

# ARE RANDOM COEFFICIENTS NEEDED IN PARTICLE SWARM OPTIMIZATION FOR SIMULATION-BASED SHIP DESIGN?

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**Abstract.** Simulation-based design optimization (SBDO) methods integrate computer simulations, design modification tools, and optimization algorithms. In hydrodynamic applications, often objective functions are computationally expensive and likely noisy, their derivatives are not directly provided, and the existence of local minima cannot be excluded a priori, which motivates the use of derivative-free global optimization algorithms. This type of algorithms (such as Particle Swarm Optimization, PSO) usually follow a stochastic formulation, requiring computationally expensive numerical experiments in order to provide statistically significant results. The objective of the present work is to investigate the effects of using (versus suppressing) random coefficients in PSO for ship hydrodynamics SBDO. A comparison is shown of 1,000 random PSO to deterministic PSO (DPSO) using 12 well-known scalable test problems, with dimensionality ranging from two to fifty. A total of 588 test functions is considered and more than 500,000 optimization runs are performed and evaluated. The results are discussed based on the probability of success of random PSO versus DPSO. Finally, a comparison of random PSO to DPSO is shown for the hull-form optimization of the DTMB 5415 model. In summary, test functions show the robustness of DPSO, which outperforms random PSO with odds of 30/1 for low-dimensional problems (indicatively  $N \leq 30$ ) and 5/1 for high-dimensional problems ( $N > 30$ ). The hull-form SBDO ( $N = 11$ ) shows how DPSO outperforms PSO with odds of 20/1. The use of DPSO in the SBDO context is therefore advised, especially if computationally expensive analyses are involved in the optimization.

## 1 INTRODUCTION

The design of marine vehicles requires accurate analyses and complex decision-making methodologies. In the last decades, the design paradigm has shifted from the build-and-test approach to more efficient and versatile simulation-based methodologies. The integration of optimization algorithms with computer simulations has led to automatic simulation-based design optimization (SBDO) procedures, with the aim of guiding the designer in the decision making process

of complex engineering applications. For shape optimization problems, SBDO consists of three main elements: (i) a geometry modification and automatic meshing tool, (ii) a simulation tool, and (iii) an optimization algorithm, which need to be integrated in an efficient and robust way.

Among numerous types and implementations of optimization algorithms, global derivative-free methods often represent a robust option for SBDO. The success of this type of methods stems from the peculiar characteristics of SBDO (in ship hydrodynamics as well as other engineering fields). The optimization objectives are often the results of complex simulations, solving systems of partial differential equations. These results are often noisy, due to the presence of solution residuals. Access to the source code is not always possible, therefore derivative-based methods (whether used) need to rely on finite differences, which may be highly inaccurate. Furthermore, in most problems the existence of multiple local optima cannot be excluded a priori, making the use of global derivative-free methods very attractive.

Among global derivative-free optimization algorithms, Particle Swarm Optimization (PSO) [1] has gained the attention of the marine engineering community, due to the ease of implementation and capability of providing approximate solutions to the optimization problem at a reasonable computational cost. PSO is based on the social-behaviour metaphor of a flock of birds or a swarm of bees searching for food. PSO belongs to the class of heuristic algorithms for single-objective evolutionary optimization. Recent applications of PSO to ship SBDO include hull-form and waterjet design optimization of fast catamarans [2, 3, 4], the optimization of unconventional multi-hull configurations [5], and surface combatant [6, 7].

The original PSO formulation uses random coefficients to sustain the swarm dynamics. This property implies that statistically significant results can be obtained only through extensive numerical campaigns. Such an approach can be too expensive in SBDO for complex industrial applications, where CPU-time expensive computer simulations are used directly as analysis tools. For these reasons, efficient deterministic approaches (such as deterministic PSO, DPSO) have been developed and investigated [8, 9], suppressing any kind of randomness in the particle position update. In this deterministic version, the swarm diversity is sustained only by the swarm dynamics. During the swarm evolution each particle is attracted by diverse positions, based on the cognitive and social experience, iteration by iteration. In most problems, this is generally sufficient to maintain the swarm sufficiently active and provide reasonable solutions.

