v)(t)\| \lesssim (1+t)^{-\frac{5}{4}} \Big(\|(n_0, v_0)\|_{L^1} + \|(n_0, v_0)\|_1 + \delta \sqrt{\mathcal{N}(t)} \Big). \tag{4.8}$$

Proof. Applying (4.3) to (4.2) with $\ell = 1$, we obtain

$$\begin{aligned}\|\nabla(n,v)(t)\| &\lesssim (1+t)^{-\frac{5}{4}}\big(\|(n_0,v_0)\|_{L^1} + \|(n_0,v_0)\|_1\big)\\ &\quad + \int_0^t (1+t-\tau)^{-\frac{5}{4}}\|(F,G)(\tau)\|_{L^1}d\tau\\ &\quad + \int_0^t \mathrm{e}^{-c(1+t-\tau)}\|(F,G)(\tau)\|_1 d\tau.\end{aligned} \tag{4.9}$$

By right of Hölder's inequality, Lemma 2.1, (3.1), (4.7) and some tedious but straightforward calculation, the nonlinear term (F,G) can be estimated as follows

$$\begin{aligned}\|(F,G)(t)\|_{L^1} &\lesssim \delta\|\nabla(n,v)(t)\|_1 \lesssim \delta(1+t)^{-\frac{5}{4}}\sqrt{\mathcal{N}(t)},\\ \|(F,G)(t)\|_1 &\lesssim \delta\|\nabla(n,v)(t)\|_1 + \delta\|\nabla^3 v(t)\|\\ &\lesssim \delta(1+t)^{-\frac{5}{4}}\sqrt{\mathcal{N}(t)} + \delta\|\nabla^3 v(t)\|.\end{aligned}$$

Substituting the above inequalities into (4.9), we arrive at

$$\begin{aligned}\|\nabla(n,v)(t)\| &\lesssim (1+t)^{-\frac{5}{4}}\big(\|(n_0,v_0)\|_{L^1} + \|(n_0,v_0)\|_1\big)\\ &\quad + \delta\int_0^t (1+t-\tau)^{-\frac{5}{4}}(1+\tau)^{-\frac{5}{4}}\sqrt{\mathcal{N}(\tau)}d\tau\\ &\quad + \delta\int_0^t \mathrm{e}^{-c(1+t-\tau)}\|\nabla^3 v(\tau)\|d\tau\\ &\lesssim (1+t)^{-\frac{5}{4}}\big(\|(n_0,v_0)\|_{L^1} + \|(n_0,v_0)\|_1\big)\\ &\quad + \delta(1+t)^{-\frac{5}{4}}\sqrt{\mathcal{N}(t)}\\ &\quad + \delta\int_0^t \mathrm{e}^{-c(1+t-\tau)}\|\nabla^3 v(\tau)\|d\tau.\end{aligned} \tag{4.10}$$

We need to estimate last term on the right hand side of (4.10). To begin with, by Hölder's inequality and (2.2), we have

$$\int_0^{\frac{t}{2}} \mathrm{e}^{-c(1+t-\tau)}\|\nabla^3 v(\tau)\|d\tau$$

$$\lesssim \Big(\int_0^{\frac{t}{2}} \mathrm{e}^{-2c(1+t-\tau)}d\tau\Big)^{\frac{1}{2}}\Big(\int_0^{\frac{t}{2}} \|\nabla^3 v(\tau)\|^2 d\tau\Big)^{\frac{1}{2}} \tag{4.11}$$
$$\lesssim \delta t^{\frac{1}{2}}\mathrm{e}^{-c(1+\frac{t}{2})} \lesssim \delta(1+t)^{-\frac{5}{4}}.$$

Furthermore, (4.1) gives rise to

$$\frac{d}{d\tau}\Big(\mathrm{e}^{-c(1+t-\tau)}H_1(\tau)\Big) + (1-c)\mathrm{e}^{-c(1+t-\tau)}H_1(\tau) + \mathrm{e}^{-c(1+t-\tau)}\|\nabla^3 v(\tau)\|^2$$
$$\lesssim \mathrm{e}^{-c(1+t-\tau)}\|\nabla(n,v)(\tau)\|^2.$$