The objective of the present work is to investigate the effects of using (versus suppressing) random coefficients of PSO in SBDO for ship hydrodynamics.

The approach includes a comparison of 1,000 random PSO to DPSO, using 12 well-known analytical test functions, with dimensionality ranging from two to fifty. The total number of functions assessed is 588 with a total number of optimization runs larger than 500,000. The numerical results are discussed based on the probability of success of random PSO versus DPSO. Finally, the comparison is shown for the hydrodynamic hull-form optimization of the DTMB 5415 model, an early and open to public version of the DDG-51 destroyer. A single-speed single-objective example is shown, aimed at the reduction of the total resistance coefficient in calm water at 18 kn, corresponding to Froude number ( $Fr$ ) equal to 0.25. The design constraints include fixed displacement and length between perpendiculars, along with a  $\pm 5\%$  maximum variation of beam and draft. The ship is free to sink and trim. An expansion of eleven orthogonal basis functions [10] is used for the modification of the hull form and the sonar dome. The solution of the problem is based on a metamodel [11], trained by a RANS solver (CFDShip-Iowa v4.5

[12]), and already used in [7].

## 2 GENERAL FORMULATION OF THE GLOBAL OPTIMIZATION PROBLEM

Consider the following objective function

$$f(\mathbf{x}) : \mathbb{R}^N \longrightarrow \mathbb{R} \quad (1)$$

and the global optimization problem

$$\min_{\mathbf{x} \in \mathcal{L}} f(\mathbf{x}), \quad \mathcal{L} \subset \mathbb{R}^N \quad (2)$$

where  $\mathcal{L}$  is a closed and bounded subset of  $\mathbb{R}^N$  and  $N$  is the number of variables, collected in  $\mathbf{x}$ . The global minimization of the objective function  $f(\mathbf{x})$  requires to find a vector  $\mathbf{a} \in \mathcal{L}$  such that:

$$\forall \mathbf{b} \in \mathcal{L} : f(\mathbf{a}) \leq f(\mathbf{b}) \quad (3)$$

Then,  $\mathbf{a}$  is a global minimum for the function  $f(\mathbf{x})$  over  $\mathcal{L}$ . Hereafter, the compact set  $\mathcal{L}$  represents box constraints.

Since the solution of Eq. 2 is in general an NP-hard problem, the exact identification of a global minimum might be very difficult. Therefore, solutions with sufficient good fitness, provided by heuristic procedures (such as PSO), are often considered acceptable for engineering purposes.

## 3 PARTICLE SWARM OPTIMIZATION

### 3.1 Stochastic formulation

The original formulation of the PSO algorithm, as presented in [13], reads

$$\begin{cases} \mathbf{v}_i^{k+1} = w\mathbf{v}_i^k + c_1 r_{1,i}^k (\mathbf{p}_i - \mathbf{x}_i^k) + c_2 r_{2,i}^k (\mathbf{g} - \mathbf{x}_i^k) \\ \mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \mathbf{v}_i^{k+1} \end{cases} \quad (4)$$

The above equations update velocity ( $\mathbf{v}_i^k$ ) and position ( $\mathbf{x}_i^k$ ) of the  $i$ -th particle at the  $k$ -th iteration, where:  $w$  is the inertia weight;  $c_1$  and  $c_2$  are respectively the social and cognitive learning rate;  $r_{1,i}^k$  and  $r_{2,i}^k$  are uniformly distributed random numbers in  $[0, 1]$ ;  $\mathbf{p}_i$  is the personal best position ever found by the  $i$ -th particle in the previous iterations and  $\mathbf{g}$  is the global best position ever found in the previous iterations by all the particles.