Integrating the above inequality from $\frac{t}{2}$ to t, we deduce

$$\int_{\frac{t}{2}}^{t} \mathrm{e}^{-c(1+t-\tau)}\|\nabla^3 v(\tau)\|^2 d\tau \lesssim \mathrm{e}^{-c(1+t-\frac{t}{2})}H_1\big(\frac{t}{2}\big) + \int_{\frac{t}{2}}^{t} \mathrm{e}^{-c(1+t-\tau)}\|\nabla(n,v)(\tau)\|_1^2 d\tau$$
$$\lesssim \delta^2\mathrm{e}^{-c(1+\frac{t}{2})} + \int_{\frac{t}{2}}^{t} \mathrm{e}^{-c(1+t-\tau)}(1+\tau)^{-3(\frac{1}{p}-\frac{1}{2})-1}\mathcal{N}(\tau)d\tau$$
$$\lesssim \delta^2\mathrm{e}^{-c(1+\frac{t}{2})} + (1+t)^{-\frac{5}{2}}\mathcal{N}(t),$$

which, together with Hölder's inequality give

$$\int_{\frac{t}{2}}^{t} \mathrm{e}^{-c(1+t-\tau)}\|\nabla^3 v(\tau)\|d\tau \lesssim \Big(\int_{\frac{t}{2}}^{t} \mathrm{e}^{-c(1+t-\tau)}d\tau\Big)^{\frac{1}{2}}\Big(\int_{\frac{t}{2}}^{t} \mathrm{e}^{-c(1+t-\tau)}\|\nabla^3 v(\tau)\|^2 d\tau\Big)^{\frac{1}{2}}$$
$$\lesssim \Big(\delta^2\mathrm{e}^{-c(1+\frac{t}{2})} + (1+t)^{-\frac{5}{4}}\mathcal{N}(t)\Big)^{\frac{1}{2}} \tag{4.12}$$
$$\lesssim (1+t)^{-\frac{5}{4}}\big(\delta + \sqrt{\mathcal{N}(t)}\big).$$

Putting (4.11) and (4.12) into (4.10) we deduce (4.8). The proof of lemma is completed. □

The proof of the time decay rate. In terms of (4.1), (4.8) and Gronwall's inequality, it is easy to get

$$H_1(t) \lesssim \mathrm{e}^{-t}H_1(0) + \int_0^t \mathrm{e}^{-(t-\tau)}\|\nabla(n,v)(\tau)\|^2 d\tau$$
$$\lesssim \mathrm{e}^{-t}H_1(0) + \int_0^t \mathrm{e}^{-(t-\tau)}(1+\tau)^{-\frac{5}{2}}\big(\|(n_0,v_0)\|_{L^1}^2 + \|(n_0,v_0)\|_1^2 + \delta^2\mathcal{N}(\tau)\big)d\tau$$

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$$\lesssim (1+t)^{-\frac{5}{2}}\big(\|(n_0,v_0)\|_{L^1}^2+\|(n_0,v_0)\|_2^2+\delta^2\mathcal{N}(t)\big),$$

that is

$$(1+t)^{\frac{5}{2}}H_1(t)\lesssim \big(\|(n_0,v_0)\|_{L^1}^2+\|(n_0,v_0)\|_2^2+\delta^2\mathcal{N}(t)\big). \tag{4.13}$$

We note that $\mathcal{N}(t)$ is non-decreasing and δ is small enough, then (4.13) implies

$$\mathcal{N}(t)\lesssim \|(n_0,v_0)\|_{L^1}^2+\|(n_0,v_0)\|_2^2. \tag{4.14}$$

Taking (4.7) into (4.14) yields (2.5).

Next, we turn to prove (2.4). Applying (4.3) to (4.2) with $\ell=0$, we have from (2.5) and (4.5) that