An overall constriction factor  $\chi$  may be used [14], in place of the inertia weight  $w$ . Accordingly, the system in Eq. 4 is recast in the following equivalent form

$$\begin{cases} \mathbf{v}_i^{k+1} = \chi \left[ \mathbf{v}_i^k + c_1 r_{1,i}^k (\mathbf{p}_i - \mathbf{x}_i^k) + c_2 r_{2,i}^k (\mathbf{g} - \mathbf{x}_i^k) \right] \\ \mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \mathbf{v}_i^{k+1} \end{cases} \quad (5)$$

### 3.2 Deterministic formulation

A deterministic version of the PSO algorithm (namely DPSO) was formulated in [8] by setting  $r_{1,i}^k = r_{2,i}^k = 1, \forall i, k$  in Eq. 5

$$\begin{cases} \mathbf{v}_i^{k+1} = \chi [\mathbf{v}_i^k + c_1(\mathbf{p}_i - \mathbf{x}_i^k) + c_2(\mathbf{g} - \mathbf{x}_i^k)] \\ \mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \mathbf{v}_i^{k+1} \end{cases} \quad (6)$$

A discussion for an effective and efficient use of DPSO for SBDO in ship hydrodynamics has been presented in [9].

### 3.3 Parameter setup

The parameter setup used for both random PSO and DPSO is selected as suggested in [9]: number of particles  $N_p = 4N$ ; particle initialization with Hammersley sequence sampling distribution on domain and bounds (for  $N < 10$ ) and domain only (for  $N \geq 10$ ) with non-null velocity [3]; set of coefficients proposed in [15], i.e.,  $\chi = 0.721, c_1 = c_2 = 1.655$ ; semi-elastic wall-type approach [9]. A limit to the number of objective function evaluations is set equal to  $400N$ , corresponding to 100 algorithm iterations.

## 4 TEST PROBLEMS

### 4.1 Analytical test functions

Twelve analytical test problems are used in the preliminary numerical experience, including a wide variety of functions, such as continuous and discontinuous, differentiable and non-differentiable, separable and non-separable, unimodal and multimodal. Each problem is studied with dimensionality  $2 \leq N \leq 50$ , resulting in a total number of test functions equal to 588. Tables 1 summarizes the test problems used in the current study, including variable bounds and global optimum.

### 4.2 Hull-form optimization of the DTMB 5415



Figure 1: A replica of the DTMB 5415 (CNR-INSEAN model 2340)

The hull-form optimization of the DTMB 5415 is used to assess the algorithm performance for ship hydrodynamic problems. Figure 1 shows the geometry of a 5.720 m length DTMB 5415 model used for towing tank experiments, as seen at CNR-INSEAN [16]. The optimization is formulated as

$$\begin{aligned} & \text{Minimize } f(\mathbf{x}) \\ & \text{subject to } g_l(\mathbf{x}) = 0, \quad \text{with } l = 1, \dots, L \\ & \quad \text{and to } h_m(\mathbf{x}) \leq 0, \quad \text{with } m = 1, \dots, M \end{aligned} \quad (7)$$