$$\begin{aligned}
\|(n,v)(t)\| &\lesssim (1+t)^{-\frac{3}{4}}\big(\|(n_0,v_0)\|_{L^1}+\|(n_0,v_0)\|\big)\\
&\quad+\int_0^t(1+t-\tau)^{-\frac{3}{4}}\big(\|(F,G)(\tau)\|_{L^1}+\|(F,G)(\tau)\|\big)d\tau\\
&\lesssim (1+t)^{-\frac{3}{4}}\big(\|(n_0,v_0)\|_{L^1}+\|(n_0,v_0)\|\big)\\
&\quad+\delta\int_0^t(1+t-\tau)^{-\frac{3}{4}}\|\nabla(n,v)(\tau)\|d\tau\\
&\lesssim \Big[(1+t)^{-\frac{3}{4}}+\delta\int_0^t(1+t-\tau)^{-\frac{3}{4}}(1+\tau)^{-\frac{5}{4}}d\tau\Big]\\
&\quad\big(\|(n_0,v_0)\|_{L^1}+\|(n_0,v_0)\|_2\big)\\
&\lesssim (1+t)^{-\frac{3}{4}}\big(\|(n_0,v_0)\|_{L^1}+\|(n_0,v_0)\|_2\big).
\end{aligned}\tag{4.15}$$

By the interpolation, it follows from (2.5) and (4.15) that for any $2\le q\le 6$

$$\|(n,v)(t)\|_{L^q}\lesssim \|(n,v)(t)\|^{\frac{6-q}{2q}}\|\nabla(n,v)(t)\|^{\frac{3q-6}{2q}}\lesssim (1+t)^{-\frac{3}{2}(1-\frac{1}{q})}\big(\|(n_0,v_0)\|_{L^1}+\|(n_0,v_0)\|_2\big),$$

which is (2.4).

Finally we turn to prove (2.6). Using (2.4), (2.5) and (2.1), it is easy to get

$$\|\partial_t(n,v,\sigma)(t)\|\lesssim \|\nabla n(t)\|+\|\nabla v(t)\|_1\lesssim (1+t)^{-\frac{5}{4}}\big(\|(n_0,v_0)\|_{L^1}+\|(n_0,v_0)\|_2\big),$$

which is (2.6) and this completes the proof of the time decay rate results of Proposition 2.2. □

5. Lower bound time decay rates

In this section, we investigate the lower bound time decay for global solutions. Define

$$\mathcal{P}=\frac{c_\nu P}{R}-\frac{c_\nu \bar{P}}{R}+\frac{1}{2}\rho|u|^2, m_0=\rho_0u_0.$$

Then the second and third equations of system (1.1) can be rewritten as

$$\begin{cases} \mathcal{P}_t+\frac{(c_\nu+R)\bar{P}}{R\bar{\rho}}\mathrm{div}m=\mathcal{F}, \\ m_t+\frac{R}{c_\nu}\nabla\mathcal{P}-\kappa_1\Delta m-\kappa_2\nabla\mathrm{div}m=\mathcal{G}, \end{cases} \tag{5.1}$$

where

$$\mathcal{F}=-\mathrm{div}\big(\mathcal{P}u+nu\big)-\frac{(c_\nu+R)\bar{P}}{R}\mathrm{div}\big[(\frac{1}{\rho}-\frac{1}{\bar{\rho}})m\big]+\mathrm{div}(u\mathcal{T}),$$

and

$$\mathcal{G}=-\mathrm{div}\Big(\frac{m\otimes m}{\rho}\Big)+\mu\Delta[(\frac{1}{\rho}-\frac{1}{\bar{\rho}})m]+(\mu+\lambda)\nabla\mathrm{div}[(\frac{1}{\rho}-\frac{1}{\bar{\rho}})m]+\frac{R}{2c_\nu}\nabla(\rho|u|^2).$$

Denote matrix-valued differential operator

$$\mathbb{A}'=\begin{pmatrix} 0 & \frac{(c_\nu+R)\bar{P}}{R\bar{\rho}}\mathrm{div} \\ \frac{R}{c_\nu}\nabla & -\kappa_1\Delta-\kappa_2\nabla\mathrm{div} \end{pmatrix},$$

then the solution semigroup $\mathrm{e}^{-t\mathbb{A}'}$ has the following property on the decay in time as the solution semigroup $\mathrm{e}^{-t\mathbb{A}}$ in Lemma 4.1.

The proof of lower bound time decay rates. By Duhamel's principle, Lemma 4.1, and the condition (1.11), we arrive at

$$\begin{aligned} \min\{\|(\mathcal{P},m)(t)\|\} &\geq \|\mathrm{e}^{-t\mathbb{A}'}(\mathcal{P}_0,m_0)\|-\int_0^t\|\mathrm{e}^{-(t-\tau)\mathbb{A}'}(\mathcal{F},\mathcal{G})(\tau)\|d\tau \\ &\geq c_3(1+t)^{-\frac{3}{4}}-\int_0^t\|\mathrm{e}^{-(t-\tau)\mathbb{A}'}(\mathcal{F},\mathcal{G})(\tau)\|d\tau. \end{aligned} \tag{5.2}$$

The second term of the right-hand side of the above inequality can be estimated as follows.

On one hand, taking the same argument in (4.15), we get

$$\int_0^t\Big\|\mathrm{e}^{-(t-\tau)\mathbb{A}}\Big(-\mathrm{div}\big(\mathcal{P}u+nu\big)+\mathrm{div}(u\mathcal{T}),-\mathrm{div}\Big(\frac{m\otimes m}{\rho}\Big)+\frac{R}{2c_\nu}\nabla(\rho|u|^2)\Big)(\tau)\Big\|d\tau$$

$$
\begin{aligned}
&\lesssim \int_0^t (1+t-\tau)^{-\frac{5}{4}} \left\| \Big(\mathcal{P}u, nu, u\mathcal{T}, \frac{m^2}{\rho}\Big)(\tau) \right\|_{L^1\cap H^1} d\tau \\
&\lesssim \int_0^t (1+t-\tau)^{-\frac{5}{4}} \|(\mathcal{P}, u)(\tau)\|_2^2 d\tau \qquad (5.3)\\
&\lesssim \int_0^t (1+t-\tau)^{-\frac{5}{4}} (1+\tau)^{-\frac{3}{2}} d\tau \\
&\lesssim (1+t)^{-\frac{5}{4}}.
\end{aligned}
$$

On the other hand, by Lemma 2.2, we have

$$
\begin{aligned}
&\int_0^t \Big\| e^{-(t-\tau)\mathbb{A}} \Big(-\frac{(c_v+R)\bar{P}}{R} \mathrm{div}\big[(\frac{1}{\rho}-\frac{1}{\bar{\rho}})m\big], \mu\Delta[(\frac{1}{\rho}-\frac{1}{\bar{\rho}})m] \\
&+ (\mu+\lambda)\nabla \mathrm{div}[(\frac{1}{\rho}-\frac{1}{\bar{\rho}})m]\Big)(\tau) \Big\| d\tau \\
&\lesssim \int_0^t (1+t-\tau)^{-\frac{7}{6}} (\| \wedge^{-\frac{4}{3}} [(\frac{1}{\rho}-\frac{1}{\bar{\rho}})m](\tau)\| + \| \wedge^2 [(\frac{1}{\rho}-\frac{1}{\bar{\rho}})m](\tau)\|) d\tau \\
&\lesssim \int_0^t (1+t-\tau)^{-\frac{7}{6}} \Big[\|(\frac{1}{\rho}-\frac{1}{\bar{\rho}})(\tau)\|_{\dot{B}^{\frac{1}{12}}_{2,1}} \|m(\tau)\|_{\dot{B}^{\frac{1}{12}}_{2,1}} + \|\nabla^2(\frac{1}{\rho}-\frac{1}{\bar{\rho}})(\tau)\| \|m(\tau)\|_{L^\infty} \\
&+ \|\nabla(\frac{1}{\rho}-\frac{1}{\bar{\rho}})(\tau)\|_{L^3} \|\nabla m(\tau)\|_{L^6} + \|(\frac{1}{\rho}-\frac{1}{\bar{\rho}})(\tau)\|_{L^\infty} \|\nabla^2 m(\tau)\| \Big] d\tau \qquad (5.4)\\
&\lesssim \int_0^t (1+t-\tau)^{-\frac{7}{6}} (\| \wedge^{\frac{1}{24}} m(\tau)\| + \|\nabla m(\tau)\|_1) d\tau \\
&\lesssim \int_0^t (1+t-\tau)^{-\frac{7}{6}} (\| \wedge^{\frac{1}{24}} m(\tau)\| + \|\nabla m(\tau)\|_1) d\tau \\
&\lesssim \int_0^t (1+t-\tau)^{-\frac{7}{6}} (1+\tau)^{-\frac{37}{48}} d\tau \\
&\lesssim (1+t)^{-\frac{37}{48}}.
\end{aligned}
$$