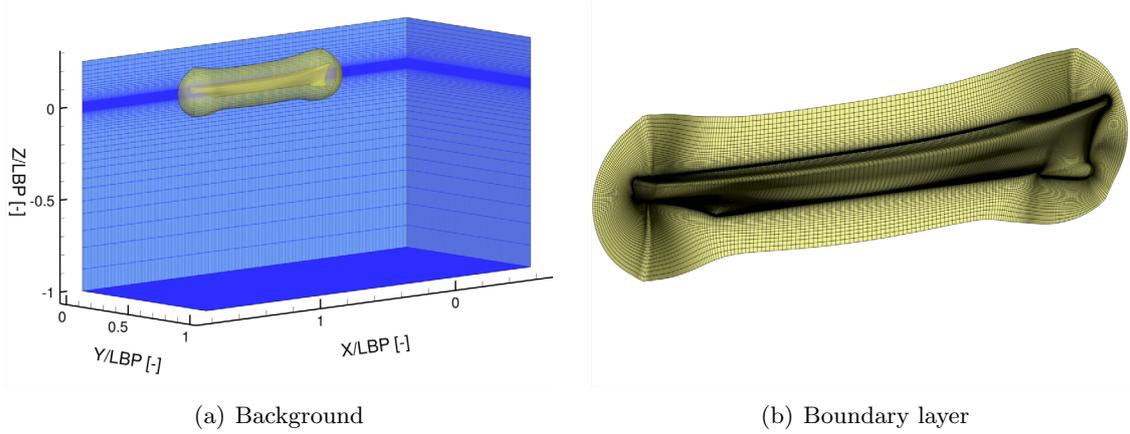
**Table 1:** Analytical test functions

Name	$f(\mathbf{x})$	Bounds $\mathbf{1} \leq \mathbf{x} \leq \mathbf{u}$	Opt. $f_{\min}^*$
Ackley	$f(x) = 20e^{-0.2\sqrt{\frac{1}{N}\sum_{i=1}^N x_i^2}} - e^{\frac{1}{N}\sum_{i=1}^N \cos(2\pi x_i)} + 20 + e$	$-5.00 \leq \mathbf{x} \leq 4.00$	0.00
Alpine	$f(x) = \sum_{i=1}^N  x_i \sin(x_i) + 0.1x_i $	$-9.00 \leq \mathbf{x} \leq 7.00$	0.00
Dixon-Price	$f(x) = (x_1 - 1)^2 + \sum_{i=2}^N [i(2x_i^2 - x_{i-1})^2]$	$-10.0 \leq \mathbf{x} \leq 10.0$	0.00
Griewank	$f(x) = 1 + \sum_{i=1}^N \frac{x_i}{4000} - \prod_{i=1}^N \cos(x_i/\sqrt{i})$	$-100. \leq \mathbf{x} \leq 90.0$	0.00
Levy n.5	$f(x) = \frac{\pi}{N} \left\{ 10 \sin^2(\pi y_1) + \sum_{i=1}^{N-1} [(y_i - 1)^2 (1 + 10 \sin^2(\pi y_{i+1}))] + (y_N - 1)^2 \right\}$ with $y_i = 1 + \frac{1}{4}(x_i - 1)$	$-10.0 \leq \mathbf{x} \leq 10.0$	0.00
Mishra n.11	$f(x) = \left[ \frac{1}{N} \sum_{i=1}^N  x_i  - \left( \prod_{i=1}^N  x_i  \right)^{\frac{1}{N}} \right]^2$	$-10.0 \leq \mathbf{x} \leq 9.00$	0.00
Rastrigin	$f(x) = 10n + \sum_{i=1}^N [x_i^2 - 10 \cos(2\pi x_i)]$	$-5.12 \leq \mathbf{x} \leq 4.12$	0.00
Rosenbrock	$f(x) = \sum_{i=1}^{N-1} [100(x_{i+1} - x_i)^2 + (x_i - 1)^2]$	$-5.00 \leq \mathbf{x} \leq 10.0$	0.00
Sphere	$f(x) = \sum_{i=1}^N x_i^2$	$-5.00 \leq \mathbf{x} \leq 4.00$	0.00
Styblinski-Tang	$f(x) = \frac{1}{2} \sum_{i=1}^N (x_i^4 - 16x_i^2 + 5x_i) + 39.2N$	$-5.00 \leq \mathbf{x} \leq 5.00$	0.00
Trigonometric n.2	$f(x) = 1 + \sum_{i=1}^N 8 \sin^2 [7(x_i - 0.9)^2] + 6 \sin^2 [14(x_i - 0.9)^2] + (x_i - 0.9)^2$	$-500. \leq \mathbf{x} \leq 500.$	1.00
Zacharov	$f(x) = \sum_{i=1}^N x_i^2 + \left( \frac{1}{2} \sum_{i=1}^N ix_i \right)^2 + \left( \frac{1}{2} \sum_{i=1}^N ix_i \right)^4$	$-5.00 \leq \mathbf{x} \leq 10.0$	0.00

where  $f(\mathbf{x})$  is the the total resistance coefficient  $C_T$  in calm water at 18 kn ( $Fr = 0.25$ ). Geometrical equality constraints  $g_l(\mathbf{x})$  include fixed length between perpendicular ( $LBP$ ) and displacement ( $\nabla$ ), whereas geometrical inequality constraints  $h_m(\mathbf{x})$  include 5% maximum variation of beam ( $B$ ) and draught ( $T$ ). Fixed  $LBP$  and  $\nabla$  are satisfied by automatic geometric scaling, while constraints for  $B$  and  $T$  are handled using a penalty function method. Modification of the parent hull are performed using a number ( $N = 11$ ) orthogonal functions, defined over surface-body patches, and dimensionality reduction based on the KLE [10].