Plugging (5.3) and (5.4) to (5.2) gives

$$
\min\{\|(\mathcal{P}, m)(t)\|\} \geq c_4 (1+t)^{-\frac{3}{4}}. \qquad (5.5)
$$

Thus by the relation between $(\mathcal{P}, m)$ and (n, u), (2.4) and (5.5), we get (2.7). Thus The proof of lower bound time decay rates results of Proposition 2.2 is completed. □

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References

[1] F. Charve, R. Danchin, A global existence result for the compressible Navier-Stokes equations in the critical L^p framework, Arch. Ration. Mech. Anal. 198 (2010) 233–271.
[2] R. Danchin, Global existence in critical spaces for compressible Navier-Stokes equations, Invent. Math. 141 (2000) 579–614.
[3] R. Danchin, Global existence in critical spaces for flows of compressible viscous and heat-conductive gases, Arch. Ration. Mech. Anal. 16 (2001) 1–39.
[4] R. Danchin, Fourier Analysis Methods for PDEs, Lecture Notes, vol. 14, 2005.
[5] K. Deckelnick, Decay estimates for the compressible Navier-Stokes equations in unbounded domains, Math. Z. 209 (1992) 115–130.
[6] R.J. Duan, H.F. Ma, Global existence and convergence rates for the 3-D compressible Navier-Stokes equations without heat conductivity, Indiana Univ. Math. J. 5 (2008) 2299–2319.
[7] R.J. Duan, H.X. Liu, S. Ukai, T. Yang, Optimal $L^p - L^q$ convergence rates for the compressible Navier-Stokes equations with potential force, J. Differ. Equ. 238 (2007) 220–233.
[8] R.J. Duan, S. Ukai, T. Yang, H.J. Zhao, Optimal convergence rates for the compressible Navier-Stokes equations with potential forces, Math. Models Methods Appl. Sci. 17 (2007) 737–758.
[9] L.C. Evans, Partial Differential Equations, Amer. Math. Soc., 1998.
[10] J.S. Fan, S. Jiang, Y.B. Ou, A blow-up criterion for compressible viscous heat-conductive flows, Ann. Inst. Henri Poincaré, Anal. Non Linéaire 27 (2010) 337–350.
[11] E. Feireisl, H. Petzeltová, Large-time behavior of solutions to the Navier-Stokes equations of compressible flow, Arch. Ration. Mech. Anal. 150 (1999) 77–96.
[12] E. Feireisl, A. Novotný, H. Petzeltová, On the existence of globally dened weak solutions to the Navier-Stokes equations, J. Math. Fluid Mech. 3 (2001) 358–392.
[13] Y. Guo, Y.J. Wang, Decay of dissipative equations and negative Sobolev spaces, Commun. Partial Differ. Equ. 37 (2012) 2165–2208.
[14] D. Hoff, Global solutions of the Navier-Stokes equations for multidimensional compressible flow with discontinuous initial data, J. Differ. Equ. 120 (1995) 215–254.
[15] D. Hoff, Strong convergence to global solutions for multidimensional flows of compressible, viscous fluids with polytropic equations of state and discontinuous initial data, Arch. Ration. Mech. Anal. 132 (1995) 1–14.
[16] D. Hoff, K. Zumbrun, Multi-dimensional diffusion waves for the Navier-Stokes equations of compressible flow, Indiana Univ. Math. J. 44 (1995) 604–676.
[17] D. Hoff, K. Zumbrun, Pointwise decay estimates for multidimensional Navier-Stokes diffusion waves, Z. Angew. Math. Phys. 48 (1997) 597–614.
[18] X.D. Huang, J. Li, Z.P. Xin, Global well-posedness of classical solutions with large oscillations and vacuum to the three-dimensional isentropic compressible Navier-Stokes equations, Commun. Pure Appl. Math. 65 (2012) 549–585.
[19] X.D. Huang, J. Li, Global classical and weak solutions to the three-dimensional full compressible Navier-Stokes system with vacuum and large oscillations, Arch. Ration. Mech. Anal. 3 (2018) 995–1059.
[20] F. Jiang, S. Jiang, J.P. Yin, Global weak solutions to the two-dimensional Navier-Stokes equations of compressible heat-conducting flows with symmetric data and forces, Discrete Contin. Dyn. Syst., Ser. A 34 (2014) 567–587.