The optimization problem is solved using a metamodel, trained by 71 RANS simulations defined using a sequential sampling procedure [11]. RANS simulations are performed with the CFDShip-Iowa v4.5 code [12], which has the capability of a 6DOF simulation and has been developed at the University of Iowa, IIHR-Hydroscience & Engineering, over the past 25 years. The SST blended  $k - \varepsilon/k - \omega$  turbulent model is used. The free-surface location is predicted by a single phase level set method. A second order upwind scheme is used to discretize the convective terms of momentum equations. The code SUGGAR runs as a separate process from the flow solver to compute interpolation coefficients for the overset grid, which enables CFDShip-Iowa to take care of 6DOF with a motion controller at every time step. Only 2DOFs are considered in the current study. Simulations are performed for the right demi hull, taking advantage of symmetry about the  $xz$ -plane. The computational grid is shown in Fig. 2. Further details are provided in [7].

In the current study, a limit to the number of function evaluations is set to 4400, i.e.,  $400N$ .



**Figure 2:** Computational volume grid for CFDSHIP-IOWA

## 5 PERFORMANCE METRICS

### 5.1 Distance to the desired optimum

In order to assess the performance of both random PSO and DPSO, an overall distance-based metric,  $\Delta_t$ , is used [17]

$$\Delta_t = \sqrt{\frac{\Delta_x^2 + \Delta_f^2}{2}} \quad \text{where} \quad \Delta_x = \sqrt{\frac{1}{N} \sum_{j=1}^N \left( \frac{g_j - x_{j,\min}^*}{Z_j} \right)^2} \quad \text{and} \quad \Delta_f = \frac{f(\mathbf{g}) - f_{\min}^*}{f_{\max}^* - f_{\min}^*} \quad (8)$$

$\Delta_x$  is a normalized Euclidean distance between the global best found by the algorithm ( $\mathbf{g}$ ) and a desired (analytical if known, otherwise a known benchmark value) minimizer ( $\mathbf{x}_{\min}^*$ ), where  $Z_j = |u_j - l_j|$  is the range of the  $j$ -th variable.  $\Delta_f$  is the associated normalized difference (error) in the function space, where  $f(\mathbf{g})$  is the minimum found by the algorithm,  $f_{\min}^*$  is the analytical minimum, and  $f_{\max}^*$  is the analytical maximum of the function  $f(\mathbf{x})$  in the variable domain.

### 5.2 Probability of success of random PSO versus DPSO

The success probability (SP) of random PSO versus DPSO is defined as

$$\text{SP} = \frac{|\mathcal{S} \cap \mathcal{T}|}{|\mathcal{T}|} \quad (9)$$

where  $\mathcal{T}$  is a random set of sets, each containing a number of  $R$  solutions obtained by random PSO. Note that  $\mathcal{T}$  has dimension  $|\mathcal{T}|$  and contains sets of dimension  $R$ .  $\mathcal{S}$  is a subset of  $\mathcal{T}$  defined as

$$\mathcal{S} = \{s \in \mathcal{T} \mid \text{argmin}[\Delta_t(s)] < \Delta_t(d)\} \quad (10)$$