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[21] F. Jiang, Nonlinear thermal instability in compressible viscous flows without heat conductivity, J. Math. Fluid Mech. 2 (2018) 1–31.
[22] S. Jiang, P. Zhang, Global spherically symmetric solutions of the compressible isentropic Navier-Stokes equations, Commun. Math. Phys. 215 (2001) 559–581.
[23] S. Jiang, P. Zhang, Axisymmetric solutions of the 3-D Navier-Stokes equations for compressible isentropic flows, J. Math. Pures Appl. 82 (2003) 949–973.
[24] S. Kawashima, Systems of a Hyperbolic-Parabolic Composite Type, with Applications to the Equations of Magnetohydrodynamics, thesis, Kyoto University, 1983.
[25] A.V. Kazhikhov, Cauchy problem for viscous gas equations, Sib. Math. J. 23 (1982) 44–49.
[26] A.V. Kazhikhov, V.V. Shelukhin, Unique global solution with respect to time of initial-boundary value problems for one-dimensional equations of a viscous gas, J. Appl. Math. Mech. 41 (1977) 273–282.
[27] T. Kobayashi, Some estimates of solutions for the equations of motion of compressible viscous fluid in an exterior domain in $\mathbb{R}^3$, J. Differ. Equ. 184 (2002) 587–619.
[28] T. Kobayashi, Y. Shibata, Decay estimates of solutions for the equations of motion of compressible viscous and heat-conductive gases in an exterior domain in $\mathbb{R}^3$, Commun. Math. Phys. 200 (1999) 621–659.
[29] H.L. Li, T. Zhang, Large time behavior of isentropic compressible Navier-Stokes system in $\mathbb{R}^3$, Math. Methods Appl. Sci. 34 (2011) 670–682.
[30] P.L. Lions, Mathematical Topics in Fluid Mechanics, vol. 2. Compressible Models, Oxford University Press, New York, 1998.
[31] T.P. Liu, W.K. Wang, The pointwise estimates of diffusion waves for the Navier-Stokes equations in odd multidimensions, Commun. Math. Phys. 196 (1998) 145–173.
[32] T.P. Liu, Y.N. Zeng, Compressible Navier-Stokes equations with zero heat conductivity, J. Differ. Equ. 153 (1999) 225–291.
[33] A. Matsumura, T. Nishida, The initial value problem for the equations of motion of compressible viscous and heat conductive fluids, Proc. Jpn. Acad., Ser. A 55 (1979) 337–342.
[34] A. Matsumura, T. Nishida, The initial value problem for the equations of motion of viscous and heat-conductive gases, J. Math. Kyoto Univ. 20 (1980) 67–104.
[35] A. Matsumura, T. Nishida, Initial boundary value problems for the equations of motion of compressible viscous and heat-conductive fluids, Commun. Math. Phys. 89 (1983) 445–464.
[36] Z. Tan, H.Q. Wang, Global existence and optimal decay rate for the strong solutions in H^2 to the 3-D compressible Navier-Stokes equations without heat conductivity, J. Math. Anal. Appl. 394 (2012) 571–580.
[37] Z. Tan, Y.J. Wang, On hyperbolic-dissipative systems of composite type, J. Differ. Equ. 260 (2016) 1091–1125.
[38] H.Y. Wen, C.J. Zhu, Global solutions to the three-dimensional full compressible Navier-Stokes equations with vacuum at infinity in some classes of large data, SIAM J. Math. Anal. 49 (2017) 162–221.
[39] G.C. Wu, Global existence and asymptotic behavior for the 3D compressible Navier-Stokes equations without heat conductivity in a bounded domain, J. Differ. Equ. 262 (2017) 844–861.
[40] G.C. Wu, Q. Chen, Z. Tan, On the outer pressure problem of the one-dimensional compressible Navier-Stokes equation with degenerate transport coefficients, J. Math. Anal. Appl. 449 (2017) 553–571.
[41] Z.P. Xin, W. Yan, On blowup of classical solutions to the compressible Navier-Stokes equations, Commun. Math. Phys. 321 (2013) 529–541.
[42] Z.P. Xin, Blowup of smooth solutions to the compressible Navier-Stokes equation with compact density, Commun. Pure Appl. Math. 51 (1998) 229–240.

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