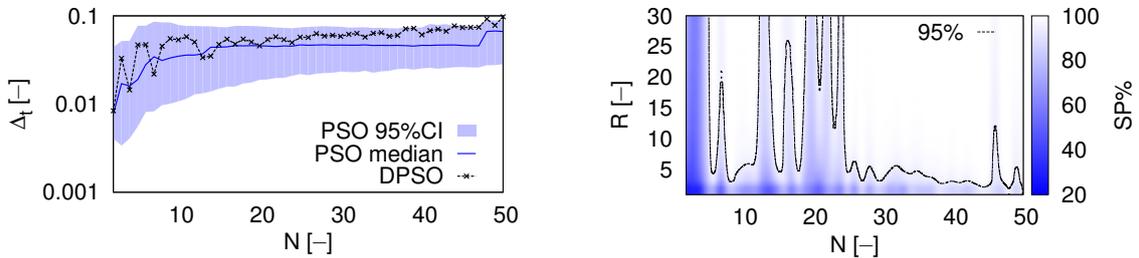
with  $\Delta_t(s) \in \mathbb{R}^R$  collecting  $\Delta_t$  values from the set of  $R$  random PSO solutions and  $\Delta_t(d)$  indicating the solution obtained by DPSO.

SR indicates the probability for random PSO to find at least one solution better than DPSO. This is evaluated using a large number  $|\mathcal{T}|$  of different random sets and comparing them to the deterministic solution. SR generally depends on the number of variables,  $N$ , and the number of random PSO runs considered,  $R$ . Hereafter,  $|\mathcal{T}| = 1,000$ .

## 6 RESULTS

### 6.1 Analytical test functions

Figure 3a shows the average  $\Delta_t$  (Eq. 8) of random PSO and DPSO, for problems of dimension  $N$ . For random PSO, the 95% confidence interval (CI) and the median is shown using 1,000 optimization runs. It can be seen how the performance of DPSO are fairly close to the median of random PSO and always falls within the 95% CI of random PSO. Both deterministic and random PSO show a more satisfactory performance for low numbers of variables.



(a) Average  $\Delta_t$  versus the number of design variables  $N$

(b) Average success probability (SP) of random PSO compared to DPSO, versus the number of design variables  $N$  and the number of random PSO runs  $R$

**Figure 3:** Analytical test functions results

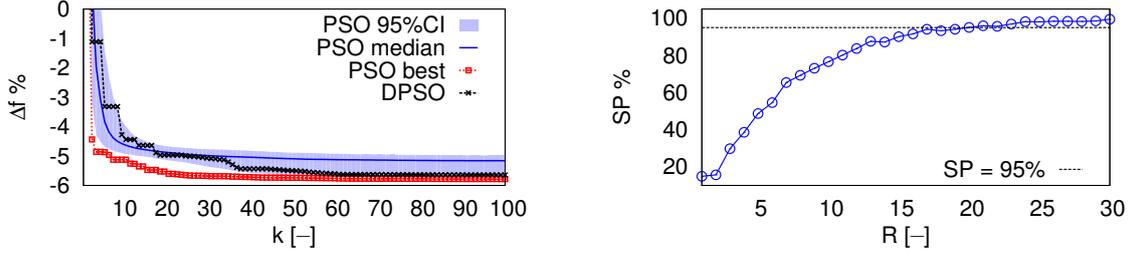
Figure 3b shows the average success probability (SP) of random PSO versus DPSO (Eq. 9). The 95% probability indicates, for each  $N$  and on average, the minimum number of random PSO runs required to obtain at least one solution better than DPSO. This number may vary significantly, depending on  $N$ . Generally, one random optimization does not guarantee a high probability of success. In order to achieve a success probability equal to 95%, at least five random PSO repetitions are needed, often significantly more. In extreme synthesis and simplifying to some extent the interpretation of the results, if random PSO is used rather than DPSO for a number of design variables  $N$  smaller or equal to 30, one is suggested to run at least 30 repetitions. For larger  $N$ , five repetitions may be enough for random PSO to outperform DPSO.

### 6.2 Hull-form optimization of the DTMB 5415

Figure 4a shows the objective function reduction versus the iteration number  $k$ , of random PSO and DPSO. For each  $k$ , the 95% confidence interval, the median and the best solution of random PSO are shown, evaluated using 1,000 optimization runs. DPSO outperforms the median of random PSO, achieving the best solution found by random PSO after 60 iterations ( $\Delta f = -5.7\%$ ). The success probability of random PSO versus DPSO is shown in Fig. 4b. For

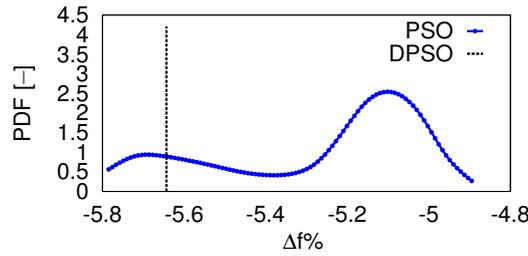
the present problem, 20 random PSO repetitions are needed to achieve at least one solution better than DPSO. The probability density function of random PSO results is shown in Fig. 4c, using a kernel density estimate technique. It may be noted how the mode (most probable outcome) is quite far from the DPSO solution, which is close to the random best.

Figure 5a shows the optimal design variables provided by all random PSO runs, their best solution, and DPSO. The solution provided by DPSO is almost coincident with the best solution by random PSO. The corresponding designs are shown in Fig. 5b and compared to the original.



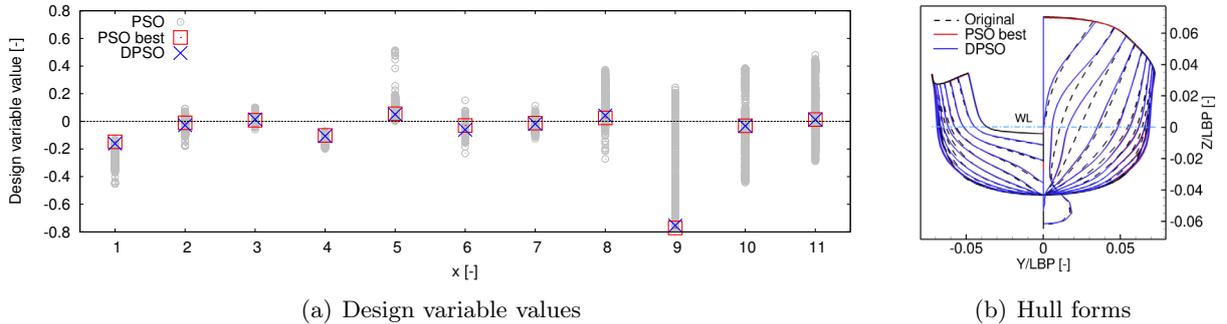
(a) Objective function reduction  $\Delta_f$  versus the iteration number  $k$

(b) Success probability (SP) of random PSO compared to DPSO, versus the number of random PSO runs  $R$



(c) Probability density function of random PSO results, compared to DPSO

**Figure 4:** Hull-form optimization of the DTMB 5415, algorithm performance

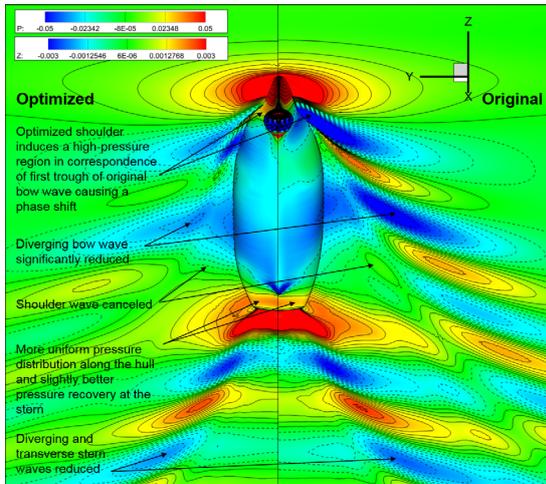


(a) Design variable values

(b) Hull forms

**Figure 5:** Hull-form optimization of the DTMB 5415, optimal design variables

In order to assess the impact of design modifications on the hydrodynamic performances, the solution provided by DPSO has been assessed with RANS and the results were presented in [7] and are shown here in Fig. 6 and Tab. 2. Figure 6 show a significant reduction of the diverging bow wave and a small reduction of the diverging and transverse stern wave. It may be also noted how the shoulder wave is cancelled. Specifically, the optimized shoulder shape induces a high pressure region in correspondence of the first trough of the original hull, causing a phase shift with the reduction of the diverging bow wave and the cancellation of the shoulder wave. The hydrodynamic coefficients for the original and the optimized hulls are finally compared in Tab. 2, confirming that a large part of the resistance reduction stems from the reduction of the piezometric pressure coefficient.



**Figure 6:** Comparison of wave elevation and pressure distribution on the hull produced by optimized and original configurations at  $Fr=0.25$ , bottom view [7]

**Table 2:** Comparison of hydrodynamic coefficients of optimized and original hulls at  $Fr=0.25$  [7] (coefficients are shown for: piezometric pressure,  $C_{pp}$ ; hydrostatic pressure,  $C_h$ ; viscous shear stress,  $C_f$ ; weight force along longitudinal axis,  $C_{mg,x}$ ; and total resistance,  $C_T$ )

Parameter	Unit	Original	Optimized	$\Delta\%$
$C_{pp}$	–	1.38E-03	9.08E-04	-34.0
$C_h$	–	0.86E-03	1.24E-03	42.0
$C_f$	–	3.16E-03	3.18E-03	0.65
$C_{mg,x}$	–	-1.19E-03	-1.35E-03	-13.4
$C_T$	–	4.21E-03	3.97E-03	-6.00
$\sigma/LBP$	–	-1.31E-03	-1.35E-03	-3.29
$\tau$	deg	-0.11	-0.12	-15.3
$S_{w,stat}/LBP^2$	–	1.48E-02	1.50E-02	0.96

## 7 CONCLUSIONS

In ship hydrodynamics SBDO, the computational cost of the optimization process is strongly affected by the simulation tools (which are usually CPU-time expensive) and by the efficiency of the optimization algorithm. PSO is a widely used global derivative-free optimization algorithm, which has been applied in SBDO for a variety of engineering applications. Its original formulation is stochastic, making use of random coefficients in the particle position update, therefore several optimization runs (repetitions) are needed in order to achieve statistical convergent results. This is often not attainable due to the computational cost of the simulation tools (especially if they are directly used by the optimization algorithm).

For this reason, efficient deterministic PSO (DPSO) formulations have been developed and assessed for SBDO applications, including ship hydrodynamics [8, 9]. The advantage of using deterministic methods is apparent, if one considers that only a single optimization run is needed. But is there any drawback for the effectiveness, efficiency, and robustness of the optimization

procedure? In order to answer this question, a systematic comparison between stochastic and deterministic versions of PSO has been presented and discussed.

The comparison is based on 588 analytical test functions (with dimensionality ranging from two to fifty) and a SBDO problem, aiming the reduction of the total resistance coefficient of the DTMB 5415 in calm water at  $Fr=0.25$ . More than 500,000 random optimization runs have been performed overall in order to assess the random PSO statistics. The discussion has been based on the probability of success of random PSO versus DPSO.

Analytical test functions have shown that on average DPSO outperforms random PSO with odds of 30/1 for low-dimensional problems (indicatively  $N \leq 30$ ) and 5/1 for high-dimensional problems ( $N > 30$ ). For the ship design SBDO problem ( $N = 11$ ), DPSO outperforms random PSO with odds of 20/1. Moreover, the effectiveness and robustness of DPSO has been further investigated, by comparison of DPSO and best random PSO optima. Design variables provided by DPSO are almost coincident with the best random PSO outcome.

In conclusion, DPSO represents a viable option for SBDO in ship hydrodynamics and its use is advisable whenever computationally expensive analyses are involved in the optimization process. The present conclusion is based on the probability of success of random PSO versus DPSO, considering a relative performance: *is random PSO better than DPSO in solving this problem?* Future work will include extensions of the current analysis, in order to investigate the probability of success of both random PSO and DPSO based on an absolute performance: *how good are random PSO and DPSO in solving this problem?* Absolute performances may represent more effectively engineering requirements addressing simulation tool accuracy, manufacturing tolerance, etc. Finally, the efficiency of the algorithms will be also addressed by analysing the statistics of data and performance profiles [18], with the aim of answering the questions: *how fast are random PSO and DPSO in solving this problem? And how computationally expensive?*

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